

# Appendix A

## Numerical Results of Experiments

This appendix presents the numerical results obtained from the experiments made in Chapter 5. The data is organized in tables and figures as follows:

- **Comparison table of optimizers:** The values of a performance indicator for 100 independent runs from 2 to 10 objective functions are summarized using descriptive statistics, such as the median (med.), mean or average (avg.), minimum (min.), maximum (max.) and standard deviation (std.). The three best values are shown in grayscale, where the darker tone corresponds to the best value. In the case of the hypervolume indicator, a zero value means that the approximation set in objective space is outside the enclosed region described by the reference point. This indicates that, the optimizer did not converge to the Pareto optimal front.
- **Box-plot of optimizers:** The information of the previous point is represented graphically using box-plots (outliers are omitted). Optimizers are sorted from the best to the worst median for each dimension. In the case of the hypervolume indicator, the widths of the boxes are proportional to the number of samples different from zero.
- **Statistical test table:** The Wilcoxon rank sum test (one-tailed) is applied to hypervolume indicator values in order to compare the performance of approximation sets. The information shown in these tables corresponds to the  $p$ -values computed in the R-project. A  $p$ -value greater than 0.05 (the significance level  $\alpha$ ) means that there is not enough evidence that one optimizer performs better than another one in a specific problem of certain dimensionality. No information is shown if the hypervolume indicator value is zero.
- **Approximation plot:** The Pareto optimal front (POF) and True Pareto front (True PF) is depicted for 2D and 3D. For higher dimensionality parallel coordinates are used for representing approximation sets.

In Table A.1, an index of all these figures and tables is presented for ease of location.

Table A.1: Relation of tables and figures of the different experiments.

Indicator	Table/Figure	Problem															
		DTLZ1	DTLZ2	DTLZ3	DTLZ4	DTLZ5	DTLZ6	DTLZ7	WFG1	WFG2	WFG3	WFG4	WFG5	WFG6	WFG7	WFG8	WFG9
Experiment 1 (Comparison of MOEAs)																	
Hypervolume	Comparison	3	25	47	69	91	113	135	157	179	201	223	245	267	289	311	333
	Box-plot	4	26	48	70	92	114	136	158	180	202	224	246	268	290	312	334
	Statistical test	5-7	27-29	49-51	71-73	93-95	115-117	137-139	159-161	181-183	203-205	225-227	247-249	269-271	291-293	313-315	335-337
$R2$	Comparison	8	30	52	74	96	118	140	162	184	206	228	250	272	294	316	338
	Box-plot	9	31	53	75	97	119	141	163	185	207	229	251	273	295	317	339
Runtime	Comparison	10	32	54	76	98	120	142	164	186	208	230	252	274	296	318	340
	Box-plot	11	33	55	77	99	121	143	165	187	209	231	253	275	297	319	341
-	Approx. plot	12-24	34-46	56-68	78-90	100-112	122-134	144-156	166-178	188-200	210-222	232-244	254-266	276-288	298-310	320-332	342-354
Experiment 2 (Performance of Utility Functions)																	
Hypervolume	Comparison	355	370	385	400	415	430	445	460	475	490	505	520	535	550	565	580
	Box-plot	356	371	386	401	416	431	446	461	476	491	506	521	536	551	566	581
	Statistical test	357-359	372-374	387-389	402-404	417-419	432-434	447-449	462-464	477-479	492-494	507-509	522-524	537-539	552-554	567-569	582-584
$R2$	Comparison	360	375	390	405	420	435	450	465	480	495	510	525	540	555	570	585
	Box-plot	361	376	391	406	421	436	451	466	481	496	511	526	541	556	571	586
-	Approx. plot	362-369	377-384	392-399	407-414	422-429	437-444	452-459	467-474	482-489	497-504	512-519	527-534	542-549	557-564	572-579	587-594
Experiment 3 (Sensitivity to the Design of Weight Vectors)																	
Hypervolume	Comparison	595	604	613	622	631	640	649	658	667	676	685	694	703	712	721	730
	Box-plot	596	605	614	623	632	641	650	659	668	677	686	695	704	713	722	731
	Statistical test	597	606	615	624	633	642	651	660	669	678	687	696	705	714	723	732
$R2$	Comparison	598	607	616	625	634	643	652	661	670	679	688	697	706	715	724	733
	Box-plot	599	608	617	626	635	644	653	662	671	680	689	698	707	716	725	734
-	Approx. plot	600-603	609-612	618-621	627-630	636-639	645-648	654-657	663-666	672-675	681-684	690-693	699-702	708-711	717-720	726-729	735-738

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Table A.2: Comparison of hypervolume indicator values for different optimizers on the DTLZ1 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	8.734346e-01	8.738003e-01	8.737079e-01	8.735956e-01	8.738857e-01	8.724069e-01	8.673199e-01	8.691769e-01	8.716020e-01	8.737985e-01	8.738110e-01	8.733910e-01	8.736873e-01
	avg.	8.733844e-01	8.737439e-01	8.732419e-01	8.735514e-01	8.738489e-01	8.722998e-01	8.670841e-01	8.686361e-01	8.704983e-01	8.737835e-01	8.737656e-01	8.733319e-01	8.736318e-01
	min.	8.722344e-01	8.713626e-01	8.531973e-01	8.725104e-01	8.733583e-01	8.704749e-01	8.574558e-01	8.563981e-01	8.618752e-01	8.734285e-01	8.731185e-01	8.720716e-01	8.729240e-01
	max.	8.737079e-01	8.739366e-01	8.739218e-01	8.739280e-01	8.739452e-01	8.739266e-01	8.714175e-01	8.721508e-01	8.727579e-01	8.738595e-01	8.739348e-01	8.735767e-01	8.738774e-01
	std.	2.440076e-04	2.748538e-04	2.206064e-03	2.654077e-04	1.068987e-04	6.338654e-04	2.603557e-03	2.998287e-03	2.662856e-03	7.160696e-05	1.549755e-04	2.136591e-04	2.153411e-04
3D	med.	9.555524e-01	9.689333e-01	8.708503e-01	9.742206e-01	9.745157e-01	9.637425e-01	9.641550e-01	9.621279e-01	9.704498e-01	9.742265e-01	9.689257e-01	9.690623e-01	9.742623e-01
	avg.	9.550525e-01	9.689145e-01	8.704594e-01	9.741546e-01	9.744879e-01	9.635953e-01	9.638756e-01	9.616669e-01	9.703763e-01	9.742197e-01	9.689603e-01	9.689667e-01	9.742264e-01
	min.	9.249775e-01	9.681165e-01	8.009318e-01	9.734604e-01	9.741681e-01	9.425283e-01	9.577672e-01	9.498465e-01	9.674370e-01	9.740395e-01	9.683805e-01	9.649935e-01	9.735498e-01
	max.	9.687004e-01	9.693121e-01	9.630539e-01	9.744246e-01	9.745695e-01	9.730126e-01	9.682085e-01	9.680136e-01	9.717142e-01	9.743164e-01	9.699062e-01	9.707590e-01	9.744226e-01
	std.	8.591776e-03	1.763741e-04	2.989390e-02	2.153037e-04	7.440985e-05	6.836488e-03	2.520996e-03	3.748257e-03	7.861322e-04	5.325901e-05	2.587439e-04	1.042212e-03	1.650704e-04
4D	med.	9.639580e-01	9.884687e-01	8.624844e-01	9.943298e-01	8.047787e-01	9.747133e-01	9.864824e-01	9.850237e-01	9.925731e-01	9.944370e-01	9.884326e-01	9.873143e-01	9.943974e-01
	avg.	8.154121e-01	9.884614e-01	8.480667e-01	9.943105e-01	8.654303e-01	9.711843e-01	9.862139e-01	9.847283e-01	9.925270e-01	9.944316e-01	9.884203e-01	9.865345e-01	9.943703e-01
	min.	1.837876e-02	9.882232e-01	3.228452e-01	9.938169e-01	7.719201e-01	9.110759e-01	9.808208e-01	9.773680e-01	9.913645e-01	9.943297e-01	9.880964e-01	9.794132e-01	9.937398e-01
	max.	9.854703e-01	9.886414e-01	9.405262e-01	9.944755e-01	9.936205e-01	9.923599e-01	9.897951e-01	9.903498e-01	9.930385e-01	9.944927e-01	9.888881e-01	9.85632e-01	9.947775e-01
	std.	2.639458e-01	7.778014e-05	6.207241e-02	1.268856e-04	9.245684e-02	1.570521e-02	1.764767e-03	2.631145e-03	3.039182e-04	2.980228e-05	1.412847e-04	2.51571e-03	1.050998e-04
5D	med.	0.000000e+00	9.967163e-01	8.537713e-01	9.985024e-01	9.918092e-01	9.814189e-01	9.935068e-01	9.933993e-01	9.980028e-01	9.987106e-01	9.967983e-01	9.939385e-01	9.986310e-01
	avg.	0.000000e+00	9.962519e-01	8.343190e-01	9.984717e-01	9.958965e-01	9.776263e-01	9.933586e-01	9.931676e-01	9.980065e-01	9.987085e-01	9.967950e-01	9.935337e-01	9.986193e-01
	min.	0.000000e+00	9.905500e-01	2.543076e-01	9.977365e-01	3.506129e-01	9.314181e-01	9.895773e-01	9.888739e-01	9.976485e-01	9.986677e-01	9.966289e-01	9.870903e-01	9.983420e-01
	max.	0.000000e+00	9.968710e-01	9.305960e-01	9.986334e-01	9.983578e-01	9.961791e-01	9.955494e-01	9.962413e-01	9.982529e-01	9.987327e-01	9.968840e-01	9.961862e-01	9.986928e-01
	std.	0.000000e+00	1.414327e-03	7.842749e-02	1.364560e-04	9.349789e-02	1.359742e-02	1.176658e-03	1.802964e-03	1.051194e-04	1.518398e-05	4.314019e-05	1.797592e-03	4.847156e-05
6D	med.	0.000000e+00	9.951489e-01	8.463574e-01	9.994771e-01	9.920169e-01	9.575644e-01	9.961916e-01	9.962213e-01	9.994027e-01	9.996664e-01	9.978145e-01	9.946612e-01	9.995881e-01
	avg.	0.000000e+00	9.932675e-01	8.211537e-01	9.994641e-01	9.938070e-01	9.560991e-01	9.959681e-01	9.959699e-01	9.994011e-01	9.996662e-01	9.977215e-01	9.944595e-01	9.995843e-01
	min.	0.000000e+00	9.616526e-01	1.221068e-01	9.991791e-01	9.921699e-02	9.080335e-01	9.919554e-01	9.899476e-01	9.992723e-01	9.996433e-01	9.963297e-01	9.821905e-01	9.994191e-01
	max.	0.000000e+00	9.980609e-01	9.330311e-01	9.996062e-01	9.995476e-01	9.929916e-01	9.981697e-01	9.979415e-01	9.995203e-01	9.996791e-01	9.986019e-01	9.980946e-01	9.996362e-01
	std.	0.000000e+00	5.651639e-03	9.257611e-02	7.989127e-05	1.330509e-01	1.823559e-02	1.109005e-03	1.308754e-03	4.010875e-05	6.978851e-06	6.360691e-04	2.108687e-03	3.396471e-05
7D	med.	0.000000e+00	9.872554e-01	8.315116e-01	9.997551e-01	9.519868e-01	9.546691e-01	9.944006e-01	9.946647e-01	9.996740e-01	9.998409e-01	9.927873e-01	9.811349e-01	9.998043e-01
	avg.	0.000000e+00	9.816736e-01	8.221327e-01	9.997416e-01	9.099563e-01	8.609945e-01	9.940195e-01	9.946996e-01	9.996599e-01	9.998402e-01	9.925764e-01	9.797401e-01	9.997975e-01
	min.	0.000000e+00	5.155551e-01	7.579787e-01	9.995612e-01	2.364616e-01	1.921871e-01	9.892251e-01	9.891611e-01	9.994752e-01	9.998041e-01	9.857445e-01	9.628000e-01	9.997339e-01
	max.	0.000000e+00	9.952074e-01	9.442175e-01	9.998397e-01	9.995339e-01	9.973798e-01	9.973009e-01	9.977685e-01	9.997396e-01	9.998633e-01	9.956641e-01	9.916458e-01	9.998370e-01
	std.	0.000000e+00	1.123620e-02	3.208858e-02	5.498313e-05	1.149606e-01	1.963424e-01	1.802111e-03	1.821607e-03	5.357032e-05	1.165437e-05	1.806478e-03	5.835605e-03	2.501178e-05
8D	med.	0.000000e+00	9.803649e-01	8.314924e-01	9.995553e-01	9.853202e-01	9.587649e-01	9.972330e-01	9.975402e-01	9.999318e-01	9.999584e-01	9.938804e-01	9.853580e-01	9.998455e-01
	avg.	0.000000e+00	9.765100e-01	8.061287e-01	9.995014e-01	8.895187e-01	6.857738e-01	9.971573e-01	9.972795e-01	9.999307e-01	9.999567e-01	9.936584e-01	9.854466e-01	9.998382e-01
	min.	0.000000e+00	9.496235e-01	5.581130e-02	9.986709e-01	1.550827e-02	5.967515e-01	9.945028e-01	9.928544e-01	9.999002e-01	9.999373e-01	9.872191e-01	9.780277e-01	9.996004e-01
	max.	0.000000e+00	9.957381e-01	8.963430e-01	9.998332e-01	9.998264e-01	9.972331e-01	9.986403e-01	9.991893e-01	9.999464e-01	9.999653e-01	9.962274e-01	9.938527e-01	9.999354e-01
	std.	0.000000e+00	1.297069e-02	1.051993e-01	2.273272e-04	1.917848e-01	1.439005e-01	8.544291e-04	1.072928e-03	7.180052e-06	6.339493e-06	1.707611e-03	3.118081e-03	5.384674e-05
9D	med.	0.000000e+00	9.777119e-01	8.332023e-01	9.971328e-01	9.508728e-01	6.016146e-01	9.986087e-01	9.987843e-01	9.999860e-01	9.999879e-01	9.942508e-01	9.861928e-01	9.996873e-01
	avg.	0.000000e+00	9.760705e-01	8.161496e-01	9.967018e-01	7.548349e-01	6.473564e-01	9.985447e-01	9.986536e-01	9.999859e-01	9.999874e-01	9.940920e-01	9.867293e-01	9.996597e-01
	min.	0.000000e+00	9.472606e-01	5.570900e-02	9.906709e-01	9.819143e-03	2.282549e-01	9.965181e-01	9.965254e-01	9.999807e-01	9.999763e-01	9.882504e-01	9.797701e-01	9.992602e-01
	max.	0.000000e+00	9.943222e-01	9.381262e-01	9.986328e-01	9.994021e-01	9.927075e-01	9.994551e-01	9.993544e-01	9.999886e-01	9.999919e-01	9.971039e-01	9.931990e-01	9.998756e-01
	std.	0.000000e+00	1.142592e-02	8.415794e-02	1.447313e-03	3.302179e-01	1.085995e-01	4.764604e-04	5.504621e-04	1.376429e-06	2.715327e-06	1.663979e-03	2.424245e-03	1.397253e-04
10D	med.	0.000000e+00	9.690008e-01	8.327973e-01	9.899974e-01	9.693538e-01	7.087199e-01	9.991662e-01	9.991788e-01	9.999971e-01	9.999944e-01	9.948660e-01	9.868172e-01	9.993001e-01
	avg.	0.000000e+00	9.694891e-01	8.124819e-01	9.889401e-01	8.809190e-01	7.087199e-01	9.990837e-01	9.991686e-01	9.999971e-01	9.999939e-01	9.937261e-01	9.874692e-01	9.992103e-01
	min.	0.000000e+00	9.022552e-01	6.700127e-02	9.865092e-01	2.171787e-01	7.087199e-01	9.974336e-01	9.979344e-01	9.999966e-01	9.999857e-01	9.883990e-01	9.841380e-01	9.976498e-01
	max.	0.000000e+00	9.934270e-01	9.91362e-01	9.947932e-01	9.999511e-01	7.087199e-01	9.997017e-01	9.997776e-01	9.999976e-01	9.999968e-01	9.976748e-01	9.958395e-01	9.998046e-01
	std.	0.000000e+00	1.552235e-02	9.067680e-02	1.792805e-03	1.792805e-03	0.000000e+00	3.235526e-04	3.235526e-04	2.327347e-07	2.583823e-06	1.074986e-02	2.291253e-03	4.031810e-04

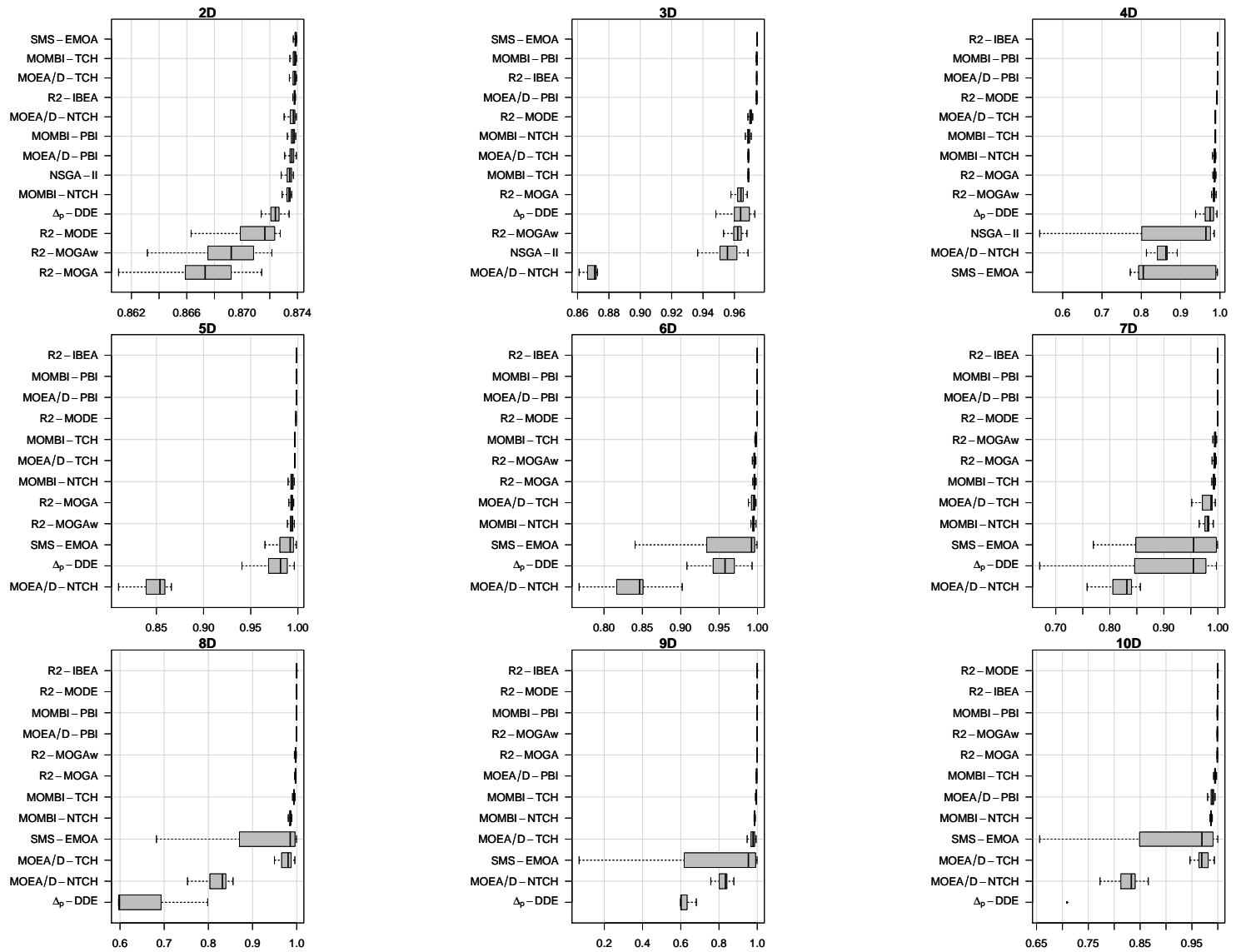


Figure A.1: Box-plot of hypervolume indicator values for different optimizers on the DTLZ1 test problem.

Table A.3: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ1 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	4.95e-29	1.28e-34	1.28e-34	4.64e-34	> 0.05	> 0.05	9.31e-03	> 0.05
MOEA/D-TCH	2.40e-26	—	4.76e-07	9.76e-12	> 0.05	3.27e-32	1.32e-34	1.89e-34	6.63e-34	> 0.05	> 0.05	6.60e-30	9.79e-08
MOEA/D-NTCH	1.70e-09	> 0.05	—	4.24e-02	> 0.05	1.86e-25	2.31e-32	1.55e-31	1.12e-29	> 0.05	> 0.05	1.35e-12	> 0.05
MOEA/D-PBI	3.54e-08	> 0.05	> 0.05	—	> 0.05	2.12e-31	1.28e-34	1.28e-34	1.78e-34	> 0.05	> 0.05	9.49e-14	> 0.05
SMS-EMOA	3.09e-32	4.06e-08	1.77e-19	1.56e-22	—	1.52e-33	1.28e-34	1.28e-34	1.28e-34	1.70e-13	1.07e-07	1.87e-33	4.30e-21
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	3.05e-34	7.61e-31	2.93e-11	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	3.29e-07	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	4.85e-18	4.55e-10	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	1.24e-32	> 0.05	6.28e-08	2.05e-16	> 0.05	2.51e-33	1.28e-34	1.28e-34	1.28e-34	—	> 0.05	4.50e-34	9.01e-11
MOMBI-TCH	3.93e-26	> 0.05	4.46e-08	2.28e-12	> 0.05	3.47e-33	1.28e-34	1.28e-34	1.28e-34	> 0.05	—	5.91e-30	1.03e-08
MOMBI-NTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	4.82e-29	1.28e-34	1.32e-34	4.12e-34	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	3.96e-15	> 0.05	> 0.05	4.34e-03	> 0.05	1.89e-32	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	1.39e-20	—
3D													
NSGA-II	—	> 0.05	5.05e-32	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	2.26e-34	—	1.28e-34	> 0.05	> 0.05	1.47e-07	1.36e-34	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	1.28e-34	1.28e-34	—	> 0.05	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.28e-34	> 0.05
SMS-EMOA	1.28e-34	1.28e-34	1.28e-34	3.25e-31	—	1.28e-34	1.28e-34	1.28e-34	1.28e-34	6.70e-33	1.28e-34	1.28e-34	1.72e-29
$\Delta_p$ -DDE	6.97e-13	> 0.05	1.61e-33	> 0.05	> 0.05	—	> 0.05	1.04e-03	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	2.44e-17	> 0.05	5.38e-34	> 0.05	> 0.05	> 0.05	—	8.29e-06	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	4.09e-10	> 0.05	2.44e-33	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	1.78e-34	1.32e-29	1.28e-34	> 0.05	> 0.05	4.58e-16	1.68e-34	1.40e-34	—	> 0.05	2.40e-28	2.62e-20	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	> 0.05
MOMBI-TCH	2.07e-34	> 0.05	1.28e-34	> 0.05	> 0.05	1.01e-07	1.28e-34	1.28e-34	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	5.71e-33	> 0.05	1.28e-34	> 0.05	> 0.05	1.97e-08	1.22e-32	1.31e-33	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	1.28e-34	1.28e-34	1.28e-34	6.80e-03	> 0.05	1.28e-34	1.28e-34	1.28e-34	1.28e-34	8.86e-03	1.28e-34	1.28e-34	—
4D													
NSGA-II	—	> 0.05	3.43e-04	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	1.28e-34	> 0.05	9.23e-09	2.82e-27	4.65e-25	2.68e-28	> 0.05	> 0.05	5.25e-04	2.03e-10	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	3.98e-02	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.28e-34	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	4.52e-08	> 0.05	1.84e-34	> 0.05	3.86e-06	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	2.23e-33	> 0.05	1.28e-34	> 0.05	5.04e-08	1.42e-17	—	5.13e-06	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	2.48e-30	> 0.05	1.28e-34	> 0.05	3.55e-07	1.76e-13	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.78e-20	2.55e-34	1.28e-34	1.28e-34	—	> 0.05	1.28e-34	1.28e-34	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	2.71e-20	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.27e-09
MOMBI-TCH	1.28e-34	> 0.05	1.28e-34	> 0.05	8.60e-09	2.54e-27	2.57e-24	7.24e-28	> 0.05	> 0.05	—	1.62e-09	> 0.05
MOMBI-NTCH	9.39e-33	> 0.05	1.28e-34	> 0.05	4.11e-08	2.19e-19	9.22e-03	8.36e-08	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	1.28e-34	1.28e-34	1.28e-34	4.40e-06	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.28e-34	—

Table A.4: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ1 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
5D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$2.82e-39$	—	$1.87e-34$	> 0.05	$2.30e-17$	$2.10e-33$	$6.17e-26$	$1.52e-26$	> 0.05	> 0.05	> 0.05	$8.19e-26$	> 0.05
MOEA/D-NTCH	$3.61e-39$	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$2.82e-39$	$1.28e-34$	$1.87e-34$	—	$2.07e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$7.34e-32$	> 0.05	$1.28e-34$	$1.28e-34$	> 0.05
SMS-EMOA	$2.82e-39$	> 0.05	$1.15e-22$	> 0.05	—	$7.46e-07$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	$2.82e-39$	> 0.05	$1.87e-34$	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$2.82e-39$	> 0.05	$1.87e-34$	> 0.05	$1.67e-03$	$1.25e-28$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	$2.82e-39$	> 0.05	$1.87e-34$	> 0.05	$2.89e-03$	$5.60e-27$	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	$2.82e-39$	$1.28e-34$	$1.87e-34$	> 0.05	$1.69e-31$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	> 0.05	$1.28e-34$	$1.28e-34$	> 0.05
R2-IBEA	$2.82e-39$	$1.28e-34$	$1.87e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$5.14e-34$
MOMBI-TCH	$2.82e-39$	$6.23e-20$	$1.87e-34$	> 0.05	$8.38e-21$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	—	$1.28e-34$	> 0.05
MOMBI-NTCH	$2.82e-39$	> 0.05	$1.87e-34$	> 0.05	$3.40e-04$	$3.70e-28$	$3.28e-02$	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	$2.82e-39$	$1.28e-34$	$1.87e-34$	$1.18e-26$	$1.32e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	$1.28e-34$	$1.28e-34$	—
6D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$2.82e-39$	—	$1.87e-34$	> 0.05	$6.27e-04$	$1.54e-32$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	$3.61e-39$	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$2.82e-39$	$1.28e-34$	$1.87e-34$	—	$1.92e-33$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.49e-11$	> 0.05	$1.28e-34$	$1.28e-34$	> 0.05
SMS-EMOA	$2.82e-39$	> 0.05	$9.09e-21$	> 0.05	—	$3.45e-08$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	$2.82e-39$	> 0.05	$2.77e-34$	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$2.82e-39$	$5.19e-07$	$1.87e-34$	> 0.05	$8.76e-10$	$1.44e-34$	—	> 0.05	> 0.05	> 0.05	> 0.05	$1.04e-11$	> 0.05
R2-MOGAw	$2.82e-39$	$7.79e-07$	$1.87e-34$	> 0.05	$4.03e-10$	$1.40e-34$	> 0.05	—	> 0.05	> 0.05	> 0.05	$3.29e-11$	> 0.05
R2-MODE	$2.82e-39$	$1.28e-34$	$1.87e-34$	> 0.05	$2.51e-33$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	> 0.05	$1.28e-34$	$1.28e-34$	> 0.05
R2-IBEA	$2.82e-39$	$1.28e-34$	$1.87e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$2.82e-39$	$7.46e-29$	$1.87e-34$	> 0.05	$2.83e-22$	$1.28e-34$	$6.33e-26$	$3.61e-25$	> 0.05	> 0.05	—	$1.30e-30$	> 0.05
MOMBI-NTCH	$2.82e-39$	> 0.05	$1.87e-34$	> 0.05	$5.30e-05$	$2.96e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	$2.82e-39$	$1.28e-34$	$1.87e-34$	$8.06e-28$	$1.78e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$4.24e-34$	> 0.05	$1.28e-34$	$1.28e-34$	—
7D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$2.82e-39$	—	$1.28e-34$	> 0.05	> 0.05	$1.50e-14$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.43e-04$	> 0.05
MOEA/D-NTCH	$2.82e-39$	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$2.82e-39$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.87e-34$	$1.28e-34$	$1.28e-34$	$1.04e-20$	> 0.05	$1.28e-34$	$1.28e-34$	> 0.05
SMS-EMOA	$2.82e-39$	> 0.05	$1.23e-19$	> 0.05	—	$8.00e-03$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	$3.61e-39$	> 0.05	$1.15e-10$	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$2.82e-39$	$6.00e-28$	$1.28e-34$	> 0.05	$1.43e-03$	$4.53e-28$	—	> 0.05	> 0.05	> 0.05	$2.26e-08$	$2.71e-34$	> 0.05
R2-MOGAw	$2.82e-39$	$4.23e-30$	$1.28e-34$	> 0.05	$2.26e-04$	$7.07e-29$	$6.82e-03$	—	> 0.05	> 0.05	$9.75e-15$	$2.13e-34$	> 0.05
R2-MODE	$2.82e-39$	$1.28e-34$	$1.28e-34$	> 0.05	$1.40e-34$	$1.87e-34$	$1.28e-34$	$1.28e-34$	—	> 0.05	$1.28e-34$	$1.28e-34$	> 0.05
R2-IBEA	$2.82e-39$	$1.28e-34$	$1.28e-34$	$8.39e-34$	$1.28e-34$	$1.87e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.48e-30$
MOMBI-TCH	$2.82e-39$	$1.01e-21$	$1.28e-34$	> 0.05	$8.24e-03$	$2.90e-27$	> 0.05	> 0.05	> 0.05	> 0.05	—	$1.00e-33$	> 0.05
MOMBI-NTCH	$2.82e-39$	> 0.05	$1.28e-34$	> 0.05	> 0.05	$3.45e-12$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	$2.82e-39$	$1.28e-34$	$1.28e-34$	$4.27e-18$	$1.28e-34$	$1.87e-34$	$1.28e-34$	$1.28e-34$	$1.53e-34$	> 0.05	$1.28e-34$	$1.28e-34$	—

Table A.5: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ1 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
8D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	2.82e-39	—	1.28e-34	> 0.05	> 0.05	4.69e-24	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	2.82e-39	> 0.05	—	> 0.05	> 0.05	5.48e-12	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	2.82e-39	1.28e-34	1.28e-34	—	3.57e-30	1.87e-34	1.28e-34	1.84e-34	> 0.05	> 0.05	1.28e-34	1.28e-34	> 0.05
SMS-EMOA	5.97e-39	> 0.05	1.70e-13	> 0.05	—	4.61e-16	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	3.61e-39	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	2.82e-39	1.53e-34	1.28e-34	> 0.05	7.98e-18	4.07e-33	—	> 0.05	> 0.05	> 0.05	7.65e-33	1.28e-34	> 0.05
R2-MOGAw	2.82e-39	1.78e-34	1.28e-34	> 0.05	2.26e-18	3.31e-33	> 0.05	—	> 0.05	> 0.05	1.30e-31	1.36e-34	> 0.05
R2-MODE	2.81e-39	1.28e-34	1.28e-34	1.28e-34	4.04e-34	1.87e-34	1.28e-34	1.28e-34	—	> 0.05	1.28e-34	1.28e-34	4.85e-33
R2-IBEA	2.82e-39	1.28e-34	1.28e-34	1.28e-34	4.04e-34	1.87e-34	1.28e-34	1.28e-34	4.42e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	2.82e-39	8.48e-30	1.28e-34	> 0.05	3.13e-05	1.01e-29	> 0.05	> 0.05	> 0.05	> 0.05	—	2.12e-32	> 0.05
MOMBI-NTCH	2.82e-39	2.67e-04	1.28e-34	> 0.05	> 0.05	4.62e-26	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	2.82e-39	1.28e-34	1.28e-34	7.45e-32	2.77e-33	1.87e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	1.28e-34	1.28e-34	—
9D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	2.82e-39	—	1.28e-34	> 0.05	1.78e-02	1.93e-32	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	2.82e-39	> 0.05	—	> 0.05	> 0.05	1.69e-22	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	2.82e-39	2.01e-34	1.28e-34	—	1.01e-21	1.93e-34	> 0.05	> 0.05	> 0.05	> 0.05	2.13e-22	1.95e-34	> 0.05
SMS-EMOA	4.64e-39	> 0.05	1.32e-03	> 0.05	—	4.31e-08	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	3.61e-39	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	2.82e-39	1.28e-34	1.28e-34	2.82e-27	5.82e-29	1.87e-34	—	> 0.05	> 0.05	> 0.05	1.44e-34	1.28e-34	> 0.05
R2-MOGAw	2.82e-39	1.28e-34	1.28e-34	1.66e-27	1.06e-29	1.87e-34	7.37e-03	—	> 0.05	> 0.05	1.44e-34	1.28e-34	> 0.05
R2-MODE	2.79e-39	1.27e-34	1.27e-34	1.27e-34	2.72e-34	1.86e-34	1.27e-34	1.27e-34	—	> 0.05	1.27e-34	1.27e-34	1.27e-34
R2-IBEA	2.80e-39	1.27e-34	1.27e-34	1.27e-34	2.73e-34	1.86e-34	1.27e-34	1.27e-34	1.66e-09	—	1.27e-34	1.27e-34	1.27e-34
MOMBI-TCH	2.82e-39	9.39e-33	1.28e-34	> 0.05	2.02e-14	3.13e-34	> 0.05	> 0.05	> 0.05	> 0.05	—	8.85e-33	> 0.05
MOMBI-NTCH	2.82e-39	3.31e-14	1.28e-34	> 0.05	6.57e-05	3.51e-33	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	2.82e-39	1.28e-34	1.28e-34	1.28e-34	3.40e-34	1.87e-34	1.95e-34	1.73e-34	> 0.05	> 0.05	1.28e-34	1.28e-34	—
10D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	7.88e-20	—	1.36e-34	> 0.05	> 0.05	4.48e-02	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	7.88e-20	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	7.88e-20	2.84e-24	1.28e-34	—	6.78e-07	4.48e-02	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.76e-07	> 0.05
SMS-EMOA	5.59e-19	> 0.05	6.89e-12	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	7.88e-20	1.28e-34	1.28e-34	1.28e-34	9.45e-29	4.48e-02	—	> 0.05	> 0.05	> 0.05	1.32e-34	1.28e-34	> 0.05
R2-MOGAw	7.88e-20	1.28e-34	1.28e-34	1.28e-34	7.70e-29	4.48e-02	> 0.05	—	> 0.05	> 0.05	1.28e-34	1.28e-34	> 0.05
R2-MODE	7.29e-20	1.11e-34	1.11e-34	1.11e-34	3.84e-33	4.36e-02	1.11e-34	1.11e-34	—	1.63e-34	1.11e-34	1.11e-34	1.11e-34
R2-IBEA	7.86e-20	1.27e-34	1.27e-34	1.27e-34	4.47e-33	4.47e-02	1.27e-34	1.27e-34	> 0.05	—	1.27e-34	1.27e-34	1.27e-34
MOMBI-TCH	7.88e-20	9.39e-33	1.36e-34	1.30e-27	1.62e-17	4.48e-02	> 0.05	> 0.05	> 0.05	> 0.05	—	6.79e-30	> 0.05
MOMBI-NTCH	7.88e-20	8.40e-22	1.28e-34	> 0.05	3.85e-04	4.48e-02	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	7.88e-20	1.28e-34	1.28e-34	1.28e-34	6.85e-29	4.48e-02	1.65e-03	4.37e-02	> 0.05	> 0.05	1.32e-34	1.28e-34	—

Table A.6: Comparison of R2 indicator values for different optimizers on the DTLZ1 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	8.385074e-02	8.372010e-02	8.375051e-02	8.376840e-02	8.370542e-02	8.368671e-02	8.440180e-02	8.410993e-02	8.416171e-02	8.373354e-02	8.371485e-02	8.383756e-02	8.373073e-02
	avg.	8.387052e-02	8.373446e-02	8.391282e-02	8.378113e-02	8.371807e-02	8.369134e-02	8.443781e-02	8.413814e-02	8.418518e-02	8.374013e-02	8.373025e-02	8.385909e-02	8.375069e-02
	min.	8.375748e-02	8.368085e-02	8.368644e-02	8.368010e-02	8.368124e-02	8.367663e-02	8.418900e-02	8.395160e-02	8.402386e-02	8.370817e-02	8.367925e-02	8.375705e-02	8.367832e-02
	max.	8.425286e-02	8.384963e-02	9.174951e-02	8.401523e-02	8.388412e-02	8.379699e-02	8.480666e-02	8.477390e-02	8.455391e-02	8.386054e-02	8.395013e-02	8.425462e-02	8.398607e-02
	std.	8.144398e-05	4.367285e-05	8.413448e-04	7.285276e-05	3.498272e-05	1.995449e-05	1.361729e-04	1.276486e-04	9.546413e-05	2.402972e-05	5.199513e-05	6.952766e-05	6.937738e-05
3D	med.	3.511511e-02	3.302707e-02	5.608644e-02	3.231443e-02	3.222944e-02	3.295353e-02	3.334104e-02	3.287088e-02	3.313518e-02	3.242773e-02	3.300843e-02	3.339101e-02	3.228973e-02
	avg.	3.571466e-02	3.304185e-02	5.625453e-02	3.233669e-02	3.223257e-02	3.301123e-02	3.332621e-02	3.289627e-02	3.314273e-02	3.243327e-02	3.302590e-02	3.344065e-02	3.230917e-02
	min.	3.336875e-02	3.295928e-02	3.480163e-02	3.223014e-02	3.210057e-02	3.248863e-02	3.292960e-02	3.256125e-02	3.280510e-02	3.227766e-02	3.292144e-02	3.299791e-02	3.222137e-02
	max.	4.206462e-02	3.337677e-02	7.329071e-02	3.258436e-02	3.240561e-02	3.366187e-02	3.378800e-02	3.376255e-02	3.351496e-02	3.259226e-02	3.322313e-02	3.426621e-02	3.257814e-02
	std.	1.886720e-03	6.001562e-05	7.064818e-03	8.352625e-05	5.590796e-05	2.004498e-04	1.901797e-04	1.998662e-04	1.212611e-04	5.020220e-05	5.538035e-05	2.292650e-04	7.026704e-05
4D	med.	2.380465e-02	2.067428e-02	4.327824e-02	1.815940e-02	3.612566e-02	1.933165e-02	1.849178e-02	1.812627e-02	1.841000e-02	1.790737e-02	2.068479e-02	2.098440e-02	1.811428e-02
	avg.	3.672688e-02	2.067669e-02	4.558573e-02	1.818322e-02	3.086051e-02	1.943022e-02	1.850923e-02	1.815813e-02	1.841859e-02	1.790918e-02	2.069069e-02	2.119110e-02	1.813441e-02
	min.	1.996967e-02	2.056859e-02	2.943675e-02	1.807106e-02	1.826573e-02	1.841516e-02	1.820051e-02	1.783888e-02	1.813560e-02	1.777702e-02	2.048579e-02	2.034299e-02	1.806582e-02
	max.	1.011482e-01	2.080705e-02	9.839260e-02	1.854938e-02	1.871796e-02	2.194369e-02	1.897667e-02	1.861518e-02	1.875237e-02	1.803716e-02	2.085206e-02	2.320697e-02	1.853196e-02
	std.	2.137609e-02	4.584309e-05	7.891262e-03	8.947101e-05	7.854645e-03	5.896343e-04	1.439326e-04	1.778563e-04	1.121107e-04	5.785786e-05	6.243442e-05	6.375727e-04	7.27515e-05
5D	med.	1.538539e+00	1.441970e-02	3.586743e-02	1.241561e-02	1.608386e-02	1.316910e-02	1.236398e-02	1.207117e-02	1.233718e-02	1.189470e-02	1.433294e-02	1.552536e-02	1.220159e-02
	avg.	1.645142e+00	1.459830e-02	3.963484e-02	1.241456e-02	1.773069e-02	1.329807e-02	1.236966e-02	1.209490e-02	1.234055e-02	1.189087e-02	1.434022e-02	1.561340e-02	1.221597e-02
	min.	6.267370e-01	1.415217e-02	2.632329e-02	1.212947e-02	1.247020e-02	1.235684e-02	1.178495e-02	1.198295e-02	1.209160e-02	1.177697e-02	1.419539e-02	1.453311e-02	1.214845e-02
	max.	2.987237e+00	1.700467e-02	1.905702e-01	1.274572e-02	4.496842e-02	1.509710e-02	1.268345e-02	1.250317e-02	1.270481e-02	1.201656e-02	1.448524e-02	1.739552e-02	1.247271e-02
	std.	5.037213e-01	5.825554e-04	1.681758e-02	1.351271e-04	4.980183e-03	5.785307e-04	1.311621e-04	1.710067e-04	1.090622e-04	4.673402e-05	4.537194e-05	5.895229e-04	5.280573e-05
6D	med.	3.368002e+00	1.263114e-02	3.055603e-02	8.819247e-03	1.251661e-02	1.099805e-02	8.720381e-03	8.516255e-03	8.676293e-03	8.338173e-03	1.145309e-02	1.252880e-02	8.624612e-03
	avg.	3.407155e+00	1.282812e-02	3.729839e-02	8.830817e-03	1.433844e-02	1.099722e-02	8.734253e-03	8.530570e-03	8.682295e-03	8.338251e-03	1.163436e-02	1.260640e-02	8.639366e-03
	min.	1.863253e+00	1.107264e-02	2.200590e-02	8.585989e-03	8.617235e-03	9.716002e-03	8.457257e-03	8.256013e-03	8.493021e-03	8.258753e-03	1.081280e-02	1.111303e-02	8.585540e-03
	max.	5.389775e+00	1.778660e-02	4.679480e-01	9.072356e-03	3.302069e-02	1.310498e-02	8.997348e-03	8.970798e-03	8.905098e-03	8.413354e-03	1.268313e-02	1.570174e-02	8.823907e-03
	std.	7.306827e-01	1.109950e-03	4.416550e-02	1.157802e-04	4.568801e-03	7.675765e-04	1.096513e-04	1.462861e-04	7.627210e-05	3.446036e-05	4.502356e-04	6.082169e-04	4.373760e-05
7D	med.	4.074695e+00	1.262226e-02	2.783701e-02	6.955768e-03	1.391703e-02	9.081126e-03	7.394306e-03	7.150244e-03	7.075159e-03	6.704738e-03	1.199106e-02	1.356857e-02	6.843416e-03
	avg.	3.982829e+00	1.326928e-02	2.883675e-02	6.970629e-03	1.281230e-02	1.513928e-02	7.404865e-03	7.167509e-03	7.099456e-03	6.707208e-03	1.212769e-02	1.371012e-02	6.853955e-03
	min.	1.238641e+00	1.094994e-02	1.798826e-02	6.814987e-03	7.538943e-03	7.390429e-03	7.069642e-03	6.556944e-03	6.884335e-03	6.630220e-03	1.084126e-02	1.263654e-02	6.812435e-03
	max.	5.676402e+00	1.728792e-02	3.489126e-02	7.241262e-03	2.654798e-02	7.160176e-02	7.898214e-03	7.870792e-03	7.556828e-03	6.833093e-03	1.381652e-02	1.552167e-02	7.000768e-03
	std.	8.833205e-01	1.480894e-03	3.135724e-03	7.816529e-05	4.004891e-03	1.282587e-02	1.540721e-04	2.597167e-04	1.305124e-04	4.343099e-05	8.185360e-04	6.455343e-04	3.606924e-05
8D	med.	4.593328e+00	1.166084e-02	2.450944e-02	5.409252e-03	1.008923e-02	2.080289e-02	5.453639e-03	5.386111e-03	5.329522e-03	5.053873e-03	1.034324e-02	1.129017e-02	5.225100e-03
	avg.	4.434247e+00	1.206464e-02	2.634852e-02	5.425904e-03	1.213666e-02	2.380381e-02	5.464507e-03	5.404871e-03	5.335763e-03	5.056324e-03	1.031187e-02	1.137417e-02	5.231620e-03
	min.	2.164358e+00	9.188365e-03	1.791018e-02	5.276984e-03	5.885885e-03	6.435851e-03	5.297021e-03	5.042237e-03	5.12752e-03	4.974724e-03	9.143791e-03	1.053461e-02	5.178599e-03
	max.	6.042133e+00	1.514940e-02	8.254573e-02	5.790345e-03	3.590651e-02	5.884447e-01	5.699419e-03	5.898154e-03	5.525670e-03	5.131232e-03	1.146526e-02	1.233845e-02	5.343359e-03
	std.	8.506108e-01	1.471225e-03	7.030882e-03	9.557108e-05	5.770139e-03	5.726075e-02	7.481486e-05	1.33366e-04	6.107140e-05	2.685240e-05	6.187582e-04	3.190228e-04	3.138622e-05
9D	med.	4.122397e+00	1.056012e-02	2.135718e-02	4.559128e-03	1.062616e-02	1.716218e-02	4.210159e-03	4.220869e-03	4.149619e-03	3.975563e-03	8.959473e-03	9.959290e-03	4.036481e-03
	avg.	3.967654e+00	1.082990e-02	2.258146e-02	4.567771e-03	1.300223e-02	1.698132e-02	4.215019e-03	4.236080e-03	4.153083e-03	3.974557e-03	8.980014e-03	9.991739e-03	4.046453e-03
	min.	2.027321e+00	8.615530e-03	1.442916e-02	4.255535e-03	5.789860e-03	5.944334e-03	4.101722e-03	3.966643e-03	4.079717e-03	3.940821e-03	7.789284e-03	9.581809e-03	3.976041e-03
	max.	5.003975e+00	1.375822e-02	7.004212e-02	4.845897e-03	3.168857e-02	8.959756e-02	4.358280e-03	4.719741e-03	4.286632e-03	4.016907e-03	1.050988e-02	1.068242e-02	4.173123e-03
	std.	6.174998e-01	1.122155e-03	5.422224e-03	1.213968e-04	6.082589e-03	8.553691e-03	4.910179e-05	1.471167e-04	3.951595e-05	1.765324e-05	5.890921e-04	2.136280e-04	4.399161e-05
10D	med.	3.297547e+00	1.017124e-02	1.935478e-02	4.180140e-03	1.083809e-02	1.360416e-02	3.359203e-03	3.358274e-03	3.320644e-03	3.203359e-03	7.844122e-03	8.813942e-03	3.290140e-03
	avg.	3.268733e+00	1.026991e-02	2.047036e-02	4.189758e-03	1.348278e-02	1.310704e-02	3.361429e-03	3.410268e-03	3.321271e-03	3.204765e-03	7.934386e-03	8.776893e-03	3.302600e-03
	min.	2.214025e+00	7.724546e-03	1.396950e-02	3.793774e-03	5.436434e-03	7.549345e-03	3.274146e-03	3.183213e-03	3.243797e-03	3.179371e-03	6.814512e-03	7.330872e-03	3.185148e-03
	max.	4.079931e+00	1.457792e-02	4.865170e-02	4.862619e-03	1.836248e-02	3.612650e-02	3.312650e-02	3.776102e-03	3.387521e-03	3.243294e-03	1.630381e-02	9.313915e-03	3.484617e-03
	std.	3.989123e-01	1.210794e-03	3.930391e-03	1.835765e-04	6.760764e-03	1.709068e-03	5.194446e-05	1.289282e-04	2.575418e-05	1.366408e-05	9.852051e-04	2.943461e-04	6.354346e-05

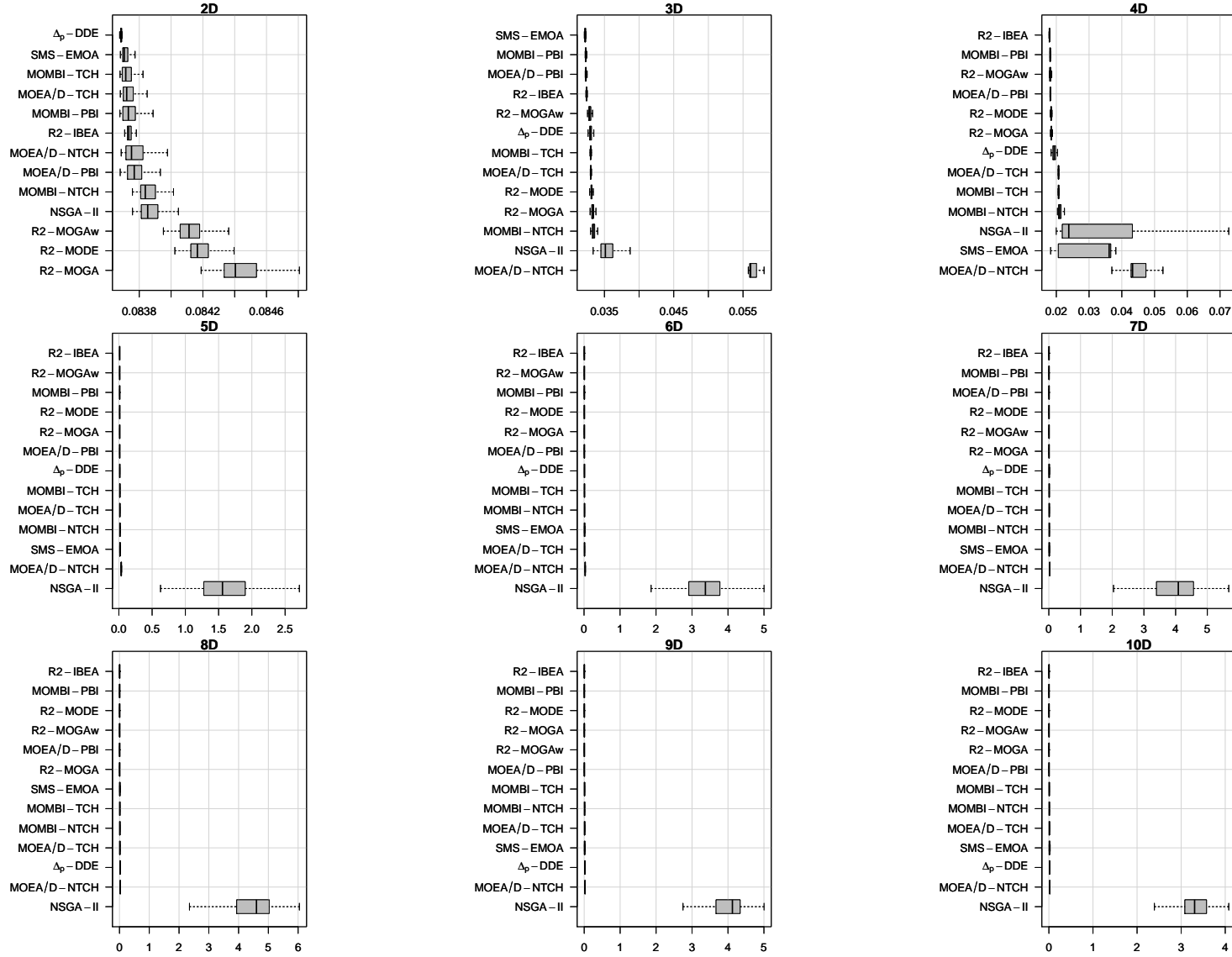


Figure A.2: Box-plot of R2 indicator values for different optimizers on the DTLZ1 test problem.

Table A.7: Comparison of runtime (in milliseconds) for different optimizers on the DTLZ1 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	1.391130e+02	3.585500e+02	8.890920e+02	8.430090e+02	2.195065e+04	1.008147e+03	2.075753e+03	1.659484e+03	2.008625e+03	5.854420e+05	1.941276e+03	1.495085e+03	2.641578e+03
	avg.	1.397804e+02	3.630220e+02	8.837353e+02	8.298641e+02	2.122329e+04	1.017312e+03	1.998703e+03	1.635794e+03	1.986146e+03	5.895935e+05	1.881835e+03	1.505769e+03	2.622540e+03
	min.	1.306990e+02	3.306930e+02	6.170960e+02	5.426120e+02	1.522593e+04	9.197430e+02	1.444663e+03	1.197029e+03	1.347716e+03	5.513610e+05	1.432049e+03	1.474689e+03	2.186127e+03
	max.	1.525670e+02	4.208590e+02	1.210603e+03	1.064243e+03	2.438248e+04	1.159957e+03	2.760808e+03	2.096304e+03	2.614321e+03	6.352280e+05	2.309959e+03	1.628995e+03	2.978548e+03
	std.	4.793493e+00	1.597084e+01	1.573511e+02	8.379470e+01	2.289796e+03	4.562875e+01	3.722775e+02	2.445412e+02	3.295403e+02	2.304065e+04	2.183163e+02	3.386895e+01	1.461823e+02
3D	med.	2.227720e+02	5.261070e+02	1.236478e+03	9.189210e+02	3.657331e+05	1.173243e+03	2.080344e+03	1.590688e+03	2.101381e+03	5.792150e+05	1.917696e+03	1.839201e+03	2.861312e+03
	avg.	2.309763e+02	5.301123e+02	1.219268e+03	9.111462e+02	3.586857e+05	1.161146e+03	2.065732e+03	1.607572e+03	2.089505e+03	5.828010e+05	1.894798e+03	1.867943e+03	2.856074e+03
	min.	1.901210e+02	3.845070e+02	9.457770e+02	6.842580e+02	2.526093e+05	7.658650e+02	1.448911e+03	1.119894e+03	1.377074e+03	5.283550e+05	1.512564e+03	1.611729e+03	2.353641e+03
	max.	3.477900e+02	8.270200e+02	1.466421e+03	1.392306e+03	4.344593e+05	1.715951e+03	2.557514e+03	2.307402e+03	2.503613e+03	6.270280e+05	2.280909e+03	2.184837e+03	3.134970e+03
	std.	3.382368e+01	9.523016e+01	1.155408e+02	9.675797e+01	3.373515e+04	1.933304e+02	2.642836e+02	1.999351e+02	2.211111e+02	2.150778e+04	1.662622e+02	1.347468e+02	1.322137e+02
4D	med.	3.030930e+02	5.257360e+02	1.342439e+03	9.522530e+02	6.526423e+05	1.211161e+03	2.343130e+03	1.783164e+03	2.487400e+03	4.838380e+05	1.926371e+03	2.118641e+03	3.177616e+03
	avg.	3.116593e+02	5.444823e+02	1.310323e+03	9.510556e+02	7.365434e+05	1.197820e+03	2.291805e+03	1.784849e+03	2.427117e+03	4.846312e+05	1.920583e+03	2.097295e+03	3.160794e+03
	min.	2.497280e+02	3.936580e+02	8.587430e+02	6.923800e+02	2.194115e+05	8.297930e+02	1.673061e+03	1.264352e+03	1.698642e+03	4.822670e+05	1.553307e+03	1.726381e+03	2.686032e+03
	max.	4.326080e+02	1.003001e+03	1.722781e+03	1.294281e+03	1.885482e+06	1.483117e+03	2.724454e+03	2.274551e+03	2.836077e+03	4.951190e+05	2.415609e+03	2.420097e+03	3.420097e+03
	std.	4.486370e+01	1.023649e+02	1.392530e+02	7.939919e+01	4.228194e+05	1.483485e+02	2.410393e+02	2.086896e+02	2.904422e+02	2.807689e+03	1.643593e+02	1.594938e+02	1.290294e+02
5D	med.	4.634650e+02	5.936830e+02	1.716056e+03	9.008690e+02	3.688071e+05	1.966702e+03	2.752027e+03	2.170170e+03	2.952324e+03	6.723700e+05	2.056078e+03	2.316038e+03	3.466887e+03
	avg.	4.656389e+02	5.994942e+02	1.646845e+03	9.293176e+02	4.758052e+05	1.952076e+03	2.728022e+03	2.182887e+03	2.910408e+03	6.719233e+05	2.091996e+03	2.318776e+03	3.470743e+03
	min.	3.604860e+02	4.508890e+02	6.825210e+02	6.944790e+02	2.277904e+05	1.357003e+03	1.965132e+03	1.696206e+03	2.031878e+03	6.638520e+05	1.674114e+03	2.037259e+03	2.865395e+03
	max.	6.387590e+02	9.130550e+02	2.092520e+03	1.910748e+03	1.774725e+06	2.566455e+03	3.170497e+03	2.770856e+03	3.378682e+03	6.787380e+05	2.885268e+03	2.657465e+03	3.845690e+03
	std.	5.204335e+01	7.438336e+01	2.534973e+02	1.595797e+02	3.174793e+05	2.471749e+02	2.888072e+02	2.265143e+02	3.242308e+02	3.054223e+03	2.487035e+02	1.450260e+02	1.994397e+02
6D	med.	6.200720e+02	6.141290e+02	1.848598e+03	9.525350e+02	4.499790e+05	2.803394e+03	2.951671e+03	2.389304e+03	3.135965e+03	7.114200e+05	2.161011e+03	2.292713e+03	3.870049e+03
	avg.	6.165837e+02	6.255701e+02	1.772065e+03	9.484281e+02	4.952557e+05	2.890935e+03	2.900353e+03	2.376442e+03	3.139869e+03	7.108565e+05	2.221818e+03	2.318226e+03	3.865938e+03
	min.	4.665750e+02	4.753340e+02	8.113770e+02	6.754810e+02	2.749547e+05	1.865957e+03	2.019626e+03	1.774967e+03	2.291378e+03	6.820630e+05	1.722768e+03	2.069538e+03	3.343604e+03
	max.	7.517850e+02	8.888790e+02	2.319126e+03	1.544135e+03	2.412645e+06	6.193999e+03	3.673554e+03	3.023985e+03	3.796013e+03	7.193500e+05	3.848331e+03	2.712226e+03	4.422082e+03
	std.	6.442564e+01	7.630287e+01	2.816327e+02	1.517815e+02	2.520657e+05	6.518767e+02	4.336122e+02	2.763189e+02	4.755295e+02	3.332070e+03	2.409933e+02	1.049623e+02	1.351849e+02
7D	med.	5.668250e+02	6.436060e+02	1.862152e+03	1.050558e+03	6.375392e+05	6.320692e+03	2.301871e+03	1.750564e+03	2.328598e+03	3.320390e+05	1.309713e+03	1.622033e+03	2.691921e+03
	avg.	5.592820e+02	6.507151e+02	1.826127e+03	1.018635e+03	9.082189e+05	7.318769e+03	2.274965e+03	1.755827e+03	2.309991e+03	3.316906e+05	1.320385e+03	1.648061e+03	2.688187e+03
	min.	4.345300e+02	4.482360e+02	8.116300e+02	7.495980e+02	3.803580e+05	3.203776e+03	1.576329e+03	1.306602e+03	1.623444e+03	3.237260e+05	1.100182e+03	1.433981e+03	2.240756e+03
	max.	6.920420e+02	9.999210e+02	2.387335e+03	1.203862e+03	3.100206e+06	8.760375e+03	2.709367e+03	2.129320e+03	2.771965e+03	3.409000e+05	1.654394e+03	1.908542e+03	2.982349e+03
	std.	6.004572e+01	8.400877e+01	2.872730e+02	1.222994e+02	5.416824e+05	6.317682e+02	3.320062e+02	1.598826e+02	2.393009e+02	3.332070e+03	1.270077e+02	1.049933e+02	1.351849e+02
8D	med.	8.159570e+02	6.783160e+02	2.077417e+03	1.125317e+03	6.283459e+05	1.500830e+04	3.738893e+03	2.894609e+03	3.572563e+03	1.286300e+06	2.149195e+03	2.437462e+03	4.128187e+03
	avg.	8.189387e+02	6.796730e+02	1.957683e+03	1.089778e+03	7.241444e+05	1.527586e+04	3.610490e+03	2.904420e+03	3.598642e+03	1.262558e+06	2.134565e+03	2.477580e+03	4.158959e+03
	min.	6.388400e+02	5.368020e+02	9.169490e+02	7.878600e+02	4.086652e+05	6.627723e+03	2.466567e+03	2.013319e+03	2.560665e+03	1.180581e+06	1.677838e+03	2.129564e+03	3.952921e+03
	max.	9.931090e+02	1.041771e+03	2.346116e+03	1.308768e+03	2.727483e+06	3.323225e+04	4.229315e+03	3.580988e+03	4.169719e+03	1.320895e+06	2.493505e+03	2.900989e+03	4.630268e+03
	std.	7.387042e+01	6.217066e+01	3.351714e+02	1.107520e+02	3.931621e+05	3.392000e+03	4.045766e+02	3.132332e+02	3.851677e+02	4.956893e+04	1.682012e+02	1.741348e+02	1.264868e+02
9D	med.	1.240255e+03	7.164020e+02	2.389168e+03	1.003383e+03	7.079355e+05	2.045421e+04	5.734188e+03	4.904337e+03	5.663101e+03	2.261923e+06	3.078124e+03	3.663268e+03	5.901198e+03
	avg.	1.231863e+03	7.140057e+02	2.276141e+03	1.019763e+03	6.680845e+05	2.155547e+04	5.572087e+03	4.919542e+03	5.663036e+03	2.397717e+06	3.048442e+03	3.663122e+03	5.931006e+03
	min.	9.860790e+02	5.258470e+02	1.209291e+03	8.057080e+02	4.253358e+05	1.241373e+04	3.492038e+03	3.734452e+03	4.143048e+03	2.255279e+06	2.282077e+03	3.157888e+03	5.006455e+03
	max.	1.460314e+03	8.503130e+02	2.769486e+03	1.342671e+03	1.120927e+06	8.158077e+04	6.801850e+03	6.086019e+03	6.824145e+03	2.732663e+06	4.003875e+03	4.148471e+03	6.785785e+03
	std.	9.385167e+01	5.617570e+01	3.953869e+02	1.361076e+02	1.442550e+05	7.382135e+03	8.530517e+02	5.129304e+02	5.648437e+02	1.906202e+05	2.279737e+02	2.399234e+02	2.892536e+02
10D	med.	1.680458e+03	7.436550e+02	2.691724e+03	1.224423e+03	9.026339e+05	2.271864e+04	9.807306e+03	7.366557e+03	9.776943e+03	4.876777e+06	3.644358e+03	5.065917e+03	8.482151e+03
	avg.	1.686060e+03	7.399302e+02	2.546968e+03	1.189198e+03	8.963058e+05	2.054734e+04	9.269635e+03	7.435923e+03	9.810816e+03	4.879721e+06	3.720732e+03	5.149782e+03	8.568786e+03
	min.	1.320334e+03	5.700380e+02	1.242019e+03	8.753670e+02	6.736666e+05	1.442056e+04	6.584946e+03	5.435974e+03	6.837352e+03	4.584848e+06	3.041339e+03	4.464239e+03	7.315172e+03
	max.	2.008135e+03	1.017033e+03	3.227710e+03	1.615639e+03	1.025446e+06	6.753073e+07	1.081728e+04	8.943583e+03	1.134960e+04	5.122731e+06	4.343668e+03	5.847625e+03	9.972161e+03
	std.	1.138808e+02	7.258330e+01	3.678156e+02	1.215940e+02	6.571153e+04	6.750438e+06	1.093257e+03	7.555923e+02	1.052918e+03	1.509802e+05	3.525844e+02	3.392520e+02	5.104703e+02



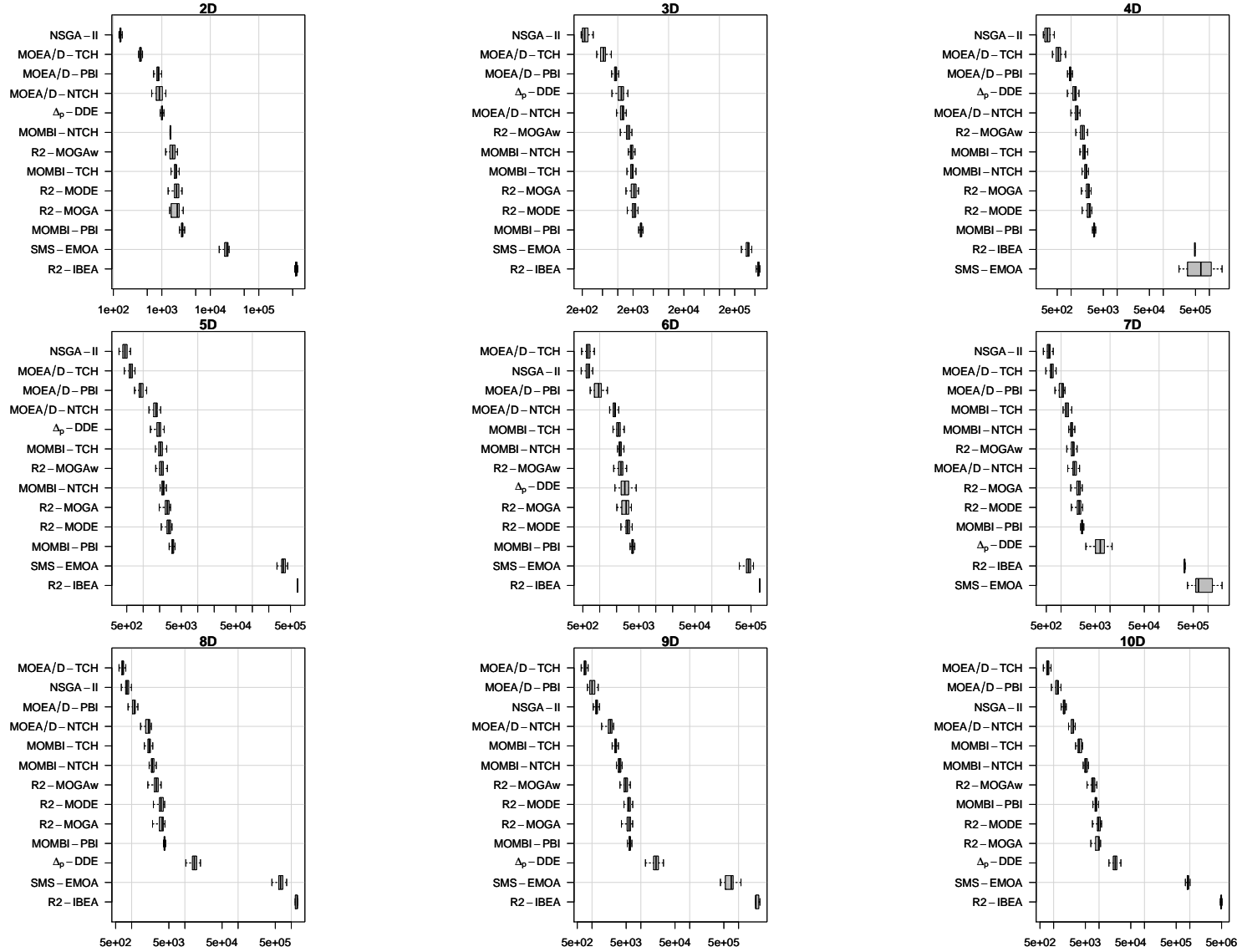


Figure A.3: Box-plot of runtime (in logarithmic scale) for different optimizers on the DTLZ1 test problem.

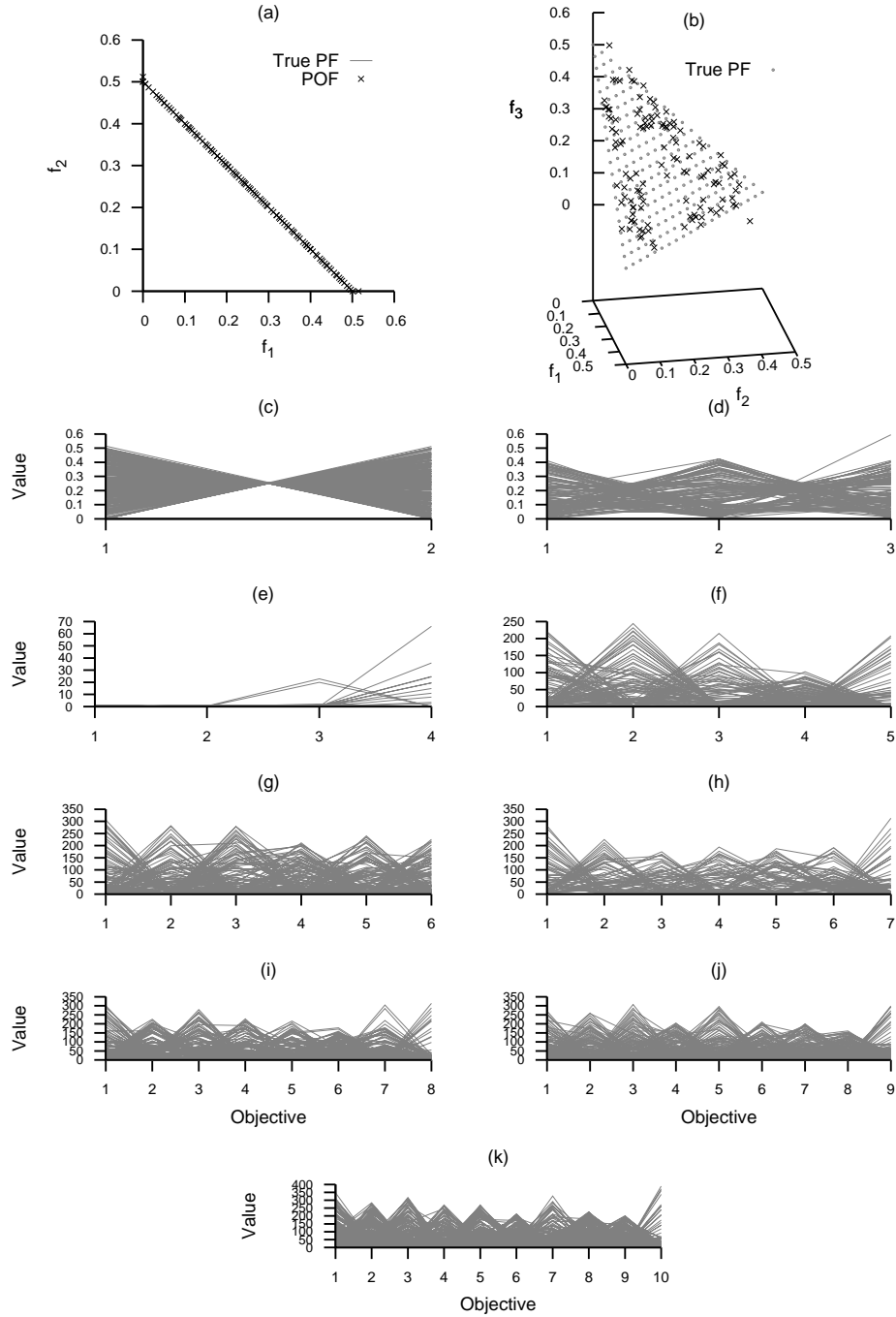


Figure A.4: Plots of the approximations obtained by NSGA-II from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

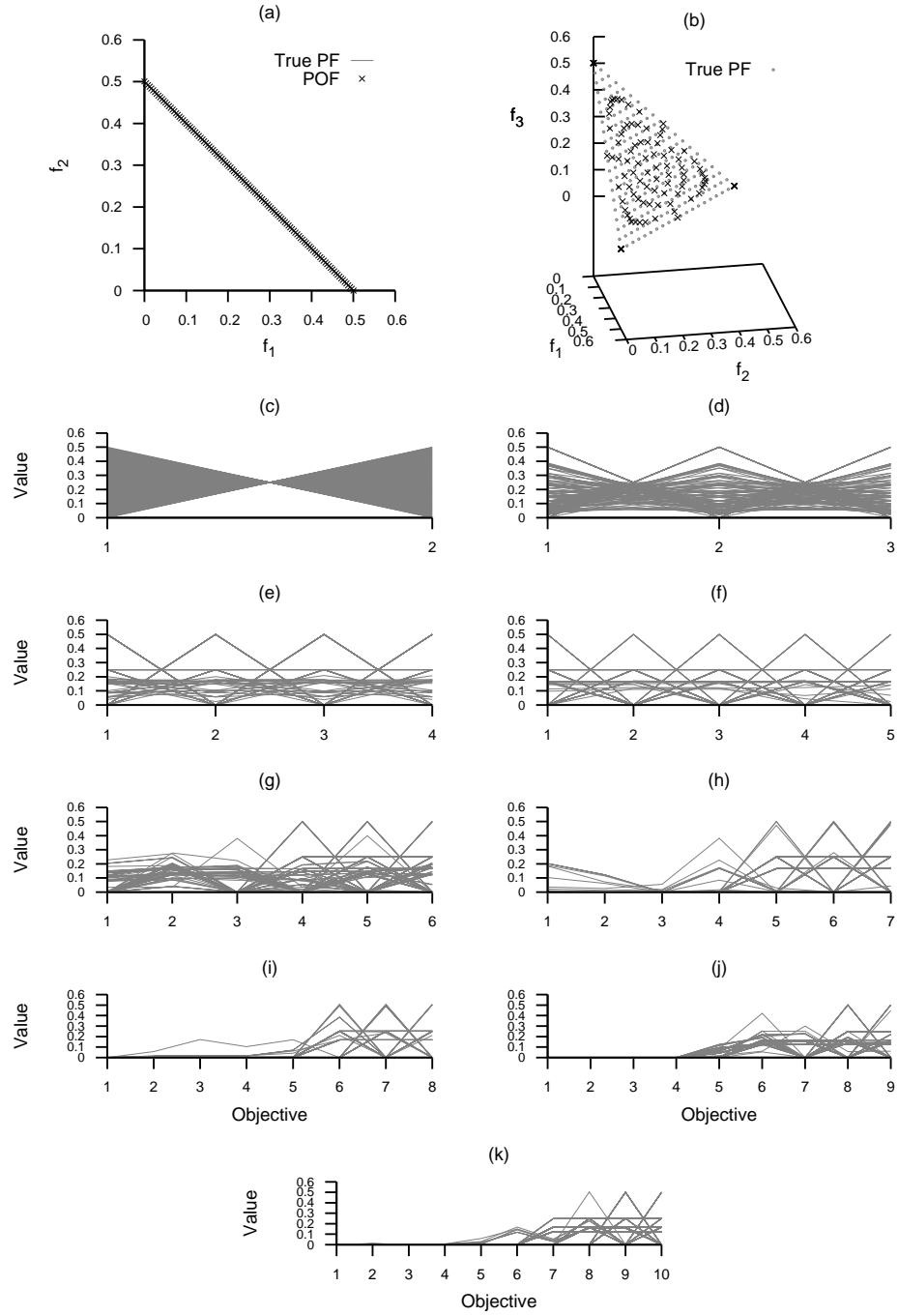


Figure A.5: Plots of the approximations obtained by MOEA/D-TCH from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

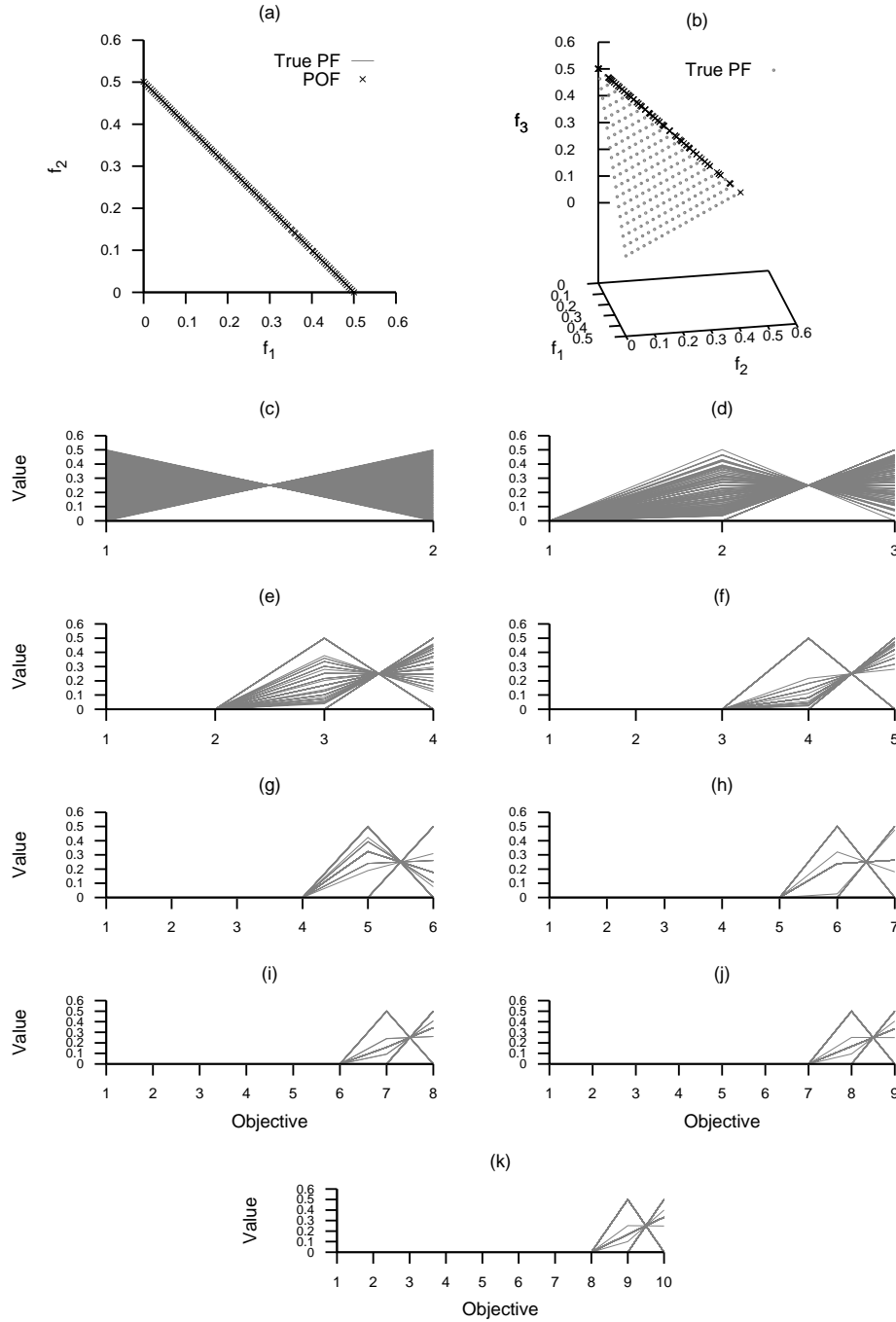


Figure A.6: Plots of the approximations obtained by MOEA/D-NTCH from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

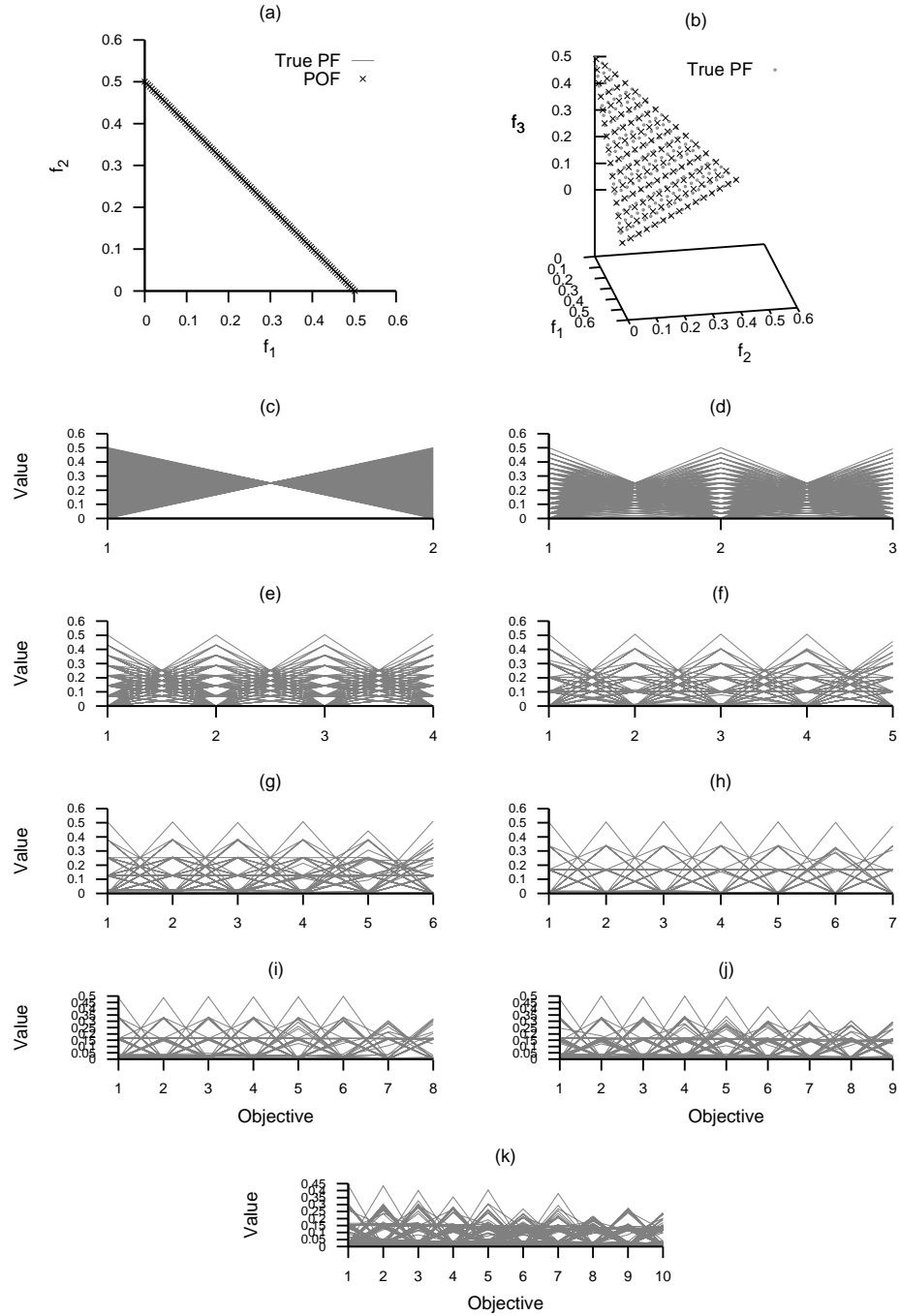


Figure A.7: Plots of the approximations obtained by MOEA/D-PBI from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

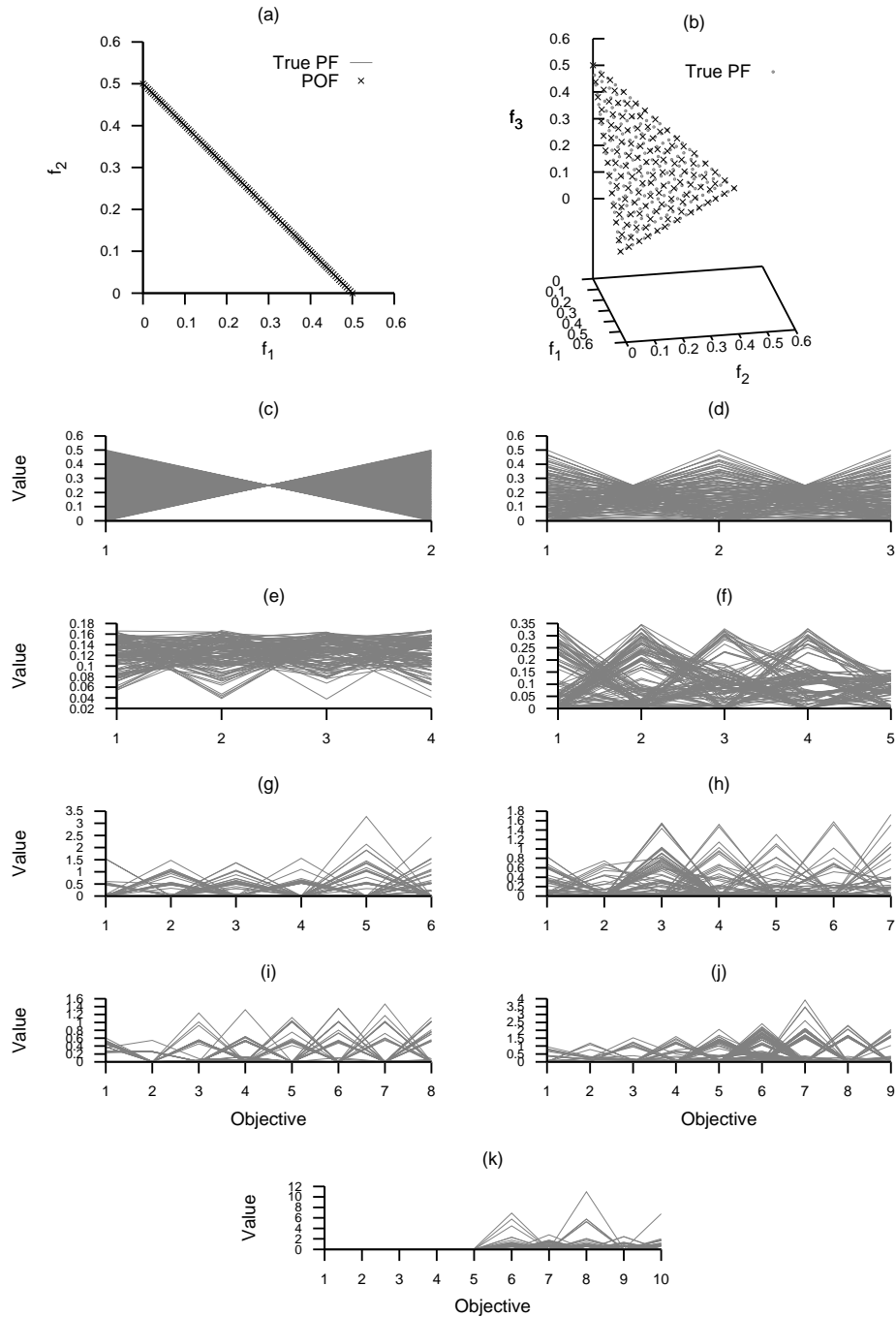


Figure A.8: Plots of the approximations obtained by SMS-EMOA from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

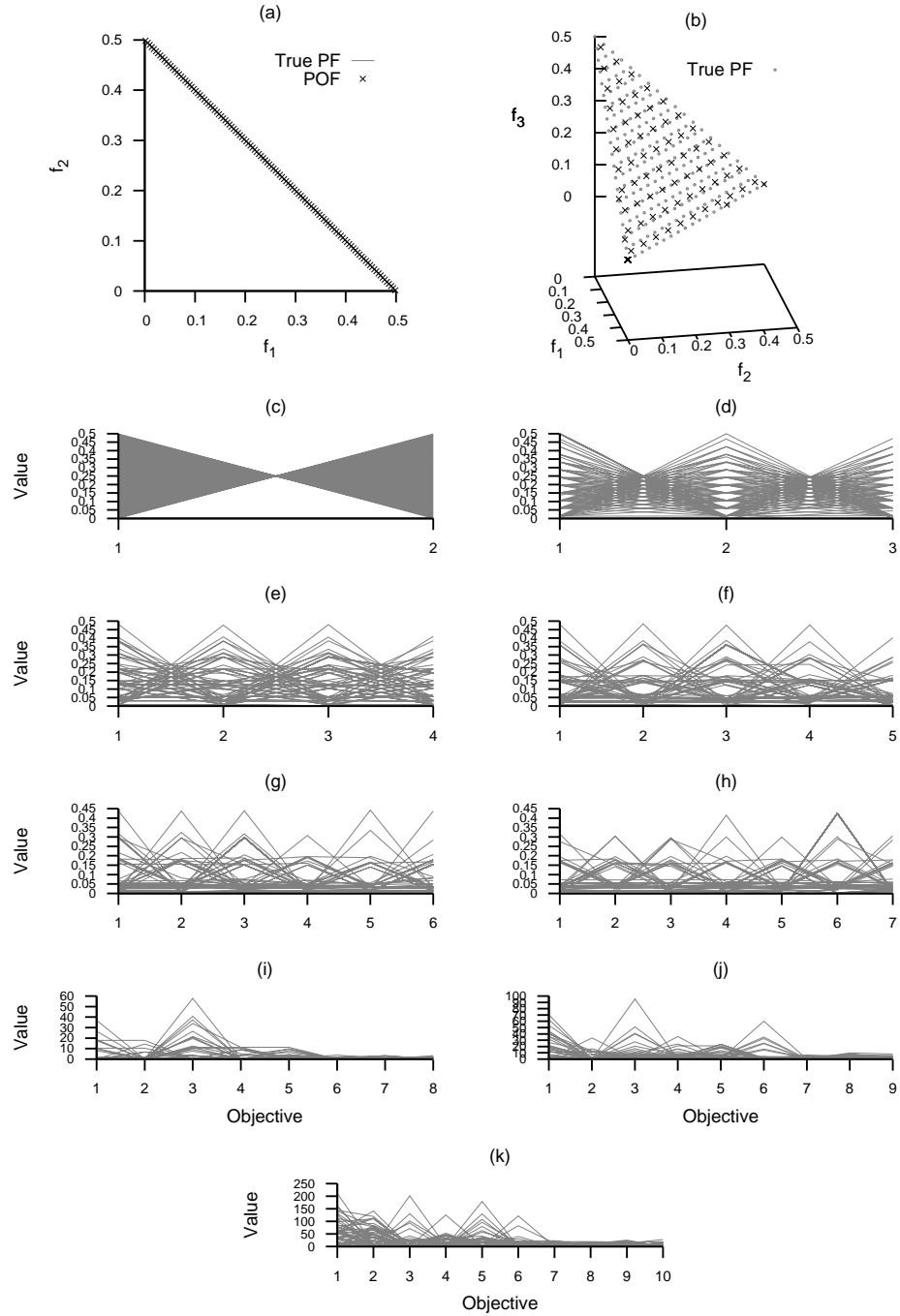


Figure A.9: Plots of the approximations obtained by  $\Delta_p$ -DDE from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

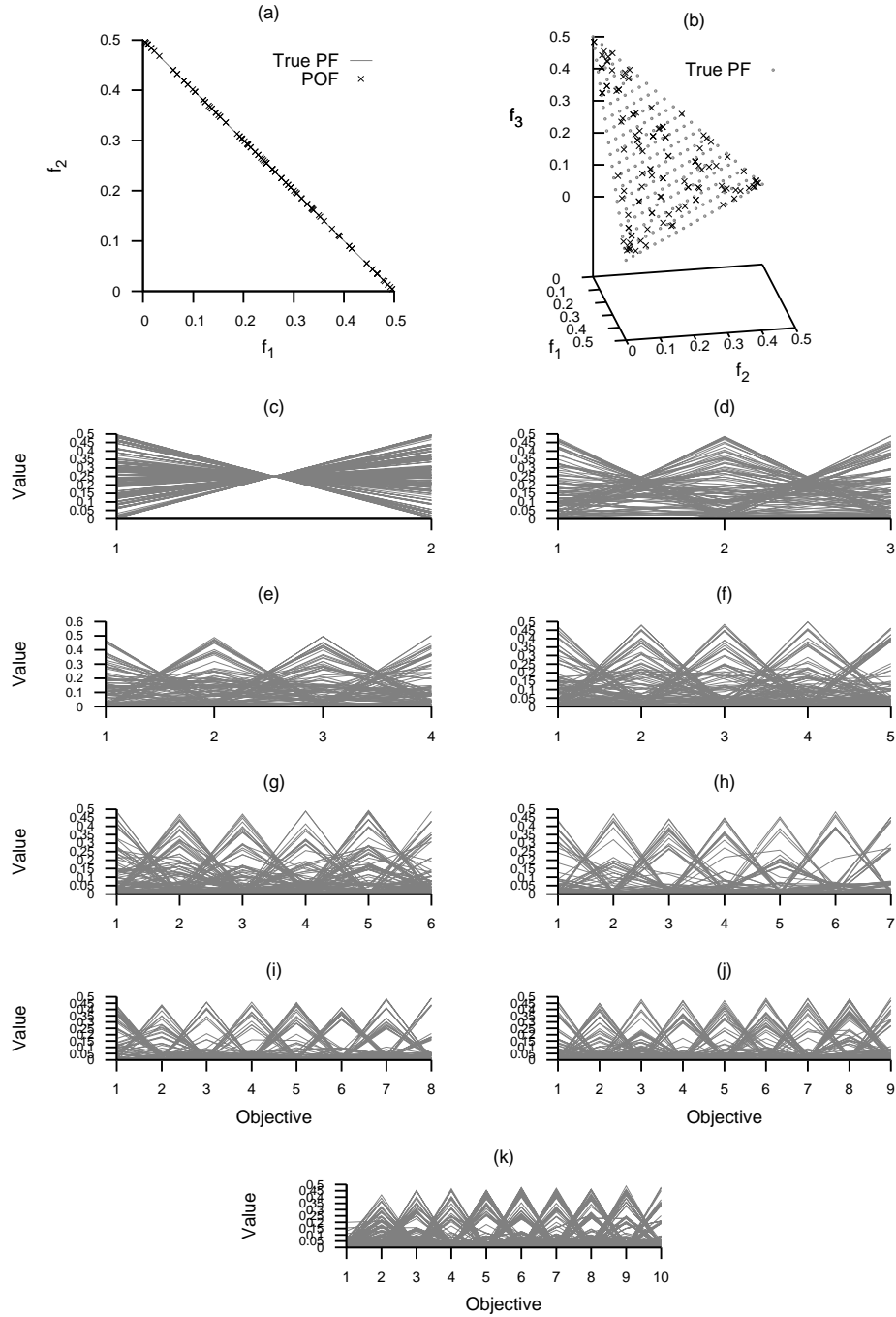


Figure A.10: Plots of the approximations obtained by *R2*-MOGA from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



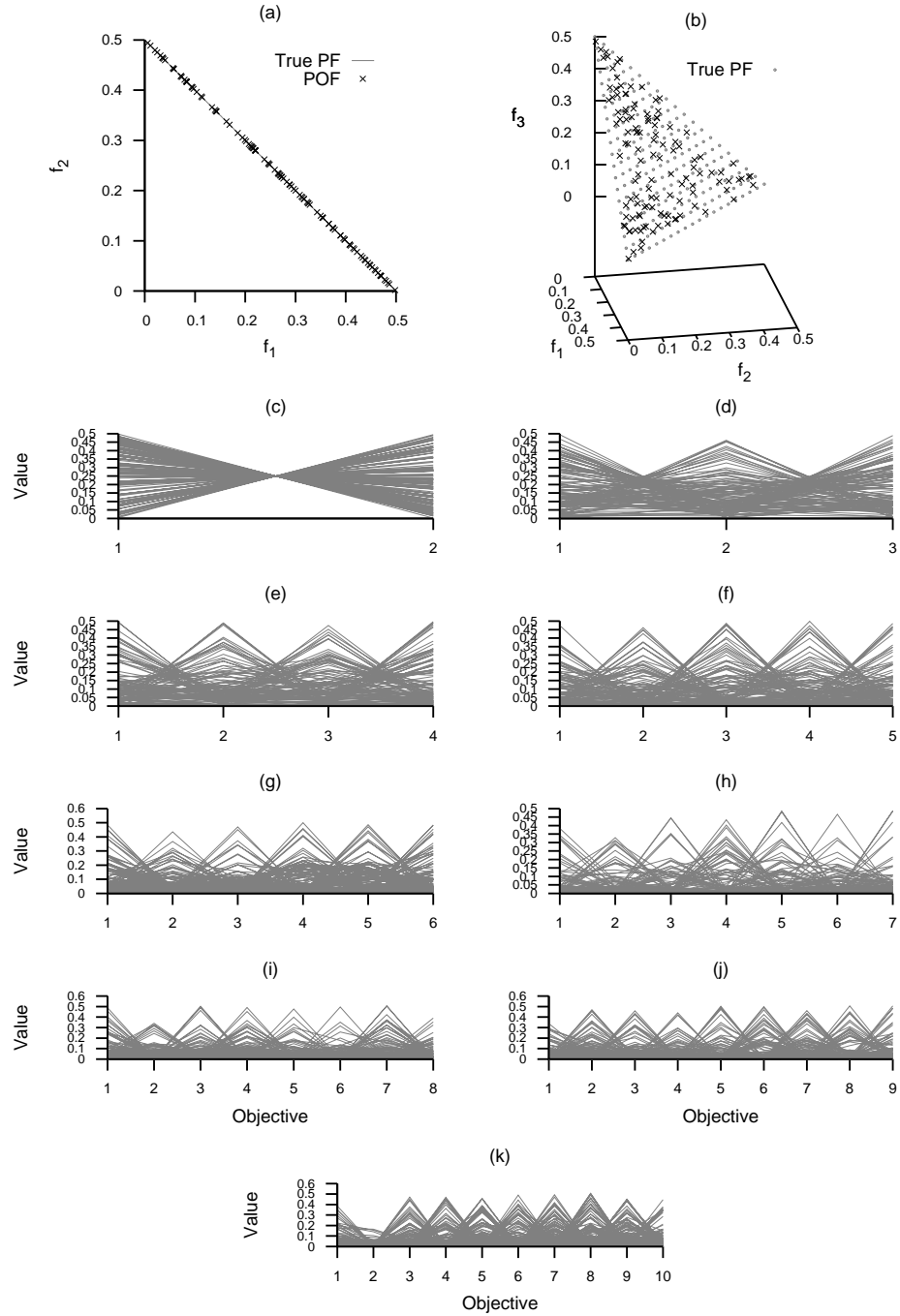


Figure A.11: Plots of the approximations obtained by *R2-MOGAw* from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

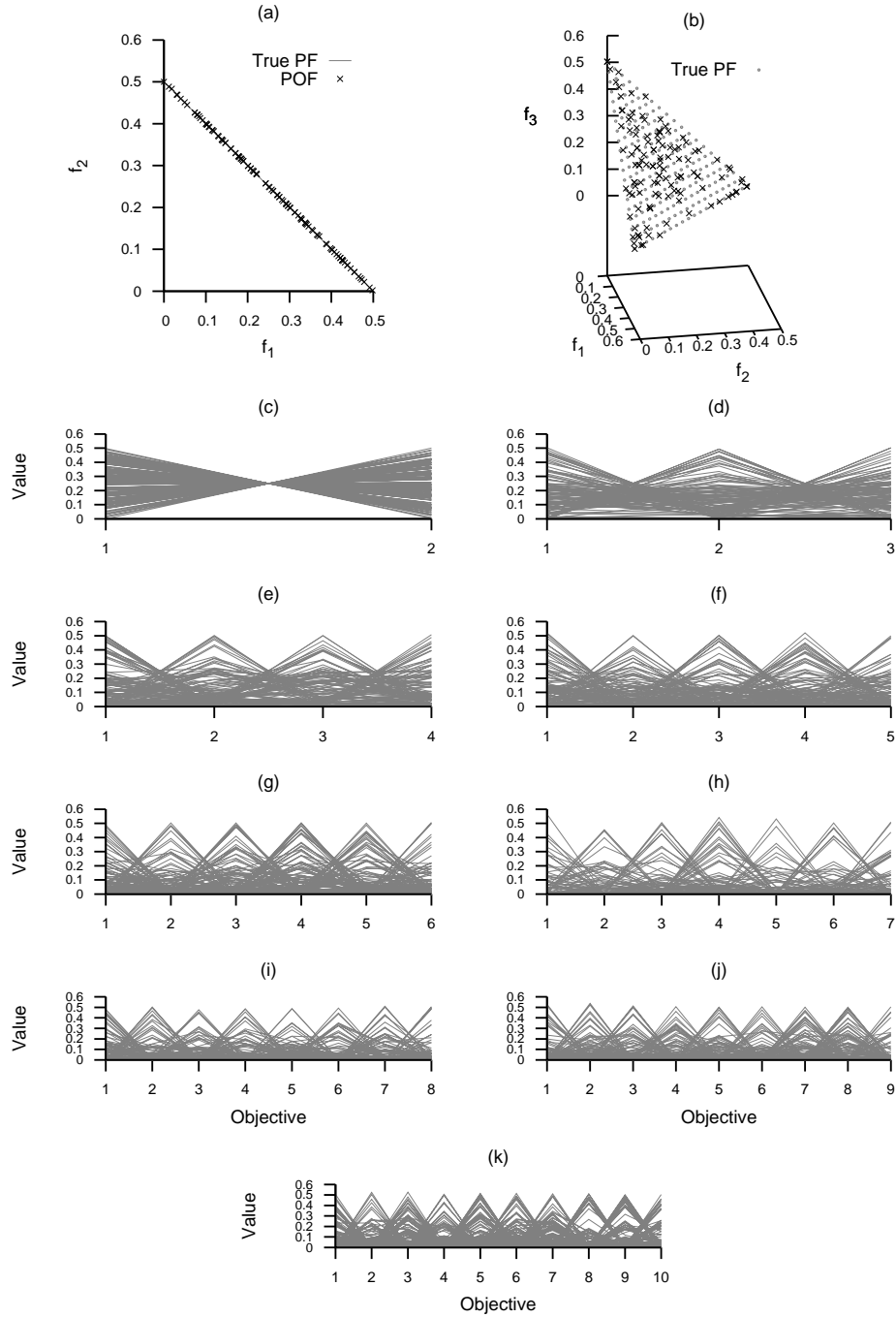


Figure A.12: Plots of the approximations obtained by *R2-MODE* from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

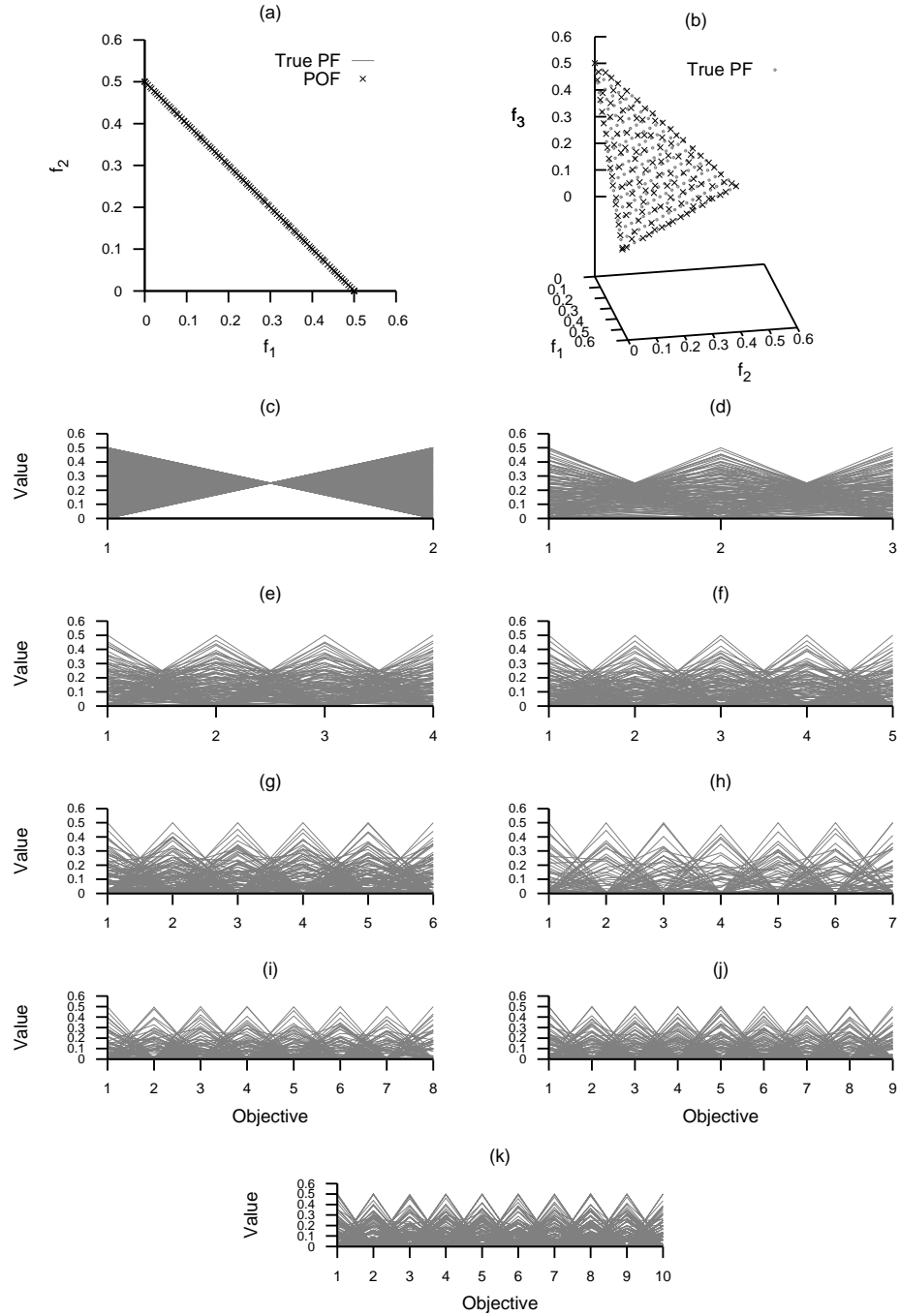


Figure A.13: Plots of the approximations obtained by R2-IBEA from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

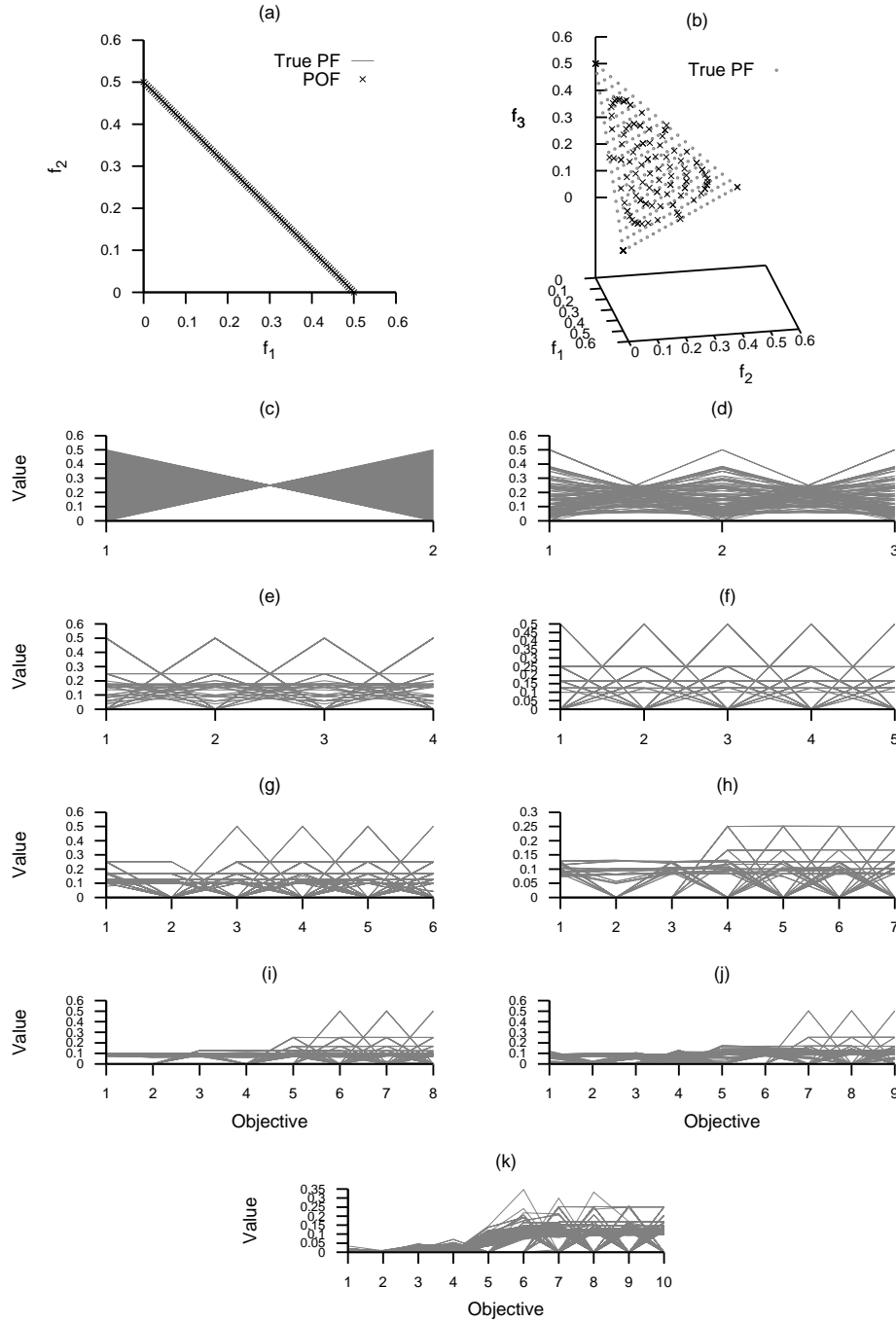


Figure A.14: Plots of the approximations obtained by MOMBI-TCH from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

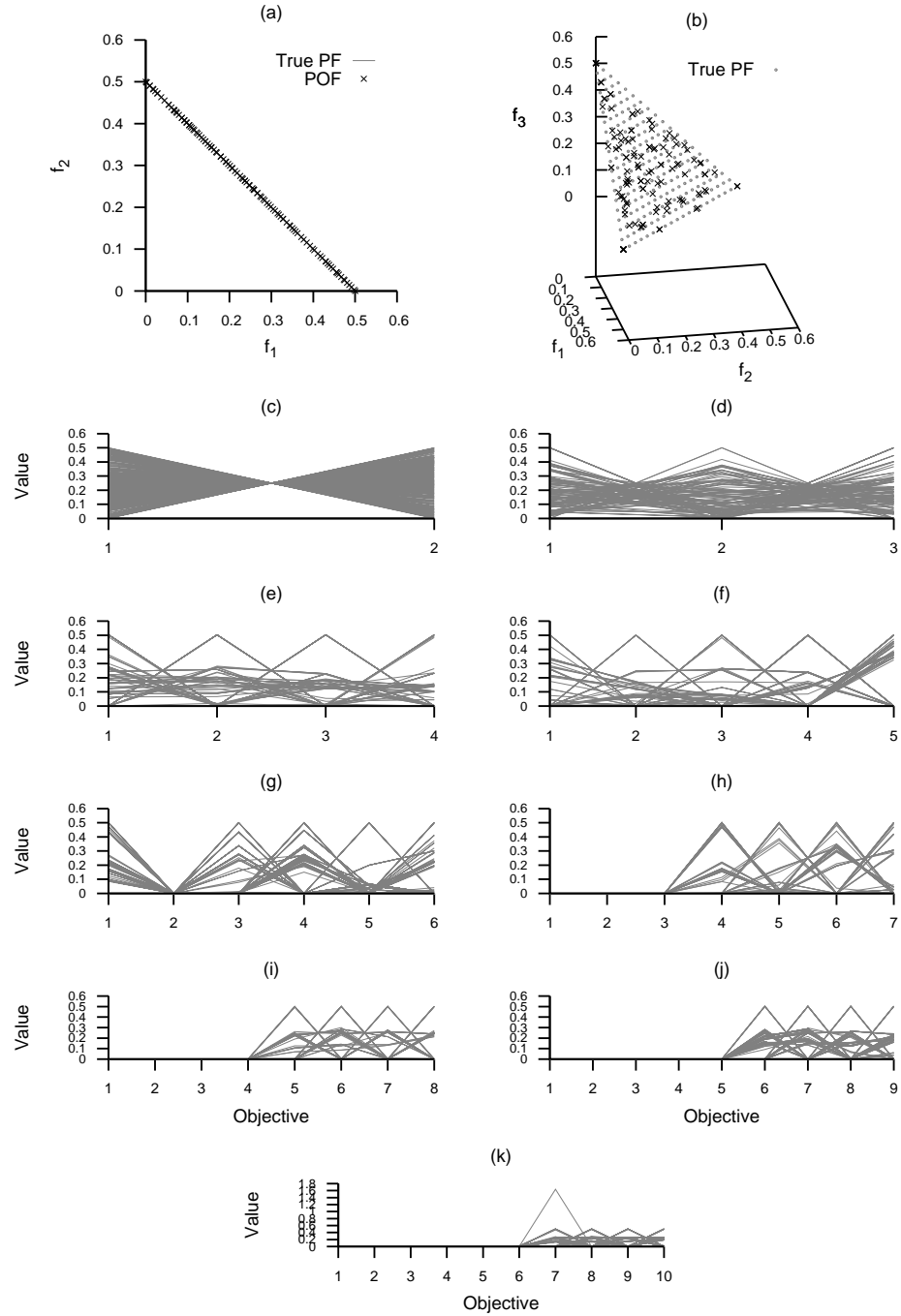


Figure A.15: Plots of the approximations obtained by MOMBI-NTCH from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

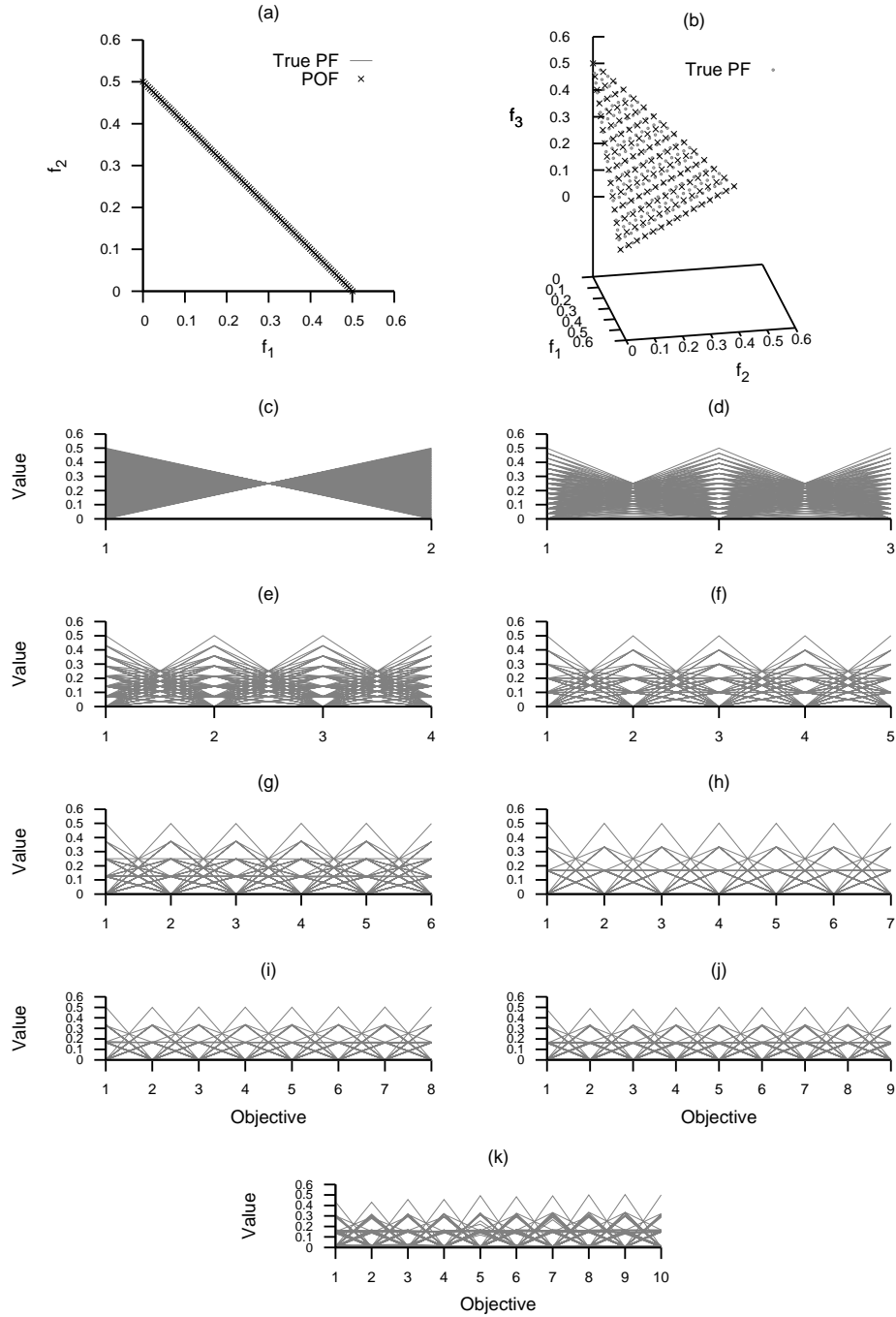


Figure A.16: Plots of the approximations obtained by MOMBI-PBI from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.8: Comparison of hypervolume indicator values for different optimizers on the DTLZ2 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	3.210041e+00	3.210867e+00	3.210768e+00	3.210853e+00	3.211608e+00	3.196187e+00	3.192914e+00	3.193951e+00	3.205840e+00	3.211129e+00	3.210818e+00	3.209403e+00	3.210499e+00
	avg.	3.210061e+00	3.210861e+00	3.210713e+00	3.210840e+00	3.211602e+00	3.196953e+00	3.190293e+00	3.192027e+00	3.205000e+00	3.211126e+00	3.210804e+00	3.209413e+00	3.210489e+00
	min.	3.209661e+00	3.210728e+00	3.209581e+00	3.210688e+00	3.211480e+00	3.155571e+00	3.147819e+00	3.158375e+00	3.175711e+00	3.210942e+00	3.210618e+00	3.208809e+00	3.209876e+00
	max.	3.210394e+00	3.210875e+00	3.210862e+00	3.210874e+00	3.211621e+00	3.210569e+00	3.204530e+00	3.206834e+00	3.207691e+00	3.211267e+00	3.210873e+00	3.209972e+00	3.210841e+00
	std.	1.678848e-04	2.242557e-05	1.883532e-04	3.708775e-05	1.951191e-05	1.274213e-02	1.083726e-02	1.085974e-02	3.957175e-03	6.865480e-05	5.254509e-05	2.074029e-04	1.823577e-04
3D	med.	7.204697e+00	7.383422e+00	6.411807e+00	7.422151e+00	7.431534e+00	7.414369e+00	7.354045e+00	7.326577e+00	7.393166e+00	7.427648e+00	7.383677e+00	7.388695e+00	7.421743e+00
	avg.	7.200007e+00	7.383686e+00	6.442565e+00	7.422149e+00	7.431539e+00	7.392500e+00	7.353170e+00	7.322264e+00	7.392679e+00	7.427598e+00	7.383834e+00	7.388654e+00	7.421723e+00
	min.	7.009836e+00	7.382357e+00	6.408709e+00	7.421998e+00	7.431399e+00	7.315550e+00	7.310730e+00	7.226704e+00	7.382249e+00	7.426628e+00	7.382309e+00	7.383618e+00	7.421200e+00
	max.	7.320675e+00	7.386898e+00	7.294744e+00	7.422207e+00	7.431686e+00	7.417668e+00	7.380991e+00	7.365824e+00	7.401333e+00	7.428481e+00	7.386836e+00	7.394248e+00	7.422053e+00
	std.	5.839806e-02	9.095784e-04	1.403833e-01	3.623787e-05	4.884768e-05	3.233107e-02	1.270901e-02	3.015986e-02	4.243701e-03	4.059323e-04	1.044390e-03	2.242911e-03	1.569161e-04
4D	med.	1.472552e+01	1.542231e+01	1.276200e+01	1.556777e+01	1.189937e+01	1.556396e+01	1.544233e+01	1.540849e+01	1.551662e+01	1.557638e+01	1.542123e+01	1.542185e+01	1.556709e+01
	avg.	1.471135e+01	1.542236e+01	1.283631e+01	1.556775e+01	1.190944e+01	1.556162e+01	1.544101e+01	1.539563e+01	1.551597e+01	1.557642e+01	1.542163e+01	1.542248e+01	1.556708e+01
	min.	1.422657e+01	1.542081e+01	1.255945e+01	1.556748e+01	1.179363e+01	1.544907e+01	1.539562e+01	1.514228e+01	1.549381e+01	1.557358e+01	1.542060e+01	1.542067e+01	1.556669e+01
	max.	1.510316e+01	1.542509e+01	1.506838e+01	1.556797e+01	1.240005e+01	1.556979e+01	1.548211e+01	1.548554e+01	1.553360e+01	1.557900e+01	1.542921e+01	1.542972e+01	1.556741e+01
	std.	1.253025e-01	7.948853e-04	3.559775e-01	1.047063e-04	7.836827e-02	1.238593e-02	1.794148e-02	5.147422e-02	8.098221e-03	1.162881e-03	1.447282e-03	1.742783e-03	1.566787e-04
5D	med.	2.941493e+01	3.153333e+01	2.542240e+01	3.166794e+01	2.430152e+01	3.164526e+01	3.147832e+01	3.144111e+01	3.160021e+01	3.167825e+01	3.153407e+01	3.153469e+01	3.166723e+01
	avg.	2.942620e+01	3.153316e+01	2.547165e+01	3.166796e+01	2.432020e+01	3.164765e+01	3.147656e+01	3.143405e+01	3.159404e+01	3.167816e+01	3.153368e+01	3.153493e+01	3.166721e+01
	min.	2.849557e+01	3.153089e+01	2.532464e+01	3.166701e+01	2.383752e+01	3.149548e+01	3.139351e+01	3.127965e+01	3.136684e+01	3.167360e+01	3.148752e+01	3.153403e+01	3.166591e+01
	max.	3.014224e+01	3.153708e+01	2.859078e+01	3.166860e+01	2.488747e+01	3.167489e+01	3.152941e+01	3.154584e+01	3.162630e+01	3.168224e+01	3.153828e+01	3.154294e+01	3.166800e+01
	std.	2.857214e-01	8.701011e-04	3.241538e-01	2.834108e-04	2.096831e-01	2.538691e-02	2.933922e-02	5.166054e-02	3.665720e-02	1.857753e-03	4.695449e-03	1.140466e-03	3.626224e-04
6D	med.	5.534950e+01	6.282535e+01	5.046712e+01	6.374009e+01	5.305381e+01	6.372645e+01	6.346045e+01	6.346204e+01	6.364523e+01	6.375161e+01	6.308353e+01	6.297146e+01	6.373806e+01
	avg.	5.515635e+01	6.276079e+01	5.069441e+01	6.374007e+01	5.305347e+01	6.370626e+01	6.345828e+01	6.344206e+01	6.357568e+01	6.375149e+01	6.304713e+01	6.297100e+01	6.373805e+01
	min.	5.041419e+01	6.136742e+01	5.045624e+01	6.373933e+01	5.109607e+01	6.231192e+01	6.333743e+01	6.322149e+01	6.233958e+01	6.374371e+01	6.283107e+01	6.285097e+01	6.373635e+01
	max.	5.866762e+01	6.317272e+01	5.736945e+01	6.374083e+01	5.432416e+01	6.375568e+01	6.354051e+01	6.359628e+01	6.368353e+01	6.375564e+01	6.318907e+01	6.317983e+01	6.373926e+01
	std.	1.748605e+00	2.481620e-01	1.057031e+00	3.540214e-04	6.497207e-01	1.433321e-01	4.170817e-02	7.973354e-02	2.460603e-01	2.278054e-03	1.006873e-01	7.957350e-02	5.375246e-04
7D	med.	7.097237e+01	1.221604e+02	1.004964e+02	1.277545e+02	1.003146e+02	1.263361e+02	1.270800e+02	1.269847e+02	1.275699e+02	1.277741e+02	1.218068e+02	1.228784e+02	1.277496e+02
	avg.	6.907730e+01	1.218160e+02	1.011378e+02	1.277545e+02	1.002544e+02	1.262013e+02	1.270752e+02	1.269319e+02	1.273958e+02	1.277739e+02	1.217793e+02	1.225544e+02	1.277494e+02
	min.	2.680668e+01	1.198434e+02	9.987591e+01	1.277538e+02	9.589385e+01	1.240811e+02	1.268092e+02	1.258948e+02	1.252577e+02	1.277628e+02	1.194244e+02	1.203942e+02	1.277455e+02
	max.	9.654712e+01	1.235530e+02	1.153471e+02	1.277548e+02	1.041526e+02	1.271519e+02	1.272888e+02	1.274816e+02	1.276417e+02	1.277822e+02	1.235331e+02	1.231983e+02	1.277521e+02
	std.	1.194559e+01	1.092969e+00	2.665939e+00	1.706577e-04	1.725065e+00	6.213868e-01	9.754212e-02	3.222027e-01	4.775350e-01	3.257479e-03	1.301388e+00	5.964771e-01	1.459325e-03
8D	med.	4.046867e+01	2.436705e+02	2.009920e+02	2.558229e+02	2.182929e+02	2.539939e+02	2.552032e+02	2.552207e+02	2.556410e+02	2.558317e+02	2.439155e+02	2.457868e+02	2.558167e+02
	avg.	4.731003e+01	2.428031e+02	2.016406e+02	2.558228e+02	2.178562e+02	2.539778e+02	2.551761e+02	2.551927e+02	2.554974e+02	2.558317e+02	2.436226e+02	2.452225e+02	2.558164e+02
	min.	1.019203e+01	2.330937e+02	1.986191e+02	2.558201e+02	2.110694e+02	2.521568e+02	2.548047e+02	2.547165e+02	2.533401e+02	2.558213e+02	2.364699e+02	2.413145e+02	2.558089e+02
	max.	1.044286e+02	2.480525e+02	2.275435e+02	2.558243e+02	2.226235e+02	2.552319e+02	2.553962e+02	2.555856e+02	2.557183e+02	2.558409e+02	2.486940e+02	2.491416e+02	2.558200e+02
	std.	2.423026e+01	3.457472e+00	3.801588e+00	8.937217e-04	2.513306e+00	6.518251e-01	1.251305e-01	2.043639e-01	4.230122e-01	3.084503e-03	3.031558e+00	1.193651e+00	2.101846e-03
9D	med.	9.291244e+01	4.878235e+02	4.019926e+02	5.118658e+02	4.624523e+02	5.098186e+02	5.113053e+02	5.113666e+02	5.116906e+02	5.118745e+02	4.881349e+02	4.916219e+02	5.118529e+02
	avg.	9.800724e+01	4.859490e+02	4.026848e+02	5.118654e+02	4.620671e+02	5.098094e+02	5.113004e+02	5.113446e+02	5.115584e+02	5.118738e+02	4.877085e+02	4.910696e+02	5.118520e+02
	min.	2.548915e+01	4.633840e+02	3.989053e+02	5.118548e+02	4.509915e+02	5.073781e+02	5.108276e+02	5.108124e+02	5.054671e+02	5.118642e+02	4.694249e+02	4.865505e+02	5.118332e+02
	max.	2.112485e+02	4.933729e+02	4.513644e+02	5.118727e+02	4.678390e+02	5.111735e+02	5.114953e+02	5.116310e+02	5.117689e+02	5.118806e+02	4.957771e+02	4.977965e+02	5.118630e+02
	std.	4.742846e+01	6.444997e+00	4.987750e+00	3.654564e-03	3.343018e+00	7.599114e-01	1.087475e-01	1.548704e-01	6.714275e-01	4.041045e-03	5.941868e+00	1.743546e+00	5.992567e-03
10D	med.	3.256503e+02	9.573993e+02	8.039549e+02	1.023867e+03	9.616218e+02	1.022179e+03	1.023398e+03	1.023392e+03	1.023824e+03	1.023884e+03	9.785497e+02	9.827967e+02	1.023844e+03
	avg.	3.263275e+02	9.608083e+02	8.063469e+02	1.023866e+03	9.637606e+02	1.022115e+03	1.023368e+03	1.023371e+03	1.023823e+03	1.023882e+03	9.781965e+02	9.815709e+02	1.023843e+03
	min.	1.464623e+02	9.271927e+02	8.008404e+02	1.023811e+03	9.520352e+02	1.020583e+03	1.022759e+03	1.022947e+03	1.023784e+03	1.023843e+03	9.497005e+02	9.729246e+02	1.023779e+03
	max.	6.107166e+02	9.857649e+02	9.076187e+02	1.023897e+03	1.023867e+03	1.023651e+03	1.023540e+03	1.023623e+03	1.023850e+03	1.023905e+03	9.972909e+02	9.949128e+02	1.023878e+03
	std.	8.937217e+01	1.582786e+01	1.460699e+01	1.562131e-02	1.292354e+01	6.017819e-01	1.310344e-01	1.389968e-01	1.352330e-02	1.303181e-02	1.037190e+01	3.926336e+00	1.818843e-02

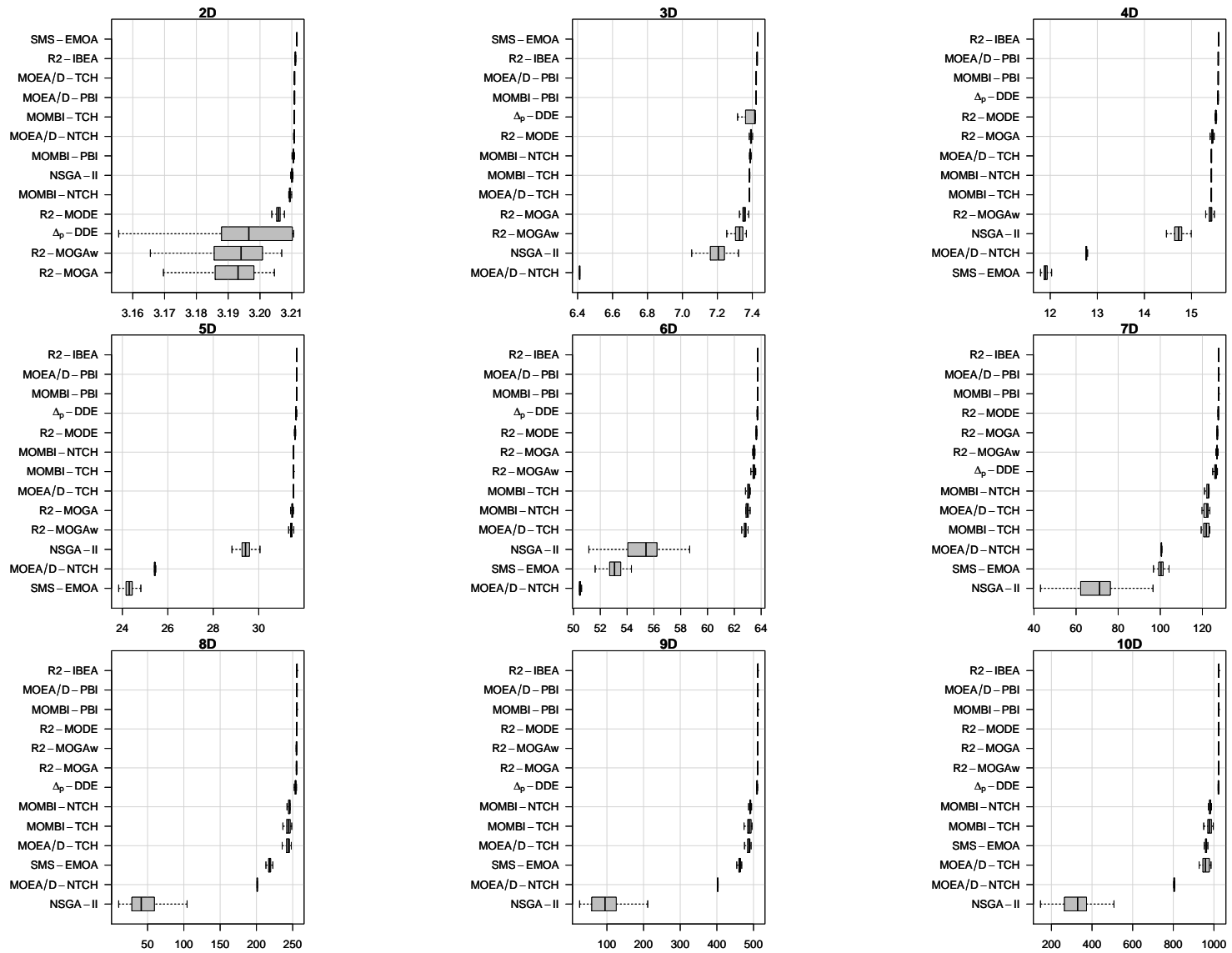


Figure A.17: Box-plot of hypervolume indicator values for different optimizers on the DTLZ2 test problem.



Table A.9: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ2 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	1.67e-09	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	1.81e-33	> 0.05
MOEA/D-TCH	1.25e-34	—	2.23e-31	8.64e-08	> 0.05	1.25e-34	1.25e-34	1.25e-34	1.25e-34	> 0.05	2.02e-21	1.25e-34	2.90e-34
MOEA/D-NTCH	5.04e-32	> 0.05	—	> 0.05	> 0.05	7.21e-33	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	2.79e-34	7.36e-20
MOEA/D-PBI	1.26e-34	> 0.05	2.05e-22	—	> 0.05	1.26e-34	1.26e-34	1.26e-34	1.26e-34	> 0.05	6.66e-09	1.26e-34	2.33e-33
SMS-EMOA	1.26e-34	1.23e-34	1.26e-34	1.24e-34	—	1.26e-34	1.26e-34	1.26e-34	1.26e-34	1.26e-34	1.26e-34	1.26e-34	1.26e-34
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	1.85e-04	1.18e-03	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	9.47e-03	3.64e-31	8.32e-25	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	1.28e-34	1.25e-34	1.28e-34	1.26e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	1.28e-34	> 0.05	1.31e-08	> 0.05	> 0.05	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	—	1.28e-34	1.08e-30
MOMBI-NTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.36e-05	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	3.97e-29	> 0.05	> 0.05	> 0.05	> 0.05	2.22e-26	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	1.36e-34	—
3D													
NSGA-II	—	> 0.05	9.39e-33	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	1.28e-34	> 0.05	> 0.05	> 0.05	1.28e-34	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	1.28e-34	1.28e-34	—	> 0.05	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.28e-34	1.44e-34
SMS-EMOA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34
$\Delta_p$ -DDE	1.32e-34	4.65e-05	1.28e-34	> 0.05	> 0.05	—	1.16e-14	1.33e-25	2.14e-04	> 0.05	4.65e-05	1.10e-04	> 0.05
R2-MOGA	1.36e-34	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	—	1.39e-17	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	2.90e-31	> 0.05	2.20e-34	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	1.28e-34	1.22e-29	1.28e-34	> 0.05	> 0.05	> 0.05	1.28e-34	1.28e-34	—	> 0.05	2.23e-29	1.23e-13	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	1.28e-34	1.28e-34	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.28e-34	4.63e-32	1.28e-34	> 0.05	> 0.05	> 0.05	1.28e-34	1.28e-34	> 0.05	> 0.05	2.55e-31	—	> 0.05
MOMBI-PBI	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.28e-34	—
4D													
NSGA-II	—	> 0.05	2.91e-33	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	5.52e-05	> 0.05	> 0.05	4.34e-17	2.44e-02	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.26e-34	1.26e-34	1.26e-34	—	1.26e-34	1.86e-22	1.26e-34	1.26e-34	1.26e-34	> 0.05	1.25e-34	1.25e-34	1.25e-34
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	—	3.05e-34	1.44e-34	2.51e-33	> 0.05	1.27e-34	1.28e-34	> 0.05
R2-MOGA	1.28e-34	7.28e-20	1.28e-34	> 0.05	1.28e-34	> 0.05	—	4.10e-17	> 0.05	> 0.05	3.13e-20	1.52e-19	> 0.05
R2-MOGAw	1.28e-34	> 0.05	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	> 0.05	1.28e-34	1.28e-34	—	> 0.05	1.27e-34	1.28e-34	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.26e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.27e-34	1.28e-34	1.27e-34
MOMBI-TCH	1.27e-34	> 0.05	1.27e-34	> 0.05	1.27e-34	> 0.05	> 0.05	8.48e-05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.28e-34	> 0.05	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	4.31e-05	> 0.05	> 0.05	4.58e-11	—	> 0.05
MOMBI-PBI	1.27e-34	1.27e-34	1.27e-34	> 0.05	1.27e-34	5.94e-20	1.27e-34	1.27e-34	1.27e-34	> 0.05	1.26e-34	1.27e-34	—

Table A.10: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ2 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
5D													
NSGA-II	—	> 0.05	1.32e-34	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	1.28e-34	> 0.05	1.28e-34	> 0.05	1.28e-34	5.62e-33	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	8.04e-31	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.27e-34	1.27e-34	5.57e-29
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e-34	2.51e-33	1.28e-34	> 0.05	1.28e-34	—	2.79e-34	1.63e-34	2.10e-28	> 0.05	2.42e-33	2.58e-33	> 0.05
R2-MOGA	1.28e-34	> 0.05	1.28e-34	> 0.05	1.28e-34	> 0.05	—	3.45e-11	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	1.28e-34	> 0.05	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	1.28e-34	1.32e-29	1.28e-34	> 0.05	1.28e-34	> 0.05	4.20e-31	6.00e-32	—	> 0.05	1.28e-29	1.32e-29	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.63e-34	1.28e-34	1.28e-34	1.28e-34	—	1.27e-34	1.28e-34	1.28e-34
MOMBI-TCH	1.27e-34	5.67e-26	1.27e-34	> 0.05	1.27e-34	> 0.05	3.52e-34	3.35e-33	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.28e-34	3.56e-33	1.28e-34	> 0.05	1.28e-34	> 0.05	1.28e-34	2.50e-33	> 0.05	> 0.05	8.58e-27	—	> 0.05
MOMBI-PBI	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.07e-30	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.27e-34	1.27e-34	—
6D													
NSGA-II	—	> 0.05	1.58e-30	> 0.05	1.35e-19	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	8.75e-31	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.28e-34	1.27e-34
SMS-EMOA	> 0.05	> 0.05	8.06e-31	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e-34	2.17e-33	1.28e-34	> 0.05	1.28e-34	—	2.82e-33	3.37e-33	4.70e-31	> 0.05	2.51e-33	2.51e-33	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	> 0.05	—	> 0.05	> 0.05	> 0.05	1.28e-34	1.28e-34	> 0.05
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	—	> 0.05	> 0.05	1.28e-34	1.28e-34	> 0.05
R2-MODE	1.28e-34	7.59e-30	1.28e-34	> 0.05	1.28e-34	> 0.05	3.06e-24	2.00e-24	—	> 0.05	1.38e-27	8.74e-28	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	2.66e-33	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	1.28e-34	1.92e-26	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	2.56e-09	> 0.05
MOMBI-NTCH	1.28e-34	1.98e-22	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	4.59e-27	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.28e-34	—
7D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	1.28e-34	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	8.58e-35	8.58e-35	8.56e-35	—	8.58e-35	8.58e-35	8.58e-35	8.58e-35	8.58e-35	> 0.05	8.58e-35	8.58e-35	8.50e-35
SMS-EMOA	1.32e-34	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	—	> 0.05	> 0.05	> 0.05	> 0.05	1.28e-34	1.28e-34	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	2.45e-31	—	1.83e-04	> 0.05	> 0.05	1.28e-34	1.28e-34	> 0.05
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	7.13e-21	> 0.05	—	> 0.05	> 0.05	1.28e-34	1.28e-34	> 0.05
R2-MODE	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	7.44e-28	5.56e-22	1.89e-22	—	> 0.05	1.28e-34	1.28e-34	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.27e-34	8.56e-35	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.27e-34
MOMBI-TCH	1.28e-34	> 0.05	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.28e-34	3.09e-07	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	6.54e-04	—	> 0.05
MOMBI-PBI	1.27e-34	1.27e-34	1.27e-34	> 0.05	1.27e-34	1.27e-34	1.27e-34	1.27e-34	1.27e-34	> 0.05	1.27e-34	1.27e-34	—

Table A.11: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ2 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
8D													
NSGA-II	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e – 34	–	1.28e – 34	> 0.05	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	1.28e – 34	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.25e – 34	1.25e – 34	1.25e – 34	–	1.25e – 34	1.25e – 34	1.25e – 34	1.25e – 34	1.25e – 34	> 0.05	1.25e – 34	1.25e – 34	1.24e – 34
SMS-EMOA	1.28e – 34	> 0.05	4.63e – 32	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	1.28e – 34	–	> 0.05	> 0.05	> 0.05	> 0.05	1.28e – 34	1.28e – 34	> 0.05
R2-MOGA	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	1.28e – 34	9.79e – 32	–	> 0.05	> 0.05	> 0.05	1.28e – 34	1.28e – 34	> 0.05
R2-MOGAw	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	1.28e – 34	8.65e – 31	> 0.05	–	> 0.05	> 0.05	1.28e – 34	1.28e – 34	> 0.05
R2-MODE	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	1.28e – 34	1.88e – 30	2.26e – 22	1.44e – 21	–	> 0.05	1.28e – 34	1.28e – 34	> 0.05
R2-IBEA	1.28e – 34	1.28e – 34	1.27e – 34	2.08e – 33	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	–	1.28e – 34	1.28e – 34	1.27e – 34
MOMBI-TCH	1.28e – 34	4.37e – 02	1.28e – 34	> 0.05	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NTCH	1.28e – 34	1.64e – 09	1.28e – 34	> 0.05	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	5.84e – 03	–	> 0.05
MOMBI-PBI	1.27e – 34	1.27e – 34	1.27e – 34	> 0.05	1.27e – 34	1.27e – 34	1.27e – 34	1.27e – 34	1.27e – 34	> 0.05	1.27e – 34	1.27e – 34	–
9D													
NSGA-II	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e – 34	–	1.28e – 34	> 0.05	9.46e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	1.28e – 34	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e – 34	1.28e – 34	1.28e – 34	–	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	1.28e – 34	1.28e – 34	2.30e – 32
SMS-EMOA	1.28e – 34	> 0.05	1.32e – 34	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	1.28e – 34	–	> 0.05	> 0.05	> 0.05	> 0.05	1.28e – 34	1.28e – 34	> 0.05
R2-MOGA	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	1.28e – 34	4.00e – 34	–	> 0.05	> 0.05	> 0.05	1.28e – 34	1.28e – 34	> 0.05
R2-MOGAw	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	1.28e – 34	7.46e – 34	3.87e – 03	–	> 0.05	> 0.05	1.28e – 34	1.28e – 34	> 0.05
R2-MODE	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	1.28e – 34	3.27e – 32	2.05e – 27	1.44e – 25	–	> 0.05	1.28e – 34	1.28e – 34	> 0.05
R2-IBEA	1.28e – 34	1.28e – 34	1.28e – 34	2.75e – 26	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	–	1.28e – 34	1.28e – 34	1.27e – 34
MOMBI-TCH	1.28e – 34	2.26e – 02	1.28e – 34	> 0.05	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NTCH	1.28e – 34	5.27e – 16	1.28e – 34	> 0.05	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	4.28e – 04	–	> 0.05
MOMBI-PBI	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	1.28e – 34	1.28e – 34	–
10D													
NSGA-II	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e – 34	–	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	1.28e – 34	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.27e – 34	1.27e – 34	1.27e – 34	–	7.49e – 34	1.27e – 34	1.27e – 34	1.27e – 34	5.38e – 31	> 0.05	1.27e – 34	1.27e – 34	1.09e – 16
SMS-EMOA	1.28e – 34	> 0.05	1.28e – 34	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	1.32e – 29	–	> 0.05	> 0.05	> 0.05	> 0.05	1.28e – 34	1.28e – 34	> 0.05
R2-MOGA	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	1.32e – 29	2.03e – 32	–	> 0.05	> 0.05	> 0.05	1.28e – 34	1.28e – 34	> 0.05
R2-MOGAw	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	1.32e – 29	2.83e – 32	> 0.05	–	> 0.05	> 0.05	1.28e – 34	1.28e – 34	> 0.05
R2-MODE	1.26e – 34	1.26e – 34	1.26e – 34	> 0.05	9.03e – 31	1.26e – 34	1.26e – 34	1.26e – 34	–	> 0.05	1.26e – 34	1.26e – 34	> 0.05
R2-IBEA	1.27e – 34	1.27e – 34	1.27e – 34	2.62e – 13	2.01e – 34	1.27e – 34	1.27e – 34	1.27e – 34	1.77e – 34	–	1.27e – 34	1.27e – 34	7.73e – 30
MOMBI-TCH	1.28e – 34	7.76e – 14	1.28e – 34	> 0.05	1.82e – 21	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NTCH	1.28e – 34	1.03e – 20	1.28e – 34	> 0.05	1.32e – 29	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	1.27e – 34	1.27e – 34	1.27e – 34	> 0.05	1.03e – 32	1.27e – 34	1.27e – 34	1.27e – 34	6.58e – 16	> 0.05	1.27e – 34	1.27e – 34	–

Table A.12: Comparison of R2 indicator values for different optimizers on the DTLZ2 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_r$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	2.181209e-01	2.180305e-01	2.180300e-01	2.180298e-01	2.178722e-01	2.181639e-01	2.192049e-01	2.186234e-01	2.187996e-01	2.179355e-01	2.180307e-01	2.182482e-01	2.180293e-01
	avg.	2.181241e-01	2.180305e-01	2.180383e-01	2.180299e-01	2.178728e-01	2.182027e-01	2.192440e-01	2.186837e-01	2.188392e-01	2.179315e-01	2.180307e-01	2.182512e-01	2.180296e-01
	min.	2.180667e-01	2.180291e-01	2.180158e-01	2.180284e-01	2.177904e-01	2.180351e-01	2.185684e-01	2.183632e-01	2.184920e-01	2.178462e-01	2.180280e-01	2.181429e-01	2.180257e-01
	max.	2.182009e-01	2.180319e-01	2.182281e-01	2.180314e-01	2.179938e-01	2.190214e-01	2.206054e-01	2.191841e-01	2.195585e-01	2.180068e-01	2.180335e-01	2.183471e-01	2.180385e-01
	std.	3.159763e-05	5.242750e-07	3.039034e-05	5.384028e-07	4.992825e-05	1.632019e-04	3.129649e-04	1.968859e-04	1.818237e-04	3.118474e-05	9.979933e-07	4.031774e-05	2.187844e-06
3D	med.	1.023371e-01	9.482117e-02	1.454396e-01	9.375328e-02	9.342105e-02	9.415902e-02	9.582775e-02	9.517195e-02	9.530193e-02	9.358387e-02	9.479937e-02	9.483855e-02	9.375496e-02
	avg.	1.022786e-01	9.481855e-02	1.437735e-01	9.375316e-02	9.343200e-02	9.519784e-02	9.583423e-02	9.525132e-02	9.529329e-02	9.358182e-02	9.479659e-02	9.483937e-02	9.375507e-02
	min.	9.647199e-02	9.476679e-02	9.838636e-02	9.374691e-02	9.326095e-02	9.392017e-02	9.492856e-02	9.452792e-02	9.478853e-02	9.334817e-02	9.470922e-02	9.466270e-02	9.374733e-02
	max.	1.115075e-01	9.485222e-02	1.457164e-01	9.375818e-02	9.361585e-02	9.945811e-02	9.690588e-02	9.750522e-02	9.609324e-02	9.376088e-02	9.483863e-02	9.501872e-02	9.376601e-02
	std.	3.028434e-03	1.856813e-05	7.622799e-03	2.382229e-06	6.554520e-05	1.627012e-03	3.605646e-04	4.506803e-04	2.468979e-04	7.116383e-05	2.534147e-05	7.504909e-05	3.637514e-06
4D	med.	7.074169e-02	5.602772e-02	1.101975e-01	5.307967e-02	1.289365e-01	5.317924e-02	5.474327e-02	5.428139e-02	5.409971e-02	5.282202e-02	5.605306e-02	5.602870e-02	5.308252e-02
	avg.	7.093535e-02	5.602909e-02	1.087218e-01	5.307922e-02	1.286607e-01	5.323301e-02	5.474460e-02	5.441906e-02	5.412399e-02	5.283081e-02	5.604940e-02	5.602042e-02	5.308295e-02
	min.	6.334403e-02	5.587891e-02	6.228230e-02	5.307062e-02	1.211009e-01	5.295442e-02	5.419531e-02	5.361693e-02	5.361331e-02	5.260711e-02	5.588959e-02	5.589694e-02	5.307332e-02
	max.	7.976062e-02	5.618449e-02	1.141300e-01	5.308667e-02	1.310058e-01	5.820804e-02	5.546428e-02	5.740379e-02	5.487234e-02	5.311785e-02	5.608170e-02	5.606377e-02	5.309419e-02
	std.	2.788847e-03	5.580329e-05	7.272083e-03	3.560463e-06	1.328579e-03	5.172538e-04	2.504692e-04	5.254275e-04	2.257599e-04	9.741346e-05	2.902193e-05	3.718042e-05	4.274056e-06
5D	med.	5.579818e-02	3.736435e-02	8.940266e-02	3.525690e-02	9.722498e-02	3.571743e-02	3.664279e-02	3.652116e-02	3.605080e-02	3.518707e-02	3.730064e-02	3.729266e-02	3.525810e-02
	avg.	5.608731e-02	3.736518e-02	8.884798e-02	3.526220e-02	9.719539e-02	3.582412e-02	3.665748e-02	3.661124e-02	3.603968e-02	3.518690e-02	3.729908e-02	3.729074e-02	3.525810e-02
	min.	4.888621e-02	3.727902e-02	6.397242e-02	3.523940e-02	9.384441e-02	3.508272e-02	3.619932e-02	3.600310e-02	3.571184e-02	3.500577e-02	3.723066e-02	3.722208e-02	3.524278e-02
	max.	6.394102e-02	3.743803e-02	9.030941e-02	3.553496e-02	1.006565e-01	4.124649e-02	3.736182e-02	3.878073e-02	3.653788e-02	3.535166e-02	3.730426e-02	3.730072e-02	3.527093e-02
	std.	2.903667e-03	3.595765e-05	2.719380e-03	3.634410e-05	1.249441e-03	7.568369e-04	2.179951e-04	4.697481e-04	1.648300e-04	8.304775e-05	8.371063e-06	1.039305e-05	4.831481e-06
6D	med.	5.676296e-02	3.413571e-02	7.485327e-02	2.432062e-02	7.222638e-02	2.440794e-02	2.552929e-02	2.550195e-02	2.489138e-02	2.420349e-02	3.280709e-02	3.348565e-02	2.432575e-02
	avg.	5.714195e-02	3.425647e-02	7.408835e-02	2.432278e-02	7.231577e-02	2.442667e-02	2.554816e-02	2.567265e-02	2.488618e-02	2.420181e-02	3.405521e-02	3.347171e-02	2.432575e-02
	min.	4.912422e-02	2.988578e-02	5.227528e-02	2.430920e-02	6.988149e-02	2.415228e-02	2.519559e-02	2.494740e-02	2.464989e-02	2.408964e-02	3.126121e-02	3.004241e-02	2.431508e-02
	max.	6.695929e-02	3.963263e-02	7.548691e-02	2.446380e-02	7.557578e-02	2.504730e-02	2.600775e-02	2.744187e-02	2.537134e-02	2.434308e-02	4.121077e-02	4.063210e-02	2.433812e-02
	std.	4.422853e-03	1.652450e-03	3.638827e-03	1.477516e-05	9.468686e-04	1.168868e-04	1.698310e-04	5.476977e-04	1.167544e-04	6.070074e-05	2.470366e-05	2.094639e-05	4.428089e-06
7D	med.	7.819665e-02	3.697359e-02	6.524712e-02	1.840519e-02	6.682984e-02	3.009487e-02	2.017335e-02	2.107372e-02	1.915314e-02	1.826765e-02	4.140467e-02	3.338075e-02	1.840757e-02
	avg.	7.793152e-02	3.711273e-02	6.431751e-02	1.840536e-02	6.678481e-02	3.006960e-02	2.020431e-02	2.117617e-02	1.918268e-02	1.827444e-02	4.238586e-02	3.579850e-02	1.840755e-02
	min.	6.062225e-02	3.197183e-02	4.456807e-02	1.840231e-02	6.481979e-02	2.587162e-02	1.968510e-02	1.914481e-02	1.883327e-02	1.818168e-02	3.483561e-02	3.226089e-02	1.840499e-02
	max.	9.279451e-02	4.192969e-02	6.611065e-02	1.840745e-02	6.876173e-02	3.433404e-02	2.109095e-02	2.433273e-02	2.041411e-02	1.840692e-02	5.248325e-02	4.557061e-02	1.841085e-02
	std.	6.930724e-03	2.181413e-03	3.871012e-03	9.849248e-07	8.612038e-04	1.950649e-03	2.221970e-04	1.038066e-03	1.993934e-04	4.875252e-05	6.099658e-03	3.957467e-03	1.192882e-06
8D	med.	8.728345e-02	3.197317e-02	5.738658e-02	1.401199e-02	5.188276e-02	2.466166e-02	1.512196e-02	1.550837e-02	1.454048e-02	1.410666e-02	3.878063e-02	2.980750e-02	1.401883e-02
	avg.	8.654701e-02	3.219225e-02	5.697015e-02	1.401212e-02	5.189920e-02	2.449135e-02	1.514345e-02	1.556463e-02	1.454183e-02	1.410804e-02	3.724851e-02	3.166334e-02	1.401892e-02
	min.	6.851175e-02	2.742824e-02	3.996683e-02	1.400372e-02	5.063527e-02	2.024345e-02	1.481509e-02	1.455729e-02	1.429283e-02	1.398735e-02	2.963748e-02	2.814630e-02	1.401085e-02
	max.	1.064165e-01	3.760257e-02	5.890231e-02	1.401898e-02	5.356215e-02	2.815615e-02	1.566246e-02	1.684875e-02	1.474048e-02	1.417913e-02	4.753252e-02	4.162819e-02	1.402840e-02
	std.	7.976934e-03	2.475900e-03	2.485675e-03	2.842770e-06	5.739246e-04	1.698071e-03	1.311135e-04	4.626930e-04	7.667117e-05	3.591441e-05	5.794050e-03	3.682743e-03	2.811523e-06
9D	med.	7.590121e-02	2.877975e-02	5.034493e-02	1.101262e-02	4.172747e-02	2.087137e-02	1.183744e-02	1.192100e-02	1.139284e-02	1.107163e-02	3.354519e-02	2.715120e-02	1.103019e-02
	avg.	7.591020e-02	2.862741e-02	5.014990e-02	1.101327e-02	4.176770e-02	2.052122e-02	1.183700e-02	1.195240e-02	1.139835e-02	1.107371e-02	3.294478e-02	2.822815e-02	1.103033e-02
	min.	6.146893e-02	2.528527e-02	3.595290e-02	1.099215e-02	4.073997e-02	1.673736e-02	1.163383e-02	1.144113e-02	1.123587e-02	1.097614e-02	2.674951e-02	2.523502e-02	1.101835e-02
	max.	8.992139e-02	3.406125e-02	5.120181e-02	1.108177e-02	4.320502e-02	2.351616e-02	1.207125e-02	1.305862e-02	1.160650e-02	1.114390e-02	4.344561e-02	3.428465e-02	1.104818e-02
	std.	5.402083e-03	2.140070e-03	1.453043e-03	9.754754e-06	4.736898e-04	1.483839e-03	7.238748e-05	2.976865e-04	5.725889e-05	2.626320e-05	4.838054e-03	2.509209e-03	5.480434e-06
10D	med.	6.466920e-02	2.704964e-02	4.561577e-02	8.843920e-03	3.444595e-02	1.734668e-02	9.397258e-03	9.406372e-03	9.097049e-03	8.863846e-03	3.126542e-02	2.498141e-02	8.879561e-03
	avg.	6.473314e-02	2.662066e-02	4.532131e-02	8.851344e-03	3.344448e-02	1.719186e-02	9.344478e-03	9.431617e-03	9.101004e-03	8.863970e-03	3.294357e-02	2.603823e-02	8.880272e-03
	min.	5.514010e-02	2.266376e-02	3.232191e-02	8.823247e-03	9.156182e-03	1.220479e-02	9.283465e-03	9.127561e-03	8.995099e-03	8.806135e-03	2.339077e-02	2.265424e-02	8.862477e-03
	max.	7.289473e-02	3.084468e-02	4.602413e-02	9.046539e-03	3.536005e-03	1.894579e-02	1.014456e-02	9.897853e-03	9.217634e-03	8.924522e-03	3.719363e-02	3.177964e-02	8.909098e-03
	std.	4.160889e-03	2.322785e-03	1.874865e-03	3.180852e-05	4.983677e-03	1.161652e-03	8.798769e-05	1.524081e-04	3.962717e-05	2.244796e-05	4.216500e-03	2.511599e-03	8.573618e-06

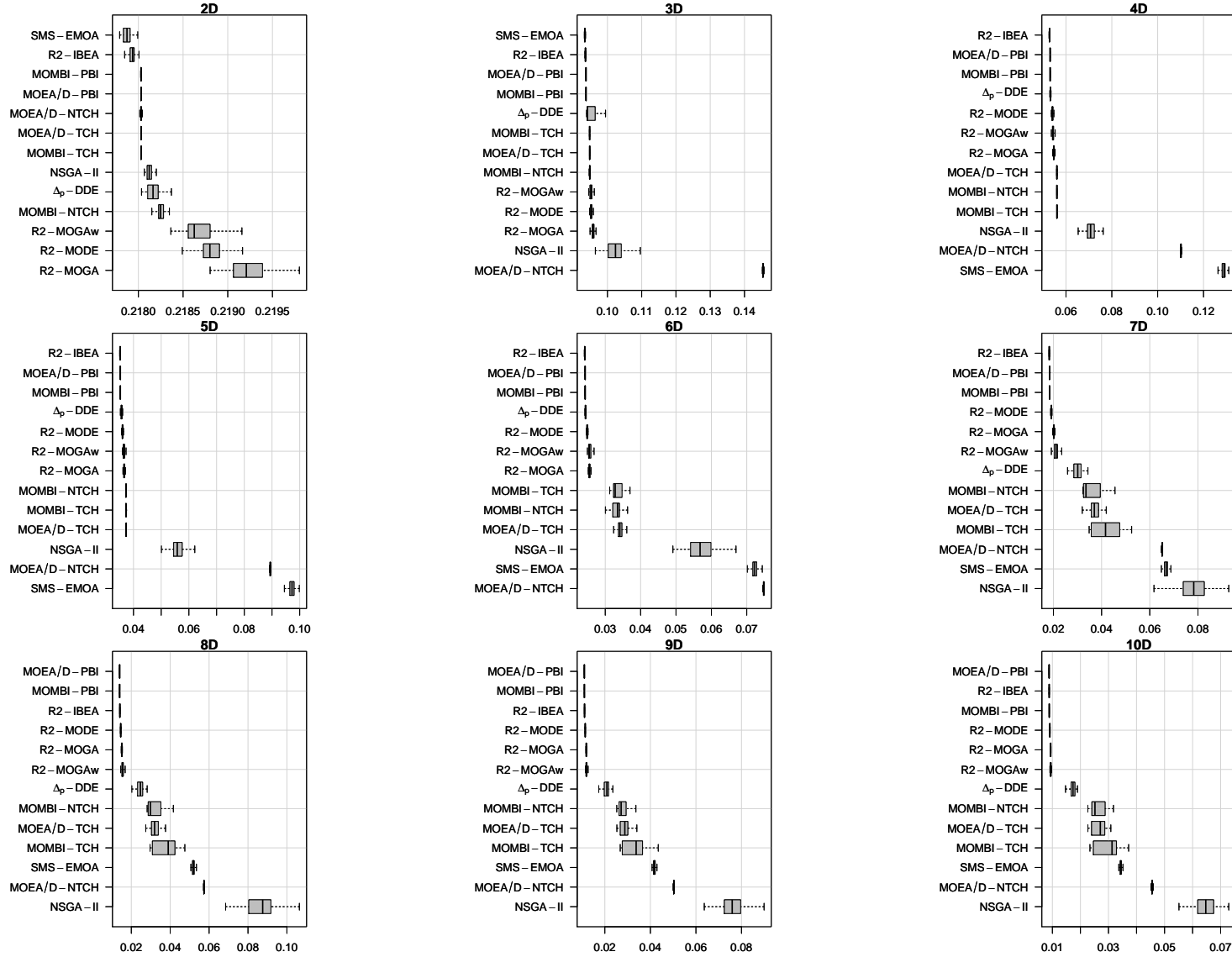


Figure A.18: Box-plot of R2 indicator values for different optimizers on the DTLZ2 test problem.

Table A.13: Comparison of runtime (in milliseconds) for different optimizers on the DTLZ2 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	2.074700e + 02	4.500550e + 02	1.076691e + 03	9.537370e + 02	2.958337e + 04	1.132920e + 03	1.927301e + 03	1.191975e + 03	1.683209e + 03	5.812800e + 05	1.977868e + 03	1.533831e + 03	2.757005e + 03
	avg.	2.141935e + 02	4.498624e + 02	1.072501e + 03	9.548878e + 02	2.924752e + 04	1.139933e + 03	1.865392e + 03	1.214643e + 03	1.630663e + 03	5.810910e + 05	1.980994e + 03	1.546273e + 03	2.726176e + 03
	min.	1.971400e + 02	3.939510e + 02	8.291010e + 02	7.775980e + 02	2.191935e + 04	1.091179e + 03	1.257416e + 03	9.235190e + 02	1.090220e + 03	5.628210e + 05	1.477719e + 03	1.513798e + 03	2.247953e + 03
	max.	3.101180e + 02	5.333810e + 02	1.577616e + 03	1.500536e + 03	3.227564e + 04	1.247232e + 03	2.217844e + 03	1.647073e + 03	2.032377e + 03	5.961390e + 05	2.285908e + 03	1.727766e + 03	2.937305e + 03
	std.	2.137256e + 01	2.120739e + 01	1.136340e + 02	9.652099e + 01	2.345296e + 03	2.745829e + 01	2.278143e + 02	1.482447e + 02	2.050088e + 02	7.916710e + 03	1.515968e + 02	3.639112e + 01	1.449306e + 02
3D	med.	3.510050e + 02	6.189780e + 02	1.427377e + 03	1.023074e + 03	7.965178e + 05	1.737692e + 03	1.803777e + 03	1.380446e + 03	1.780555e + 03	5.919690e + 05	1.927228e + 03	1.934403e + 03	2.963238e + 03
	avg.	3.585376e + 02	6.391001e + 02	1.417185e + 03	1.023079e + 03	7.890844e + 05	1.729206e + 03	1.810124e + 03	1.406407e + 03	1.774009e + 03	5.873984e + 05	1.929530e + 03	1.935020e + 03	2.947963e + 03
	min.	3.124090e + 02	4.473650e + 02	9.515440e + 02	7.676610e + 02	6.242753e + 05	1.237338e + 03	1.315554e + 03	9.730960e + 02	1.267621e + 03	5.015410e + 05	1.577270e + 03	1.648765e + 03	2.410177e + 03
	max.	4.527420e + 02	1.082518e + 03	1.858178e + 03	1.551150e + 03	8.633572e + 05	2.125169e + 03	2.174824e + 03	1.883685e + 03	2.158304e + 03	6.325940e + 05	2.433342e + 03	2.306781e + 03	4.323502e + 03
	std.	3.606974e + 01	1.041213e + 02	1.032697e + 02	8.989792e + 01	5.585213e + 04	1.792191e + 02	1.905674e + 02	1.778513e + 02	2.227152e + 02	2.668369e + 04	1.757975e + 02	1.488875e + 02	2.177957e + 02
4D	med.	4.777870e + 02	6.397230e + 02	1.507425e + 03	1.075618e + 03	5.322348e + 06	2.000680e + 03	2.248439e + 03	1.749306e + 03	2.450478e + 03	5.602750e + 05	1.973408e + 03	2.156706e + 03	3.246707e + 03
	avg.	4.832409e + 02	6.565200e + 02	1.495917e + 03	1.058178e + 03	5.227424e + 06	1.970200e + 03	2.196461e + 03	1.728366e + 03	2.356078e + 03	5.815754e + 05	1.996059e + 03	2.168070e + 03	3.225242e + 03
	min.	4.020090e + 02	4.786390e + 02	1.027785e + 03	7.865220e + 02	3.996153e + 06	1.315054e + 03	1.507212e + 03	1.222341e + 03	1.557650e + 03	5.475640e + 05	1.598446e + 03	1.846072e + 03	2.551011e + 03
	max.	6.443410e + 02	1.193542e + 03	2.013019e + 03	1.339113e + 03	5.923131e + 06	2.557097e + 03	2.647249e + 03	2.192487e + 03	2.650751e + 03	6.478380e + 05	2.903341e + 03	2.477433e + 03	3.643692e + 03
	std.	5.659159e + 01	1.094520e + 02	1.433818e + 02	1.034013e + 02	4.468217e + 05	2.377781e + 02	2.288333e + 02	1.898203e + 02	2.618227e + 02	3.135202e + 04	2.107079e + 02	1.396777e + 02	1.890557e + 02
5D	med.	6.066930e + 02	7.029810e + 02	1.888551e + 03	1.000718e + 03	7.184289e + 06	3.517904e + 03	2.546160e + 03	2.044287e + 03	2.540204e + 03	6.882570e + 05	2.139482e + 03	3.611488e + 03	3.611488e + 03
	avg.	6.062500e + 02	7.037198e + 02	1.861200e + 03	1.020079e + 03	7.180499e + 06	3.465801e + 03	2.550704e + 03	2.063314e + 03	2.534391e + 03	6.856159e + 05	2.171287e + 03	2.431896e + 03	3.593344e + 03
	min.	4.911900e + 02	5.597040e + 02	1.383954e + 03	7.818830e + 02	5.886636e + 06	2.292314e + 03	1.806819e + 03	1.369367e + 03	1.889477e + 03	5.760050e + 05	1.754543e + 03	1.974069e + 03	3.003344e + 03
	max.	7.874610e + 02	9.130500e + 02	2.357637e + 03	1.893930e + 03	8.180238e + 06	4.669418e + 03	3.044620e + 03	2.688198e + 03	2.990853e + 03	6.960810e + 05	2.805239e + 03	2.777320e + 03	3.980042e + 03
	std.	6.878202e + 01	7.889327e + 01	1.584030e + 02	1.623830e + 02	4.720488e + 05	4.572738e + 02	2.613692e + 02	2.326844e + 02	2.661179e + 02	1.629854e + 04	2.301752e + 02	1.565599e + 02	1.794716e + 02
6D	med.	7.176290e + 02	7.547080e + 02	1.928525e + 03	1.154995e + 03	8.363134e + 06	3.222051e + 03	2.910430e + 03	2.304308e + 03	3.106425e + 03	7.233670e + 05	2.417512e + 03	2.455089e + 03	4.504307e + 03
	avg.	7.269043e + 02	7.523845e + 02	1.927544e + 03	1.125126e + 03	8.336195e + 06	3.350753e + 03	2.932968e + 03	2.334994e + 03	3.071652e + 03	7.209765e + 05	2.528904e + 03	2.454137e + 03	4.508076e + 03
	min.	5.701260e + 02	5.430030e + 02	1.467040e + 03	7.998570e + 02	7.002680e + 06	2.253775e + 03	2.341845e + 03	1.720727e + 03	2.183168e + 03	6.077870e + 05	1.698737e + 03	2.137317e + 03	4.188553e + 03
	max.	9.809240e + 02	9.596960e + 02	2.659240e + 03	1.562921e + 03	9.455414e + 06	5.162030e + 03	3.491809e + 03	3.098755e + 03	3.588838e + 03	7.303570e + 05	4.112852e + 03	2.778615e + 03	5.165124e + 03
	std.	7.394543e + 01	7.756259e + 01	1.782492e + 02	1.295692e + 02	5.864085e + 05	5.322571e + 02	2.727809e + 02	2.850022e + 02	3.136326e + 02	1.521496e + 04	2.522260e + 02	1.602121e + 02	1.395889e + 02
7D	med.	6.392690e + 02	7.555860e + 02	2.193459e + 03	1.085008e + 03	5.890605e + 06	9.764956e + 03	2.286065e + 03	1.707900e + 03	2.475593e + 03	3.404190e + 05	1.537448e + 03	1.778482e + 03	2.691145e + 03
	avg.	6.365474e + 02	7.614592e + 02	2.159474e + 03	1.076551e + 03	5.897522e + 06	9.798683e + 03	2.296683e + 03	1.710727e + 03	2.420847e + 03	3.403531e + 05	1.550923e + 03	1.777040e + 03	2.671983e + 03
	min.	4.948780e + 02	6.069060e + 02	1.504740e + 03	8.192580e + 02	4.802051e + 06	5.645965e + 03	1.577861e + 03	1.186473e + 03	1.734169e + 03	3.217540e + 05	1.244277e + 03	1.565041e + 03	2.314124e + 03
	max.	8.186940e + 02	9.963570e + 02	2.716670e + 03	1.361310e + 03	6.926549e + 06	1.408771e + 04	2.719669e + 03	2.235725e + 03	2.811054e + 03	3.460630e + 05	2.403048e + 03	2.016532e + 03	2.972907e + 03
	std.	6.256572e + 01	7.756259e + 01	1.782492e + 02	1.231653e + 02	4.727547e + 05	1.818246e + 03	2.471967e + 02	1.870606e + 02	3.667481e + 03	1.579047e + 03	1.579047e + 03	1.180433e + 02	1.064277e + 02
8D	med.	9.299930e + 02	8.013470e + 02	2.305803e + 03	1.250306e + 03	9.627089e + 06	2.300695e + 04	3.488386e + 03	2.730879e + 03	4.253363e + 03	1.303820e + 06	1.962357e + 03	2.720097e + 03	4.261500e + 03
	avg.	9.192480e + 02	8.044541e + 02	2.280242e + 03	1.226569e + 03	9.659067e + 06	2.340397e + 04	3.424897e + 03	2.731048e + 03	4.099246e + 03	1.279270e + 06	1.996265e + 03	2.702201e + 03	4.219470e + 03
	min.	7.284520e + 02	6.297060e + 02	1.640631e + 03	9.007450e + 02	7.460711e + 06	1.599244e + 04	2.296221e + 03	1.814365e + 03	2.630932e + 03	1.129057e + 06	1.701519e + 03	2.304383e + 03	3.609797e + 03
	max.	1.108605e + 03	1.091534e + 03	2.613574e + 03	1.519496e + 03	1.221669e + 07	3.469389e + 04	4.038619e + 03	3.317309e + 03	4.693221e + 03	1.339068e + 06	2.420167e + 03	3.034450e + 03	4.832630e + 03
	std.	6.859895e + 01	9.098976e + 01	1.818140e + 02	1.459852e + 02	8.926812e + 05	4.431057e + 03	4.273867e + 02	3.023916e + 02	4.068116e + 02	5.131117e + 04	1.679192e + 03	1.623303e + 02	2.227954e + 02
9D	med.	1.350123e + 03	8.753390e + 02	2.865164e + 03	1.215326e + 03	1.332673e + 07	4.486758e + 04	4.957234e + 03	4.426710e + 03	5.930642e + 03	2.303842e + 06	3.085579e + 03	3.649456e + 03	6.170885e + 03
	avg.	1.345417e + 03	8.697272e + 02	2.816501e + 03	1.233979e + 03	1.314114e + 07	4.609000e + 04	4.907627e + 03	4.382579e + 03	5.869136e + 03	2.438959e + 06	3.081794e + 03	3.714717e + 03	6.153838e + 03
	min.	1.065767e + 03	6.444450e + 02	2.176793e + 03	9.245660e + 02	9.266595e + 06	2.419317e + 04	3.364570e + 03	3.045644e + 03	4.038137e + 03	2.297009e + 06	2.690070e + 03	3.327990e + 03	5.140197e + 03
	max.	1.517403e + 03	1.135059e + 03	3.257388e + 03	1.609702e + 03	1.713069e + 07	7.462168e + 04	6.158964e + 03	5.184795e + 03	6.989584e + 03	2.772456e + 06	3.483969e + 03	4.316874e + 03	7.077234e + 03
	std.	8.227482e + 01	8.189488e + 01	1.835904e + 02	1.650239e + 02	1.971054e + 06	1.076589e + 04	6.933944e + 02	4.361557e + 02	6.310494e + 02	1.901533e + 05	1.873561e + 02	2.568235e + 02	3.743277e + 02
10D	med.	1.893461e + 03	9.335940e + 02	2.926926e + 03	1.416443e + 03	1.656502e + 07	6.776357e + 04	8.415186e + 03	6.547831e + 03	9.728189e + 03	4.948306e + 06	3.894685e + 03	5.496740e + 03	8.609003e + 03
	avg.	1.853120e + 03	9.313530e + 02	2.863252e + 03	1.386218e + 03	1.601058e + 07	6.893924e + 04	8.107985e + 03	6.527245e + 03	9.508061e + 03	4.941090e + 06	3.919252e + 03	5.558902e + 03	8.626210e + 03
	min.	1.460570e + 03	7.523810e + 02	1.953837e + 03	1.016419e + 03	3.327978e + 06	4.851528e + 04	5.283979e + 03	6.637782e + 03	6.214258e + 03	4.363138e + 06	3.259511e + 03	4.659270e + 03	7.672297e + 03
	max.	2.088380e + 03	1.291860e + 03	3.385182e + 03	1.766892e + 03	2.230458e + 07	1.044417e + 05	9.386380e + 03	8.110081e + 03	1.072414e + 04	5.186879e + 06	4.531511e + 03	6.254607e + 03	9.889969e + 03
	std.	1.320571e + 02	8.289835e + 01	2.568084e + 02	1.596078e + 02	3.902053e + 06	1.227658e + 04	8.434033e + 02	6.409404e + 02	9.065463e + 02	1.601282e + 05	3.426281e + 02	3.326357e + 02	4.514517e + 02

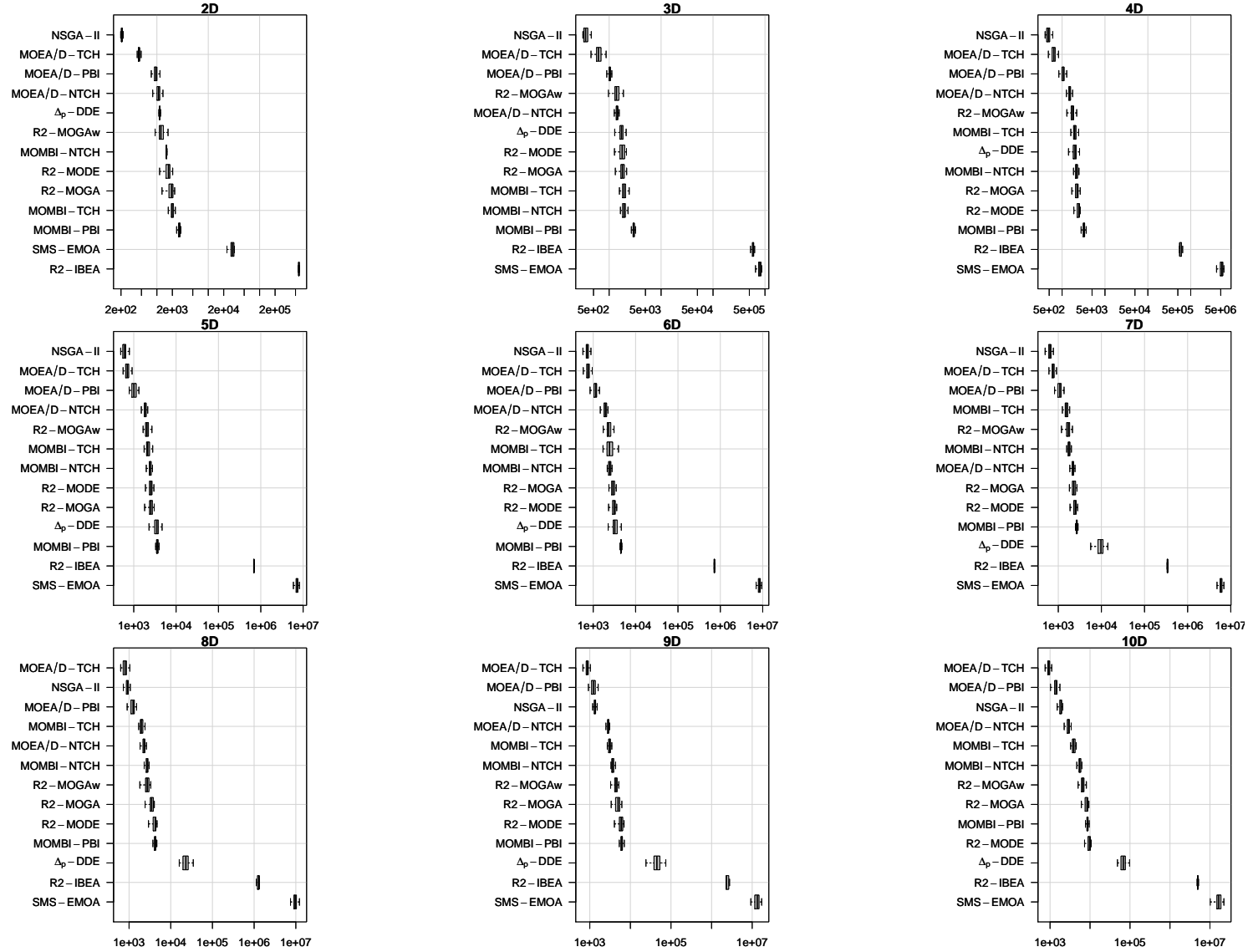


Figure A.19: Box-plot of runtime (in logarithmic scale) for different optimizers on the DTLZ2 test problem.

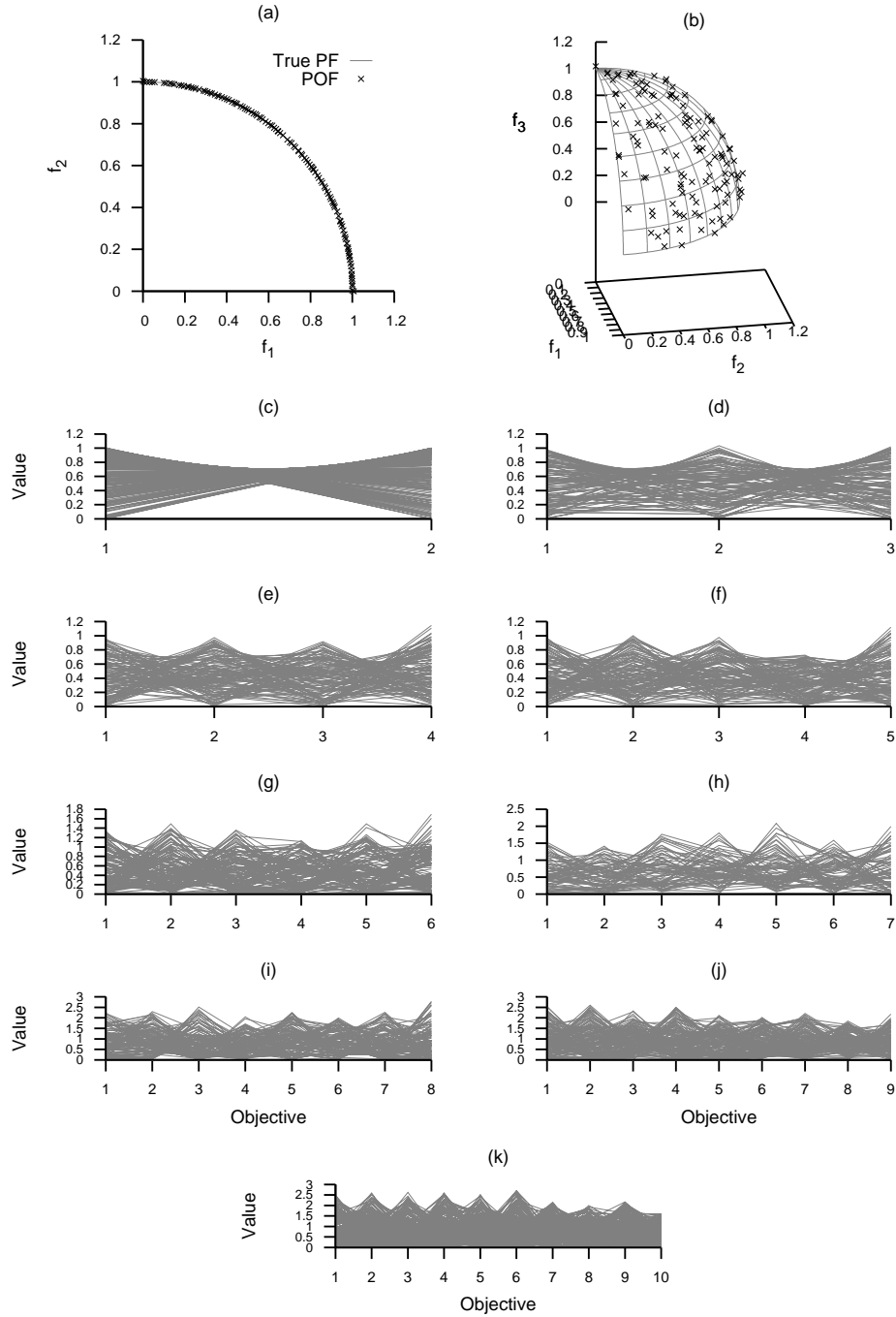


Figure A.20: Plots of the approximations obtained by NSGA-II from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



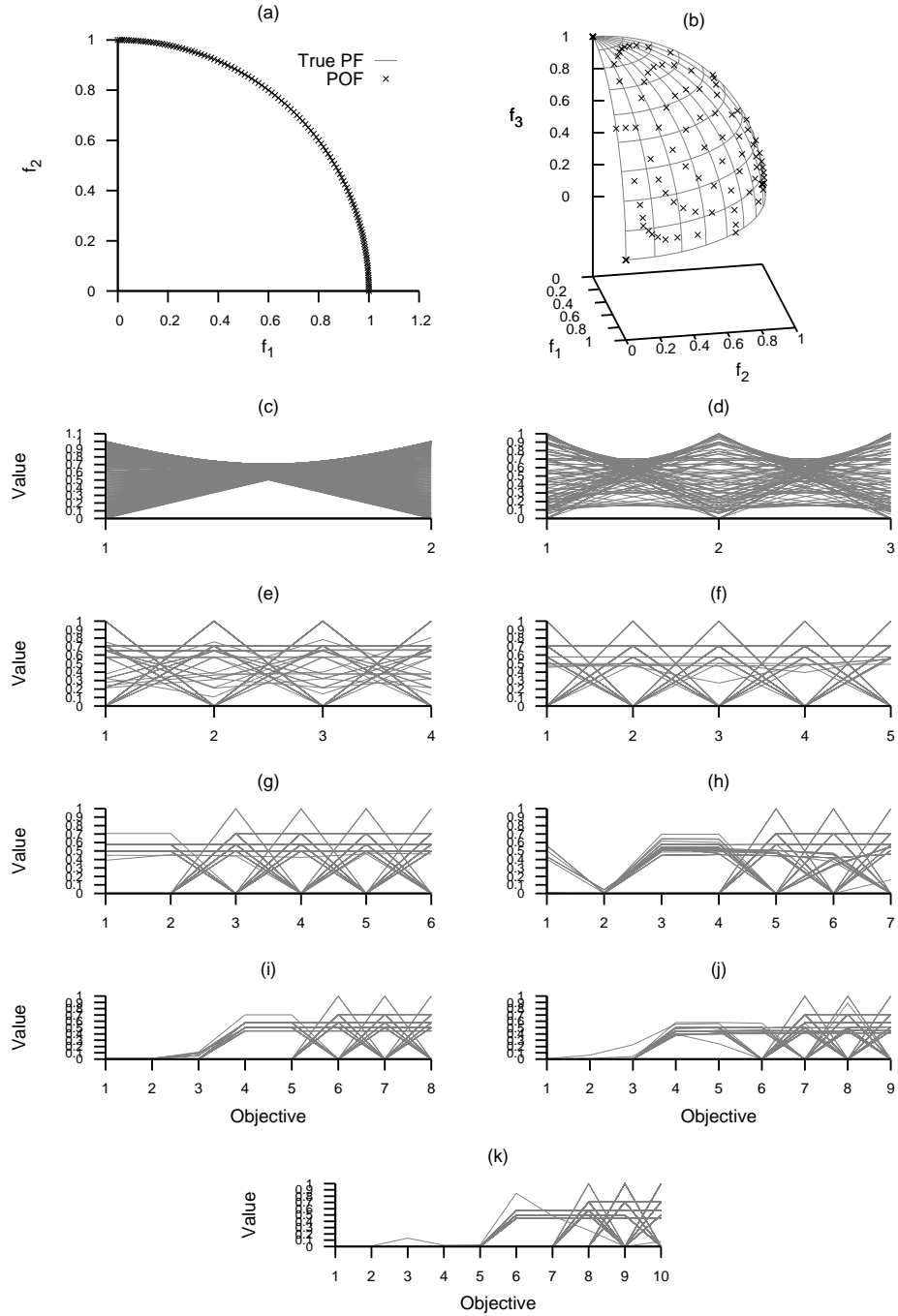


Figure A.21: Plots of the approximations obtained by MOEA/D-TCH from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

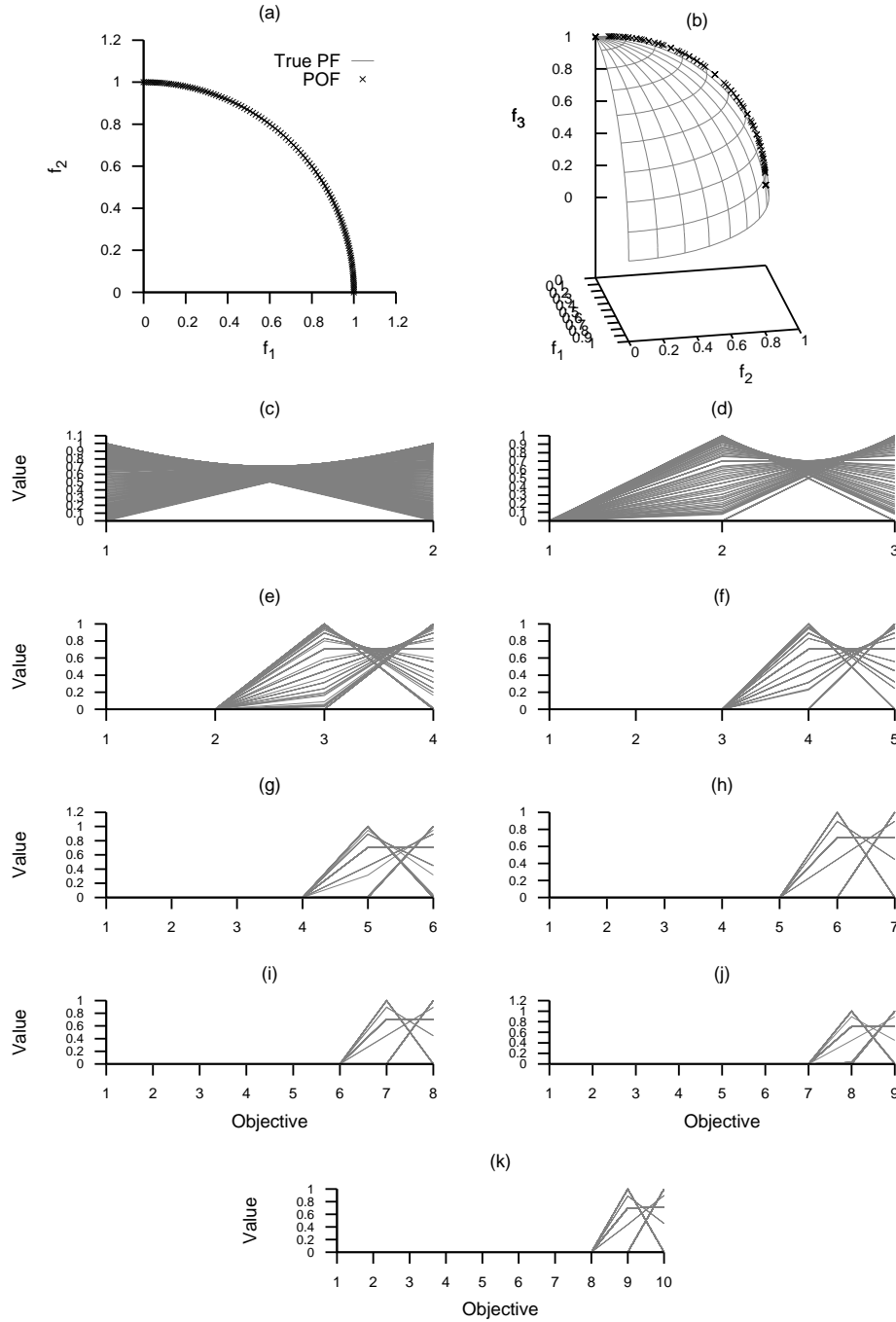


Figure A.22: Plots of the approximations obtained by MOEA/D-NTCH from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

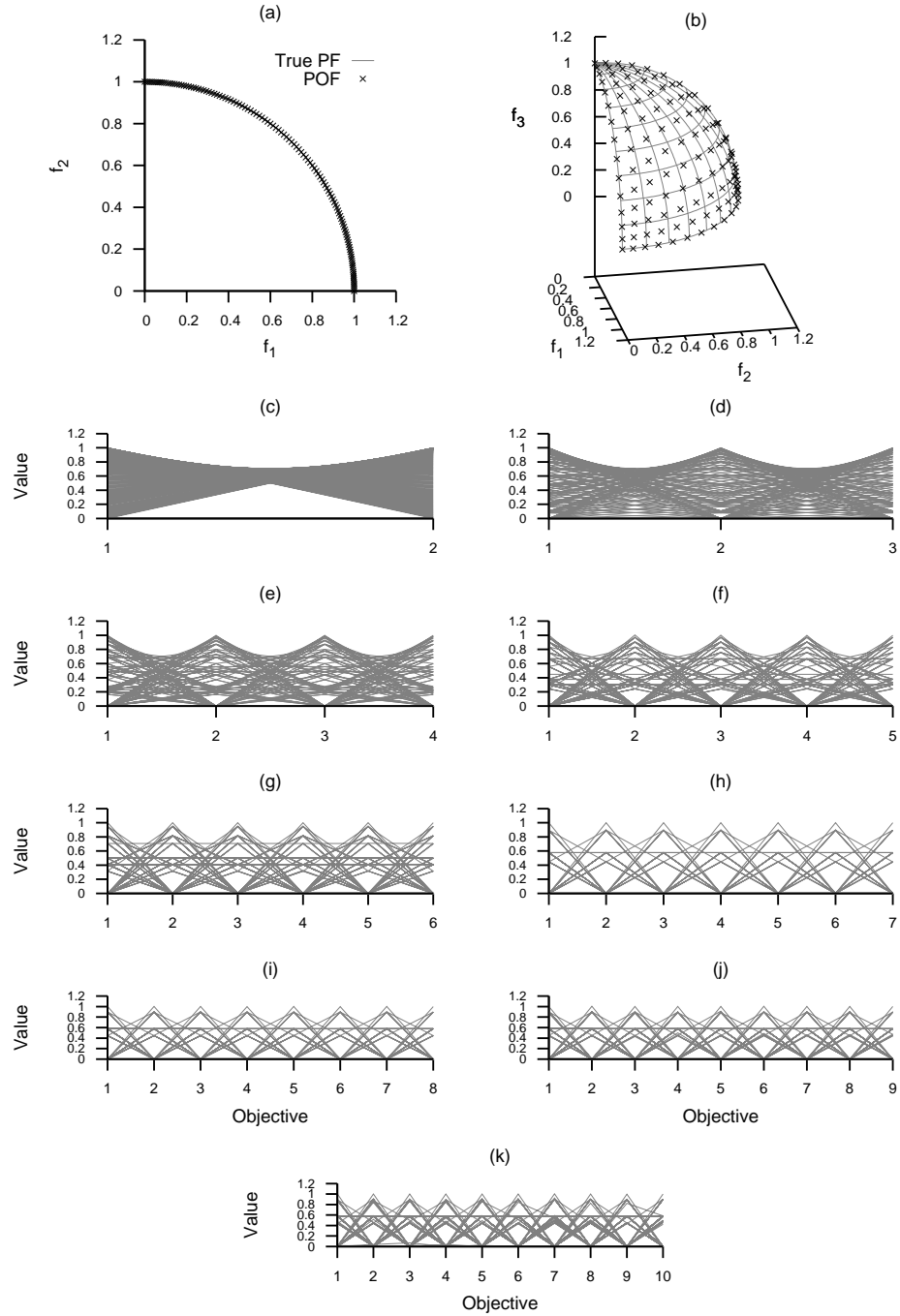


Figure A.23: Plots of the approximations obtained by MOEA/D-PBI from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

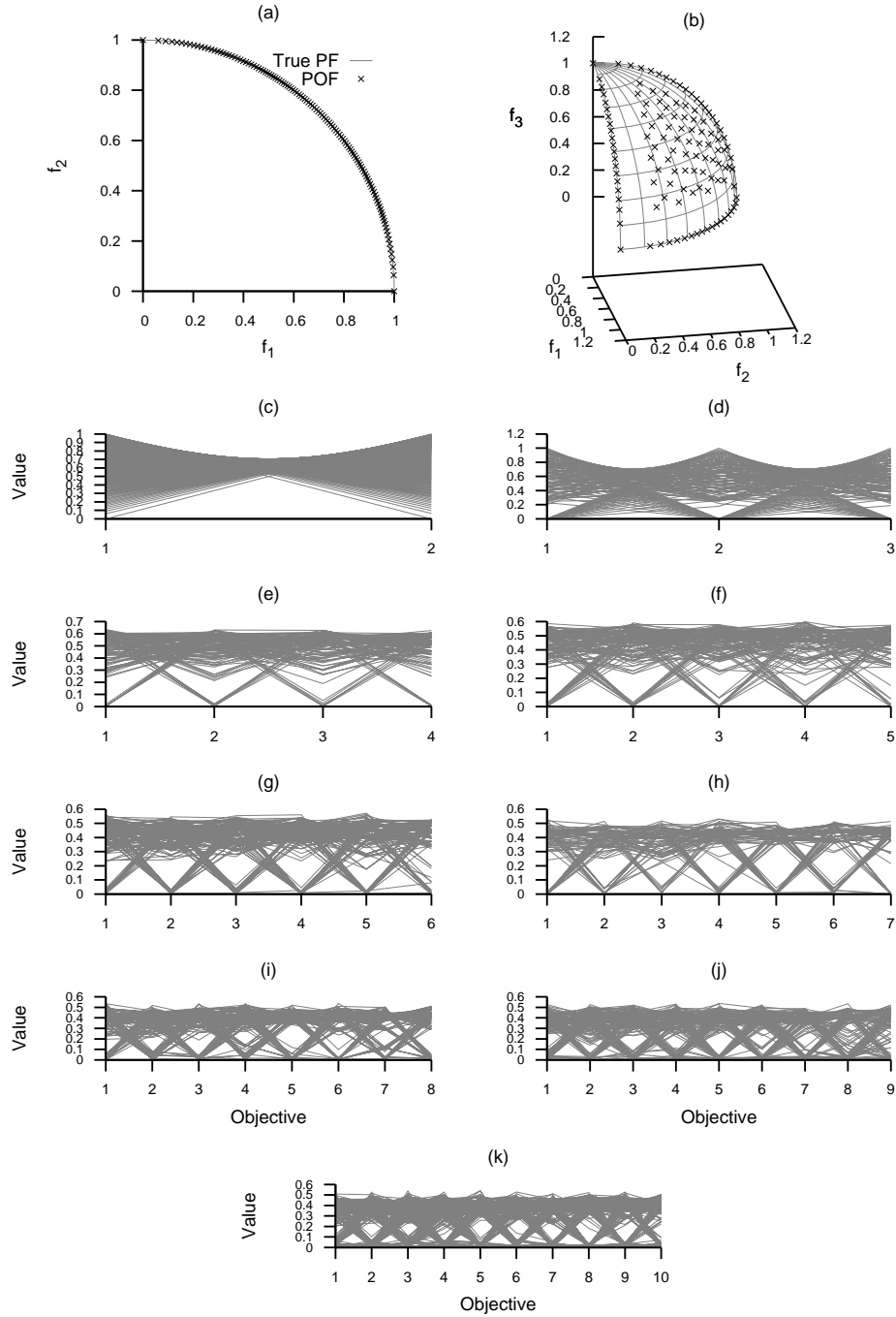


Figure A.24: Plots of the approximations obtained by SMS-EMOA from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

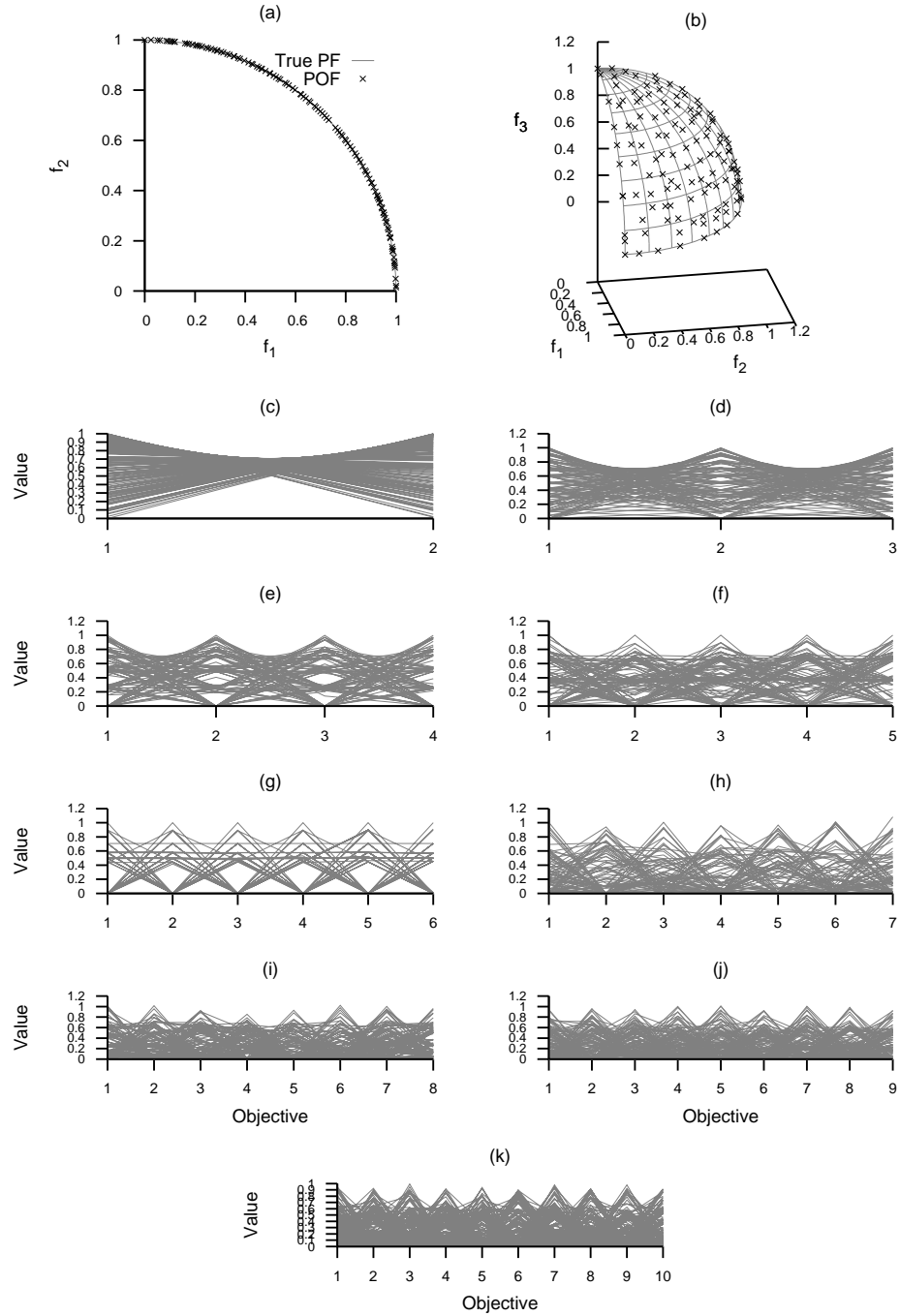


Figure A.25: Plots of the approximations obtained by  $\Delta_p$ -DDE from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

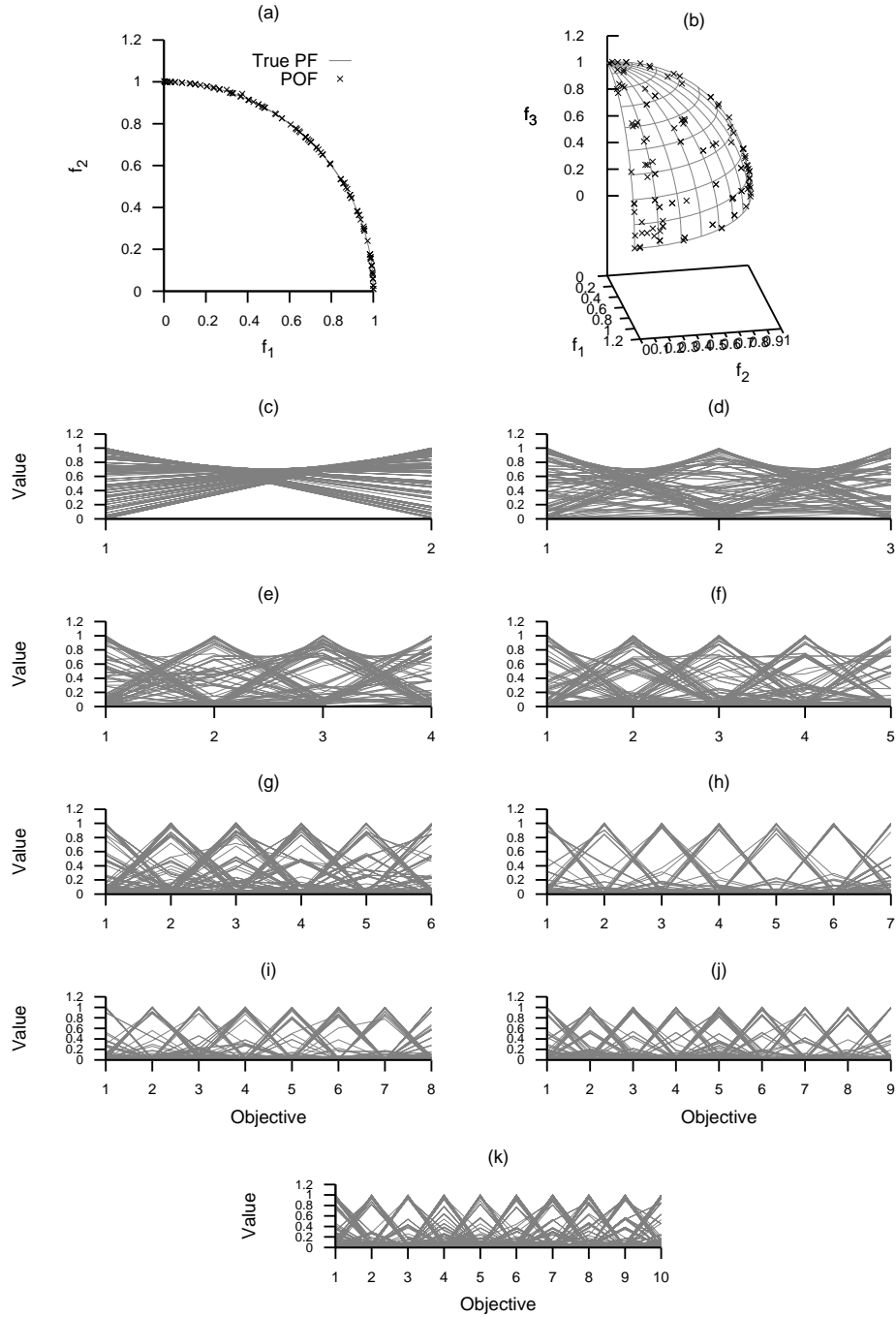


Figure A.26: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

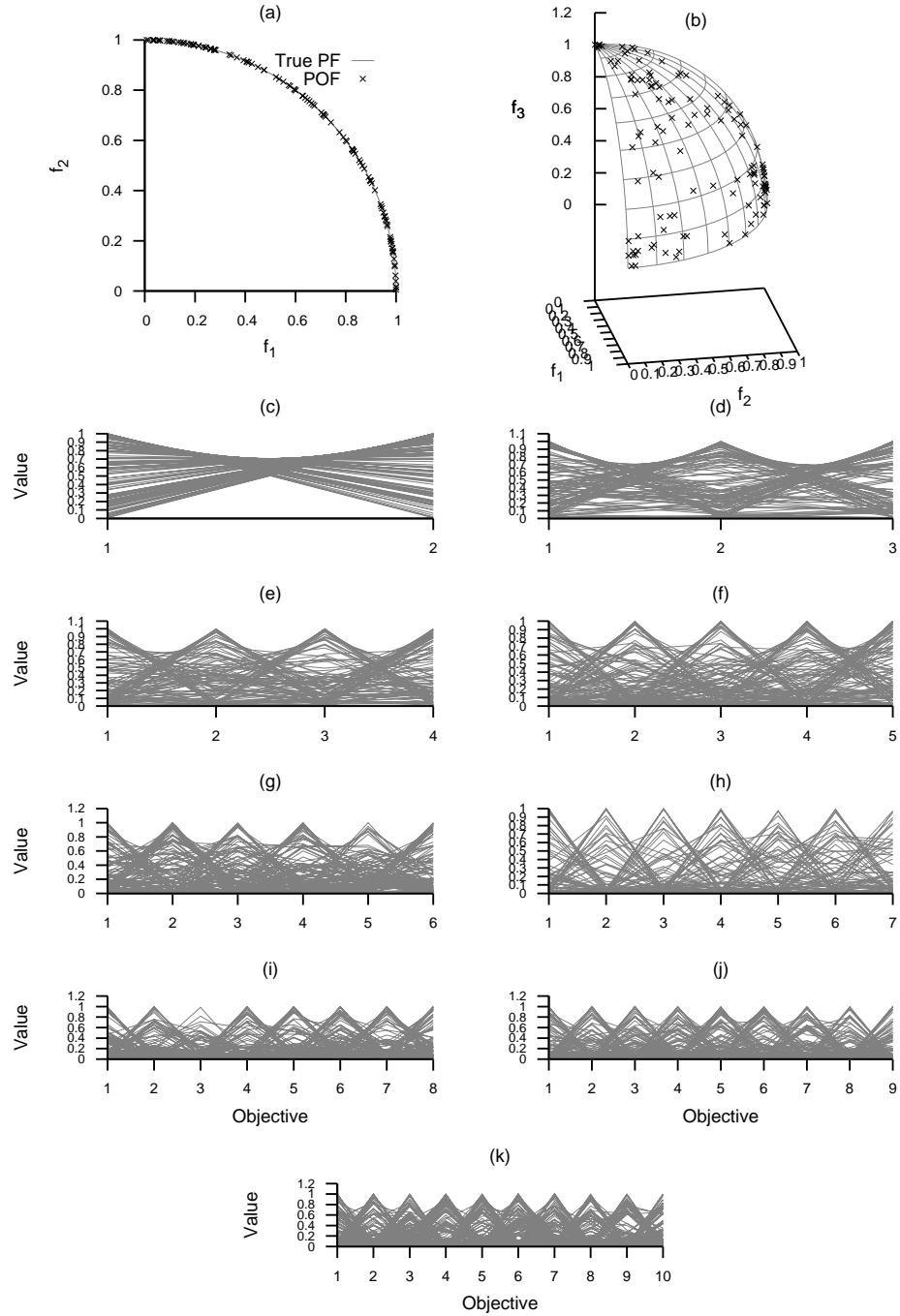


Figure A.27: Plots of the approximations obtained by *R2-MOGAw* from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

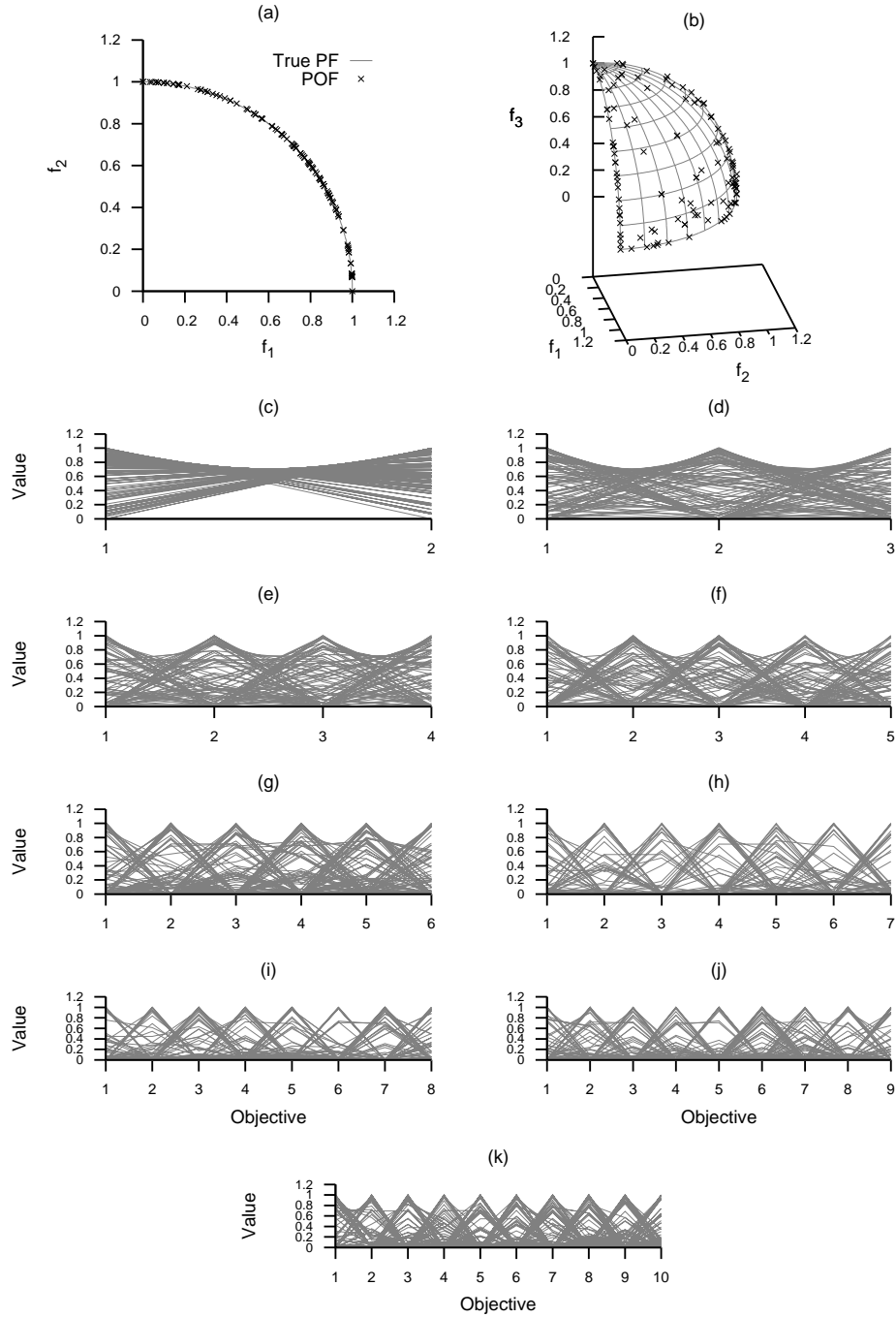


Figure A.28: Plots of the approximations obtained by *R2-MODE* from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



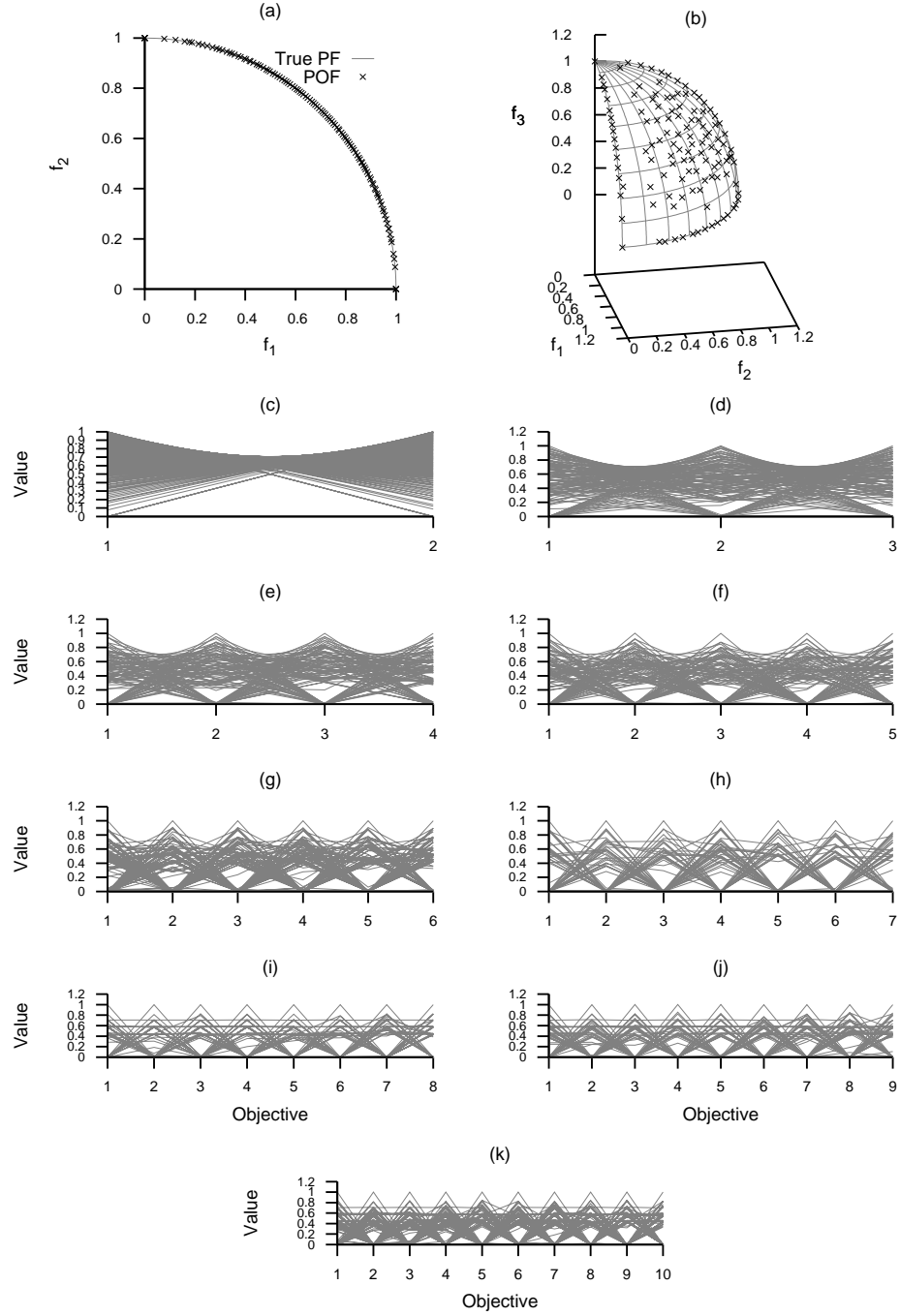


Figure A.29: Plots of the approximations obtained by R2-IBEA from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

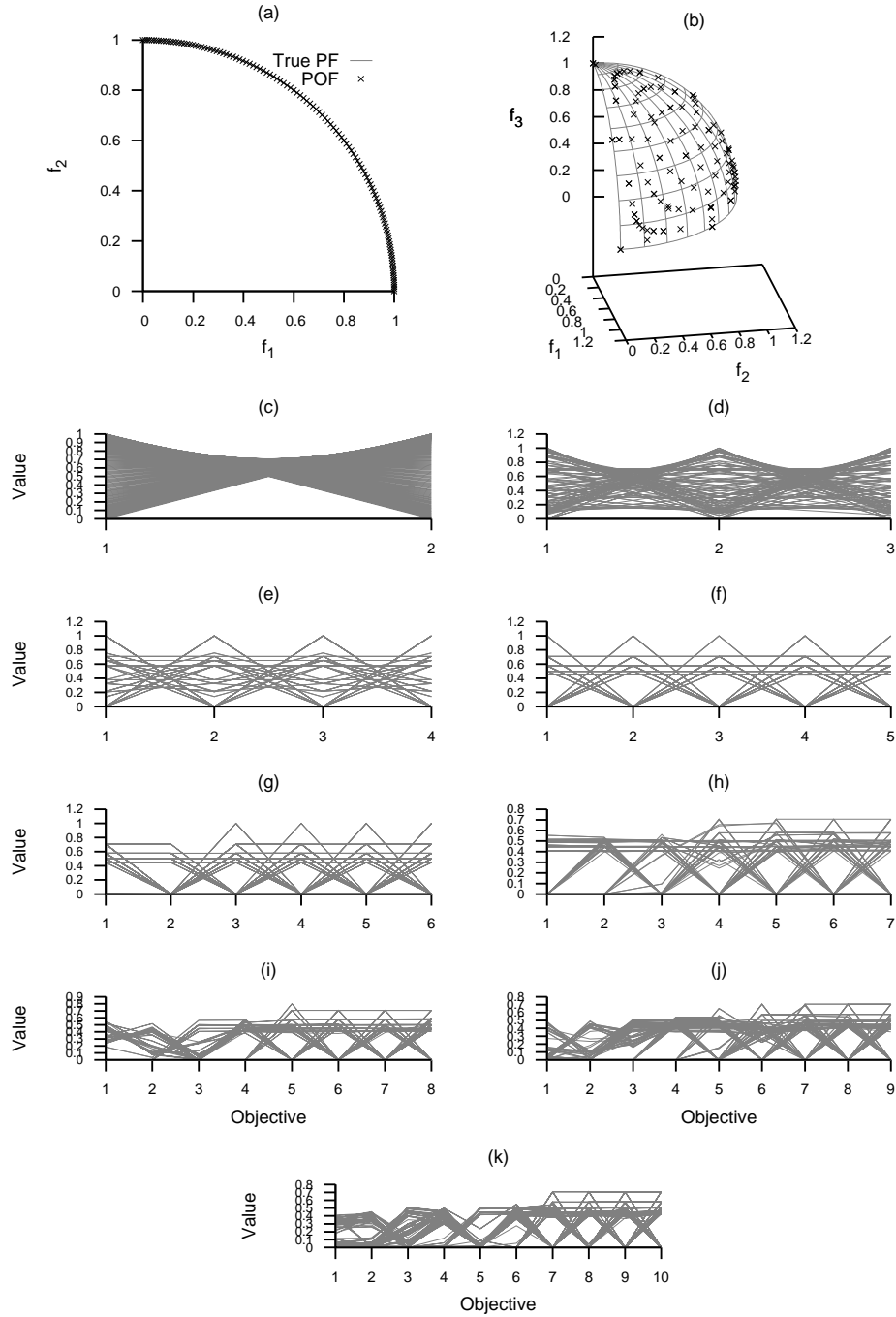


Figure A.30: Plots of the approximations obtained by MOMBI-TCH from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

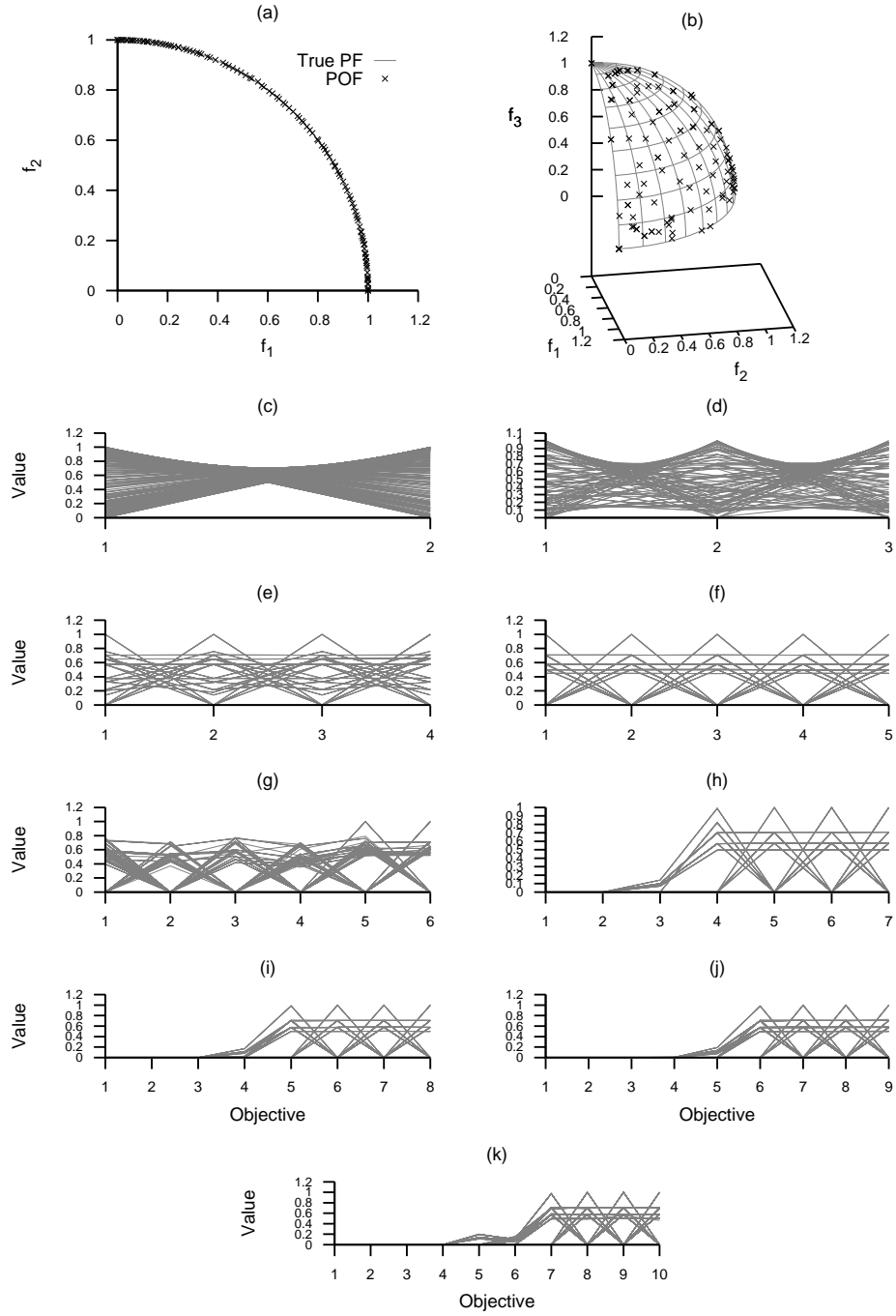


Figure A.31: Plots of the approximations obtained by MOMBI-NTCH from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

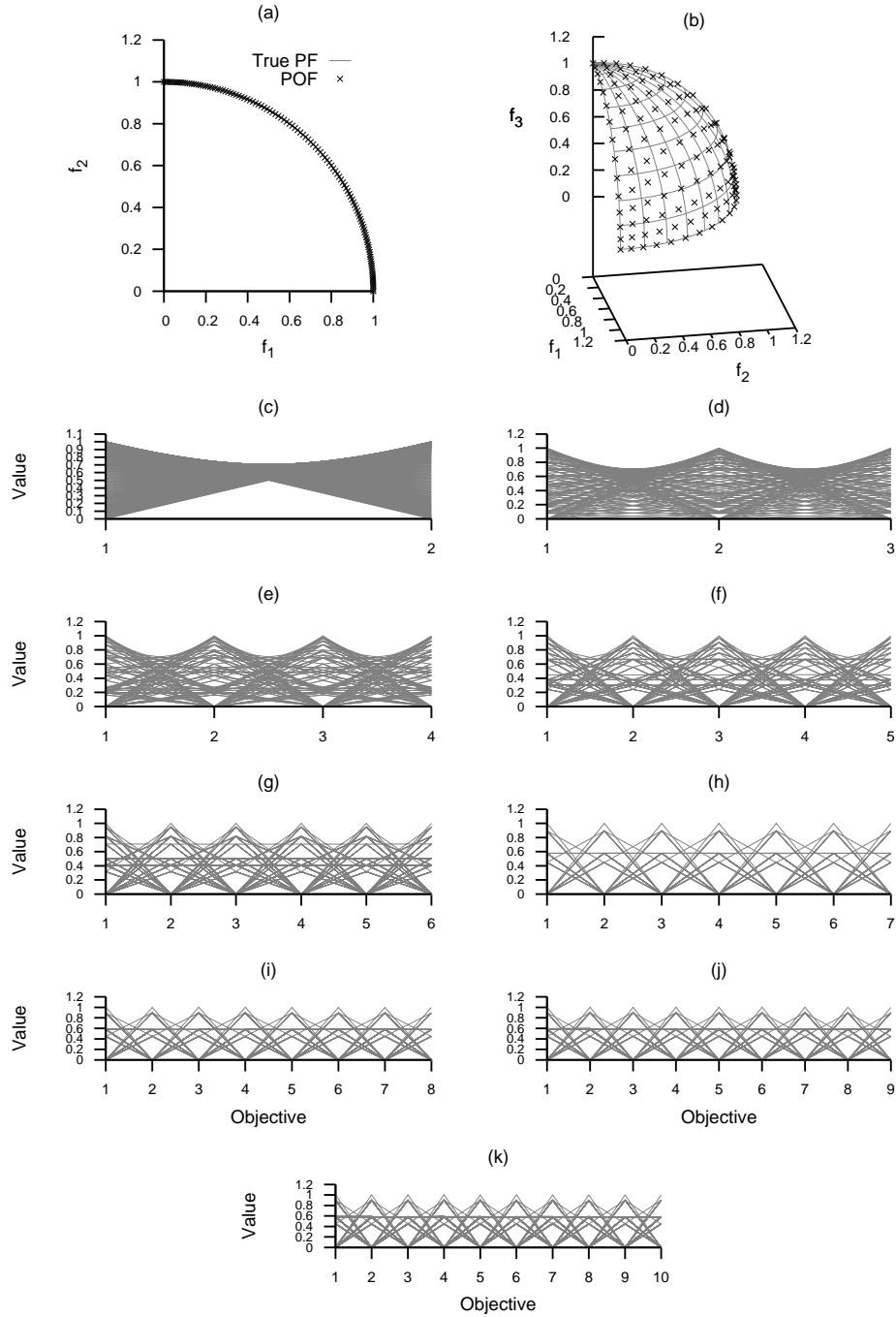


Figure A.32: Plots of the approximations obtained by MOMBI-PBI from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.14: Comparison of hypervolume indicator values for different optimizers on the DTLZ3 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	4.793872e+01	4.820301e+01	4.820073e+01	4.818855e+01	4.820349e+01	4.815838e+01	4.810939e+01	4.813783e+01	4.820299e+01	4.820412e+01	4.820382e+01	4.820261e+01	4.819491e+01
	avg.	4.712444e+01	4.820218e+01	4.816813e+01	4.816728e+01	4.820226e+01	4.812366e+01	4.808735e+01	4.812894e+01	4.624848e+01	4.817794e+01	4.820281e+01	4.820219e+01	4.819223e+01
	min.	4.032887e+01	4.817661e+01	4.537692e+01	4.713437e+01	4.818234e+01	4.559775e+01	4.778582e+01	4.787813e+01	2.041612e+01	4.581234e+01	4.819127e+01	4.818335e+01	4.813612e+01
	max.	4.819656e+01	4.820897e+01	4.820735e+01	4.820704e+01	4.821009e+01	4.821055e+01	4.819145e+01	4.819984e+01	4.820720e+01	4.820994e+01	4.820949e+01	4.820809e+01	4.820610e+01
	std.	1.641499e+00	4.947107e-03	2.823876e-01	1.146260e-01	6.000856e-03	2.638653e-01	7.358689e-02	6.025181e-02	4.310835e+00	2.390952e-01	4.236134e-03	4.143913e-03	1.035987e-02
3D	med.	3.213374e+02	3.423777e+02	3.373545e+02	3.423414e+02	3.422325e+02	3.423023e+02	3.420924e+02	3.421533e+02	3.423912e+02	3.424009e+02	3.423709e+02	3.423696e+02	3.423748e+02
	avg.	3.052842e+02	3.423744e+02	3.368608e+02	3.421748e+02	3.401175e+02	3.421819e+02	3.420638e+02	3.420966e+02	3.420053e+02	3.423971e+02	3.423677e+02	3.423247e+02	3.423669e+02
	min.	1.411289e+00	3.423316e+02	2.974115e+02	3.374949e+02	3.024061e+02	3.370165e+02	3.415497e+02	3.412183e+02	3.380363e+02	3.423097e+02	3.422914e+02	3.380038e+02	3.422218e+02
	max.	3.421537e+02	3.423923e+02	3.381744e+02	3.424071e+02	3.424178e+02	3.424147e+02	3.422924e+02	3.423524e+02	3.424005e+02	3.424205e+02	3.423867e+02	3.423871e+02	3.424101e+02
	std.	4.926037e+01	1.065280e-02	3.997101e+00	7.917299e-01	5.826830e+00	5.555253e-01	1.593543e-01	2.185725e-01	1.230164e+00	1.635944e-02	1.565883e-02	4.366046e-01	3.080946e-02
4D	med.	5.349303e+02	2.400411e+03	2.360720e+03	2.399898e+03	1.895105e+03	2.400530e+03	2.399598e+03	2.399739e+03	2.400514e+03	2.400475e+03	2.400392e+03	2.400311e+03	2.400418e+03
	avg.	8.269671e+02	2.400141e+03	2.352650e+03	2.289692e+03	1.594025e+03	2.400462e+03	2.399354e+03	2.399609e+03	2.400297e+03	2.400376e+03	2.400375e+03	2.400203e+03	2.400077e+03
	min.	0.000000e+00	2.390138e+03	2.053514e+03	3.552413e+01	1.376684e+01	2.399290e+03	2.386875e+03	2.397582e+03	2.393233e+03	2.393348e+03	2.399581e+03	2.390753e+03	2.372014e+03
	max.	2.398314e+03	2.400430e+03	2.394949e+03	2.400511e+03	2.396303e+03	2.400560e+03	2.400142e+03	2.400263e+03	2.400533e+03	2.400541e+03	2.400418e+03	2.400423e+03	2.400535e+03
	std.	8.756001e+02	1.521141e+00	4.290223e+01	4.089104e+02	7.515604e+02	1.845585e-01	1.371752e+00	5.899891e-01	1.243124e+00	7.158601e-01	8.521183e-02	9.606724e-01	2.840605e+00
5D	med.	0.000000e+00	1.680644e+04	1.651858e+04	1.678493e+04	4.275212e+03	1.680493e+04	1.680328e+04	1.680401e+04	1.680660e+04	1.680608e+04	1.680645e+04	1.680599e+04	1.680579e+04
	avg.	0.000000e+00	1.676627e+04	1.645687e+04	1.593028e+04	6.268773e+03	1.612477e+04	1.680275e+04	1.680338e+04	1.680641e+04	1.680556e+04	1.680471e+04	1.680385e+04	1.680202e+04
	min.	0.000000e+00	1.336326e+04	1.456703e+04	6.149889e+03	2.341716e+02	6.141943e+02	1.679356e+04	1.678979e+04	1.679428e+04	1.679028e+04	1.676616e+04	1.677309e+04	1.660730e+04
	max.	0.000000e+00	1.680653e+04	1.652714e+04	1.680628e+04	1.599316e+04	1.680645e+04	1.680578e+04	1.680580e+04	1.680662e+04	1.680652e+04	1.680652e+04	1.680617e+04	1.680651e+04
	std.	0.000000e+00	3.442933e+02	2.768688e+02	1.633556e+03	4.792609e+03	2.619019e+03	2.125738e+00	2.357944e+00	1.347691e+00	2.097801e+00	5.950424e+00	7.413873e+00	2.033024e+01
6D	med.	0.000000e+00	1.176217e+05	1.155231e+05	1.152287e+05	3.929994e+04	1.173290e+05	1.176315e+05	1.176355e+05	1.176486e+05	1.176458e+05	1.176438e+05	1.176417e+05	1.176435e+05
	avg.	0.000000e+00	1.175576e+05	1.148434e+05	1.080861e+05	3.784408e+04	1.002488e+05	1.176278e+05	1.176335e+05	1.176412e+05	1.176446e+05	1.176206e+05	1.175901e+05	1.175517e+05
	min.	0.000000e+00	1.168421e+05	9.633443e+04	1.151553e+03	3.913995e+03	8.325118e+03	1.175867e+05	1.175684e+05	1.173995e+05	1.176025e+05	1.169391e+05	1.161017e+05	1.144221e+05
	max.	0.000000e+00	1.176446e+05	1.156564e+05	1.176478e+05	1.058375e+05	1.176487e+05	1.176448e+05	1.176447e+05	1.176487e+05	1.176482e+05	1.176463e+05	1.176430e+05	1.176481e+05
	std.	0.000000e+00	1.266218e+02	2.795486e+03	1.497795e+04	2.923682e+04	3.003094e+04	1.275138e+01	1.108892e+01	3.268303e+01	5.393422e+00	8.758718e+01	2.229975e+02	4.167394e+02
7D	med.	0.000000e+00	8.221654e+05	8.085397e+05	7.082812e+05	2.183438e+05	8.192135e+05	8.232860e+05	8.234046e+05	8.235410e+05	8.235376e+05	8.231668e+05	8.231992e+05	8.235289e+05
	avg.	0.000000e+00	8.213314e+05	8.075167e+05	7.539703e+05	3.469506e+05	8.192135e+05	8.232340e+05	8.233721e+05	8.214184e+05	8.235348e+05	8.225843e+05	8.226020e+05	8.234634e+05
	min.	0.000000e+00	7.681267e+05	7.313920e+05	7.026605e+05	3.696587e+04	8.192135e+05	8.225989e+05	8.229368e+05	6.450634e+05	8.235005e+05	8.050887e+05	7.959358e+05	8.200723e+05
	max.	0.000000e+00	8.233655e+05	8.211467e+05	8.235416e+05	7.641955e+05	8.192135e+05	8.234662e+05	8.235152e+05	8.235426e+05	8.235423e+05	8.233655e+05	8.232577e+05	8.235414e+05
	std.	0.000000e+00	5.935662e+03	7.850325e+03	5.705895e+04	2.708601e+05	0.000000e+00	1.866183e+02	1.060150e+02	1.788382e+04	7.747027e+00	2.209381e+03	2.949610e+03	3.614195e+02
8D	med.	0.000000e+00	5.753074e+06	5.658651e+06	4.951553e+06	1.399778e+06	0.000000e+00	5.764095e+06	5.764350e+06	5.764797e+06	5.764708e+06	5.762570e+06	5.762418e+06	5.764668e+06
	avg.	0.000000e+00	5.753649e+06	5.649521e+06	5.172251e+06	2.028621e+06	0.000000e+00	5.763932e+06	5.764276e+06	5.756736e+06	5.764653e+06	5.758753e+06	5.758742e+06	5.763755e+06
	min.	0.000000e+00	5.661550e+06	5.300696e+06	4.223887e+06	3.163269e+04	0.000000e+00	5.761008e+06	5.763060e+06	5.234734e+06	5.762581e+06	5.635654e+06	5.631064e+06	5.744668e+06
	max.	0.000000e+00	5.763305e+06	5.665631e+06	5.764786e+06	5.464338e+06	0.000000e+00	5.764685e+06	5.764724e+06	5.764801e+06	5.764782e+06	5.763474e+06	5.762808e+06	5.764777e+06
	std.	0.000000e+00	1.425340e+04	4.793877e+04	3.719995e+05	1.923115e+06	0.000000e+00	7.049237e+02	3.384521e+02	5.664051e+04	2.361244e+02	1.485226e+04	1.544533e+04	3.295671e+03
9D	med.	0.000000e+00	4.028434e+07	3.959925e+07	3.463346e+07	3.863997e+06	0.000000e+00	4.035174e+07	4.035195e+07	3.855149e+06	4.035250e+07	4.033735e+07	4.033694e+07	4.035237e+07
	avg.	0.000000e+00	4.025899e+07	3.955216e+07	3.642906e+07	5.542757e+06	0.000000e+00	4.035128e+07	4.035182e+07	3.697830e+06	4.035019e+07	4.022370e+07	4.025918e+07	4.034382e+07
	min.	0.000000e+00	3.965149e+07	3.554514e+07	3.424600e+07	1.417671e+05	0.000000e+00	4.034653e+07	4.034207e+07	2.508055e+03	4.025475e+07	3.662127e+07	3.897948e+07	3.988873e+07
	max.	0.000000e+00	4.034176e+07	4.025057e+07	4.035342e+07	1.108295e+07	0.000000e+00	4.035326e+07	4.035335e+07	3.991437e+07	4.035338e+07	4.034770e+07	4.034056e+07	4.035347e+07
	std.	0.000000e+00	1.422278e+05	4.142387e+05	2.652132e+06	4.656641e+06	0.000000e+00	1.490800e+03	1.035168e+03	1.043684e+04	5.059664e+05	2.115673e+05	4.891929e+04	
10D	med.	0.000000e+00	2.818423e+08	2.771458e+08	2.423332e+08	0.000000e+00	0.000000e+00	2.824655e+08	2.824677e+08	0.000000e+00	2.824480e+08	2.823592e+08	2.823644e+08	2.824593e+08
	avg.	0.000000e+00	2.815766e+08	2.766366e+08	2.521595e+08	0.000000e+00	0.000000e+00	2.824640e+08	2.824660e+08	0.000000e+00	2.823974e+08	2.814701e+08	2.820167e+08	2.824391e+08
	min.	0.000000e+00	2.734404e+08	2.494457e+08	2.413170e+08	0.000000e+00	0.000000e+00	2.824320e+08	2.824320e+08	0.000000e+00	2.812081e+08	2.545363e+08	2.743435e+08	2.819267e+08
	max.	0.000000e+00	2.847144e+08	2.775625e+08	2.824715e+08	0.000000e+00	0.000000e+00	2.824730e+08	2.824737e+08	0.000000e+00	2.824733e+08	2.824485e+08	2.824587e+08	2.824735e+08
	std.	0.000000e+00	1.222892e+06	3.247150e+06	1.721338e+07	0.000000e+00	0.000000e+00	7.606590e+03	6.706675e+03	0.000000e+00	1.688795e+05	3.285153e+06	1.313959e+06	8.197293e+04

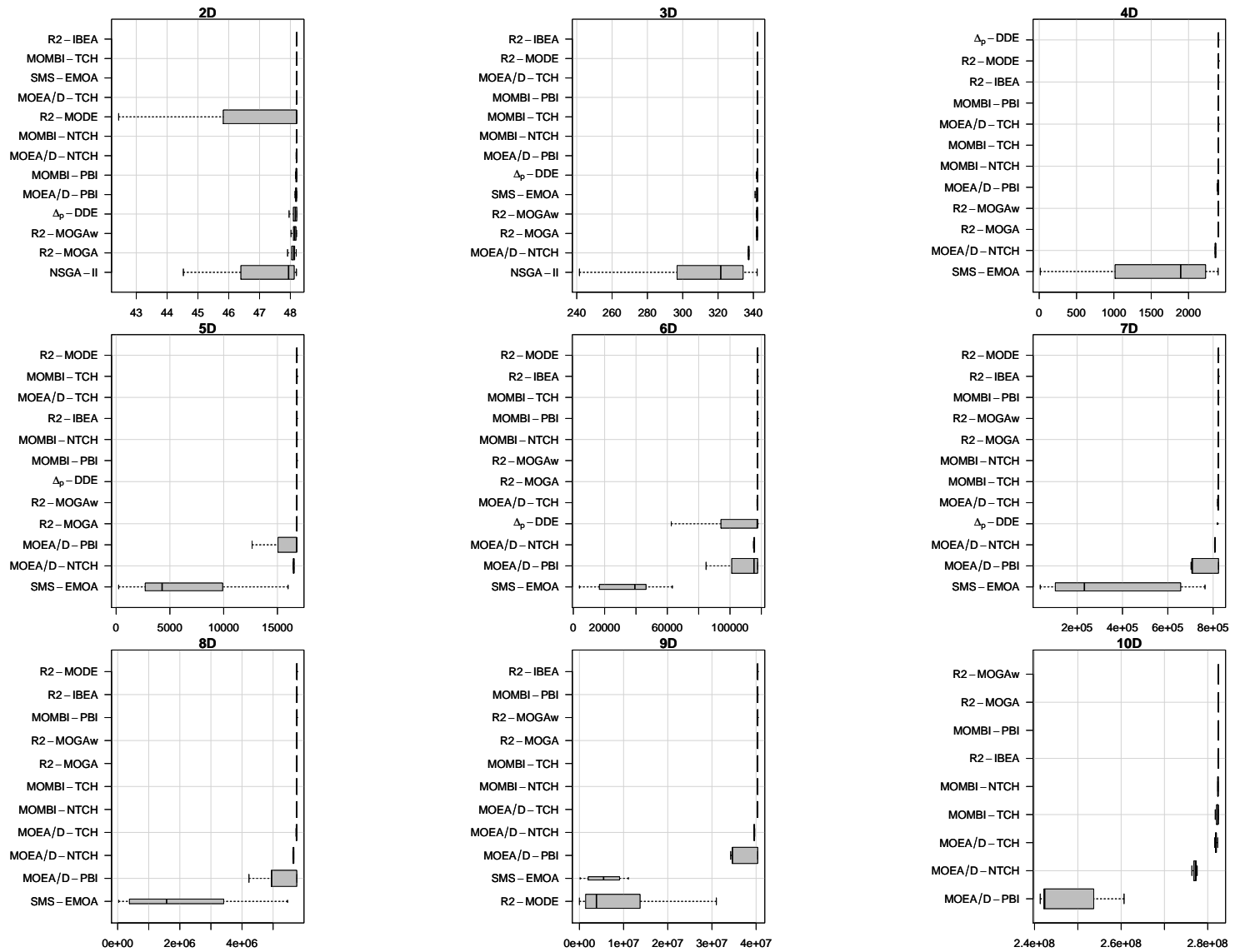


Figure A.33: Box-plot of hypervolume indicator values for different optimizers on the DTLZ3 test problem.

Table A.15: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ3 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$2.26e-34$	—	$5.16e-04$	$1.97e-27$	> 0.05	> 0.05	$1.53e-34$	$3.37e-33$	$1.75e-03$	> 0.05	> 0.05	> 0.05	$1.84e-17$
MOEA/D-NTCH	$1.74e-31$	> 0.05	—	$1.29e-16$	> 0.05	> 0.05	$2.44e-31$	$7.48e-27$	> 0.05	> 0.05	> 0.05	> 0.05	$8.70e-08$
MOEA/D-PBI	$3.98e-27$	> 0.05	> 0.05	—	> 0.05	> 0.05	$3.15e-26$	$1.45e-14$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	$2.71e-34$	> 0.05	$5.89e-05$	$1.44e-25$	—	$3.54e-02$	$1.78e-34$	$1.43e-32$	$1.03e-04$	> 0.05	> 0.05	> 0.05	$9.22e-17$
$\Delta_p$ -DDE	$2.30e-17$	> 0.05	> 0.05	> 0.05	> 0.05	—	$6.94e-09$	$1.09e-03$	$3.70e-06$	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$7.27e-08$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	$5.47e-14$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.30e-06$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	$4.96e-04$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	$6.81e-33$	> 0.05	$3.68e-05$	$4.09e-23$	> 0.05	$4.84e-02$	$6.42e-33$	$1.11e-30$	$6.58e-06$	—	> 0.05	> 0.05	$1.74e-15$
MOMBI-TCH	$1.68e-34$	> 0.05	$2.44e-05$	$1.22e-28$	> 0.05	> 0.05	$1.32e-34$	$1.20e-33$	$5.09e-04$	> 0.05	—	> 0.05	$2.44e-19$
MOMBI-NTCH	$1.84e-34$	> 0.05	$7.81e-04$	$2.15e-28$	> 0.05	> 0.05	$1.40e-34$	$1.54e-33$	$1.30e-02$	> 0.05	> 0.05	—	$2.91e-18$
MOMBI-PBI	$3.68e-32$	> 0.05	> 0.05	$5.37e-06$	> 0.05	> 0.05	$4.93e-33$	$4.61e-24$	> 0.05	> 0.05	> 0.05	> 0.05	—
3D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$1.28e-34$	—	$1.28e-34$	$1.16e-13$	$8.95e-10$	> 0.05	$1.28e-34$	$1.83e-34$	> 0.05	> 0.05	$9.44e-05$	$1.34e-05$	> 0.05
MOEA/D-NTCH	$5.76e-20$	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$5.71e-34$	> 0.05	$1.40e-34$	—	$6.21e-04$	> 0.05	$2.17e-25$	$7.38e-21$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	$2.16e-27$	> 0.05	$5.29e-21$	> 0.05	—	> 0.05	$1.74e-03$	$3.41e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	$1.76e-33$	> 0.05	$1.71e-33$	> 0.05	$8.85e-04$	—	$6.51e-09$	$2.43e-06$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$1.29e-32$	> 0.05	$1.28e-34$	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	$6.05e-33$	> 0.05	$1.28e-34$	> 0.05	> 0.05	> 0.05	$6.96e-03$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	$1.87e-33$	$5.38e-19$	$1.68e-34$	$3.08e-19$	$5.05e-11$	> 0.05	$7.28e-24$	$1.00e-23$	—	> 0.05	$9.06e-21$	$7.72e-21$	$3.37e-10$
R2-IBEA	$1.28e-34$	$8.85e-24$	$1.28e-34$	$3.23e-28$	$2.09e-19$	$4.77e-06$	$1.28e-34$	$1.83e-34$	$2.87e-09$	—	$1.77e-26$	$9.47e-27$	$5.03e-20$
MOMBI-TCH	$1.28e-34$	> 0.05	$1.28e-34$	$5.48e-09$	$1.67e-08$	> 0.05	$1.32e-34$	$5.38e-34$	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	$1.73e-34$	> 0.05	$1.32e-34$	$1.51e-08$	$2.73e-08$	> 0.05	$2.51e-33$	$4.45e-33$	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	$1.28e-34$	> 0.05	$1.28e-34$	$2.10e-09$	$3.21e-09$	> 0.05	$3.88e-34$	$1.15e-32$	> 0.05	> 0.05	> 0.05	> 0.05	—
4D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$1.02e-34$	—	$1.52e-34$	$5.70e-23$	$7.01e-32$	> 0.05	$7.33e-31$	$7.98e-31$	> 0.05	> 0.05	$4.01e-12$	$8.13e-25$	> 0.05
MOEA/D-NTCH	$8.87e-33$	> 0.05	—	> 0.05	$9.53e-20$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$3.44e-30$	> 0.05	$4.90e-16$	—	$5.15e-23$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	$4.15e-10$	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	$9.38e-35$	$1.28e-17$	$1.28e-34$	$8.41e-28$	$5.81e-32$	—	$7.19e-33$	$1.94e-31$	$1.73e-03$	$1.18e-09$	$2.19e-18$	$2.06e-21$	$3.39e-15$
R2-MOGA	$1.09e-34$	> 0.05	$1.36e-34$	> 0.05	$6.23e-32$	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	$1.13e-34$	> 0.05	$1.28e-34$	> 0.05	$5.83e-32$	> 0.05	$3.02e-03$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	$9.98e-35$	$6.02e-31$	$1.49e-34$	$2.21e-31$	$6.84e-32$	> 0.05	$7.20e-31$	$7.83e-31$	—	$5.51e-17$	$7.78e-31$	$7.19e-31$	$2.16e-26$
R2-IBEA	$9.67e-35$	$8.20e-13$	$1.36e-34$	$5.55e-27$	$6.21e-32$	> 0.05	$4.00e-33$	$1.54e-32$	> 0.05	—	$8.19e-16$	$6.82e-24$	$6.71e-07$
MOMBI-TCH	$9.34e-35$	> 0.05	$1.27e-34$	$7.27e-23$	$5.78e-32$	> 0.05	$5.84e-34$	$9.82e-34$	> 0.05	> 0.05	—	$7.15e-20$	> 0.05
MOMBI-NTCH	$9.68e-35$	> 0.05	$1.36e-34$	$1.41e-14$	$6.22e-32$	> 0.05	$2.31e-32$	$4.90e-31$	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	$9.69e-35$	> 0.05	$1.40e-34$	$4.46e-20$	$7.31e-32$	> 0.05	$4.31e-31$	$1.11e-28$	> 0.05	> 0.05	$1.26e-03$	$2.30e-10$	—

Table A.16: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ3 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
5D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$2.66e-39$	—	$2.40e-33$	$2.52e-20$	$9.80e-16$	$2.44e-11$	$7.17e-13$	$2.01e-10$	> 0.05	$2.53e-05$	> 0.05	$5.43e-07$	$1.22e-06$
MOEA/D-NTCH	$2.82e-39$	> 0.05	—	> 0.05	$1.34e-15$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$2.82e-39$	> 0.05	$1.16e-05$	—	$3.54e-14$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	$5.30e-29$	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	$4.64e-39$	> 0.05	$1.30e-18$	$5.85e-07$	$5.66e-14$	—	$1.22e-04$	$4.18e-03$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$2.82e-39$	> 0.05	$1.28e-34$	$1.32e-07$	$9.17e-16$	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	$2.82e-39$	> 0.05	$1.28e-34$	$5.68e-10$	$9.17e-16$	> 0.05	$1.47e-03$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	$1.26e-39$	$5.65e-32$	$6.90e-35$	$8.36e-34$	$3.19e-16$	$1.19e-32$	$1.53e-32$	$1.85e-32$	—	$2.58e-32$	$9.48e-32$	$1.61e-32$	$1.87e-32$
R2-IBEA	$2.81e-39$	> 0.05	$1.28e-34$	$1.51e-25$	$9.14e-16$	$3.42e-17$	$3.01e-28$	$1.21e-26$	> 0.05	—	> 0.05	$3.10e-03$	$8.00e-04$
MOMBI-TCH	$2.70e-39$	> 0.05	$1.24e-34$	$1.49e-28$	$8.68e-16$	$2.26e-24$	$9.81e-24$	$1.32e-23$	> 0.05	$4.45e-16$	—	$1.63e-21$	$5.53e-17$
MOMBI-NTCH	$2.75e-39$	> 0.05	$1.26e-34$	$3.04e-22$	$8.90e-16$	$1.10e-18$	$8.63e-25$	$1.31e-24$	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	$2.82e-39$	> 0.05	$1.28e-34$	$2.45e-18$	$9.16e-16$	$3.04e-07$	$1.09e-14$	$3.92e-12$	> 0.05	> 0.05	> 0.05	> 0.05	—
6D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$2.82e-39$	—	$1.28e-34$	$3.17e-10$	$2.92e-08$	$4.50e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	$2.82e-39$	> 0.05	—	> 0.05	$3.45e-08$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$2.82e-39$	> 0.05	> 0.05	—	$4.55e-07$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	$6.61e-26$	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	$3.05e-30$	> 0.05	$4.50e-02$	> 0.05	$3.14e-05$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$2.82e-39$	$4.52e-03$	$1.28e-34$	$2.31e-13$	$2.93e-08$	$1.85e-02$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	$2.82e-39$	$1.75e-06$	$1.28e-34$	$3.56e-15$	$2.92e-08$	$6.96e-03$	$5.68e-05$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	$1.29e-39$	$7.31e-28$	$7.05e-35$	$7.07e-31$	$1.41e-08$	$3.85e-10$	$5.87e-26$	$1.03e-24$	—	$2.16e-19$	$4.70e-23$	$4.24e-24$	$1.46e-22$
R2-IBEA	$2.80e-39$	$2.48e-26$	$1.27e-34$	$3.75e-26$	$2.91e-08$	$2.03e-05$	$2.96e-30$	$1.40e-26$	> 0.05	—	$1.11e-10$	$6.21e-22$	$1.30e-05$
MOMBI-TCH	$2.76e-39$	$2.50e-14$	$1.26e-34$	$1.44e-20$	$2.88e-08$	$4.55e-04$	$2.34e-19$	$7.75e-16$	> 0.05	> 0.05	—	$1.49e-18$	> 0.05
MOMBI-NTCH	$2.74e-39$	$6.37e-07$	$1.25e-34$	$1.13e-15$	$2.85e-08$	$9.84e-04$	$1.67e-15$	$1.43e-08$	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	$2.81e-39$	$8.45e-11$	$1.18e-32$	$1.24e-18$	$2.92e-08$	$9.31e-04$	$5.25e-09$	$9.62e-06$	> 0.05	> 0.05	> 0.05	$5.12e-03$	—
7D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$2.82e-39$	—	$3.68e-33$	$1.84e-03$	$5.86e-17$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	$2.82e-39$	> 0.05	—	$7.30e-03$	$6.14e-17$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$2.82e-39$	> 0.05	> 0.05	—	$1.54e-13$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	$1.53e-29$	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	$2.08e-23$	> 0.05	$4.81e-02$	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$2.82e-39$	$1.29e-16$	$1.28e-34$	$8.86e-05$	$5.86e-17$	$4.48e-02$	—	> 0.05	> 0.05	> 0.05	$5.11e-08$	$1.54e-07$	> 0.05
R2-MOGAw	$2.82e-39$	$1.05e-27$	$1.28e-34$	$3.82e-06$	$5.86e-17$	$4.48e-02$	$1.47e-10$	—	> 0.05	> 0.05	$1.79e-25$	$1.05e-25$	> 0.05
R2-MODE	$2.62e-39$	$4.69e-26$	$2.67e-33$	$1.72e-21$	$8.09e-17$	> 0.05	$7.49e-23$	$2.48e-20$	—	$3.44e-03$	$3.17e-24$	$1.02e-23$	$5.75e-07$
R2-IBEA	$2.81e-39$	$1.28e-34$	$1.28e-34$	$7.45e-24$	$5.84e-17$	$4.47e-02$	$1.28e-34$	$1.62e-34$	> 0.05	—	$1.28e-34$	$1.28e-34$	$1.34e-08$
MOMBI-TCH	$2.82e-39$	$8.86e-05$	$3.17e-33$	$6.71e-04$	$5.86e-17$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	$2.82e-39$	$6.61e-06$	$2.91e-33$	$4.32e-04$	$5.86e-17$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	$2.82e-39$	$3.00e-32$	$1.32e-34$	$3.46e-16$	$5.86e-17$	$4.48e-02$	$3.76e-29$	$2.35e-24$	> 0.05	> 0.05	$3.75e-31$	$3.96e-31$	—



Table A.17: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ3 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI	
8D														
NSGA-II	–	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	
MOEA/D-TCH	2.82e – 39	–	2.96e – 34	7.19e – 10	1.39e – 06		> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	2.82e – 39	> 0.05	–	2.41e – 09	1.56e – 06		> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	2.82e – 39	> 0.05	> 0.05	–	6.10e – 05		> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	2.83e – 25	> 0.05	> 0.05	> 0.05	–		> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE														
R2-MOGA	2.82e – 39	2.31e – 32	1.28e – 34	3.92e – 12	1.39e – 06		–	> 0.05	> 0.05	> 0.05	> 0.05	1.87e – 27	1.76e – 28	> 0.05
R2-MOGAw	2.82e – 39	1.53e – 34	1.28e – 34	5.49e – 13	1.39e – 06		6.42e – 06	–	> 0.05	> 0.05	> 0.05	2.47e – 34	1.28e – 34	> 0.05
R2-MODE	2.20e – 39	1.06e – 27	3.67e – 32	6.83e – 29	1.22e – 06		4.44e – 23	1.36e – 21	–	1.45e – 14	1.02e – 25	1.25e – 25	5.43e – 17	
R2-IBEA	2.82e – 39	1.89e – 34	1.28e – 34	3.03e – 21	1.39e – 06		2.16e – 30	1.38e – 25	> 0.05	–	5.37e – 34	2.07e – 34	1.30e – 03	
MOMBI-TCH	2.82e – 39	3.30e – 12	2.37e – 33	4.86e – 11	1.39e – 06		> 0.05	> 0.05	> 0.05	> 0.05	–	1.06e – 02	> 0.05	
MOMBI-NTCH	2.82e – 39	2.04e – 11	2.36e – 33	3.75e – 11	1.39e – 06		> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	
MOMBI-PBI	2.82e – 39	2.17e – 29	1.28e – 34	1.87e – 18	1.39e – 06		5.03e – 14	1.73e – 09	> 0.05	> 0.05	5.75e – 27	7.68e – 27	–	
9D														
NSGA-II	–	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	
MOEA/D-TCH	2.82e – 39	–	1.95e – 34	4.44e – 07	3.73e – 04		> 0.05	> 0.05	7.34e – 30	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	2.82e – 39	> 0.05	–	1.75e – 06	3.73e – 04		> 0.05	> 0.05	1.83e – 28	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	2.82e – 39	> 0.05	> 0.05	–	3.73e – 04		> 0.05	> 0.05	6.50e – 28	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	2.23e – 24	> 0.05	> 0.05	> 0.05	–		> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE														
R2-MOGA	2.82e – 39	1.28e – 34	1.28e – 34	2.32e – 16	3.73e – 04		–	> 0.05	6.18e – 30	> 0.05	1.44e – 34	1.28e – 34	> 0.05	
R2-MOGAw	2.82e – 39	1.28e – 34	1.28e – 34	7.88e – 18	3.73e – 04		4.52e – 02	–	6.18e – 30	> 0.05	2.87e – 34	1.28e – 34	> 0.05	
R2-MODE	2.41e – 36	> 0.05	> 0.05	> 0.05	> 0.05		> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	2.82e – 39	1.29e – 32	1.28e – 34	2.90e – 20	3.73e – 04		7.04e – 05	5.84e – 03	6.18e – 30	–	2.63e – 30	4.57e – 31	> 0.05	
MOMBI-TCH	2.82e – 39	1.90e – 12	1.99e – 28	2.30e – 07	3.73e – 04		> 0.05	> 0.05	7.86e – 30	> 0.05	–	7.40e – 03	> 0.05	
MOMBI-NTCH	2.82e – 39	2.86e – 06	1.13e – 30	2.97e – 07	3.73e – 04		> 0.05	> 0.05	6.85e – 30	> 0.05	> 0.05	–	> 0.05	
MOMBI-PBI	2.82e – 39	3.02e – 30	1.36e – 34	4.57e – 19	3.73e – 04		8.11e – 04	4.26e – 02	6.39e – 30	> 0.05	1.84e – 27	1.60e – 28	–	
10D														
NSGA-II	–	> 0.05	> 0.05	> 0.05		–	> 0.05	> 0.05		> 0.05	> 0.05	> 0.05	> 0.05	
MOEA/D-TCH	> 0.05	–	4.12e – 34	4.63e – 11			> 0.05	> 0.05		> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	–	1.10e – 10			> 0.05	> 0.05		> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	–			> 0.05	> 0.05		> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA					–									
$\Delta_p$ -DDE														
R2-MOGA	> 0.05	4.63e – 32	1.28e – 34	2.13e – 26			–	> 0.05		3.13e – 11	1.83e – 34	2.79e – 34	2.98e – 06	
R2-MOGAw	> 0.05	4.62e – 32	1.28e – 34	5.75e – 28			7.01e – 03	–		5.02e – 14	1.67e – 34	2.26e – 34	2.73e – 10	
R2-MODE									–					
R2-IBEA	> 0.05	2.82e – 28	1.28e – 34	8.14e – 18			> 0.05	> 0.05		–	4.09e – 18	1.77e – 18	> 0.05	
MOMBI-TCH	> 0.05	6.77e – 10	1.60e – 28	2.79e – 11			> 0.05	> 0.05		> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	> 0.05	4.50e – 18	6.80e – 31	4.86e – 11			> 0.05	> 0.05		> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-PBI	> 0.05	4.84e – 31	1.28e – 34	3.11e – 21			> 0.05	> 0.05		4.59e – 04	1.99e – 28	3.95e – 28	–	

Table A.18: Comparison of R2 indicator values for different optimizers on the DTLZ3 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	2.534435e-01	2.189979e-01	2.192593e-01	2.201693e-01	2.188420e-01	2.182232e-01	2.202455e-01	2.195334e-01	2.191348e-01	2.187822e-01	2.189338e-01	2.190678e-01	2.197580e-01
	avg.	3.121469e-01	2.190945e-01	2.206797e-01	2.206998e-01	2.191278e-01	2.204752e-01	2.205016e-01	2.198022e-01	3.680813e-01	2.212878e-01	2.190640e-01	2.191797e-01	2.200500e-01
	min.	2.199432e-01	2.182376e-01	2.183404e-01	2.183640e-01	2.179974e-01	2.180441e-01	2.191681e-01	2.185493e-01	2.185676e-01	2.180461e-01	2.181491e-01	2.183834e-01	2.182039e-01
	max.	7.209124e-01	2.212156e-01	2.964901e-01	2.336864e-01	2.218850e-01	4.364665e-01	2.230673e-01	2.231349e-01	2.193640e+00	4.379704e-01	2.206511e-01	2.217431e-01	2.279011e-01
	std.	1.192213e-01	5.707806e-04	7.989600e-03	1.953993e-03	8.197638e-04	2.181871e-02	8.322547e-04	8.404226e-04	2.782190e-01	2.191637e-02	5.718315e-04	5.581106e-04	1.388241e-03
3D	med.	3.038220e-01	9.543888e-02	1.464513e-01	9.620735e-02	1.019326e-01	9.552355e-02	9.685216e-02	9.578850e-02	9.536861e-02	9.428331e-02	9.553230e-02	9.601772e-02	9.554039e-02
	avg.	3.248367e-01	9.558200e-02	1.493891e-01	9.968260e-02	1.289810e-01	9.708731e-02	9.688769e-02	9.592682e-02	1.039246e-01	9.439314e-02	9.569516e-02	9.708726e-02	9.574405e-02
	min.	1.060750e-01	9.479067e-02	1.386715e-01	9.429154e-02	9.366416e-02	9.400982e-02	9.579034e-02	9.493475e-02	9.485959e-02	9.368044e-02	9.478987e-02	9.509783e-02	9.405553e-02
	max.	8.934903e-01	9.781555e-02	3.111810e-01	1.932599e-01	3.812414e-01	1.934791e-01	9.940108e-02	9.798402e-02	1.914986e-01	9.691315e-02	9.805544e-02	1.915747e-01	1.006897e-01
	std.	1.353418e-01	5.493857e-04	1.683236e-02	1.653887e-02	5.947540e-02	9.921987e-03	5.911354e-04	6.220611e-04	2.744381e-02	5.197902e-04	6.892597e-04	9.560862e-03	1.092871e-03
4D	med.	4.741005e-01	5.644784e-02	1.109579e-01	5.821206e-02	3.376360e-01	5.382734e-02	5.546176e-02	5.462900e-02	5.414703e-02	5.347033e-02	5.666008e-02	5.824084e-02	5.458754e-02
	avg.	5.136372e-01	5.822274e-02	1.154879e-01	1.080231e-01	3.423791e-01	5.479911e-02	5.604692e-02	5.476135e-02	5.579915e-02	5.409520e-02	5.694517e-02	5.894658e-02	5.571283e-02
	min.	7.690973e-02	5.596905e-02	7.904252e-02	5.361981e-02	9.116198e-02	5.319518e-02	5.448788e-02	5.395212e-02	5.377303e-02	5.285570e-02	5.600573e-02	5.582599e-02	5.347014e-02
	max.	1.374152e+00	1.158479e-01	2.540144e-01	8.019087e-01	8.004016e-01	6.961531e-02	1.106995e-01	5.672507e-02	1.084872e-01	1.075609e-01	6.805376e-02	1.146414e-01	1.441670e-01
	std.	2.422833e-01	9.512707e-03	1.916022e-02	1.214743e-01	1.495994e-01	2.458510e-03	5.537692e-03	5.339573e-04	9.297025e-03	5.416447e-03	1.290994e-03	5.790615e-03	8.986336e-03
5D	med.	9.118191e+00	3.806062e-02	9.041865e-02	6.327305e-02	4.069869e-01	4.423419e-02	3.717946e-02	3.657166e-02	3.615677e-02	3.587901e-02	3.830710e-02	4.077853e-02	3.680353e-02
	avg.	9.110114e+00	4.509720e-02	9.434139e-02	1.043847e-01	4.338395e-01	1.381997e-01	3.722573e-02	3.663955e-02	3.656510e-02	3.693520e-02	4.214972e-02	4.393917e-02	4.028222e-02
	min.	4.308300e+00	3.702141e-02	8.471922e-02	3.563599e-02	1.549554e-01	3.784764e-02	3.660196e-02	3.602084e-02	3.588015e-02	3.537235e-02	3.744154e-02	3.962312e-02	3.562016e-02
	max.	1.589912e+01	2.210218e-01	2.006837e-01	3.115430e-01	1.016579e+00	3.766331e+00	3.815818e-02	3.813745e-02	7.219476e-02	7.177319e-02	9.071191e-02	8.249684e-02	1.194674e-01
	std.	2.645600e+00	2.180605e-02	1.440418e-02	7.702109e-02	1.578237e-01	4.344587e-01	3.036697e-04	4.290748e-04	3.613290e-03	5.235764e-03	1.139040e-02	1.029083e-02	1.168429e-02
6D	med.	2.167352e+01	3.823696e-02	7.663345e-02	1.074962e-01	3.933449e-01	2.991398e+00	2.596112e-02	2.553553e-02	2.504731e-02	2.471982e-02	3.501408e-02	3.391691e-02	2.585557e-02
	avg.	2.242140e+01	4.215478e-02	8.193893e-02	1.076348e-01	4.105772e-01	2.647086e+00	2.597840e-02	2.562552e-02	2.609856e-02	2.492890e-02	3.881922e-02	3.843004e-02	3.309099e-02
	min.	1.240108e+01	3.198547e-02	7.408973e-02	2.492989e-02	1.283734e-01	2.446639e-02	2.490290e-02	2.468477e-02	2.438502e-02	3.019834e-02	3.277205e-02	3.277205e-02	2.459622e-02
	max.	3.389051e+01	8.660957e-02	2.133709e-01	4.155591e-01	8.617669e-01	7.447821e+00	2.694183e-02	2.712873e-02	7.434693e-02	3.648837e-02	9.245655e-02	9.923996e-02	1.125859e-01
	std.	4.778335e+00	9.737474e-03	1.936903e-02	7.126628e-02	1.403840e-01	2.161201e+00	2.294358e-04	4.462890e-04	6.245590e-03	1.304833e-03	1.175701e-02	1.317493e-02	1.807752e-02
7D	med.	2.737843e+01	4.216640e-02	6.659990e-02	1.395452e-01	2.342571e-01	4.327962e+00	2.068360e-02	2.070959e-02	1.920313e-02	1.868847e-02	3.895710e-02	3.533923e-02	1.915929e-02
	avg.	2.698645e+01	4.272950e-02	6.778630e-02	9.476442e-02	2.427828e-01	4.408180e+00	2.106937e-02	2.101911e-02	2.473137e-02	1.872301e-02	4.355280e-02	3.864660e-02	2.207244e-02
	min.	1.254730e+01	3.209653e-02	4.753682e-02	1.854506e-02	9.810784e-02	8.752458e-02	2.010209e-02	1.910632e-02	1.890853e-02	1.832459e-02	3.203894e-02	3.399197e-02	1.852762e-02
	max.	3.679202e+01	1.029114e-01	1.138067e-01	1.459153e-01	5.106783e-01	6.643314e+00	2.365648e-02	2.686820e-02	1.151725e-01	1.916018e-02	1.078793e-01	1.058153e-01	6.603099e-02
	std.	5.064414e+00	1.036301e-02	5.505442e-03	5.666960e-02	9.331352e-02	1.056000e-01	9.956317e-04	1.376060e-03	1.447837e-02	1.853942e-04	1.318164e-02	1.099638e-02	8.279257e-03
8D	med.	3.373213e+01	3.765963e-02	5.891383e-02	1.222728e-01	2.989157e-01	3.517555e+00	1.544983e-02	1.545915e-02	1.482020e-02	1.436434e-02	3.309843e-02	3.105702e-02	1.478367e-02
	avg.	3.355964e+01	3.654328e-02	6.083889e-02	9.427210e-02	3.103681e-01	3.631409e+00	1.612654e-02	1.586228e-02	1.840135e-02	1.473056e-02	3.701040e-02	3.382422e-02	1.903612e-02
	min.	1.730270e+01	2.850603e-02	5.656972e-02	1.412444e-02	8.262558e-02	2.089775e+00	1.508712e-02	1.463887e-02	1.445836e-02	1.409980e-02	3.007506e-02	2.996743e-02	1.423507e-02
	max.	4.998989e+01	5.829897e-02	1.242444e-01	1.264168e-01	7.167236e-01	7.711384e+00	3.102896e-02	2.935385e-02	8.510391e-02	2.441578e-02	9.590351e-02	8.400154e-02	5.763398e-02
	std.	5.335187e+00	5.005359e-03	8.924535e-03	4.562273e-02	1.178321e-01	8.529478e-01	1.928071e-03	1.739632e-03	1.070544e-02	1.658758e-03	1.089627e-02	9.185594e-03	9.769349e-03
9D	med.	3.444643e+01	3.328485e-02	5.204943e-02	1.107512e-01	3.522006e-01	2.748059e+00	1.210614e-02	1.190794e-02	1.417455e-01	1.159048e-02	2.998792e-02	2.762875e-02	1.195477e-02
	avg.	3.436791e+01	3.328263e-02	5.280486e-02	8.075270e-02	3.729356e-01	2.924286e+00	1.248062e-02	1.246854e-02	1.446087e-01	1.323214e-02	3.620810e-02	3.422358e-02	1.460409e-02
	min.	2.317815e+01	2.598454e-02	3.535480e-02	1.144887e-02	1.355072e-01	1.559264e+00	1.186071e-02	1.148280e-02	5.238750e-02	1.109295e-02	2.480137e-02	2.645049e-02	1.129861e-02
	max.	4.471013e+01	6.596033e-02	9.450264e-02	1.185793e-01	8.446383e-01	6.553590e+00	2.228860e-02	2.837623e-02	2.271523e-01	4.155743e-02	1.465284e-01	8.324073e-02	5.754194e-02
	std.	4.324147e+00	5.664349e-03	4.912228e-03	4.435472e-02	1.265072e-01	7.518427e-01	1.258821e-03	2.081493e-03	3.596465e-02	4.892486e-03	1.924251e-02	1.325085e-02	7.905266e-03
10D	med.	3.155006e+01	3.092150e-02	4.753518e-02	1.002899e-01	4.269917e-01	2.299017e+00	9.703468e-03	9.569916e-03	2.346680e-01	1.093277e-02	2.788000e-02	2.466513e-02	1.021599e-02
	avg.	3.087945e+01	3.245881e-02	4.867012e-02	7.879313e-02	4.388562e-01	2.508743e+00	1.031165e-02	1.031489e-02	2.418489e-01	1.516467e-02	3.544536e-02	2.809372e-02	1.179215e-02
	min.	1.914282e+01	2.469648e-02	4.534674e-02	9.678078e-03	1.531296e-01	1.746037e+00	9.455865e-03	9.158191e-03	1.515953e-01	9.074685e-03	2.183090e-02	1.976523e-02	9.360645e-03
	max.	4.066007e+01	6.188936e-02	9.807325e-02	1.031166e-01	8.161843e-01	6.651248e+00	2.077905e-02	2.349050e-02	3.316850e-01	4.554508e-02	1.085131e-01	7.267182e-02	3.308821e-02
	std.	3.742363e+00	6.495386e-03	6.373926e-03	3.791140e-02	1.431547e-01	8.437257e-01	1.516962e-03	2.143990e-03	3.542055e-02	8.031043e-03	1.733357e-02	9.985260e-03	5.732066e-03

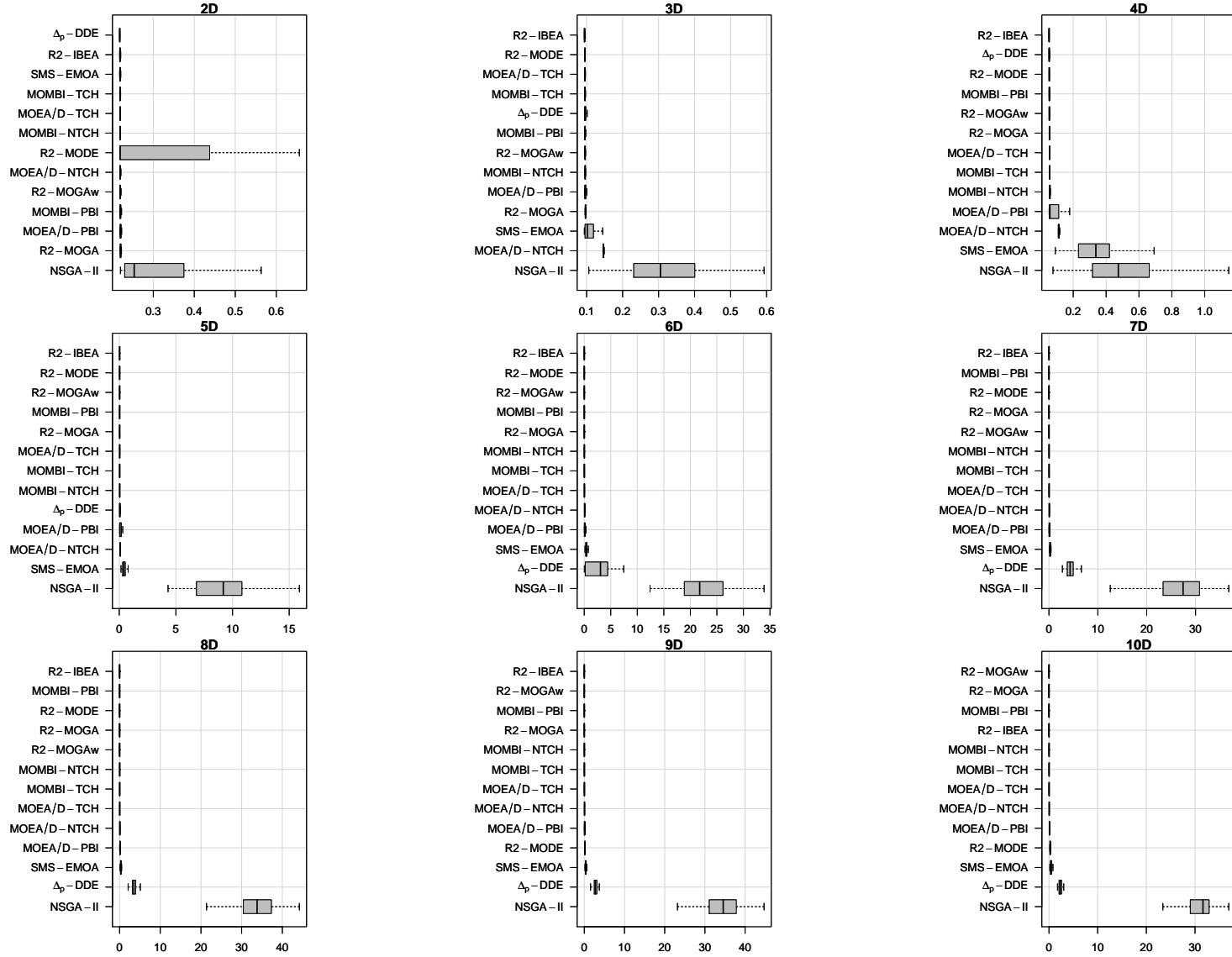


Figure A.34: Box-plot of R2 indicator values for different optimizers on the DTLZ3 test problem.

Table A.19: Comparison of runtime (in milliseconds) for different optimizers on the DTLZ3 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_\rho$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	1.763520e + 02	4.313300e + 02	1.027224e + 03	8.994050e + 02	1.759987e + 04	1.149649e + 03	3.203308e + 03	2.691313e + 03	3.659356e + 03	5.565290e + 05	1.976790e + 03	1.533901e + 03	2.717598e + 03
	avg.	1.875328e + 02	4.641572e + 02	1.002084e + 03	9.152862e + 02	1.737862e + 04	1.152751e + 03	3.235827e + 03	2.703682e + 03	3.676680e + 03	5.560140e + 05	1.939938e + 03	1.550754e + 03	2.713842e + 03
	min.	1.627390e + 02	3.467400e + 02	7.188890e + 02	6.423500e + 02	1.387278e + 04	1.099563e + 03	2.209946e + 03	1.929803e + 03	2.673340e + 03	5.416180e + 05	1.484714e + 03	1.513105e + 03	2.350529e + 03
	max.	3.035110e + 02	7.672440e + 02	1.420089e + 03	1.500859e + 03	1.983761e + 04	1.315052e + 03	4.090946e + 03	3.501610e + 03	4.738543e + 03	5.703600e + 05	2.381633e + 03	1.717857e + 03	3.102198e + 03
	std.	2.441464e + 01	8.430913e + 01	1.225059e + 02	1.164124e + 02	1.116235e + 03	3.249124e + 01	4.134826e + 02	3.011916e + 02	4.391265e + 02	6.943923e + 03	1.894784e + 02	4.745801e + 01	1.277541e + 02
3D	med.	2.496040e + 02	5.846790e + 02	1.283340e + 03	1.042511e + 03	5.408360e + 04	1.521902e + 03	2.849965e + 03	2.373552e + 03	3.689351e + 03	4.819210e + 05	1.938724e + 03	2.050027e + 03	2.900877e + 03
	avg.	2.610904e + 02	6.007067e + 02	1.294265e + 03	1.052975e + 03	6.082279e + 04	1.538102e + 03	2.902516e + 03	2.381135e + 03	3.705518e + 03	5.047939e + 05	1.944205e + 03	2.057678e + 03	2.909932e + 03
	min.	2.211690e + 02	4.373190e + 02	1.023453e + 03	7.848050e + 02	2.322219e + 04	1.157991e + 03	2.150164e + 03	1.759676e + 03	2.720465e + 03	4.792450e + 05	1.627584e + 03	1.646705e + 03	2.401424e + 03
	max.	3.267370e + 02	1.037117e + 03	1.561138e + 03	1.896538e + 03	1.667841e + 05	2.070037e + 03	3.671753e + 03	2.989433e + 03	5.021010e + 03	5.992030e + 05	2.509332e + 03	2.552543e + 03	3.276858e + 03
	std.	2.920225e + 01	1.008886e + 02	1.286971e + 02	1.317364e + 02	3.234010e + 04	1.927921e + 02	3.316553e + 02	2.646076e + 02	4.197698e + 02	4.033095e + 04	1.476720e + 02	1.456887e + 02	1.602532e + 02
4D	med.	3.618520e + 02	6.275480e + 02	1.442081e + 03	1.080103e + 03	2.531493e + 05	1.669793e + 03	3.053058e + 03	2.569495e + 03	4.177030e + 03	6.313050e + 05	1.992847e + 03	2.208017e + 03	3.145675e + 03
	avg.	3.697283e + 02	6.454905e + 02	1.431611e + 03	1.078016e + 03	2.602373e + 05	1.656180e + 03	3.042902e + 03	2.575498e + 03	4.240993e + 03	6.293684e + 05	2.003216e + 03	2.184205e + 03	3.126197e + 03
	min.	2.944410e + 02	4.644980e + 02	1.025869e + 03	8.293060e + 02	1.847885e + 05	1.084155e + 03	2.157356e + 03	1.898385e + 03	3.012685e + 03	5.959940e + 05	1.640958e + 03	1.795916e + 03	2.667040e + 03
	max.	5.801380e + 02	1.129495e + 03	1.643431e + 03	1.633102e + 03	3.318402e + 05	2.164433e + 03	3.662899e + 03	3.162601e + 03	5.315099e + 03	6.543580e + 05	2.377018e + 03	2.569622e + 03	3.459949e + 03
	std.	5.441604e + 01	1.099495e + 02	1.198542e + 02	1.106982e + 02	4.742532e + 04	2.345296e + 02	3.181175e + 02	2.601677e + 02	4.684691e + 02	1.199462e + 04	1.624501e + 02	1.413550e + 02	1.749536e + 02
5D	med.	5.286110e + 02	6.508400e + 02	1.847371e + 03	1.033602e + 03	3.187073e + 05	2.391271e + 03	3.370302e + 03	2.907887e + 03	4.872421e + 03	6.737230e + 05	2.218371e + 03	2.382497e + 03	3.653095e + 03
	avg.	5.328942e + 02	6.638628e + 02	1.830440e + 03	1.023795e + 03	3.145896e + 05	2.406305e + 03	3.429423e + 03	2.876603e + 03	4.847570e + 03	6.599707e + 05	2.258180e + 03	2.401203e + 03	3.681069e + 03
	min.	4.195480e + 02	4.823740e + 02	1.331433e + 03	2.085810e + 02	2.085810e + 05	1.207813e + 03	3.526018e + 03	2.101816e + 03	3.569231e + 03	5.656990e + 05	1.817973e + 03	2.099065e + 03	3.001708e + 03
	max.	6.628710e + 02	9.371660e + 02	2.313649e + 03	1.329789e + 03	3.584526e + 05	3.495515e + 03	4.267081e + 03	3.530259e + 03	6.089270e + 03	6.811580e + 05	3.674845e + 03	3.189315e + 03	3.955379e + 03
	std.	6.122760e + 01	8.564185e + 01	1.597107e + 02	1.202212e + 02	2.893185e + 04	4.867223e + 02	3.755037e + 02	3.087224e + 02	5.496815e + 02	3.663690e + 04	2.986195e + 02	1.510928e + 02	1.811166e + 02
6D	med.	6.893700e + 02	6.940470e + 02	1.993405e + 03	1.059493e + 03	4.249203e + 05	2.591387e + 03	3.954406e + 03	3.168759e + 03	6.135587e + 03	7.122710e + 05	2.266826e + 03	2.513818e + 03	3.861636e + 03
	avg.	6.900029e + 02	6.972897e + 02	1.967002e + 03	1.064481e + 03	4.188183e + 05	2.538747e + 03	3.853961e + 03	3.187908e + 03	6.071432e + 03	7.116606e + 05	2.342744e + 03	2.515173e + 03	3.870683e + 03
	min.	5.502480e + 02	5.013930e + 02	1.547718e + 03	8.337700e + 02	2.535254e + 05	1.621630e + 03	2.680336e + 03	2.502214e + 03	4.413109e + 03	6.666170e + 05	1.887866e + 03	2.108058e + 03	3.157382e + 03
	max.	9.021000e + 02	9.861480e + 02	2.402098e + 03	1.527131e + 03	4.576643e + 05	3.814141e + 03	4.624147e + 03	3.785684e + 03	7.516480e + 03	7.193940e + 05	3.338645e + 03	3.825166e + 03	4.298278e + 03
	std.	6.425767e + 01	8.557646e + 01	1.362665e + 02	1.354495e + 02	3.053337e + 04	4.020109e + 02	4.547802e + 02	2.881033e + 02	6.966151e + 02	5.700491e + 03	2.877472e + 02	1.463074e + 02	2.230179e + 02
7D	med.	6.317740e + 02	7.254970e + 02	2.206610e + 03	1.091756e + 03	5.260631e + 05	8.002749e + 03	2.505266e + 03	2.206539e + 03	3.630959e + 03	3.778330e + 05	1.569242e + 03	1.746828e + 03	2.775399e + 03
	avg.	6.323113e + 02	7.300662e + 02	2.159910e + 03	1.084882e + 03	5.176266e + 05	8.041690e + 03	2.534442e + 03	2.212152e + 03	3.582609e + 03	3.772377e + 05	1.575222e + 03	1.751871e + 03	2.789812e + 03
	min.	5.026420e + 02	5.722610e + 02	1.542945e + 03	7.448190e + 02	3.121827e + 05	4.331615e + 03	1.758355e + 03	1.578847e + 03	2.594129e + 03	3.581730e + 05	1.253040e + 03	1.469645e + 03	2.345287e + 03
	max.	8.101730e + 02	1.009733e + 03	2.484869e + 03	1.340985e + 03	6.546967e + 05	1.217405e + 03	3.593748e + 03	2.764215e + 03	4.292507e + 03	3.943380e + 05	2.391971e + 03	2.019613e + 03	3.250372e + 03
	std.	5.965177e + 01	7.869625e + 01	1.673313e + 02	1.319592e + 02	5.796171e + 04	1.617126e + 03	4.0202587e + 02	2.170896e + 02	3.917954e + 02	8.127250e + 03	1.568366e + 02	1.850489e + 02	1.535017e + 02
8D	med.	9.222970e + 02	7.899700e + 02	2.316924e + 03	1.206236e + 03	7.040209e + 05	1.551025e + 04	4.387386e + 03	3.469968e + 03	7.229980e + 03	1.290604e + 06	1.939251e + 03	2.561168e + 03	4.026857e + 03
	avg.	9.186689e + 02	7.876377e + 02	2.274293e + 03	1.182458e + 03	6.812087e + 05	1.566760e + 04	4.421540e + 03	3.538731e + 03	7.168665e + 03	1.267026e + 06	1.976233e + 03	2.617889e + 03	4.051922e + 03
	min.	7.229920e + 02	6.054850e + 02	1.591679e + 03	7.997570e + 02	4.222269e + 05	8.923047e + 03	3.132842e + 03	2.652315e + 03	5.514690e + 03	1.183137e + 06	1.680163e + 03	2.308105e + 03	3.358023e + 03
	max.	1.132980e + 03	1.052175e + 03	2.650323e + 03	1.654210e + 03	8.222670e + 05	2.329742e + 04	5.558083e + 03	4.319577e + 03	8.441378e + 03	1.318684e + 06	2.456274e + 03	2.983980e + 03	4.727843e + 03
	std.	7.405567e + 01	8.495209e + 01	1.936666e + 02	1.615570e + 02	9.193145e + 04	3.445227e + 03	5.031177e + 02	3.523056e + 02	7.134733e + 02	4.847738e + 04	1.743363e + 02	1.850489e + 02	2.202960e + 02
9D	med.	1.303708e + 03	8.606690e + 02	2.842053e + 03	1.248795e + 03	7.471263e + 05	2.620779e + 04	7.012121e + 03	5.920453e + 03	1.115388e + 04	2.277616e + 06	2.768060e + 03	4.001719e + 03	6.165591e + 03
	avg.	1.304876e + 03	8.593396e + 02	2.778459e + 03	1.236649e + 03	7.430479e + 05	2.740275e + 04	6.756194e + 03	5.879314e + 03	1.111421e + 04	2.411845e + 06	2.801403e + 03	3.992696e + 03	6.181256e + 03
	min.	1.040736e + 03	6.423880e + 02	2.005092e + 03	8.664320e + 02	4.577413e + 05	1.280743e + 04	4.135407e + 03	4.455348e + 03	8.137782e + 03	2.265163e + 06	2.354867e + 03	3.343626e + 03	4.905652e + 03
	max.	1.615518e + 03	1.084302e + 03	3.052959e + 03	1.617860e + 03	9.044256e + 05	5.911146e + 04	8.778931e + 03	6.974627e + 03	1.421751e + 04	2.771035e + 06	3.649379e + 03	4.398722e + 03	7.273613e + 03
	std.	9.687226e + 01	7.630736e + 01	2.207731e + 02	1.395091e + 02	8.320324e + 04	8.038898e + 03	1.307246e + 03	4.983901e + 02	1.199243e + 03	1.931713e + 05	2.622927e + 02	2.204968e + 02	3.510749e + 02
10D	med.	1.853199e + 03	9.038650e + 02	2.943851e + 03	1.258377e + 03	1.004808e + 06	3.346232e + 04	1.066151e + 04	9.567503e + 03	1.699925e + 04	4.891300e + 06	4.180048e + 03	5.732968e + 03	8.471751e + 03
	avg.	1.839762e + 03	9.109797e + 02	2.886850e + 03	1.233265e + 03	9.920375e + 05	3.521108e + 04	1.053065e + 04	9.501753e + 03	1.659123e + 04	4.890951e + 06	4.140006e + 03	5.661167e + 03	8.458672e + 03
	min.	1.586971e + 03	7.432230e + 02	2.180545e + 03	8.926830e + 02	6.796744e + 05	1.965395e + 04	7.135179e + 03	7.283547e + 03	1.189923e + 04	4.544581e + 06	3.244395e + 03	4.852559e + 03	7.054695e + 03
	max.	2.052998e + 03	1.233443e + 03	3.402277e + 03	1.581194e + 03	1.188224e + 06	7.579678e + 04	1.460404e + 04	1.155894e + 04	2.084213e + 04	5.129757e + 06	4.611476e + 03	6.343282e + 03	9.830118e + 03
	std.	1.016036e + 02	8.174393e + 01	2.403856e + 02	1.676835e + 02	7.942381e + 04	9.451294e + 03	1.896890e + 03	1.055262e + 03	1.931551e + 03	1.498001e + 05	3.329473e + 02	3.157129e + 02	3.888934e + 02

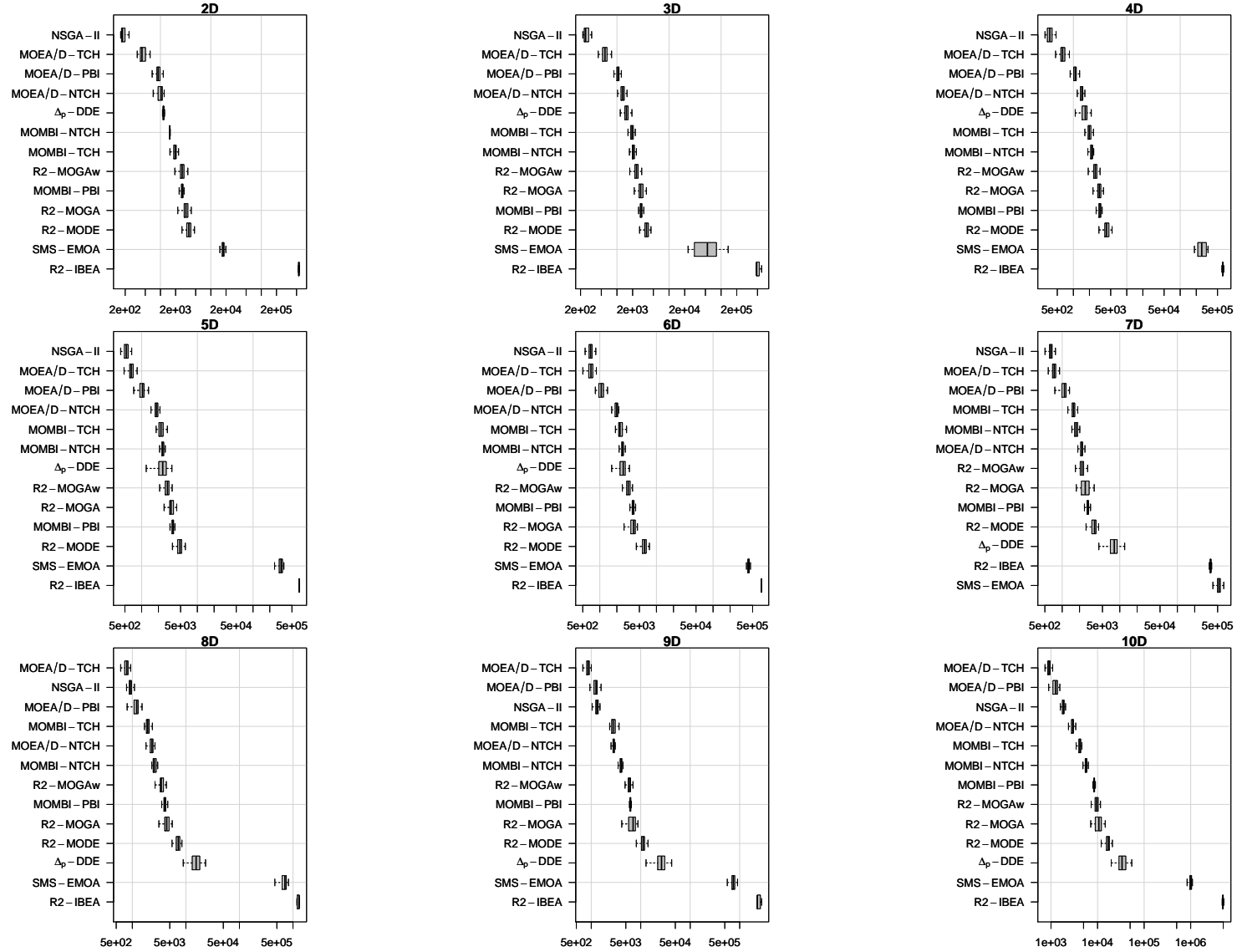


Figure A.35: Box-plot of runtime (in logarithmic scale) for different optimizers on the DTLZ3 test problem.

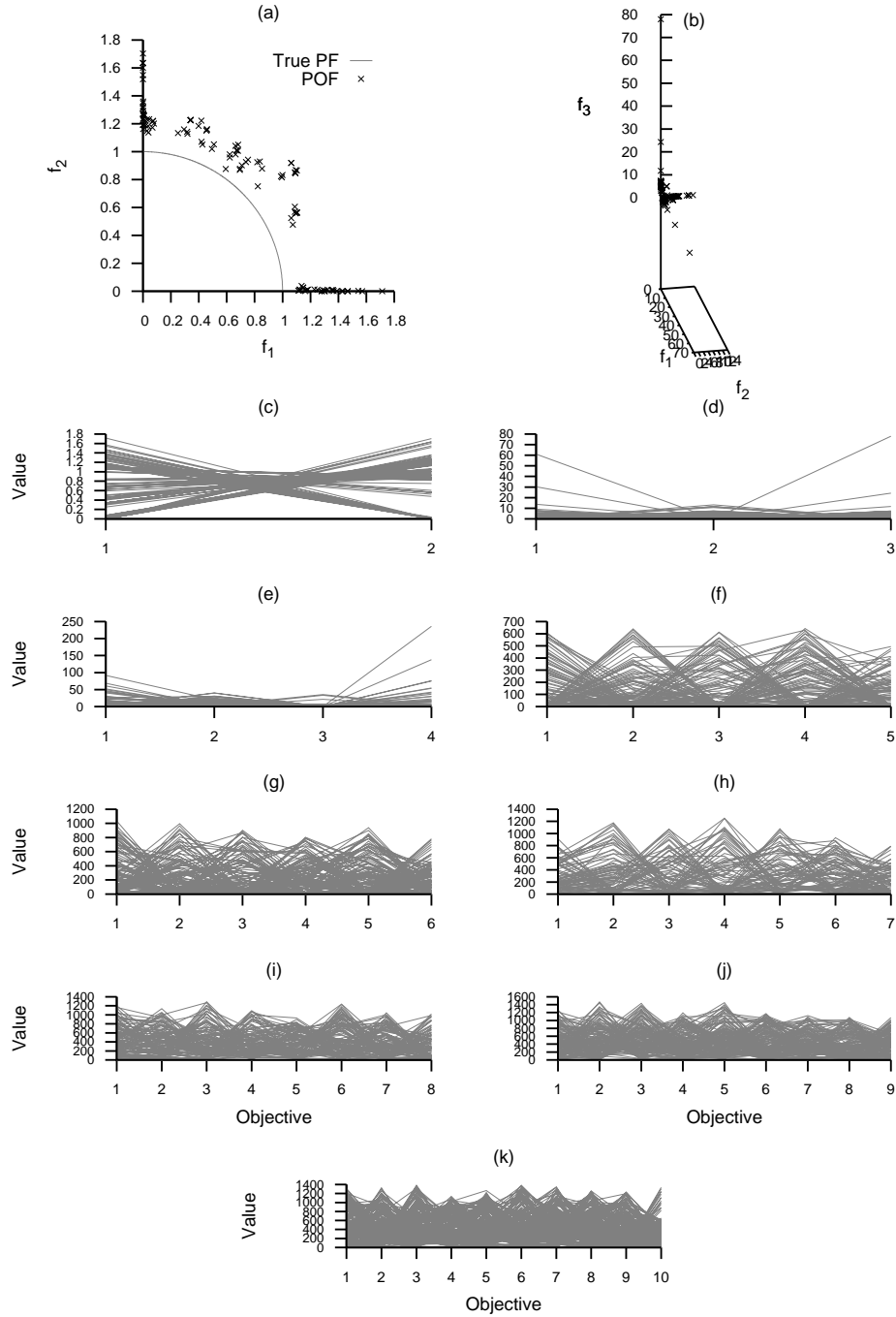


Figure A.36: Plots of the approximations obtained by NSGA-II from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

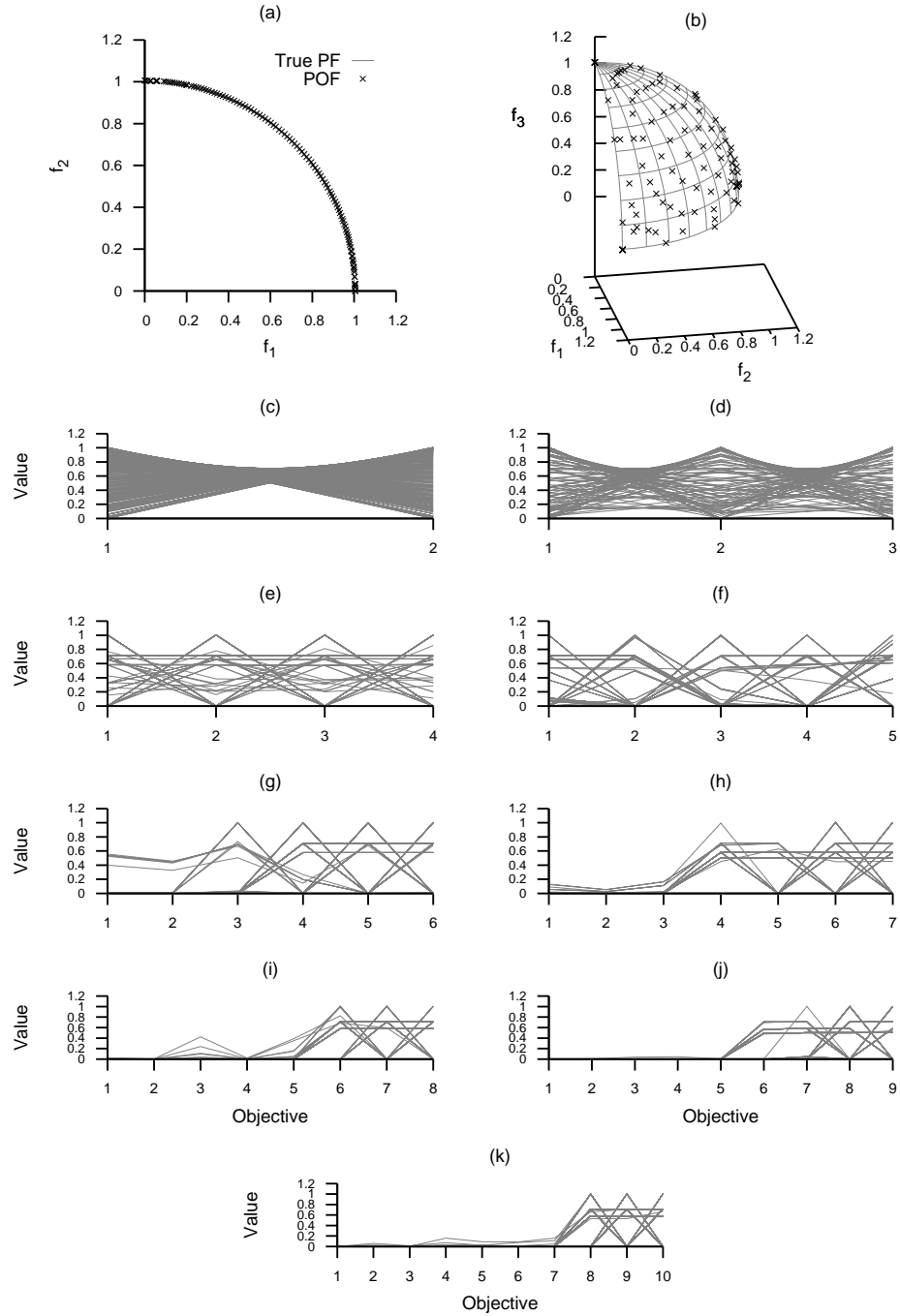


Figure A.37: Plots of the approximations obtained by MOEA/D-TCH from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

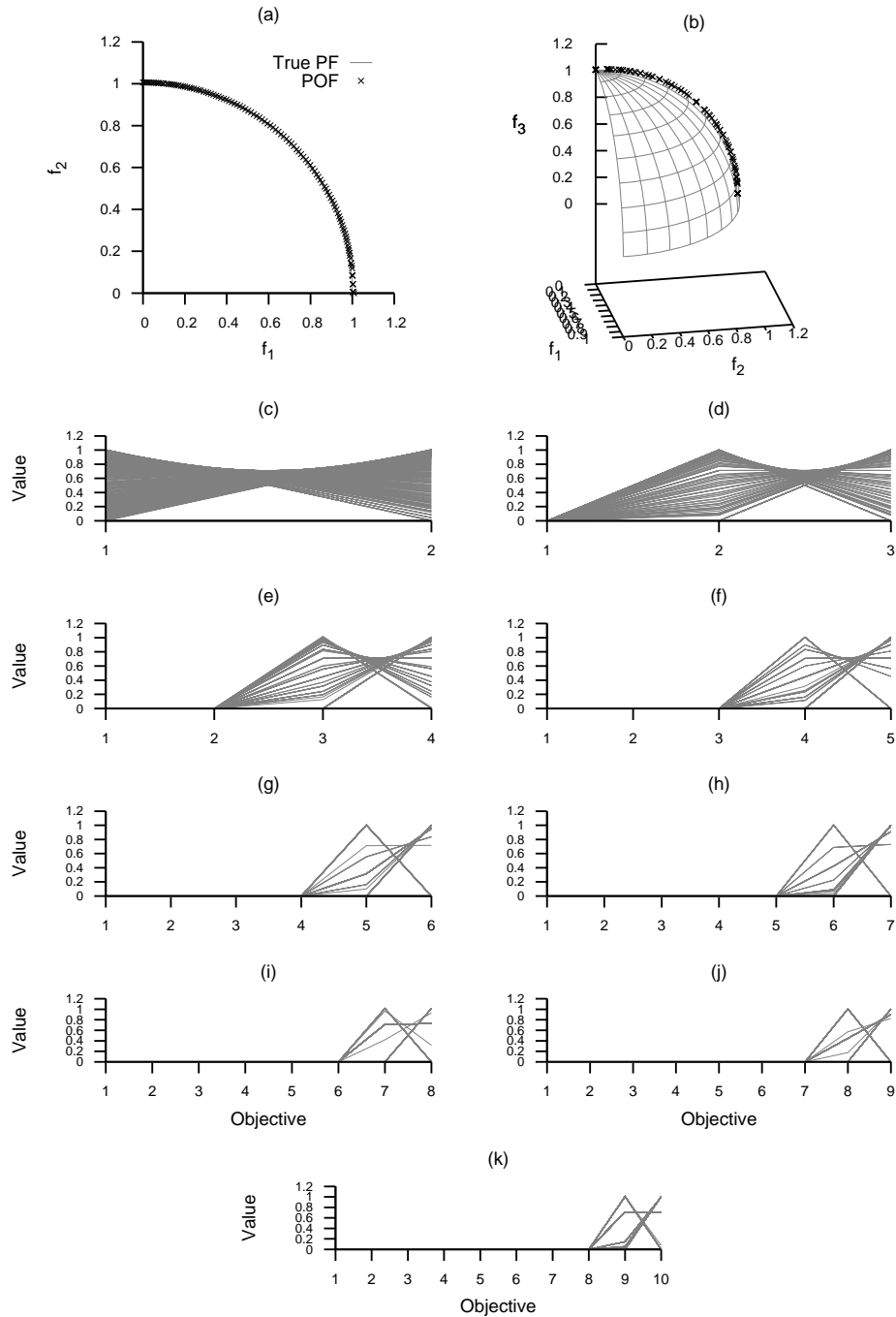


Figure A.38: Plots of the approximations obtained by MOEA/D-NTCH from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



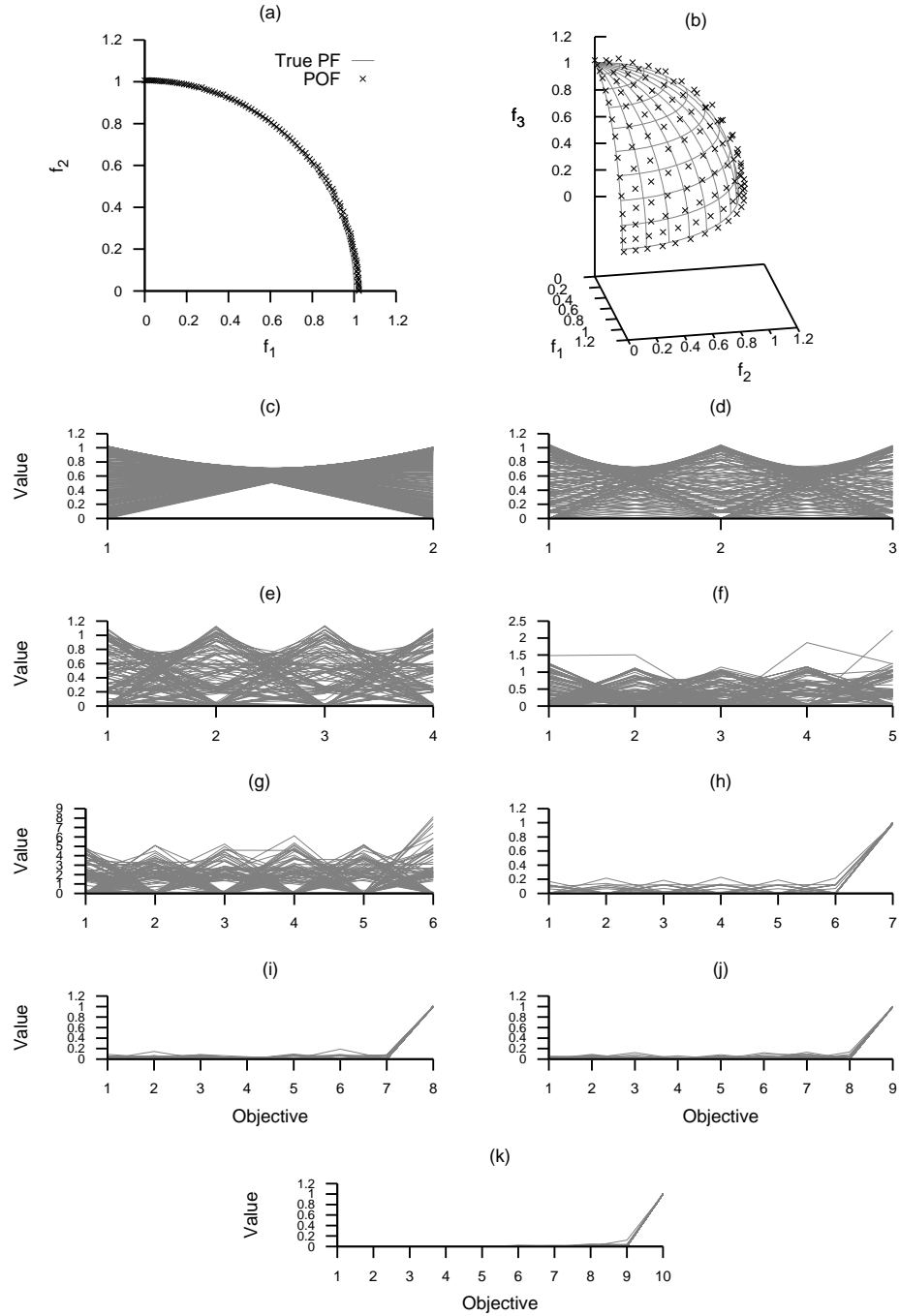


Figure A.39: Plots of the approximations obtained by MOEA/D-PBI from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

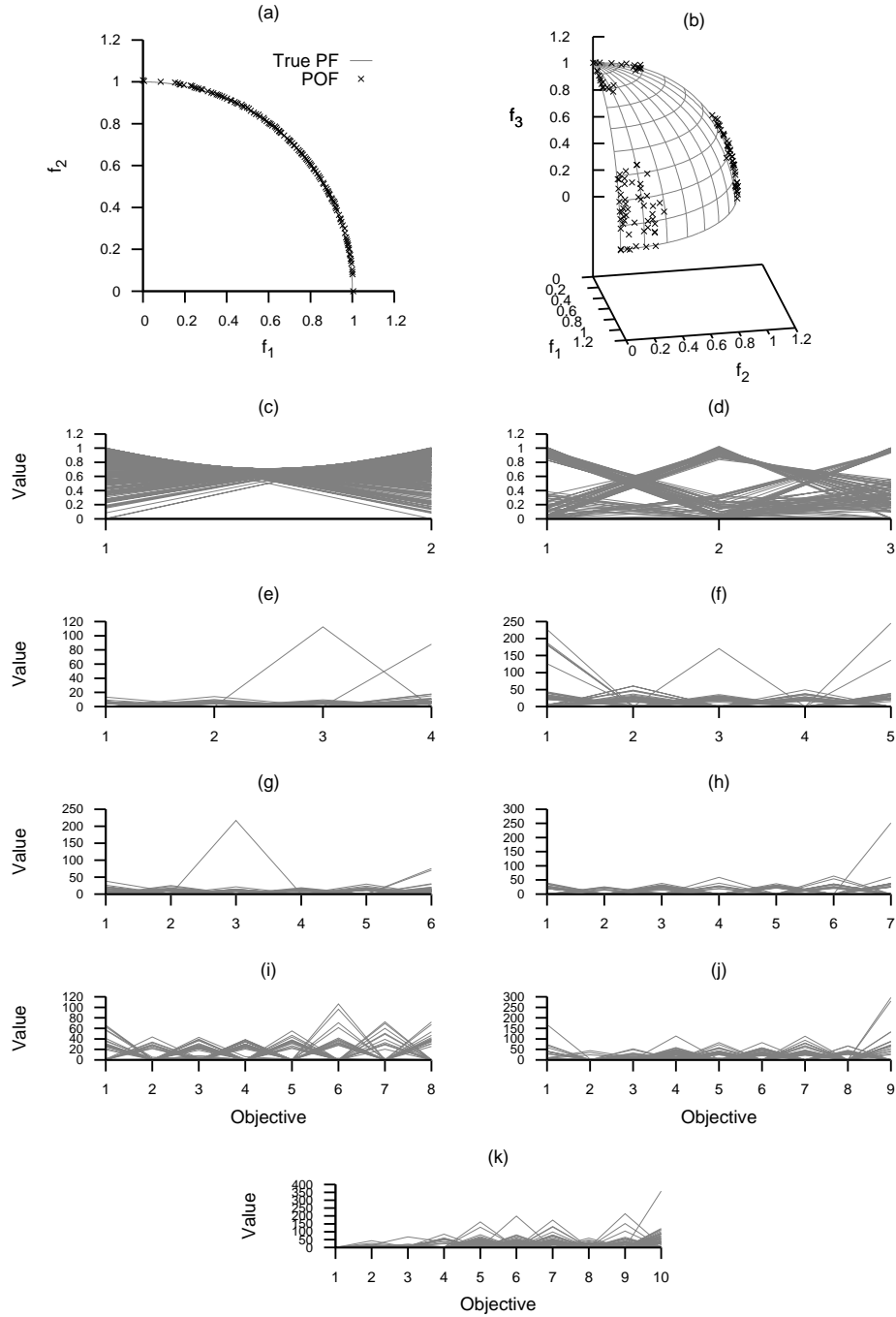


Figure A.40: Plots of the approximations obtained by SMS-EMOA from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

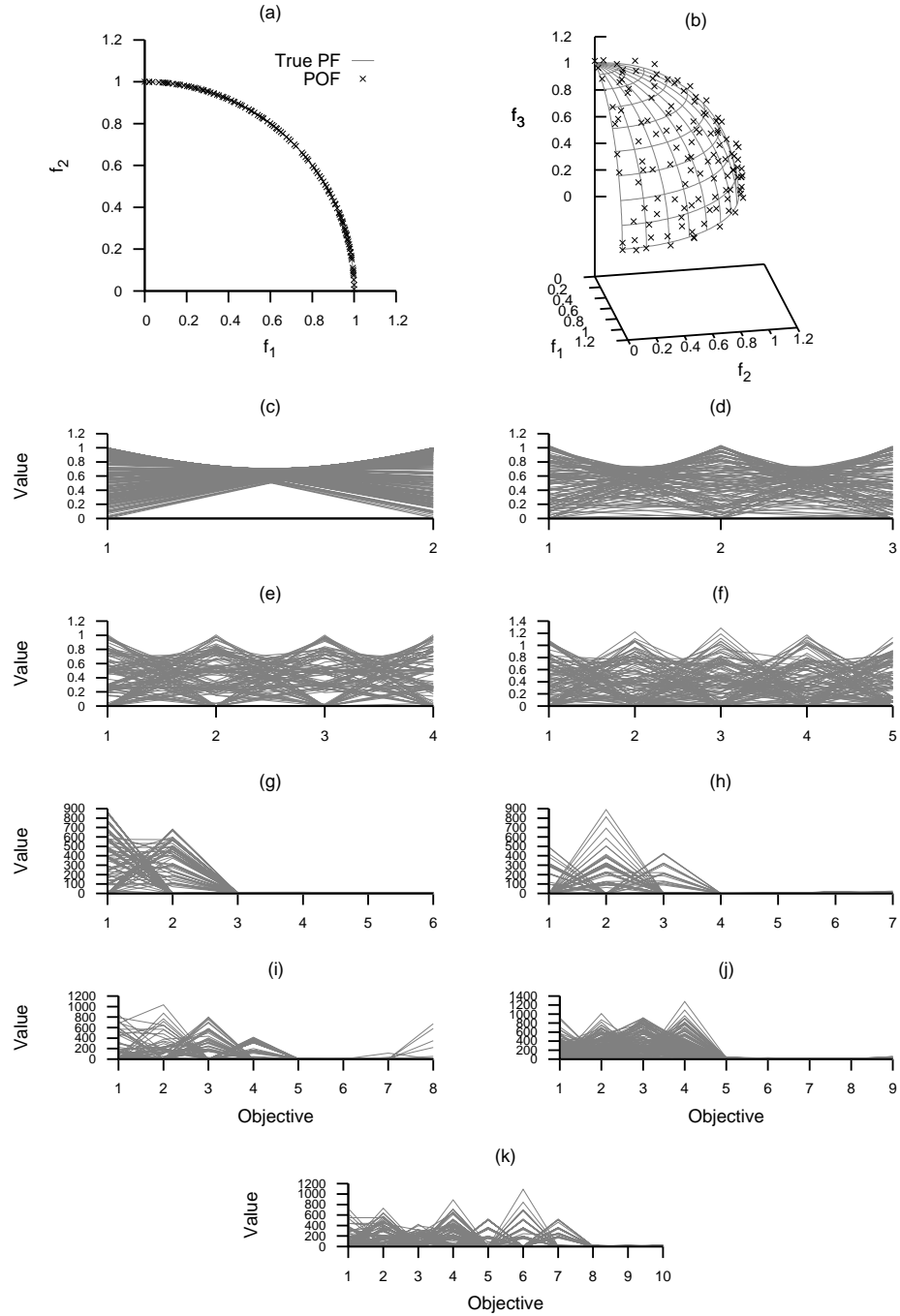


Figure A.41: Plots of the approximations obtained by  $\Delta_p$ -DDE from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

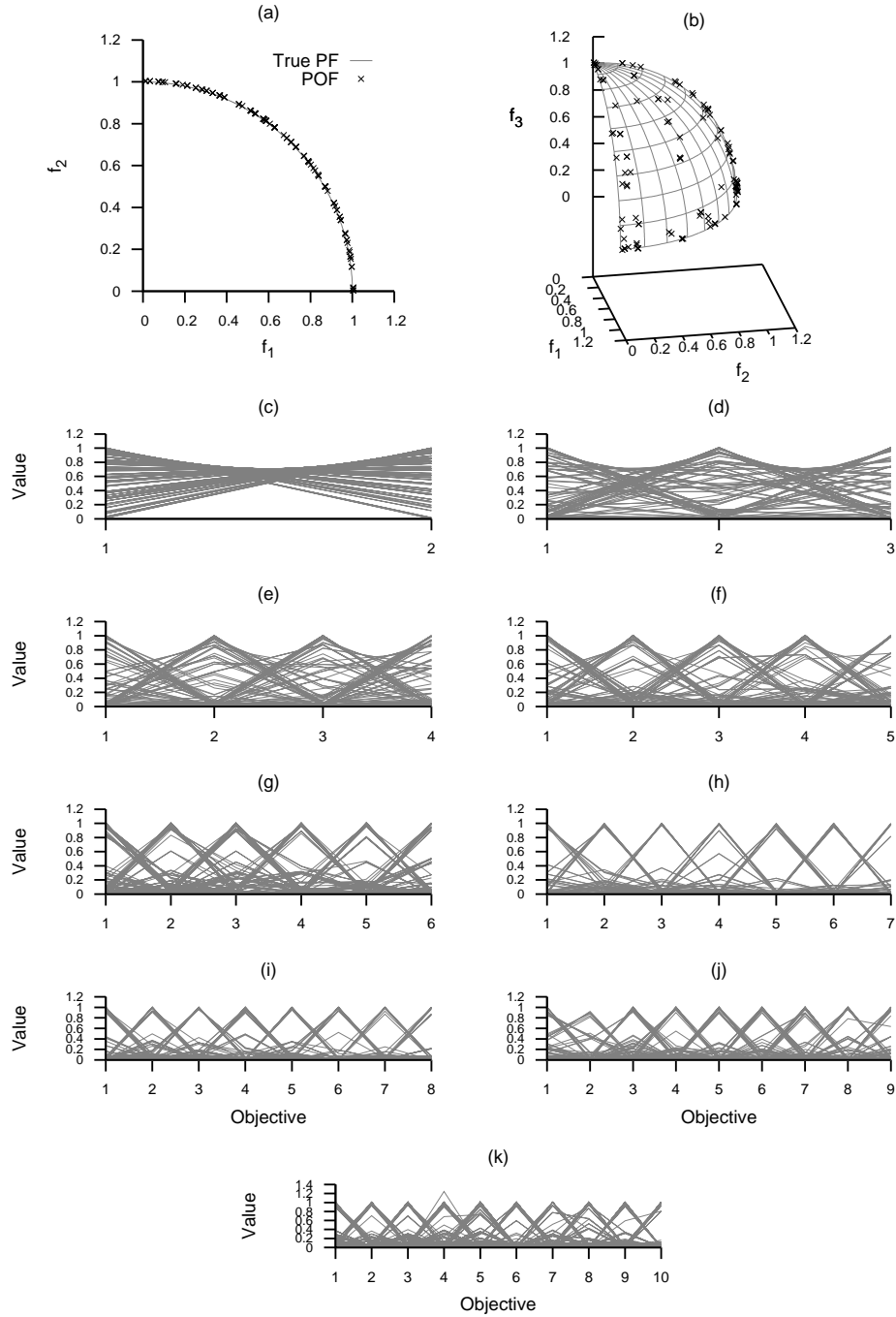


Figure A.42: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

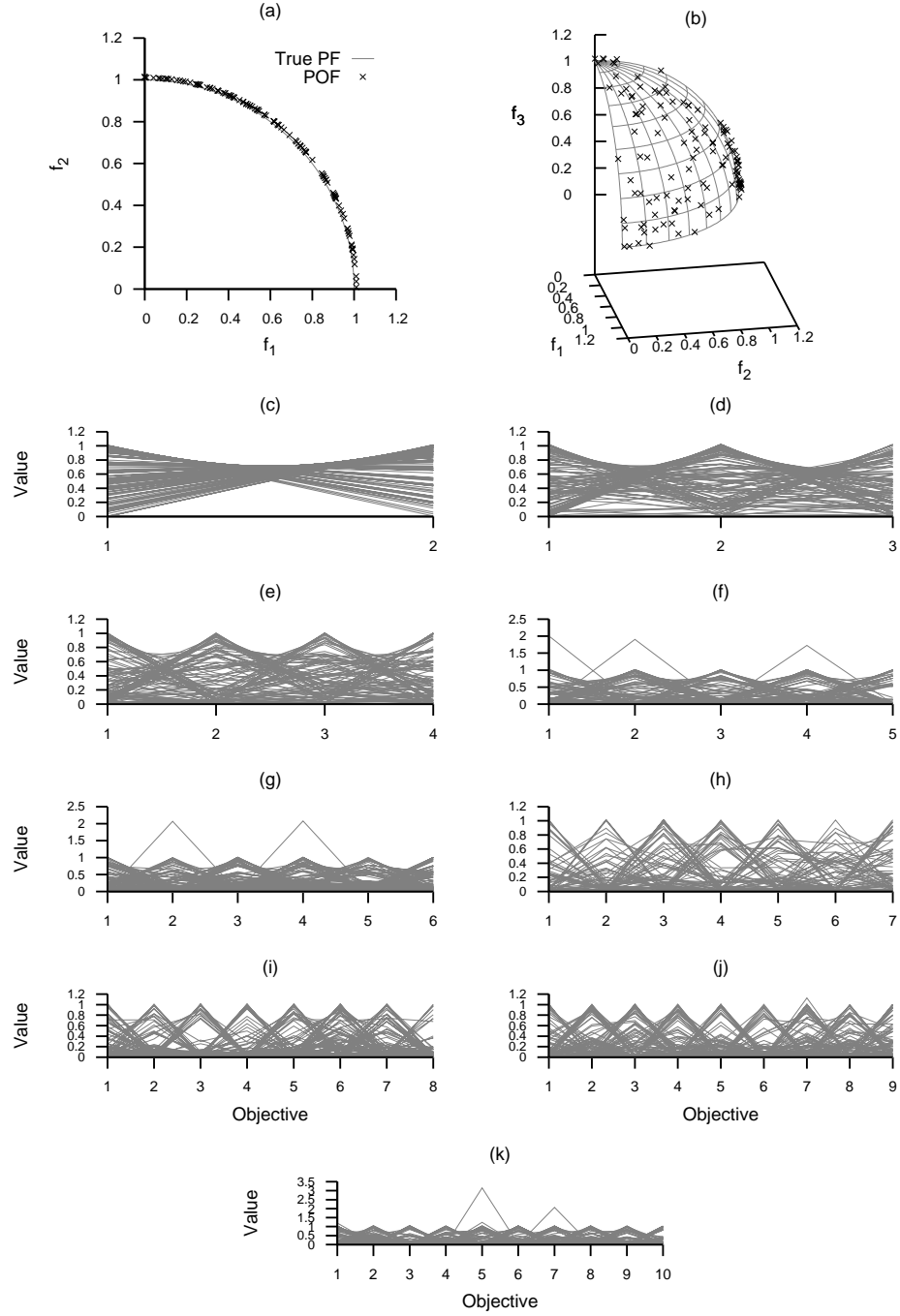


Figure A.43: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

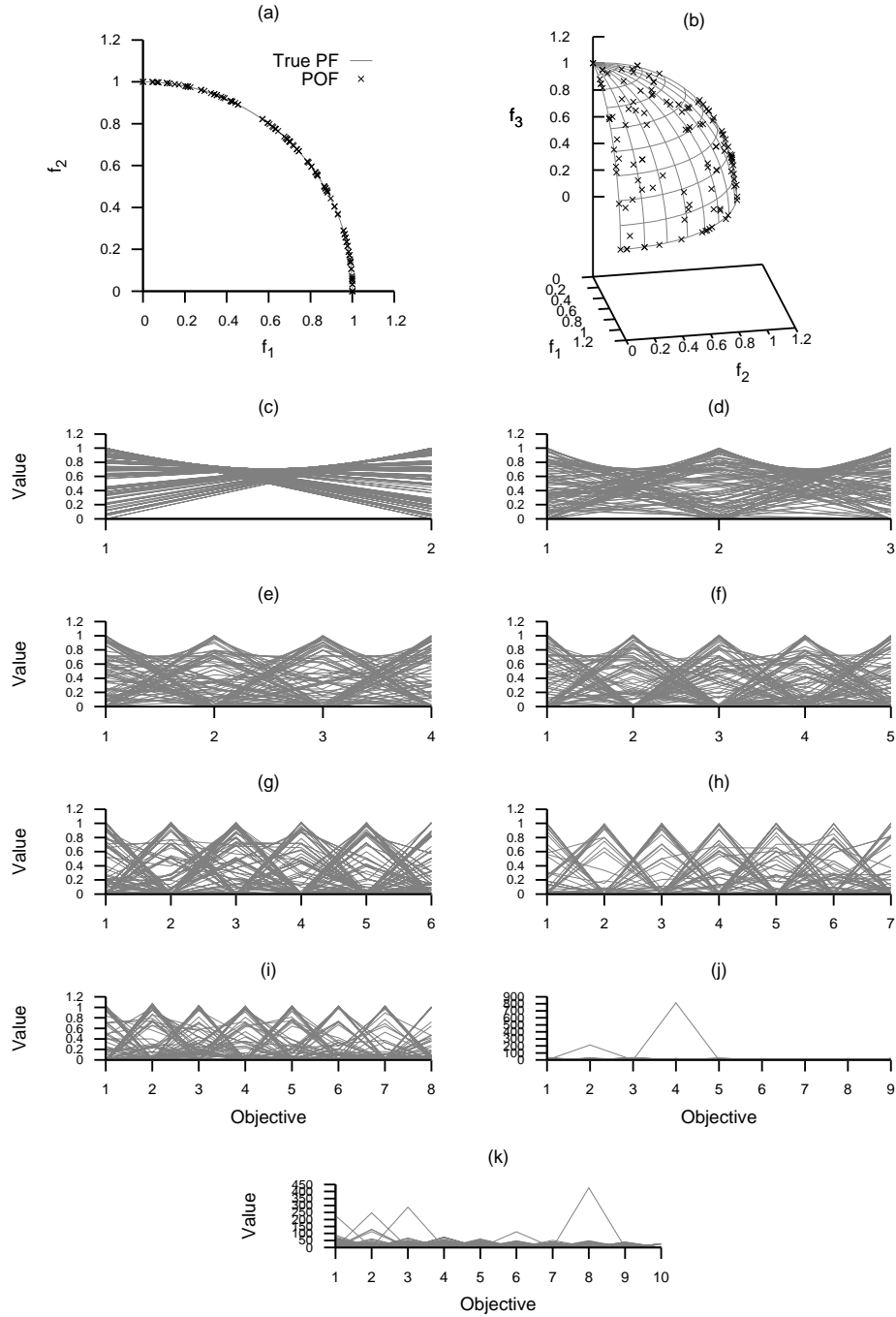


Figure A.44: Plots of the approximations obtained by *R2-MODE* from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

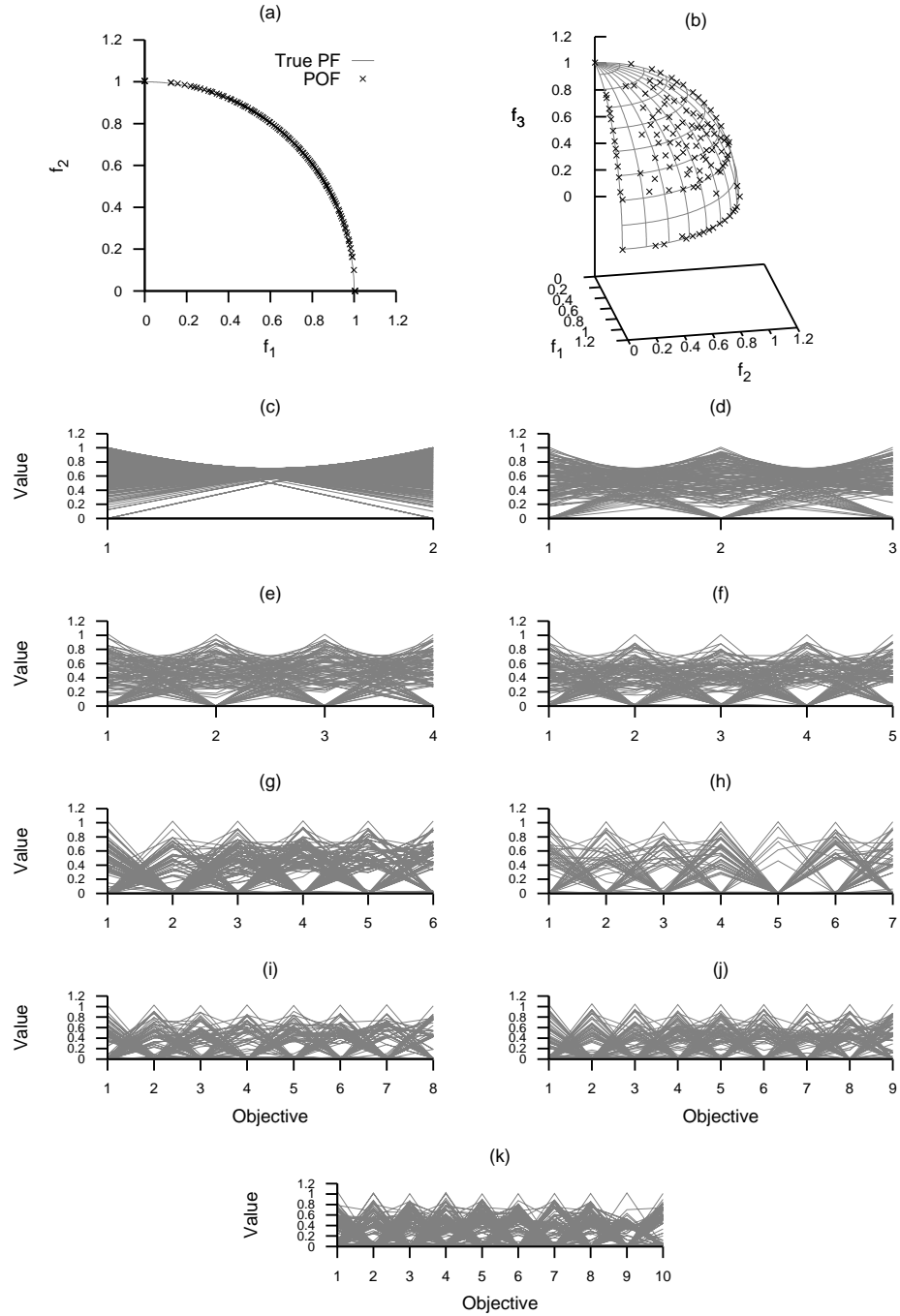


Figure A.45: Plots of the approximations obtained by *R2*-IBEA from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

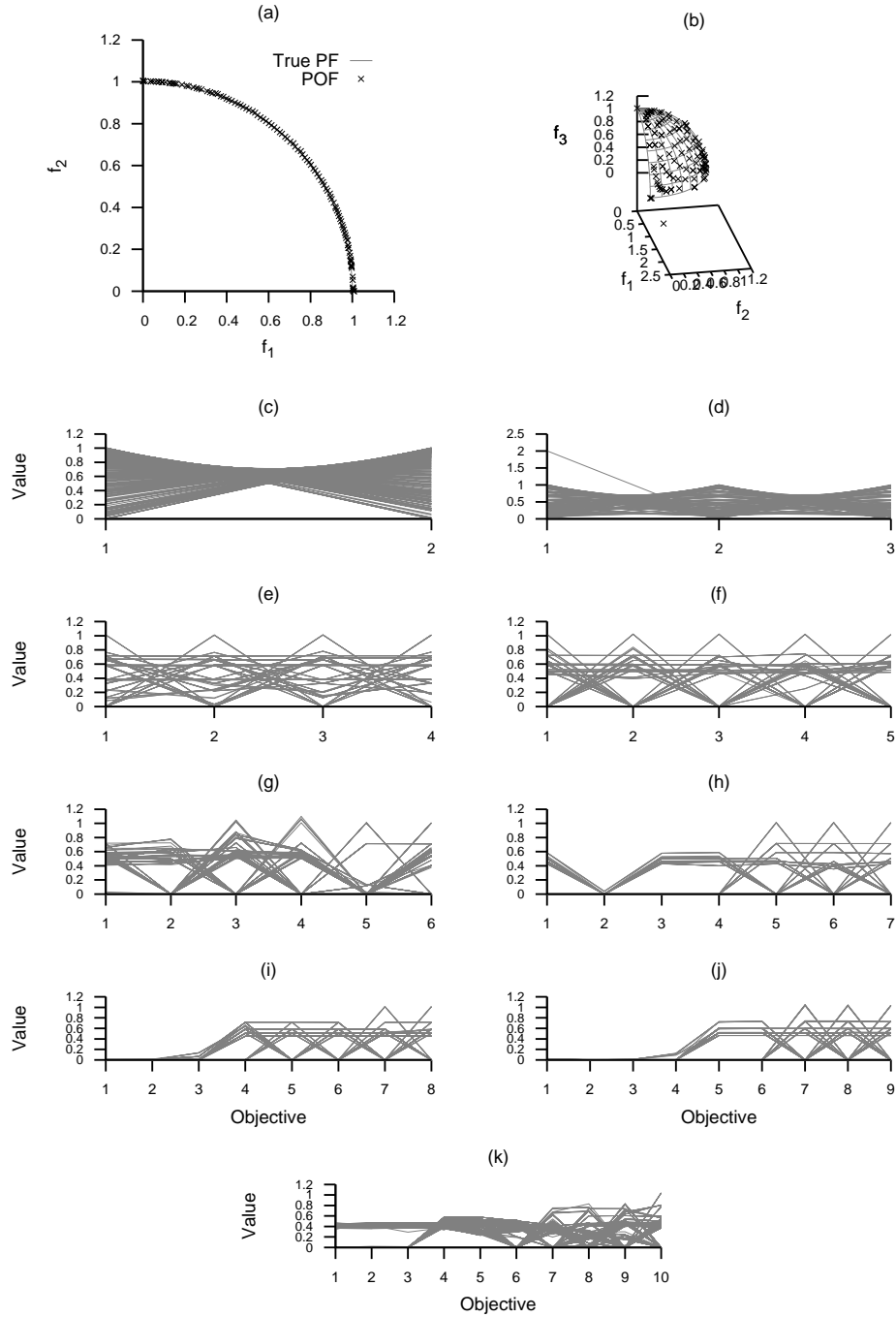


Figure A.46: Plots of the approximations obtained by MOMBI-TCH from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



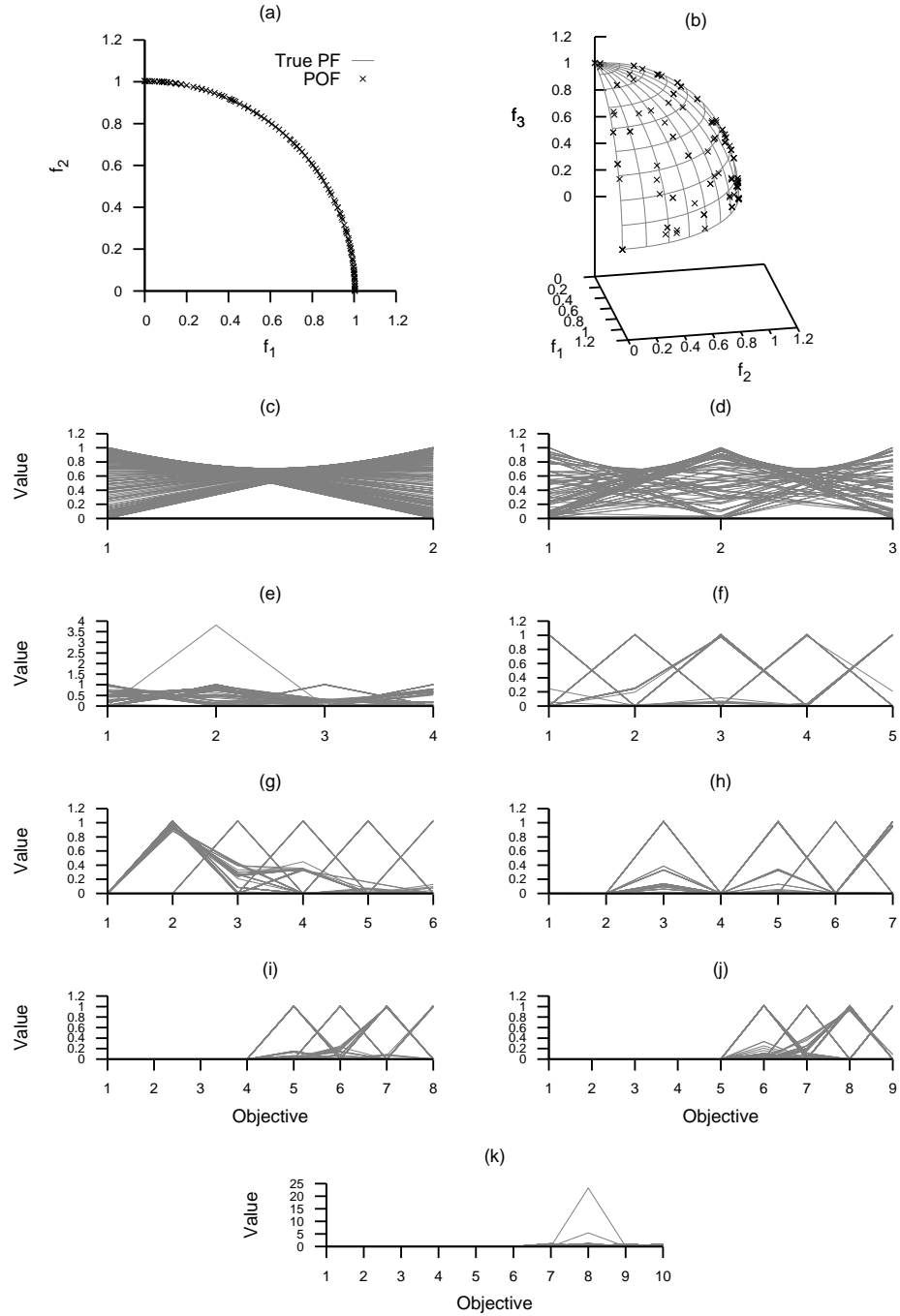


Figure A.47: Plots of the approximations obtained by MOMBI-NTCH from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

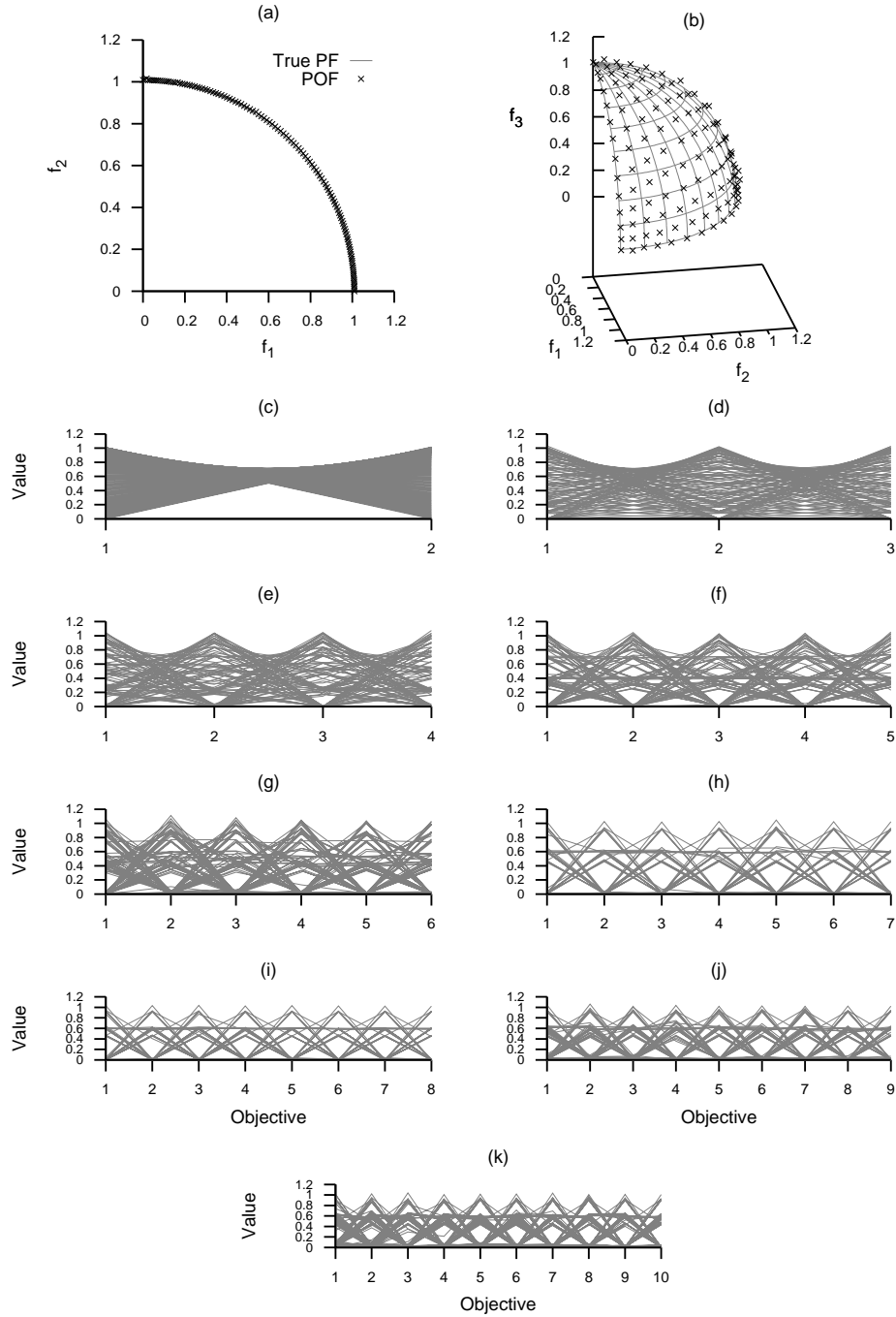


Figure A.48: Plots of the approximations obtained by MOMBI-PBI from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.20: Comparison of hypervolume indicator values for different optimizers on the DTLZ4 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	3.210149e+00	3.210860e+00	1.999708e+00	3.210851e+00	3.211608e+00	3.201619e+00	3.198168e+00	3.195479e+00	3.201593e+00	3.211102e+00	3.210831e+00	3.209385e+00	3.210601e+00
	avg.	3.210122e+00	2.702303e+00	2.396899e+00	2.739264e+00	2.969280e+00	3.199920e+00	3.195330e+00	3.191610e+00	3.198665e+00	3.090001e+00	3.210826e+00	3.161013e+00	3.198485e+00
	min.	3.209658e+00	2.000000e+00	1.671832e+00	2.000000e+00	2.000000e+00	3.149873e+00	3.162008e+00	3.147484e+00	3.152955e+00	2.000000e+00	3.210672e+00	2.000000e+00	2.000000e+00
	max.	3.210479e+00	3.210875e+00	3.210869e+00	3.210876e+00	3.211620e+00	3.210550e+00	3.204615e+00	3.208349e+00	3.206501e+00	3.211253e+00	3.210875e+00	3.210033e+00	3.210844e+00
	std.	1.853330e-04	6.006445e-01	5.882748e-01	5.927049e-01	4.870815e-01	1.100418e-02	8.890213e-03	1.182593e-02	8.803606e-03	3.651640e-01	3.962216e-05	3.281848e-01	1.210591e-01
3D	med.	7.277431e+00	7.383355e+00	6.409801e+00	6.412261e+00	7.431505e+00	7.413040e+00	7.378895e+00	7.348439e+00	7.387845e+00	7.427891e+00	7.384547e+00	7.391404e+00	7.421943e+00
	avg.	7.173554e+00	6.407321e+00	5.359066e+00	6.400387e+00	6.923149e+00	7.386701e+00	7.358296e+00	7.278159e+00	7.386808e+00	7.309029e+00	7.375186e+00	7.264244e+00	7.421934e+00
	min.	6.419276e+00	4.000000e+00	3.983272e+00	4.000000e+00	4.000000e+00	7.270192e+00	6.393456e+00	6.321684e+00	7.366738e+00	4.000000e+00	6.411823e+00	6.407630e+00	7.421693e+00
	max.	7.389143e+00	7.397226e+00	7.382987e+00	7.422278e+00	7.431620e+00	7.416357e+00	7.394331e+00	7.383941e+00	7.397521e+00	7.428706e+00	7.388772e+00	7.397662e+00	7.422126e+00
	std.	2.827282e-01	1.345481e+00	1.246715e+00	1.318655e+00	8.721746e-01	3.794521e-02	1.381646e-01	2.484379e-01	6.101679e-03	5.236683e-01	9.732370e-02	3.321381e-01	9.456045e-05
4D	med.	1.502053e+01	1.465528e+01	1.274663e+01	1.482292e+01	1.224131e+01	1.556324e+01	1.549269e+01	1.545411e+01	1.551190e+01	1.557719e+01	1.542131e+01	1.542161e+01	1.556774e+01
	avg.	1.498084e+01	1.386932e+01	1.160564e+01	1.375895e+01	1.231870e+01	1.556109e+01	1.546275e+01	1.540696e+01	1.551077e+01	1.554099e+01	1.536473e+01	1.525327e+01	1.556028e+01
	min.	1.447353e+01	8.000000e+00	7.999968e+00	8.000000e+00	1.196911e+01	1.554723e+01	1.474858e+01	1.464395e+01	1.548646e+01	1.485645e+01	1.461214e+01	1.373597e+01	1.482287e+01
	max.	1.519870e+01	1.543181e+01	1.463081e+01	1.556821e+01	1.399390e+01	1.557086e+01	1.551459e+01	1.550128e+01	1.552811e+01	1.557910e+01	1.542950e+01	1.542837e+01	1.556794e+01
	std.	1.749098e-01	2.084705e+00	2.491119e+00	2.970671e-01	1.444305e-01	5.811339e-03	1.817467e-01	1.817467e-01	9.567039e-03	1.577170e-01	1.775341e-01	3.356692e-01	7.448562e-02
5D	med.	3.028211e+01	3.071708e+01	2.535720e+01	3.112059e+01	2.475294e+01	3.165155e+01	3.157379e+01	3.154159e+01	3.158805e+01	3.167999e+01	3.152377e+01	3.151715e+01	3.166883e+01
	avg.	3.025828e+01	2.944722e+01	2.404397e+01	2.947164e+01	2.480758e+01	3.161426e+01	3.152457e+01	3.149010e+01	3.157764e+01	3.164863e+01	3.142473e+01	3.131007e+01	3.166883e+01
	min.	2.936246e+01	1.600000e+01	1.600000e+01	1.600000e+01	2.395866e+01	2.919050e+01	2.952463e+01	3.084945e+01	3.057649e+01	3.116446e+01	3.011126e+01	3.009258e+01	3.166828e+01
	max.	3.084981e+01	3.153403e+01	2.902022e+01	3.166948e+01	2.735323e+01	3.167397e+01	3.160514e+01	3.158061e+01	3.161411e+01	3.168399e+01	3.153602e+01	3.153664e+01	3.166923e+01
	std.	2.653464e-01	2.997700e+00	3.812113e+00	3.361294e+00	5.199166e-01	2.507203e-01	2.396994e-01	1.624858e-01	1.017825e-01	1.227367e-01	2.511012e-01	3.391787e-01	1.961366e-04
6D	med.	5.739084e+01	6.153080e+01	5.032599e+01	6.209191e+01	5.328989e+01	6.370963e+01	6.360839e+01	6.358864e+01	6.361504e+01	6.375348e+01	6.290226e+01	6.297911e+01	6.374110e+01
	avg.	5.721177e+01	5.925554e+01	4.898291e+01	6.046982e+01	5.209563e+01	6.368325e+01	6.356596e+01	6.352445e+01	6.351476e+01	6.372305e+01	6.290487e+01	6.270649e+01	6.374002e+01
	min.	5.236490e+01	3.200000e+01	3.200000e+01	3.200000e+01	4.822283e+01	6.195952e+01	6.199693e+01	6.191744e+01	6.074476e+01	6.336856e+01	6.225694e+01	5.794332e+01	6.371496e+01
	max.	6.062179e+01	6.312065e+01	6.080083e+01	6.374193e+01	5.531635e+01	6.373619e+01	6.365563e+01	6.365519e+01	6.366653e+01	6.375749e+01	6.312389e+01	6.307829e+01	6.374149e+01
	std.	1.758783e+00	7.072223e+00	7.608370e+00	6.741727e+00	2.506627e+00	1.825711e-01	1.944233e-01	2.228918e-01	4.001178e-01	1.034776e-01	1.863104e-01	6.967011e-01	4.224970e-03
7D	med.	3.726546e+01	1.225806e+02	1.000291e+02	1.263956e+02	9.948648e+01	1.270706e+02	1.274621e+02	1.273411e+02	1.274318e+02	1.277739e+02	1.210226e+02	1.221638e+02	1.277534e+02
	avg.	3.960844e+01	1.189622e+02	1.010195e+02	1.218551e+02	9.958144e+01	1.267717e+02	1.271365e+02	1.272256e+02	1.259330e+02	1.276300e+02	1.210806e+02	1.217699e+02	1.272956e+02
	min.	1.268244e+01	6.400000e+01	6.399830e+01	6.400000e+01	7.740984e+01	1.207549e+02	1.170288e+02	1.255314e+02	9.632143e+01	1.265529e+02	1.193990e+02	1.089168e+02	1.273791e+02
	max.	7.404039e+01	1.234743e+02	1.219218e+02	1.277564e+02	1.076220e+02	1.277477e+02	1.275627e+02	1.275759e+02	1.275590e+02	1.277802e+02	1.234812e+02	1.231732e+02	1.277542e+02
	std.	1.366799e+01	1.091385e+01	1.543154e+01	1.441631e+01	5.080616e+00	9.354019e-01	1.136780e+00	3.061829e-01	4.449080e+00	2.540410e-01	9.926296e-01	2.025386e+00	8.329525e-02
8D	med.	3.063069e+01	2.445554e+02	2.002306e+02	2.527888e+02	2.219828e+02	2.553495e+02	2.556052e+02	2.556407e+02	2.552505e+02	2.558344e+02	2.393977e+02	2.439254e+02	2.558094e+02
	avg.	3.508411e+01	2.424790e+02	2.093058e+02	2.467548e+02	2.210593e+02	2.550720e+02	2.555434e+02	2.555261e+02	2.533218e+02	2.557864e+02	2.395485e+02	2.432014e+02	2.557819e+02
	min.	8.595970e+00	1.993214e+02	1.280000e+02	1.280000e+02	1.966670e+02	2.484290e+02	2.549959e+02	2.562925e+02	2.248775e+02	2.550271e+02	2.348143e+02	2.278456e+02	2.552753e+02
	max.	1.111926e+02	2.469791e+02	2.429424e+02	2.558267e+02	2.294180e+02	2.557566e+02	2.556891e+02	2.557270e+02	2.555971e+02	2.558414e+02	2.464383e+02	2.462729e+02	2.558256e+02
	std.	2.012999e+01	7.566822e+00	2.361322e+01	2.096668e+01	5.436193e+00	1.121006e+00	1.526685e-01	3.687652e-01	5.960340e+00	1.182879e-01	2.435533e+00	2.783348e+00	7.755240e-02
9D	med.	8.104468e+01	4.890328e+02	4.453757e+02	5.085828e+02	4.728898e+02	5.108284e+02	5.116154e+02	5.117584e+02	5.105300e+02	5.118498e+02	4.745343e+02	4.850100e+02	5.117262e+02
	avg.	8.866927e+01	4.870021e+02	4.303082e+02	5.043512e+02	4.713741e+02	5.104330e+02	5.115577e+02	5.117049e+02	5.015850e+02	5.117757e+02	4.747901e+02	4.830957e+02	5.116711e+02
	min.	1.365448e+01	4.588531e+02	2.560000e+02	4.533062e+02	4.330758e+02	5.003058e+02	5.109369e+02	5.107048e+02	3.626788e+02	5.106257e+02	4.512683e+02	4.395552e+02	5.111166e+02
	max.	2.378389e+02	4.939586e+02	4.900513e+02	5.118650e+02	5.118513e+02	5.116471e+02	5.117740e+02	5.118092e+02	5.114906e+02	5.118911e+02	4.900741e+02	4.925824e+02	5.118768e+02
	std.	4.858997e+01	6.967582e+00	4.452209e+01	1.143426e+01	1.041781e+01	1.518490e+00	1.771542e-01	1.475158e-01	2.761388e+01	2.110266e-01	6.762019e+00	8.245416e+00	1.752481e-01
10D	med.	5.695858e+02	9.797368e+02	9.175904e+02	1.019569e+03	1.023838e+03	1.023763e+03	1.023772e+03	1.023841e+03	1.023780e+03	1.023920e+03	9.429039e+02	9.703514e+02	1.023911e+03
	avg.	5.764488e+02	9.785347e+02	9.014572e+02	1.017506e+03	1.009283e+03	1.023763e+03	1.023764e+03	1.023830e+03	1.023780e+03	1.023918e+03	9.442357e+02	9.708227e+02	1.023910e+03
	min.	4.291681e+02	9.588862e+02	9.29478e+02	9.361663e+02	9.705047e+02	1.023627e+03	1.023525e+03	1.023587e+03	1.023737e+03	1.023814e+03	9.235966e+02	9.526820e+02	1.023904e+03
	max.	8.044108e+02	9.899378e+02	9.789942e+02	1.023813e+03	1.023904e+03	1.023874e+03	1.023818e+03	1.023864e+03	1.023819e+03	1.023925e+03	9.722101e+02	9.925361e+02	1.023913e+03
	std.	7.907505e+01	6.513080e+00	6.128623e+01	1.117045e+01	2.047838e+01	6.334860e-02	4.902870e-02	4.569562e-02	1.830458e-02	1.495436e-02	1.204980e+01	7.530500e+00	1.812367e-03

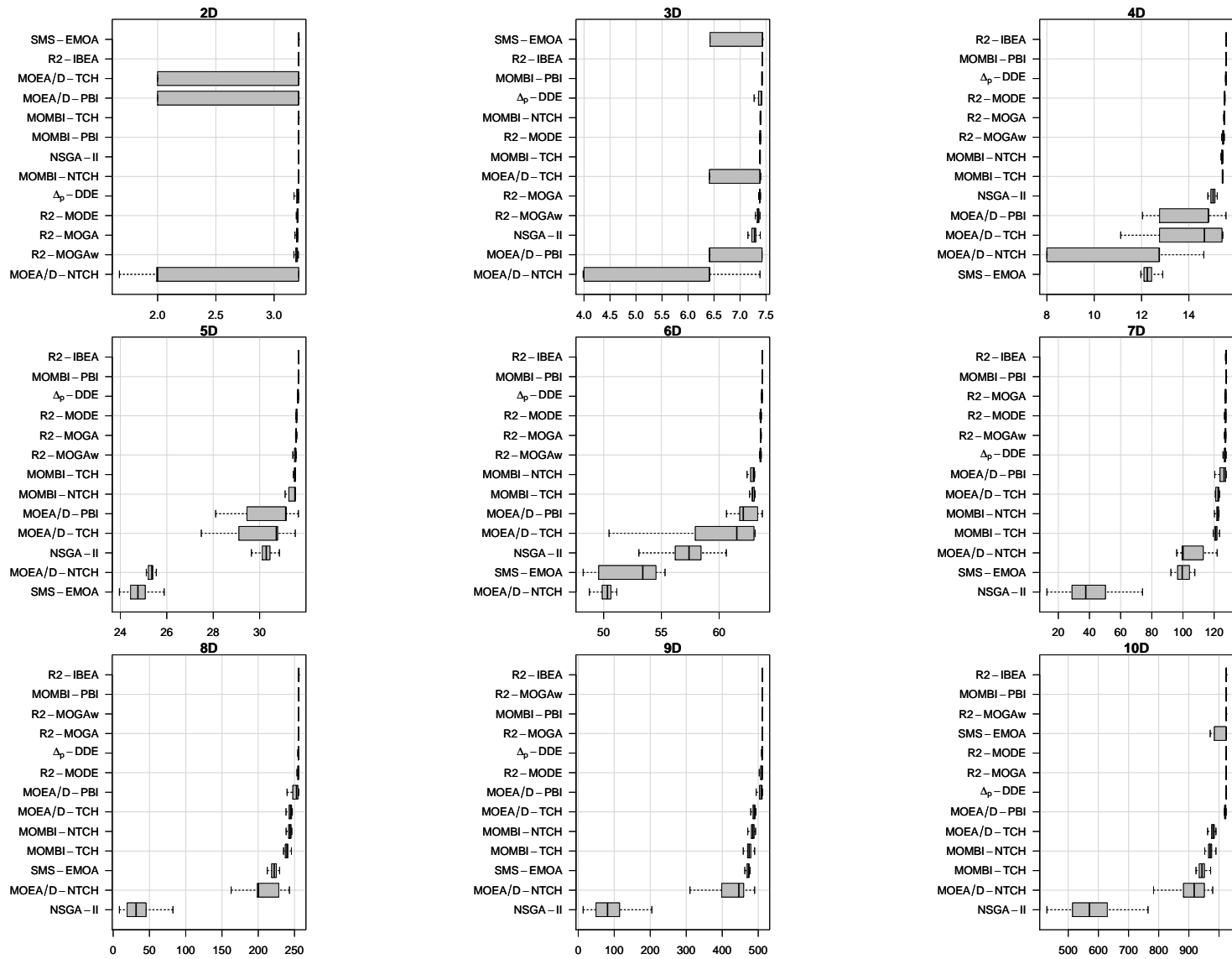


Figure A.49: Box-plot of hypervolume indicator values for different optimizers on the DTLZ4 test problem.

Table A.21: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ4 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D													
NSGA-II	—	> 0.05	3.32e-08	> 0.05	> 0.05	9.61e-11	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	1.47e-33	> 0.05
MOEA/D-TCH	2.48e-02	—	3.27e-16	> 0.05	> 0.05	2.48e-02	2.48e-02	2.48e-02	2.48e-02	> 0.05	> 0.05	1.49e-02	2.25e-02
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	7.13e-03	> 0.05	1.40e-16	—	> 0.05	4.66e-03	3.51e-03	3.59e-03	3.54e-03	> 0.05	> 0.05	3.91e-03	9.62e-03
SMS-EMOA	1.12e-13	1.09e-17	4.21e-25	3.11e-17	—	1.12e-13	1.12e-13	1.12e-13	1.12e-13	1.64e-14	1.12e-13	5.28e-14	9.31e-14
$\Delta_p$ -DDE	> 0.05	> 0.05	1.10e-06	> 0.05	> 0.05	—	1.30e-04	1.10e-07	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	> 0.05	> 0.05	4.64e-05	> 0.05	> 0.05	> 0.05	—	1.05e-02	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	> 0.05	> 0.05	4.64e-05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	> 0.05	> 0.05	4.64e-05	> 0.05	> 0.05	> 0.05	7.42e-06	1.23e-08	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	7.35e-23	1.56e-25	1.21e-29	3.22e-25	> 0.05	7.35e-23	7.35e-23	7.35e-23	7.35e-23	—	7.34e-23	4.48e-23	6.50e-23
MOMBI-TCH	1.28e-34	> 0.05	8.58e-30	> 0.05	> 0.05	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	—	1.28e-34	2.18e-32
MOMBI-NTCH	> 0.05	> 0.05	5.17e-05	> 0.05	> 0.05	1.18e-03	1.32e-29	1.32e-29	1.32e-29	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	1.80e-32	> 0.05	2.21e-20	> 0.05	> 0.05	1.69e-31	2.51e-33	2.51e-33	2.51e-33	> 0.05	> 0.05	2.36e-33	—
3D													
NSGA-II	—	> 0.05	6.97e-33	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	> 0.05	—	1.75e-16	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	1.61e-04	6.52e-17	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	8.57e-06	1.67e-12	5.57e-28	1.23e-13	—	3.14e-04	1.81e-04	4.20e-05	3.14e-04	3.95e-05	2.39e-04	6.14e-06	3.14e-04
$\Delta_p$ -DDE	5.74e-31	8.47e-13	2.67e-34	> 0.05	> 0.05	—	1.01e-04	1.35e-14	2.27e-03	> 0.05	2.56e-03	6.28e-05	> 0.05
R2-MOGA	7.61e-31	> 0.05	1.78e-32	> 0.05	> 0.05	> 0.05	—	1.40e-25	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	8.62e-20	> 0.05	5.96e-29	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	7.46e-34	1.20e-13	1.65e-34	> 0.05	> 0.05	> 0.05	3.33e-16	2.04e-33	—	> 0.05	4.88e-07	> 0.05	> 0.05
R2-IBEA	8.54e-27	4.44e-29	1.07e-32	2.33e-29	> 0.05	4.08e-26	3.15e-26	1.64e-26	4.08e-26	—	3.58e-26	7.48e-27	4.08e-26
MOMBI-TCH	4.50e-32	6.62e-07	1.38e-34	> 0.05	> 0.05	> 0.05	8.35e-14	4.93e-33	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	9.64e-20	1.57e-17	4.69e-27	> 0.05	> 0.05	> 0.05	1.74e-18	1.24e-20	2.24e-06	> 0.05	3.63e-19	—	> 0.05
MOMBI-PBI	1.28e-34	1.16e-34	8.24e-35	2.50e-02	> 0.05	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.28e-34	—
4D													
NSGA-II	—	1.47e-06	2.18e-34	1.21e-06	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	> 0.05	—	9.28e-21	> 0.05	4.09e-21	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	1.16e-05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	5.32e-04	1.47e-16	—	2.59e-16	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e-34	1.27e-34	1.12e-34	5.79e-13	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.28e-34	> 0.05
R2-MOGA	3.89e-30	2.89e-33	1.13e-34	1.84e-09	1.28e-34	> 0.05	—	1.64e-23	> 0.05	> 0.05	7.58e-30	1.77e-30	> 0.05
R2-MOGAw	5.11e-28	3.48e-28	1.13e-34	8.31e-09	1.28e-34	> 0.05	> 0.05	—	> 0.05	> 0.05	6.58e-17	1.85e-18	> 0.05
R2-MODE	1.28e-34	1.27e-34	1.12e-34	1.06e-10	1.28e-34	> 0.05	9.76e-23	4.78e-34	—	> 0.05	1.28e-34	1.28e-34	> 0.05
R2-IBEA	2.63e-29	6.01e-33	1.12e-34	4.40e-33	1.28e-34	2.04e-28	1.19e-28	9.03e-29	2.04e-28	—	1.03e-28	1.52e-29	1.77e-28
MOMBI-TCH	2.53e-28	9.73e-12	1.16e-34	4.36e-09	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.36e-15	1.28e-10	1.43e-34	1.35e-05	1.32e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	1.64e-33	2.74e-34	1.11e-34	2.29e-15	1.27e-34	9.13e-28	2.21e-33	2.08e-33	2.48e-33	> 0.05	2.14e-33	1.46e-33	—

Table A.22: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ4 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
5D													
NSGA-II	—	> 0.05	1.22e-34	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	3.48e-03	—	2.12e-27	> 0.05	7.67e-29	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	5.25e-07	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	1.34e-02	7.78e-27	—	2.99e-26	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	2.51e-33	1.29e-32	1.22e-34	2.56e-18	1.28e-34	—	6.30e-25	1.44e-26	1.49e-23	> 0.05	9.46e-28	5.68e-29	> 0.05
R2-MOGA	2.44e-33	1.63e-32	1.22e-34	1.07e-15	1.28e-34	> 0.05	—	1.88e-19	> 0.05	> 0.05	1.02e-27	1.56e-28	> 0.05
R2-MOGAw	1.32e-34	1.14e-27	1.22e-34	1.82e-15	1.28e-34	> 0.05	> 0.05	—	> 0.05	> 0.05	9.99e-07	4.15e-10	> 0.05
R2-MODE	1.78e-34	7.46e-34	1.22e-34	1.42e-17	1.28e-34	> 0.05	3.66e-12	2.55e-31	—	> 0.05	2.36e-33	2.36e-33	> 0.05
R2-IBEA	1.28e-34	4.51e-33	1.22e-34	2.90e-33	1.28e-34	2.69e-26	1.14e-27	9.72e-28	2.54e-27	—	6.00e-28	6.16e-29	4.06e-26
MOMBI-TCH	2.29e-33	6.10e-23	1.22e-34	1.72e-14	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	9.95e-33	1.26e-19	1.22e-34	2.49e-10	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	1.27e-34	1.27e-34	1.22e-34	2.41e-20	1.27e-34	4.06e-26	1.27e-34	1.27e-34	1.27e-34	> 0.05	1.27e-34	1.27e-34	—
6D													
NSGA-II	—	> 0.05	7.87e-23	> 0.05	2.56e-29	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	7.75e-14	—	1.64e-23	> 0.05	3.42e-22	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.49e-22	2.12e-05	3.38e-27	—	5.56e-26	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e-34	5.38e-34	1.26e-34	3.06e-24	1.28e-34	—	7.59e-30	3.11e-30	7.38e-30	> 0.05	2.51e-33	1.71e-33	> 0.05
R2-MOGA	1.28e-34	5.88e-34	1.26e-34	4.77e-22	1.28e-34	> 0.05	—	2.64e-05	> 0.05	> 0.05	4.02e-33	1.71e-33	> 0.05
R2-MOGAw	1.28e-34	7.65e-33	1.26e-34	7.47e-21	1.28e-34	> 0.05	> 0.05	—	> 0.05	> 0.05	3.97e-31	1.30e-31	> 0.05
R2-MODE	1.28e-34	7.83e-31	1.30e-34	1.66e-21	1.28e-34	> 0.05	> 0.05	3.25e-05	—	> 0.05	4.35e-28	1.43e-28	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.26e-34	8.65e-34	1.28e-34	3.52e-25	1.27e-25	6.84e-26	1.03e-25	—	1.28e-34	1.28e-34	5.27e-25
MOMBI-TCH	1.28e-34	1.41e-07	1.26e-34	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	3.65e-34	3.18e-07	1.30e-34	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	1.28e-34	1.28e-34	1.26e-34	2.70e-29	1.28e-34	4.01e-33	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.28e-34	—
7D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.84e-34	—	1.05e-26	> 0.05	1.83e-28	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	4.07e-07	> 0.05	> 0.05
MOEA/D-NTCH	5.32e-34	> 0.05	—	> 0.05	4.02e-04	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	2.33e-34	6.79e-20	1.24e-26	—	9.97e-27	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.59e-20	2.98e-19	> 0.05
SMS-EMOA	1.28e-34	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e-34	1.43e-33	1.34e-34	7.21e-04	1.28e-34	—	> 0.05	> 0.05	> 0.05	> 0.05	8.40e-34	1.57e-33	> 0.05
R2-MOGA	1.28e-34	1.43e-33	1.47e-34	6.81e-14	1.28e-34	3.58e-09	—	3.84e-03	> 0.05	> 0.05	2.51e-33	2.37e-33	> 0.05
R2-MOGAw	1.28e-34	1.28e-34	1.27e-34	6.27e-10	1.28e-34	7.09e-09	> 0.05	—	> 0.05	> 0.05	1.28e-34	1.28e-34	> 0.05
R2-MODE	1.28e-34	4.67e-23	4.99e-32	4.51e-09	1.24e-33	2.14e-08	> 0.05	1.23e-02	—	> 0.05	7.28e-24	5.90e-22	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.26e-34	1.08e-29	1.28e-34	1.15e-28	3.91e-14	3.13e-23	9.30e-17	—	1.28e-34	1.28e-34	4.40e-05
MOMBI-TCH	1.28e-34	> 0.05	3.53e-32	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.28e-34	> 0.05	7.34e-33	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.02e-09	—	> 0.05
MOMBI-PBI	1.24e-34	1.24e-34	1.22e-34	4.18e-26	1.24e-34	4.36e-34	1.15e-29	1.73e-32	3.78e-30	> 0.05	1.24e-34	1.24e-34	—

Table A.23: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ4 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
8D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	5.58e-30	> 0.05	4.20e-31	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	8.40e-18	5.10e-03	> 0.05
MOEA/D-NTCH	1.28e-34	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	7.71e-20	1.05e-27	—	3.15e-26	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	3.19e-22	8.97e-21	> 0.05
SMS-EMOA	1.28e-34	> 0.05	2.17e-03	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e-34	1.28e-34	1.28e-34	4.73e-19	1.28e-34	—	> 0.05	> 0.05	7.42e-04	> 0.05	1.28e-34	1.28e-34	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	2.60e-28	1.28e-34	1.12e-11	—	> 0.05	1.98e-22	> 0.05	1.28e-34	1.28e-34	> 0.05
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	5.54e-28	1.28e-34	1.96e-13	1.53e-05	—	1.64e-21	> 0.05	1.28e-34	1.28e-34	> 0.05
R2-MODE	1.28e-34	1.82e-26	7.01e-33	2.23e-10	5.38e-34	> 0.05	> 0.05	> 0.05	—	> 0.05	2.10e-28	1.33e-26	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	2.58e-33	1.28e-34	1.37e-30	3.17e-25	9.29e-23	5.22e-33	—	1.28e-34	1.28e-34	4.89e-11
MOMBI-TCH	1.28e-34	> 0.05	6.10e-23	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.28e-34	> 0.05	1.13e-30	> 0.05	1.40e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.18e-19	—	> 0.05
MOMBI-PBI	1.28e-34	1.28e-34	1.28e-34	8.71e-32	1.28e-34	2.00e-32	7.60e-31	1.79e-29	1.03e-33	> 0.05	1.28e-34	1.28e-34	—
9D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	3.68e-30	> 0.05	2.33e-24	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	5.95e-23	3.09e-08	> 0.05
MOEA/D-NTCH	1.28e-34	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	1.20e-26	5.08e-33	—	1.27e-25	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	4.71e-28	2.16e-27	> 0.05
SMS-EMOA	1.28e-34	> 0.05	3.33e-15	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e-34	1.28e-34	1.28e-34	1.97e-10	1.30e-31	—	> 0.05	> 0.05	4.22e-03	> 0.05	1.28e-34	1.28e-34	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	1.78e-28	4.63e-32	1.96e-28	—	> 0.05	3.57e-30	> 0.05	1.28e-34	1.28e-34	> 0.05
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.11e-31	4.63e-32	5.88e-33	1.13e-16	—	1.24e-33	> 0.05	1.28e-34	1.28e-34	> 0.05
R2-MODE	1.28e-34	3.61e-25	1.99e-28	1.87e-03	1.16e-23	> 0.05	> 0.05	> 0.05	—	> 0.05	4.89e-26	1.20e-25	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	3.37e-32	2.58e-33	6.54e-32	2.54e-22	7.46e-15	2.45e-32	—	1.28e-34	1.28e-34	1.21e-11
MOMBI-TCH	1.28e-34	> 0.05	9.14e-18	> 0.05	7.94e-04	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.28e-34	> 0.05	5.28e-25	> 0.05	6.88e-23	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.37e-16	—	> 0.05
MOMBI-PBI	1.28e-34	1.28e-34	1.28e-34	1.13e-30	2.45e-32	1.86e-31	4.65e-08	> 0.05	1.68e-32	> 0.05	1.28e-34	1.28e-34	—
10D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	4.08e-31	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.39e-33	1.51e-12	> 0.05
MOEA/D-NTCH	2.48e-34	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	2.50e-33	3.23e-34	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.06e-33	2.90e-33	> 0.05
SMS-EMOA	1.28e-34	3.33e-17	6.62e-34	2.38e-03	—	> 0.05	4.43e-02	> 0.05	4.90e-02	> 0.05	1.49e-34	5.84e-28	> 0.05
$\Delta_p$ -DDE	1.28e-34	1.28e-34	1.28e-34	5.40e-31	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	1.28e-34	1.28e-34	> 0.05
R2-MOGA	1.27e-34	1.27e-34	1.27e-34	1.08e-29	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	1.27e-34	1.27e-34	> 0.05
R2-MOGAw	1.27e-34	1.27e-34	1.27e-34	4.07e-34	> 0.05	4.65e-16	3.61e-28	—	2.23e-27	> 0.05	1.27e-34	1.27e-34	> 0.05
R2-MODE	1.27e-34	1.27e-34	1.27e-34	6.84e-30	> 0.05	4.38e-02	1.80e-03	> 0.05	—	> 0.05	1.27e-34	1.27e-34	> 0.05
R2-IBEA	1.07e-34	1.07e-34	1.07e-34	1.07e-34	2.84e-33	5.17e-34	1.13e-34	1.92e-32	1.21e-34	—	1.07e-34	1.07e-34	2.88e-32
MOMBI-TCH	1.28e-34	> 0.05	1.05e-08	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.28e-34	> 0.05	6.60e-24	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	5.43e-30	—	> 0.05
MOMBI-PBI	9.15e-35	9.15e-35	9.15e-35	9.13e-35	9.28e-35	9.14e-35	9.10e-35	9.08e-35	9.11e-35	> 0.05	9.15e-35	9.15e-35	—

Table A.24: Comparison of R2 indicator values for different optimizers on the DTLZ4 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	2.181109e-01	2.180311e-01	5.000710e-01	2.180303e-01	2.179028e-01	2.181296e-01	2.191952e-01	2.186593e-01	2.188815e-01	2.179277e-01	2.180308e-01	2.182619e-01	2.180293e-01
	avg.	2.181153e-01	3.364577e-01	4.078119e-01	3.278136e-01	2.743078e-01	2.181700e-01	2.192214e-01	2.187260e-01	2.189306e-01	2.461335e-01	2.180309e-01	2.295246e-01	2.208493e-01
	min.	2.180319e-01	2.180294e-01	2.180186e-01	2.180290e-01	2.177901e-01	2.180400e-01	2.187997e-01	2.182493e-01	2.184986e-01	2.178455e-01	2.180284e-01	2.181610e-01	2.180263e-01
	max.	2.181968e-01	5.000000e-01	5.820420e-01	5.000000e-01	5.000000e-01	2.195456e-01	2.199710e-01	2.195639e-01	2.199638e-01	5.000000e-01	2.180349e-01	5.000000e-01	5.000000e-01
	std.	3.532806e-05	1.398695e-01	1.372407e-01	1.379971e-01	1.134146e-01	1.648578e-04	2.309654e-04	2.569590e-04	2.423835e-04	8.504849e-02	1.153158e-06	5.548872e-02	2.819704e-02
3D	med.	9.932134e-02	9.485464e-02	1.464326e-01	1.452995e-01	9.347956e-02	9.420030e-02	9.557796e-02	9.513824e-02	9.534903e-02	9.357825e-02	9.479172e-02	9.481448e-02	9.375457e-02
	avg.	1.047196e-01	1.595259e-01	2.284315e-01	1.597440e-01	1.233614e-01	9.543160e-02	9.659819e-02	9.867380e-02	9.540996e-02	1.009804e-01	9.528571e-02	1.014601e-01	9.375466e-02
	min.	9.477372e-02	9.471380e-02	9.514411e-02	9.374650e-02	9.326184e-02	9.392801e-02	9.503155e-02	9.445760e-02	9.468084e-02	9.343283e-02	9.460031e-02	9.463529e-02	9.374574e-02
	max.	1.460005e-01	3.333219e-01	3.347158e-01	3.333219e-01	3.333219e-01	1.021297e-01	1.468410e-01	1.462489e-01	9.608330e-02	3.333219e-01	1.453195e-01	1.466526e-01	9.376612e-02
	std.	1.519210e-02	9.508744e-02	9.579668e-02	9.275296e-02	5.827579e-02	1.912946e-03	7.185944e-03	1.293841e-02	2.751509e-04	3.526504e-02	5.054097e-03	1.729525e-02	4.014625e-06
4D	med.	6.565551e-02	7.323478e-02	1.109502e-01	7.090491e-02	1.254356e-01	5.316044e-02	5.438134e-02	5.437082e-02	5.424691e-02	5.283065e-02	5.604642e-02	5.603002e-02	5.307599e-02
	avg.	6.632572e-02	9.175926e-02	1.450766e-01	9.685774e-02	1.223915e-01	5.317847e-02	5.508547e-02	5.542096e-02	5.429103e-02	5.370510e-02	5.689603e-02	5.910994e-02	5.325436e-02
	min.	6.090483e-02	5.586354e-02	7.344601e-02	5.304338e-02	8.705922e-02	5.293401e-02	5.383039e-02	5.353629e-02	5.371257e-02	5.265448e-02	5.588148e-02	5.583542e-02	5.306280e-02
	max.	7.728888e-02	2.499807e-01	2.499817e-01	2.499807e-01	1.278310e-01	5.358500e-02	7.203622e-02	7.185877e-02	5.516131e-02	7.034214e-02	7.327382e-02	7.326807e-02	7.082957e-02
	std.	3.602884e-03	5.329580e-02	6.296159e-02	6.432685e-02	7.029036e-03	1.323357e-04	3.425257e-03	4.080346e-03	2.467376e-04	3.815952e-03	3.759033e-03	6.640829e-03	1.775284e-03
5D	med.	5.095152e-02	4.669788e-02	9.011635e-02	4.474513e-02	9.529303e-02	3.562287e-02	3.619422e-02	3.646459e-02	3.615337e-02	3.516792e-02	3.730128e-02	3.729699e-02	3.524690e-02
	avg.	5.097909e-02	5.699625e-02	1.056179e-01	5.752984e-02	9.592968e-02	3.618274e-02	3.681788e-02	3.709588e-02	3.614867e-02	3.565247e-02	3.782900e-02	3.913477e-02	3.524785e-02
	min.	4.588915e-02	3.727817e-02	5.843982e-02	3.523668e-02	7.560815e-02	3.507400e-02	3.574406e-02	3.578921e-02	3.580771e-02	3.500369e-02	3.723660e-02	3.727847e-02	3.523171e-02
	max.	6.035428e-02	1.999784e-01	1.999784e-01	1.999784e-01	1.038044e-01	4.551376e-02	6.013469e-02	4.621815e-02	3.648340e-02	4.443575e-02	4.670870e-02	4.834132e-02	3.526909e-02
	std.	2.694053e-03	2.965633e-02	4.367751e-02	3.292582e-02	4.724021e-03	1.984839e-03	2.945428e-03	2.187866e-03	1.553360e-04	1.875673e-03	2.116798e-03	3.553379e-03	7.588246e-06
6D	med.	5.201511e-02	3.803085e-02	7.488431e-02	3.589359e-02	7.175447e-02	2.446820e-02	2.509960e-02	2.571676e-02	2.498450e-02	2.422763e-02	4.095558e-02	3.052540e-02	2.431059e-02
	avg.	5.229739e-02	4.436849e-02	8.311196e-02	4.055035e-02	7.481533e-02	2.449702e-02	2.545889e-02	2.616459e-02	2.501000e-02	2.456455e-02	3.751708e-02	3.375568e-02	2.431058e-02
	min.	4.154497e-02	3.001077e-02	3.848671e-02	2.429175e-02	6.909321e-02	2.425846e-02	2.475394e-02	2.486946e-02	2.469342e-02	2.407789e-02	3.003494e-02	2.999627e-02	2.428454e-02
	max.	6.652217e-02	1.666457e-01	1.666457e-01	1.666457e-01	8.532236e-02	2.538504e-02	3.620003e-02	3.610628e-02	2.549074e-02	2.868892e-02	4.121972e-02	5.057456e-02	2.434597e-02
	std.	4.737968e-03	2.751713e-02	3.515331e-02	2.791266e-02	4.693758e-03	1.541457e-04	1.521774e-03	1.614677e-03	1.619354e-04	1.172382e-03	4.340830e-03	4.527668e-03	8.316941e-06
7D	med.	8.603369e-02	3.797988e-02	6.569821e-02	2.512502e-02	6.803512e-02	2.632041e-02	1.961688e-02	2.133076e-02	1.936138e-02	1.832367e-02	4.730353e-02	3.774843e-02	1.840356e-02
	avg.	8.687320e-02	4.257140e-02	6.587376e-02	3.209232e-02	6.885115e-02	2.654853e-02	2.090951e-02	2.158686e-02	1.938185e-02	1.931824e-02	4.705198e-02	3.825706e-02	1.858478e-02
	min.	6.567925e-02	3.232019e-02	3.377924e-02	1.839422e-02	6.311501e-02	1.829462e-02	1.913760e-02	1.935421e-02	1.906474e-02	1.815814e-02	3.473460e-02	3.218904e-02	1.839918e-02
	max.	1.118808e-01	1.428378e-01	1.428378e-01	1.428378e-01	8.534856e-02	3.232548e-02	4.110215e-02	2.598696e-02	1.990045e-02	2.505156e-02	5.326928e-02	4.948000e-02	2.116968e-02
	std.	8.262972e-03	1.908659e-02	2.910764e-02	2.666020e-02	4.178324e-03	2.620569e-03	2.782280e-03	1.249405e-03	1.575124e-04	1.607688e-03	4.273072e-03	5.139502e-03	6.657206e-04
8D	med.	8.396231e-02	3.535165e-02	5.732648e-02	2.193172e-02	5.171780e-02	1.905257e-02	1.474859e-02	1.539178e-02	1.465557e-02	1.410810e-02	4.587114e-02	3.778721e-02	1.400611e-02
	avg.	8.421105e-02	3.547831e-02	5.122879e-02	2.442625e-02	5.203548e-02	1.913999e-02	1.497737e-02	1.570552e-02	1.465246e-02	1.434590e-02	4.529347e-02	3.668661e-02	1.400611e-02
	min.	6.601461e-02	2.819963e-02	2.910730e-02	1.399487e-02	4.916165e-02	1.397767e-02	1.441102e-02	1.449863e-02	1.446989e-02	1.401546e-02	3.369859e-02	2.819431e-02	1.399948e-02
	max.	1.023194e-01	5.791308e-02	1.249825e-01	1.249825e-01	5.948959e-02	2.379956e-02	1.661206e-02	1.971367e-02	1.488504e-02	1.784043e-02	4.901654e-02	4.514837e-02	1.401461e-02
	std.	7.831681e-03	5.411301e-03	1.669993e-02	1.367974e-02	1.803118e-03	2.234312e-03	6.180873e-04	9.763277e-04	8.629246e-05	6.531897e-04	3.094019e-03	4.948798e-03	2.154532e-06
9D	med.	6.962331e-02	3.141262e-02	3.636652e-02	1.622640e-02	4.078176e-02	1.524394e-02	1.150194e-02	1.177801e-02	1.148119e-02	1.108132e-02	4.181480e-02	3.658450e-02	1.100302e-02
	avg.	6.896708e-02	3.168188e-02	4.163491e-02	1.730469e-02	4.003908e-02	1.528190e-02	1.166904e-02	1.192648e-02	1.148842e-02	1.118930e-02	4.130181e-02	3.586174e-02	1.100264e-02
	min.	4.718589e-02	2.488281e-02	2.586602e-02	1.098960e-02	1.114606e-02	1.107610e-02	1.138418e-02	1.130729e-02	1.133126e-02	1.100928e-02	3.248976e-02	2.730087e-02	1.098896e-02
	max.	7.943875e-02	4.038922e-02	1.110954e-01	3.317028e-02	4.377961e-02	1.921034e-02	1.300028e-02	1.453751e-02	1.189307e-02	1.384017e-02	4.446658e-02	4.052581e-02	1.101057e-02
	std.	6.263515e-03	3.799306e-03	1.361743e-02	4.628033e-03	4.822576e-03	1.933409e-03	4.485063e-04	4.900559e-04	7.989834e-05	4.022579e-04	2.602904e-03	2.908497e-03	3.576938e-06
10D	med.	5.584544e-02	2.928869e-02	3.169440e-02	1.448415e-02	9.298942e-03	1.226390e-02	9.161396e-03	9.327195e-03	9.119090e-03	8.881663e-03	3.874897e-02	3.364977e-02	8.836364e-03
	avg.	5.576483e-02	2.900642e-02	3.320910e-02	1.426273e-02	1.797608e-02	1.252944e-02	9.193552e-03	9.405776e-03	9.114951e-03	8.898063e-03	3.815925e-02	3.326006e-02	8.837331e-03
	min.	4.392982e-02	2.278581e-02	2.369401e-02	9.599718e-03	8.986856e-03	9.790824e-03	9.059148e-03	9.040597e-03	9.011282e-03	8.812961e-03	3.271440e-02	2.388680e-02	8.821964e-03
	max.	6.562622e-02	3.668954e-02	4.822122e-02	2.964632e-02	3.395310e-02	1.528294e-02	1.010024e-02	1.063545e-02	9.214035e-03	9.805728e-03	4.068299e-02	3.749278e-02	8.856218e-03
	std.	4.923768e-03	3.652895e-03	7.869802e-03	3.325356e-03	1.085282e-02	1.588001e-03	1.779937e-04	2.890842e-04	4.369764e-05	1.225199e-04	1.944446e-03	2.625061e-03	6.331738e-06



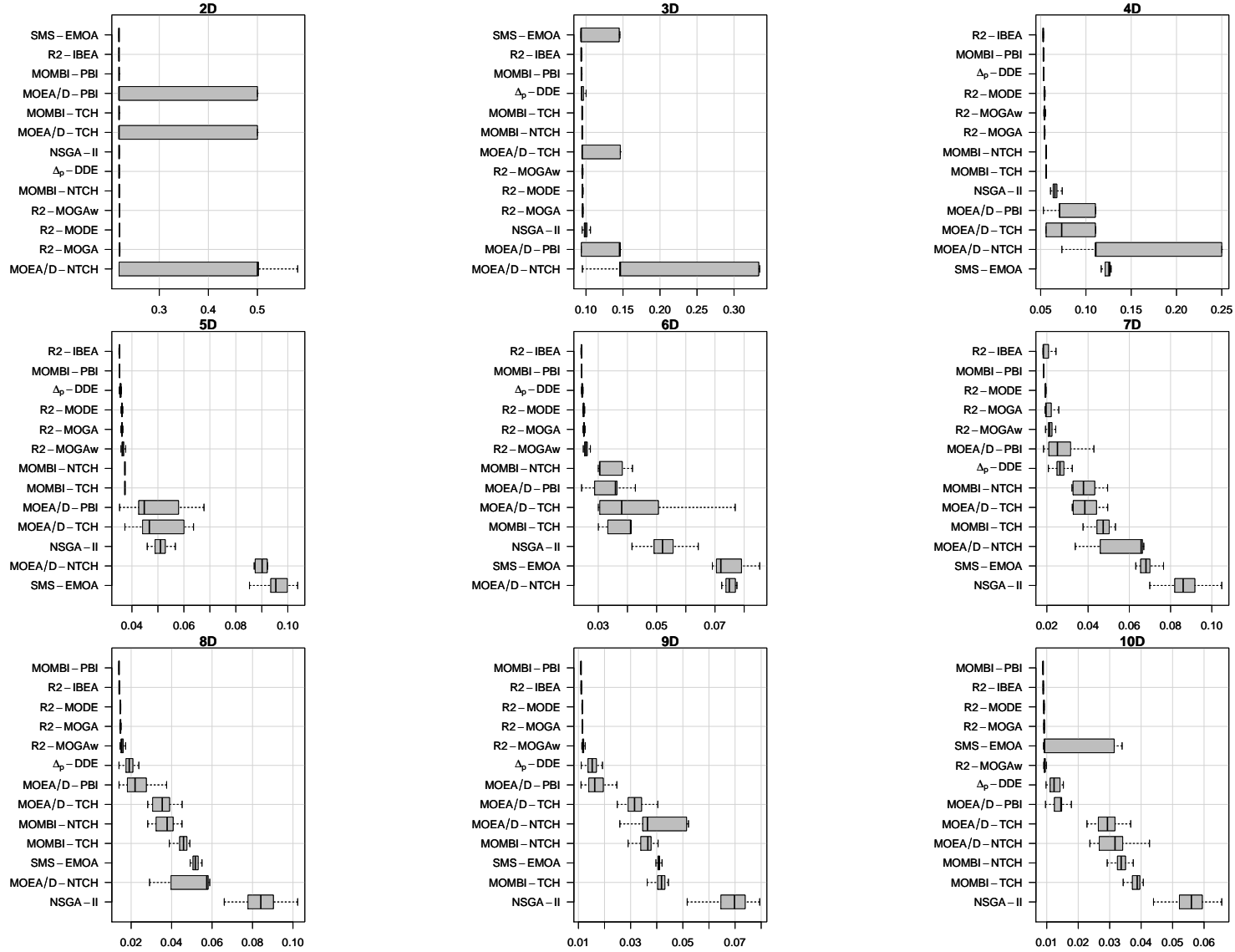


Figure A.50: Box-plot of R2 indicator values for different optimizers on the DTLZ4 test problem.

Table A.25: Comparison of runtime (in milliseconds) for different optimizers on the DTLZ4 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_\rho$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	2.238980e+02	5.235420e+02	9.521250e+02	8.945350e+02	2.925300e+04	1.147394e+03	2.288378e+03	1.500054e+03	2.059394e+03	5.713280e+05	1.973231e+03	1.545370e+03	2.724754e+03
	avg.	2.394357e+02	5.292782e+02	9.697243e+02	9.135135e+02	3.008610e+04	1.148654e+03	2.720408e+03	1.677287e+03	2.054638e+03	5.712752e+05	1.956811e+03	1.555177e+03	2.705220e+03
	min.	1.982860e+02	3.182230e+02	6.102940e+02	6.374690e+02	2.030335e+04	1.084115e+03	1.395747e+03	9.884520e+02	1.314826e+03	5.487210e+05	1.519996e+03	1.460010e+03	2.373141e+03
	max.	3.711710e+02	8.988220e+02	1.703289e+03	1.447273e+03	4.466103e+04	1.250140e+03	2.227299e+04	1.323728e+04	2.634543e+03	6.013460e+05	2.408123e+03	1.715531e+03	2.964506e+03
	std.	3.733393e+01	1.082383e+02	1.642161e+02	1.488565e+02	5.265383e+03	2.766706e+01	2.525788e+03	1.336761e+03	2.672743e+02	9.647165e+03	1.751721e+02	3.887397e+01	1.455034e+02
3D	med.	3.664250e+02	6.492930e+02	1.402631e+03	1.055523e+03	7.524482e+05	1.765240e+03	2.115591e+03	1.669359e+03	2.277192e+03	6.216330e+05	2.038512e+03	2.086150e+03	3.015392e+03
	avg.	3.751196e+02	6.548702e+02	1.390280e+03	1.068613e+03	7.064784e+05	1.752266e+03	2.107992e+03	1.693105e+03	2.287380e+03	6.078599e+05	2.022466e+03	2.078123e+03	2.994019e+03
	min.	3.150200e+02	4.280720e+02	8.187670e+02	7.743950e+02	4.580445e+04	1.238533e+03	1.505369e+03	1.204048e+03	1.644478e+03	4.917660e+05	1.673856e+03	1.737341e+03	2.406989e+03
	max.	4.863170e+02	1.085767e+03	2.283293e+03	1.897013e+03	9.646064e+05	2.232833e+03	2.746950e+03	2.249874e+03	2.849348e+03	6.792840e+05	2.733262e+03	2.447027e+03	3.461945e+03
	std.	3.536894e+01	1.183690e+02	2.269678e+02	1.646824e+02	1.868673e+05	1.938812e+02	2.327860e+02	2.119261e+02	2.686105e+02	4.375860e+04	1.601426e+02	1.493662e+02	1.565308e+02
4D	med.	5.435500e+02	6.828000e+02	1.511035e+03	1.112857e+03	5.061483e+06	1.960175e+03	2.453445e+03	2.054984e+03	2.863312e+03	6.476050e+05	2.087179e+03	2.236848e+03	3.266097e+03
	avg.	5.489562e+02	6.982311e+02	1.524908e+03	1.122272e+03	4.871683e+06	1.932609e+03	2.436191e+03	2.046994e+03	2.826714e+03	6.464093e+05	2.078942e+03	2.232879e+03	3.288528e+03
	min.	4.461530e+02	4.871690e+02	1.076922e+03	7.526740e+02	2.191563e+06	1.236245e+03	1.637359e+03	1.581650e+03	1.911739e+03	5.988970e+05	1.699828e+03	1.784623e+03	2.845420e+03
	max.	7.449660e+02	1.191570e+03	1.939699e+03	1.623551e+03	6.029823e+06	3.104464e+03	3.289903e+03	2.597601e+03	3.430948e+03	6.926720e+05	2.530597e+03	2.561267e+03	4.155213e+03
	std.	6.296100e+01	1.103371e+02	1.840159e+02	1.380416e+02	8.880771e+05	2.633484e+02	3.193876e+02	2.391728e+02	3.401330e+02	1.662650e+04	1.620312e+02	1.658433e+02	1.927358e+02
5D	med.	7.148140e+02	7.405505e+02	1.986914e+03	1.170216e+03	3.183457e+06	3.183457e+03	2.959064e+03	2.410487e+03	3.539580e+03	7.268410e+05	2.276846e+03	2.352869e+03	3.872409e+03
	avg.	7.193112e+02	7.597536e+02	2.054094e+03	1.185511e+03	6.591771e+06	3.152838e+03	2.920535e+03	2.478926e+03	3.465552e+03	7.577452e+05	2.357016e+03	2.380508e+03	3.857084e+03
	min.	5.708120e+02	5.904680e+02	1.200811e+03	8.534420e+02	3.137785e+06	2.051270e+03	2.048838e+03	1.926126e+03	3.627193e+03	6.877690e+05	1.879851e+03	2.056829e+03	3.130968e+03
	max.	9.188570e+02	1.110529e+03	3.180439e+03	1.731299e+03	8.246303e+06	4.157759e+03	3.473162e+03	3.134407e+03	4.035730e+03	9.224040e+05	3.798115e+03	2.886484e+03	4.569025e+03
	std.	6.914586e+01	1.055594e+02	3.176249e+02	1.650168e+02	9.667691e+05	4.957447e+02	3.240712e+02	2.602587e+02	3.561184e+02	6.370754e+04	3.457035e+02	1.731407e+02	2.195072e+02
6D	med.	8.456560e+02	8.229840e+02	2.092866e+03	1.263523e+03	7.589107e+06	2.795610e+03	3.266387e+03	2.741318e+03	3.918215e+03	7.446410e+05	2.445724e+03	2.510570e+03	4.730425e+03
	avg.	8.569799e+02	8.400044e+02	2.168731e+03	1.310863e+03	7.592169e+06	2.867512e+03	3.265484e+03	2.748718e+03	3.889721e+03	7.478676e+05	2.502371e+03	2.513248e+03	4.743076e+03
	min.	6.895250e+02	6.133370e+02	1.500723e+03	9.321670e+02	5.828039e+06	1.750354e+03	2.340125e+03	2.121356e+03	2.946511e+03	6.020710e+05	1.827574e+03	2.178519e+03	3.891407e+03
	max.	1.084959e+03	1.306234e+03	3.468866e+03	2.072611e+03	9.196889e+06	4.223999e+03	3.957713e+03	3.563761e+03	4.694476e+03	9.524980e+05	4.799847e+03	2.925228e+03	5.365772e+03
	std.	8.059400e+01	1.122483e+02	3.359338e+02	2.168000e+02	7.675764e+05	3.465634e+02	3.249605e+02	3.533862e+02	4.352448e+04	5.359052e+02	1.622775e+02	2.239136e+02	2.239136e+02
7D	med.	7.969800e+02	9.033100e+02	2.304674e+03	1.335321e+03	5.705744e+06	8.334704e+03	2.527202e+03	1.998674e+03	2.963760e+03	4.002730e+05	1.534164e+03	2.129565e+03	3.066998e+03
	avg.	7.942981e+02	9.138936e+02	2.422438e+03	1.359704e+03	5.624348e+06	8.272511e+03	2.553552e+03	1.997627e+03	2.943565e+03	4.047269e+05	1.542741e+03	2.118241e+03	3.080763e+03
	min.	6.409410e+02	6.667620e+02	1.640264e+03	9.461390e+02	4.078655e+06	3.455383e+03	1.725619e+03	1.355517e+03	1.998442e+03	3.664380e+05	1.247081e+03	1.668205e+03	2.643112e+03
	max.	9.478630e+02	1.266471e+03	4.117256e+03	1.889670e+03	7.369707e+06	1.321418e+04	3.280703e+03	2.683796e+03	3.617451e+03	4.824890e+05	1.897960e+03	2.426956e+03	3.938268e+03
	std.	8.192784e+01	1.351860e+02	3.952223e+02	1.886339e+02	6.792173e+05	1.610296e+03	2.823025e+02	2.537928e+02	3.358376e+02	2.027888e+02	1.388103e+02	1.390182e+02	1.715801e+02
8D	med.	1.170769e+03	1.030281e+03	2.537800e+03	1.392204e+03	7.981664e+06	1.521575e+04	4.048949e+03	3.218810e+03	4.570354e+03	1.318880e+06	2.454494e+03	3.051419e+03	4.479519e+03
	avg.	1.158949e+03	1.048005e+03	2.649611e+03	1.454795e+03	7.897961e+06	1.561711e+04	4.033221e+03	3.217324e+03	4.576225e+03	1.316451e+06	2.465028e+03	3.045889e+03	4.519798e+03
	min.	9.374300e+02	7.788640e+02	1.797219e+03	1.024715e+03	5.155009e+06	6.622931e+03	3.018444e+03	2.221154e+03	3.346295e+03	1.141363e+06	2.054566e+03	2.522530e+03	3.717660e+03
	max.	1.348263e+03	1.828988e+03	4.749194e+03	3.147082e+03	9.711206e+06	2.610117e+04	4.680619e+03	4.177868e+03	5.429467e+03	1.595988e+06	3.155944e+03	3.734629e+03	5.733673e+03
	std.	8.192784e+01	1.351860e+02	4.749655e+02	3.058848e+02	8.887488e+05	4.159844e+03	3.817584e+02	3.714233e+02	4.716665e+02	7.372422e+04	1.955362e+02	1.926053e+02	3.178422e+02
9D	med.	1.634709e+03	1.151795e+03	3.174860e+03	1.641475e+03	7.605896e+06	2.154273e+04	6.284858e+03	5.219358e+03	8.729320e+03	2.756731e+06	3.057723e+03	4.018811e+03	6.916434e+03
	avg.	1.636210e+03	1.204423e+03	3.470398e+03	1.677895e+03	7.699434e+06	2.237332e+04	6.110137e+03	5.182371e+03	8.349243e+03	2.777624e+06	3.146269e+03	4.114685e+03	6.965394e+03
	min.	1.421843e+03	9.385040e+02	2.669767e+03	1.158773e+03	2.360965e+06	9.544950e+03	3.937028e+03	3.531697e+03	5.717613e+03	2.335304e+06	2.573945e+03	3.444169e+03	5.362117e+03
	max.	1.929733e+03	2.356971e+03	6.133338e+03	2.622317e+03	1.226775e+07	4.524607e+04	7.484202e+03	6.237259e+03	9.209407e+03	3.147546e+06	3.800885e+03	4.796347e+03	8.283652e+03
	std.	1.067744e+02	1.906724e+02	6.595368e+02	2.732916e+02	1.927264e+06	5.659930e+03	9.893260e+02	5.583311e+02	8.156751e+02	1.320693e+05	2.859835e+02	3.346629e+02	4.767233e+02
10D	med.	2.241558e+03	1.340845e+03	3.322293e+03	1.826781e+03	3.434711e+06	3.035914e+04	9.748760e+03	8.151331e+03	1.317103e+04	5.329119e+06	4.258169e+03	6.228018e+03	9.615143e+03
	avg.	2.225589e+03	1.385576e+03	3.580004e+03	1.845826e+03	3.127430e+06	3.124782e+04	9.463759e+03	8.037545e+03	1.296020e+04	5.362673e+06	4.325690e+03	6.210821e+03	9.716742e+03
	min.	1.903785e+03	1.017049e+03	2.158976e+03	1.225885e+03	1.965579e+06	1.945685e+04	6.326321e+03	8.401615e+03	8.946032e+03	4.629100e+06	3.535367e+03	5.206930e+03	8.363867e+03
	max.	2.529180e+03	2.780046e+03	5.839279e+03	3.174877e+03	3.342862e+07	5.047969e+04	1.175653e+04	1.016138e+04	1.580210e+04	6.202438e+06	5.802612e+03	7.554765e+03	1.279538e+04
	std.	1.362092e+02	2.136221e+02	8.369199e+02	3.535545e+02	2.127218e+06	6.780088e+03	1.388758e+03	8.222806e+02	1.499867e+03	3.451693e+05	4.620856e+02	4.183967e+02	7.306391e+02

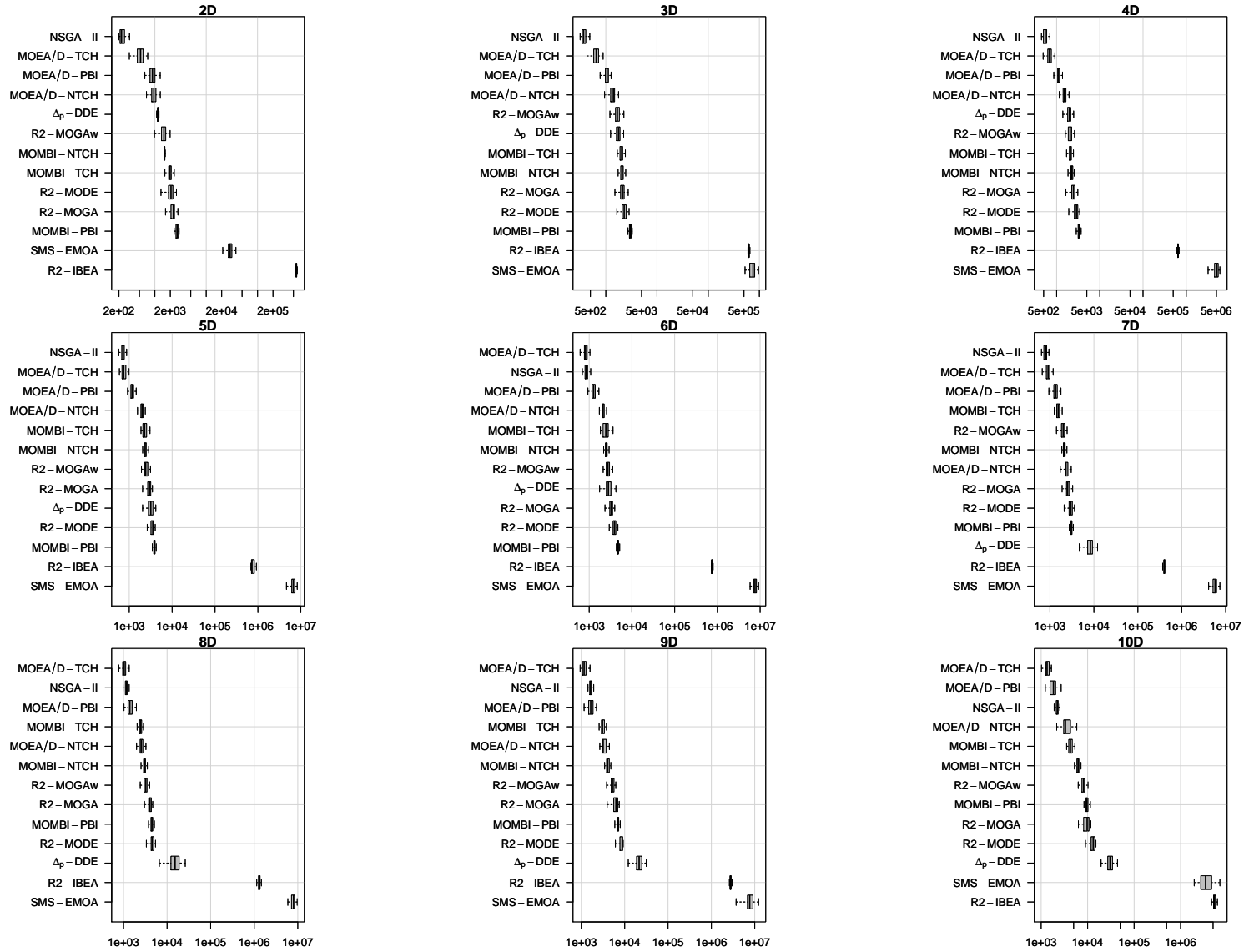


Figure A.51: Box-plot of runtime (in logarithmic scale) for different optimizers on the DTLZ4 test problem.

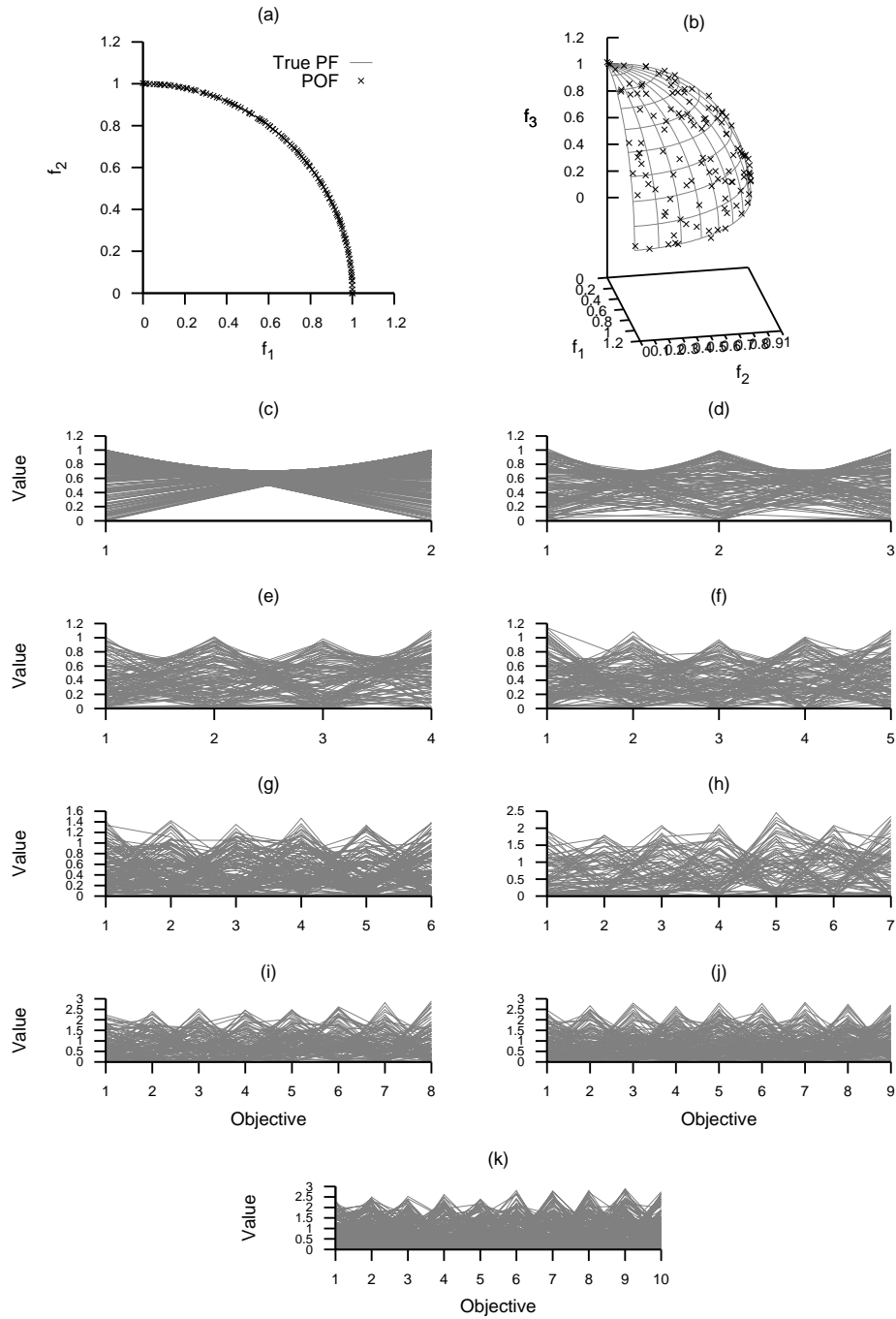


Figure A.52: Plots of the approximations obtained by NSGA-II from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

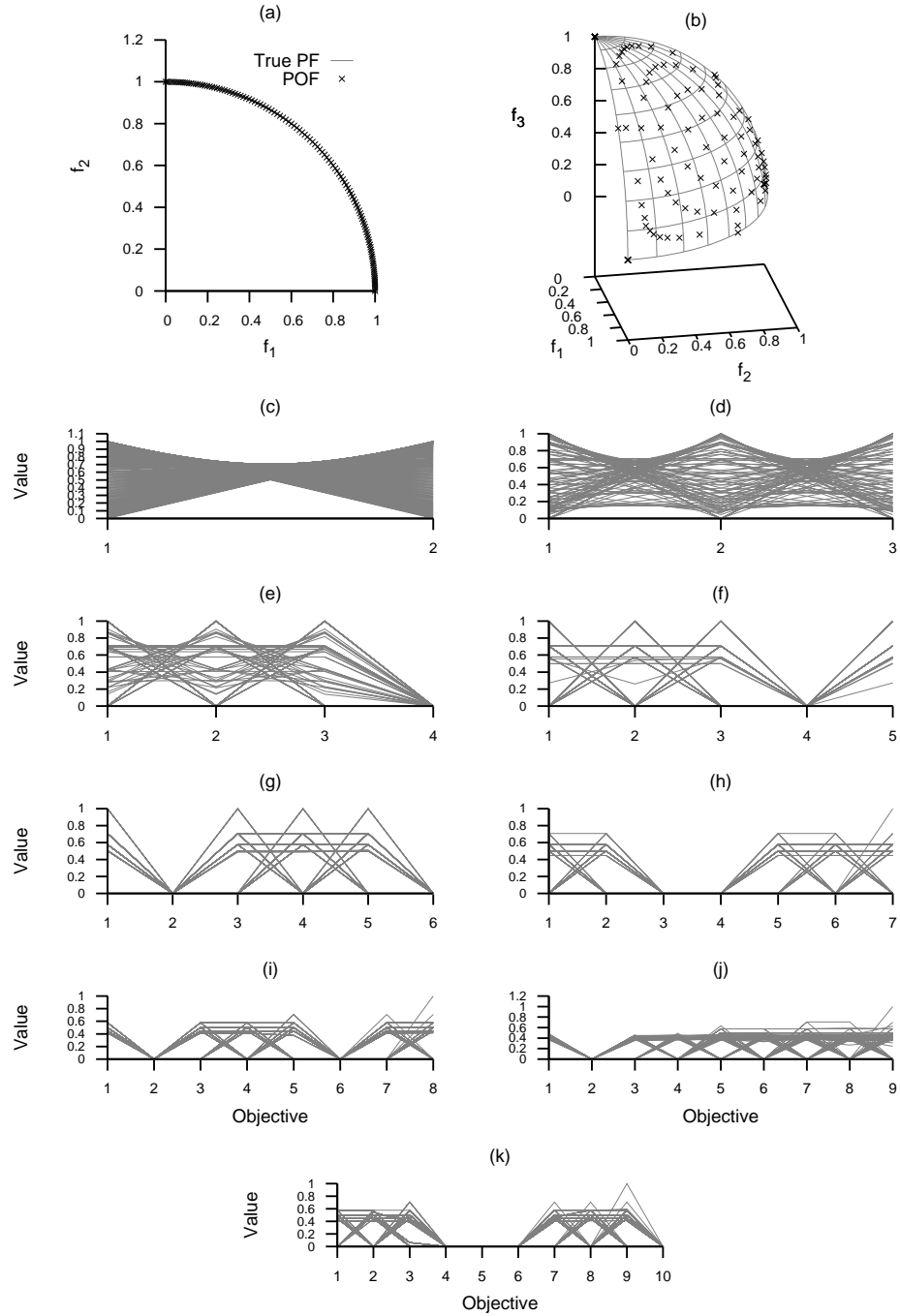


Figure A.53: Plots of the approximations obtained by MOEA/D-TCH from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

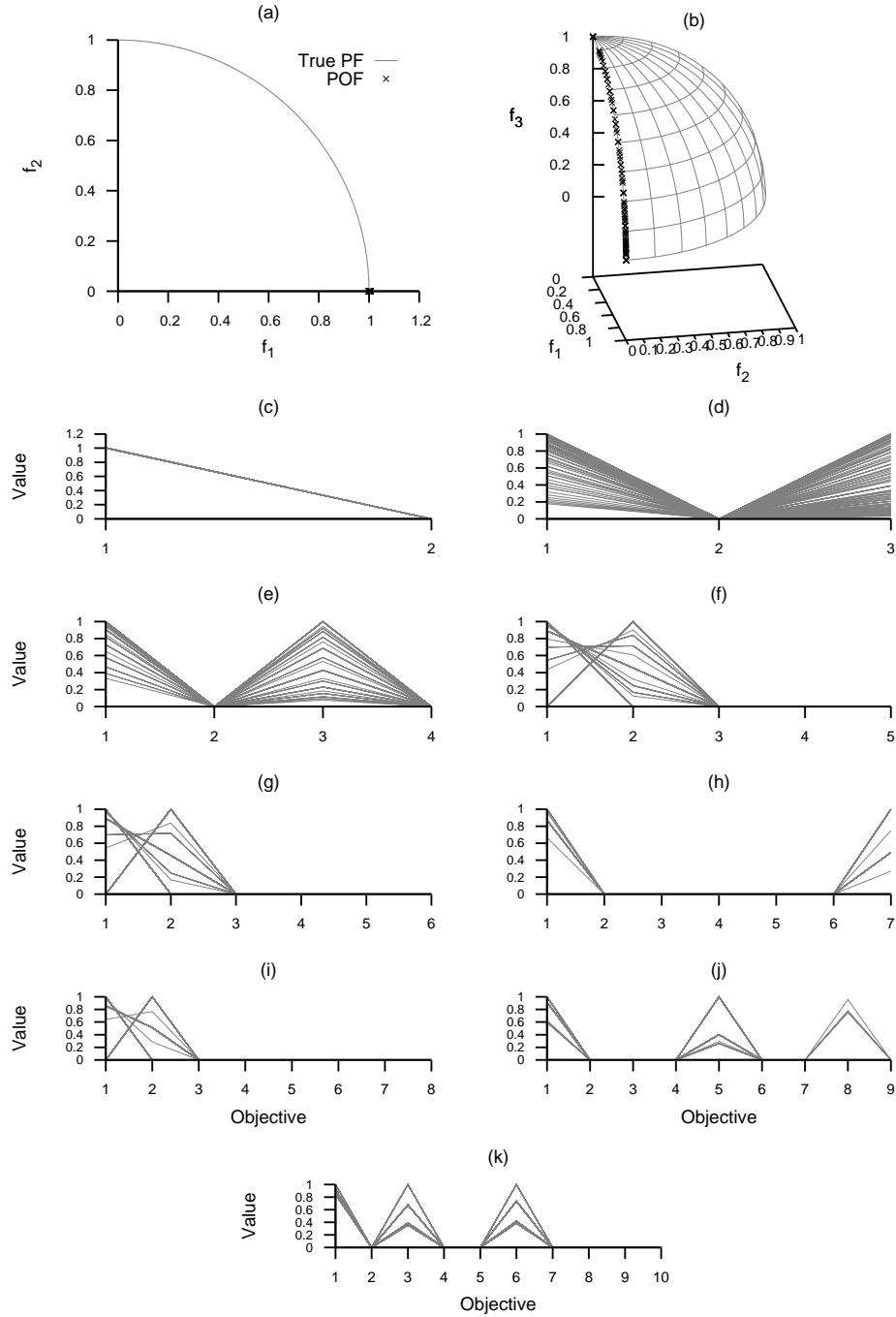


Figure A.54: Plots of the approximations obtained by MOEA/D-NTCH from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

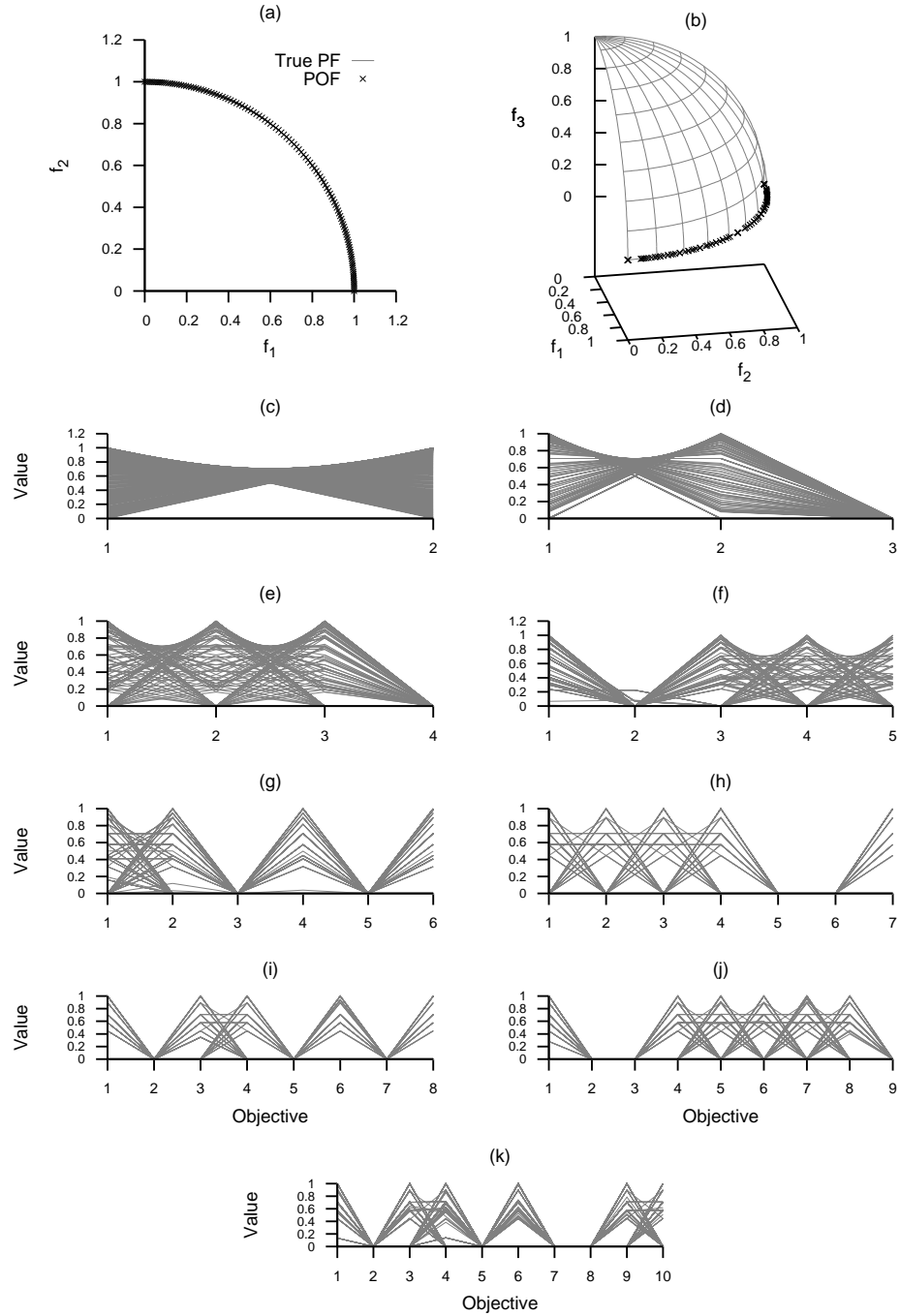


Figure A.55: Plots of the approximations obtained by MOEA/D-PBI from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

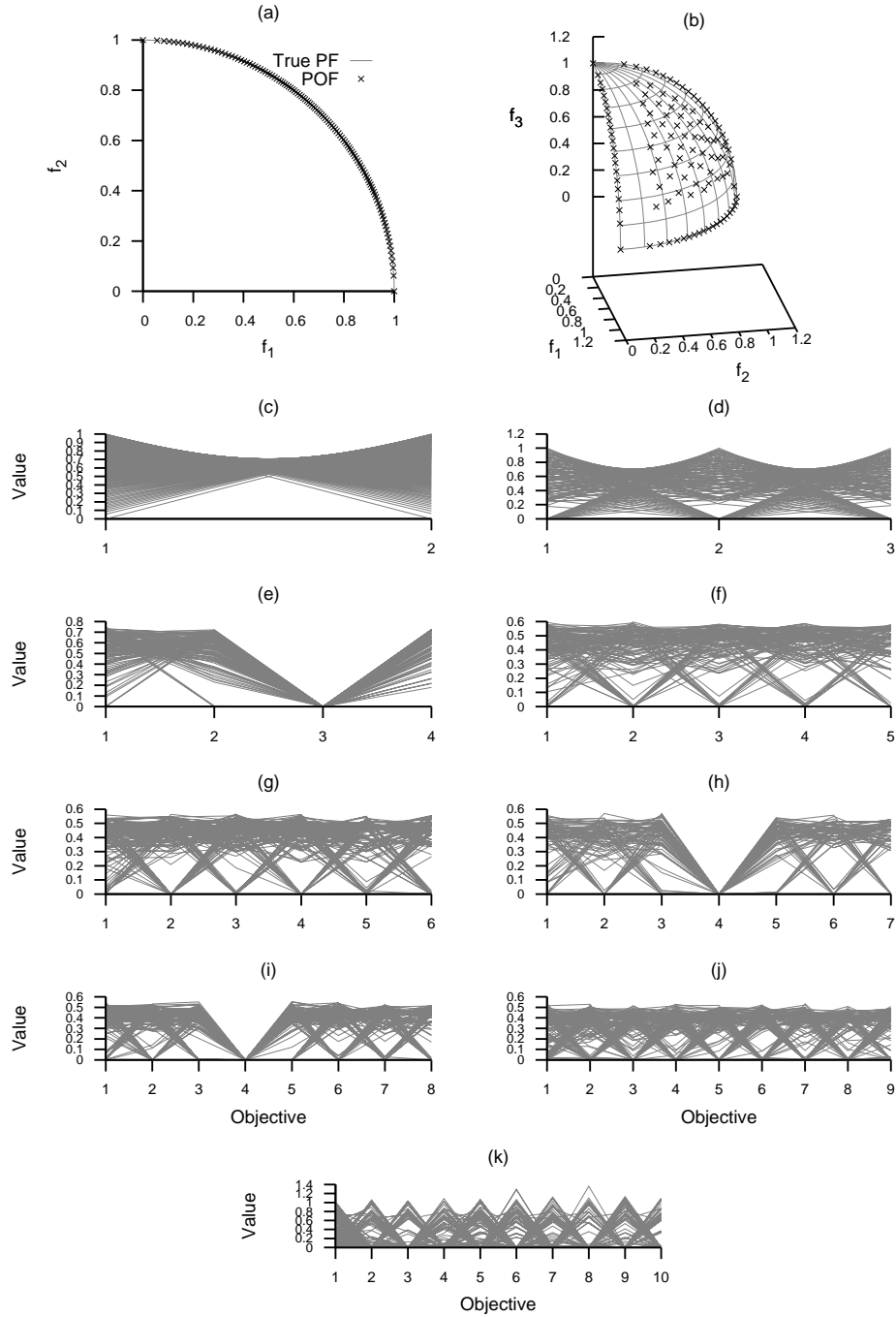


Figure A.56: Plots of the approximations obtained by SMS-EMOA from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



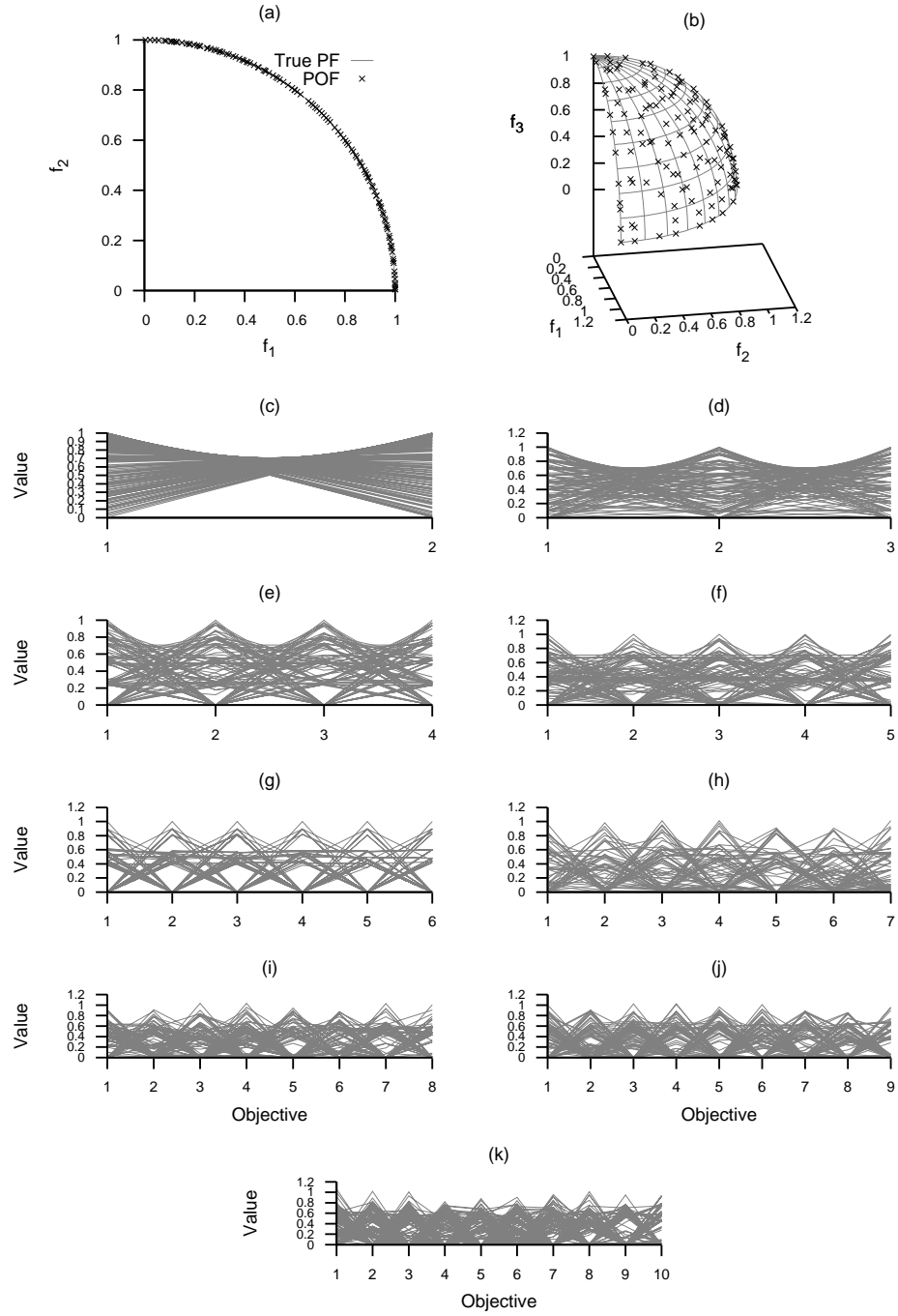


Figure A.57: Plots of the approximations obtained by  $\Delta_p$ -DDE from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

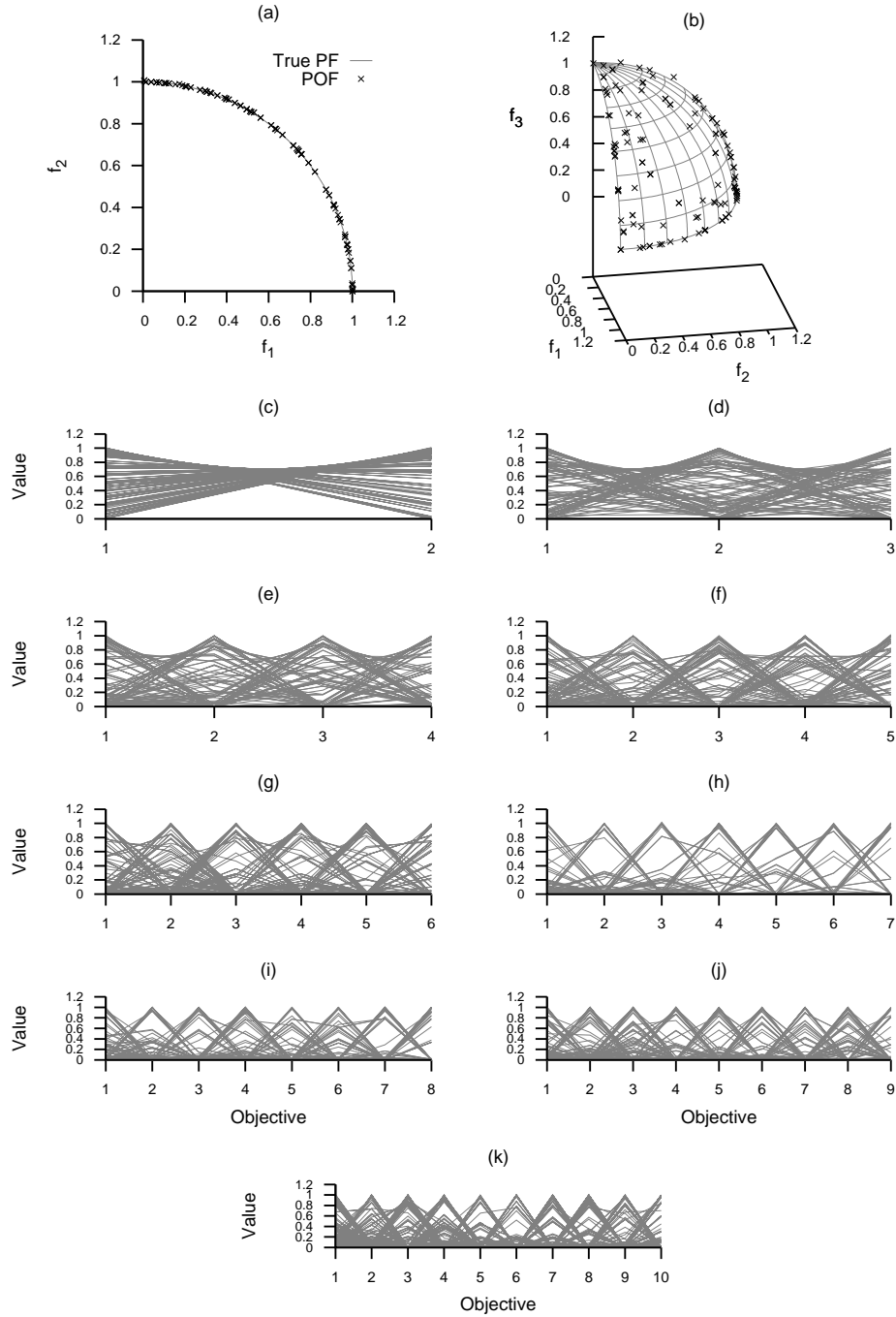


Figure A.58: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

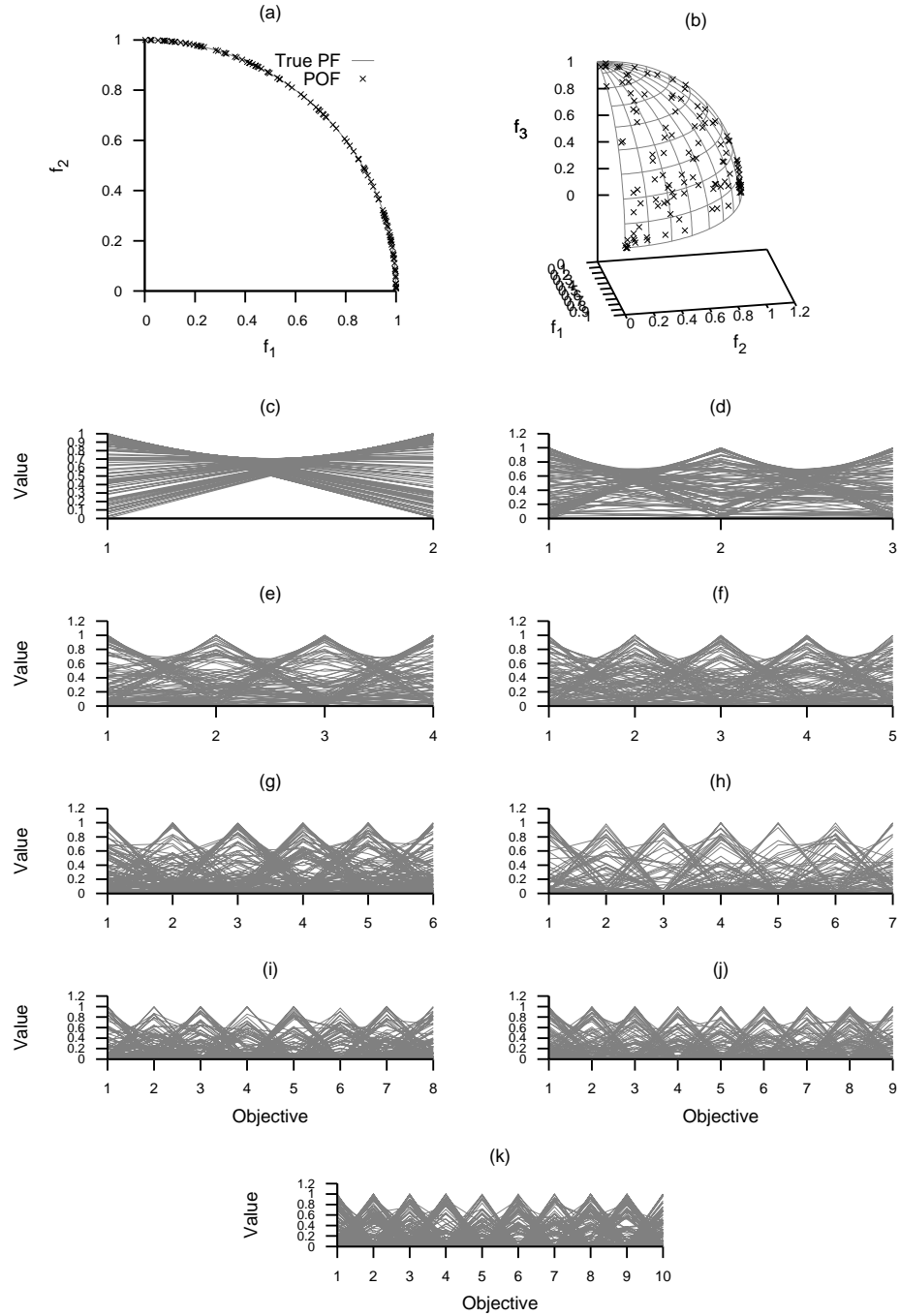


Figure A.59: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

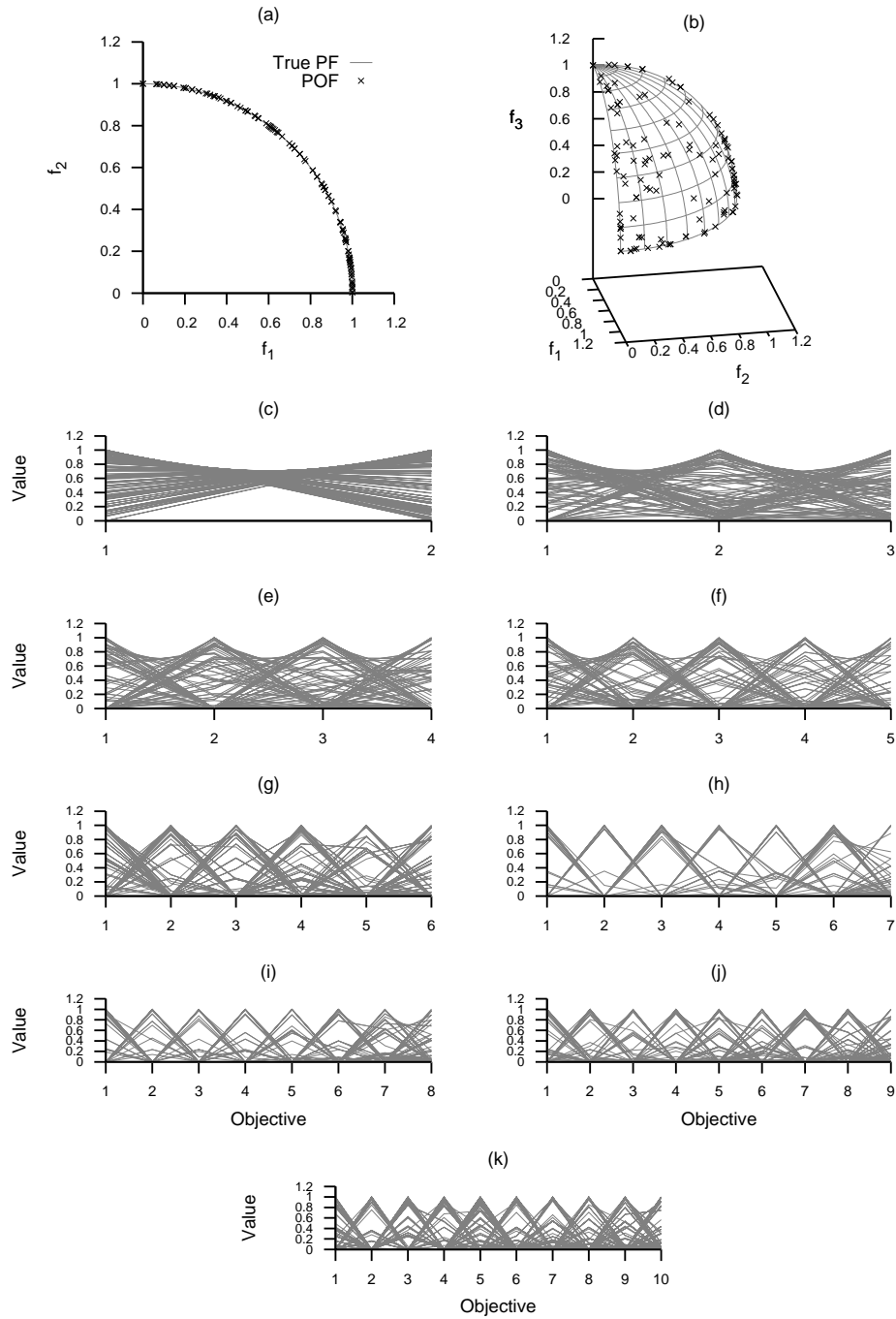


Figure A.60: Plots of the approximations obtained by *R2-MODE* from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

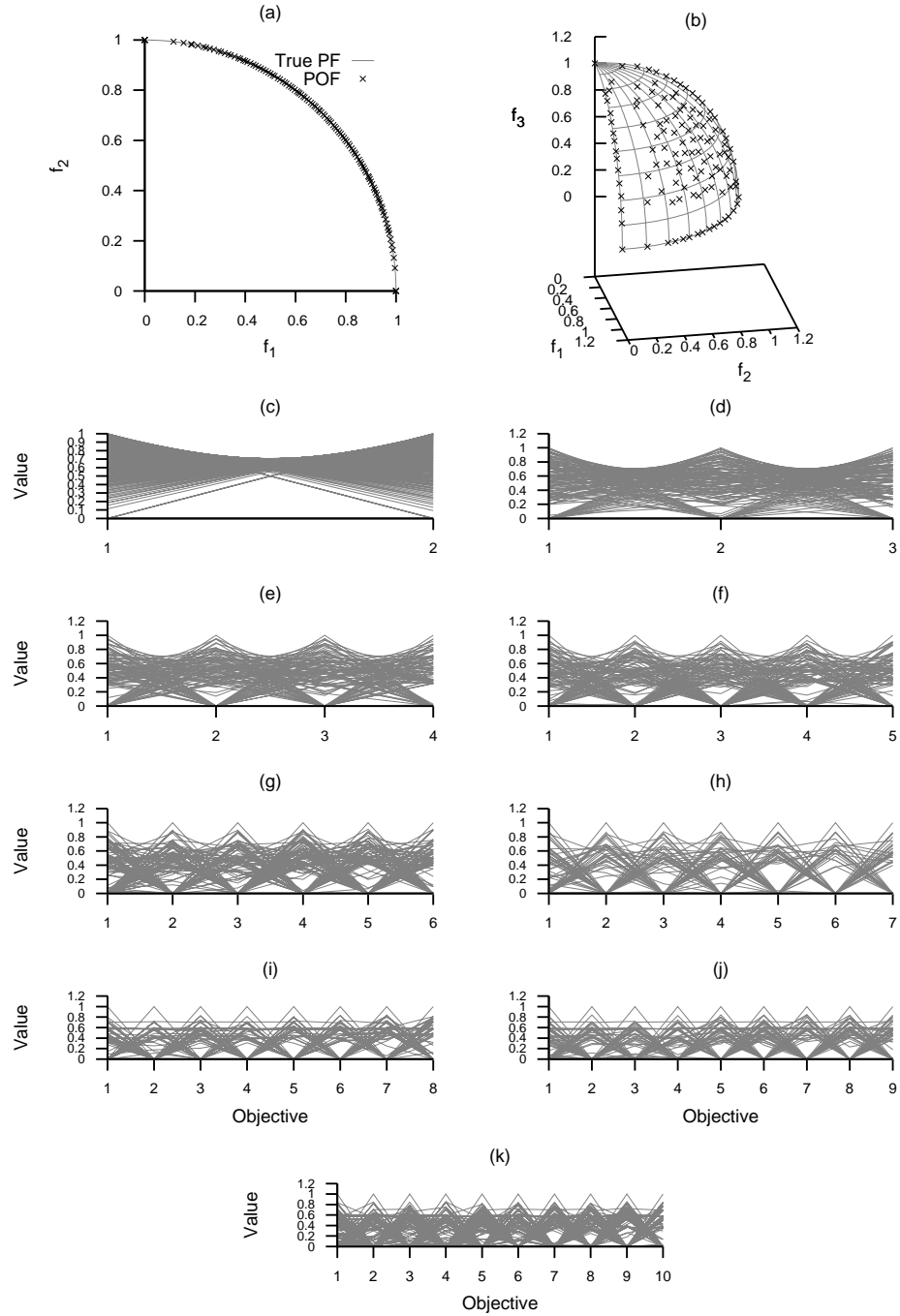


Figure A.61: Plots of the approximations obtained by R2-IBEA from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

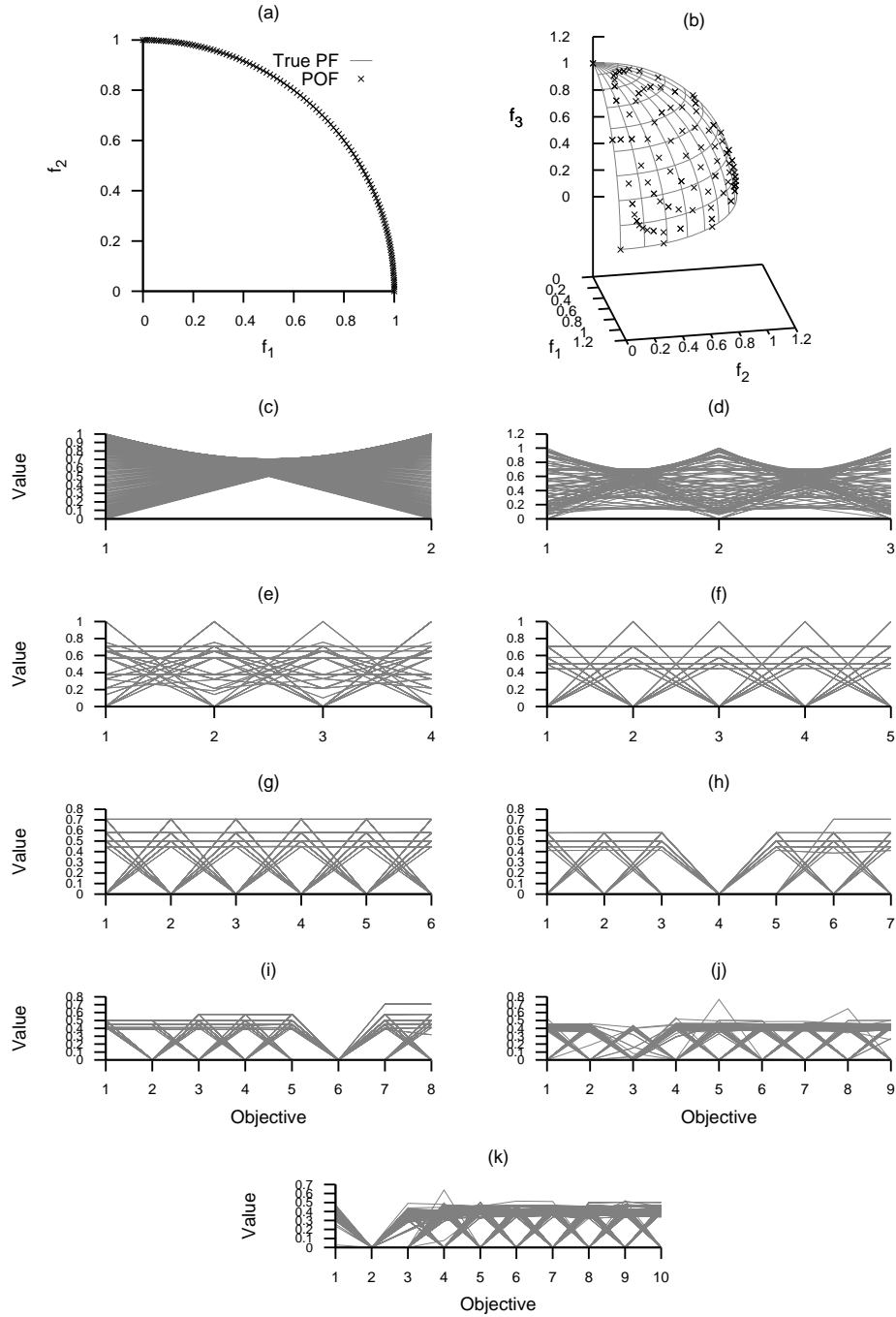


Figure A.62: Plots of the approximations obtained by MOMBI-TCH from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

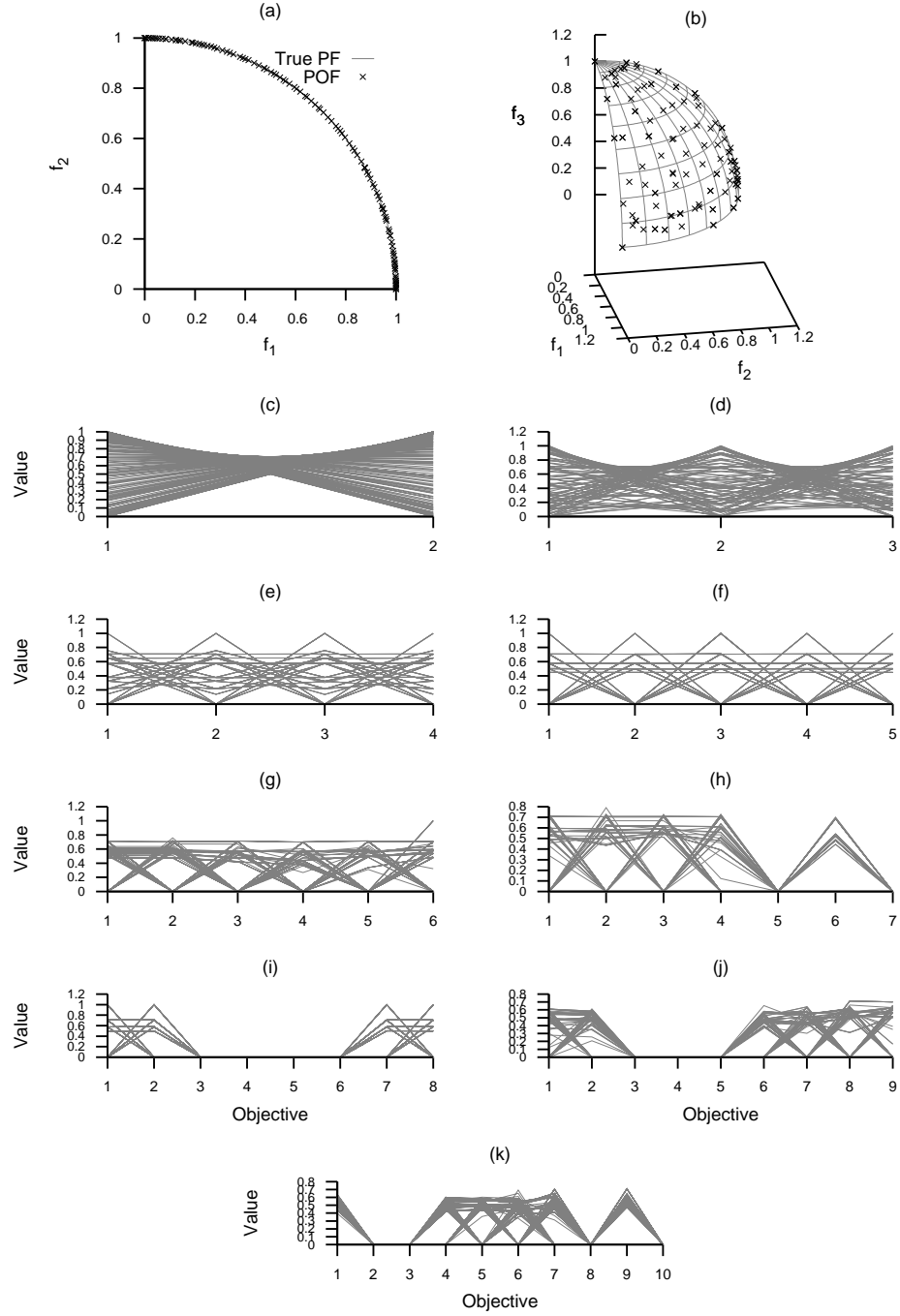


Figure A.63: Plots of the approximations obtained by MOMBI-NTCH from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

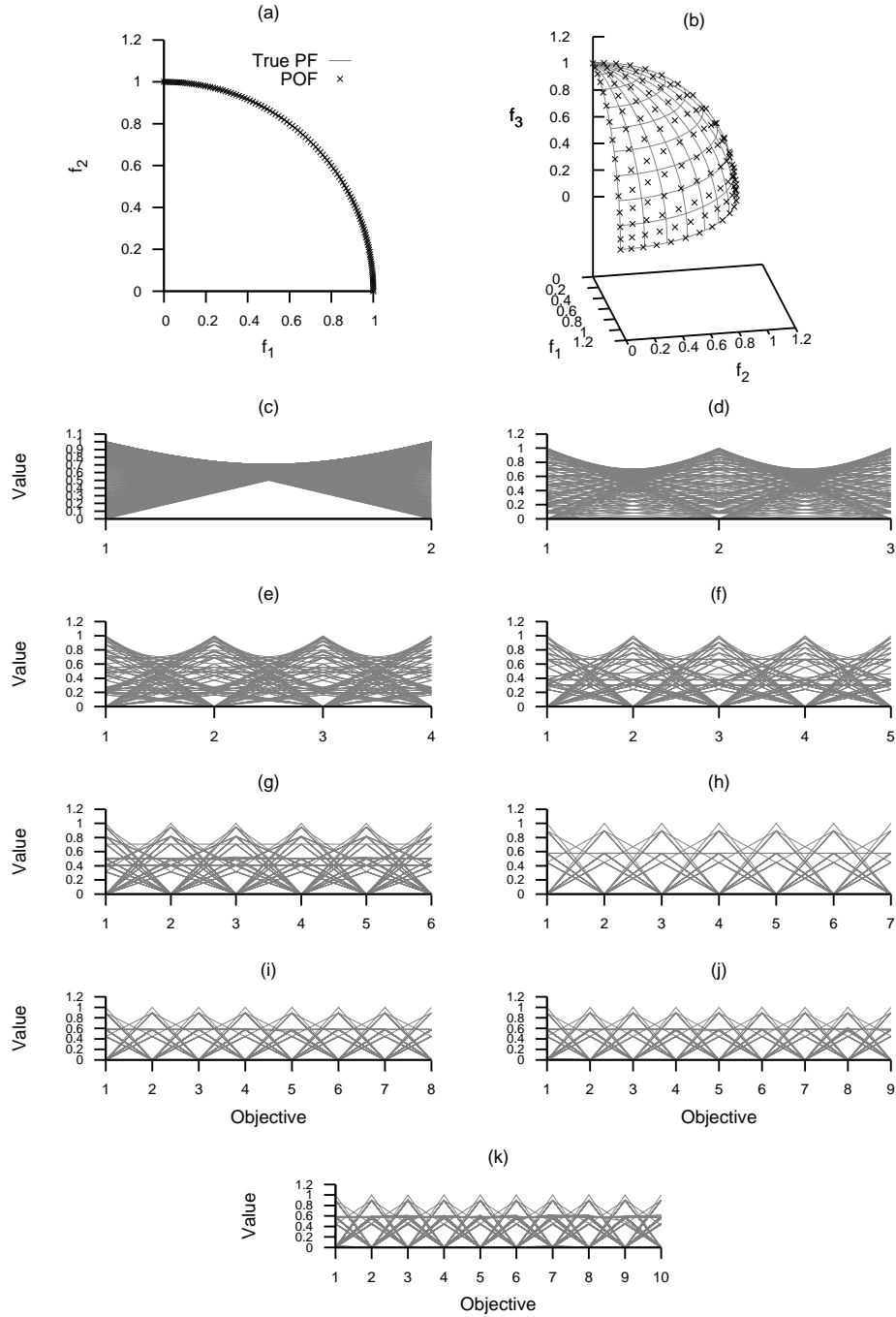


Figure A.64: Plots of the approximations obtained by MOMBI-PBI from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



Table A.26: Comparison of hypervolume indicator values for different optimizers on the DTLZ5 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	1.520999e+01	1.521085e+01	1.521056e+01	1.521081e+01	1.521160e+01	1.516908e+01	1.516627e+01	1.516524e+01	1.520575e+01	1.521113e+01	1.521070e+01	1.520922e+01	1.520974e+01
	avg.	1.520999e+01	1.521084e+01	1.521054e+01	1.521077e+01	1.521158e+01	1.516954e+01	1.516244e+01	1.515895e+01	1.520023e+01	1.521113e+01	1.521067e+01	1.520922e+01	1.520971e+01
	min.	1.520920e+01	1.521070e+01	1.520942e+01	1.521032e+01	1.521135e+01	1.504341e+01	1.505132e+01	1.503765e+01	1.504976e+01	1.521092e+01	1.521008e+01	1.520835e+01	1.520828e+01
	max.	1.521038e+01	1.521087e+01	1.521085e+01	1.521087e+01	1.521162e+01	1.521069e+01	1.520421e+01	1.520678e+01	1.520716e+01	1.521125e+01	1.521087e+01	1.520973e+01	1.521085e+01
3D	med.	1.998792e-04	3.519189e-05	2.165744e-04	1.100434e-04	5.125677e-05	3.812974e-02	2.755311e-02	3.404771e-02	2.402171e-02	6.876960e-05	1.637167e-04	2.541724e-04	5.479907e-04
	avg.	5.986497e+01	5.984301e+01	5.984079e+01	5.973712e+01	5.987523e+01	5.837975e+01	5.937567e+01	5.926090e+01	5.984303e+01	5.987225e+01	5.984261e+01	5.984361e+01	5.973376e+01
	avg.	5.986478e+01	5.984286e+01	5.984208e+01	5.973648e+01	5.987512e+01	5.809331e+01	5.933040e+01	5.920509e+01	5.970159e+01	5.987216e+01	5.984232e+01	5.984356e+01	5.973364e+01
	min.	5.986062e+01	5.983897e+01	5.983662e+01	5.972660e+01	5.987343e+01	5.557799e+01	5.845189e+01	5.850461e+01	5.826771e+01	5.987068e+01	5.983906e+01	5.983326e+01	5.972221e+01
4D	med.	5.986745e+01	5.984317e+01	5.985897e+01	5.973802e+01	5.987540e+01	5.924392e+01	5.983309e+01	5.978325e+01	5.985181e+01	5.987284e+01	5.984422e+01	5.985318e+01	5.973823e+01
	std.	1.414396e-03	5.839059e-04	4.073780e-03	1.847066e-03	3.554225e-04	9.973197e-01	2.993668e-01	2.936249e-01	3.069391e-01	4.307426e-04	9.635920e-04	4.171840e-03	3.067849e-03
	med.	2.381492e+02	2.388869e+02	2.384235e+02	2.381687e+02	2.385305e+02	2.255927e+02	2.377670e+02	2.367137e+02	2.387219e+02	2.393174e+02	2.388254e+02	2.392075e+02	2.381392e+02
	avg.	2.381213e+02	2.388775e+02	2.381588e+02	2.381681e+02	2.384699e+02	2.265212e+02	2.376615e+02	2.365548e+02	2.387022e+02	2.393029e+02	2.388414e+02	2.391977e+02	2.381417e+02
5D	min.	2.375795e+02	2.378144e+02	2.364138e+02	2.381181e+02	2.374674e+02	2.157342e+02	2.352136e+02	2.330736e+02	2.368650e+02	2.389353e+02	2.387546e+02	2.387306e+02	2.380980e+02
	max.	2.385441e+02	2.391163e+02	2.390895e+02	2.382059e+02	2.390961e+02	2.357597e+02	2.388759e+02	2.390029e+02	2.393266e+02	2.395660e+02	2.391098e+02	2.395256e+02	2.381848e+02
	std.	2.201643e-01	1.577649e-01	6.918479e-01	1.362063e-02	3.177071e-01	4.885508e+00	6.545821e-01	1.107709e+00	3.267352e-01	1.355417e-01	6.435098e-02	1.587000e-01	1.945659e-02
6D	med.	9.476034e+02	9.454845e+02	9.450054e+02	9.463834e+02	9.117782e+02	8.663383e+02	9.490793e+02	9.458106e+02	9.541181e+02	9.586628e+02	9.457171e+02	9.497518e+02	9.471577e+02
	avg.	9.471790e+02	9.446491e+02	9.450232e+02	9.464136e+02	9.135127e+02	8.695492e+02	9.488449e+02	9.454277e+02	9.539682e+02	9.586229e+02	9.459043e+02	9.497604e+02	9.470515e+02
	min.	9.387188e+02	9.380837e+02	9.374721e+02	9.450435e+02	8.440617e+02	8.088966e+02	9.427675e+02	9.337932e+02	9.490681e+02	9.570900e+02	9.452696e+02	9.451472e+02	9.461580e+02
	max.	9.524148e+02	9.504251e+02	9.519282e+02	9.473757e+02	9.549613e+02	9.339269e+02	9.544960e+02	9.533241e+02	9.569942e+02	9.597234e+02	9.484153e+02	9.53358e+02	9.482842e+02
7D	std.	2.325150e+00	2.061564e+00	3.023611e+00	3.932463e-01	3.371316e+01	2.526097e+01	2.537586e+00	4.524202e+00	1.623245e+00	5.152468e-01	5.100080e-01	1.790231e+00	4.143114e-01
	med.	3.736509e+03	3.752916e+03	3.754141e+03	3.759985e+03	3.295496e+03	3.702004e+03	3.784296e+03	3.782469e+03	3.806124e+03	3.832566e+03	3.752983e+03	3.770434e+03	3.770343e+03
	avg.	3.733598e+03	3.746428e+03	3.756554e+03	3.758882e+03	3.305928e+03	3.599989e+03	3.781805e+03	3.778322e+03	3.806603e+03	3.832533e+03	3.752934e+03	3.770655e+03	3.770362e+03
	min.	3.657452e+03	3.689267e+03	3.707190e+03	3.741322e+03	3.145706e+03	3.180297e+03	3.742643e+03	3.720706e+03	3.793585e+03	3.825664e+03	3.749763e+03	3.749975e+03	3.763968e+03
8D	max.	3.777694e+03	3.761177e+03	3.793439e+03	3.773582e+03	3.499108e+03	3.780341e+03	3.803193e+03	3.810248e+03	3.819079e+03	3.837932e+03	3.763958e+03	3.798445e+03	3.777086e+03
	std.	2.241531e+01	1.285490e+01	1.530130e+01	6.537366e+00	7.605954e+01	1.743838e+02	1.291392e+01	1.965152e+01	5.437745e+00	2.335078e+00	1.983541e+00	1.120980e+01	2.767205e+00
	med.	1.473506e+04	1.491922e+04	1.489100e+04	1.493145e+04	1.164738e+04	1.197908e+04	1.490074e+04	1.499258e+04	1.508832e+04	1.527910e+04	1.492062e+04	1.494703e+04	1.493659e+04
	avg.	1.468789e+04	1.491383e+04	1.488319e+04	1.492906e+04	1.164894e+04	1.194205e+04	1.489161e+04	1.497953e+04	1.508390e+04	1.527346e+04	1.492060e+04	1.495188e+04	1.491422e+04
9D	min.	1.385514e+04	1.481252e+04	1.474773e+04	1.487578e+04	1.135994e+04	1.038981e+04	1.454943e+04	1.465079e+04	1.497670e+04	1.521042e+04	1.491637e+04	1.482753e+04	1.228801e+04
	max.	1.498080e+04	1.493584e+04	1.501803e+04	1.497385e+04	1.195748e+04	1.313440e+04	1.508953e+04	1.513865e+04	1.517323e+04	1.530555e+04	1.492518e+04	1.506193e+04	1.500021e+04
	std.	2.044862e+02	2.188129e+01	5.296301e+01	1.995835e+01	1.152769e+02	5.653585e+02	1.074693e+02	1.049027e+02	3.554445e+01	2.329846e+01	1.836667e+00	4.883167e+01	2.656117e+02
	med.	5.757684e+04	5.954174e+04	5.948930e+04	5.948859e+04	4.972209e+04	4.739453e+04	6.018290e+04	6.023941e+04	6.068430e+04	6.119227e+04	5.978587e+04	5.981579e+04	5.962648e+04
10D	avg.	5.718632e+04	5.941622e+04	5.947072e+04	5.949243e+04	4.969802e+04	4.774374e+04	6.016760e+04	6.018728e+04	6.067652e+04	6.116308e+04	5.978473e+04	5.983358e+04	5.964046e+04
	min.	4.924415e+04	5.787595e+04	5.839421e+04	5.934399e+04	4.788538e+04	4.371038e+04	5.943169e+04	5.875184e+04	5.995163e+04	6.085203e+04	5.976953e+04	5.947990e+04	5.957605e+04
	max.	5.904669e+04	5.966074e+04	6.010016e+04	5.971451e+04	5.124033e+04	5.853645e+04	6.061351e+04	6.082294e+04	6.088010e+04	6.128942e+04	5.980834e+04	6.038215e+04	5.990995e+04
	std.	1.426610e+03	3.024604e+02	2.704078e+02	7.145102e+01	7.247018e+02	2.726543e+03	2.532080e+02	3.606234e+02	1.339872e+02	1.006032e+02	6.507176e+00	1.747192e+02	5.226583e+01
9D	med.	2.280304e+05	2.376416e+05	2.380527e+05	2.373668e+05	2.407108e+05	1.904470e+05	2.420702e+05	2.417916e+05	2.435827e+05	2.451339e+05	2.398837e+05	2.392650e+05	2.381565e+05
	avg.	2.276276e+05	2.371870e+05	2.380792e+05	2.374341e+05	2.396177e+05	1.924328e+05	2.420771e+05	2.416817e+05	2.434721e+05	2.450340e+05	2.398836e+05	2.392163e+05	2.382060e+05
	min.	2.124206e+05	2.337533e+05	2.284056e+05	2.370485e+05	2.154540e+05	1.697838e+05	2.404748e+05	2.377301e+05	2.405608e+05	2.437626e+05	2.398028e+05	2.359967e+05	2.379745e+05
	max.	2.353520e+05	2.384989e+05	2.408822e+05	2.382538e+05	2.449261e+05	2.346567e+05	2.434202e+05	2.433761e+05	2.445810e+05	2.454298e+05	2.401026e+05	2.410058e+05	2.389368e+05
10D	std.	4.298323e+03	9.518471e+02	1.522618e+03	4.888284e+02	4.888284e+02	1.260591e+05	7.139773e+02	6.074697e+02	3.412859e+02	3.109416e+01	8.033314e+02	1.909000e+02	1.909000e+02
	med.	9.096627e+05	9.487965e+05	9.519180e+05	9.476140e+05	9.770070e+05	7.754288e+05	9.713438e+05	9.693654e+05	9.765996e+05	9.813316e+05	9.613636e+05	9.575009e+05	9.518069e+05
	avg.	9.075072e+05	9.480562e+05	9.504779e+05	9.477215e+05	9.770444e+05	7.847471e+05	9.711617e+05	9.687502e+05	9.764990e+05	9.810410e+05	9.610642e+05	9.575279e+05	9.521429e+05
	min.	8.642879e+05	9.392637e+05	9.304636e+05	9.465297e+05	9.725713e+05	6.951348e+05	9.656803e+05	9.735714e+05	9.735714e+05	9.751037e+05	9.587309e+05	9.469802e+05	9.507105e+05
10D	max.	9.340905e+05	9.543243e+05	9.615215e+05	9.498431e+05	9.797380e+05	9.321077e+05	9.746233e+05	9.743584e+05	9.782059e+05	9.828043e+05	9.617521e+05	9.647401e+05	9.560001e+05
	std.	1.303655e+04	3.972481e+03	6.303247e+03	6.929602e+02	1.395012e+03	4.865684e+04	2.193105e+03	3.129130e+03	9.817994e+02	1.266587e+03	8.329607e+02	3.208374e+03	1.085091e+03

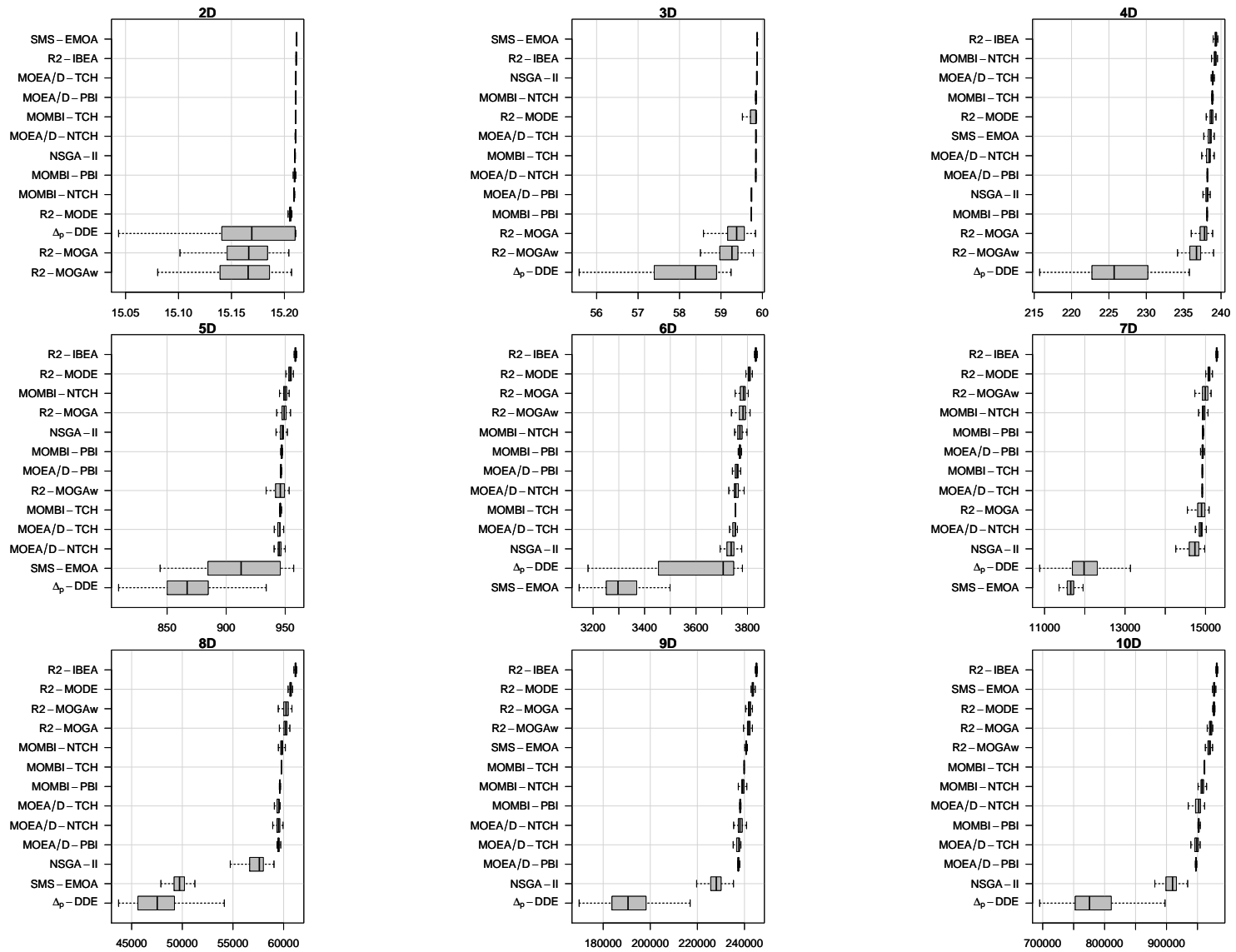


Figure A.65: Box-plot of hypervolume indicator values for different optimizers on the DTLZ5 test problem.

Table A.27: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ5 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	2.83e-09	1.27e-34	1.27e-34	1.27e-34	> 0.05	> 0.05	8.25e-33	2.12e-04
MOEA/D-TCH	9.95e-35	—	1.99e-32	5.80e-07	> 0.05	1.00e-34	1.00e-34	1.00e-34	1.00e-34	> 0.05	1.29e-20	9.99e-35	4.32e-34
MOEA/D-NTCH	5.10e-31	> 0.05	—	> 0.05	> 0.05	9.57e-29	1.27e-34	1.27e-34	1.27e-34	> 0.05	> 0.05	2.44e-34	3.88e-26
MOEA/D-PBI	1.29e-34	> 0.05	5.77e-21	—	> 0.05	4.94e-34	1.21e-34	1.21e-34	1.21e-34	> 0.05	6.21e-08	1.20e-34	1.10e-32
SMS-EMOA	7.35e-35	5.79e-35	7.34e-35	7.00e-35	—	7.41e-35	7.42e-35	7.42e-35	7.41e-35	7.23e-35	7.34e-35	7.38e-35	7.39e-35
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	2.55e-02	9.78e-03	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	3.45e-04	1.71e-28	1.26e-26	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	1.24e-34	9.79e-35	1.24e-34	1.18e-34	> 0.05	1.25e-34	1.25e-34	1.25e-34	1.25e-34	—	1.24e-34	1.24e-34	1.25e-34
MOMBI-TCH	5.53e-34	> 0.05	7.02e-07	> 0.05	> 0.05	5.07e-32	1.27e-34	1.27e-34	1.27e-34	> 0.05	—	1.26e-34	3.25e-30
MOMBI-NTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	5.49e-06	1.28e-34	1.28e-34	1.27e-34	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	2.46e-09	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	9.56e-12	—
3D													
NSGA-II	—	1.26e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34
MOEA/D-TCH	> 0.05	—	1.01e-09	1.26e-34	> 0.05	1.26e-34	1.26e-34	1.26e-34	> 0.05	> 0.05	9.08e-11	> 0.05	1.26e-34
MOEA/D-NTCH	> 0.05	> 0.05	—	1.28e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	1.28e-34
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	> 0.05	1.28e-34	2.48e-30	4.76e-32	> 0.05	> 0.05	> 0.05	> 0.05	2.24e-15
SMS-EMOA	1.27e-34	1.24e-34	1.27e-34	1.26e-34	—	1.27e-34	1.27e-34	1.27e-34	1.27e-34	1.26e-34	1.26e-34	1.27e-34	1.27e-34
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	7.10e-27	—	1.11e-03	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	2.08e-22	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	> 0.05	> 0.05	> 0.05	1.88e-09	> 0.05	9.79e-32	8.06e-20	6.25e-23	—	> 0.05	> 0.05	> 0.05	8.89e-10
R2-IBEA	1.28e-34	1.26e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	> 0.05	> 0.05	3.27e-07	1.28e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	—	> 0.05	1.28e-34
MOMBI-NTCH	> 0.05	> 0.05	2.38e-04	1.28e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	6.30e-03	—	1.28e-34
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.28e-34	1.09e-29	5.05e-32	> 0.05	> 0.05	> 0.05	> 0.05	—
4D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	1.28e-34	1.85e-09	2.56e-26	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	3.27e-33	—	4.71e-28	2.51e-33	2.42e-23	1.28e-34	5.54e-33	2.66e-33	9.35e-07	> 0.05	1.56e-08	> 0.05	2.51e-33
MOEA/D-NTCH	9.87e-07	> 0.05	—	6.77e-10	> 0.05	1.28e-34	9.88e-10	2.40e-22	> 0.05	> 0.05	> 0.05	> 0.05	5.23e-10
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	> 0.05	1.28e-34	6.59e-15	3.23e-28	> 0.05	> 0.05	> 0.05	> 0.05	2.53e-19
SMS-EMOA	7.83e-17	> 0.05	1.26e-03	3.84e-20	—	1.28e-34	2.19e-21	1.93e-30	> 0.05	> 0.05	> 0.05	> 0.05	7.64e-21
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.49e-34	—	2.30e-14	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	2.82e-33	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	9.72e-28	> 0.05	1.03e-14	1.94e-29	8.97e-09	1.28e-34	3.06e-28	6.18e-32	—	> 0.05	> 0.05	> 0.05	1.36e-29
R2-IBEA	1.28e-34	5.71e-33	1.68e-34	1.28e-34	1.89e-34	1.28e-34	1.28e-34	1.44e-34	9.39e-33	—	4.37e-34	8.80e-07	1.28e-34
MOMBI-TCH	1.28e-34	> 0.05	5.18e-28	1.28e-34	9.19e-23	1.28e-34	1.52e-33	2.23e-33	1.16e-03	> 0.05	—	> 0.05	1.28e-34
MOMBI-NTCH	1.28e-34	7.85e-28	5.22e-34	1.28e-34	1.27e-33	1.28e-34	1.40e-34	1.84e-34	3.66e-29	> 0.05	1.54e-30	—	1.28e-34
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.28e-34	1.12e-13	2.00e-27	> 0.05	> 0.05	> 0.05	> 0.05	—

Table A.28: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ5 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
5D													
NSGA-II	–	4.54e – 15	1.67e – 09	3.68e – 07	9.74e – 18	1.28e – 34	> 0.05	9.10e – 03	> 0.05	> 0.05	2.53e – 10	> 0.05	1.60e – 03
MOEA/D-TCH	> 0.05	–	> 0.05	> 0.05	6.57e – 06	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	–	> 0.05	4.67e – 07	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	1.90e – 23	2.92e – 09	–	9.02e – 22	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	1.66e – 14	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	–	1.19e – 16	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	2.22e – 06	1.42e – 22	6.19e – 17	1.35e – 14	1.04e – 21	1.28e – 34	–	2.73e – 08	> 0.05	> 0.05	3.45e – 18	> 0.05	7.75e – 11
R2-MOGAw	> 0.05	1.42e – 02	> 0.05	> 0.05	1.66e – 10	1.32e – 34	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	7.92e – 34	1.40e – 34	4.50e – 34	1.28e – 34	5.14e – 30	1.28e – 34	2.23e – 29	1.33e – 32	–	> 0.05	1.28e – 34	4.09e – 29	1.28e – 34
R2-IBEA	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.32e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	–	1.28e – 34	1.28e – 34	1.28e – 34
MOMBI-TCH	> 0.05	1.31e – 08	8.28e – 05	> 0.05	4.55e – 11	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NTCH	4.12e – 15	3.89e – 32	8.32e – 25	1.03e – 29	1.44e – 26	1.28e – 34	5.10e – 03	2.27e – 13	> 0.05	> 0.05	5.83e – 32	–	1.17e – 25
MOMBI-PBI	> 0.05	9.21e – 30	6.11e – 14	3.80e – 19	1.08e – 24	1.28e – 34	> 0.05	1.79e – 02	> 0.05	> 0.05	4.78e – 28	> 0.05	–
6D													
NSGA-II	–	> 0.05	> 0.05	> 0.05	1.28e – 34	1.77e – 05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	6.21e – 07	–	> 0.05	> 0.05	1.28e – 34	7.38e – 11	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	1.72e – 14	1.89e – 04	–	> 0.05	1.28e – 34	1.02e – 17	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	8.72e – 20	6.05e – 15	1.61e – 02	–	1.28e – 34	7.22e – 23	> 0.05	> 0.05	> 0.05	> 0.05	1.10e – 10	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	6.60e – 24	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	8.73e – 32	2.18e – 32	2.70e – 22	3.87e – 27	1.28e – 34	2.83e – 32	–	> 0.05	> 0.05	> 0.05	8.00e – 32	8.25e – 10	1.29e – 13
R2-MOGAw	2.91e – 26	4.67e – 24	1.82e – 15	9.90e – 17	1.28e – 34	1.83e – 28	> 0.05	–	> 0.05	> 0.05	4.14e – 22	3.98e – 06	2.98e – 09
R2-MODE	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.09e – 32	5.68e – 29	–	> 0.05	1.28e – 34	1.68e – 34	1.28e – 34
R2-IBEA	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	–	1.28e – 34	1.28e – 34	1.28e – 34
MOMBI-TCH	3.13e – 14	2.05e – 02	> 0.05	> 0.05	1.28e – 34	9.36e – 16	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NTCH	1.17e – 27	6.07e – 31	2.21e – 12	3.28e – 14	1.28e – 34	1.06e – 29	> 0.05	> 0.05	> 0.05	> 0.05	3.56e – 29	–	> 0.05
MOMBI-PBI	2.70e – 30	1.28e – 34	4.80e – 15	2.55e – 30	1.28e – 34	2.12e – 31	> 0.05	> 0.05	> 0.05	> 0.05	1.28e – 34	> 0.05	–
7D													
NSGA-II	–	> 0.05	> 0.05	> 0.05	1.28e – 34	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	5.39e – 28	–	1.94e – 08	> 0.05	1.28e – 34	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	2.13e – 20	> 0.05	–	> 0.05	1.28e – 34	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	6.25e – 31	4.20e – 08	1.93e – 14	–	1.28e – 34	1.28e – 34	7.47e – 03	> 0.05	> 0.05	> 0.05	2.22e – 06	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	2.20e – 10	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	6.86e – 17	> 0.05	> 0.05	> 0.05	1.28e – 34	1.28e – 34	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	9.99e – 27	2.15e – 11	9.21e – 15	1.22e – 09	1.28e – 34	1.28e – 34	6.02e – 09	–	> 0.05	> 0.05	1.19e – 10	9.86e – 05	5.32e – 08
R2-MODE	1.32e – 34	1.28e – 34	1.58e – 34	1.28e – 34	1.28e – 34	1.28e – 34	5.35e – 32	2.39e – 17	–	> 0.05	1.28e – 34	1.37e – 32	1.49e – 34
R2-IBEA	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	–	1.28e – 34	1.28e – 34	1.28e – 34
MOMBI-TCH	1.32e – 29	4.89e – 05	1.79e – 11	> 0.05	1.28e – 34	1.28e – 34	2.83e – 02	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NTCH	6.61e – 31	1.43e – 11	1.93e – 16	7.11e – 05	1.28e – 34	1.28e – 34	2.38e – 05	> 0.05	> 0.05	> 0.05	3.56e – 10	–	2.57e – 02
MOMBI-PBI	5.27e – 31	3.18e – 32	8.00e – 21	2.10e – 04	1.28e – 34	2.79e – 34	4.36e – 04	> 0.05	> 0.05	> 0.05	2.51e – 33	> 0.05	–

Table A.29: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ5 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
8D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	1.10e-33	5.23e-33	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.81e-33	—	> 0.05	> 0.05	1.28e-34	1.44e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	2.79e-34	> 0.05	—	> 0.05	1.28e-34	1.32e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	> 0.05	> 0.05	—	1.28e-34	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	1.41e-12	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	6.00e-32	3.29e-30	3.18e-32	1.28e-34	1.28e-34	—	> 0.05	> 0.05	> 0.05	4.77e-25	3.80e-19	4.56e-29
R2-MOGAw	1.49e-34	1.36e-28	3.98e-27	9.74e-30	1.28e-34	1.28e-34	> 0.05	—	> 0.05	> 0.05	7.15e-19	3.14e-15	7.96e-23
R2-MODE	1.28e-34	1.28e-34	1.44e-34	1.28e-34	1.28e-34	1.28e-34	3.07e-31	7.58e-26	—	> 0.05	1.28e-34	2.71e-34	1.28e-34
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.44e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	1.28e-34	1.28e-34	4.24e-22	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	4.08e-31
MOMBI-NTCH	1.28e-34	4.73e-30	3.49e-21	3.54e-31	1.28e-34	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	1.55e-02	—	1.66e-18
MOMBI-PBI	1.28e-34	2.52e-31	3.57e-12	4.08e-31	1.28e-34	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—
9D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	1.15e-32	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	2.01e-34	—	> 0.05	> 0.05	> 0.05	1.40e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	6.06e-34	4.17e-08	—	1.94e-08	> 0.05	1.36e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	> 0.05	> 0.05	—	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	3.23e-28	4.07e-26	6.62e-25	4.07e-26	—	2.40e-34	> 0.05	> 0.05	> 0.05	> 0.05	5.82e-24	1.41e-23	4.07e-26
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.78e-34	1.28e-34	5.41e-25	1.28e-34	—	6.73e-03	> 0.05	> 0.05	1.28e-34	2.01e-34	1.28e-34
R2-MOGAw	1.28e-34	5.07e-34	4.01e-32	1.68e-34	9.73e-16	1.28e-34	> 0.05	—	> 0.05	> 0.05	3.05e-27	4.32e-29	4.39e-33
R2-MODE	1.28e-34	1.28e-34	1.36e-34	1.28e-34	1.28e-28	1.28e-34	2.47e-27	3.98e-29	—	> 0.05	1.28e-34	1.40e-34	1.28e-34
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.10e-33	1.28e-34	1.28e-34	1.28e-34	1.81e-33	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	1.28e-34	1.28e-34	8.32e-29	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	—	5.46e-14	1.28e-34
MOMBI-NTCH	1.28e-34	1.54e-30	9.60e-12	4.37e-32	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	3.85e-22
MOMBI-PBI	1.28e-34	2.90e-30	> 0.05	8.11e-33	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—
10D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	1.20e-31	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	> 0.05	2.09e-03	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	1.49e-34	1.22e-05	—	1.69e-08	> 0.05	1.36e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	> 0.05	> 0.05	—	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	5.21e-34	1.83e-34	1.34e-03	> 0.05	1.28e-34	1.28e-34	1.28e-34
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	—	8.01e-09	> 0.05	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-MOGAw	1.28e-34	1.28e-34	1.78e-34	1.28e-34	> 0.05	1.28e-34	> 0.05	—	> 0.05	> 0.05	3.25e-31	2.37e-33	1.28e-34
R2-MODE	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	3.76e-34	2.13e-34	—	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	6.60e-31	1.28e-34	1.28e-34	1.28e-34	2.82e-31	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	1.28e-34	1.28e-34	2.82e-33	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	—	1.53e-18	1.28e-34
MOMBI-NTCH	1.28e-34	1.64e-31	3.05e-19	1.81e-33	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	3.40e-26
MOMBI-PBI	1.28e-34	1.15e-17	> 0.05	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—

Table A.30: Comparison of R2 indicator values for different optimizers on the DTLZ5 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	2.181334e-01	2.180305e-01	2.180300e-01	2.180298e-01	2.178756e-01	2.181498e-01	2.192151e-01	2.186528e-01	2.188078e-01	2.179243e-01	2.180309e-01	2.182556e-01	2.180292e-01
	avg.	2.181335e-01	2.180305e-01	2.180383e-01	2.180299e-01	2.178848e-01	2.181930e-01	2.192464e-01	2.187008e-01	2.188621e-01	2.179261e-01	2.180309e-01	2.182577e-01	2.180295e-01
	min.	2.180473e-01	2.180291e-01	2.180158e-01	2.180284e-01	2.178041e-01	2.180211e-01	2.187412e-01	2.183425e-01	2.185785e-01	2.178593e-01	2.180284e-01	2.181632e-01	2.180246e-01
	max.	2.182700e-01	2.180319e-01	2.182281e-01	2.180314e-01	2.180133e-01	2.188717e-01	2.206341e-01	2.201386e-01	2.199213e-01	2.180256e-01	2.180335e-01	2.183864e-01	2.180347e-01
	std.	3.536143e-05	5.242750e-07	3.039034e-05	5.384028e-07	4.102978e-05	1.395526e-04	3.015797e-04	2.359799e-04	2.023252e-04	3.327049e-05	1.100903e-06	4.327053e-05	1.975545e-06
3D	med.	1.631667e-01	1.637211e-01	1.637662e-01	1.665221e-01	1.629258e-01	1.790041e-01	1.639361e-01	1.636758e-01	1.636697e-01	1.629652e-01	1.637222e-01	1.637317e-01	1.665135e-01
	avg.	1.631724e-01	1.637212e-01	1.637213e-01	1.665215e-01	1.629260e-01	1.800490e-01	1.639793e-01	1.637210e-01	1.637672e-01	1.629647e-01	1.637191e-01	1.637324e-01	1.665111e-01
	min.	1.630833e-01	1.637181e-01	1.632671e-01	1.664893e-01	1.628890e-01	1.736858e-01	1.635299e-01	1.632965e-01	1.634564e-01	1.629301e-01	1.636905e-01	1.634764e-01	1.664409e-01
	max.	1.632940e-01	1.637256e-01	1.638876e-01	1.665297e-01	1.629566e-01	1.901311e-01	1.652802e-01	1.645740e-01	1.664236e-01	1.629988e-01	1.637249e-01	1.640047e-01	1.665325e-01
	std.	3.915609e-05	1.073608e-06	1.083072e-04	5.439140e-06	1.364267e-05	3.504393e-03	2.739455e-04	2.778189e-04	4.087744e-04	1.476912e-05	6.858297e-06	1.251259e-04	1.451684e-05
4D	med.	1.226013e-01	1.197294e-01	1.210779e-01	1.228343e-01	1.211205e-01	1.354966e-01	1.208644e-01	1.197552e-01	1.207654e-01	1.191263e-01	1.198075e-01	1.198472e-01	1.228671e-01
	avg.	1.225991e-01	1.197921e-01	1.220546e-01	1.228389e-01	1.212039e-01	1.359217e-01	1.209234e-01	1.198199e-01	1.208679e-01	1.191829e-01	1.198069e-01	1.198833e-01	1.228654e-01
	min.	1.212066e-01	1.195628e-01	1.199213e-01	1.227059e-01	1.197871e-01	1.291156e-01	1.197141e-01	1.186779e-01	1.194442e-01	1.185106e-01	1.196204e-01	1.191031e-01	1.227110e-01
	max.	1.239841e-01	1.229385e-01	1.276558e-01	1.229659e-01	1.239044e-01	1.471371e-01	1.227153e-01	1.211800e-01	1.222199e-01	1.201630e-01	1.199375e-01	1.211633e-01	1.229535e-01
	std.	5.594708e-04	3.875948e-04	2.222687e-03	5.807994e-05	8.033765e-04	3.327945e-03	6.365193e-04	5.350187e-04	5.483773e-04	3.611154e-04	7.068570e-05	3.689532e-04	4.805820e-05
5D	med.	9.844921e-02	9.727494e-02	9.722167e-02	9.660812e-02	9.757746e-02	1.101376e-01	9.315964e-02	9.349985e-02	9.286899e-02	9.115182e-02	9.713715e-02	9.648070e-02	9.648276e-02
	avg.	9.880209e-02	9.745485e-02	9.723088e-02	9.655220e-02	9.789445e-02	1.103936e-01	9.324738e-02	9.360004e-02	9.295802e-02	9.115991e-02	9.706757e-02	9.648801e-02	9.646228e-02
	min.	9.539402e-02	9.527937e-02	9.428158e-02	9.581019e-02	9.119817e-02	1.021372e-01	9.233559e-02	9.216778e-02	9.185306e-02	9.088545e-02	9.592742e-02	9.381806e-02	9.566936e-02
	max.	1.068938e-01	9.884548e-02	1.021029e-01	9.684517e-02	1.051115e-01	1.220571e-01	9.486220e-02	9.565858e-02	9.493756e-02	9.156444e-02	9.736310e-02	9.847785e-02	9.684557e-02
	std.	1.850465e-03	5.134977e-04	1.715263e-03	1.982045e-04	2.819346e-03	4.156328e-03	5.485049e-04	8.551662e-04	6.303419e-04	1.261112e-04	2.216038e-04	1.077435e-03	1.883399e-04
6D	med.	9.142075e-02	8.210479e-02	8.272963e-02	8.228143e-02	9.108066e-02	9.032442e-02	7.869928e-02	7.801879e-02	7.818461e-02	7.703090e-02	8.209204e-02	8.219615e-02	8.180775e-02
	avg.	9.119737e-02	8.238266e-02	8.316259e-02	8.235240e-02	9.097077e-02	9.142776e-02	7.868547e-02	7.808743e-02	7.821519e-02	7.702608e-02	8.219481e-02	8.233194e-02	8.187355e-02
	min.	8.527641e-02	8.201980e-02	8.038458e-02	8.160440e-02	8.406156e-02	8.597147e-02	7.764547e-02	7.716123e-02	7.750883e-02	7.654629e-02	8.194015e-02	8.018092e-02	8.125119e-02
	max.	9.992168e-02	8.400212e-02	8.690557e-02	8.393781e-02	9.866174e-02	1.104898e-01	7.956150e-02	7.930541e-02	7.912412e-02	7.812348e-02	8.278909e-02	8.462744e-02	8.237186e-02
	std.	3.160383e-03	4.898701e-04	1.699255e-03	4.713869e-04	3.178654e-03	4.205474e-03	4.063599e-04	4.477641e-04	2.969001e-04	2.251335e-04	2.327451e-04	2.471861e-04	2.471861e-04
7D	med.	8.082692e-02	7.181844e-02	7.322342e-02	7.230490e-02	9.636790e-02	1.001280e-01	6.972536e-02	6.774630e-02	6.888882e-02	6.672498e-02	7.159369e-02	7.401381e-02	7.205100e-02
	avg.	8.207820e-02	7.190164e-02	7.328229e-02	7.235988e-02	9.644044e-02	1.016412e-01	6.985346e-02	6.783788e-02	6.896592e-02	6.698715e-02	7.161944e-02	7.408474e-02	7.274581e-02
	min.	7.518849e-02	7.155601e-02	7.133590e-02	7.176971e-02	9.167611e-02	8.568655e-02	6.837788e-02	6.673609e-02	6.789185e-02	6.635524e-02	7.144155e-02	7.144322e-02	7.131994e-02
	max.	9.930509e-02	7.352782e-02	7.617273e-02	7.322365e-02	1.007862e-01	1.289852e-01	7.195107e-02	6.958077e-02	7.065201e-02	6.921605e-02	7.189978e-02	7.698535e-02	1.424996e-01
	std.	4.682316e-03	3.501008e-04	1.014240e-03	3.087942e-04	1.878282e-03	9.179097e-03	7.207816e-04	5.904033e-04	4.782342e-04	7.532798e-04	1.067733e-04	1.243592e-03	7.048391e-03
8D	med.	7.823848e-02	6.334000e-02	6.474856e-02	6.424670e-02	7.453901e-02	8.794863e-02	5.966377e-02	5.865928e-02	5.915376e-02	5.845994e-02	6.283453e-02	6.442458e-02	6.373463e-02
	avg.	7.926682e-02	6.361295e-02	6.485239e-02	6.423840e-02	7.471140e-02	8.858844e-02	5.968625e-02	5.876862e-02	5.920997e-02	5.864765e-02	6.284840e-02	6.475150e-02	6.371979e-02
	min.	6.889689e-02	6.292461e-02	6.209759e-02	6.374467e-02	7.131173e-02	7.357422e-02	5.888699e-02	5.800079e-02	5.845661e-02	5.799262e-02	6.264790e-02	6.282630e-02	6.313520e-02
	max.	1.037126e-01	6.598624e-02	7.374135e-02	6.482383e-02	8.022358e-02	1.033207e-01	6.079451e-02	6.031660e-02	6.039654e-02	6.026574e-02	6.306590e-02	6.700108e-02	6.391991e-02
	std.	5.952748e-03	6.106602e-04	1.458642e-03	2.600259e-04	1.910019e-03	6.641234e-03	3.743210e-04	4.320206e-04	3.877345e-04	5.282206e-04	9.802047e-05	1.008969e-03	1.343925e-04
9D	med.	7.024303e-02	5.577090e-02	5.675220e-02	5.585061e-02	5.310075e-02	7.549161e-02	5.140701e-02	5.123125e-02	5.106484e-02	5.087591e-02	5.486080e-02	5.663287e-02	5.543774e-02
	avg.	7.028454e-02	5.592783e-02	5.682332e-02	5.583406e-02	5.316262e-02	7.583208e-02	5.140778e-02	5.123718e-02	5.107277e-02	5.102596e-02	5.488387e-02	5.666391e-02	5.543213e-02
	min.	5.992171e-02	5.516127e-02	5.492581e-02	5.533994e-02	5.116385e-02	6.304190e-02	5.088688e-02	5.059644e-02	5.062853e-02	5.058006e-02	5.471816e-02	5.401640e-02	5.499232e-02
	max.	8.299126e-02	5.753665e-02	6.027745e-02	5.624203e-02	5.718706e-02	9.026150e-02	5.202188e-02	5.185417e-02	5.198640e-02	5.228133e-02	5.512090e-02	5.911128e-02	5.561937e-02
	std.	4.363042e-03	5.979971e-04	1.016722e-03	2.000502e-04	7.578486e-04	6.124185e-03	2.175487e-04	2.635083e-04	1.944015e-04	3.744282e-04	9.698606e-05	8.570220e-04	1.036402e-04
10D	med.	6.369176e-02	5.101759e-02	5.174689e-02	5.096546e-02	4.652352e-02	6.601135e-02	4.608052e-02	4.607441e-02	4.579801e-02	4.576019e-02	4.988812e-02	5.134029e-02	5.023981e-02
	avg.	6.382425e-02	5.111607e-02	5.187417e-02	5.095366e-02	4.653956e-02	6.639123e-02	4.607268e-02	4.607972e-02	4.578894e-02	4.588130e-02	4.990233e-02	5.131143e-02	5.020642e-02
	min.	5.742419e-02	5.021353e-02	4.954568e-02	5.053002e-02	4.631966e-02	5.548214e-02	4.571020e-02	4.571457e-02	4.560232e-02	4.558482e-02	4.972606e-02	4.932539e-02	4.983454e-02
	max.	7.159107e-02	5.274406e-02	5.538864e-02	5.130834e-02	4.710799e-02	8.298633e-02	6.635981e-02	4.671091e-02	4.612002e-02	4.709687e-02	5.015120e-02	5.336310e-02	5.043609e-02
	std.	2.871961e-03	5.628751e-04	1.155519e-03	1.713727e-04	1.451387e-04	5.093501e-03	1.143775e-04	1.775748e-04	9.606347e-05	2.843421e-04	9.750860e-05	7.527188e-04	1.444112e-04

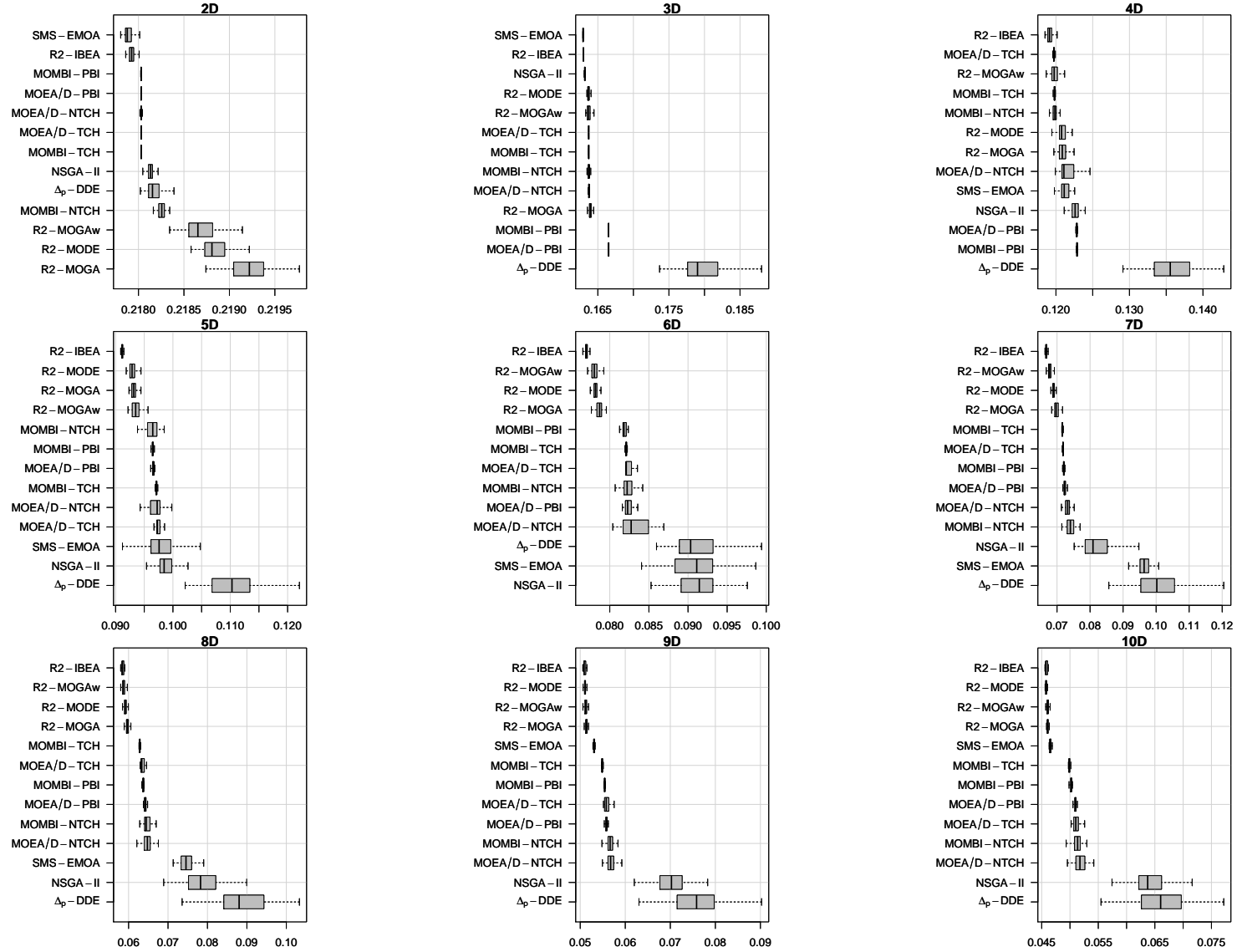


Figure A.66: Box-plot of R2 indicator values for different optimizers on the DTLZ5 test problem.

Table A.31: Comparison of runtime (in milliseconds) for different optimizers on the DTLZ5 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	2.232770e + 02	5.584210e + 02	1.116694e + 03	9.337450e + 02	2.799686e + 04	1.150029e + 03	1.835821e + 03	1.228260e + 03	1.537194e + 03	5.693810e + 05	1.944833e + 03	1.543641e + 03	2.741022e + 03
	avg.	2.392229e + 02	5.802404e + 02	1.110664e + 03	9.227708e + 02	2.781917e + 04	1.209043e + 03	1.836980e + 03	1.234741e + 03	1.535599e + 03	5.692251e + 05	1.929115e + 03	1.593375e + 03	2.734148e + 03
	min.	1.968380e + 02	4.029160e + 02	8.571710e + 02	7.106890e + 02	2.037865e + 04	1.083424e + 03	1.357348e + 03	8.406060e + 02	1.094640e + 03	5.471100e + 05	1.473874e + 03	1.517033e + 03	2.343060e + 03
	max.	3.741330e + 02	9.271390e + 02	1.707662e + 03	1.265728e + 03	3.217385e + 04	1.708024e + 03	2.320167e + 03	1.631173e + 03	1.982425e + 03	5.959530e + 05	2.209237e + 03	2.215422e + 03	3.139513e + 03
	std.	4.431087e + 01	1.009191e + 02	1.103173e + 02	9.597199e + 01	3.127722e + 03	1.475898e + 02	2.024121e + 02	1.588263e + 02	1.903568e + 02	7.956540e + 03	1.698519e + 02	1.310940e + 02	1.278091e + 02
3D	med.	2.748550e + 02	6.274840e + 02	1.419746e + 03	1.035723e + 03	8.045436e + 05	2.251899e + 03	2.275795e + 03	1.578171e + 03	2.081004e + 03	6.064750e + 05	1.926433e + 03	2.014053e + 03	3.027025e + 03
	avg.	2.824660e + 02	6.360369e + 02	1.418172e + 03	1.046373e + 03	7.938087e + 05	2.181699e + 03	2.272512e + 03	1.606012e + 03	2.048753e + 03	5.933956e + 05	1.934730e + 03	2.038804e + 03	2.986343e + 03
	min.	2.332970e + 02	4.769260e + 02	9.610290e + 02	7.853140e + 02	5.359883e + 05	1.453139e + 03	1.688285e + 03	1.145041e + 03	1.520893e + 03	4.808900e + 05	1.505288e + 03	1.689082e + 03	2.427722e + 03
	max.	4.049480e + 02	8.929390e + 02	1.785420e + 03	1.789430e + 03	9.318544e + 05	2.692024e + 03	2.747102e + 03	2.001256e + 03	2.453706e + 03	6.278620e + 05	2.273037e + 03	2.598379e + 03	3.210746e + 03
	std.	4.119731e + 01	8.047658e + 01	1.082544e + 02	1.222755e + 02	9.827267e + 04	2.776247e + 02	2.335807e + 02	1.912156e + 02	2.131150e + 02	4.009651e + 04	1.660162e + 02	1.642401e + 02	1.429901e + 02
4D	med.	4.647770e + 02	6.509560e + 02	1.451979e + 03	1.041959e + 03	3.575752e + 06	2.452485e + 03	2.791112e + 03	2.122888e + 03	2.867206e + 03	6.349130e + 05	1.969801e + 03	2.173891e + 03	3.207919e + 03
	avg.	4.804161e + 02	6.559875e + 02	1.443758e + 03	1.028325e + 03	3.576286e + 06	2.461870e + 03	2.796892e + 03	2.094343e + 03	2.856876e + 03	6.351132e + 05	1.974878e + 03	2.181204e + 03	3.212483e + 03
	min.	3.779720e + 02	4.979510e + 02	1.136605e + 03	7.634990e + 02	2.278885e + 06	1.767556e + 03	2.164691e + 03	1.725935e + 03	2.223118e + 03	6.084550e + 05	1.571133e + 03	1.878561e + 03	2.850033e + 03
	max.	7.080480e + 02	8.954090e + 02	1.694470e + 03	1.549018e + 03	5.760827e + 06	3.199797e + 03	3.356950e + 03	2.421737e + 03	3.252945e + 03	6.522630e + 05	2.629191e + 03	2.505888e + 03	3.612532e + 03
	std.	7.410117e + 01	8.619185e + 01	1.175171e + 02	1.119539e + 02	6.476780e + 05	3.083582e + 02	2.623412e + 02	1.612345e + 02	2.308818e + 02	8.596848e + 03	2.162655e + 02	1.440440e + 02	1.349551e + 02
5D	med.	6.005900e + 02	6.819820e + 02	1.902182e + 03	1.027313e + 03	6.888291e + 06	4.937857e + 03	3.547637e + 03	2.733534e + 03	3.693451e + 03	6.854810e + 05	2.125683e + 03	2.246523e + 03	3.606172e + 03
	avg.	6.036660e + 02	6.938498e + 02	1.855312e + 03	1.028432e + 03	6.680422e + 06	4.840776e + 03	3.520110e + 03	2.733956e + 03	3.737532e + 03	6.544081e + 05	2.198548e + 03	2.268575e + 03	3.576433e + 03
	min.	4.782990e + 02	4.966990e + 02	1.267369e + 03	7.549720e + 02	3.586146e + 06	2.897110e + 03	2.616144e + 03	2.091014e + 03	2.873922e + 03	5.644260e + 05	1.766184e + 03	1.969256e + 03	2.951974e + 03
	max.	8.730180e + 02	1.293746e + 03	2.243496e + 03	1.624885e + 03	8.250534e + 06	6.098073e + 03	4.119195e + 03	3.211267e + 03	4.448730e + 03	7.046800e + 05	3.844083e + 03	2.889630e + 03	3.998704e + 03
	std.	6.771477e + 01	1.051159e + 02	1.747488e + 02	1.450821e + 02	9.777809e + 05	6.559412e + 02	3.112080e + 02	1.991280e + 02	3.502765e + 02	5.458163e + 04	2.999651e + 02	1.661612e + 02	2.179869e + 02
6D	med.	6.950440e + 02	6.883100e + 02	1.933412e + 03	1.042360e + 03	9.042010e + 06	4.282529e + 03	4.119173e + 03	3.152502e + 03	4.783278e + 03	7.052760e + 05	2.206915e + 03	2.376808e + 03	3.912522e + 03
	avg.	6.885234e + 02	7.024481e + 02	1.915494e + 03	1.040472e + 03	8.996729e + 06	4.376907e + 03	4.101566e + 03	3.120202e + 03	4.721710e + 03	7.049369e + 05	2.267056e + 03	2.404412e + 03	3.899217e + 03
	min.	5.592050e + 02	5.247740e + 02	1.476058e + 03	7.646840e + 02	7.061990e + 06	2.334263e + 03	3.158353e + 03	2.316927e + 03	3.756868e + 03	6.959400e + 05	1.684359e + 03	2.134895e + 03	3.264518e + 03
	max.	9.032810e + 02	1.005648e + 03	2.479092e + 03	1.793787e + 03	1.010519e + 07	6.989747e + 03	4.653825e + 03	3.648167e + 03	5.208870e + 03	7.129500e + 05	3.466047e + 03	2.767180e + 03	4.387652e + 03
	std.	7.523176e + 01	7.888249e + 01	1.716044e + 02	1.416140e + 02	6.312002e + 05	9.319491e + 02	3.259763e + 02	2.614930e + 02	3.358128e + 02	3.429952e + 03	3.557086e + 02	1.648283e + 02	2.200981e + 02
7D	med.	6.023460e + 02	7.121370e + 02	2.187502e + 03	1.094674e + 03	6.967259e + 06	1.706163e + 04	3.143219e + 03	2.255191e + 03	3.552386e + 03	3.495120e + 05	1.382394e + 03	1.722185e + 03	2.827721e + 03
	avg.	6.047381e + 02	7.280459e + 02	2.154314e + 03	1.088127e + 03	6.868738e + 06	1.875165e + 04	3.136863e + 03	2.260903e + 03	3.492930e + 03	3.485374e + 05	1.399272e + 03	1.748574e + 03	2.822203e + 03
	min.	4.695710e + 02	5.537530e + 02	1.616859e + 03	7.859930e + 02	5.280123e + 06	1.110350e + 04	2.417285e + 03	1.784243e + 03	2.621860e + 03	3.319940e + 05	1.184428e + 03	1.479432e + 03	2.444043e + 03
	max.	7.703030e + 02	1.114305e + 03	2.410261e + 03	1.584383e + 03	7.957733e + 06	4.865693e + 04	3.593090e + 03	2.818784e + 03	3.893323e + 03	3.664620e + 05	2.361646e + 03	2.060909e + 03	3.179217e + 03
	std.	6.584482e + 01	9.412810e + 01	1.219873e + 02	1.361071e + 02	6.373314e + 05	6.277456e + 03	2.548148e + 02	2.465178e + 02	2.771365e + 02	7.906849e + 03	1.567366e + 02	1.313743e + 02	1.298750e + 02
8D	med.	8.447710e + 02	7.684690e + 02	2.289729e + 03	1.172202e + 03	1.069570e + 07	4.921090e + 04	4.917784e + 03	3.757891e + 03	5.539910e + 03	1.282579e + 06	1.950193e + 03	2.525076e + 03	4.061270e + 03
	avg.	8.486489e + 02	7.698981e + 02	2.247312e + 03	1.145511e + 03	1.060379e + 07	5.212745e + 04	4.886927e + 03	3.715264e + 03	5.448025e + 03	1.258567e + 06	1.966995e + 03	2.560792e + 03	4.105767e + 03
	min.	6.775960e + 02	5.369970e + 02	1.568778e + 03	8.371020e + 02	8.326615e + 06	3.231590e + 04	3.748448e + 03	2.812182e + 03	4.001691e + 03	1.172844e + 06	1.675307e + 03	2.278088e + 03	3.670217e + 03
	max.	1.022407e + 03	9.813180e + 02	2.510254e + 03	1.608656e + 03	1.248394e + 07	1.199766e + 05	5.641407e + 03	4.253659e + 03	6.032006e + 03	1.313754e + 06	2.394985e + 03	3.334297e + 03	4.793849e + 03
	std.	7.633646e + 01	8.534191e + 01	1.440514e + 02	1.550443e + 02	1.231555e + 06	1.320906e + 04	3.826207e + 02	3.156199e + 02	3.696102e + 02	4.839403e + 04	1.674999e + 02	1.850882e + 02	2.310561e + 02
9D	med.	1.236690e + 03	8.262000e + 02	2.737051e + 03	1.225391e + 03	1.573793e + 07	9.739916e + 04	7.433572e + 03	6.062308e + 03	9.574687e + 03	2.616051e + 06	2.996688e + 03	3.675146e + 03	5.913384e + 03
	avg.	1.222690e + 03	8.328476e + 02	2.684827e + 03	1.213767e + 03	1.507935e + 07	1.106414e + 05	7.017707e + 03	5.999063e + 03	9.305631e + 03	2.616339e + 06	3.037356e + 03	3.732402e + 03	5.952075e + 03
	min.	9.985220e + 02	6.411170e + 02	2.078025e + 03	8.698730e + 02	7.899017e + 06	6.262578e + 04	5.282001e + 03	4.734259e + 03	7.540295e + 03	2.595661e + 06	2.452366e + 03	3.183295e + 03	4.999886e + 03
	max.	1.450314e + 03	1.036555e + 03	3.004484e + 03	1.436665e + 03	1.749397e + 07	2.730805e + 05	8.409863e + 03	6.783960e + 03	9.986686e + 03	2.636566e + 06	3.639291e + 03	4.399073e + 03	7.255952e + 03
	std.	9.904251e + 01	7.467858e + 01	1.762381e + 02	1.259116e + 02	1.919883e + 06	3.391719e + 04	1.062649e + 03	4.102577e + 02	5.656687e + 02	8.283264e + 03	2.206972e + 02	2.606718e + 02	3.986576e + 02
10D	med.	1.702236e + 03	8.671030e + 02	2.858695e + 03	1.408060e + 03	1.158571e + 07	3.160125e + 05	1.004784e + 04	8.366351e + 03	1.364294e + 04	4.838952e + 06	4.243072e + 03	5.250750e + 03	8.489312e + 03
	avg.	1.678871e + 03	8.782399e + 02	2.815394e + 03	1.396157e + 03	1.177820e + 07	3.578621e + 05	1.016252e + 04	8.445965e + 03	1.337213e + 04	4.843409e + 06	4.265320e + 03	5.358515e + 03	8.516631e + 03
	min.	1.310847e + 03	6.757330e + 02	1.987748e + 03	9.965320e + 02	8.312446e + 06	1.523375e + 05	8.192458e + 03	7.320963e + 03	1.027212e + 04	4.536107e + 06	3.349404e + 03	4.478053e + 03	7.068243e + 03
	max.	2.013704e + 03	1.342272e + 03	3.365806e + 03	1.893267e + 03	1.542105e + 07	1.205993e + 06	1.272436e + 04	9.949472e + 03	1.476683e + 04	5.098417e + 06	5.294857e + 03	6.170480e + 03	9.692153e + 03
	std.	1.280051e + 02	9.684048e + 01	2.227088e + 02	1.082502e + 02	1.723142e + 06	1.545897e + 05	1.524845e + 03	5.877902e + 02	1.066859e + 03	1.483676e + 05	3.021680e + 02	4.699844e + 02	5.632525e + 02



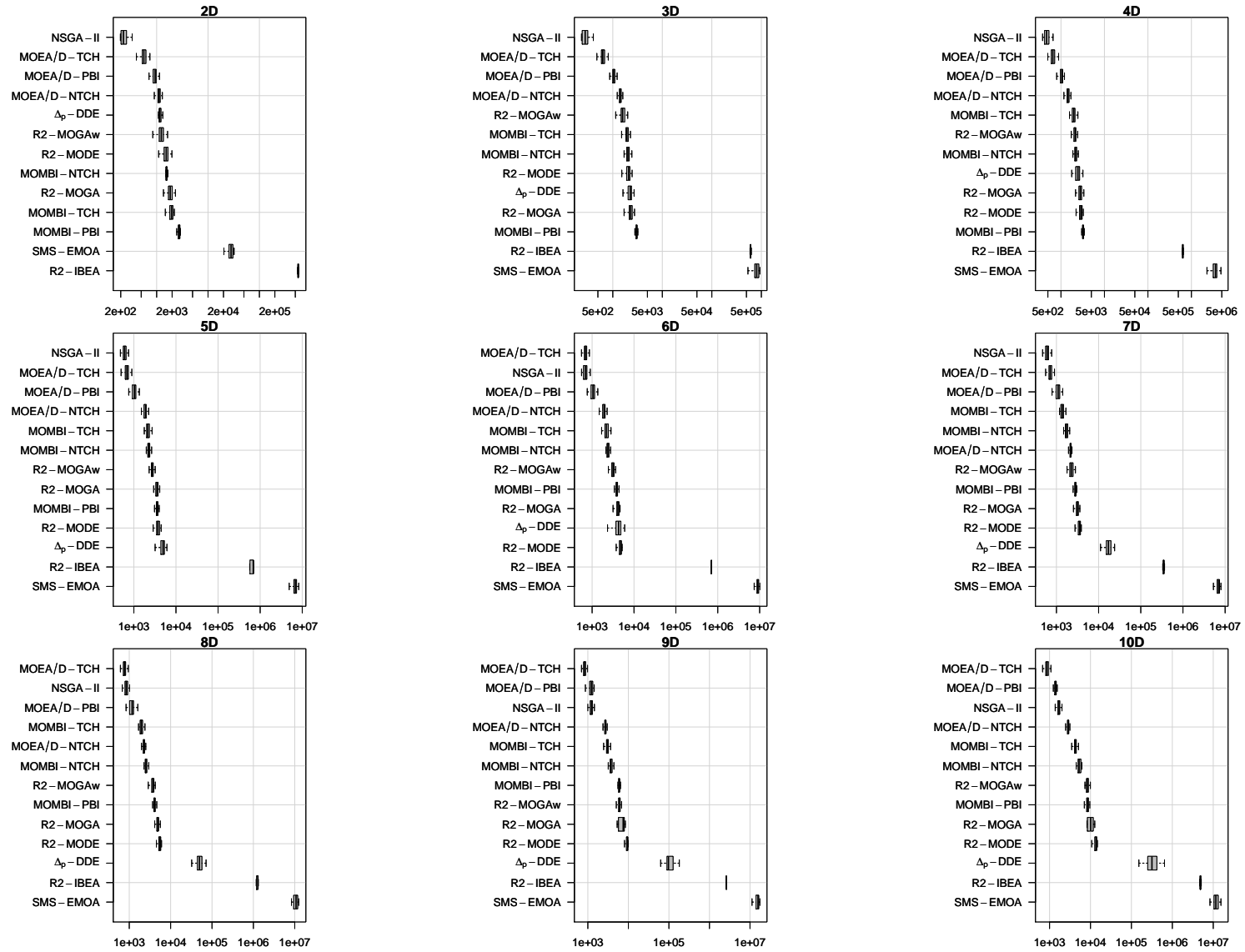


Figure A.67: Box-plot of runtime (in logarithmic scale) for different optimizers on the DTLZ5 test problem.

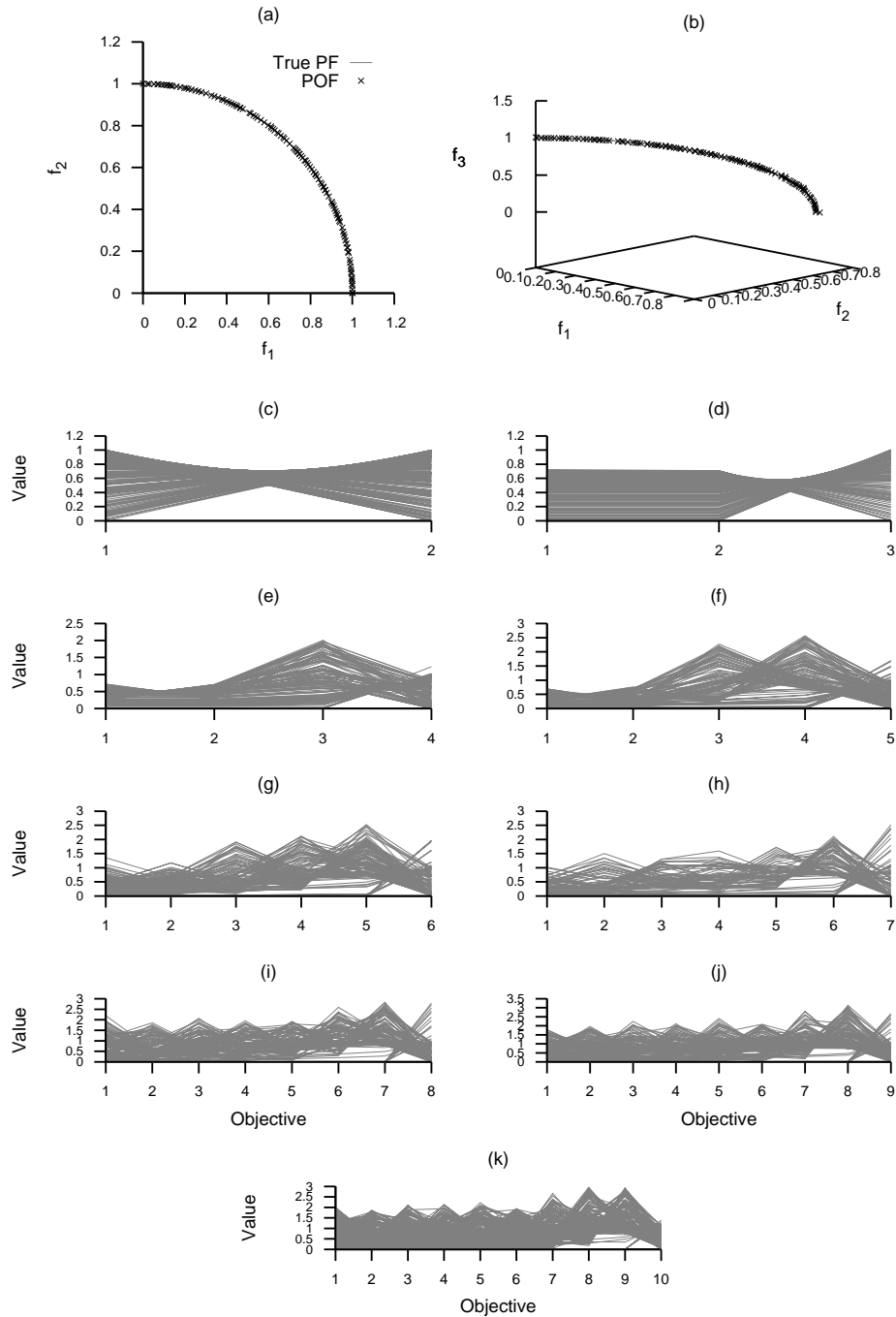


Figure A.68: Plots of the approximations obtained by NSGA-II from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

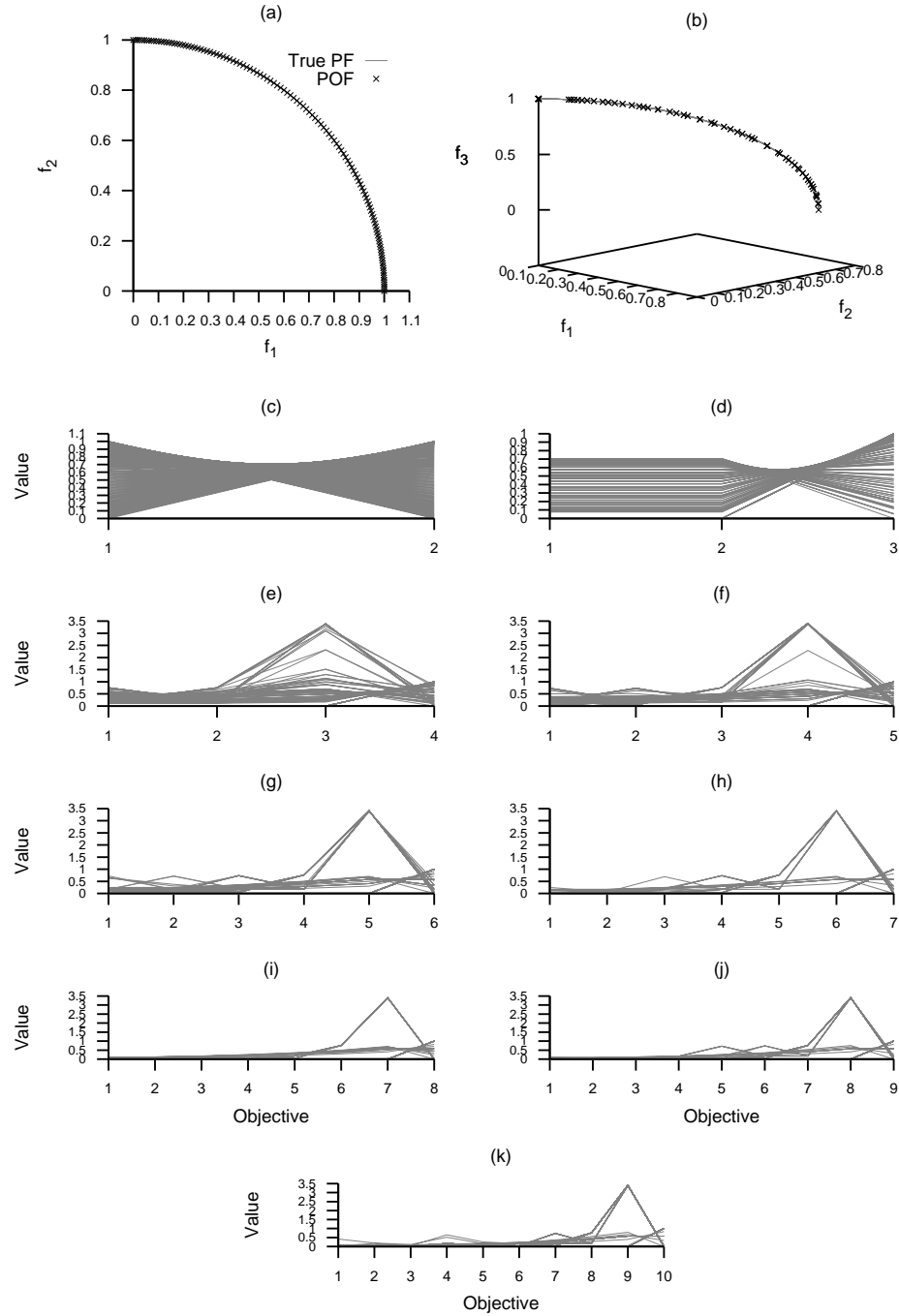


Figure A.69: Plots of the approximations obtained by MOEA/D-TCH from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

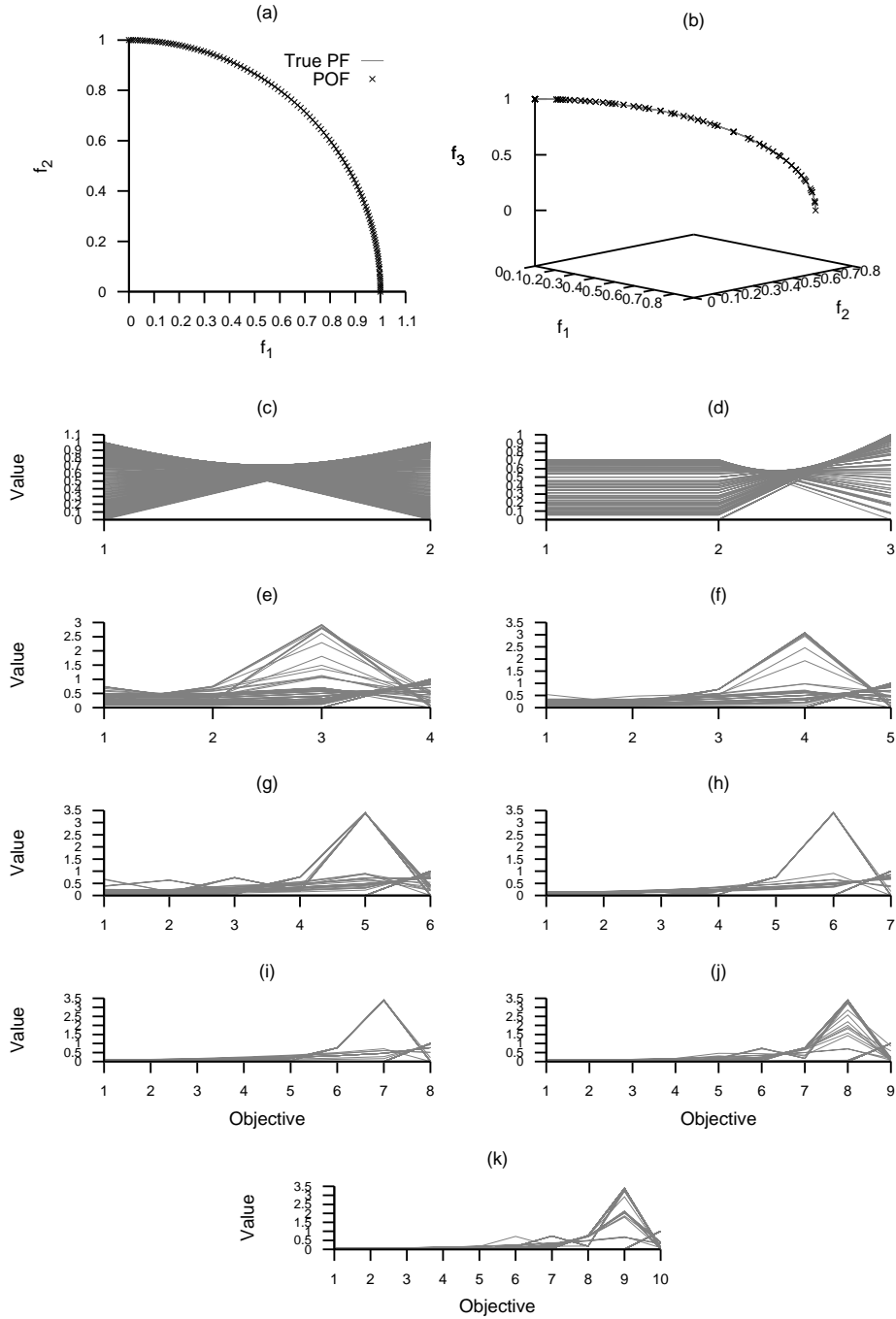


Figure A.70: Plots of the approximations obtained by MOEA/D-NTCH from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

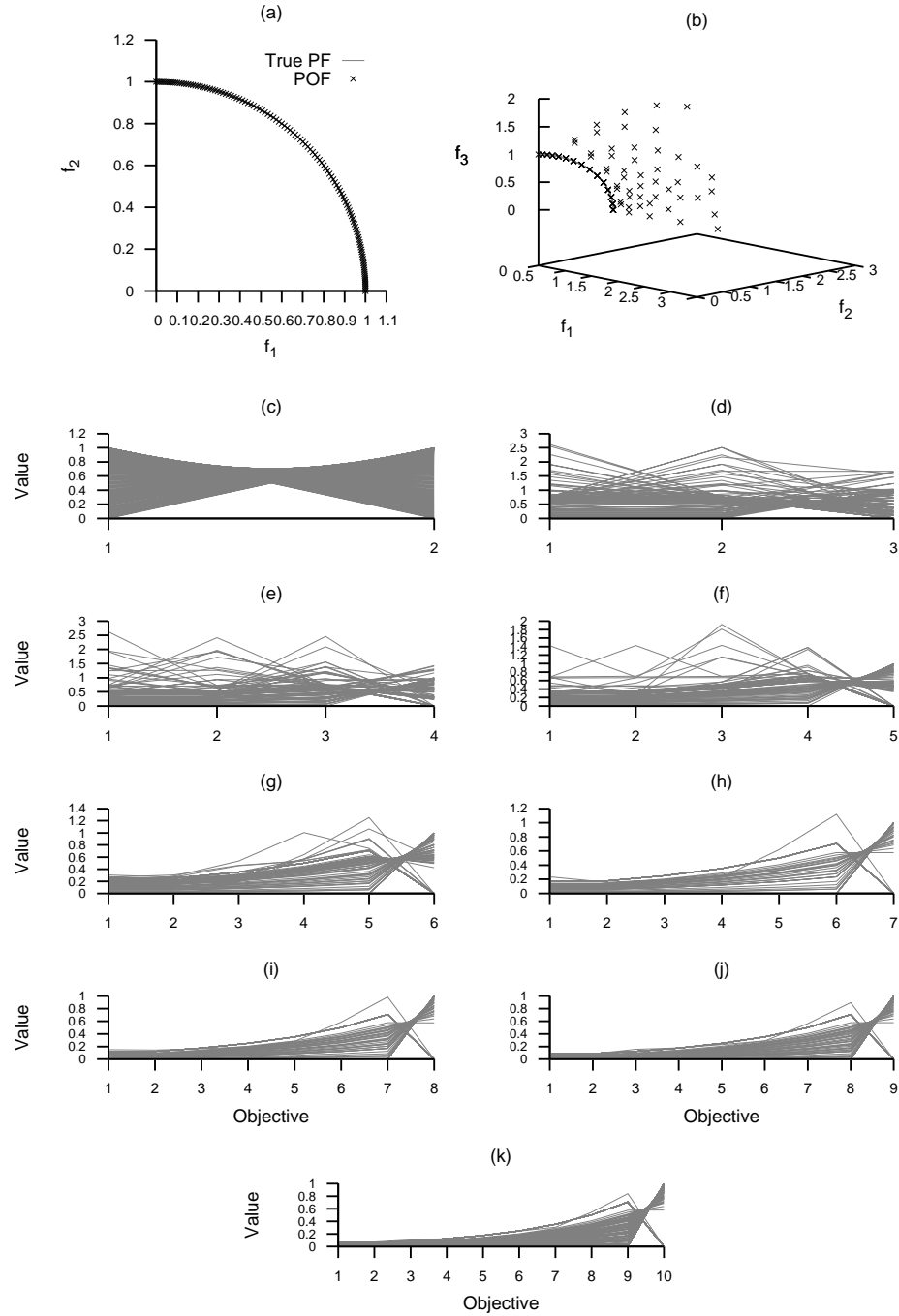


Figure A.71: Plots of the approximations obtained by MOEA/D-PBI from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

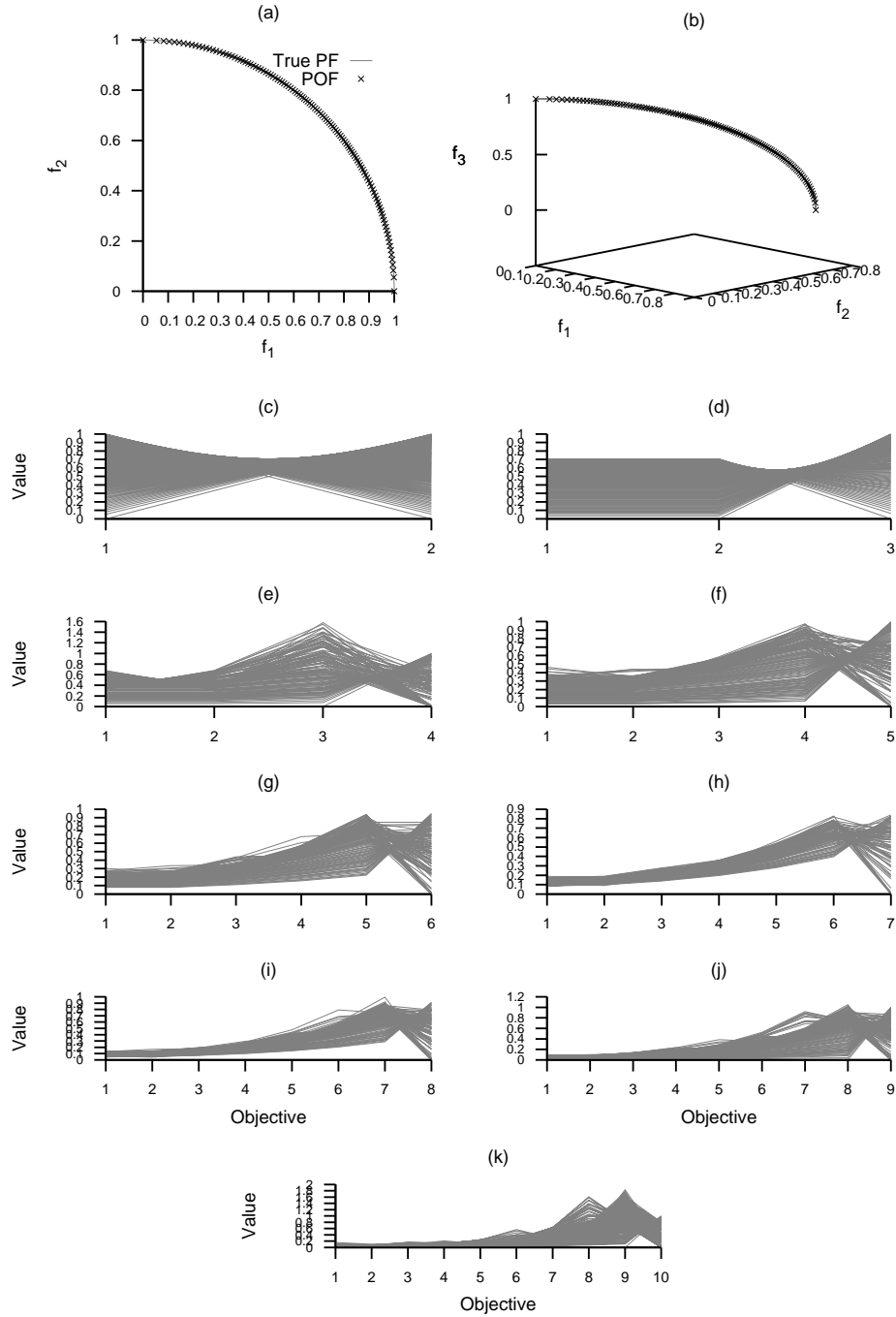


Figure A.72: Plots of the approximations obtained by SMS-EMOA from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

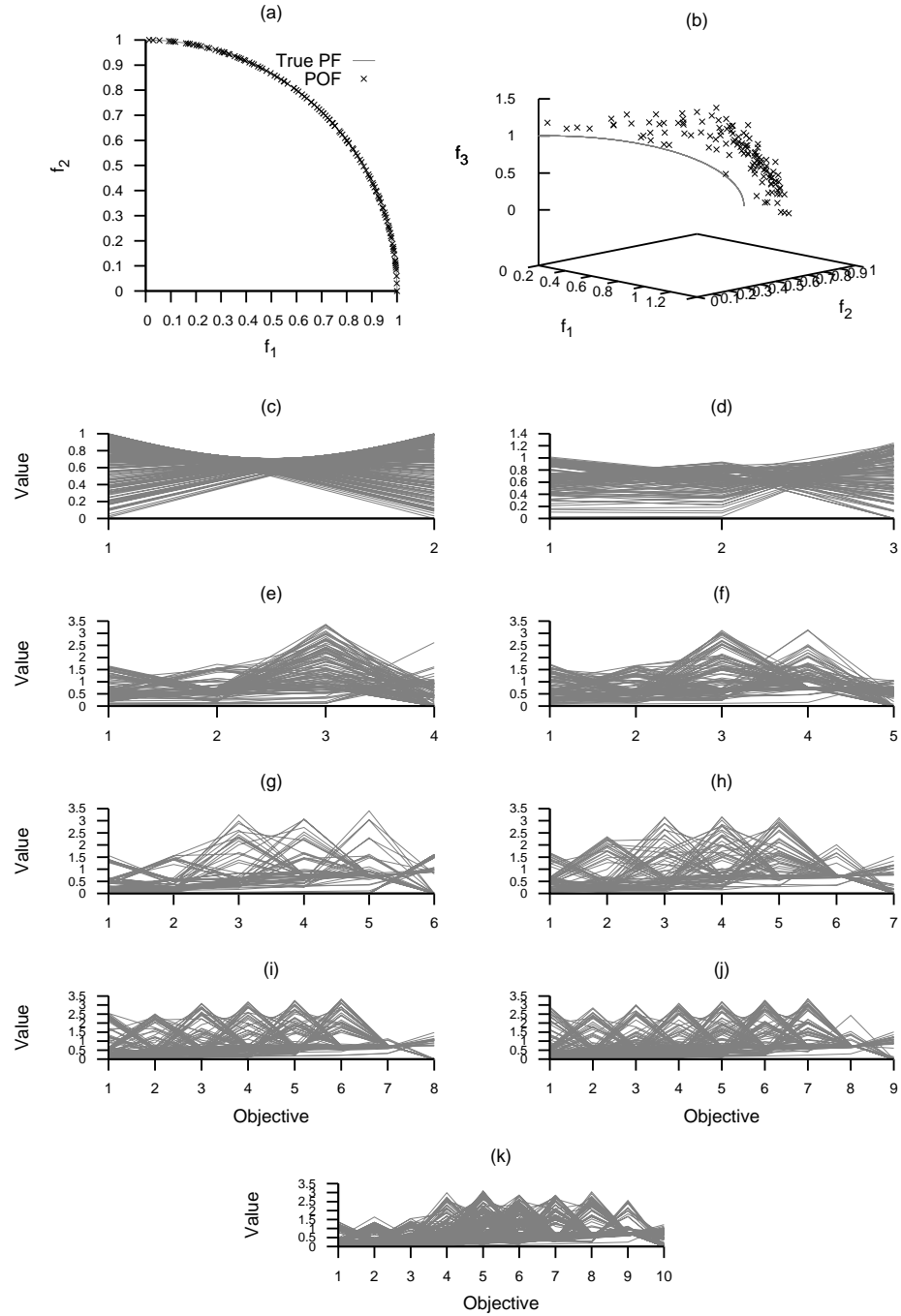


Figure A.73: Plots of the approximations obtained by  $\Delta_p$ -DDE from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

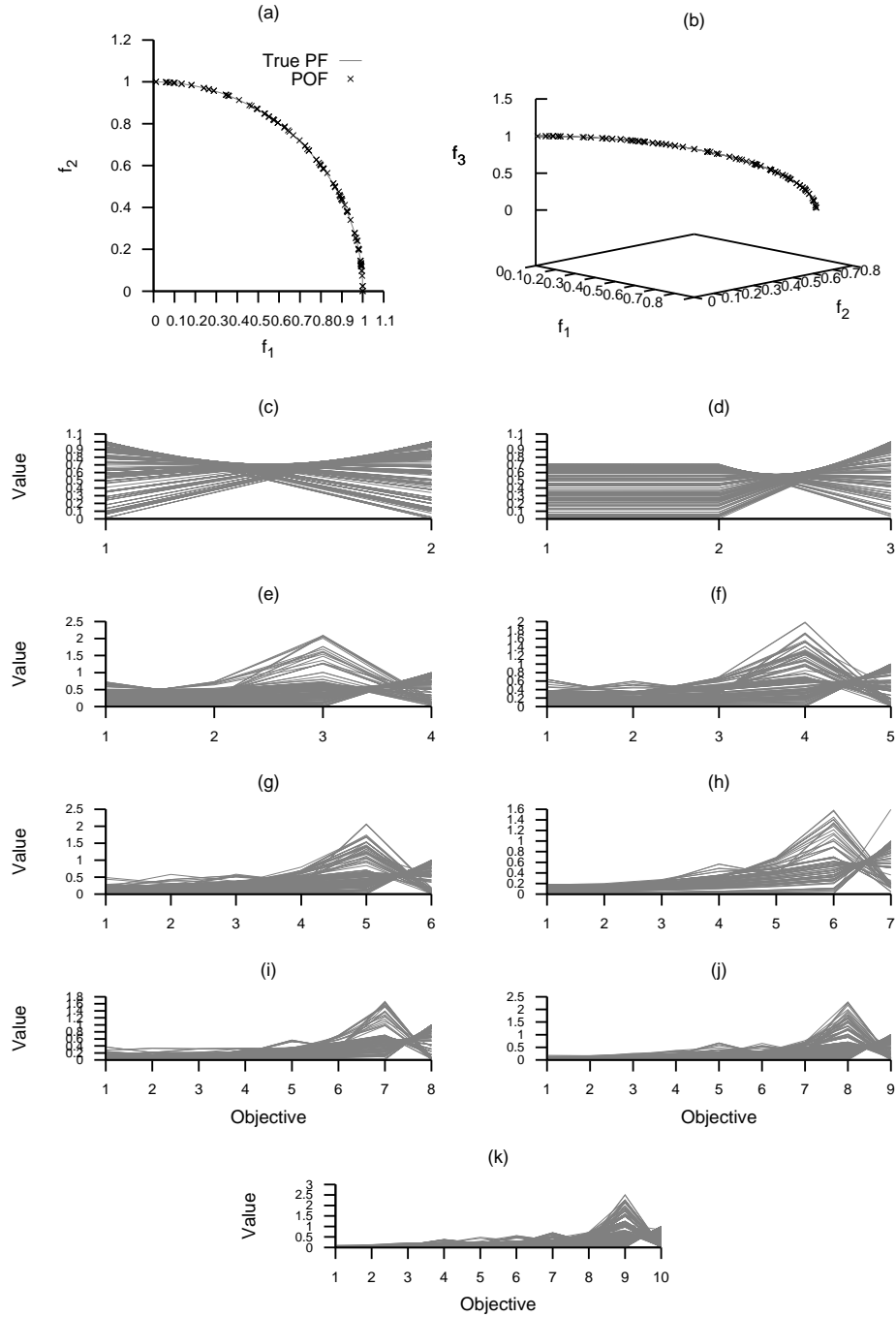


Figure A.74: Plots of the approximations obtained by *R2*-MOGA from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



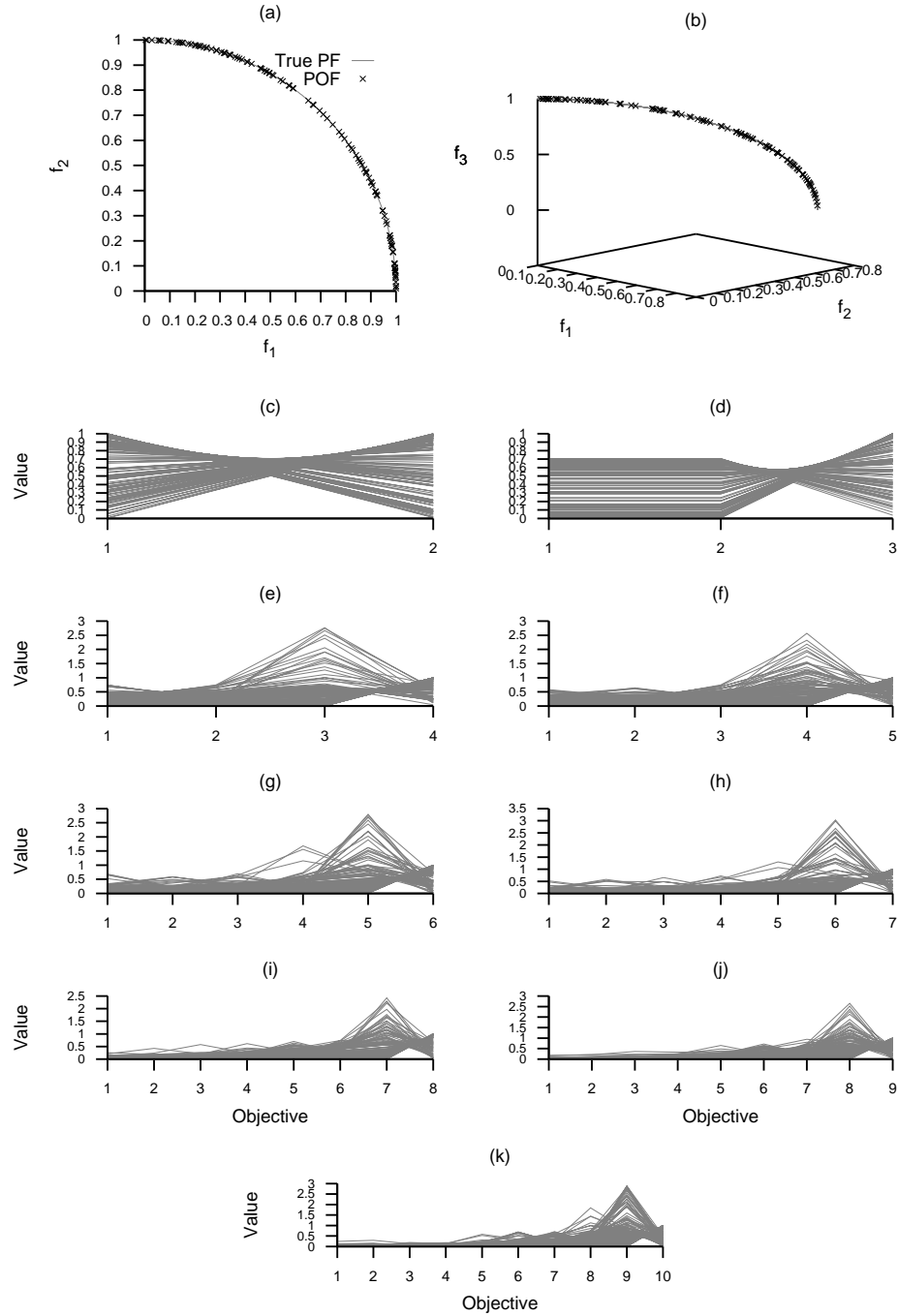


Figure A.75: Plots of the approximations obtained by *R2-MOGAw* from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

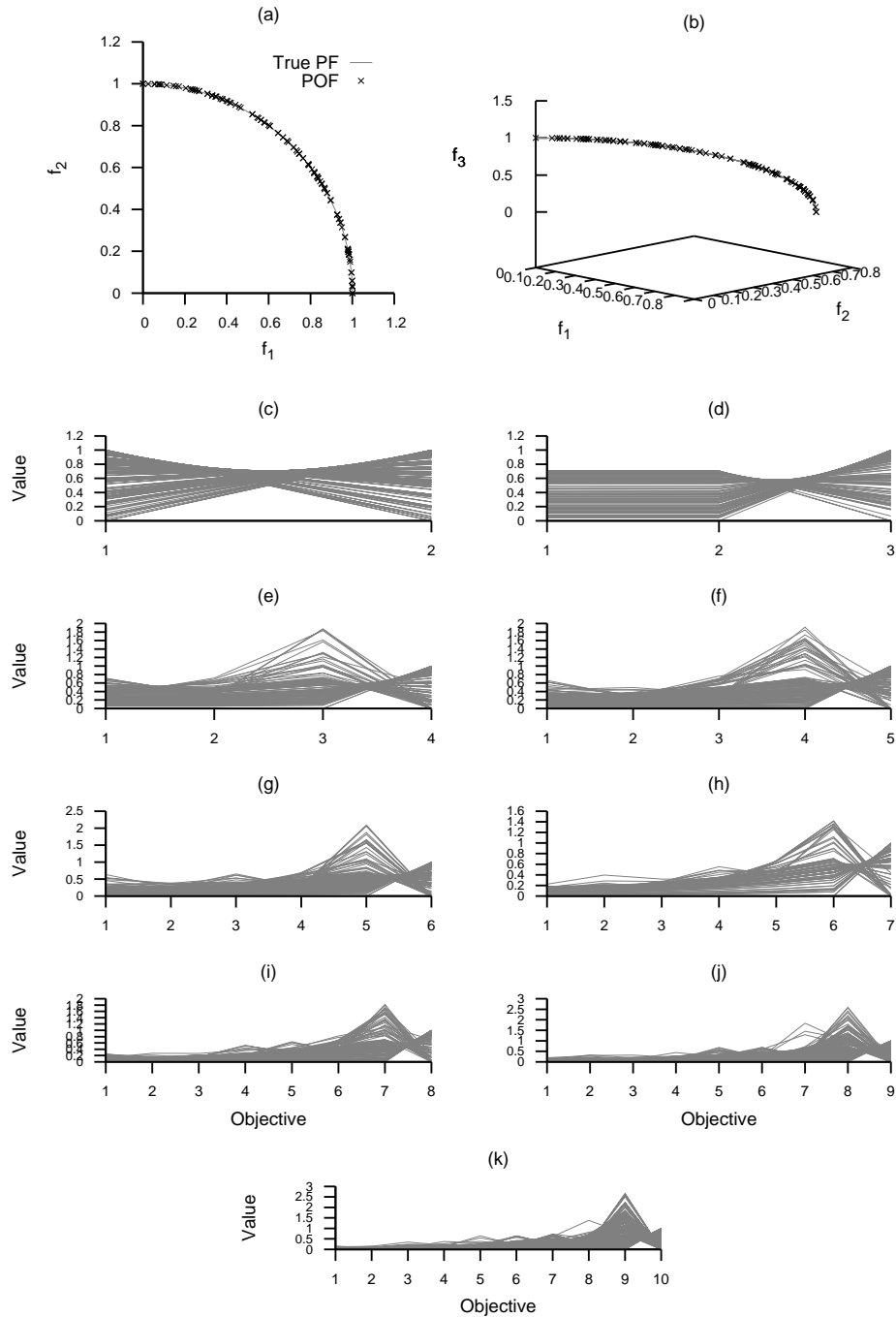


Figure A.76: Plots of the approximations obtained by *R2-MODE* from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

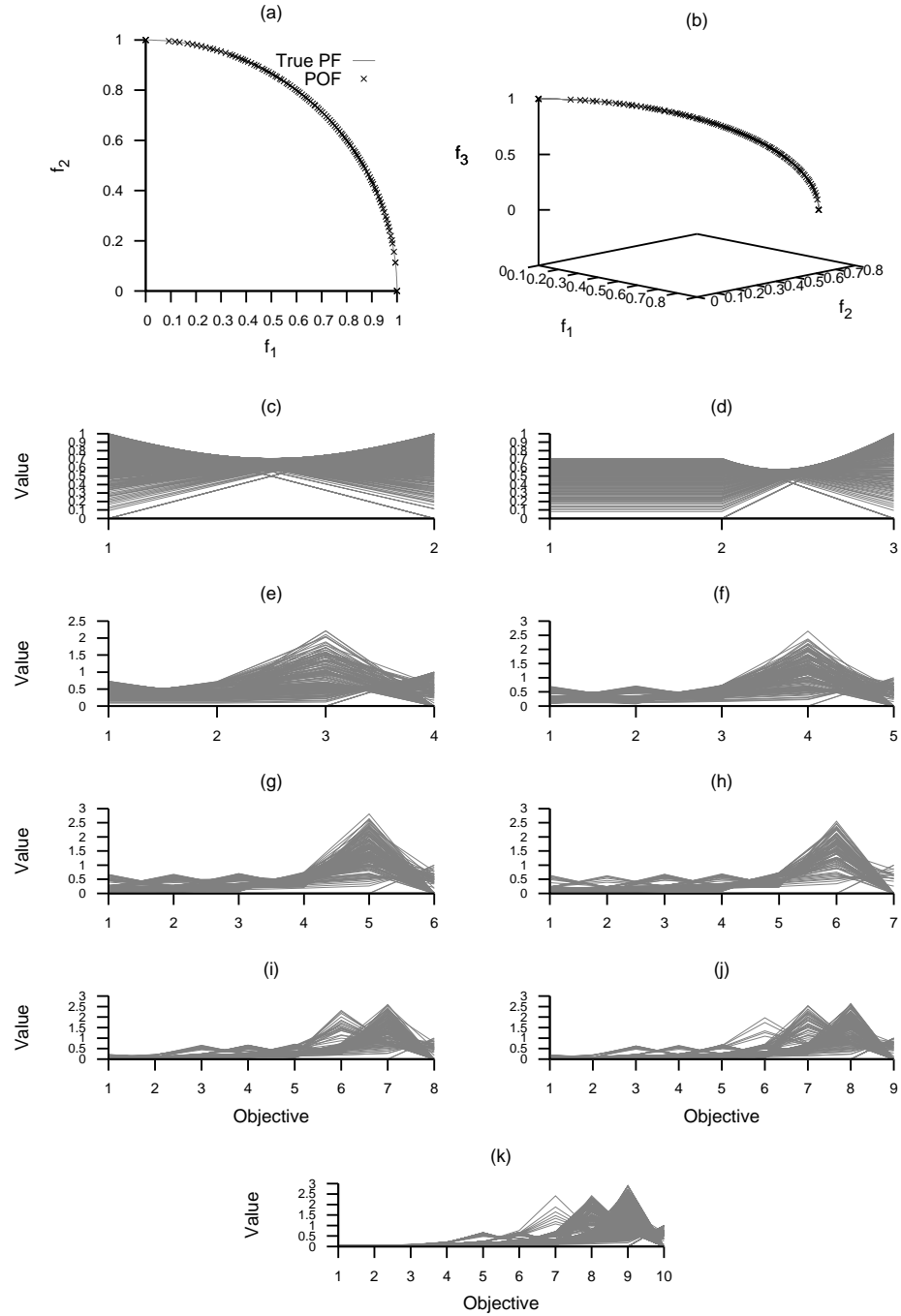


Figure A.77: Plots of the approximations obtained by R2-IBEA from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

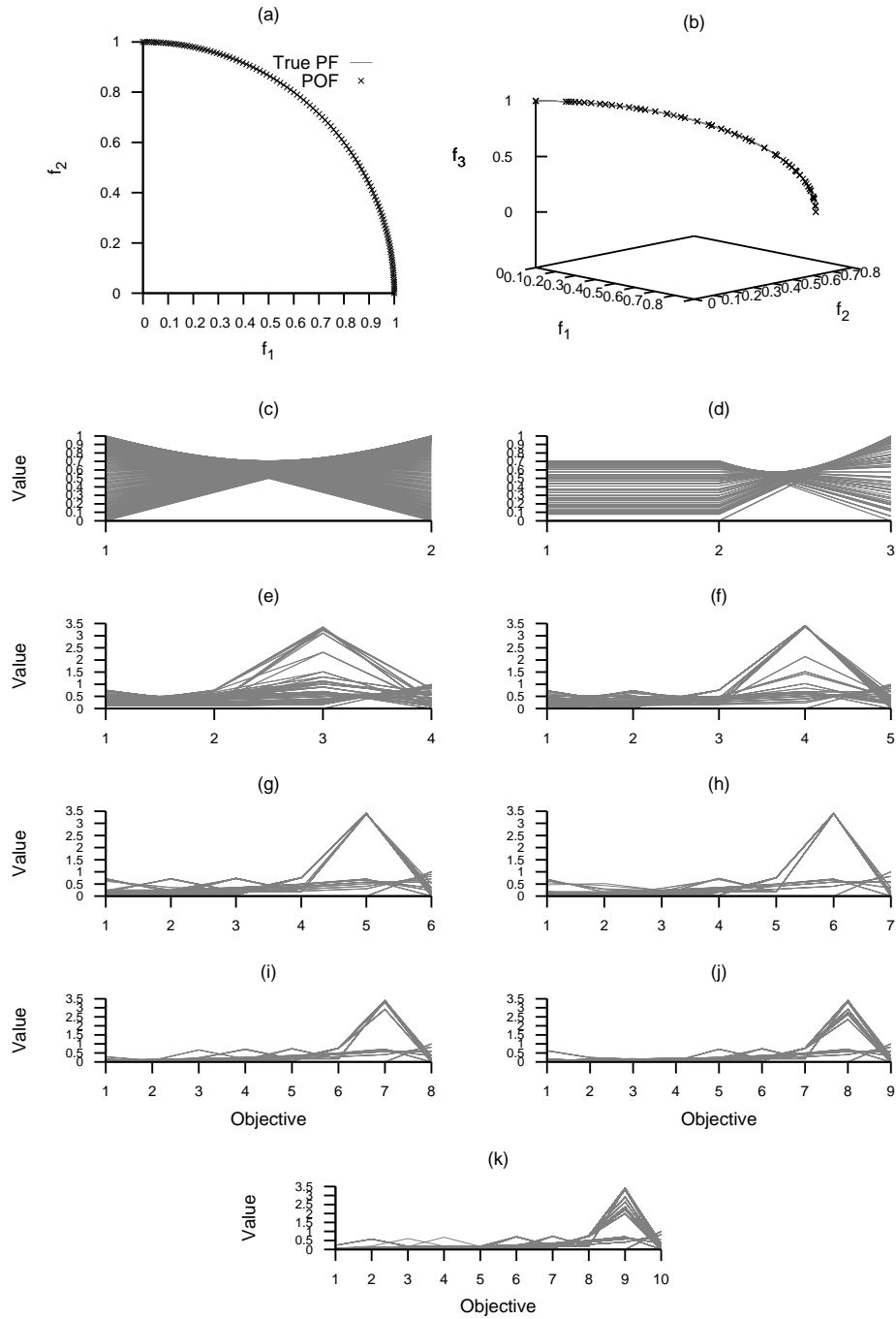


Figure A.78: Plots of the approximations obtained by MOMBI-TCH from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

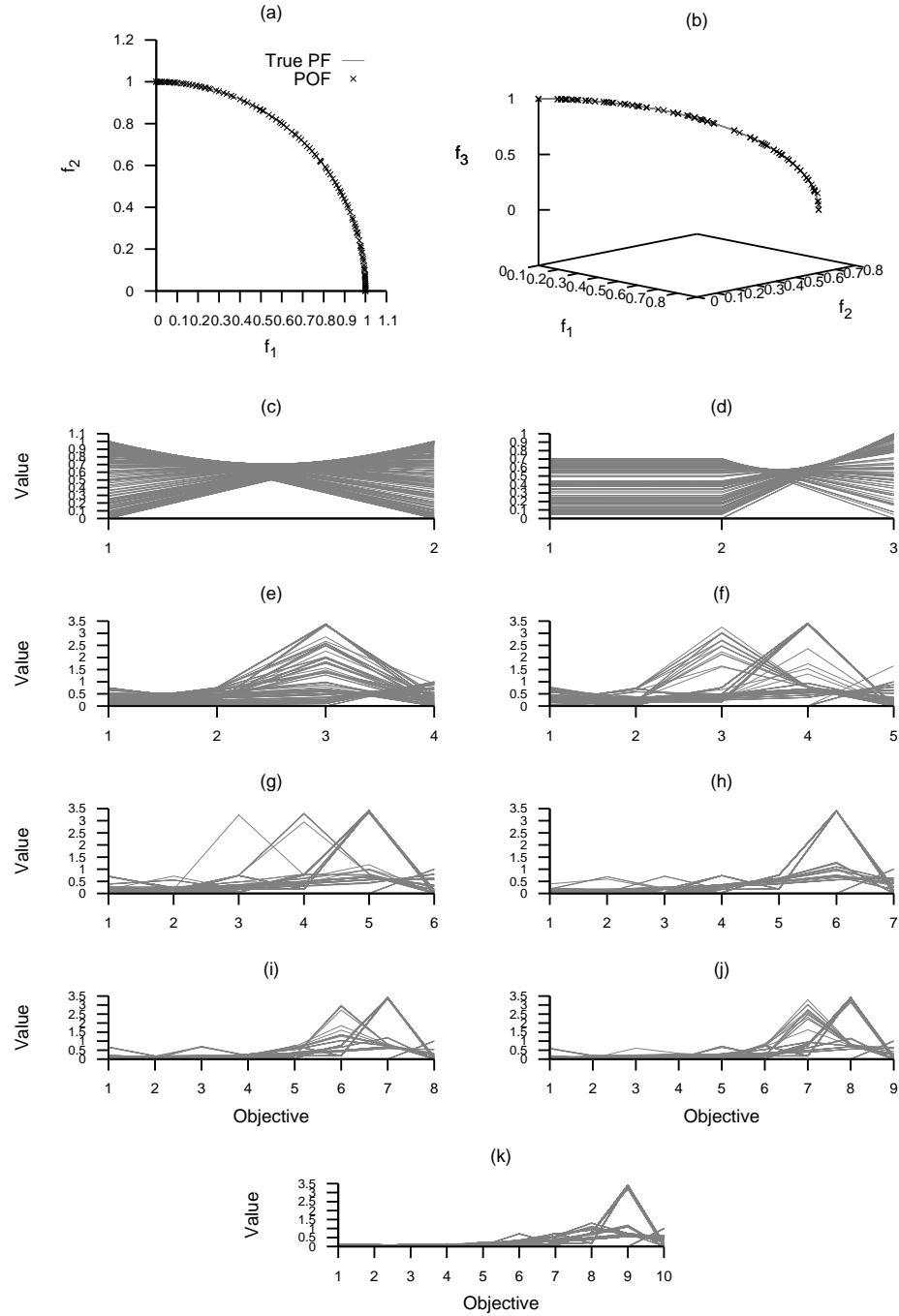


Figure A.79: Plots of the approximations obtained by MOMBI-NTCH from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

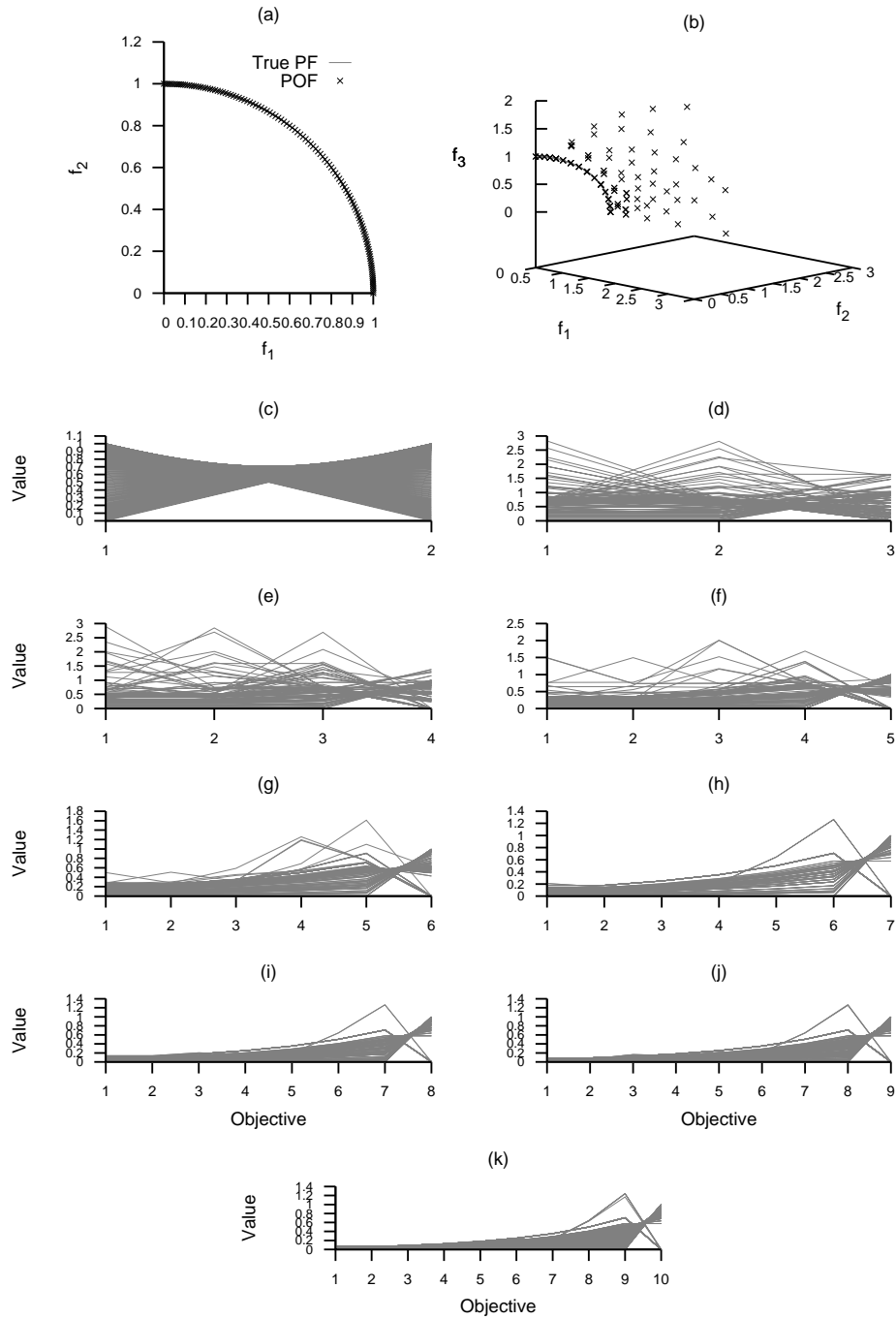


Figure A.80: Plots of the approximations obtained by MOMBI-PBI from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.32: Comparison of hypervolume indicator values for different optimizers on the DTLZ6 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	1.201020e+02	1.200317e+02	1.200285e+02	1.200294e+02	1.201051e+02	1.202101e+02	1.199172e+02	1.199622e+02	1.202007e+02	1.202110e+02	1.201022e+02	1.201041e+02	1.201152e+02
	avg.	1.201032e+02	1.200245e+02	1.200340e+02	1.200291e+02	1.200979e+02	1.202101e+02	1.199037e+02	1.199219e+02	1.201576e+02	1.201983e+02	1.201013e+02	1.201021e+02	1.201159e+02
	min.	1.200082e+02	1.198607e+02	1.199063e+02	1.198989e+02	1.200020e+02	1.202093e+02	1.191403e+02	1.195468e+02	1.198362e+02	1.201394e+02	1.199868e+02	1.199514e+02	1.200145e+02
	max.	1.202069e+02	1.201453e+02	1.201816e+02	1.201756e+02	1.202092e+02	1.202105e+02	1.201220e+02	1.201626e+02	1.202089e+02	1.202112e+02	1.202105e+02	1.202079e+02	1.202093e+02
	std.	4.116030e-02	6.205293e-02	5.813202e-02	6.025583e-02	4.614133e-02	2.356806e-04	1.403922e-01	1.408713e-01	7.874017e-02	1.953278e-02	4.511892e-02	4.744676e-02	4.083221e-02
3D	med.	1.316194e+03	1.316780e+03	1.316992e+03	1.316475e+03	1.318068e+03	1.319014e+03	1.313263e+03	1.312541e+03	1.316484e+03	1.319059e+03	1.317911e+03	1.317839e+03	1.313426e+03
	avg.	1.316229e+03	1.316800e+03	1.317035e+03	1.316194e+03	1.318126e+03	1.311892e+03	1.312923e+03	1.312129e+03	1.316260e+03	1.318970e+03	1.317903e+03	1.317828e+03	1.313456e+03
	min.	1.314691e+03	1.315464e+03	1.314953e+03	1.312472e+03	1.317192e+03	1.286301e+03	1.304209e+03	1.301357e+03	1.308908e+03	1.318038e+03	1.316875e+03	1.317078e+03	1.312225e+03
	max.	1.317339e+03	1.318279e+03	1.318605e+03	1.318241e+03	1.319044e+03	1.319039e+03	1.317872e+03	1.316570e+03	1.319034e+03	1.319062e+03	1.318703e+03	1.318646e+03	1.315432e+03
	std.	5.204294e-01	6.015902e-01	6.281976e-01	1.127131e+00	3.854136e-01	9.581243e+00	2.700341e+00	2.936565e+00	2.550088e+00	1.981351e-01	4.157073e-01	3.518532e-01	5.971971e-01
4D	med.	1.389381e+04	1.448145e+04	1.447167e+04	1.446337e+04	1.451076e+04	1.452931e+04	1.447799e+04	1.447177e+04	1.450534e+04	1.452917e+04	1.446003e+04	1.448216e+04	1.442520e+04
	avg.	1.389362e+04	1.448153e+04	1.446330e+04	1.445388e+04	1.444119e+04	1.452900e+04	1.447721e+04	1.446604e+04	1.450271e+04	1.452907e+04	1.445999e+04	1.448211e+04	1.442449e+04
	min.	1.372099e+04	1.446059e+04	1.441610e+04	1.430828e+04	1.355867e+04	1.452397e+04	1.442947e+04	1.436575e+04	1.446963e+04	1.452117e+04	1.443849e+04	1.445944e+04	1.440541e+04
	max.	1.403360e+04	1.450294e+04	1.450888e+04	1.448447e+04	1.452391e+04	1.455290e+04	1.451532e+04	1.450382e+04	1.451469e+04	1.453449e+04	1.447877e+04	1.450095e+04	1.444214e+04
	std.	5.933168e+01	9.387593e+00	4.380031e+01	3.082246e+01	2.213570e+02	9.265028e-01	1.736762e+01	2.658315e+01	9.758201e+00	3.153544e+00	7.547882e+00	9.244019e+00	7.328462e+00
5D	med.	1.337831e+05	1.581836e+05	1.587838e+05	1.587183e+05	1.514058e+05	1.595525e+05	1.593137e+05	1.590080e+05	1.595858e+05	1.597496e+05	1.566880e+05	1.571681e+05	1.586208e+05
	avg.	1.334224e+05	1.581705e+05	1.585674e+05	1.583583e+05	1.528779e+05	1.583533e+05	1.593022e+05	1.589476e+05	1.595276e+05	1.597313e+05	1.565806e+05	1.570593e+05	1.586049e+05
	min.	1.254850e+05	1.573987e+05	1.524036e+05	1.544122e+05	1.468078e+05	1.419973e+05	1.586688e+05	1.577514e+05	1.585021e+05	1.594583e+05	1.543905e+05	1.538183e+05	1.582441e+05
	max.	1.390749e+05	1.589530e+05	1.595374e+05	1.592571e+05	1.597809e+05	1.598039e+05	1.596459e+05	1.593738e+05	1.597127e+05	1.599085e+05	1.574090e+05	1.581485e+05	1.588557e+05
	std.	2.826994e+03	2.803259e+02	9.209921e+02	8.963776e+02	3.995871e+03	4.060067e+03	2.118988e+02	2.784262e+02	1.928830e+02	8.917166e+01	5.250783e+02	7.430711e+02	1.313005e+02
6D	med.	1.210968e+06	1.732205e+06	1.741453e+06	1.743351e+06	1.649778e+06	1.593341e+06	1.752872e+06	1.747305e+06	1.755173e+06	1.755393e+06	1.706020e+06	1.708308e+06	1.741812e+06
	avg.	1.215285e+06	1.732053e+06	1.740146e+06	1.738818e+06	1.658341e+06	1.598306e+06	1.752371e+06	1.747082e+06	1.754668e+06	1.754939e+06	1.703816e+06	1.707671e+06	1.741468e+06
	min.	1.135294e+06	1.722783e+06	1.692235e+06	1.681721e+06	1.602003e+06	1.552980e+06	1.745158e+06	1.739490e+06	1.744455e+06	1.750687e+06	1.677046e+06	1.666214e+06	1.731337e+06
	max.	1.319016e+06	1.741580e+06	1.751741e+06	1.750400e+06	1.755433e+06	1.747672e+06	1.755821e+06	1.751798e+06	1.757194e+06	1.757296e+06	1.721637e+06	1.728577e+06	1.744787e+06
	std.	4.415965e+04	3.792582e+03	8.568077e+03	1.235798e+04	3.987790e+04	3.754295e+04	2.268159e+03	2.208225e+03	1.870405e+03	1.416366e+03	1.010515e+04	9.898368e+03	2.050546e+03
7D	med.	1.168007e+07	1.909011e+07	1.904541e+07	1.920948e+07	1.738885e+07	1.741972e+07	1.920979e+07	1.922346e+07	1.927005e+07	1.925550e+07	1.893897e+07	1.874329e+07	1.921673e+07
	avg.	1.175591e+07	1.907949e+07	1.900003e+07	1.920478e+07	1.743038e+07	1.750680e+07	1.920494e+07	1.921286e+07	1.926325e+07	1.925762e+07	1.890595e+07	1.869999e+07	1.921573e+07
	min.	1.027179e+07	1.890719e+07	1.794389e+07	1.910799e+07	1.714364e+07	1.623055e+07	1.904504e+07	1.905088e+07	1.913544e+07	1.918992e+07	1.843171e+07	1.797550e+07	1.919679e+07
	max.	1.319674e+07	1.919176e+07	1.920868e+07	1.922990e+07	1.822878e+07	1.909564e+07	1.928380e+07	1.927980e+07	1.929755e+07	1.930782e+07	1.905040e+07	1.891535e+07	1.923390e+07
	std.	5.724149e+05	5.460275e+04	2.066098e+05	2.283205e+04	2.138554e+05	6.874916e+05	4.757208e+04	4.537574e+04	2.737833e+04	2.286155e+04	1.244126e+05	1.631819e+05	7.808895e+03
8D	med.	1.198509e+08	2.103817e+08	2.094805e+08	2.107424e+08	1.961256e+08	1.924939e+08	2.118353e+08	2.12575e+08	2.122648e+08	2.119485e+08	2.066746e+08	2.047652e+08	2.102798e+08
	avg.	1.198592e+08	2.102487e+08	2.087880e+08	2.103144e+08	1.965255e+08	1.935899e+08	2.117849e+08	2.111999e+08	2.121740e+08	2.118617e+08	2.066310e+08	2.045215e+08	2.102659e+08
	min.	1.080740e+08	2.090471e+08	2.001751e+08	2.065091e+08	1.912261e+08	1.792699e+08	2.102655e+08	2.104823e+08	2.107168e+08	2.109519e+08	2.033939e+08	1.999911e+08	2.094643e+08
	max.	1.309974e+08	2.112283e+08	2.115651e+08	2.115388e+08	2.125192e+08	2.117038e+08	2.122784e+08	2.118611e+08	2.124617e+08	2.123065e+08	2.095003e+08	2.081151e+08	2.107782e+08
	std.	4.373778e+06	4.762804e+05	2.284221e+06	1.161841e+06	3.385985e+06	9.258986e+06	3.235957e+05	3.271154e+05	3.129269e+05	3.225402e+05	1.225256e+06	1.594303e+06	2.730634e+05
9D	med.	1.322960e+09	2.314140e+09	2.287429e+09	2.306547e+09	2.182929e+09	2.101791e+09	2.328692e+09	2.317241e+09	2.337242e+09	2.328549e+09	2.260390e+09	2.235726e+09	2.291314e+09
	avg.	1.325013e+09	2.313602e+09	2.277142e+09	2.305552e+09	2.184065e+09	2.108699e+09	2.328394e+09	2.316845e+09	2.336784e+09	2.327612e+09	2.259795e+09	2.231659e+09	2.291353e+09
	min.	1.251015e+09	2.296709e+09	2.140775e+09	2.267034e+09	2.147368e+09	2.003190e+09	2.310279e+09	2.303943e+09	2.331822e+09	2.310682e+09	2.237133e+09	2.157961e+09	2.278601e+09
	max.	1.410060e+09	2.326582e+09	2.324737e+09	2.324737e+09	2.278063e+09	2.326528e+09	2.333922e+09	2.325372e+09	2.338853e+09	2.332595e+09	2.281124e+09	2.265226e+09	2.304158e+09
	std.	3.614170e+07	6.314110e+06	3.957895e+07	1.748808e+07	2.446246e+07	8.038022e+07	3.198094e+06	4.111177e+06	1.585609e+06	3.724599e+06	7.551349e+06	2.061718e+07	5.535784e+06
10D	med.	1.506882e+10	2.548168e+10	2.504475e+10	2.553986e+10	2.396791e+10	2.338596e+10	2.558130e+10	2.543025e+10	2.572329e+10	2.555653e+10	2.476779e+10	2.432235e+10	2.492524e+10
	avg.	1.511206e+10	2.546630e+10	2.485604e+10	2.536970e+10	2.402331e+10	2.357800e+10	2.557610e+10	2.542392e+10	2.572129e+10	2.554594e+10	2.475435e+10	2.432592e+10	2.491579e+10
	min.	1.403183e+10	2.522269e+10	2.331958e+10	2.418818e+10	2.367331e+10	2.212991e+10	2.548321e+10	2.520742e+10	2.566648e+10	2.527497e+10	2.440655e+10	2.353210e+10	2.466304e+10
	max.	1.628967e+10	2.557659e+10	2.554893e+10	2.557207e+10	2.475394e+10	2.570769e+10	2.562975e+10	2.551541e+10	2.573830e+10	2.560977e+10	2.491899e+10	2.475823e+10	2.510424e+10
	std.	4.081312e+08	6.705519e+07	5.824512e+08	2.827193e+08	2.189607e+08	1.003616e+09	2.616204e+07	5.253210e+07	1.190040e+07	5.728045e+07	9.075315e+07	2.122051e+08	8.095296e+07

Figure A.81: Box-plot of hypervolume indicator values for different optimizers on the DTLZ6 test problem.



Table A.33: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ6 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D													
NSGA-II	—	$2.70e-18$	$1.33e-15$	$1.07e-16$	$> 0.05$	$> 0.05$	$1.60e-29$	$2.76e-26$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-TCH	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.42e-13$	$3.82e-09$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-NTCH	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$6.55e-16$	$1.44e-10$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-PBI	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$7.61e-15$	$6.98e-10$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
SMS-EMOA	$> 0.05$	$1.36e-15$	$3.37e-14$	$1.00e-14$	—	$> 0.05$	$5.46e-28$	$7.52e-25$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
$\Delta_p$ -DDE	$9.89e-35$	$9.89e-35$	$9.89e-35$	$9.88e-35$	$9.88e-35$	—	$9.89e-35$	$9.89e-35$	$9.85e-35$	$> 0.05$	$1.93e-33$	$9.89e-35$	$1.03e-34$
R2-MOGA	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
R2-MOGAw	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
R2-MODE	$2.76e-13$	$1.05e-22$	$8.89e-22$	$3.94e-22$	$2.22e-13$	$> 0.05$	$2.11e-29$	$1.27e-27$	—	$> 0.05$	$7.15e-13$	$6.43e-13$	$1.68e-10$
R2-IBEA	$7.86e-33$	$1.09e-34$	$2.18e-34$	$1.41e-34$	$4.78e-33$	$4.39e-05$	$9.67e-35$	$1.57e-34$	$4.32e-12$	—	$3.00e-32$	$6.99e-33$	$3.88e-31$
MOMBI-TCH	$> 0.05$	$1.24e-17$	$4.62e-15$	$6.95e-16$	$> 0.05$	$> 0.05$	$7.46e-29$	$8.51e-26$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$
MOMBI-NTCH	$> 0.05$	$2.42e-17$	$4.24e-15$	$5.11e-16$	$> 0.05$	$> 0.05$	$1.93e-28$	$1.25e-25$	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$
MOMBI-PBI	$1.82e-02$	$1.78e-22$	$3.25e-19$	$1.47e-20$	$2.39e-02$	$> 0.05$	$4.98e-31$	$3.55e-28$	$> 0.05$	$> 0.05$	$1.42e-02$	$> 0.05$	—
3D													
NSGA-II	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.33e-21$	$2.74e-29$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.53e-34$
MOEA/D-TCH	$1.84e-10$	—	$> 0.05$	$2.01e-05$	$> 0.05$	$> 0.05$	$7.20e-27$	$9.66e-33$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$
MOEA/D-NTCH	$7.17e-18$	$3.38e-03$	—	$8.59e-11$	$> 0.05$	$> 0.05$	$3.61e-29$	$1.43e-33$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.36e-34$
MOEA/D-PBI	$4.29e-02$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$1.95e-20$	$7.01e-27$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$9.80e-29$
SMS-EMOA	$1.40e-34$	$2.41e-31$	$1.97e-27$	$1.73e-32$	—	$> 0.05$	$3.76e-34$	$1.28e-34$	$2.45e-06$	$> 0.05$	$4.43e-04$	$2.57e-07$	$1.28e-34$
$\Delta_p$ -DDE	$7.07e-03$	$8.49e-03$	$1.02e-02$	$6.70e-03$	$> 0.05$	—	$1.09e-03$	$6.48e-04$	$3.62e-02$	$> 0.05$	$3.36e-02$	$2.39e-02$	$1.91e-03$
R2-MOGA	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$2.74e-02$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
R2-MOGAw	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
R2-MODE	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$3.17e-15$	$2.61e-19$	—	$> 0.05$	$> 0.05$	$> 0.05$	$2.32e-17$
R2-IBEA	$1.04e-34$	$1.11e-34$	$1.79e-34$	$1.09e-34$	$6.27e-31$	$4.49e-22$	$1.04e-34$	$1.04e-34$	$2.74e-28$	—	$2.03e-33$	$4.89e-34$	$1.04e-34$
MOMBI-TCH	$4.85e-34$	$1.69e-26$	$6.29e-21$	$1.46e-30$	$> 0.05$	$> 0.05$	$8.92e-34$	$1.28e-34$	$6.19e-05$	$> 0.05$	—	$> 0.05$	$1.28e-34$
MOMBI-NTCH	$3.49e-34$	$1.50e-26$	$2.71e-20$	$2.45e-30$	$> 0.05$	$> 0.05$	$1.24e-33$	$1.28e-34$	$1.57e-04$	$> 0.05$	$> 0.05$	—	$1.28e-34$
MOMBI-PBI	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.60e-04$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—
4D													
NSGA-II	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-TCH	$1.28e-34$	—	$3.67e-04$	$5.90e-22$	$> 0.05$	$> 0.05$	$3.93e-02$	$1.48e-06$	$> 0.05$	$> 0.05$	$1.13e-30$	$> 0.05$	$1.28e-34$
MOEA/D-NTCH	$1.28e-34$	$> 0.05$	—	$5.37e-04$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.37e-04$	$> 0.05$	$1.24e-20$
MOEA/D-PBI	$1.28e-34$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$3.15e-19$
SMS-EMOA	$1.05e-26$	$2.70e-22$	$8.97e-21$	$7.40e-23$	—	$> 0.05$	$1.91e-20$	$3.51e-22$	$1.05e-06$	$> 0.05$	$7.40e-23$	$2.18e-22$	$7.40e-23$
$\Delta_p$ -DDE	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MOGA	$1.28e-34$	$> 0.05$	$1.90e-02$	$3.27e-12$	$> 0.05$	$> 0.05$	—	$1.77e-03$	$> 0.05$	$> 0.05$	$1.58e-16$	$> 0.05$	$6.62e-34$
R2-MOGAw	$1.28e-34$	$> 0.05$	$> 0.05$	$1.68e-04$	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$1.04e-05$	$> 0.05$	$1.33e-26$
R2-MODE	$1.28e-34$	$6.33e-26$	$7.02e-24$	$2.04e-33$	$> 0.05$	$> 0.05$	$1.95e-23$	$2.75e-28$	—	$> 0.05$	$2.07e-34$	$9.56e-26$	$1.28e-34$
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.49e-34$	$> 0.05$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$	$1.36e-34$
MOMBI-NTCH	$1.28e-34$	$> 0.05$	$1.28e-04$	$1.05e-22$	$> 0.05$	$> 0.05$	$1.75e-02$	$3.13e-07$	$> 0.05$	$> 0.05$	$1.85e-30$	—	$1.28e-34$
MOMBI-PBI	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—

Table A.34: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ6 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
5D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	> 0.05	> 0.05	6.97e-13	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.32e-34	7.34e-32	> 0.05
MOEA/D-NTCH	1.28e-34	8.46e-13	—	2.55e-02	3.61e-13	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.01e-30	7.46e-29	7.76e-03
MOEA/D-PBI	1.28e-34	5.62e-07	> 0.05	—	1.31e-12	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.37e-26	2.83e-21	> 0.05
SMS-EMOA	1.28e-34	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e-34	9.43e-25	4.45e-23	2.91e-24	6.43e-16	—	1.99e-11	1.38e-23	> 0.05	> 0.05	5.28e-25	5.28e-25	6.13e-24
R2-MOGA	1.28e-34	1.73e-34	5.87e-21	1.06e-28	1.97e-16	> 0.05	—	6.23e-20	> 0.05	> 0.05	1.28e-34	1.28e-34	1.00e-33
R2-MOGAw	1.28e-34	5.14e-30	1.20e-04	2.78e-10	3.75e-13	> 0.05	> 0.05	—	> 0.05	> 0.05	1.28e-34	2.33e-34	1.98e-22
R2-MODE	1.28e-34	2.01e-34	1.23e-30	1.37e-32	8.35e-23	> 0.05	6.38e-17	4.73e-30	—	> 0.05	1.28e-34	1.28e-34	1.31e-33
R2-IBEA	1.28e-34	1.28e-34	1.53e-34	1.28e-34	4.98e-31	2.86e-22	1.15e-32	1.28e-34	1.04e-23	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	1.28e-34	> 0.05	> 0.05	> 0.05	1.56e-10	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.28e-34	> 0.05	> 0.05	> 0.05	1.18e-11	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	3.49e-13	—	> 0.05
MOMBI-PBI	1.28e-34	1.33e-25	> 0.05	> 0.05	6.97e-13	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.28e-34	1.28e-34	—
6D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	> 0.05	> 0.05	8.66e-24	1.46e-31	> 0.05	> 0.05	> 0.05	> 0.05	1.28e-34	2.40e-34	> 0.05
MOEA/D-NTCH	1.28e-34	4.90e-18	—	> 0.05	2.05e-24	6.23e-33	> 0.05	> 0.05	> 0.05	> 0.05	1.76e-33	5.39e-33	> 0.05
MOEA/D-PBI	1.28e-34	2.57e-11	> 0.05	—	3.55e-24	9.12e-33	> 0.05	> 0.05	> 0.05	> 0.05	9.55e-31	9.21e-30	4.02e-02
SMS-EMOA	1.28e-34	> 0.05	> 0.05	> 0.05	—	2.87e-25	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.30e-31	4.38e-31	8.84e-30	1.53e-34	—	2.75e-27	> 0.05	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-MOGAw	1.28e-34	1.44e-34	2.70e-14	7.59e-10	1.91e-25	7.69e-34	> 0.05	—	> 0.05	> 0.05	1.28e-34	1.28e-34	1.13e-30
R2-MODE	1.28e-34	1.28e-34	1.27e-33	1.10e-33	2.18e-32	1.36e-34	8.33e-19	1.15e-32	—	> 0.05	1.28e-34	1.28e-34	1.36e-34
R2-IBEA	1.28e-34	1.28e-34	1.44e-34	1.28e-34	7.22e-33	1.28e-34	7.56e-18	1.44e-34	> 0.05	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	1.28e-34	> 0.05	> 0.05	> 0.05	2.85e-18	2.40e-28	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.28e-34	> 0.05	> 0.05	> 0.05	7.31e-19	1.22e-28	> 0.05	> 0.05	> 0.05	> 0.05	5.49e-03	—	> 0.05
MOMBI-PBI	1.28e-34	2.18e-32	> 0.05	> 0.05	2.63e-24	3.08e-33	> 0.05	> 0.05	> 0.05	> 0.05	1.28e-34	1.28e-34	—
7D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	9.65e-03	> 0.05	1.28e-34	8.11e-33	> 0.05	> 0.05	> 0.05	> 0.05	7.07e-29	1.36e-34	> 0.05
MOEA/D-NTCH	1.28e-34	> 0.05	—	> 0.05	1.49e-34	4.20e-31	> 0.05	> 0.05	> 0.05	> 0.05	2.23e-10	1.10e-24	> 0.05
MOEA/D-PBI	1.28e-34	2.91e-33	8.89e-31	—	1.28e-34	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	1.28e-34	1.28e-34	> 0.05
SMS-EMOA	1.28e-34	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	3.11e-29	3.67e-27	> 0.05	1.28e-34	1.68e-34	—	> 0.05	> 0.05	> 0.05	1.40e-34	1.28e-34	> 0.05
R2-MOGAw	1.28e-34	3.20e-30	1.25e-28	3.87e-04	1.28e-34	1.58e-34	> 0.05	—	> 0.05	> 0.05	1.28e-34	1.28e-34	3.51e-02
R2-MODE	1.28e-34	2.13e-34	5.88e-34	1.19e-28	1.28e-34	1.28e-34	1.58e-21	2.09e-20	—	3.12e-03	1.28e-34	1.28e-34	1.61e-27
R2-IBEA	1.28e-34	1.32e-34	1.84e-34	2.12e-31	1.28e-34	1.28e-34	1.23e-18	3.54e-16	> 0.05	—	1.28e-34	1.28e-34	1.13e-30
MOMBI-TCH	1.28e-34	> 0.05	> 0.05	> 0.05	1.28e-34	1.92e-29	> 0.05	> 0.05	> 0.05	> 0.05	—	6.07e-21	> 0.05
MOMBI-NTCH	1.28e-34	> 0.05	> 0.05	> 0.05	1.49e-34	2.43e-26	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	1.28e-34	1.28e-34	4.78e-34	3.03e-05	1.28e-34	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	1.28e-34	1.28e-34	—

Table A.35: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ6 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
8D													
NSGA-II	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e – 34	–	4.39e – 07	> 0.05	2.51e – 33	1.51e – 28	> 0.05	> 0.05	> 0.05	> 0.05	1.63e – 34	1.28e – 34	> 0.05
MOEA/D-NTCH	1.28e – 34	> 0.05	–	> 0.05	1.26e – 32	1.77e – 24	> 0.05	> 0.05	> 0.05	> 0.05	4.20e – 15	1.37e – 25	> 0.05
MOEA/D-PBI	1.28e – 34	4.90e – 03	3.38e – 09	–	2.99e – 33	4.95e – 29	> 0.05	> 0.05	> 0.05	> 0.05	8.24e – 32	2.13e – 34	1.11e – 02
SMS-EMOA	1.28e – 34	> 0.05	> 0.05	> 0.05	–	2.39e – 04	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e – 34	1.13e – 33	1.27e – 33	1.42e – 30	2.51e – 33	5.07e – 34	–	7.84e – 24	> 0.05	> 0.05	1.28e – 34	1.28e – 34	6.06e – 34
R2-MOGAw	1.28e – 34	1.79e – 29	2.34e – 28	4.99e – 10	2.51e – 33	3.89e – 32	> 0.05	–	> 0.05	> 0.05	1.28e – 34	1.28e – 34	2.44e – 33
R2-MODE	1.28e – 34	2.33e – 34	2.88e – 34	7.65e – 33	2.51e – 33	1.95e – 34	1.01e – 18	9.82e – 31	–	4.72e – 16	1.28e – 34	1.28e – 34	1.40e – 34
R2-IBEA	1.28e – 34	2.13e – 34	5.80e – 34	2.28e – 30	2.51e – 33	4.12e – 34	1.70e – 02	8.11e – 25	> 0.05	–	1.28e – 34	1.28e – 34	1.28e – 34
MOMBI-TCH	1.28e – 34	> 0.05	> 0.05	> 0.05	7.14e – 32	5.88e – 19	> 0.05	> 0.05	> 0.05	> 0.05	–	3.63e – 19	> 0.05
MOMBI-NTCH	1.28e – 34	> 0.05	> 0.05	> 0.05	3.11e – 30	1.16e – 15	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	1.28e – 34	> 0.05	6.98e – 07	> 0.05	2.51e – 33	2.15e – 28	> 0.05	> 0.05	> 0.05	> 0.05	1.32e – 34	1.28e – 34	–
9D													
NSGA-II	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e – 34	–	1.09e – 21	> 0.05	1.28e – 34	1.69e – 31	> 0.05	> 0.05	> 0.05	> 0.05	1.28e – 34	1.28e – 34	6.63e – 34
MOEA/D-NTCH	1.28e – 34	> 0.05	–	> 0.05	1.08e – 27	2.25e – 26	> 0.05	> 0.05	> 0.05	> 0.05	3.61e – 10	1.59e – 20	> 0.05
MOEA/D-PBI	1.28e – 34	> 0.05	5.03e – 10	–	1.83e – 34	1.95e – 29	> 0.05	> 0.05	> 0.05	> 0.05	4.00e – 34	1.28e – 34	2.26e – 08
SMS-EMOA	1.28e – 34	> 0.05	> 0.05	> 0.05	–	1.71e – 17	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e – 34	9.95e – 33	3.44e – 34	1.13e – 31	1.28e – 34	2.88e – 34	–	1.50e – 32	> 0.05	> 0.05	1.28e – 34	1.28e – 34	1.28e – 34
R2-MOGAw	1.28e – 34	5.35e – 05	5.75e – 27	3.92e – 02	1.28e – 34	5.83e – 32	> 0.05	–	> 0.05	> 0.05	1.28e – 34	1.28e – 34	1.32e – 34
R2-MODE	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	2.26e – 34	1.28e – 34	–	1.73e – 34	1.28e – 34	1.28e – 34	1.28e – 34
R2-IBEA	1.28e – 34	6.54e – 32	6.63e – 34	1.60e – 28	1.28e – 34	5.38e – 34	> 0.05	3.86e – 31	> 0.05	–	1.28e – 34	1.28e – 34	1.28e – 34
MOMBI-TCH	1.28e – 34	> 0.05	> 0.05	> 0.05	4.12e – 32	4.67e – 24	> 0.05	> 0.05	> 0.05	> 0.05	–	6.74e – 27	> 0.05
MOMBI-NTCH	1.28e – 34	> 0.05	> 0.05	> 0.05	2.94e – 25	2.28e – 22	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	1.28e – 34	> 0.05	> 0.05	> 0.05	1.28e – 34	9.73e – 27	> 0.05	> 0.05	> 0.05	> 0.05	1.53e – 34	1.28e – 34	–
10D													
NSGA-II	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e – 34	–	4.67e – 23	> 0.05	1.27e – 34	3.65e – 21	> 0.05	3.27e – 08	> 0.05	> 0.05	1.28e – 34	1.28e – 34	1.28e – 34
MOEA/D-NTCH	1.28e – 34	> 0.05	–	> 0.05	7.45e – 19	5.17e – 16	> 0.05	> 0.05	> 0.05	> 0.05	2.03e – 05	2.27e – 13	1.07e – 02
MOEA/D-PBI	1.28e – 34	1.50e – 03	1.23e – 15	–	2.25e – 34	2.14e – 21	> 0.05	4.18e – 06	> 0.05	> 0.05	2.38e – 31	1.66e – 33	7.47e – 24
SMS-EMOA	1.27e – 34	> 0.05	> 0.05	> 0.05	–	9.30e – 14	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e – 34	1.50e – 30	4.24e – 34	1.16e – 23	1.27e – 34	4.08e – 26	–	2.13e – 34	> 0.05	1.04e – 06	1.28e – 34	1.28e – 34	1.28e – 34
R2-MOGAw	1.28e – 34	> 0.05	4.52e – 19	> 0.05	1.27e – 34	4.81e – 20	> 0.05	–	> 0.05	> 0.05	1.28e – 34	1.28e – 34	1.28e – 34
R2-MODE	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.27e – 34	1.89e – 34	1.28e – 34	1.28e – 34	–	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34
R2-IBEA	1.28e – 34	5.39e – 20	1.55e – 31	1.24e – 09	1.27e – 34	1.21e – 24	> 0.05	1.09e – 28	> 0.05	–	1.28e – 34	1.28e – 34	1.28e – 34
MOMBI-TCH	1.28e – 34	> 0.05	> 0.05	> 0.05	7.18e – 33	7.99e – 16	> 0.05	> 0.05	> 0.05	> 0.05	–	2.92e – 32	> 0.05
MOMBI-NTCH	1.28e – 34	> 0.05	> 0.05	> 0.05	8.68e – 19	6.45e – 14	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	1.28e – 34	> 0.05	> 0.05	> 0.05	2.00e – 34	3.91e – 16	> 0.05	> 0.05	> 0.05	> 0.05	1.38e – 24	1.58e – 34	–

Table A.36: Comparison of R2 indicator values for different optimizers on the DTLZ6 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	2.325781e-01	2.413114e-01	2.415995e-01	2.414071e-01	2.320597e-01	2.181492e-01	2.330036e-01	2.332262e-01	2.186384e-01	2.179528e-01	2.322777e-01	2.320757e-01	2.302493e-01
	avg.	2.323240e-01	2.418777e-01	2.408894e-01	2.413475e-01	2.328899e-01	2.181527e-01	2.326445e-01	2.329520e-01	2.186722e-01	2.196646e-01	2.323820e-01	2.322783e-01	2.300776e-01
	min.	2.185097e-01	2.268994e-01	2.218910e-01	2.226065e-01	2.178937e-01	2.180526e-01	2.190074e-01	2.191498e-01	2.183243e-01	2.178600e-01	2.180302e-01	2.182308e-01	2.180247e-01
	max.	2.443978e-01	2.610203e-01	2.564188e-01	2.564765e-01	2.451558e-01	2.182652e-01	2.435965e-01	2.457102e-01	2.192253e-01	2.276276e-01	2.468941e-01	2.511159e-01	2.429304e-01
	std.	5.315264e-03	7.715790e-03	7.279960e-03	7.459733e-03	5.949713e-03	3.979403e-05	5.218545e-03	5.536078e-03	1.882701e-04	2.655011e-03	5.855286e-03	6.102577e-03	5.297032e-03
3D	med.	1.815756e-01	1.745983e-01	1.758020e-01	1.768277e-01	1.696638e-01	1.632764e-01	1.715456e-01	1.707585e-01	1.635261e-01	1.629806e-01	1.698687e-01	1.708182e-01	1.969856e-01
	avg.	1.811822e-01	1.745380e-01	1.751858e-01	1.771219e-01	1.692386e-01	1.664186e-01	1.716523e-01	1.707367e-01	1.635625e-01	1.635882e-01	1.698610e-01	1.709445e-01	1.970330e-01
	min.	1.740099e-01	1.654527e-01	1.660952e-01	1.678016e-01	1.629319e-01	1.631602e-01	1.639356e-01	1.651987e-01	1.632143e-01	1.629289e-01	1.639019e-01	1.657095e-01	1.861699e-01
	max.	1.892434e-01	1.805636e-01	1.865244e-01	1.923885e-01	1.754518e-01	1.771385e-01	1.765268e-01	1.775654e-01	1.643002e-01	1.699160e-01	1.769780e-01	1.756563e-01	2.064697e-01
	std.	3.306331e-03	2.990702e-03	3.731293e-03	3.576343e-03	2.592399e-03	3.837192e-03	2.621960e-03	2.669734e-03	1.895264e-04	1.351785e-03	2.688443e-03	2.230917e-03	3.122067e-03
4D	med.	2.972586e-01	1.272329e-01	1.336763e-01	1.331527e-01	1.231958e-01	1.235224e-01	1.240627e-01	1.279120e-01	1.201686e-01	1.190107e-01	1.428564e-01	1.374783e-01	1.491895e-01
	avg.	2.972812e-01	1.275862e-01	1.354508e-01	1.338352e-01	1.255383e-01	1.237468e-01	1.241449e-01	1.279151e-01	1.204228e-01	1.188418e-01	1.431329e-01	1.374572e-01	1.494388e-01
	min.	2.746272e-01	1.218999e-01	1.248812e-01	1.261214e-01	1.202041e-01	1.228845e-01	1.193791e-01	1.232062e-01	1.191836e-01	1.173691e-01	1.373470e-01	1.305615e-01	1.449161e-01
	max.	3.233790e-01	1.338917e-01	1.640363e-01	1.482972e-01	1.653658e-01	1.274566e-01	1.285842e-01	1.360068e-01	1.224242e-01	1.219392e-01	1.530385e-01	1.510157e-01	1.550212e-01
	std.	9.614638e-03	2.315851e-03	6.934410e-03	3.923749e-03	8.764819e-03	6.838689e-04	1.758236e-03	2.622738e-03	7.824218e-04	1.351907e-03	2.699371e-03	3.492735e-03	2.158835e-03
5D	med.	3.844634e-01	1.104396e-01	1.108484e-01	1.031483e-01	1.088941e-01	1.022460e-01	9.664418e-02	1.079287e-01	9.260846e-02	9.533236e-02	1.542526e-01	1.550501e-01	1.138597e-01
	avg.	3.838996e-01	1.103666e-01	1.125711e-01	1.036137e-01	1.076813e-01	1.082122e-01	9.673275e-02	1.077947e-01	9.270295e-02	9.521273e-02	1.565943e-01	1.569850e-01	1.139842e-01
	min.	3.551946e-01	1.045163e-01	1.007985e-01	9.851765e-02	9.284954e-02	9.696992e-02	9.182077e-02	1.006110e-01	9.182077e-02	9.178930e-02	1.412003e-01	1.347315e-01	1.092831e-01
	max.	4.306229e-01	1.162181e-01	1.702462e-01	1.309597e-01	1.283671e-01	1.813281e-01	1.000075e-01	1.153181e-01	9.435366e-02	9.733029e-02	1.869604e-01	2.061781e-01	1.197589e-01
	std.	1.635599e-02	2.516774e-03	9.692462e-03	3.634069e-03	9.160378e-03	1.984652e-02	1.344709e-03	2.939586e-03	5.032651e-04	1.004717e-03	8.337187e-03	1.335219e-02	2.264822e-03
6D	med.	4.125908e-01	9.531148e-02	9.601437e-02	8.727830e-02	9.707592e-02	1.535692e-01	8.141965e-02	9.322746e-02	7.788644e-02	8.317296e-02	1.541540e-01	1.541974e-01	9.949796e-02
	avg.	4.119394e-01	9.518331e-02	9.759693e-02	9.002258e-02	9.599741e-02	1.511133e-01	8.144923e-02	9.367781e-02	7.790866e-02	8.315926e-02	1.562358e-01	1.551024e-01	9.969474e-02
	min.	3.621554e-01	8.830715e-02	8.805149e-02	8.412683e-02	7.774856e-02	9.736908e-02	7.907058e-02	8.868650e-02	7.694293e-02	8.063370e-02	1.351852e-01	1.260991e-01	9.561093e-02
	max.	4.504450e-01	1.008165e-01	1.410234e-01	1.172656e-01	1.115958e-01	1.628528e-01	8.440050e-02	1.014344e-01	7.875667e-02	8.510868e-02	1.833097e-01	1.967272e-01	1.049560e-01
	std.	1.775380e-02	2.741939e-03	7.088553e-03	6.893338e-03	8.713049e-03	1.223702e-02	1.265488e-02	2.522523e-03	3.018995e-04	9.290858e-04	1.058557e-02	1.106068e-02	2.052198e-03
7D	med.	3.974563e-01	8.564184e-02	8.711263e-02	7.598931e-02	1.018506e-01	1.242309e-01	7.089743e-02	7.486704e-02	6.857745e-02	7.894717e-02	1.174683e-01	1.404759e-01	7.492667e-02
	avg.	3.946921e-01	8.550425e-02	9.116148e-02	7.632527e-02	1.005445e-01	1.257574e-01	7.098187e-02	7.500868e-02	6.858323e-02	7.895809e-02	1.207613e-01	1.427013e-01	7.493868e-02
	min.	3.473819e-01	7.916079e-02	7.720972e-02	7.292449e-02	7.787447e-02	9.545529e-02	6.929429e-02	6.731051e-02	6.745978e-02	7.324807e-02	1.055482e-01	1.201915e-01	7.269659e-02
	max.	4.406548e-01	9.055887e-02	1.396575e-01	8.448139e-02	1.083291e-01	1.805813e-01	7.364910e-02	8.136754e-02	7.000342e-02	8.348605e-02	1.648656e-01	1.902759e-01	7.752731e-02
	std.	1.847803e-02	2.469057e-03	1.145918e-02	1.720919e-03	5.303990e-03	1.724606e-02	9.437533e-04	2.576271e-03	5.309097e-04	2.176225e-03	1.170334e-02	1.308825e-02	1.050576e-03
8D	med.	3.462935e-01	7.453638e-02	7.822240e-02	6.873458e-02	7.937805e-02	1.121518e-01	6.159254e-02	7.136465e-02	5.881399e-02	6.700870e-02	1.184106e-01	1.329654e-01	7.573530e-02
	avg.	3.458863e-01	7.440794e-02	8.427586e-02	7.245067e-02	7.825727e-02	1.097147e-01	6.159441e-02	7.172110e-02	5.883954e-02	6.706528e-02	1.185353e-01	1.344356e-01	7.571184e-02
	min.	3.175730e-01	6.952804e-02	6.978040e-02	6.501313e-02	5.835966e-02	7.186317e-02	5.931855e-02	6.594418e-02	5.820202e-02	6.557011e-02	1.004080e-01	1.106222e-01	6.845996e-02
	max.	3.797101e-01	7.959419e-02	1.181954e-01	9.632416e-02	8.867480e-02	1.580720e-01	6.335799e-02	7.515252e-02	5.982026e-02	6.837545e-02	1.417491e-01	1.571976e-01	8.246397e-02
	std.	1.492261e-02	2.161134e-03	1.287124e-02	7.110625e-03	5.518521e-03	1.900406e-02	7.844971e-04	2.907332e-03	3.205268e-04	6.349274e-04	8.830048e-03	9.451751e-03	2.273088e-03
9D	med.	2.954788e-01	6.530593e-02	8.257864e-02	6.491818e-02	6.637197e-02	1.035217e-01	5.771489e-02	6.959244e-02	5.084375e-02	6.137344e-02	1.136163e-01	1.264458e-01	8.395944e-02
	avg.	2.954450e-01	6.548733e-02	8.270661e-02	6.875399e-02	6.642829e-02	1.007564e-01	5.777784e-02	6.989296e-02	5.085801e-02	6.143374e-02	1.137627e-01	1.280527e-01	8.371034e-02
	min.	2.738477e-01	6.103095e-02	6.284653e-02	5.705970e-02	5.594700e-02	6.023720e-02	5.618706e-02	6.464928e-02	5.050788e-02	6.034368e-02	1.014287e-01	1.095774e-01	7.497116e-02
	max.	3.195918e-01	7.236797e-02	1.147916e-01	9.337415e-02	7.393350e-02	1.322949e-01	5.947255e-02	7.617545e-02	5.130572e-02	6.317971e-02	1.246588e-01	1.555439e-01	8.997600e-02
	std.	9.063090e-03	2.016683e-03	1.274262e-02	1.110414e-02	3.437385e-03	1.248588e-02	7.115792e-04	2.128731e-03	1.806049e-04	4.436310e-04	4.157600e-03	8.940958e-03	2.860479e-03
10D	med.	2.524678e-01	5.875288e-02	8.035382e-02	5.328734e-02	6.484831e-02	9.182545e-02	5.678925e-02	6.882119e-02	4.558154e-02	5.884326e-02	1.070970e-01	1.233950e-01	9.002050e-02
	avg.	2.526014e-01	5.892092e-02	8.040680e-02	6.174237e-02	6.452201e-02	8.749801e-02	5.689232e-02	6.910183e-02	4.559627e-02	5.882222e-02	1.075368e-01	1.237978e-01	9.042035e-02
	min.	2.342608e-01	5.490293e-02	5.539515e-02	5.113569e-02	5.693264e-02	5.283133e-02	5.538369e-02	6.386616e-02	4.539541e-02	5.739578e-02	1.006049e-01	1.067906e-01	8.282035e-02
	max.	2.708281e-01	6.521897e-02	1.074635e-01	9.798370e-02	6.956513e-02	1.008886e-01	5.911648e-02	7.535545e-02	4.601880e-02	6.011647e-02	1.213341e-01	1.067307e-01	9.865370e-02
	std.	7.925620e-03	1.953951e-03	1.345814e-02	1.408209e-02	2.640281e-03	1.335962e-02	7.708182e-04	2.455446e-03	1.132523e-04	5.617354e-04	3.718249e-03	7.209935e-03	2.699900e-03

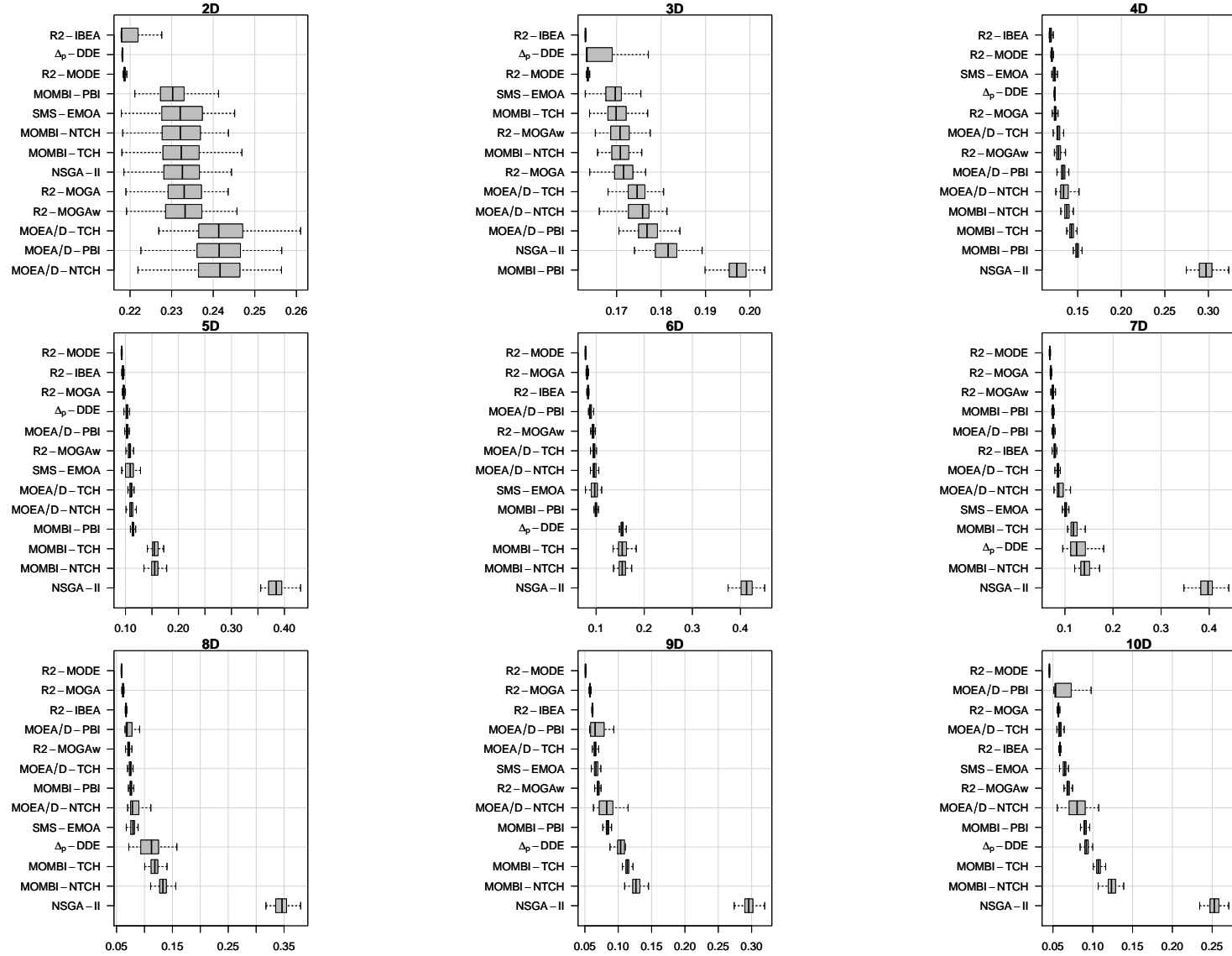


Figure A.82: Box-plot of R2 indicator values for different optimizers on the DTLZ6 test problem.

Table A.37: Comparison of runtime (in milliseconds) for different optimizers on the DTLZ6 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_\rho$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	2.143810e + 02	4.848680e + 02	1.004216e + 03	8.808460e + 02	2.777929e + 04	1.523883e + 03	2.259686e + 03	1.756553e + 03	1.647200e + 03	5.613840e + 05	1.984542e + 03	2.004427e + 03	2.768610e + 03
	avg.	2.279385e + 02	5.081756e + 02	9.873554e + 02	8.734920e + 02	2.721453e + 04	1.518439e + 03	2.263384e + 03	1.746744e + 03	1.655839e + 03	5.541640e + 05	1.968021e + 03	1.961313e + 03	2.743152e + 03
	min.	1.930730e + 02	3.556440e + 02	7.254720e + 02	6.731290e + 02	1.970310e + 04	1.117324e + 03	1.637740e + 03	1.346108e + 03	1.133238e + 03	4.828620e + 05	1.494069e + 03	1.577116e + 03	2.285580e + 03
	max.	3.599950e + 02	7.942190e + 02	1.331142e + 03	1.260292e + 03	3.020344e + 04	1.855111e + 03	2.695465e + 03	2.186825e + 03	2.061994e + 03	5.785690e + 05	2.338856e + 03	2.397291e + 03	3.069830e + 03
	std.	3.388499e + 01	9.917708e + 01	1.189263e + 02	8.887342e + 01	1.996099e + 03	1.555689e + 02	2.579519e + 02	2.046394e + 02	1.847682e + 02	2.041980e + 04	1.678273e + 02	2.147438e + 02	1.538496e + 02
3D	med.	3.324990e + 02	5.511080e + 02	1.350732e + 03	9.849690e + 02	4.930363e + 05	2.376026e + 03	2.395817e + 03	1.853324e + 03	2.117173e + 03	6.120190e + 05	1.950651e + 03	2.097182e + 03	3.094441e + 03
	avg.	3.486869e + 02	5.646628e + 02	1.337404e + 03	9.881362e + 02	4.812829e + 05	2.343932e + 03	2.361311e + 03	1.834748e + 03	2.047367e + 03	5.695276e + 05	1.952726e + 03	2.084959e + 03	3.065790e + 03
	min.	2.876200e + 02	3.944710e + 02	9.619120e + 02	7.798950e + 02	3.158965e + 05	1.754420e + 03	1.654024e + 03	1.432413e + 03	1.419162e + 03	4.826010e + 05	1.646162e + 03	1.742471e + 03	2.697403e + 03
	max.	5.124360e + 02	8.641720e + 02	1.812280e + 03	1.181837e + 03	6.576509e + 05	3.044041e + 03	2.758240e + 03	2.269441e + 03	2.417897e + 03	6.348050e + 05	2.252036e + 03	2.462935e + 03	3.280753e + 03
	std.	5.226791e + 01	1.000402e + 02	1.257540e + 02	9.517739e + 01	5.388809e + 04	2.332453e + 02	2.544954e + 02	1.785797e + 02	2.625252e + 02	6.620324e + 04	1.370465e + 02	1.584915e + 02	1.014484e + 02
4D	med.	5.141440e + 02	6.399460e + 02	1.470512e + 03	1.078614e + 03	3.153174e + 06	1.515165e + 03	2.829956e + 03	2.071193e + 03	2.598639e + 03	7.243870e + 05	2.013471e + 03	2.202395e + 03	3.189653e + 03
	avg.	5.158964e + 02	6.512690e + 02	1.446769e + 03	1.072203e + 03	3.163952e + 06	1.488558e + 03	2.800672e + 03	2.068784e + 03	2.586441e + 03	7.221896e + 05	2.033150e + 03	2.226523e + 03	3.165905e + 03
	min.	4.176760e + 02	4.613640e + 02	1.167274e + 03	8.423540e + 02	2.275307e + 06	1.108308e + 03	2.078138e + 03	1.564654e + 03	2.031199e + 03	6.657200e + 05	1.640169e + 03	1.828939e + 03	2.591691e + 03
	max.	6.813480e + 02	1.012374e + 03	1.674026e + 03	1.363891e + 03	4.5272284e + 06	1.840992e + 03	3.362544e + 03	2.592636e + 03	2.989356e + 03	7.515660e + 05	2.595439e + 03	3.589867e + 03	3.589867e + 03
	std.	6.565495e + 01	8.953294e + 01	1.016344e + 02	9.963185e + 01	4.430363e + 05	1.645328e + 02	2.815709e + 02	2.080303e + 02	2.331268e + 02	1.659414e + 04	1.945353e + 02	1.474971e + 02	2.208250e + 02
5D	med.	6.614170e + 02	6.900360e + 02	1.835899e + 03	1.110426e + 03	5.964980e + 06	1.821439e + 03	3.420356e + 03	2.587521e + 03	3.384833e + 03	5.696380e + 05	2.520824e + 03	2.255357e + 03	3.660971e + 03
	avg.	6.648878e + 02	6.941192e + 02	1.807271e + 03	1.083995e + 03	5.877695e + 06	1.803710e + 03	3.344727e + 03	2.596147e + 03	3.324151e + 03	6.043743e + 05	2.557990e + 03	2.290734e + 03	3.660804e + 03
	min.	5.409530e + 02	5.306260e + 02	1.377786e + 03	7.794730e + 02	3.741047e + 06	1.216133e + 03	2.266766e + 03	2.095146e + 03	2.478251e + 03	5.677400e + 05	1.874537e + 03	2.053389e + 03	2.948996e + 03
	max.	8.923610e + 02	1.235207e + 03	2.222136e + 03	1.255765e + 03	6.666147e + 06	2.268170e + 03	3.905929e + 03	3.158608e + 03	3.747696e + 03	7.108060e + 05	4.140232e + 03	2.825798e + 03	4.201511e + 03
	std.	6.882040e + 01	9.817956e + 01	1.487712e + 02	1.120627e + 02	4.669861e + 05	2.029910e + 02	3.041044e + 02	2.271839e + 02	2.812663e + 02	5.037462e + 04	4.313043e + 02	1.564834e + 02	2.118332e + 02
6D	med.	7.886300e + 02	7.448480e + 02	1.984038e + 03	1.142337e + 03	7.653706e + 06	2.631554e + 03	3.511442e + 03	2.963602e + 03	3.908919e + 03	7.101960e + 05	2.213100e + 03	2.444886e + 03	3.913478e + 03
	avg.	7.947273e + 02	7.374611e + 02	1.965848e + 03	1.135887e + 03	7.613974e + 06	2.610599e + 03	3.514854e + 03	2.960192e + 03	3.907869e + 03	7.099275e + 05	2.334755e + 03	2.469335e + 03	3.942019e + 03
	min.	6.488770e + 02	5.351890e + 02	1.542136e + 03	8.916150e + 02	5.911308e + 06	1.451781e + 03	2.791379e + 03	2.230105e + 03	3.018432e + 03	6.893490e + 05	1.731844e + 03	2.135933e + 03	3.200231e + 03
	max.	1.090907e + 03	9.706990e + 02	2.531285e + 03	1.946540e + 03	8.656365e + 06	3.173561e + 03	4.079463e + 03	3.504377e + 03	4.355629e + 03	7.179320e + 05	3.705149e + 03	2.909742e + 03	4.530931e + 03
	std.	7.419073e + 01	7.887051e + 01	1.879653e + 02	1.471655e + 02	5.038291e + 05	3.418073e + 02	3.388396e + 02	2.624127e + 02	2.839913e + 02	4.263008e + 03	4.375112e + 02	1.755028e + 02	2.554041e + 02
7D	med.	6.695730e + 02	7.639650e + 02	2.151514e + 03	1.176462e + 03	6.242462e + 06	9.215148e + 03	2.881655e + 03	2.414870e + 03	3.059815e + 03	3.548100e + 05	1.588921e + 03	1.772946e + 03	2.843825e + 03
	avg.	6.723130e + 02	7.736229e + 02	2.108172e + 03	1.146861e + 03	6.189223e + 06	9.073060e + 03	2.889632e + 03	2.390596e + 03	3.040751e + 03	3.540896e + 05	1.599826e + 03	1.786063e + 03	2.840141e + 03
	min.	5.397330e + 02	5.656180e + 02	1.720048e + 03	8.619220e + 02	4.839640e + 06	5.708068e + 03	2.116865e + 03	1.971997e + 03	2.288429e + 03	3.302350e + 05	1.240816e + 03	1.577441e + 03	2.325456e + 03
	max.	9.260210e + 02	9.977790e + 02	2.419813e + 03	1.525151e + 03	7.021359e + 06	1.219393e + 04	3.278625e + 03	2.887099e + 03	3.414971e + 03	3.697980e + 05	2.479728e + 03	2.091107e + 03	3.151430e + 03
	std.	7.155943e + 01	8.590354e + 01	1.607607e + 02	1.277992e + 02	5.239203e + 05	1.120880e + 03	2.437584e + 02	2.100500e + 02	2.268479e + 02	8.228417e + 03	1.620532e + 02	1.253739e + 02	1.338500e + 02
8D	med.	9.806600e + 02	8.162310e + 02	2.336657e + 03	1.220131e + 03	1.007779e + 07	1.567566e + 04	4.623835e + 03	3.605023e + 03	4.938200e + 03	1.380671e + 06	2.037694e + 03	2.826263e + 03	4.250914e + 03
	avg.	9.771752e + 02	8.084388e + 02	2.294164e + 03	1.193256e + 03	1.008517e + 07	1.556897e + 04	4.526672e + 03	3.616136e + 03	4.827612e + 03	1.293984e + 06	2.047179e + 03	2.819794e + 03	4.270755e + 03
	min.	7.729620e + 02	6.117600e + 02	1.678077e + 03	8.847750e + 02	7.747436e + 06	8.369756e + 03	3.585066e + 03	2.821381e + 03	3.581416e + 03	1.117714e + 06	1.714877e + 03	2.317708e + 03	3.708878e + 03
	max.	1.239650e + 03	9.891810e + 02	2.761451e + 03	1.490058e + 03	1.163950e + 07	2.454791e + 04	5.100631e + 03	4.315781e + 03	5.438933e + 03	1.540572e + 06	2.430093e + 03	3.382233e + 03	4.826294e + 03
	std.	8.546542e + 01	8.022071e + 01	1.689135e + 02	1.560753e + 02	7.946719e + 05	2.756967e + 03	3.717872e + 02	3.129350e + 02	3.511608e + 02	1.228337e + 05	1.832812e + 02	1.554836e + 02	1.807407e + 02
9D	med.	1.392152e + 03	8.876540e + 02	2.726383e + 03	1.321455e + 03	1.658700e + 07	2.428138e + 04	6.675601e + 03	5.510329e + 03	7.737849e + 03	2.645520e + 06	2.753264e + 03	4.007572e + 03	6.121675e + 03
	avg.	1.380561e + 03	8.842951e + 02	2.695904e + 03	1.261046e + 03	1.618095e + 07	2.476227e + 04	6.296032e + 03	5.469811e + 03	7.731597e + 03	2.643912e + 06	2.811512e + 03	3.981479e + 03	6.204807e + 03
	min.	1.112587e + 03	6.753650e + 02	2.043459e + 03	8.801670e + 02	1.215208e + 07	1.352027e + 04	4.356671e + 03	4.240834e + 03	6.168007e + 03	2.502544e + 06	2.389185e + 03	3.300911e + 03	5.491155e + 03
	max.	1.569936e + 03	1.295471e + 03	3.059364e + 03	1.678547e + 03	1.809890e + 07	3.724021e + 04	7.407341e + 03	6.419370e + 03	8.787391e + 03	2.663629e + 06	3.315710e + 03	4.340382e + 03	7.118678e + 03
	std.	1.071810e + 02	8.774767e + 01	2.001201e + 02	1.788580e + 02	1.479727e + 06	5.787442e + 03	9.877343e + 02	3.633586e + 02	5.284834e + 02	1.585035e + 04	2.498228e + 02	2.155503e + 02	3.571787e + 02
10D	med.	1.937013e + 03	9.329060e + 02	2.812992e + 03	1.310222e + 03	2.479444e + 07	3.592136e + 04	9.461879e + 03	7.766144e + 03	1.133482e + 04	5.051583e + 06	4.383625e + 03	5.247584e + 03	8.602520e + 03
	avg.	1.927330e + 03	9.347587e + 02	2.789765e + 03	1.302238e + 03	2.400086e + 07	3.783095e + 04	9.229568e + 03	7.812085e + 03	1.107899e + 04	4.984486e + 06	4.324453e + 03	5.366644e + 03	8.758707e + 03
	min.	1.602810e + 03	6.870250e + 02	2.113104e + 03	9.488030e + 02	1.879605e + 07	2.120286e + 04	6.614847e + 03	5.894180e + 03	7.994523e + 03	4.348486e + 06	3.510814e + 03	4.467501e + 03	7.044648e + 03
	max.	2.143727e + 03	1.364466e + 03	3.516755e + 03	1.883686e + 03	2.686941e + 07	6.605109e + 04	1.111553e + 04	9.149689e + 03	1.198737e + 04	5.098693e + 06	4.981583e + 03	5.621619e + 03	1.027576e + 04
	std.	1.199976e + 02	9.372953e + 01	2.176428e + 02	1.852932e + 02	2.184776e + 06	8.626992e + 03	1.333232e + 03	6.896112e + 02	7.820627e + 02	1.276313e + 05	2.926435e + 02	4.292478e + 02	5.646745e + 02

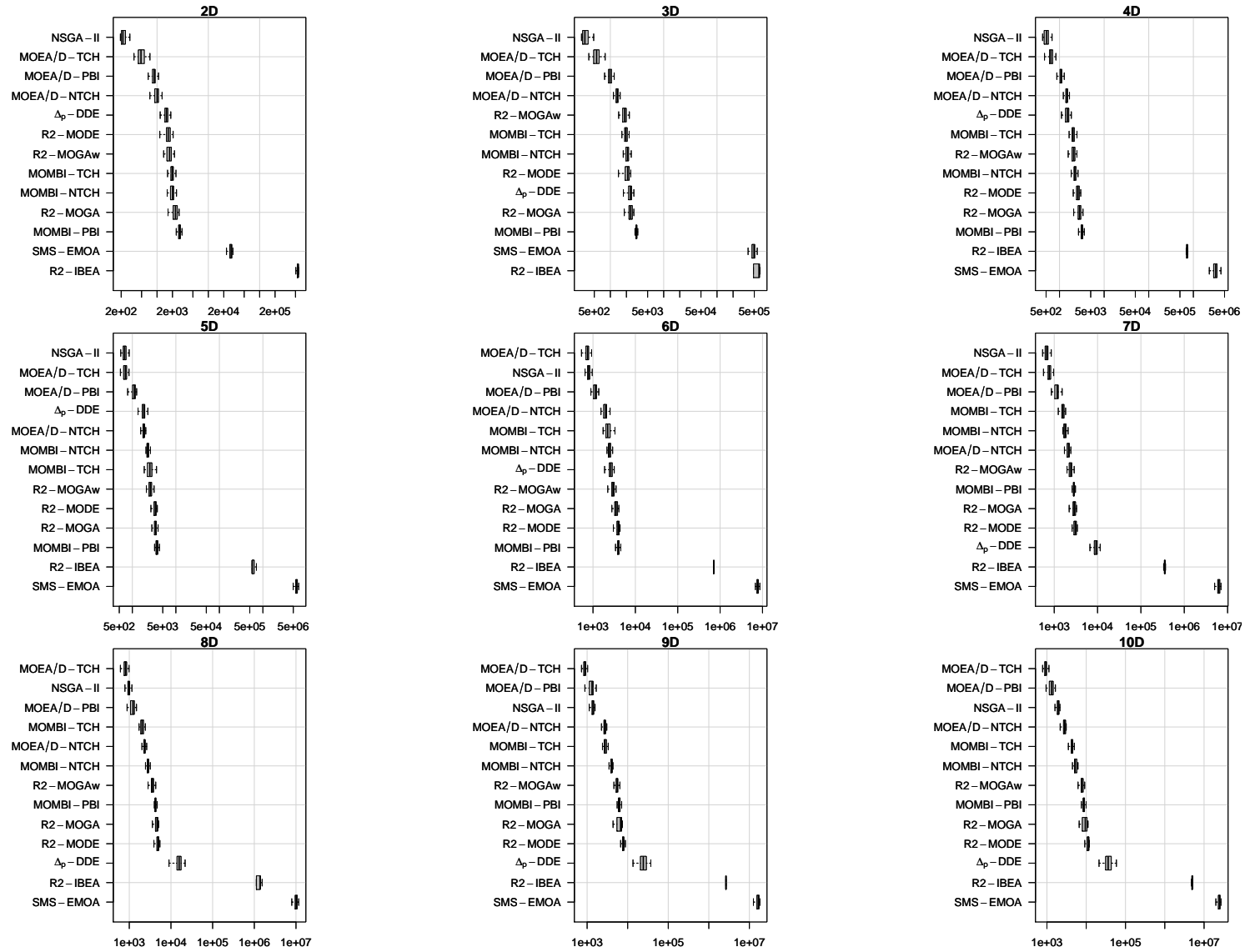


Figure A.83: Box-plot of runtime (in logarithmic scale) for different optimizers on the DTLZ6 test problem.

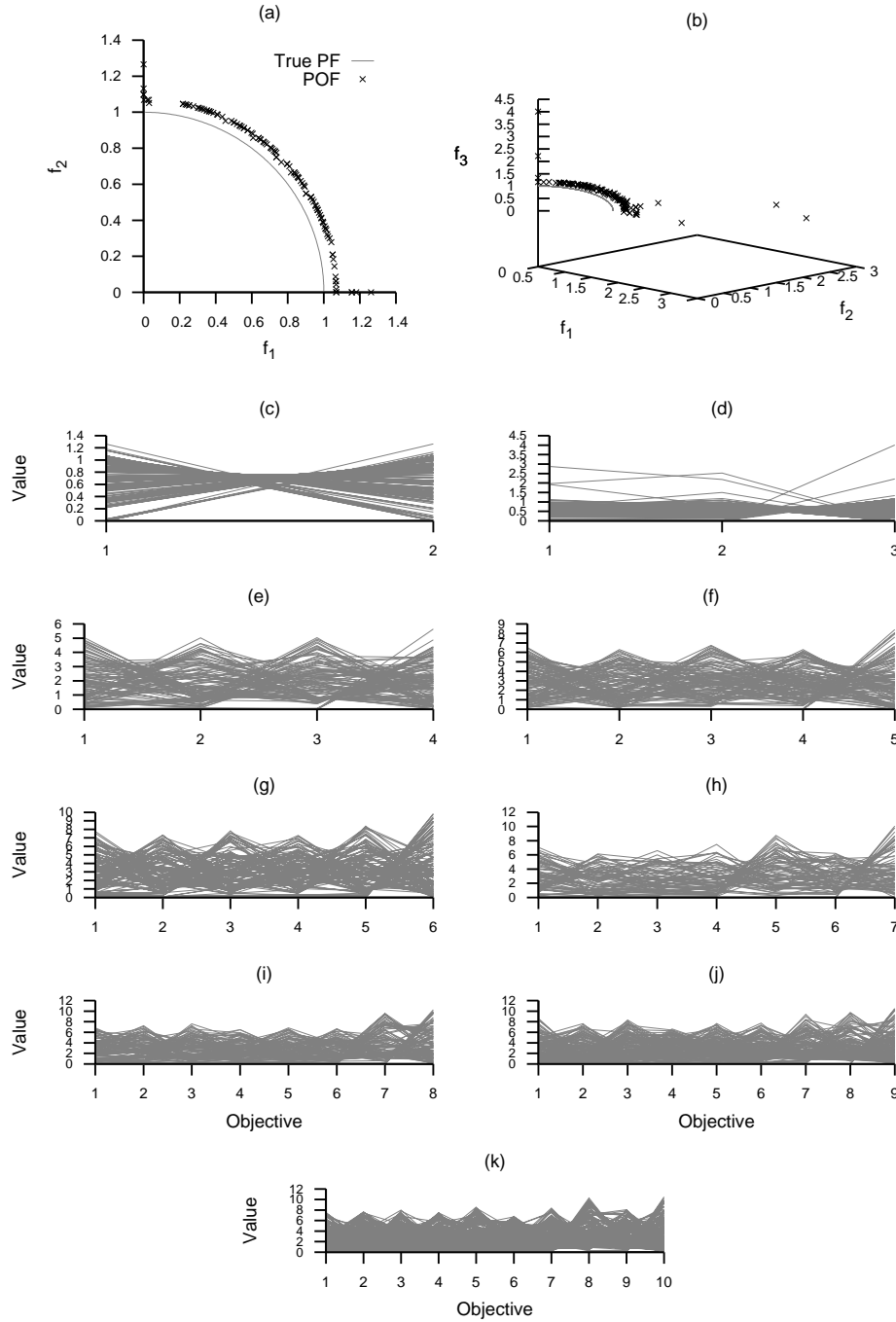


Figure A.84: Plots of the approximations obtained by NSGA-II from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



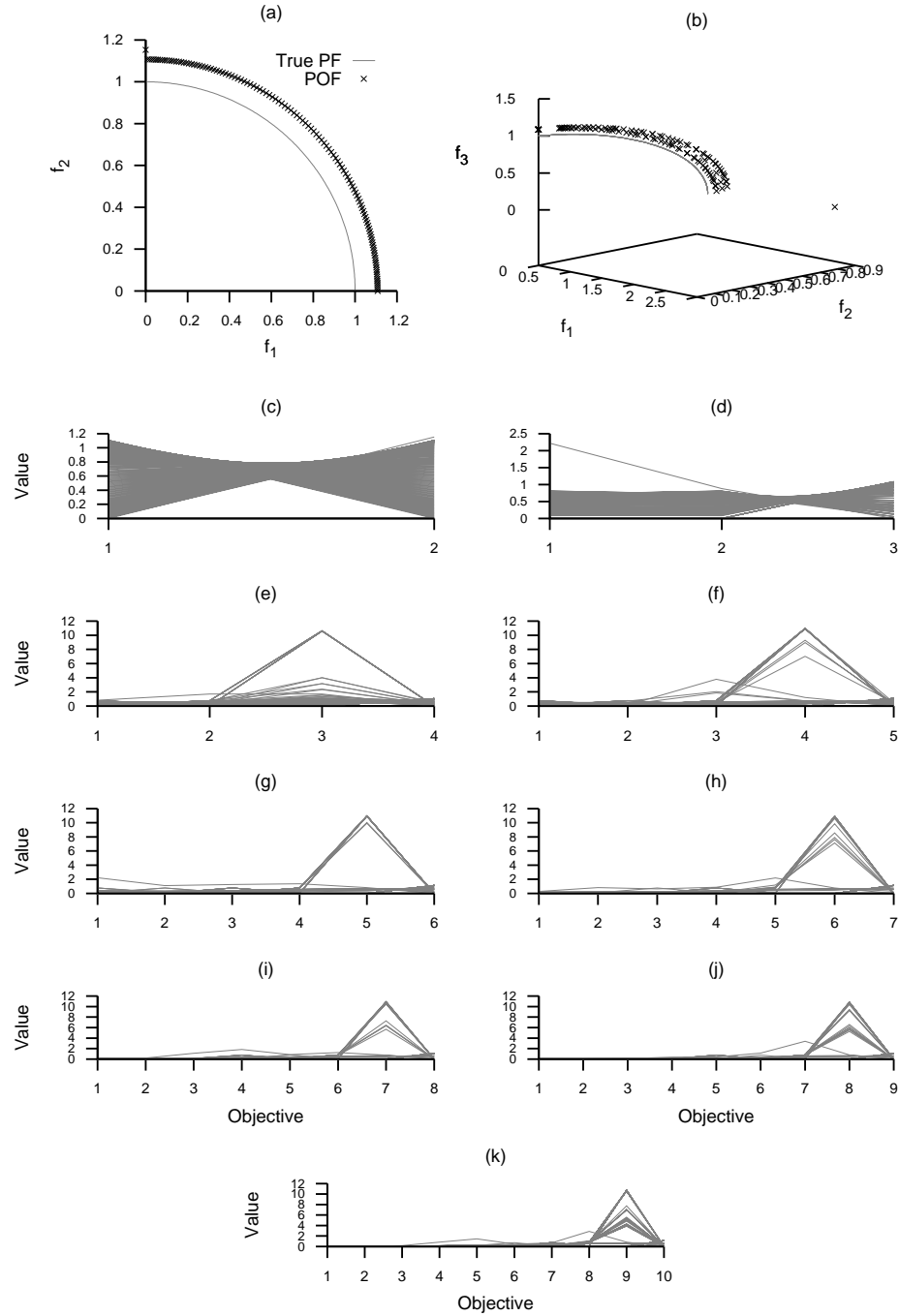


Figure A.85: Plots of the approximations obtained by MOEA/D-TCH from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

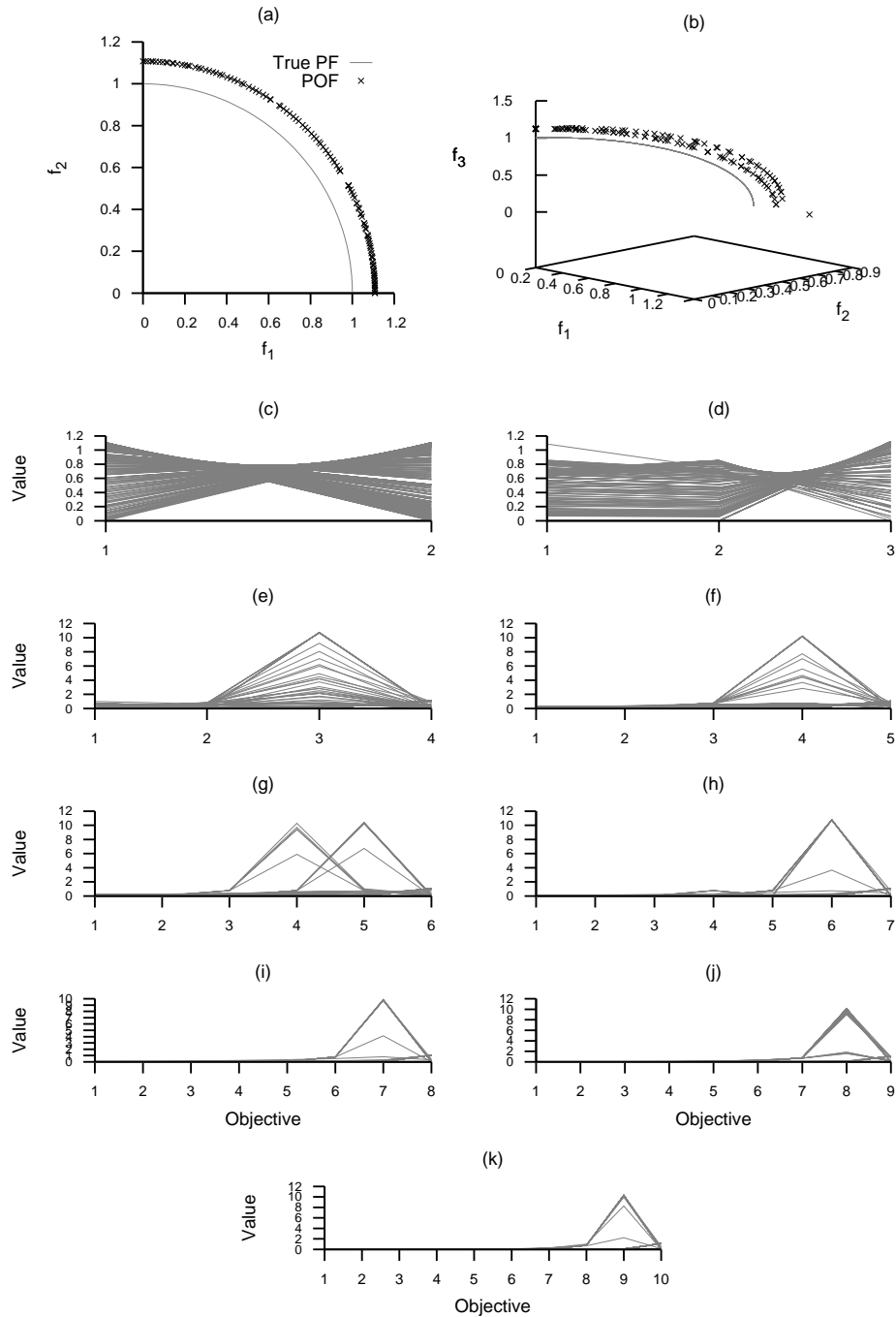


Figure A.86: Plots of the approximations obtained by MOEA/D-NTCH from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

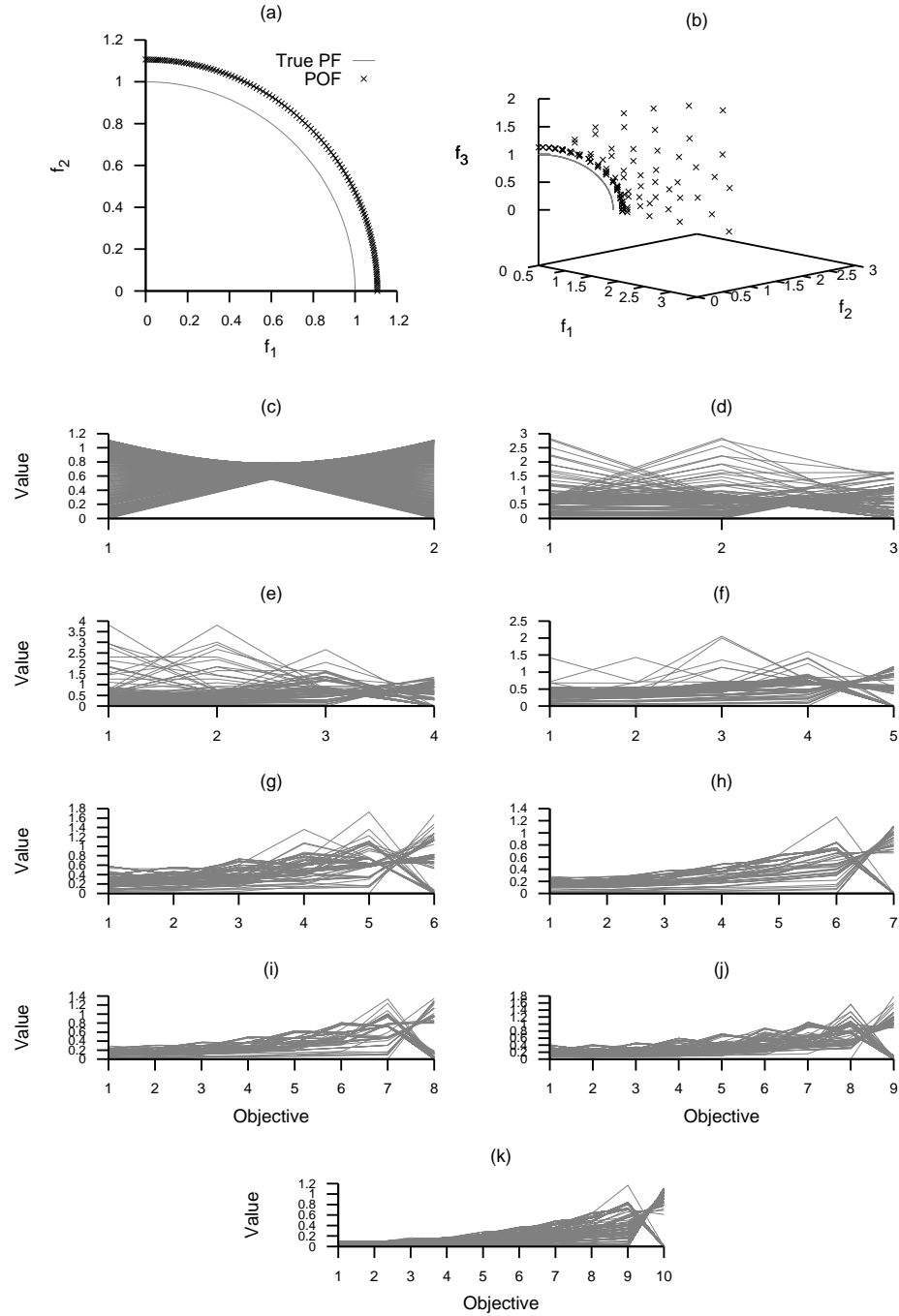


Figure A.87: Plots of the approximations obtained by MOEA/D-PBI from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

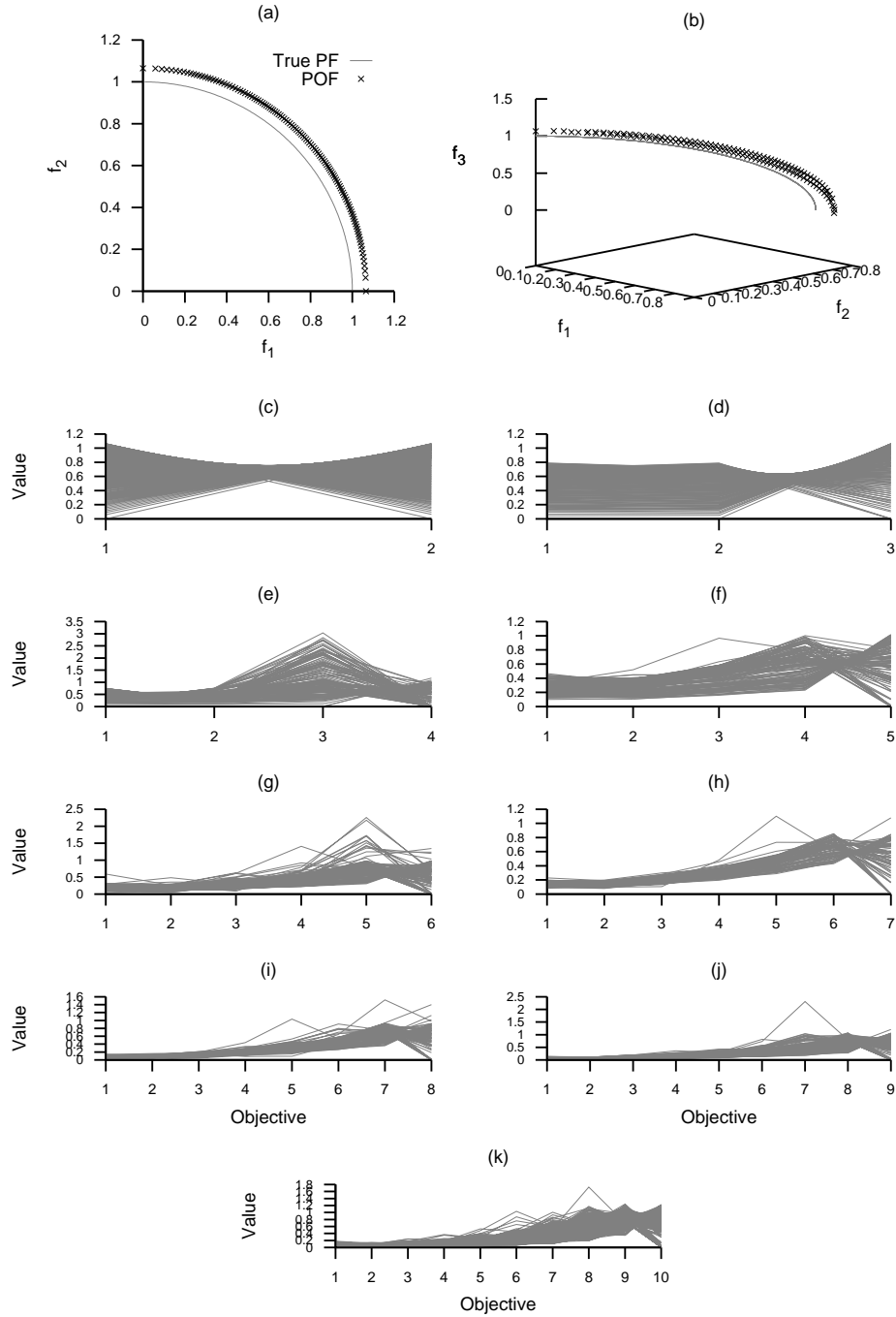


Figure A.88: Plots of the approximations obtained by SMS-EMOA from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

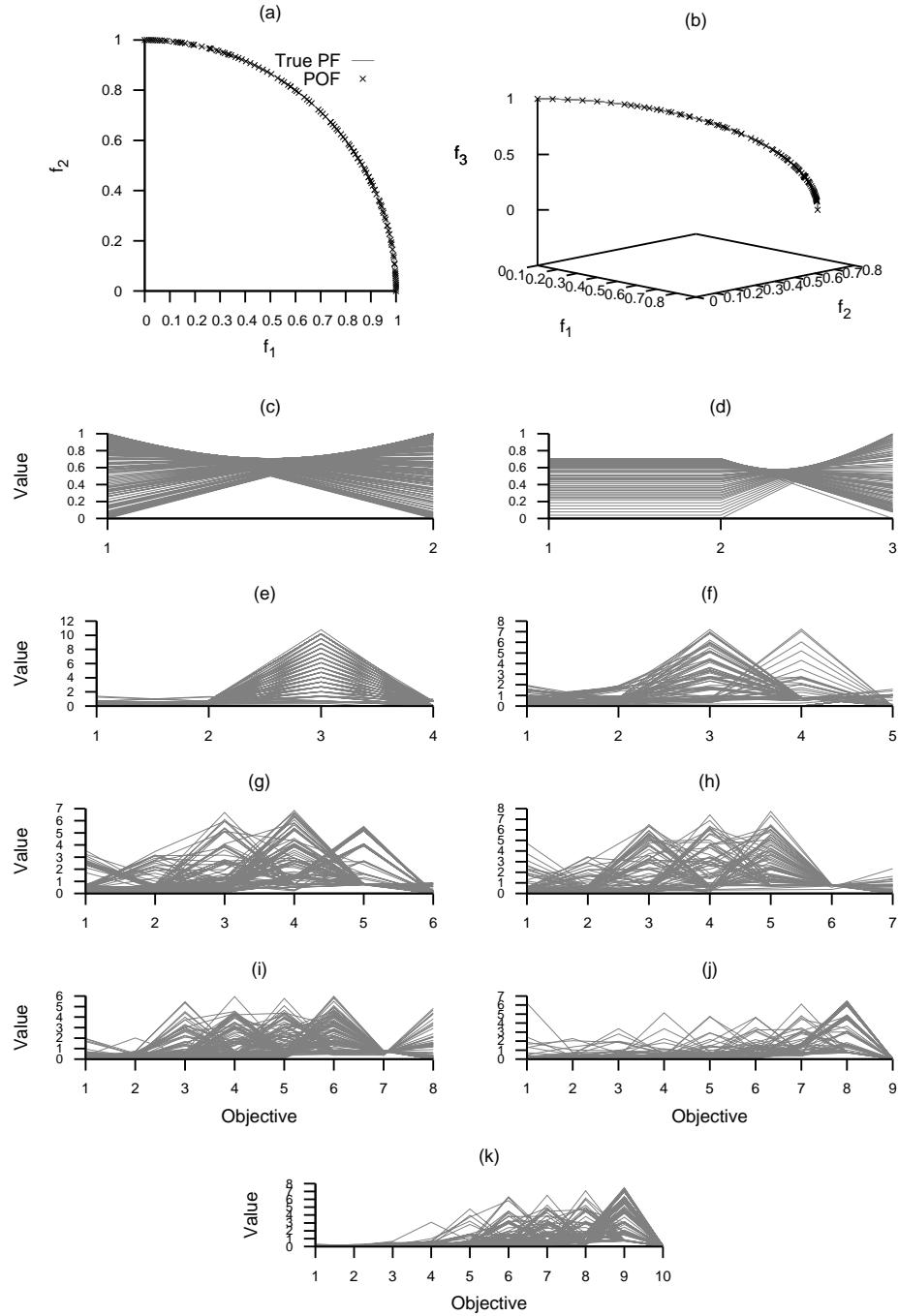


Figure A.89: Plots of the approximations obtained by  $\Delta_p$ -DDE from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

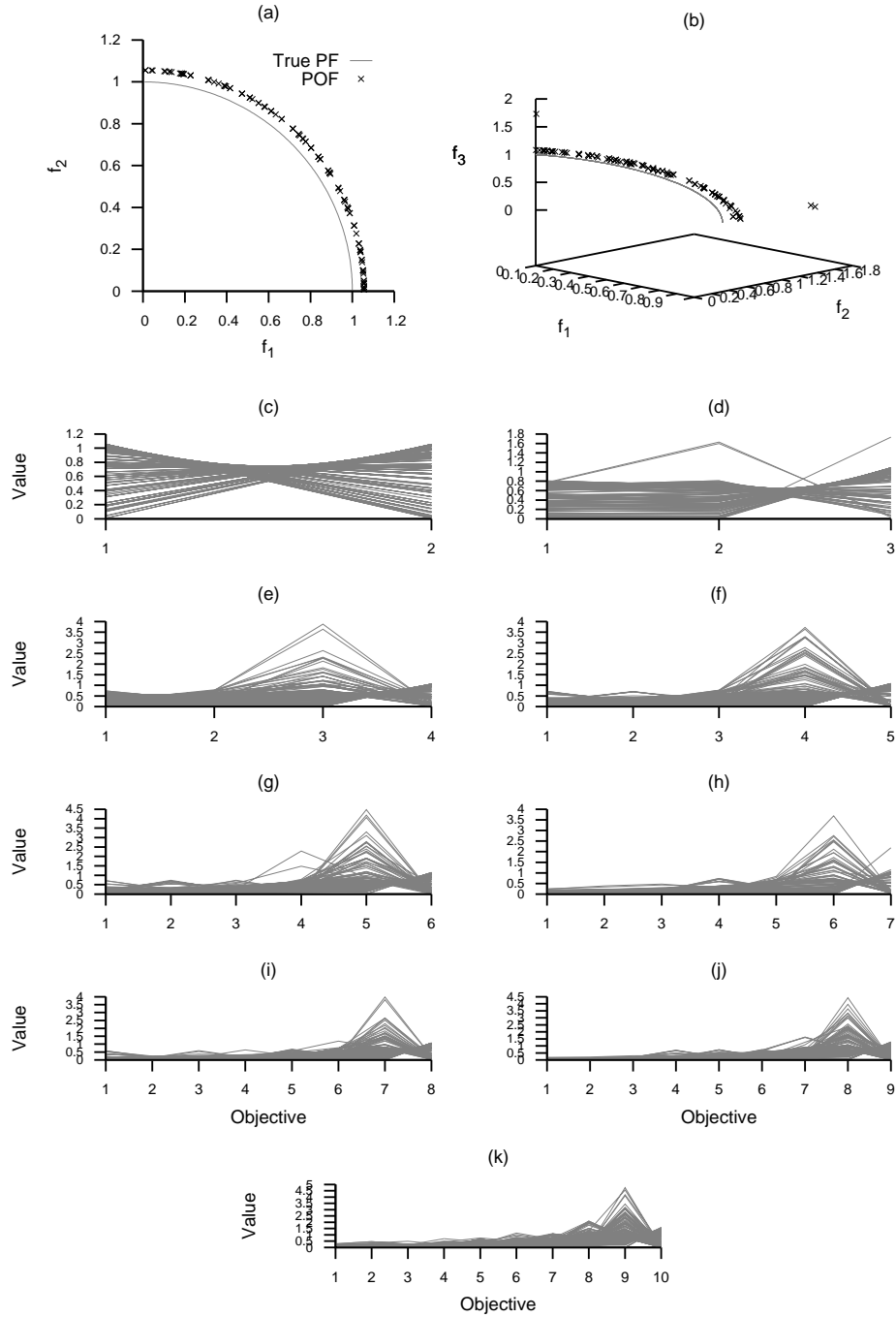


Figure A.90: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

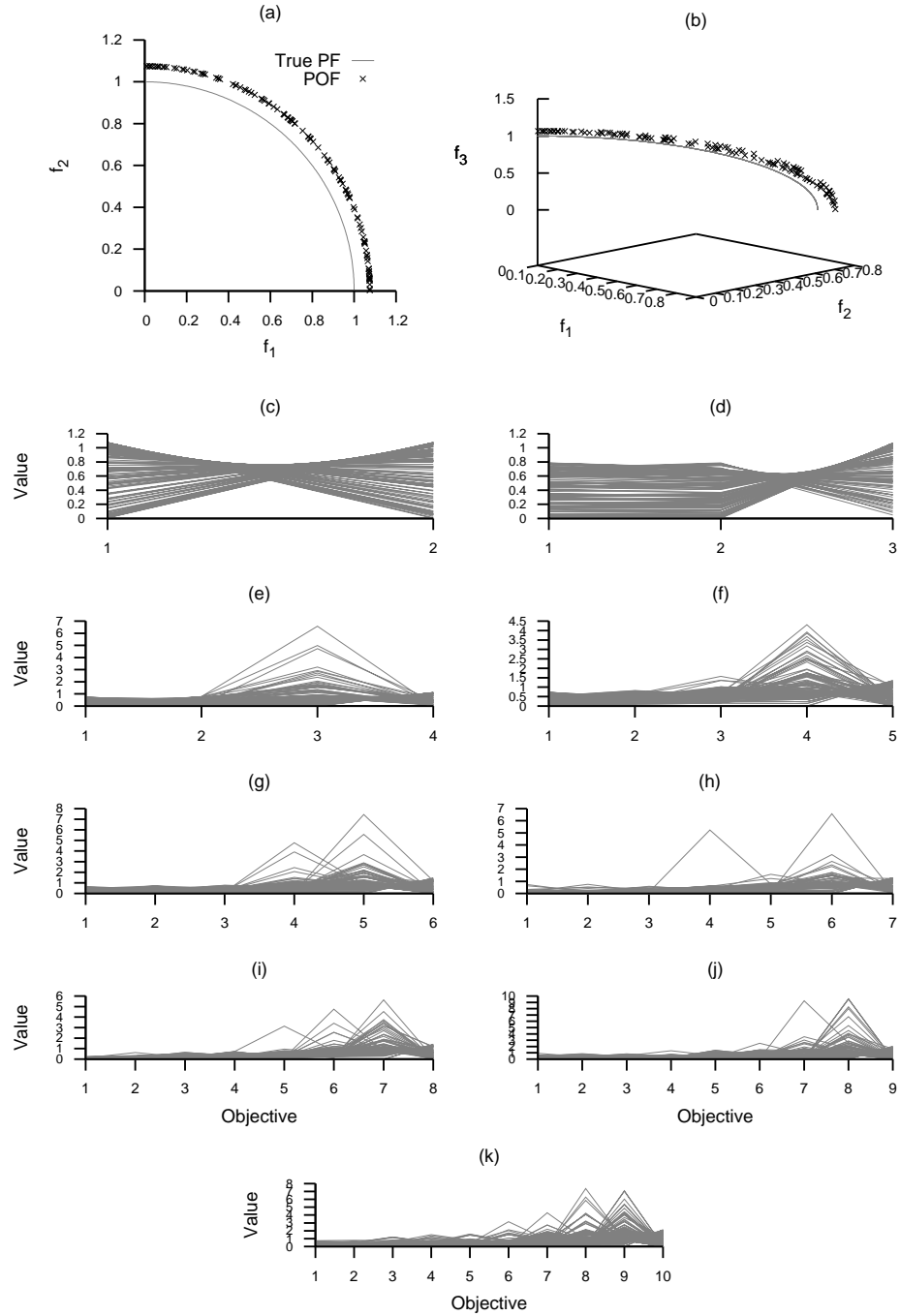


Figure A.91: Plots of the approximations obtained by *R2-MOGAw* from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

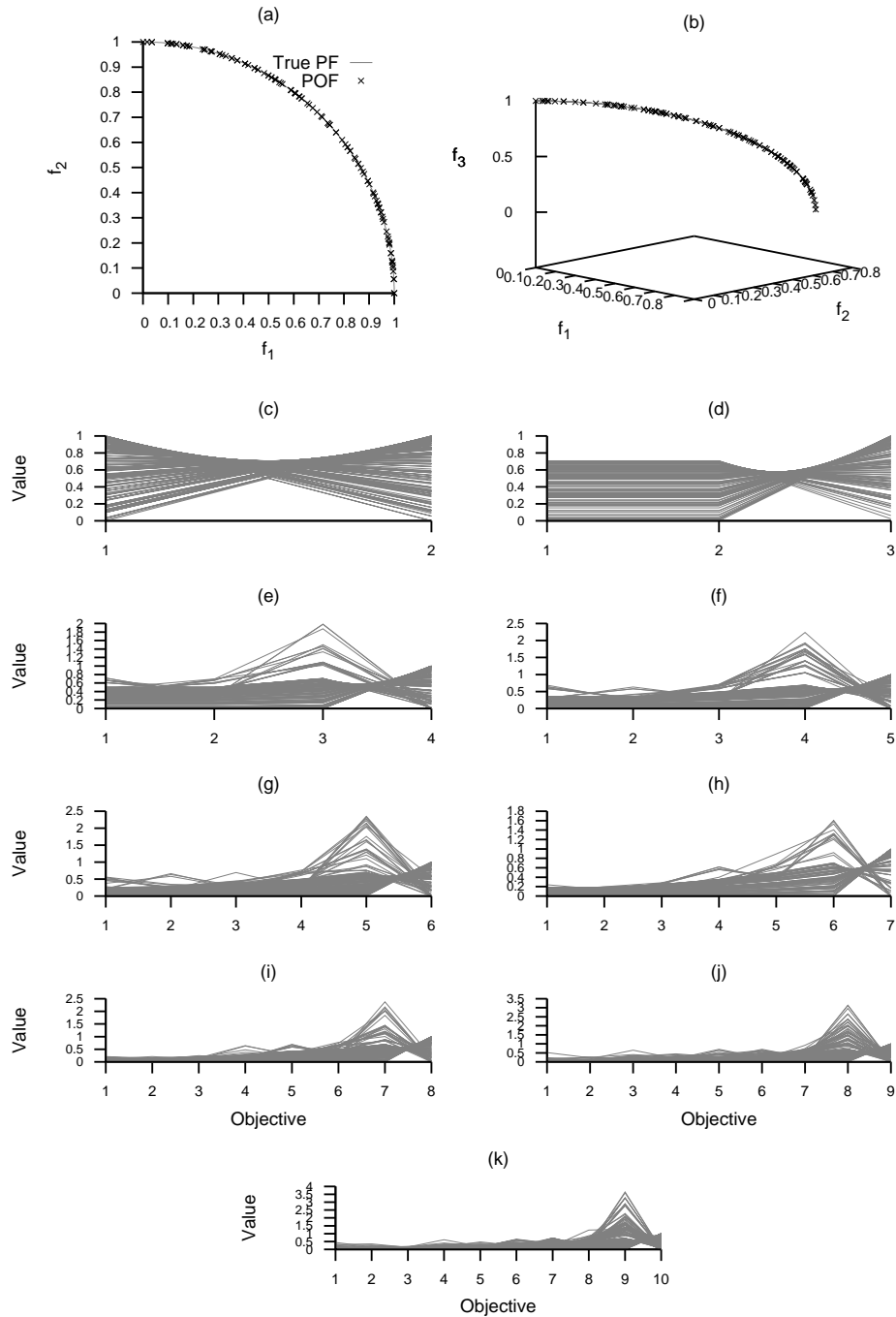


Figure A.92: Plots of the approximations obtained by *R2-MODE* from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



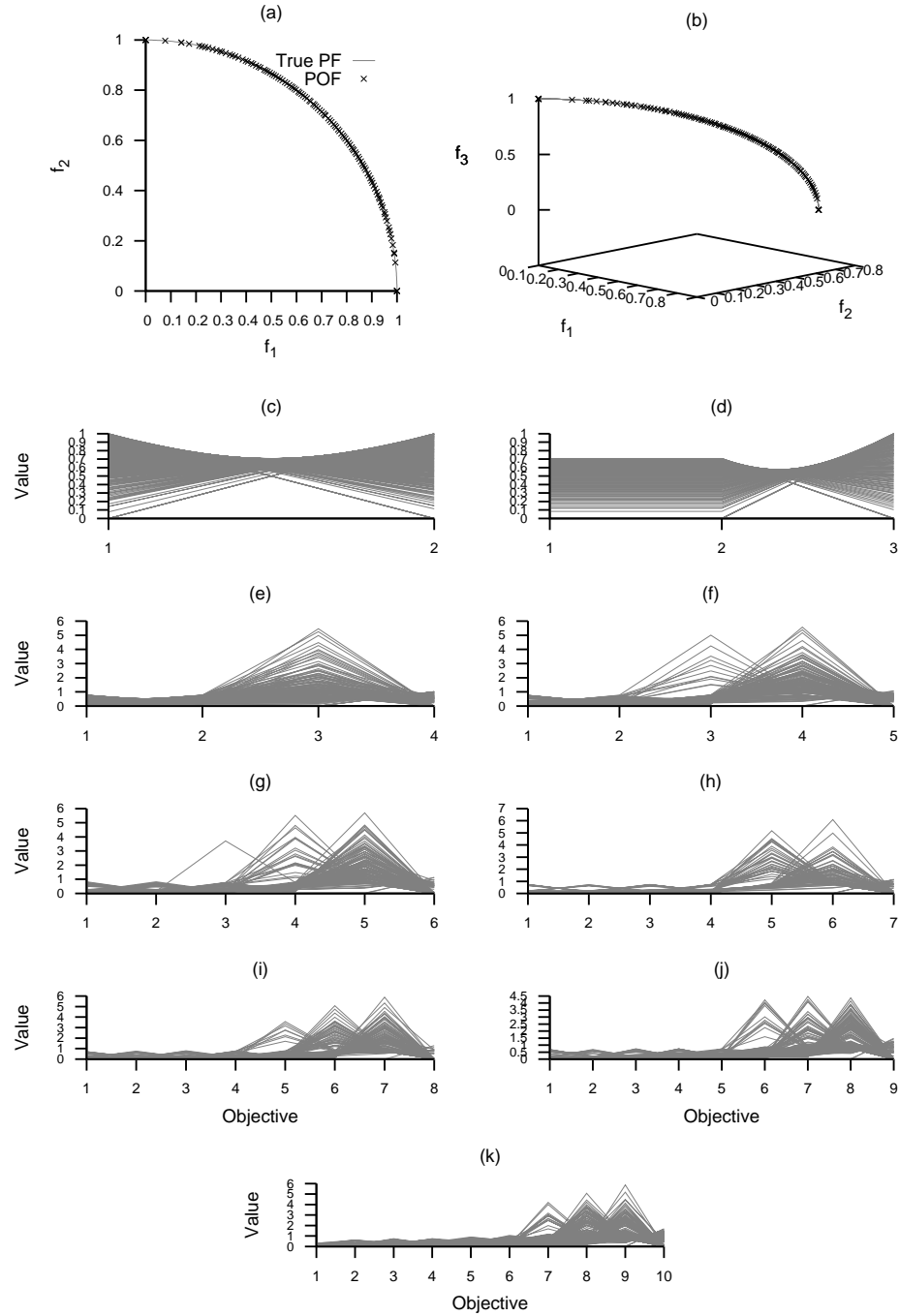


Figure A.93: Plots of the approximations obtained by R2-IBEA from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

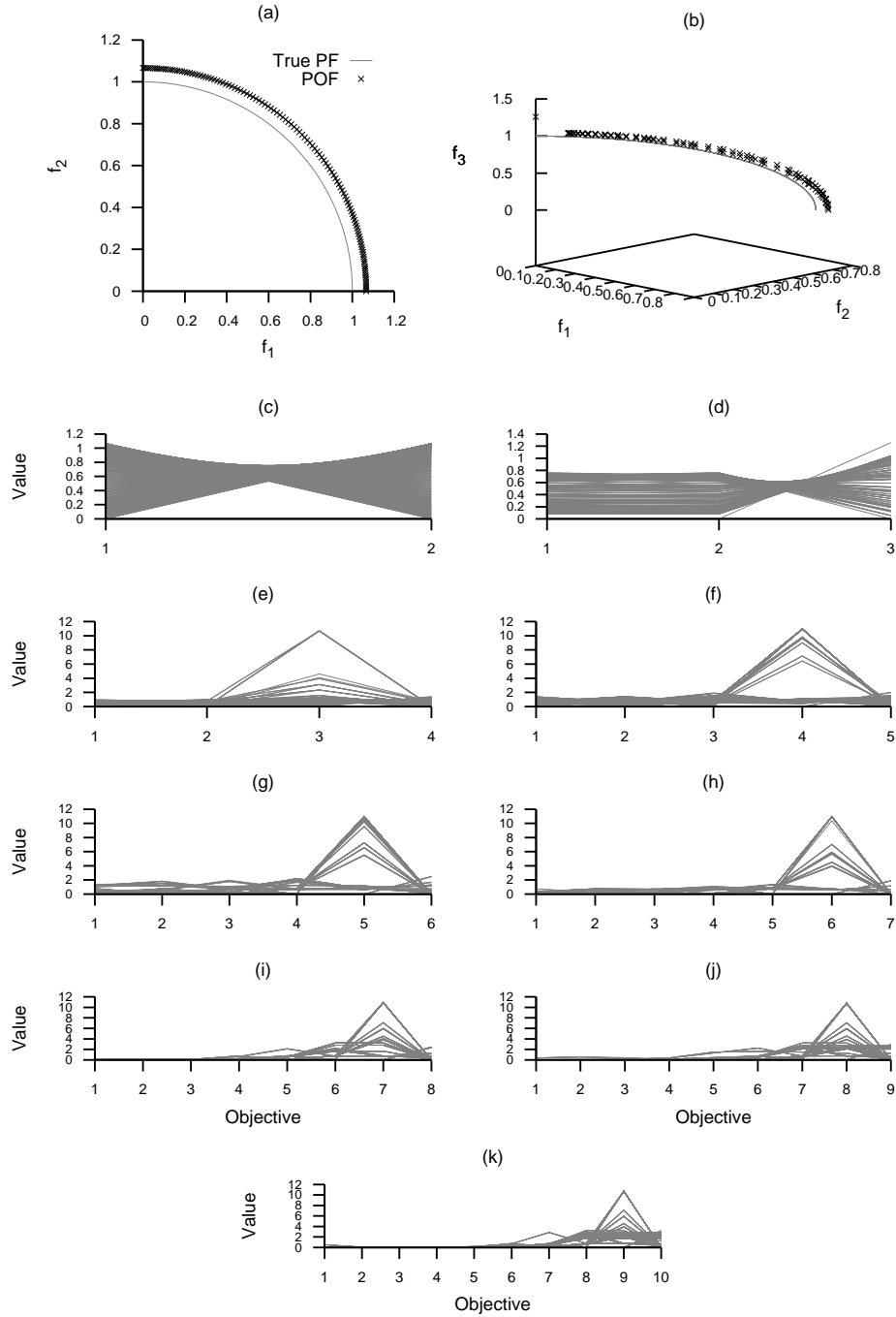


Figure A.94: Plots of the approximations obtained by MOMBI-TCH from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

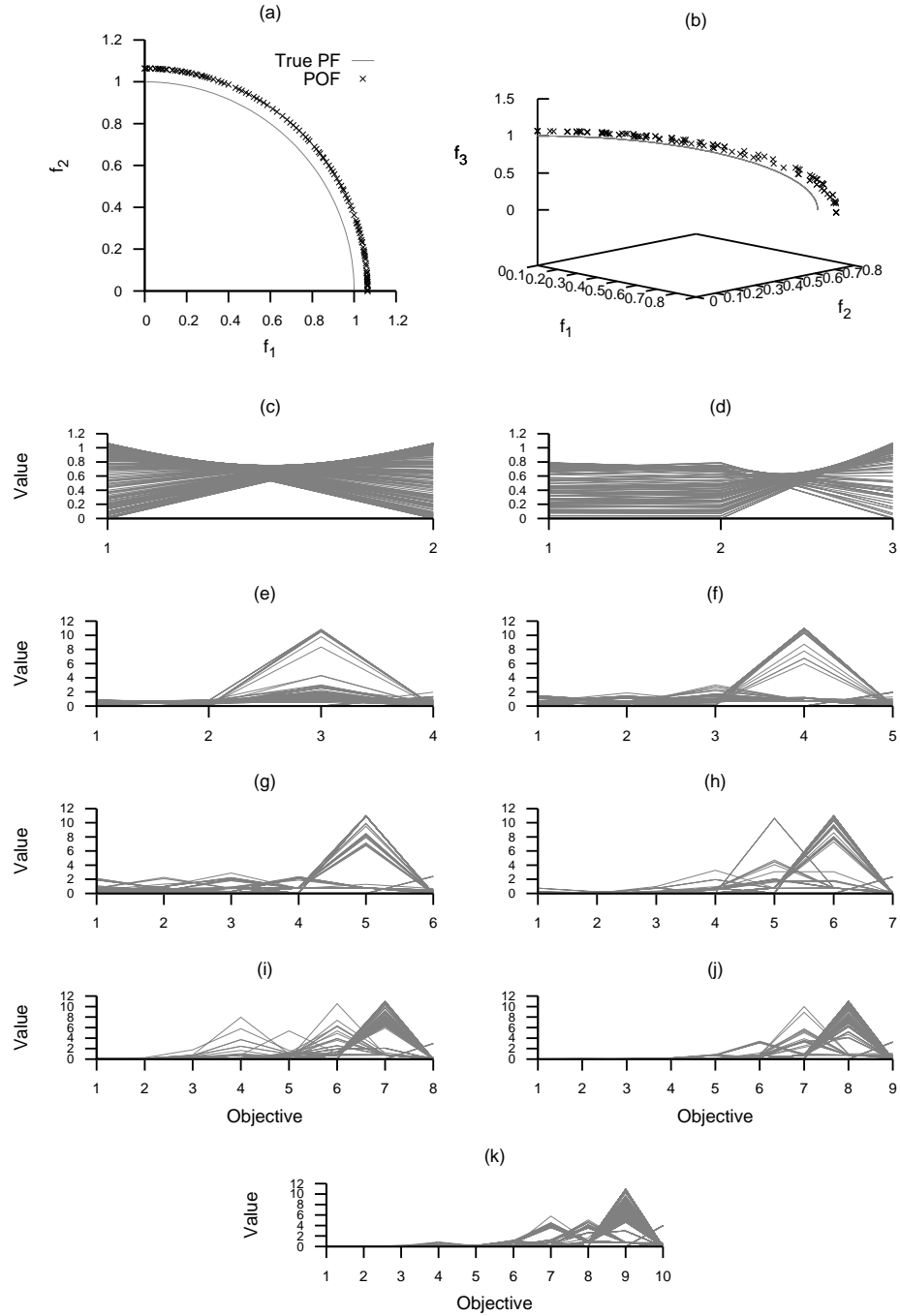


Figure A.95: Plots of the approximations obtained by MOMBI-NTCH from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

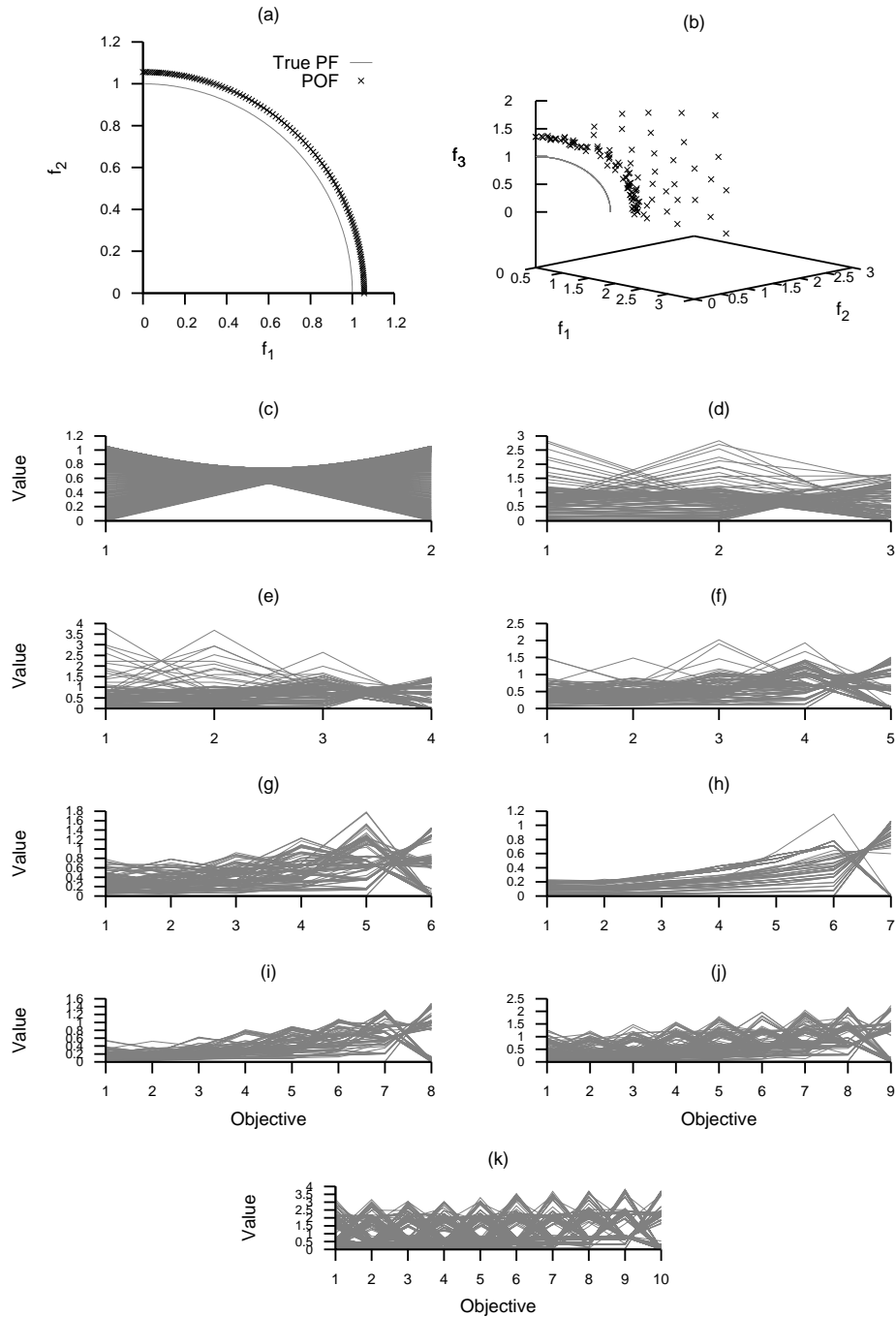


Figure A.96: Plots of the approximations obtained by MOMBI-PBI from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.38: Comparison of hypervolume indicator values for different optimizers on the DTLZ7 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_P$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	1.772480e+01	1.737751e+01	1.737719e+01	1.771931e+01	1.772581e+01	1.761423e+01	1.738405e+01	1.728211e+01	1.770631e+01	1.772491e+01	1.772192e+01	1.772291e+01	1.771474e+01
	avg.	1.772479e+01	1.750186e+01	1.744287e+01	1.766095e+01	1.772574e+01	1.760818e+01	1.728308e+01	1.710696e+01	1.758817e+01	1.772492e+01	1.772198e+01	1.772275e+01	1.771466e+01
	min.	1.772444e+01	1.737699e+01	1.737639e+01	1.737531e+01	1.772478e+01	1.701471e+01	1.588911e+01	1.505963e+01	1.648050e+01	1.772430e+01	1.771780e+01	1.772070e+01	1.772228e+01
	max.	1.772510e+01	1.772406e+01	1.772460e+01	1.772181e+01	1.772584e+01	1.772257e+01	1.770816e+01	1.769911e+01	1.771637e+01	1.772521e+01	1.772410e+01	1.772409e+01	1.772230e+01
	std.	1.418413e-04	1.667318e-01	1.365132e-01	1.294566e-01	1.812639e-04	1.171247e-01	3.712635e-01	5.811945e-01	2.570454e-01	1.634515e-04	1.413655e-03	8.078977e-04	4.464758e-03
3D	med.	1.625727e+01	1.611884e+01	1.574321e+01	1.624352e+01	1.616077e+01	1.568592e+01	1.522768e+01	1.510171e+01	1.619958e+01	1.636467e+01	1.611713e+01	1.630053e+01	1.624761e+01
	avg.	1.621093e+01	1.603450e+01	1.586085e+01	1.623135e+01	1.616356e+01	1.562234e+01	1.512256e+01	1.466061e+01	1.604402e+01	1.636393e+01	1.611807e+01	1.628683e+01	1.624675e+01
	min.	1.585559e+01	1.574287e+01	1.572072e+01	1.574385e+01	1.590868e+01	1.400619e+01	1.255174e+01	9.719387e+00	1.347709e+01	1.633119e+01	1.611037e+01	1.605853e+01	1.622681e+01
	max.	1.630807e+01	1.612751e+01	1.630188e+01	1.625842e+01	1.637037e+01	1.630684e+01	1.616368e+01	1.612044e+01	1.630300e+01	1.637204e+01	1.618206e+01	1.632061e+01	1.625708e+01
	std.	1.016939e-01	1.204839e-01	1.731456e-01	7.079816e-02	1.155159e-01	4.323448e-01	6.824288e-01	1.406949e+00	4.312167e-01	5.900564e-03	7.286148e-03	5.769655e-02	5.312461e-03
4D	med.	1.423275e+01	1.405896e+01	1.408219e+01	1.403204e+01	1.451449e+01	1.220876e+01	1.214944e+01	1.187276e+01	1.440554e+01	1.480566e+01	1.383563e+01	1.449625e+01	1.410244e+01
	avg.	1.417787e+01	1.401840e+01	1.417979e+01	1.400633e+01	1.431565e+01	1.224598e+01	1.209790e+01	1.144165e+01	1.424559e+01	1.479669e+01	1.384247e+01	1.449576e+01	1.409655e+01
	min.	1.334970e+01	1.383230e+01	1.403648e+01	1.343959e+01	1.015739e+01	8.335459e+00	8.878037e+00	4.785225e+00	1.155970e+01	1.435243e+01	1.382334e+01	1.343534e+01	1.399082e+01
	max.	1.442015e+01	1.419204e+01	1.452598e+01	1.414190e+01	1.469439e+01	1.402565e+01	1.406742e+01	1.418329e+01	1.461041e+01	1.484578e+01	1.406102e+01	1.455359e+01	1.413556e+01
	std.	1.959550e-01	1.402125e-01	1.481472e-01	1.110643e-01	7.982446e-01	1.159265e+01	1.042412e+00	2.162405e+00	5.284867e-01	5.235353e-02	3.373519e-02	3.373519e-02	2.672768e-02
5D	med.	1.128330e+01	1.207228e+01	1.239050e+01	6.632180e+00	1.254955e+01	9.530219e+00	9.327853e+00	9.002057e+00	1.204837e+01	1.301004e+01	1.195363e+01	1.246622e+01	7.930343e+00
	avg.	1.127591e+01	1.215301e+01	1.245269e+01	6.582571e+00	1.254674e+01	9.112810e+00	9.271454e+00	8.485852e+00	1.159792e+01	1.300513e+01	1.199014e+01	1.249499e+01	6.397375e+00
	min.	9.938904e+00	1.195956e+01	1.233506e+01	1.942670e+00	1.237415e+01	6.004364e+00	4.295885e+00	1.752453e+00	5.004023e+00	1.283646e+01	1.193280e+01	1.238484e+01	3.957258e-01
	max.	1.218175e+01	1.243445e+01	1.271663e+01	1.214119e+01	1.266316e+01	1.212565e+01	1.136384e+01	1.202672e+01	1.265419e+01	1.312772e+01	1.222778e+01	1.267276e+01	1.147891e+01
	std.	4.292879e-01	1.555332e-01	1.098797e-01	1.421676e+00	5.516576e-02	1.681988e+00	9.929234e-01	2.485358e+00	1.318756e+00	6.379519e-02	6.862621e-02	7.419639e-02	3.662227e+00
6D	med.	7.201025e+00	1.001897e+01	1.071389e+01	3.010195e-01	1.049732e+01	8.707716e+00	6.587967e+00	6.483835e+00	9.483471e+00	1.087549e+01	6.026100e+00	9.679229e+00	1.689105e-01
	avg.	7.198799e+00	9.709019e+00	1.070585e+01	7.559983e-01	1.049120e+01	8.295555e+00	6.524449e+00	6.328746e+00	8.973341e+00	1.081639e+01	7.514874e+00	9.013127e+00	3.009163e-01
	min.	5.398210e+00	5.944880e+00	1.006647e+01	1.680153e-02	1.020933e+01	2.221281e+00	2.322481e+00	4.254753e-01	1.761578e+00	9.721319e+00	5.884775e+00	7.209575e+00	1.101620e-02
	max.	8.373582e+00	1.066714e+01	1.087544e+01	4.611917e+00	1.070468e+01	9.000100e+00	8.727890e+00	1.021762e+01	1.058413e+01	1.129017e+01	1.040089e+01	1.082052e+01	8.685237e+00
	std.	4.721324e-01	1.157947e+00	1.119831e-01	1.106340e+00	1.116773e-01	1.142310e+00	1.126137e+00	2.519902e+00	1.452061e+00	3.095007e-01	1.743132e+00	1.192963e+00	8.880978e-01
7D	med.	3.419147e+00	5.386171e+00	8.744941e+00	3.270779e-02	8.255152e+00	4.921470e+00	3.521101e+00	2.171392e+00	4.634357e+00	3.943842e+00	1.297963e+00	5.000797e+00	5.457545e-03
	avg.	3.492283e+00	5.079091e+00	8.715724e+00	3.418151e-01	7.660584e+00	4.815163e+00	3.224791e+00	2.646788e+00	4.675548e+00	4.464070e+00	1.518496e+00	4.949603e+00	1.160390e-02
	min.	2.342515e+00	4.903421e-01	8.001329e+00	1.734397e-03	9.536540e-03	1.919996e+00	4.283396e-01	4.177557e-02	4.454769e-01	2.956974e+00	4.805222e-01	2.261152e+00	4.624768e-04
	max.	5.141579e+00	8.694043e+00	8.892895e+00	3.355103e+00	8.822642e+00	6.826996e+00	5.238408e+00	7.291774e+00	8.121388e+00	8.932015e+00	5.412484e+00	8.485949e+00	2.014949e-01
	std.	5.137960e-01	2.323108e+00	1.171019e-01	7.387445e-01	2.206066e+00	1.032515e+00	1.227144e+00	2.079665e+00	1.961746e+00	1.411136e+00	1.216228e+00	1.640278e+00	2.287819e-02
8D	med.	5.449157e-01	2.009202e+00	6.861234e+00	5.760527e-03	5.982937e+00	1.000560e+00	2.895890e+00	2.772100e+00	3.902094e+00	1.943097e+00	1.212241e-01	2.681736e+00	7.834286e-04
	avg.	5.408725e-01	2.639309e+00	6.841700e+00	8.569387e-02	5.970082e+00	1.137232e+00	2.766674e+00	2.553062e+00	3.707429e+00	2.968247e+00	3.207260e-01	2.894473e+00	1.156260e-03
	min.	1.546198e-01	1.132322e-01	6.459977e+00	2.953331e-04	5.113759e+00	1.509022e-02	2.548662e-01	3.858361e-02	2.795593e-01	1.874660e+00	1.114375e-01	1.182214e+00	8.665252e-05
	max.	1.099832e+00	6.861088e+00	7.082651e+00	9.749990e-01	6.721674e+00	3.374391e+00	3.901195e+00	6.106034e+00	6.060959e+00	7.419353e+00	1.993311e+00	6.812438e+00	4.352052e-03
	std.	1.971527e-01	1.825037e+00	1.269969e-01	1.908364e-01	3.488228e-01	8.292815e-01	7.788717e-01	1.628632e+00	1.469107e+00	1.576627e+00	3.075117e-01	1.284603e+00	1.040938e-03
9D	med.	4.573806e-03	1.319000e+00	4.906740e+00	4.076489e-03	3.130287e+00	2.875505e-01	2.163168e+00	1.391094e+00	3.177093e+00	1.398052e+00	2.660244e-02	1.683408e+00	1.196427e-04
	avg.	8.350265e-04	1.587307e+00	4.902256e+00	3.022224e-02	3.099550e+00	3.234025e-01	2.149656e+00	1.473555e+00	3.022422e+00	1.627963e+00	6.174390e-02	2.046943e+00	1.156431e-04
	min.	3.866863e-03	2.597906e-02	4.534856e+00	5.539845e-05	2.048496e+00	1.454994e-02	6.716342e-01	2.273063e-02	8.084795e-01	1.351538e+00	2.265383e-02	6.732394e-01	1.761664e-05
	max.	5.837259e-02	4.521861e+00	5.200088e+00	2.425406e-01	3.693270e+00	5.875595e-01	3.085832e+00	3.434968e+00	4.175770e+00	4.566778e+00	5.938563e-01	5.258115e+00	6.197188e-04
	std.	8.977934e-03	1.271320e+00	1.627951e-01	5.959820e-02	3.378674e-01	2.400987e-01	4.680234e-01	1.014844e+00	7.508740e-01	5.543930e+00	1.121133e+00	7.605415e-02	8.070427e-05
10D	med.	5.249787e+00	3.903390e-01	2.893759e+00	2.857973e-03	9.623705e-01	1.216161e+01	1.443151e+00	6.934844e-01	1.585825e+00	1.002269e+00	5.590796e-03	9.478212e-01	1.786547e-05
	avg.	5.455100e+00	7.091379e-01	2.877315e+00	1.188268e-02	1.041105e+00	1.220766e+01	1.409198e+00	8.144383e-01	1.572889e+00	1.063874e+00	8.637017e-03	1.045174e+00	1.488646e-05
	min.	2.614008e+00	5.438431e-03	2.248070e+00	1.134181e-05	4.539230e-02	5.765086e+00	2.811418e-01	1.449959e-02	1.866188e-01	9.574546e-01	4.489874e-03	3.297817e-01	1.965790e-06
	max.	9.633890e+00	2.919172e+00	3.337563e+00	1.453774e-01	2.053183e+00	1.944170e+01	2.187026e+00	3.068195e+00	2.004756e+00	1.823787e+00	5.655958e-02	2.582186e+00	2.649139e-05
	std.	1.606683e+00	6.755171e-01	2.509435e-01	2.371467e-02	4.555894e-01	2.530226e+00	3.256343e-01	5.947873e-01	2.845953e-01	1.594379e-01	9.093379e-03	4.681787e-01	6.120255e-06

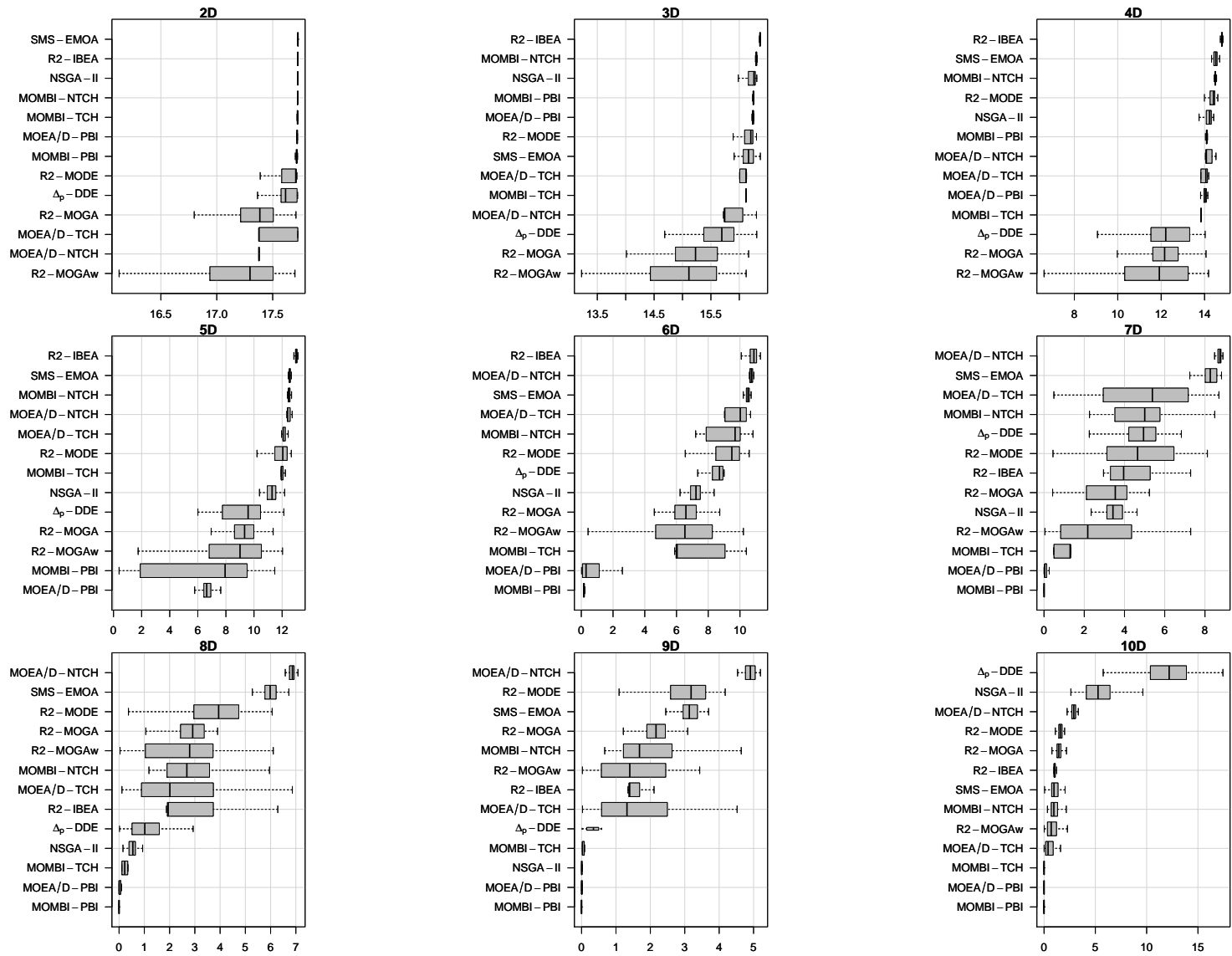


Figure A.97: Box-plot of hypervolume indicator values for different optimizers on the DTLZ7 test problem.

Table A.39: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ7 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D													
NSGA-II	–	1.24e – 34	1.81e – 34	1.27e – 34	> 0.05	1.27e – 34	1.27e – 34	1.27e – 34	1.27e – 34	> 0.05	1.27e – 34	1.27e – 34	1.27e – 34
MOEA/D-TCH	> 0.05	–	2.09e – 17	> 0.05	> 0.05	> 0.05	1.14e – 05	1.27e – 09	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	1.69e – 02	8.57e – 06	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	> 0.05	3.71e – 06	–	> 0.05	1.02e – 04	2.84e – 24	2.43e – 26	1.79e – 17	> 0.05	> 0.05	> 0.05	5.10e – 06
SMS-EMOA	5.33e – 34	1.00e – 34	1.02e – 34	1.02e – 34	–	1.02e – 34	1.02e – 34	1.02e – 34	1.02e – 34	3.31e – 33	1.02e – 34	1.02e – 34	1.02e – 34
$\Delta_p$ -DDE	> 0.05	2.02e – 03	3.88e – 12	> 0.05	> 0.05	–	6.70e – 19	9.74e – 18	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	> 0.05	3.14e – 02	3.40e – 08	> 0.05	> 0.05	> 0.05	2.30e – 18	3.43e – 20	–	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	5.65e – 09	1.25e – 34	1.41e – 34	1.27e – 34	> 0.05	1.27e – 34	1.27e – 34	1.27e – 34	1.27e – 34	–	1.27e – 34	1.27e – 34	1.27e – 34
MOMBI-TCH	> 0.05	9.55e – 07	5.05e – 18	3.02e – 25	> 0.05	1.98e – 24	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	–	> 0.05	1.64e – 28
MOMBI-NTCH	> 0.05	1.99e – 06	1.54e – 18	1.23e – 33	> 0.05	2.51e – 31	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	4.53e – 05	–	9.24e – 33
MOMBI-PBI	> 0.05	1.23e – 04	1.37e – 14	> 0.05	> 0.05	2.00e – 10	1.68e – 34	1.28e – 34	2.62e – 20	> 0.05	> 0.05	> 0.05	–
3D													
NSGA-II	–	6.74e – 20	5.28e – 26	2.77e – 03	1.16e – 03	1.64e – 24	1.07e – 33	1.57e – 33	2.58e – 07	> 0.05	7.89e – 13	> 0.05	2.18e – 02
MOEA/D-TCH	> 0.05	–	1.15e – 12	> 0.05	> 0.05	9.36e – 16	2.36e – 29	2.23e – 29	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	–	> 0.05	> 0.05	6.07e – 06	1.91e – 20	3.43e – 20	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	2.31e – 32	8.10e – 29	–	2.42e – 07	2.59e – 25	3.76e – 34	3.76e – 34	3.12e – 10	> 0.05	4.76e – 32	> 0.05	> 0.05
SMS-EMOA	> 0.05	4.84e – 10	4.14e – 23	> 0.05	–	2.61e – 23	3.57e – 33	7.88e – 33	> 0.05	> 0.05	2.78e – 02	> 0.05	> 0.05
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	6.19e – 09	1.23e – 10	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	> 0.05	3.93e – 11	4.89e – 14	> 0.05	> 0.05	4.62e – 15	9.81e – 26	1.26e – 26	–	> 0.05	1.78e – 07	> 0.05	> 0.05
R2-IBEA	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	2.31e – 32	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	–	1.28e – 34	1.28e – 34	1.28e – 34
MOMBI-TCH	> 0.05	> 0.05	1.32e – 29	> 0.05	> 0.05	7.47e – 22	4.12e – 32	1.76e – 33	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NTCH	1.10e – 24	9.54e – 31	1.30e – 30	2.16e – 27	8.05e – 14	1.13e – 30	1.84e – 34	2.13e – 34	9.47e – 28	> 0.05	2.97e – 27	–	2.97e – 27
MOMBI-PBI	> 0.05	1.28e – 34	1.32e – 29	2.01e – 05	1.24e – 09	4.08e – 26	1.28e – 34	1.28e – 34	6.38e – 13	> 0.05	1.28e – 34	> 0.05	–
4D													
NSGA-II	–	2.13e – 12	> 0.05	1.21e – 15	> 0.05	2.04e – 33	4.78e – 34	1.07e – 31	> 0.05	> 0.05	1.06e – 25	> 0.05	1.35e – 09
MOEA/D-TCH	> 0.05	–	> 0.05	> 0.05	> 0.05	2.44e – 33	1.03e – 33	3.19e – 29	> 0.05	> 0.05	3.22e – 12	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	3.42e – 12	–	5.60e – 15	> 0.05	1.28e – 34	2.33e – 34	2.18e – 31	> 0.05	> 0.05	2.55e – 34	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	–	> 0.05	4.93e – 33	1.48e – 33	8.74e – 28	> 0.05	> 0.05	5.55e – 24	> 0.05	> 0.05
SMS-EMOA	1.95e – 21	3.34e – 25	3.76e – 20	2.80e – 25	–	1.48e – 29	5.74e – 30	2.04e – 30	9.63e – 09	> 0.05	3.13e – 27	> 0.05	3.09e – 23
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	1.76e – 02	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	2.03e – 10	6.28e – 19	8.31e – 10	5.76e – 20	> 0.05	1.23e – 30	3.86e – 31	1.07e – 30	–	> 0.05	8.87e – 24	> 0.05	3.92e – 18
R2-IBEA	2.07e – 34	1.28e – 34	1.58e – 34	1.28e – 34	1.48e – 33	1.28e – 34	1.28e – 34	1.28e – 34	7.92e – 34	–	1.28e – 34	2.37e – 33	1.28e – 34
MOMBI-TCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.07e – 30	2.51e – 33	4.20e – 25	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NTCH	3.55e – 34	1.28e – 34	6.07e – 30	1.28e – 34	> 0.05	1.28e – 34	1.28e – 34	1.28e – 34	1.76e – 13	> 0.05	1.28e – 34	–	1.28e – 34
MOMBI-PBI	> 0.05	1.02e – 07	> 0.05	2.95e – 17	> 0.05	1.36e – 34	1.95e – 34	3.78e – 30	> 0.05	> 0.05	2.01e – 34	> 0.05	–

Table A.40: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ7 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
5D													
NSGA-II	—	> 0.05	> 0.05	$7.40e-31$	> 0.05	$5.69e-24$	$1.19e-32$	$1.18e-20$	> 0.05	> 0.05	> 0.05	> 0.05	$2.74e-31$
MOEA/D-TCH	$1.09e-32$	—	> 0.05	$1.54e-32$	> 0.05	$6.43e-34$	$1.28e-34$	$8.40e-34$	$2.95e-02$	> 0.05	$2.47e-20$	> 0.05	$1.28e-34$
MOEA/D-NTCH	$1.28e-34$	$6.48e-22$	—	$1.28e-34$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.30e-18$	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	$1.28e-34$	$2.26e-34$	$1.60e-08$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$	$6.34e-29$	> 0.05	$1.28e-34$	$6.33e-08$	$1.28e-34$
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	$1.63e-20$	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$7.65e-07$
R2-MOGA	> 0.05	> 0.05	> 0.05	$2.86e-29$	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$6.00e-08$
R2-MOGAw	> 0.05	> 0.05	> 0.05	$3.00e-10$	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	$4.60e-05$
R2-MODE	$2.96e-10$	> 0.05	> 0.05	$1.28e-29$	> 0.05	$1.28e-23$	$3.98e-27$	$5.28e-24$	—	> 0.05	> 0.05	> 0.05	$3.98e-29$
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$3.87e-29$	> 0.05	> 0.05	$5.42e-31$	> 0.05	$2.17e-33$	$1.28e-34$	$7.34e-32$	> 0.05	> 0.05	—	> 0.05	$1.28e-34$
MOMBI-NTCH	$1.28e-34$	$1.13e-31$	$1.97e-06$	$1.28e-34$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	$6.44e-24$	> 0.05	$1.28e-34$	—	$1.28e-34$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	$1.81e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—
6D													
NSGA-II	—	> 0.05	> 0.05	$1.28e-34$	> 0.05	> 0.05	$1.55e-07$	$9.65e-03$	> 0.05	> 0.05	> 0.05	> 0.05	$2.51e-33$
MOEA/D-TCH	$2.84e-26$	—	> 0.05	$1.28e-34$	> 0.05	$1.97e-26$	$2.53e-28$	$3.06e-24$	$8.01e-09$	> 0.05	$1.01e-19$	$5.03e-07$	$1.58e-34$
MOEA/D-NTCH	$1.28e-34$	$3.68e-30$	—	$1.28e-34$	$1.17e-26$	$1.28e-34$	$1.28e-34$	$1.32e-34$	$2.26e-34$	> 0.05	$1.53e-34$	$1.30e-31$	$1.28e-34$
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	$1.28e-34$	$1.63e-18$	> 0.05	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.32e-34$	$5.23e-33$	> 0.05	$8.92e-34$	$1.18e-24$	$1.28e-34$
$\Delta_p$ -DDE	$9.23e-22$	> 0.05	> 0.05	$2.33e-34$	> 0.05	—	$2.10e-24$	$1.48e-09$	> 0.05	> 0.05	> 0.05	> 0.05	$5.38e-34$
R2-MOGA	> 0.05	> 0.05	> 0.05	$3.24e-34$	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.44e-33$
R2-MOGAw	> 0.05	> 0.05	> 0.05	$7.14e-32$	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	$2.04e-33$
R2-MODE	$8.55e-23$	> 0.05	> 0.05	$2.07e-34$	> 0.05	$1.49e-08$	$2.73e-25$	$3.54e-16$	—	> 0.05	$5.21e-09$	> 0.05	$3.24e-34$
R2-IBEA	$1.28e-34$	$1.08e-27$	$6.01e-06$	$1.28e-34$	$1.59e-18$	$1.28e-34$	$1.28e-34$	$2.26e-34$	$6.61e-33$	—	$7.03e-34$	$3.11e-30$	$1.28e-34$
MOMBI-TCH	> 0.05	> 0.05	> 0.05	$1.28e-34$	> 0.05	> 0.05	$6.37e-03$	$2.20e-03$	> 0.05	> 0.05	—	> 0.05	$6.63e-34$
MOMBI-NTCH	$1.41e-27$	> 0.05	> 0.05	$1.28e-34$	> 0.05	$2.89e-03$	$4.59e-28$	$4.49e-16$	> 0.05	> 0.05	$1.77e-09$	—	$5.22e-34$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—
7D													
NSGA-II	—	> 0.05	> 0.05	$6.23e-33$	> 0.05	> 0.05	> 0.05	$3.95e-05$	> 0.05	> 0.05	$7.58e-26$	> 0.05	$1.28e-34$
MOEA/D-TCH	$3.35e-05$	—	> 0.05	$1.95e-31$	> 0.05	> 0.05	$4.52e-08$	$9.24e-13$	> 0.05	> 0.05	$7.65e-22$	> 0.05	$1.28e-34$
MOEA/D-NTCH	$1.28e-34$	$5.22e-34$	—	$1.28e-34$	$4.82e-21$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.32e-34$	$4.77e-32$	$1.28e-34$	$1.53e-34$	$1.28e-34$
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$4.45e-23$
SMS-EMOA	$4.54e-25$	$7.71e-20$	> 0.05	$8.06e-28$	—	$4.77e-25$	$2.59e-25$	$1.17e-25$	$1.24e-24$	$2.08e-22$	$7.20e-26$	$1.17e-22$	$1.50e-32$
$\Delta_p$ -DDE	$1.01e-19$	> 0.05	> 0.05	$3.44e-34$	> 0.05	—	$2.16e-17$	$2.23e-13$	> 0.05	$1.02e-03$	$1.84e-29$	> 0.05	$1.28e-34$
R2-MOGA	> 0.05	> 0.05	> 0.05	$3.25e-31$	> 0.05	> 0.05	—	$2.68e-03$	> 0.05	> 0.05	$1.20e-17$	> 0.05	$1.28e-34$
R2-MOGAw	> 0.05	> 0.05	> 0.05	$6.30e-25$	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	$3.52e-04$	> 0.05	$1.63e-34$
R2-MODE	$9.87e-07$	> 0.05	> 0.05	$2.06e-32$	> 0.05	> 0.05	$7.72e-08$	$1.17e-10$	—	> 0.05	$9.88e-23$	> 0.05	$1.28e-34$
R2-IBEA	$6.60e-07$	> 0.05	> 0.05	$7.03e-34$	> 0.05	> 0.05	$3.82e-06$	$3.84e-10$	> 0.05	—	$9.47e-30$	> 0.05	$1.28e-34$
MOMBI-TCH	> 0.05	> 0.05	> 0.05	$4.66e-22$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	$1.28e-34$
MOMBI-NTCH	$2.41e-11$	> 0.05	> 0.05	$9.75e-34$	> 0.05	> 0.05	$2.04e-11$	$1.76e-13$	> 0.05	$1.09e-02$	$9.28e-29$	—	$1.28e-34$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—



Table A.41: Wilcoxon rank sum test applied to hypervolume indicator values on the DTLZ7 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
8D													
NSGA-II	—	> 0.05	> 0.05	1.30e-27	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	7.07e-14	> 0.05	1.28e-34
MOEA/D-TCH	6.81e-21	—	> 0.05	2.00e-32	> 0.05	1.55e-08	> 0.05	> 0.05	> 0.05	> 0.05	1.90e-24	> 0.05	1.28e-34
MOEA/D-NTCH	1.28e-34	2.37e-33	—	1.28e-34	6.24e-34	7.82e-29	1.28e-34	1.28e-34	1.28e-34	8.96e-30	1.28e-34	4.00e-34	1.28e-34
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	7.99e-26
SMS-EMOA	1.28e-34	8.24e-32	> 0.05	1.28e-34	—	7.82e-29	1.28e-34	2.91e-33	3.86e-31	3.38e-24	1.28e-34	9.82e-31	1.28e-34
$\Delta_p$ -DDE	5.91e-07	> 0.05	> 0.05	6.88e-24	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	2.34e-14	> 0.05	7.82e-29
R2-MOGA	5.88e-33	> 0.05	> 0.05	2.13e-34	> 0.05	1.16e-19	—	> 0.05	> 0.05	> 0.05	1.98e-33	> 0.05	1.28e-34
R2-MOGAw	4.45e-17	> 0.05	> 0.05	7.34e-32	> 0.05	1.82e-08	> 0.05	—	> 0.05	> 0.05	6.93e-24	> 0.05	1.28e-34
R2-MODE	1.00e-29	1.18e-05	> 0.05	4.37e-34	> 0.05	3.33e-20	5.73e-10	2.62e-07	—	2.48e-05	7.65e-33	2.86e-06	1.28e-34
R2-IBEA	1.28e-34	> 0.05	> 0.05	1.28e-34	> 0.05	1.46e-19	> 0.05	> 0.05	> 0.05	> 0.05	7.69e-34	> 0.05	1.28e-34
MOMBI-TCH	> 0.05	> 0.05	> 0.05	2.45e-22	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	1.28e-34
MOMBI-NTCH	1.28e-34	> 0.05	> 0.05	1.28e-34	> 0.05	9.28e-19	> 0.05	> 0.05	> 0.05	> 0.05	3.34e-34	—	1.28e-34
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—
9D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.49e-34
MOEA/D-TCH	1.40e-34	—	> 0.05	4.02e-33	> 0.05	9.61e-03	> 0.05	> 0.05	> 0.05	> 0.05	4.44e-31	> 0.05	1.28e-34
MOEA/D-NTCH	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	3.73e-04	1.28e-34	1.28e-34	1.28e-34	1.36e-34	1.28e-34	6.43e-31	1.28e-34
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	2.18e-31
SMS-EMOA	1.28e-34	1.05e-15	> 0.05	1.28e-34	—	3.73e-04	7.88e-29	1.78e-26	> 0.05	5.59e-30	1.28e-34	2.71e-17	1.28e-34
$\Delta_p$ -DDE	1.35e-03	> 0.05	> 0.05	4.29e-03	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	3.73e-04
R2-MOGA	1.28e-34	7.02e-06	> 0.05	1.28e-34	> 0.05	3.73e-04	—	7.02e-07	> 0.05	5.27e-16	1.28e-34	1.69e-03	1.28e-34
R2-MOGAw	1.63e-34	> 0.05	> 0.05	7.88e-33	> 0.05	8.38e-03	> 0.05	—	> 0.05	> 0.05	1.63e-30	> 0.05	1.28e-34
R2-MODE	1.28e-34	6.29e-15	> 0.05	1.28e-34	> 0.05	3.73e-04	1.45e-17	2.46e-21	—	3.09e-23	1.28e-34	8.17e-13	1.28e-34
R2-IBEA	1.28e-34	1.09e-05	> 0.05	1.28e-34	> 0.05	3.73e-04	> 0.05	> 0.05	> 0.05	—	1.28e-34	> 0.05	1.28e-34
MOMBI-TCH	7.14e-32	> 0.05	> 0.05	1.58e-17	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	1.28e-34
MOMBI-NTCH	1.28e-34	2.24e-04	> 0.05	1.28e-34	> 0.05	3.73e-04	> 0.05	4.92e-04	> 0.05	1.16e-02	1.28e-34	—	1.28e-34
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—
10D													
NSGA-II	—	1.32e-34	6.81e-33	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.40e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34
MOEA/D-TCH	> 0.05	—	> 0.05	1.00e-33	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.16e-33	> 0.05	1.28e-34
MOEA/D-NTCH	> 0.05	7.92e-34	—	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.87e-33	1.28e-34	1.28e-34	1.28e-34	1.84e-34	1.28e-34
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	9.19e-34
SMS-EMOA	> 0.05	7.52e-08	> 0.05	1.44e-34	—	> 0.05	> 0.05	3.19e-04	> 0.05	> 0.05	1.53e-34	> 0.05	1.28e-34
$\Delta_p$ -DDE	5.39e-33	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34
R2-MOGA	> 0.05	7.91e-15	> 0.05	1.28e-34	7.76e-10	> 0.05	—	1.93e-16	> 0.05	9.51e-17	1.28e-34	1.77e-10	1.28e-34
R2-MOGAw	> 0.05	2.72e-02	> 0.05	8.66e-34	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	2.71e-34	> 0.05	1.28e-34
R2-MODE	> 0.05	8.33e-19	> 0.05	1.28e-34	3.20e-16	> 0.05	3.22e-05	9.23e-22	—	1.12e-25	1.28e-34	2.01e-16	1.28e-34
R2-IBEA	> 0.05	2.86e-13	> 0.05	1.28e-34	3.98e-02	> 0.05	> 0.05	9.26e-06	> 0.05	—	1.28e-34	3.09e-02	1.28e-34
MOMBI-TCH	> 0.05	> 0.05	> 0.05	1.48e-06	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	1.28e-34
MOMBI-NTCH	> 0.05	2.35e-08	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	2.69e-04	> 0.05	> 0.05	1.28e-34	—	1.28e-34
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—

Table A.42: Comparison of R2 indicator values for different optimizers on the DTLZ7 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	1.179937e+00	1.786868e+00	1.786896e+00	1.182085e+00	1.179842e+00	1.182613e+00	1.180555e+00	1.180789e+00	1.180311e+00	1.180128e+00	1.179869e+00	1.179956e+00	1.180611e+00
	avg.	1.179947e+00	1.568591e+00	1.671611e+00	1.285043e+00	1.179842e+00	1.183137e+00	1.180618e+00	1.180951e+00	1.180376e+00	1.180134e+00	1.179870e+00	1.179953e+00	1.180637e+00
	min.	1.179833e+00	1.179923e+00	1.179963e+00	1.180440e+00	1.179823e+00	1.180040e+00	1.180252e+00	1.180254e+00	1.180073e+00	1.179932e+00	1.179843e+00	1.179885e+00	1.180190e+00
	max.	1.180110e+00	1.786918e+00	1.786986e+00	1.788203e+00	1.179868e+00	1.203232e+00	1.181985e+00	1.183592e+00	1.181253e+00	1.180444e+00	1.179918e+00	1.180060e+00	1.181395e+00
	std.	4.432445e-05	2.925224e-01	2.392607e-01	2.284965e-01	7.838451e-06	3.077904e-03	2.878758e-04	5.518873e-04	2.172620e-04	8.975852e-05	1.900410e-05	3.840361e-05	2.481531e-04
3D	med.	9.105150e-01	9.023231e-01	1.719430e+00	9.022473e-01	9.018081e-01	9.227691e-01	9.033373e-01	9.036735e-01	9.031125e-01	9.070615e-01	9.021330e-01	9.026220e-01	9.024360e-01
	avg.	9.145614e-01	1.149327e+00	1.558343e+00	9.186219e-01	9.662611e-01	1.064761e+00	9.034058e-01	9.079791e-01	9.031539e-01	9.071196e-01	9.021313e-01	9.267638e-01	9.024809e-01
	min.	9.068760e-01	9.021398e-01	9.025000e-01	9.020978e-01	9.015657e-01	9.029940e-01	9.026931e-01	9.027760e-01	9.024963e-01	9.034406e-01	9.020466e-01	9.022789e-01	9.021231e-01
	max.	1.308981e+00	1.719455e+00	1.719572e+00	1.719416e+00	1.305008e+00	1.730082e+00	9.046733e-01	1.305968e+00	9.050991e-01	9.120033e-01	9.021773e-01	1.305237e+00	9.029526e-01
	std.	3.989133e-02	2.826846e-01	2.334404e-01	1.149730e-01	1.485094e-01	2.519502e-01	4.093314e-04	4.021316e-02	4.086715e-04	1.469453e-03	2.283653e-05	9.607498e-02	1.915384e-04
4D	med.	7.978610e-01	1.061216e+00	1.685097e+00	7.773646e-01	7.631761e-01	7.852692e-01	7.615652e-01	7.621087e-01	7.627692e-01	7.723461e-01	7.607875e-01	7.614659e-01	7.786316e-01
	avg.	8.127212e-01	1.070899e+00	1.544450e+00	8.230971e-01	8.994921e-01	9.395490e-01	7.617171e-01	7.653827e-01	7.627515e-01	7.787559e-01	7.667841e-01	7.946496e-01	7.783849e-01
	min.	7.808361e-01	7.607606e-01	7.615603e-01	7.766405e-01	7.612463e-01	7.736068e-01	7.603358e-01	7.606367e-01	7.608730e-01	7.679971e-01	7.605869e-01	7.608634e-01	7.661221e-01
	max.	1.100646e+00	1.685065e+00	1.685314e+00	1.692976e+00	1.687137e+00	1.703338e+00	7.637346e-01	1.061851e+00	7.640322e-01	1.389115e+00	1.061154e+00	1.371117e+00	7.862126e-01
	std.	6.429554e-02	2.718892e-01	2.048346e-01	2.005421e-01	2.353453e-01	3.190101e-01	6.190947e-04	2.996882e-02	5.735707e-04	6.169102e-02	4.225517e-02	1.130642e-01	3.370626e-03
5D	med.	8.255344e-01	9.789583e-01	1.782290e+00	7.444157e-01	9.884858e-01	7.489179e-01	7.187843e-01	7.193478e-01	7.250309e-01	7.313486e-01	7.181131e-01	7.196319e-01	7.495162e-01
	avg.	9.003653e-01	1.102126e+00	1.658393e+00	8.138922e-01	1.085949e+00	1.030755e+00	7.188968e-01	7.221959e-01	7.251069e-01	7.423237e-01	7.962599e-01	7.794498e-01	7.505898e-01
	min.	7.808765e-01	7.180002e-01	7.979187e-01	7.324778e-01	7.215001e-01	7.291743e-01	7.167621e-01	7.169441e-01	7.205696e-01	7.261553e-01	7.179575e-01	7.185500e-01	7.356886e-01
	max.	1.554211e+00	1.782245e+00	1.782715e+00	1.795735e+00	1.784654e+00	1.802385e+00	7.212957e-01	9.796775e-01	7.299203e-01	9.984589e-01	1.243930e+00	1.244228e+00	7.630907e-01
	std.	1.552572e-01	2.855511e-01	1.849634e-01	2.335864e-01	2.824762e-01	3.822418e-01	6.752607e-04	2.605567e-02	1.992941e-03	5.217337e-02	1.363184e-01	1.381012e-01	6.258993e-03
6D	med.	8.230155e-01	1.027689e+00	1.446895e+00	8.633805e-01	1.047773e+00	2.056295e+00	6.208116e-01	6.209390e-01	6.324764e-01	6.346088e-01	6.204506e-01	6.212806e-01	6.662648e-01
	avg.	8.302108e-01	1.058205e+00	1.472650e+00	9.139287e-01	1.141632e+00	2.034764e+00	6.248528e-01	6.271552e-01	6.326523e-01	6.551737e-01	7.375487e-01	6.958884e-01	6.652439e-01
	min.	7.101958e-01	6.212198e-01	8.221716e-01	6.665399e-01	6.263920e-01	6.425527e-01	6.195591e-01	6.192426e-01	6.245112e-01	6.276309e-01	6.198617e-01	6.207663e-01	6.398337e-01
	max.	1.073839e+00	1.446741e+00	1.658143e+00	1.477842e+00	1.671313e+00	2.396530e+00	8.215365e-01	8.216755e-01	6.412907e-01	8.436947e-01	1.236101e+00	1.446828e+00	6.752264e-01
	std.	4.807858e-02	2.179933e-01	2.238304e-01	1.721898e-01	2.985123e-01	2.985087e-01	2.821902e-02	3.435685e-02	2.717094e-03	6.259473e-02	1.589077e-01	1.281717e-01	6.478164e-03
7D	med.	8.453603e-01	1.094508e+00	1.272571e+00	9.698184e-01	9.242213e-01	2.193919e+00	5.763581e-01	5.770239e-01	5.902604e-01	6.082768e-01	7.432152e-01	7.433255e-01	6.227190e-01
	avg.	8.589496e-01	1.095907e+00	1.329868e+00	1.033026e+00	1.017853e+00	2.197537e+00	6.416127e-01	6.255764e-01	5.905378e-01	6.939871e-01	7.346715e-01	7.764020e-01	6.271506e-01
	min.	7.543771e-01	5.747679e-01	7.429452e-01	6.283648e-01	5.765450e-01	5.951431e-01	5.735785e-01	5.736197e-01	5.837564e-01	5.890769e-01	5.737913e-01	5.739566e-01	6.074921e-01
	max.	1.074269e+00	1.630503e+00	1.630459e+00	1.650469e+00	1.642147e+00	2.629504e+00	1.094044e+00	9.191784e-01	5.993154e-01	1.305114e+00	1.094444e+00	1.094355e+00	7.903155e-01
	std.	6.771311e-02	2.286286e-01	2.132811e-01	2.053330e-01	2.746215e-01	2.031691e-01	1.050269e-01	8.763302e-02	3.690719e-03	1.288246e-01	1.445205e-01	1.673548e-01	3.262953e-02
8D	med.	1.136645e+00	9.933930e-01	1.301285e+00	8.924892e-01	1.170976e+00	2.499709e+00	5.384329e-01	5.384759e-01	5.602524e-01	5.530382e-01	5.414790e-01	6.867166e-01	5.829302e-01
	avg.	1.137393e+00	1.024127e+00	1.289584e+00	9.748803e-01	1.167375e+00	2.507264e+00	5.561587e-01	5.813400e-01	5.613424e-01	6.338024e-01	6.383830e-01	7.006625e-01	5.847107e-01
	min.	8.934709e-01	5.411287e-01	8.380886e-01	5.905969e-01	5.577829e-01	1.919300e+00	5.368120e-01	5.359857e-01	5.491698e-01	5.441649e-01	5.389975e-01	5.377593e-01	5.712798e-01
	max.	1.386381e+00	1.614015e+00	1.613281e+00	1.487742e+00	1.768100e+00	3.100618e+00	8.378923e-01	8.380311e-01	5.761203e-01	1.169729e+00	9.932212e-01	1.302256e+00	7.297351e-01
	std.	1.044740e-01	2.245850e-01	1.951928e-01	2.074460e-01	2.921388e-01	1.970774e-01	5.301560e-02	8.544818e-02	5.436965e-03	1.234029e-01	1.128227e-01	1.674717e-01	2.069480e-02
9D	med.	1.461354e+00	1.076793e+00	1.351867e+00	9.768687e-01	1.398352e+00	2.803059e+00	5.228673e-01	5.221589e-01	5.541475e-01	5.459909e-01	5.323663e-01	6.609044e-01	5.625289e-01
	avg.	1.477043e+00	1.064177e+00	1.322797e+00	1.011421e+00	1.401646e+00	2.802625e+00	5.348349e-01	5.355970e-01	5.551193e-01	5.885956e-01	5.932487e-01	7.502436e-01	5.623514e-01
	min.	1.090667e+00	5.358059e-01	9.354764e-01	5.823057e-01	7.358572e-01	2.218523e+00	5.209155e-01	5.206443e-01	5.402516e-01	5.324627e-01	5.262768e-01	5.243111e-01	5.533662e-01
	max.	1.797065e+00	1.498110e+00	1.673611e+00	1.382851e+00	2.070098e+00	3.452116e+00	6.571163e-01	6.568340e-01	5.739757e-01	8.287145e-01	9.348155e-01	1.355110e+00	5.660584e-01
	std.	1.385118e-01	2.133593e-01	1.476799e-01	1.977839e-01	3.372734e-01	2.398807e-01	3.840357e-02	4.017127e-02	6.722440e-03	7.251062e-02	8.976539e-02	2.074597e-01	2.018425e-03
10D	med.	1.787790e+00	1.015518e+00	1.294517e+00	1.039640e+00	1.850105e+00	3.024109e+00	5.039954e-01	5.033465e-01	5.484811e-01	5.403990e-01	5.264453e-01	6.392316e-01	5.411776e-01
	avg.	1.821733e+00	1.049885e+00	1.284002e+00	1.029040e+00	1.812476e+00	3.026925e+00	5.167326e-01	5.167326e-01	5.491172e-01	5.475679e-01	5.475575e-01	6.872960e-01	5.411856e-01
	min.	1.511053e+00	5.331461e-01	5.125133e-01	5.790614e-01	1.010157e+00	2.474830e+00	5.018259e-01	5.013242e-01	5.284950e-01	5.168519e-01	5.166154e-01	5.099909e-01	5.370029e-01
	max.	2.216734e+00	1.5414380e+00	1.678305e+00	1.408391e+00	2.298259e+00	3.815702e+00	6.270867e-01	8.732136e-01	5.768302e-01	7.984906e-01	7.641360e-01	1.010456e+00	5.473246e-01
	std.	1.611152e-01	1.844531e-01	2.079230e-01	1.802812e-01	2.803527e-01	2.381074e-01	2.669451e-02	4.880970e-02	8.354531e-03	6.967820e-02	5.458146e-02	1.380444e-01	1.842465e-03

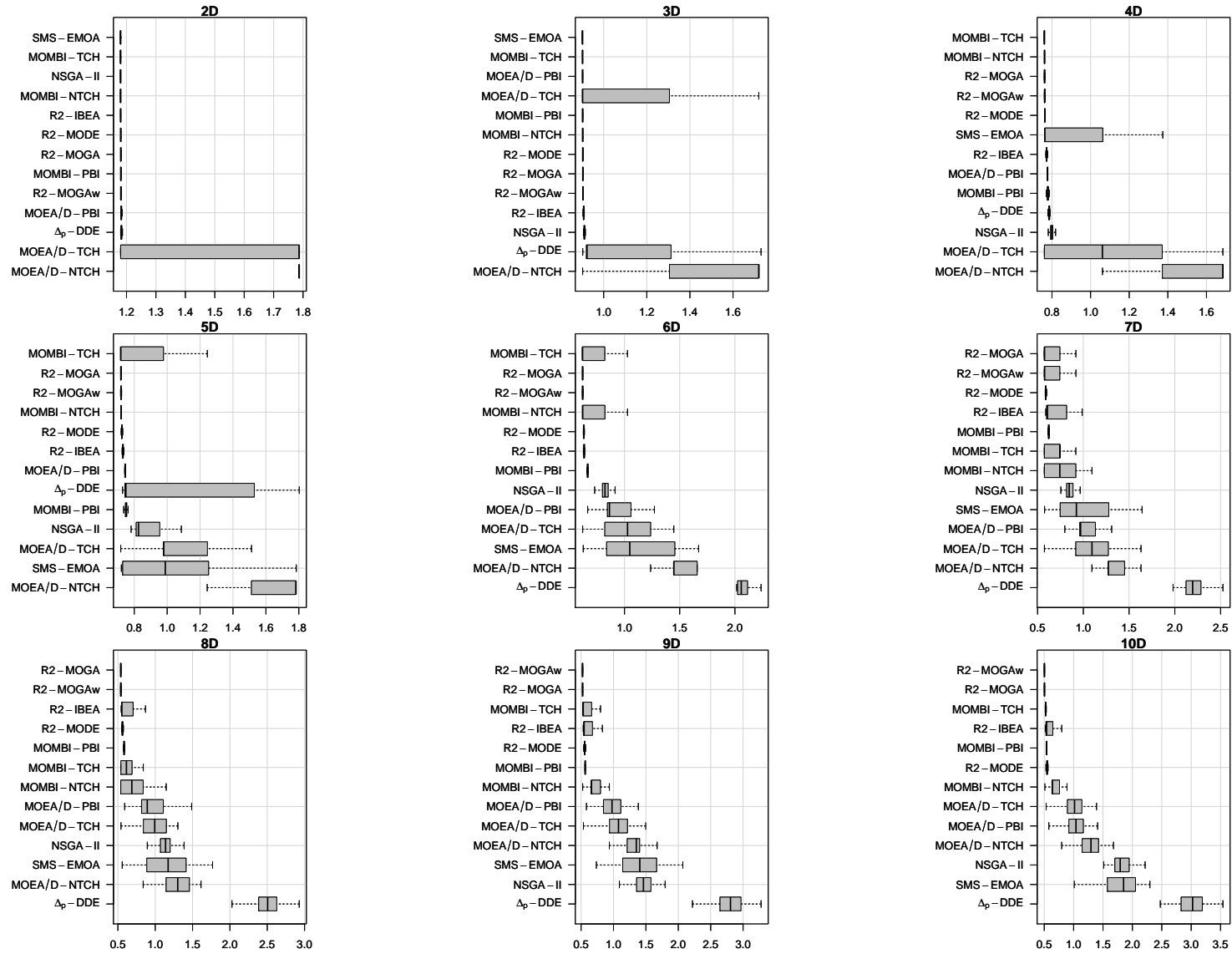


Figure A.98: Box-plot of R2 indicator values for different optimizers on the DTLZ7 test problem.

Table A.43: Comparison of runtime (in milliseconds) for different optimizers on the DTLZ7 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	2.811190e+02	6.754000e+02	1.151354e+03	1.114118e+03	2.624621e+04	1.635419e+03	4.183782e+03	2.984271e+03	3.853687e+03	5.488910e+05	2.009183e+03	2.217712e+03	2.795632e+03
	avg.	2.953192e+02	6.882262e+02	1.162477e+03	1.105119e+03	2.617149e+04	1.631249e+03	4.205810e+03	2.960047e+03	3.924522e+03	5.519567e+05	2.020341e+03	2.152531e+03	2.783598e+03
	min.	2.497580e+02	4.965760e+02	8.811280e+02	8.378820e+02	1.999633e+04	1.292518e+03	3.243191e+03	1.860290e+03	2.826134e+03	5.181290e+05	1.606053e+03	1.699483e+03	2.433940e+03
	max.	4.304780e+02	1.095096e+03	1.982606e+03	1.308780e+03	2.926806e+04	2.024764e+03	4.967604e+03	4.251266e+03	4.811150e+03	5.943020e+05	2.347845e+03	2.722122e+03	3.066668e+03
	std.	4.202192e+01	1.170923e+02	1.188055e+02	8.959736e+01	2.385478e+03	1.453771e+02	3.798066e+02	4.625152e+02	4.445108e+02	1.804121e+04	1.611094e+02	2.028072e+02	1.319910e+02
3D	med.	4.044230e+02	7.201740e+02	1.587347e+03	1.192109e+03	6.055298e+05	1.758176e+03	4.228433e+03	3.275147e+03	4.322159e+03	6.617440e+05	2.062605e+03	2.197790e+03	3.112612e+03
	avg.	4.112642e+02	7.318692e+02	1.566311e+03	1.180490e+03	5.901533e+05	1.740856e+03	4.243093e+03	3.250277e+03	4.298414e+03	6.360155e+05	2.054746e+03	2.214450e+03	3.097550e+03
	min.	3.521080e+02	5.512620e+02	1.105262e+03	9.172740e+02	4.308608e+05	1.215653e+03	3.062422e+03	2.362928e+03	3.320763e+03	5.272290e+05	1.685527e+03	1.768236e+03	2.706939e+03
	max.	5.839770e+02	1.122677e+03	1.896444e+03	1.611144e+03	6.434164e+05	2.098894e+03	4.985942e+03	4.429459e+03	5.258951e+03	6.874770e+05	2.368428e+03	2.598508e+03	3.447964e+03
	std.	4.834570e+01	9.178064e+01	1.293021e+02	1.102200e+02	4.808560e+04	1.857161e+02	4.143588e+02	4.290554e+02	5.000563e+02	5.191579e+04	1.622034e+02	1.338525e+02	1.404520e+02
4D	med.	5.268380e+02	7.584550e+02	1.648052e+03	1.200581e+03	3.341098e+06	2.172667e+03	4.742172e+03	3.578924e+03	5.395666e+03	7.193590e+05	1.595654e+03	2.173111e+03	3.335474e+03
	avg.	5.290439e+02	7.702625e+02	1.635892e+03	1.189576e+03	3.383540e+06	2.184014e+03	4.734528e+03	3.674412e+03	5.243218e+03	7.173055e+05	1.613039e+03	2.198935e+03	3.300194e+03
	min.	4.273910e+02	6.243150e+02	1.311208e+03	9.042040e+02	2.188735e+06	1.687815e+03	3.464897e+03	2.806438e+03	3.919789e+03	6.648850e+05	1.571302e+03	1.915624e+03	2.987352e+03
	max.	7.512900e+02	1.034630e+03	1.919926e+03	1.432729e+03	4.410046e+06	2.957117e+03	5.787919e+03	5.013840e+03	5.904078e+03	7.439860e+05	1.776052e+03	2.751402e+03	3.593407e+03
	std.	6.365405e+01	7.954185e+01	9.821567e+01	1.041877e+02	4.087474e+05	2.143415e+02	4.559487e+02	5.191158e+02	4.693797e+02	1.506568e+04	4.537701e+01	1.487086e+02	1.256761e+02
5D	med.	6.497950e+02	8.336650e+02	2.060368e+03	1.263683e+03	5.560614e+06	4.277886e+03	5.595404e+03	6.643104e+03	5.638253e+03	5.720390e+05	2.665033e+03	2.438998e+03	3.695332e+03
	avg.	6.497381e+02	8.496291e+02	2.045210e+03	1.251295e+03	4.506080e+06	4.263317e+03	5.625164e+03	6.686044e+03	5.723600e+03	6.040571e+05	2.725863e+03	2.460444e+03	3.674662e+03
	min.	5.159080e+02	6.304420e+02	1.596011e+03	9.797330e+02	2.638975e+06	1.902138e+03	4.423118e+03	3.380401e+03	4.522500e+03	5.657940e+05	1.972073e+03	2.106123e+03	3.137521e+03
	max.	8.519310e+02	1.214826e+03	2.427550e+03	2.130276e+03	6.033273e+06	6.990601e+03	6.639719e+03	6.475804e+03	6.841225e+03	7.201320e+05	4.133881e+03	2.839213e+03	4.139397e+03
	std.	6.755902e+01	9.885938e+01	1.347604e+02	1.427811e+02	7.302167e+05	9.423235e+02	5.371825e+02	5.346888e+02	4.589383e+02	5.093905e+04	4.622083e+02	1.786889e+02	1.884627e+02
6D	med.	7.576550e+02	9.205310e+02	2.146415e+03	1.300746e+03	5.560614e+06	9.812360e+02	6.379274e+03	5.169986e+03	7.220320e+03	7.107250e+05	2.291623e+03	2.715624e+03	4.020445e+03
	avg.	7.576418e+02	9.250577e+02	2.115712e+03	1.277239e+03	5.617722e+06	1.161388e+03	6.240546e+03	5.217036e+03	7.227384e+03	7.103629e+05	2.403079e+03	2.713406e+03	3.989618e+03
	min.	6.036300e+02	7.032990e+02	1.680399e+03	9.415050e+02	3.354561e+06	6.211540e+02	4.852315e+03	4.260735e+03	5.756640e+03	6.795940e+05	1.796881e+03	2.273929e+03	3.209057e+03
	max.	9.987510e+02	1.321688e+03	2.480845e+03	1.563666e+03	7.397077e+06	6.080552e+03	8.040504e+03	6.663570e+03	8.023899e+03	7.196650e+05	3.621057e+03	3.185109e+03	4.422090e+03
	std.	7.582944e+01	9.600582e+01	1.794216e+02	1.296062e+02	9.709183e+05	8.890532e+02	5.798216e+02	5.061664e+02	5.181704e+02	4.959971e+03	4.406428e+02	1.664707e+02	2.455763e+02
7D	med.	6.860640e+02	9.516790e+02	2.310485e+03	1.272499e+03	5.149201e+06	1.292558e+03	4.900318e+03	4.053942e+03	4.928059e+03	3.293220e+05	1.629782e+03	1.828531e+03	3.327889e+03
	avg.	6.840090e+02	9.516593e+02	2.267304e+03	1.255789e+03	5.237410e+06	1.580691e+03	5.008003e+03	4.167798e+03	4.878612e+03	3.290561e+05	1.636817e+03	1.841760e+03	3.321618e+03
	min.	5.334500e+02	7.512730e+02	1.792618e+03	9.280460e+02	3.442233e+06	1.027878e+03	3.755296e+03	3.101683e+03	3.583780e+03	3.162100e+05	1.360531e+03	1.586597e+03	2.990732e+03
	max.	8.733320e+02	1.241032e+03	2.524168e+03	1.542711e+03	7.167812e+06	3.031463e+03	7.851106e+03	6.344482e+03	5.394616e+03	3.374000e+05	1.936619e+03	2.096305e+03	3.659567e+03
	std.	7.582944e+01	9.600582e+01	1.794216e+02	1.344241e+02	9.037634e+05	2.904017e+03	7.384435e+02	6.240318e+02	3.710554e+02	3.185369e+03	1.228401e+02	1.049567e+02	1.108125e+02
8D	med.	9.316540e+02	1.008495e+03	2.500788e+03	1.236758e+03	6.969216e+06	1.808933e+03	7.970919e+03	6.449625e+03	8.316483e+03	1.382271e+06	2.153809e+03	2.699455e+03	4.231280e+03
	avg.	9.355335e+02	1.010106e+03	2.440098e+03	1.256586e+03	6.930952e+06	1.790503e+03	7.930368e+03	6.589572e+03	8.305784e+03	1.296938e+06	2.148890e+03	2.722786e+03	4.192757e+03
	min.	7.860010e+02	8.082680e+02	1.824538e+03	9.134960e+02	3.613568e+06	1.436093e+03	5.852243e+03	4.650839e+03	6.317410e+03	1.121014e+06	1.829838e+03	2.371755e+03	3.429561e+03
	max.	1.134564e+03	1.270099e+03	2.931283e+03	1.641999e+03	1.060429e+07	2.188707e+03	1.040707e+04	1.005622e+04	9.332004e+03	1.549732e+06	2.708899e+03	3.092664e+03	4.674187e+03
	std.	7.596785e+01	8.423078e+01	1.732070e+02	1.656651e+02	1.867688e+06	1.222682e+02	8.921054e+02	9.566067e+02	7.671048e+02	1.218940e+05	1.861165e+02	1.682541e+02	2.147167e+02
9D	med.	1.365926e+03	1.028262e+03	2.942071e+03	1.479264e+03	9.895728e+06	2.418298e+03	8.713669e+03	1.082471e+04	1.376129e+04	2.647340e+06	3.263801e+03	4.133845e+03	6.049630e+03
	avg.	1.349597e+03	1.032707e+03	2.857797e+03	1.432204e+03	9.740079e+06	2.293798e+03	1.015784e+04	1.085542e+04	1.380127e+04	2.645840e+06	3.262646e+03	4.134694e+03	6.119982e+03
	min.	1.124203e+03	8.485090e+02	2.267015e+03	1.001181e+03	5.966212e+06	1.595743e+03	7.930373e+03	8.322451e+03	1.061605e+04	2.478218e+06	2.655385e+03	3.458200e+03	5.255124e+03
	max.	1.521468e+03	1.408187e+03	3.135003e+03	1.624715e+03	1.425566e+07	2.873306e+03	1.640096e+04	1.427623e+04	1.578101e+04	2.662383e+06	3.598443e+03	4.556226e+03	7.229120e+03
	std.	9.512944e+01	9.236130e+01	1.919010e+02	1.366873e+02	2.059481e+06	3.108851e+02	2.266440e+03	1.248983e+03	1.148175e+03	1.805152e+04	1.875213e+02	2.109076e+02	3.851583e+02
10D	med.	1.913441e+03	1.102492e+03	2.945301e+03	1.516279e+03	1.243347e+07	2.301849e+03	1.447647e+04	1.690284e+04	2.188142e+04	5.046108e+06	4.030048e+03	5.691283e+03	8.432366e+03
	avg.	1.893191e+03	1.097632e+03	2.924841e+03	1.496449e+03	1.241360e+07	2.310183e+03	1.636666e+04	1.716302e+04	2.096612e+04	4.980186e+06	4.114413e+03	5.682148e+03	8.387644e+03
	min.	1.614522e+03	8.666740e+02	2.206195e+03	1.072342e+03	7.870573e+06	2.134857e+03	1.320829e+04	1.239735e+04	1.573816e+04	4.340605e+06	3.356384e+03	4.929018e+03	6.978411e+03
	max.	2.127138e+03	1.374465e+03	3.675104e+03	1.925534e+03	1.926229e+07	2.624911e+03	2.285963e+04	2.486369e+04	2.605228e+04	5.099551e+06	4.829143e+03	6.382486e+03	1.048817e+04
	std.	9.460065e+01	8.152846e+01	2.546024e+02	1.521574e+02	2.860982e+06	9.129827e+01	3.305113e+03	2.009931e+03	3.654970e+03	1.271311e+05	3.513096e+02	3.312131e+02	4.822998e+02

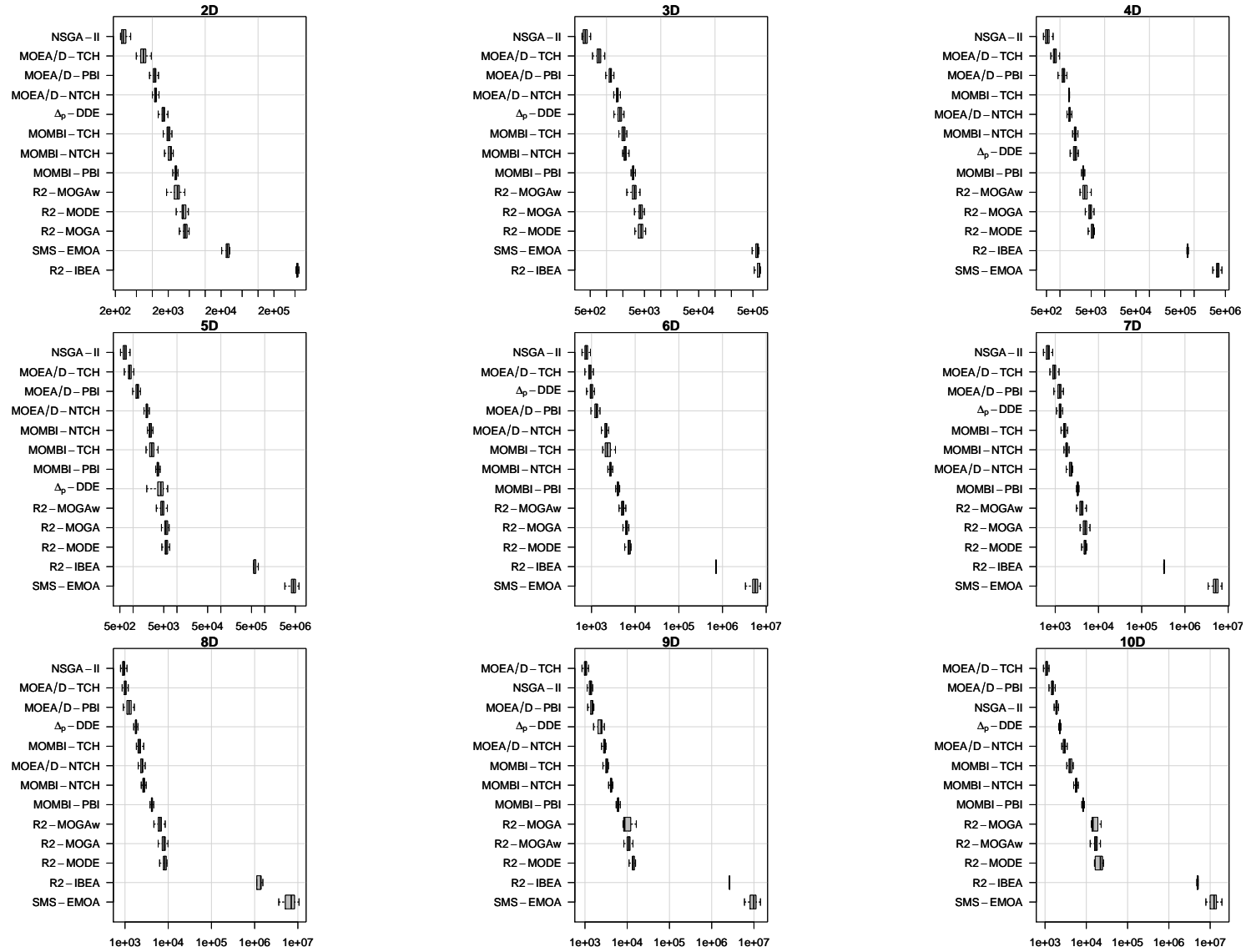


Figure A.99: Box-plot of runtime (in logarithmic scale) for different optimizers on the DTLZ7 test problem.

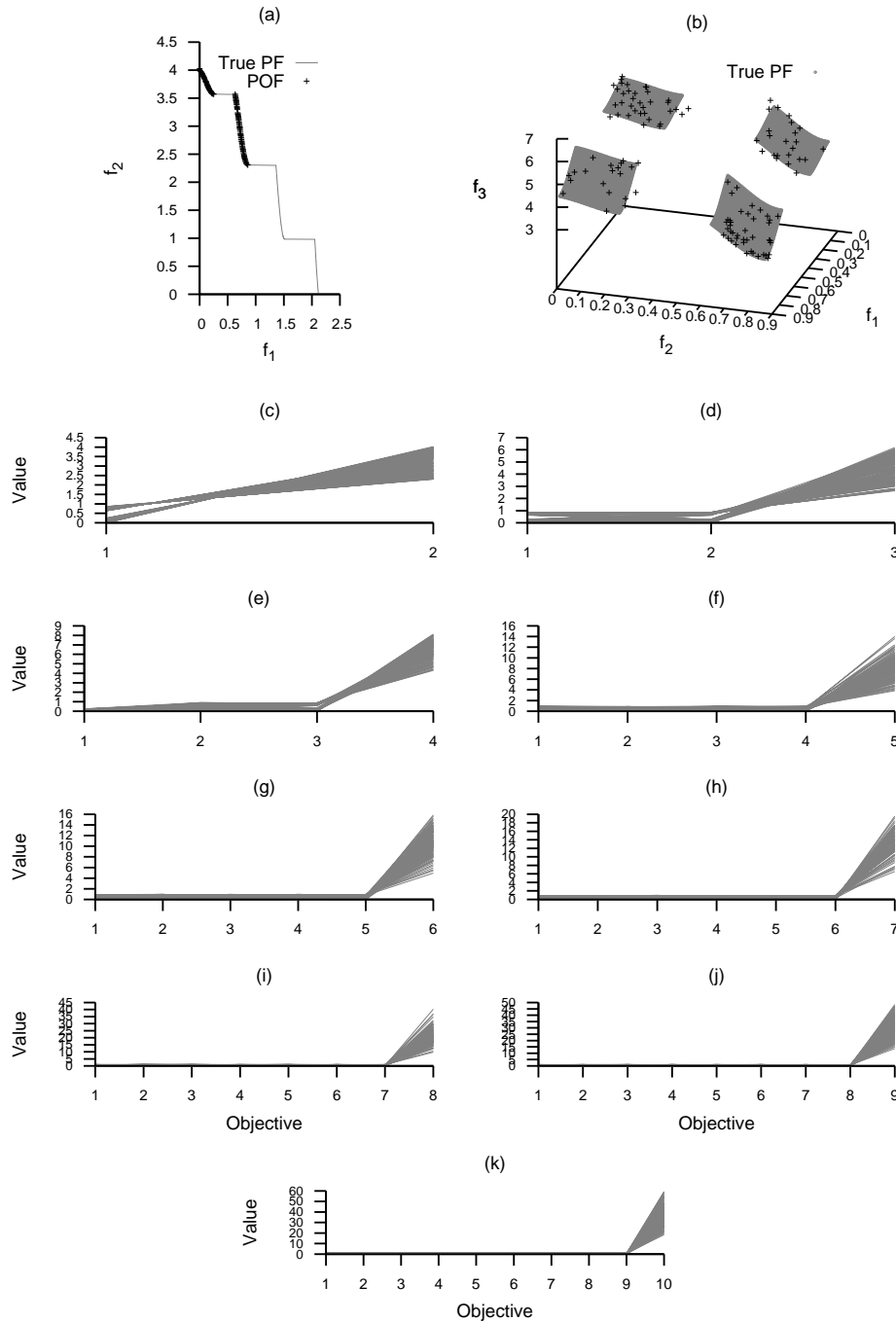


Figure A.100: Plots of the approximations obtained by NSGA-II from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

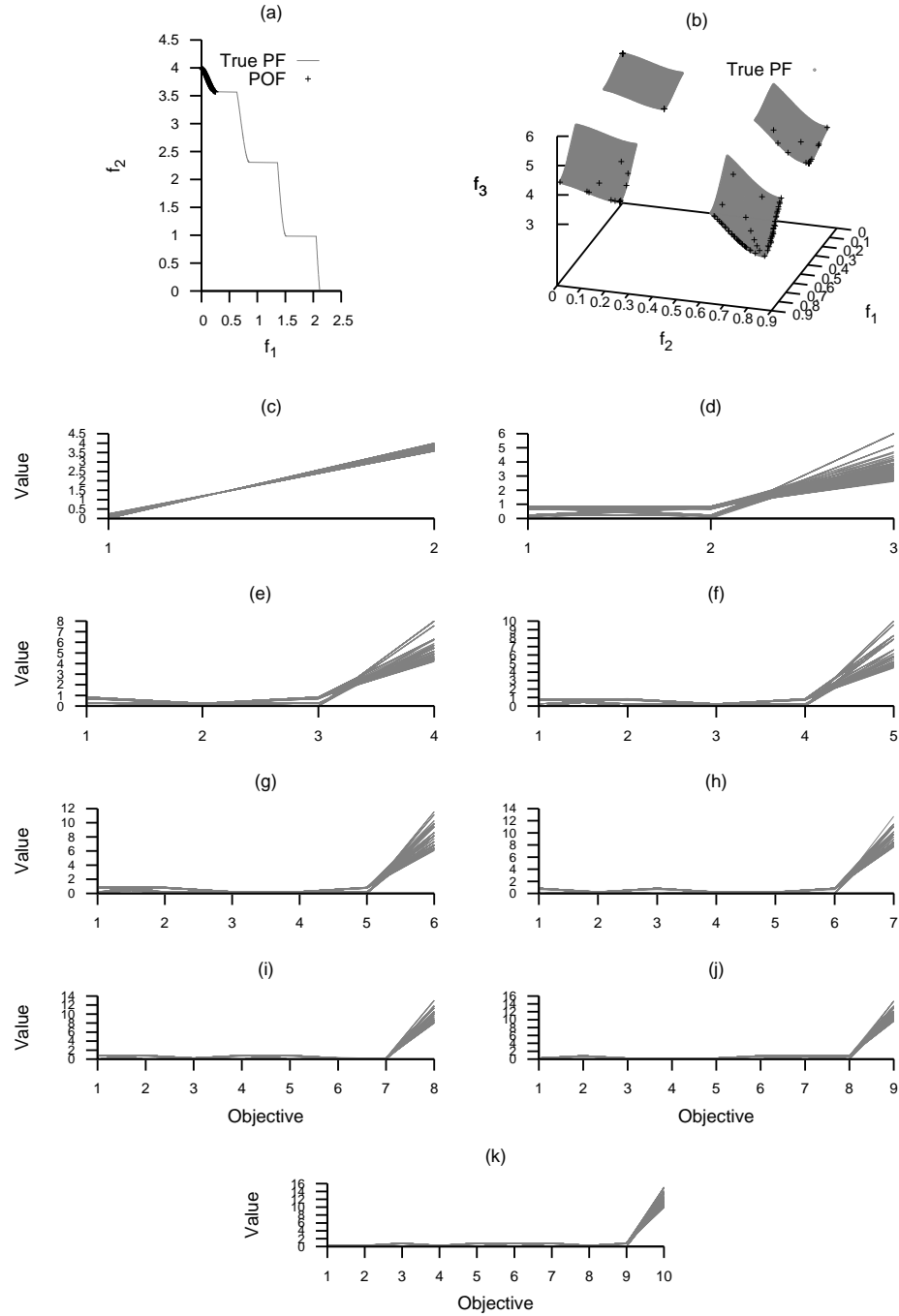


Figure A.101: Plots of the approximations obtained by MOEA/D-TCH from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

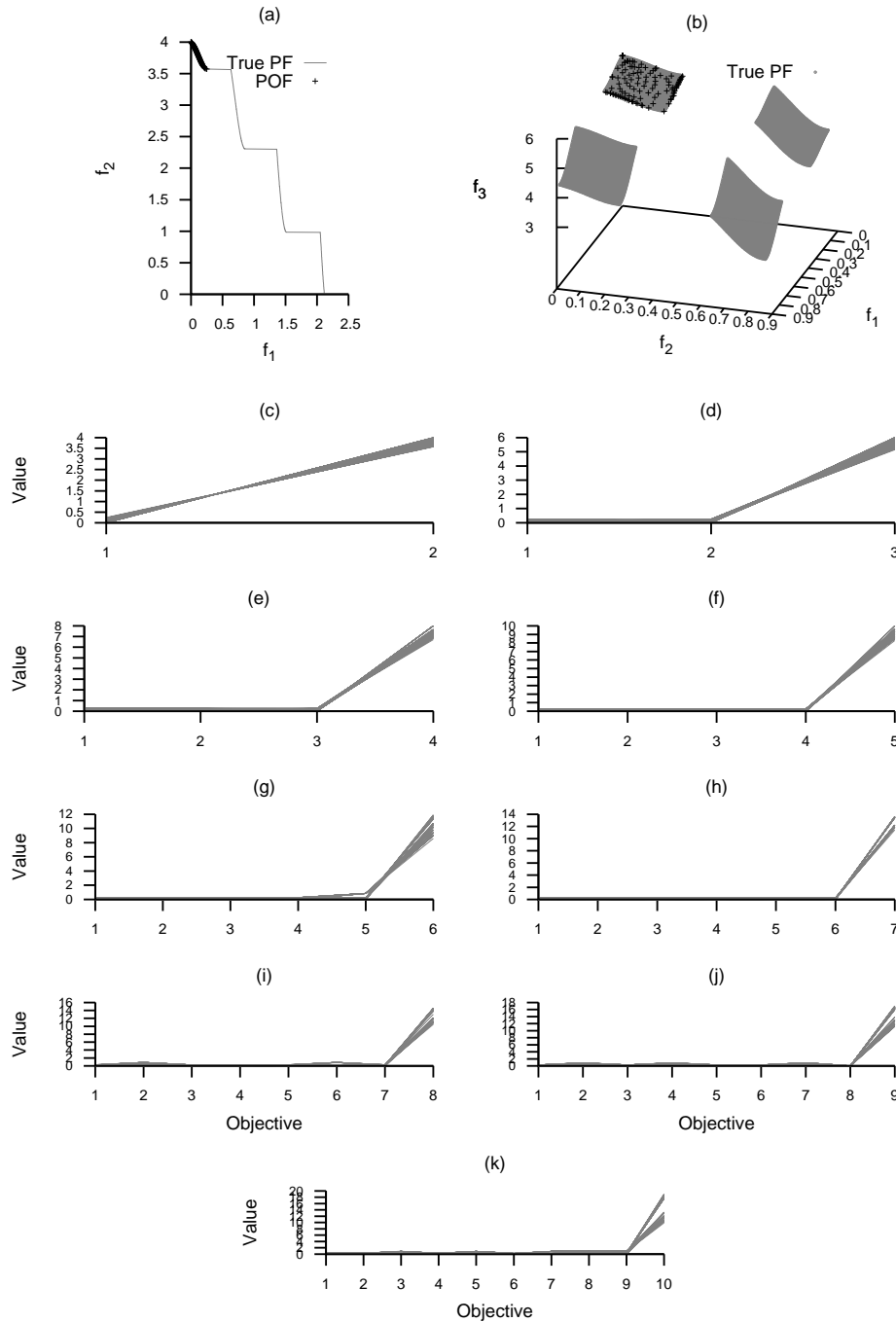


Figure A.102: Plots of the approximations obtained by MOEA/D-NTCH from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



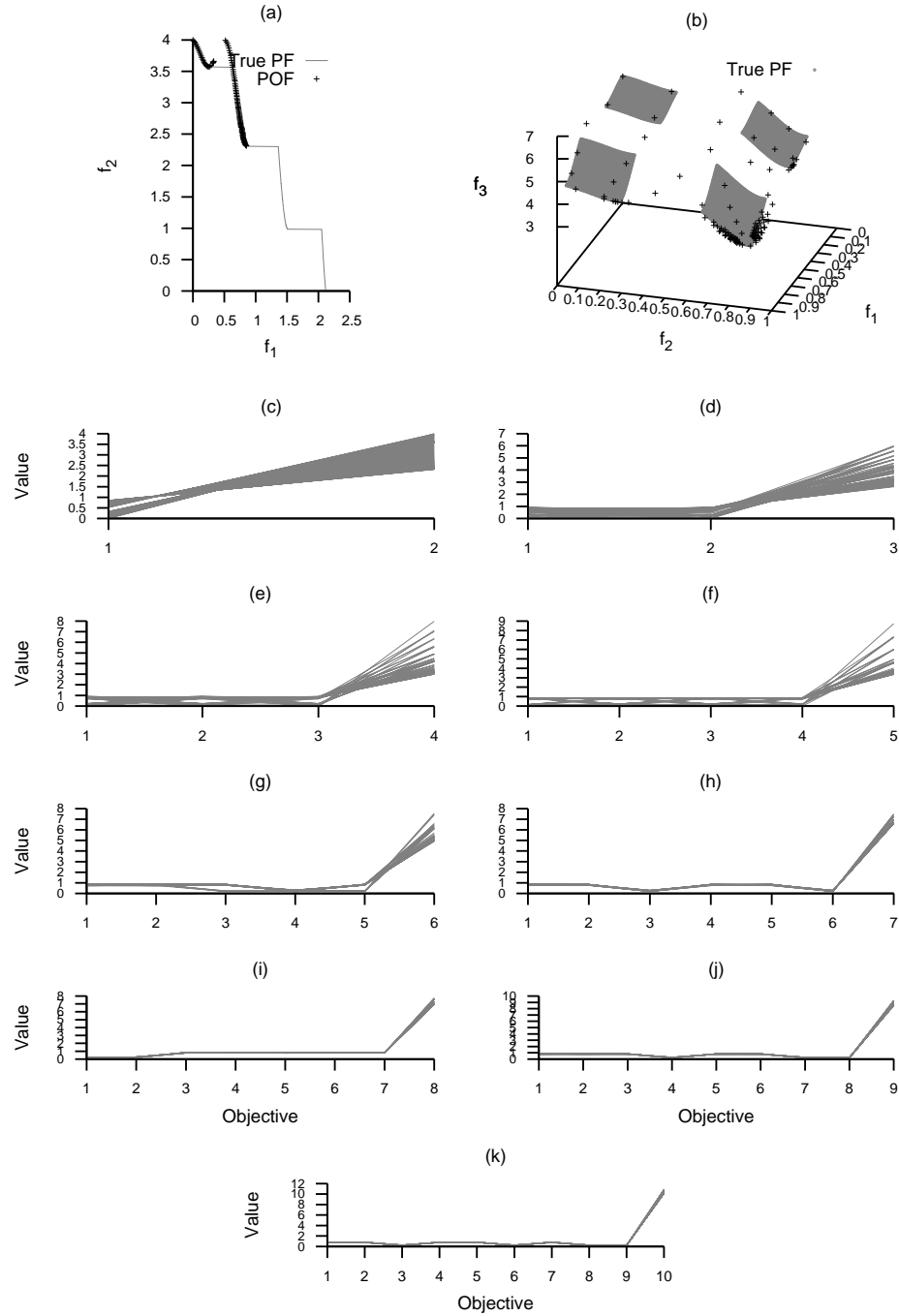


Figure A.103: Plots of the approximations obtained by MOEA/D-PBI from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

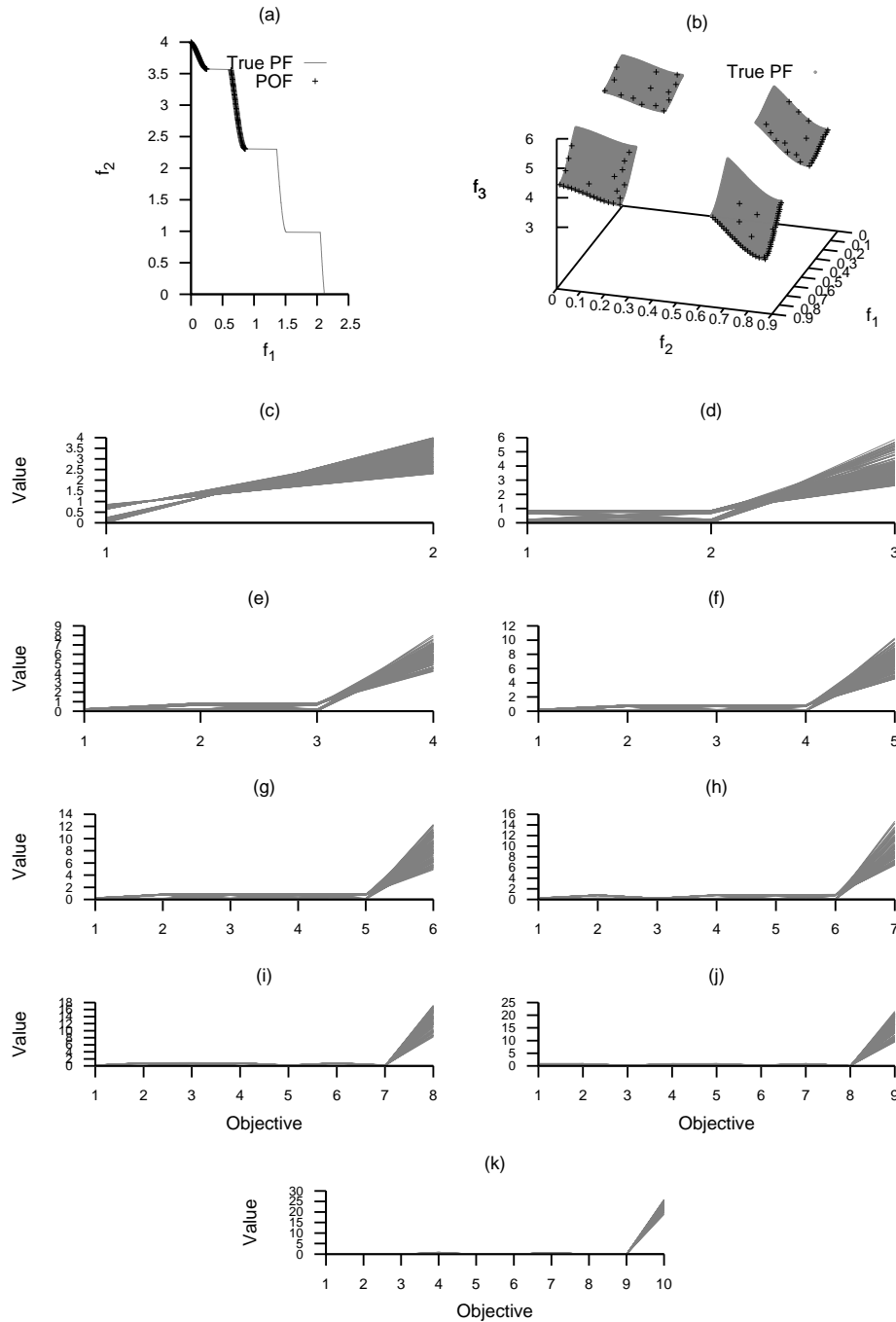


Figure A.104: Plots of the approximations obtained by SMS-EMOA from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

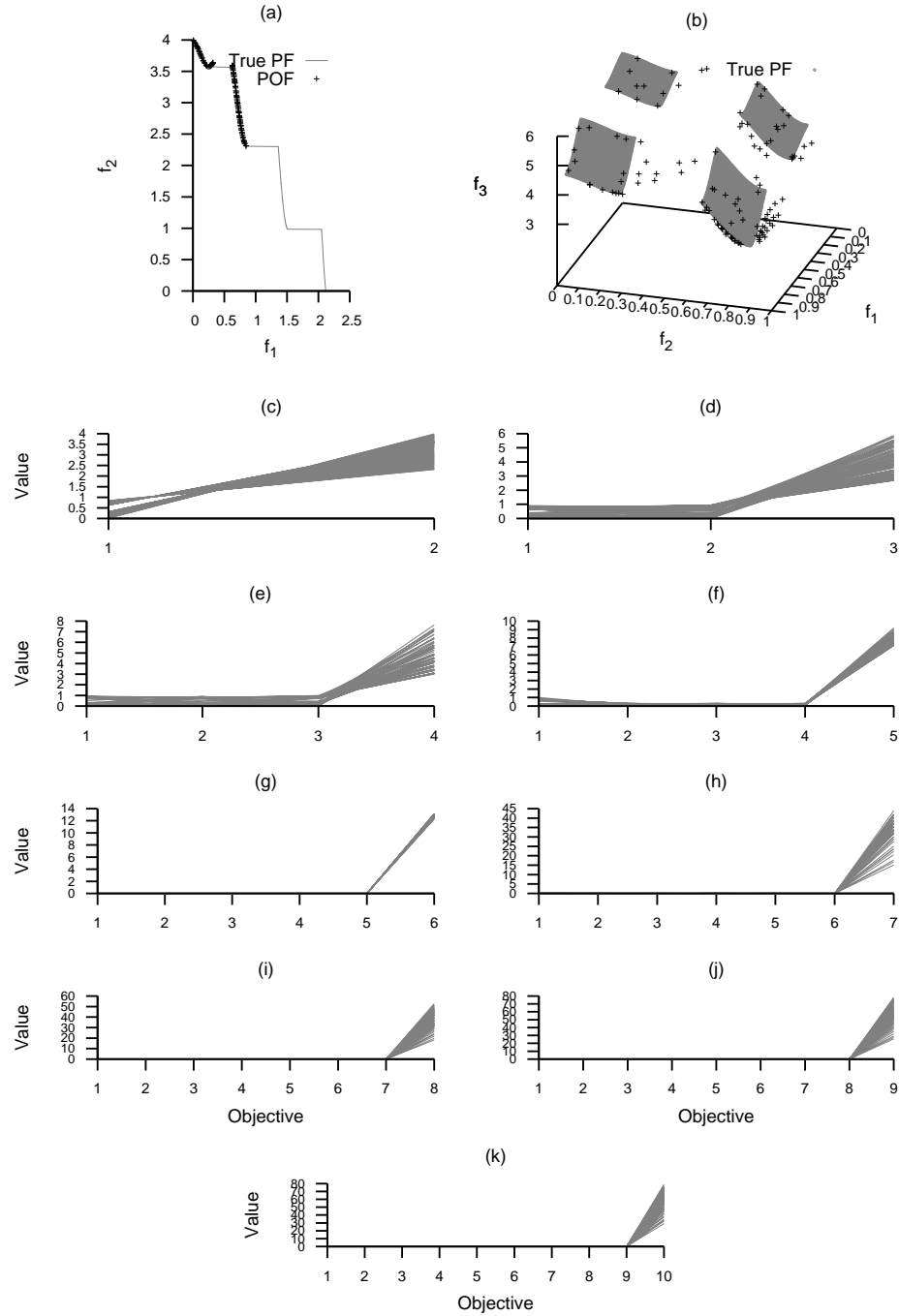


Figure A.105: Plots of the approximations obtained by  $\Delta_p$ -DDE from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

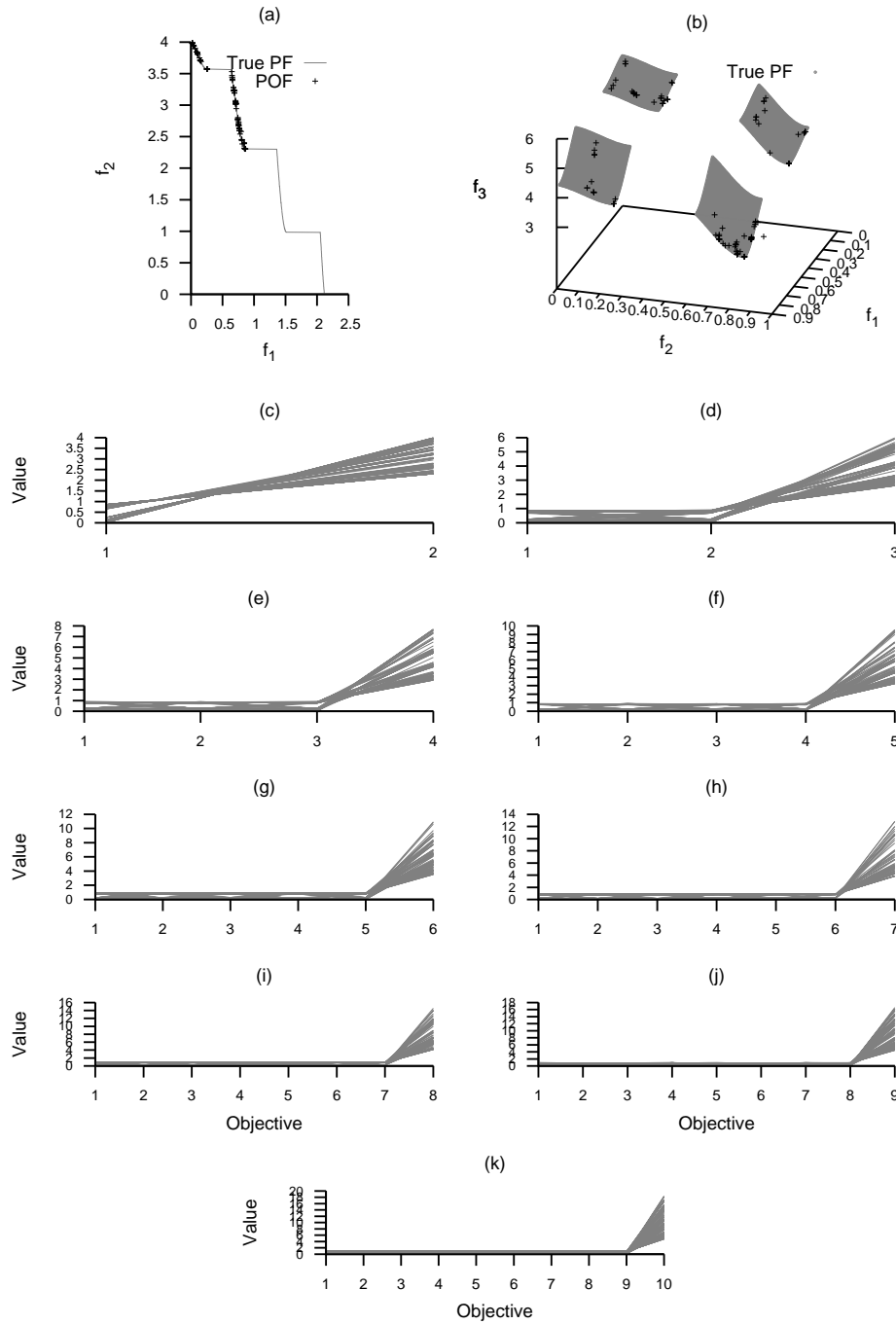


Figure A.106: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

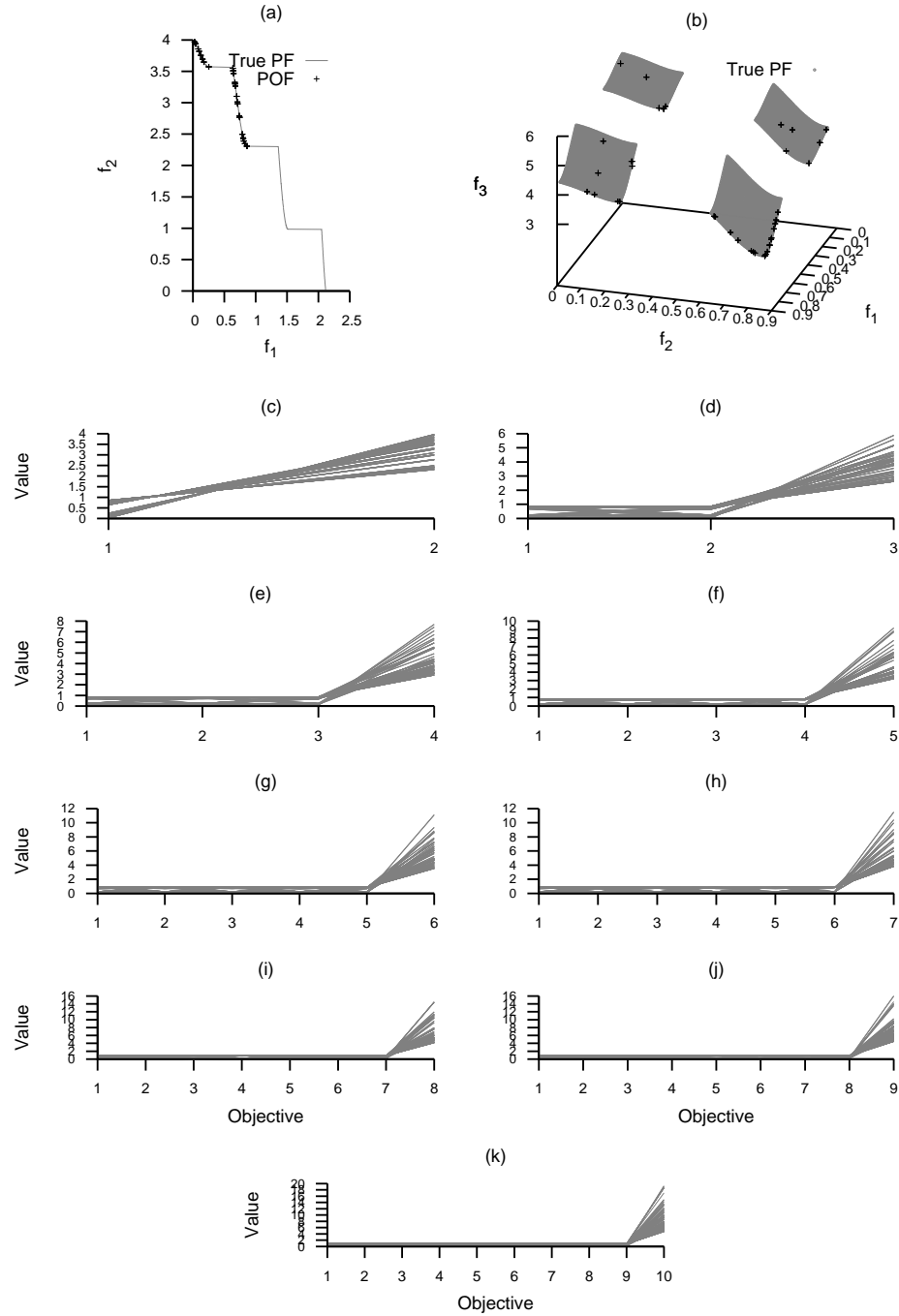


Figure A.107: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

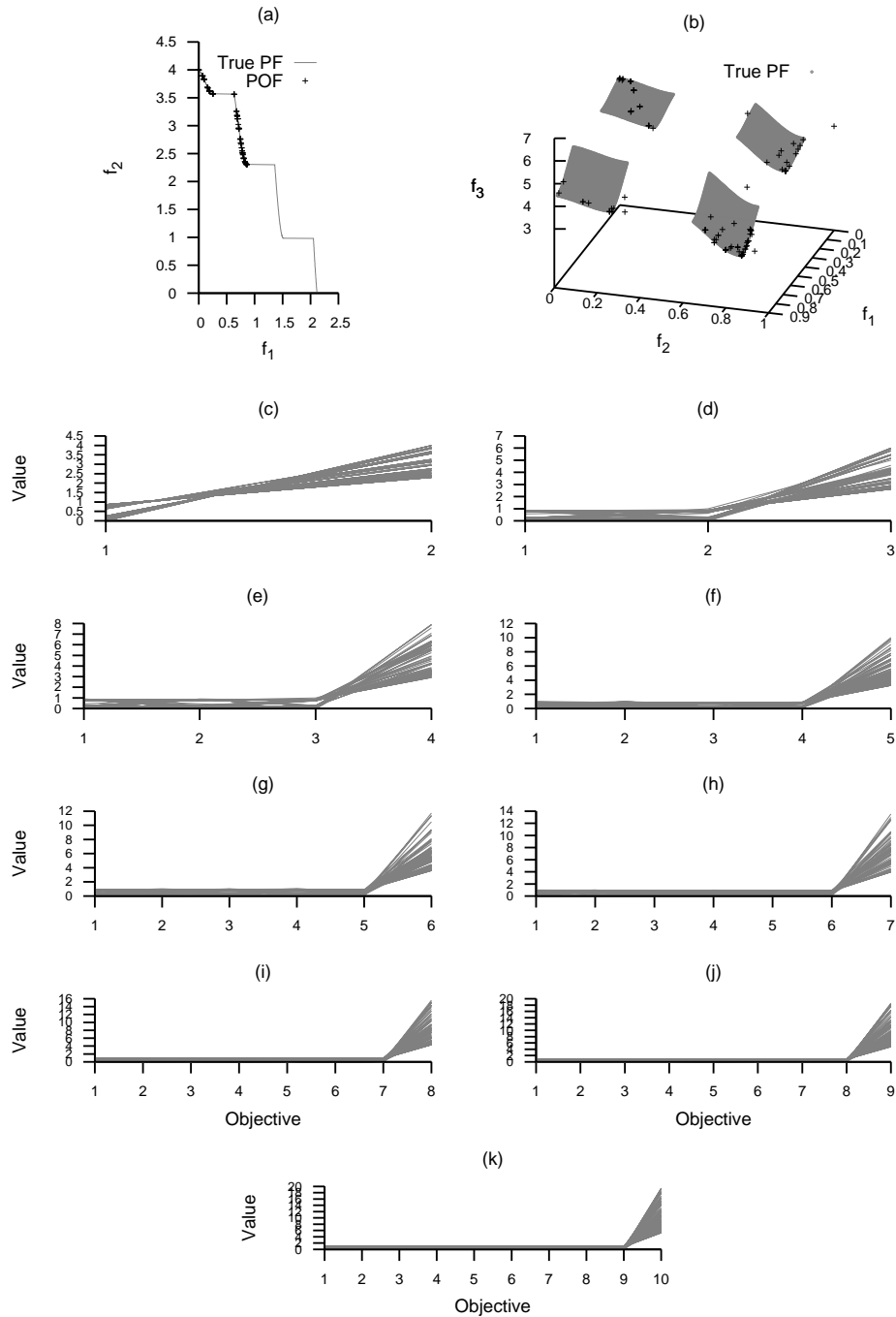


Figure A.108: Plots of the approximations obtained by *R2-MODE* from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

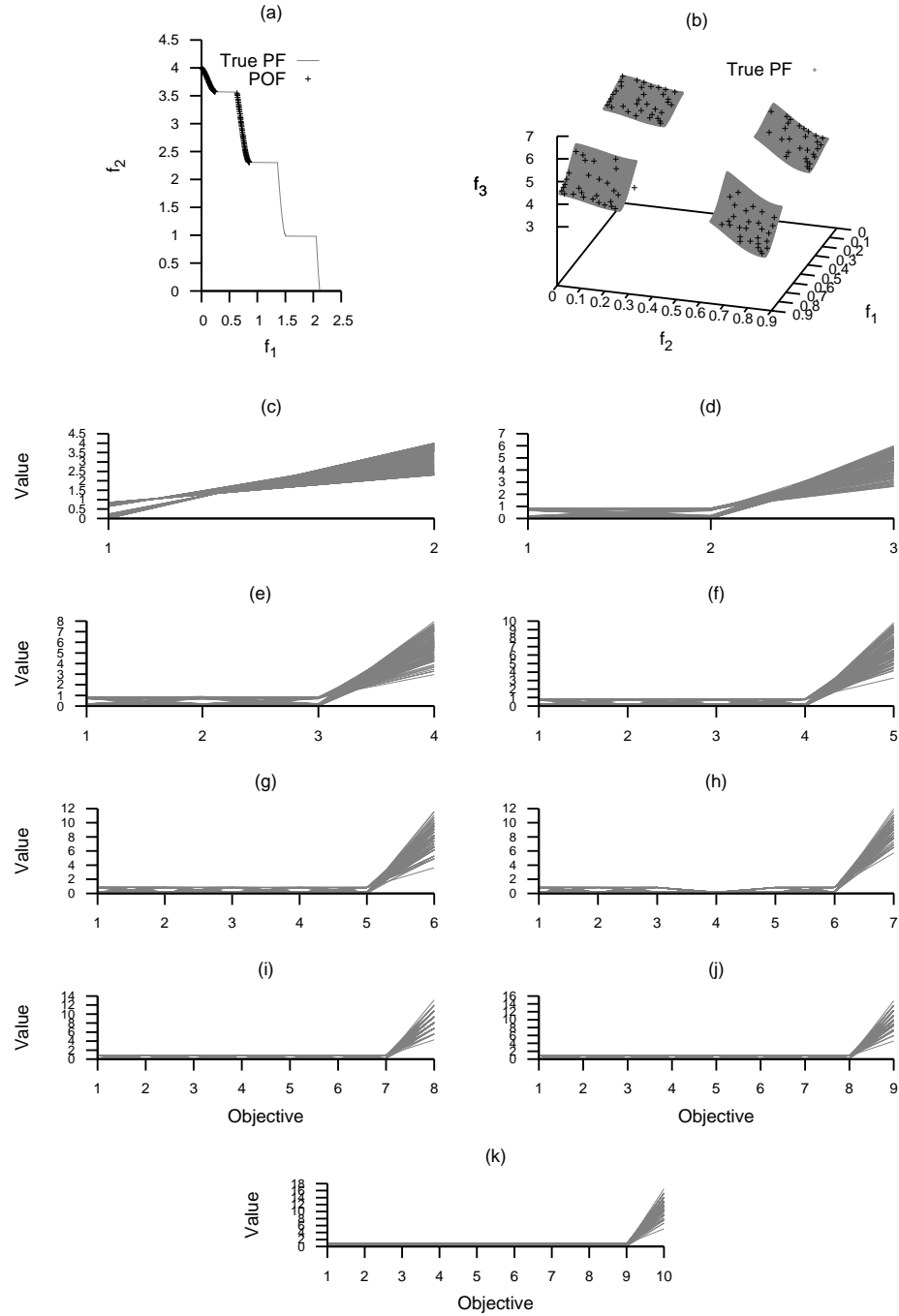


Figure A.109: Plots of the approximations obtained by R2-IBEA from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

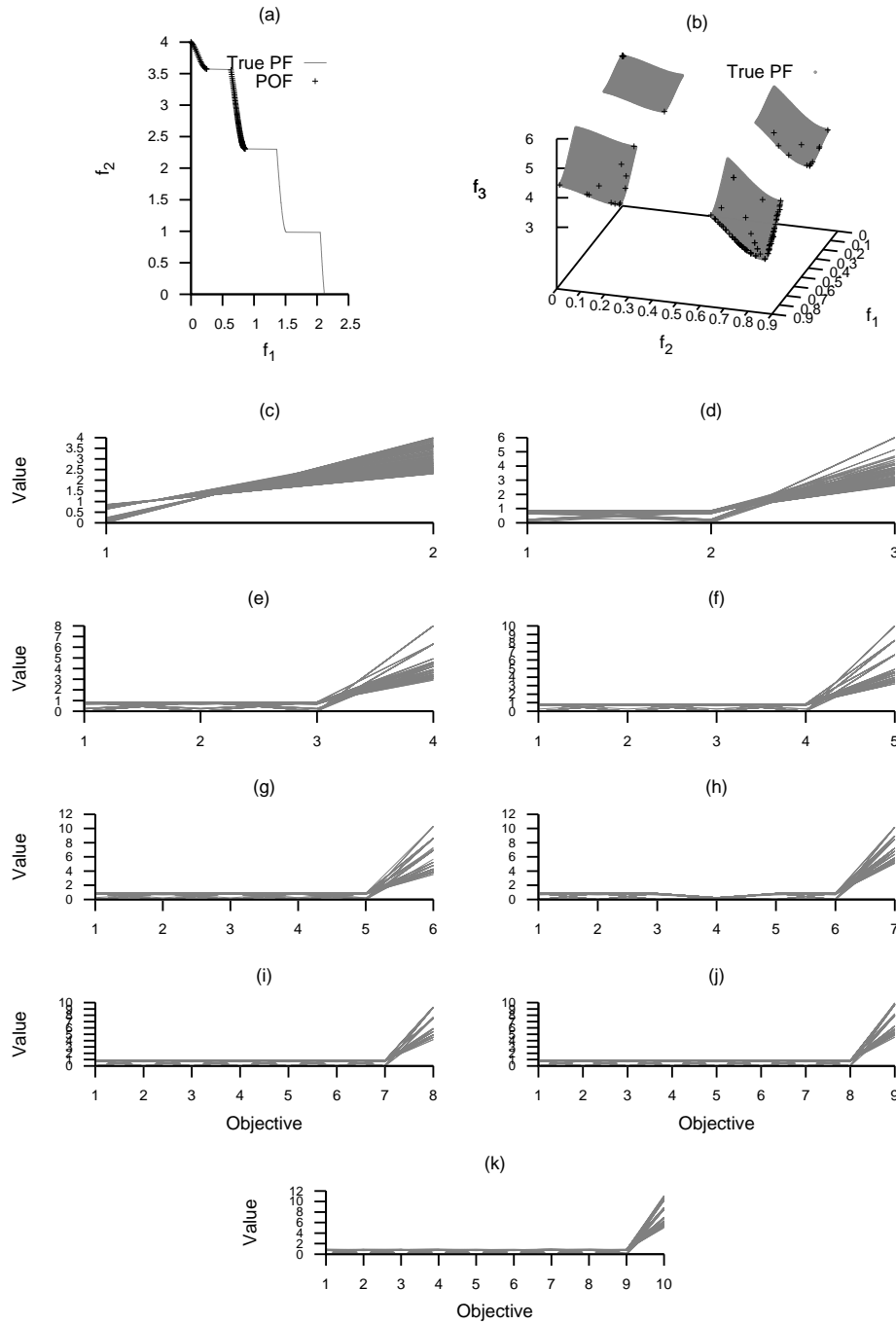


Figure A.110: Plots of the approximations obtained by MOMBI-TCH from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



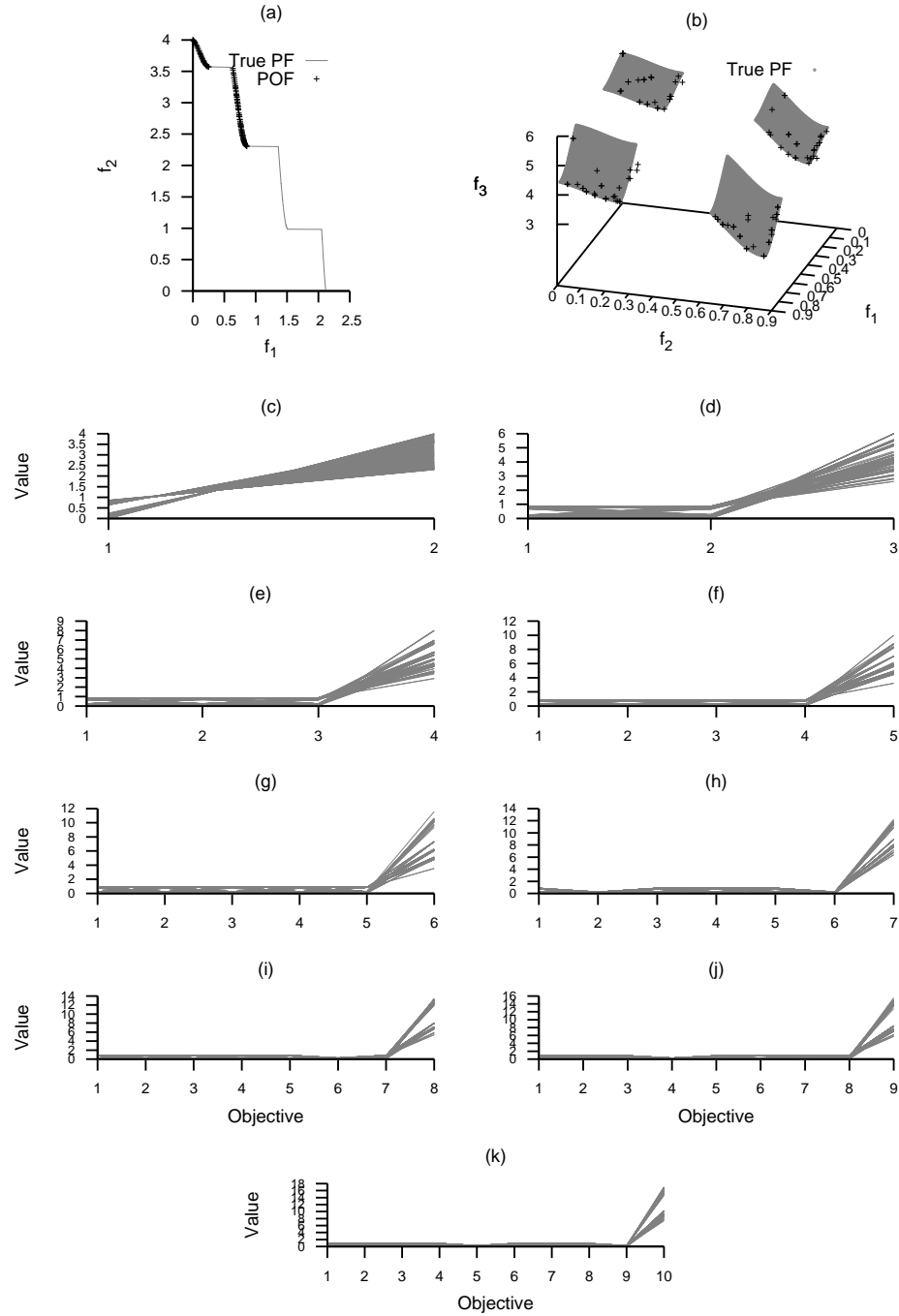


Figure A.111: Plots of the approximations obtained by MOMBI-NTCH from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

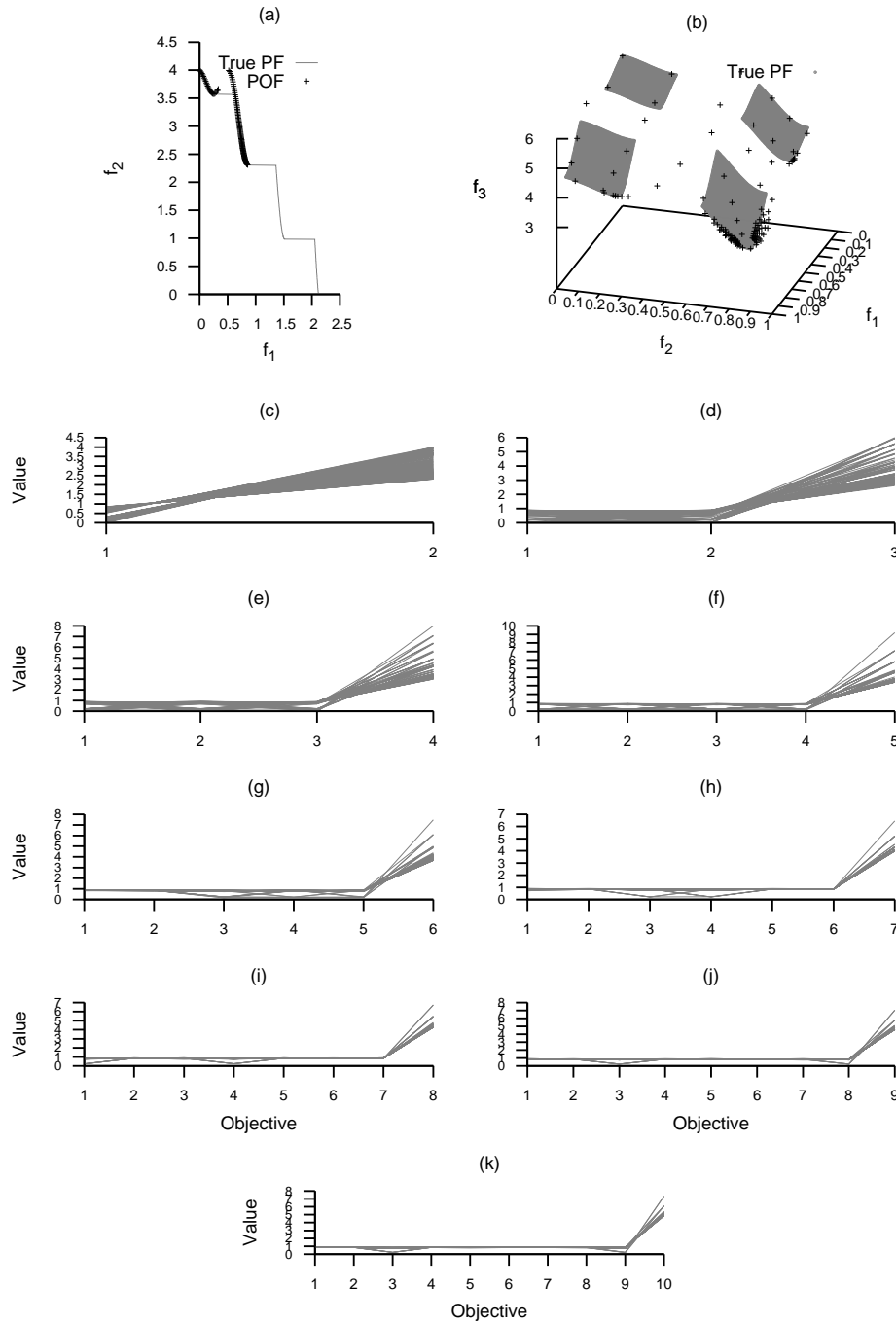


Figure A.112: Plots of the approximations obtained by MOMBI-PBI from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.44: Comparison of hypervolume indicator values for different optimizers on the WFG1 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	5.462359e+00	4.914358e+00	5.364344e+00	4.299120e+00	5.216719e+00	5.875994e+00	5.846619e+00	5.912147e+00	5.676851e+00	8.532078e+00	5.638118e+00	5.842622e+00	4.906127e+00
	avg.	5.613198e+00	4.994187e+00	5.449888e+00	4.295479e+00	5.218911e+00	5.878426e+00	5.957711e+00	6.005061e+00	5.684949e+00	8.698179e+00	5.626208e+00	6.000237e+00	4.968546e+00
	min.	4.871588e+00	3.265890e+00	3.971556e+00	3.617274e+00	4.506204e+00	5.769845e+00	5.067192e+00	5.067505e+00	5.537496e+00	7.754294e+00	4.793036e+00	5.174677e+00	4.398853e+00
	max.	7.120354e+00	6.259757e+00	6.538528e+00	4.817146e+00	7.409844e+00	5.983787e+00	7.188923e+00	7.484200e+00	5.849240e+00	1.038463e+01	6.392600e+00	7.452735e+00	6.386459e+00
	std.	5.029624e-01	5.607255e-01	5.744606e-01	2.055168e-01	3.931786e-01	4.617552e-02	5.528027e-01	5.470447e-01	5.805579e-02	7.636584e-01	3.025477e-01	5.286265e-01	2.928271e-01
3D	med.	3.568445e+01	5.298790e+01	4.864920e+01	4.969971e+01	6.018661e+01	4.763078e+01	5.459438e+01	5.695421e+01	4.910925e+01	7.553449e+01	5.391138e+01	5.449829e+01	4.917298e+01
	avg.	3.573018e+01	5.227857e+01	4.853772e+01	4.847948e+01	5.971890e+01	4.766870e+01	5.463634e+01	5.702944e+01	4.909904e+01	7.665978e+01	5.385996e+01	5.439026e+01	4.850851e+01
	min.	3.355509e+01	4.756529e+01	3.959740e+01	4.296551e+01	5.274862e+01	4.687834e+01	5.281628e+01	5.474343e+01	4.793777e+01	6.881641e+01	5.055970e+01	5.016687e+01	4.257459e+01
	max.	3.878649e+01	5.469985e+01	5.354379e+01	5.176109e+01	6.357477e+01	4.847397e+01	5.775135e+01	5.903375e+01	5.035301e+01	8.392603e+01	5.578139e+01	5.693552e+01	5.126336e+01
	std.	9.222034e-01	1.915141e+00	2.910133e+00	2.715035e+00	2.338678e+00	3.185352e-01	9.476713e-01	9.201671e-01	4.561858e-01	4.231603e+00	7.929417e-01	1.243932e+00	2.191201e+00
4D	med.	2.999472e+02	4.363328e+02	3.752584e+02	3.773955e+02	3.843418e+02	3.936088e+02	4.547109e+02	4.676028e+02	4.335961e+02	5.972767e+02	4.335852e+02	4.328992e+02	3.758139e+02
	avg.	3.000457e+02	4.325936e+02	3.743511e+02	3.872948e+02	3.852523e+02	3.951974e+02	4.549730e+02	4.672755e+02	4.335653e+02	5.997024e+02	4.324732e+02	4.322864e+02	3.840830e+02
	min.	2.822502e+02	4.008668e+02	3.178595e+02	3.559593e+02	3.675130e+02	3.826978e+02	4.330471e+02	4.447273e+02	4.261006e+02	5.663855e+02	4.136064e+02	4.106206e+02	3.574447e+02
	max.	3.177772e+02	4.545648e+02	4.322106e+02	4.299096e+02	4.284461e+02	4.121265e+02	4.757274e+02	4.841930e+02	4.435653e+02	6.722141e+02	4.449113e+02	4.460360e+02	4.140905e+02
	std.	6.853279e+00	1.300987e+01	2.430402e+01	2.169376e+01	1.103903e+01	5.704333e+00	8.068783e+00	7.401526e+00	3.409073e+00	2.039213e+01	6.627654e+00	2.181924e+00	1.843906e+01
5D	med.	3.165284e+03	4.205915e+03	3.449449e+03	3.785514e+03	3.637982e+03	3.997044e+03	4.439048e+03	4.551241e+03	4.420813e+03	5.908312e+03	4.346138e+03	4.283159e+03	3.713781e+03
	avg.	3.164572e+03	4.239323e+03	3.442911e+03	3.837411e+03	3.644455e+03	3.992746e+03	4.447962e+03	4.556431e+03	4.424050e+03	5.920511e+03	4.350517e+03	4.299735e+03	3.754181e+03
	min.	3.014939e+03	4.022654e+03	2.783251e+03	3.584182e+03	3.391319e+03	3.822199e+03	4.322882e+03	4.377570e+03	4.363041e+03	5.517745e+03	4.170580e+03	4.167911e+03	3.594959e+03
	max.	3.295570e+03	4.515805e+03	4.152484e+03	4.493660e+03	3.888046e+03	4.126977e+03	4.618256e+03	4.736161e+03	4.498066e+03	6.495744e+03	4.582562e+03	4.491752e+03	4.179526e+03
	std.	6.297992e+01	1.184932e+02	2.327068e+02	1.836751e+02	8.322119e+01	5.933607e+01	7.429734e+01	9.237423e+01	3.024078e+01	1.638093e+02	6.802183e+01	7.933194e+01	1.291893e+02
6D	med.	3.961896e+04	5.058502e+04	4.075742e+04	4.537424e+04	4.178292e+04	4.745718e+04	5.276427e+04	5.397905e+04	5.287953e+04	7.143837e+04	5.261463e+04	5.178997e+04	4.549634e+04
	avg.	3.962700e+04	5.077014e+04	4.079408e+04	4.628078e+04	4.170160e+04	4.749803e+04	5.283112e+04	5.414936e+04	5.291284e+04	7.153553e+04	5.259595e+04	5.186736e+04	4.570715e+04
	min.	3.808494e+04	4.837625e+04	3.428403e+04	4.340534e+04	3.833528e+04	4.640198e+04	5.084747e+04	5.253975e+04	5.191570e+04	6.672165e+04	4.925144e+04	5.047884e+04	4.337861e+04
	max.	4.099992e+04	5.370963e+04	4.701061e+04	5.194246e+04	4.452259e+04	4.937755e+04	5.537359e+04	5.700767e+04	5.356893e+04	7.811093e+04	5.500136e+04	5.445637e+04	5.040109e+04
	std.	6.426644e+02	9.744763e+02	2.550162e+03	2.280993e+03	1.174205e+03	5.115224e+02	9.125991e+02	8.821086e+02	2.894650e+02	2.130271e+03	9.485106e+02	6.750439e+02	1.126724e+03
7D	med.	5.675085e+05	7.103932e+05	5.507385e+05	6.592825e+05	5.947915e+05	6.645683e+05	7.317450e+05	7.710217e+05	7.747603e+05	1.138909e+06	7.601295e+05	7.547361e+05	6.828833e+05
	avg.	5.671659e+05	7.100479e+05	5.483226e+05	6.640954e+05	5.932660e+05	6.651603e+05	7.340535e+05	7.711226e+05	7.750353e+05	1.137462e+06	7.587543e+05	7.553591e+05	6.826636e+05
	min.	5.345314e+05	6.827213e+05	4.355632e+05	6.246522e+05	5.279304e+05	6.542456e+05	7.120322e+05	7.355933e+05	7.596985e+05	1.011569e+06	7.097780e+05	7.325436e+05	6.434942e+05
	max.	5.836994e+05	7.475913e+05	6.510853e+05	7.375575e+05	6.505399e+05	6.797777e+05	7.671301e+05	8.056107e+05	7.948739e+05	1.222339e+06	7.872074e+05	7.887441e+05	7.536961e+05
	std.	8.653643e+03	1.230891e+04	4.182843e+04	2.390291e+04	2.030456e+04	5.905857e+03	1.096246e+04	1.273553e+04	6.319036e+03	4.066803e+04	1.441700e+04	1.079752e+04	1.887342e+04
8D	med.	9.328351e+06	1.136470e+07	8.575436e+06	1.065207e+07	8.692666e+06	1.061006e+07	1.183472e+07	1.210138e+07	1.172177e+07	1.691765e+07	1.190240e+07	1.191409e+07	1.135197e+07
	avg.	9.339379e+06	1.133524e+07	8.693511e+06	1.070343e+07	8.705381e+06	1.060913e+07	1.182022e+07	1.213613e+07	1.172530e+07	1.691214e+07	1.191395e+07	1.191702e+07	1.134547e+07
	min.	9.104162e+06	1.083173e+07	6.739908e+06	1.010184e+07	8.097936e+06	1.044831e+07	1.146003e+07	1.175905e+07	1.156896e+07	1.544364e+07	1.142539e+07	1.160367e+07	1.088232e+07
	max.	9.649505e+06	1.181545e+07	1.048732e+07	1.185216e+07	9.354137e+06	1.070902e+07	1.212068e+07	1.282519e+07	1.194497e+07	1.791833e+07	1.229190e+07	1.231945e+07	1.189698e+07
	std.	9.730284e+04	2.058054e+05	6.597119e+05	3.352670e+05	2.702971e+05	4.808386e+04	1.376781e+05	1.779200e+05	6.899932e+04	4.272451e+05	1.928349e+05	1.354452e+05	1.888264e+05
9D	med.	1.708704e+08	2.009463e+08	1.511786e+08	1.909083e+08	1.504182e+08	1.911000e+08	2.131923e+08	2.166479e+08	2.049415e+08	2.691906e+08	2.110870e+08	2.118505e+08	2.050601e+08
	avg.	1.710400e+08	2.009571e+08	1.525088e+08	1.912840e+08	1.504794e+08	1.912819e+08	2.132641e+08	2.168482e+08	2.050181e+08	2.694135e+08	2.112527e+08	2.117147e+08	2.050124e+08
	min.	1.680926e+08	1.892866e+08	1.298587e+08	1.835622e+08	1.363577e+08	1.901592e+08	2.086256e+08	2.120742e+08	2.035420e+08	2.553186e+08	2.051766e+08	2.052986e+08	1.995332e+08
	max.	1.754480e+08	2.112792e+08	1.835066e+08	2.091752e+08	1.614670e+08	1.936965e+08	2.218463e+08	2.225252e+08	2.078647e+08	2.832295e+08	2.163483e+08	2.227167e+08	2.101744e+08
	std.	1.464659e+06	3.571941e+06	1.073714e+07	3.856771e+06	7.473478e+05	1.093847e+05	2.098197e+06	1.973211e+06	8.098268e+05	5.995060e+06	2.354342e+06	2.214950e+06	2.467371e+06
10D	med.	3.449011e+09	4.000843e+09	3.043455e+09	3.801957e+09	2.836797e+09	3.820934e+09	4.259319e+09	4.295204e+09	4.025032e+09	4.722387e+09	4.163104e+09	4.173288e+09	4.058654e+09
	avg.	3.447654e+09	4.006121e+09	3.041157e+09	3.804224e+09	2.848816e+09	3.824122e+09	4.261389e+09	4.298027e+09	4.026265e+09	4.726504e+09	4.167185e+09	4.176136e+09	4.062747e+09
	min.	3.386511e+09	3.830700e+09	2.558320e+09	3.674781e+09	2.546466e+09	3.798558e+09	4.191595e+09	4.212989e+09	4.000947e+09	4.559714e+09	4.048004e+09	4.104300e+09	3.940567e+09
	max.	3.517417e+09	4.169867e+09	3.572187e+09	4.149556e+09	3.143816e+09	3.910851e+09	4.333448e+09	4.379027e+09	4.059071e+09	4.925270e+09	4.339987e+09	4.412840e+09	4.191077e+09
	std.	2.552201e+07	6.444808e+07	1.629448e+08	6.856123e+07	9.937826e+07	1.559904e+07	2.840457e+07	3.222883e+07	1.081034e+07	8.109985e+07	4.070563e+07	4.16258e+07	4.038116e+07

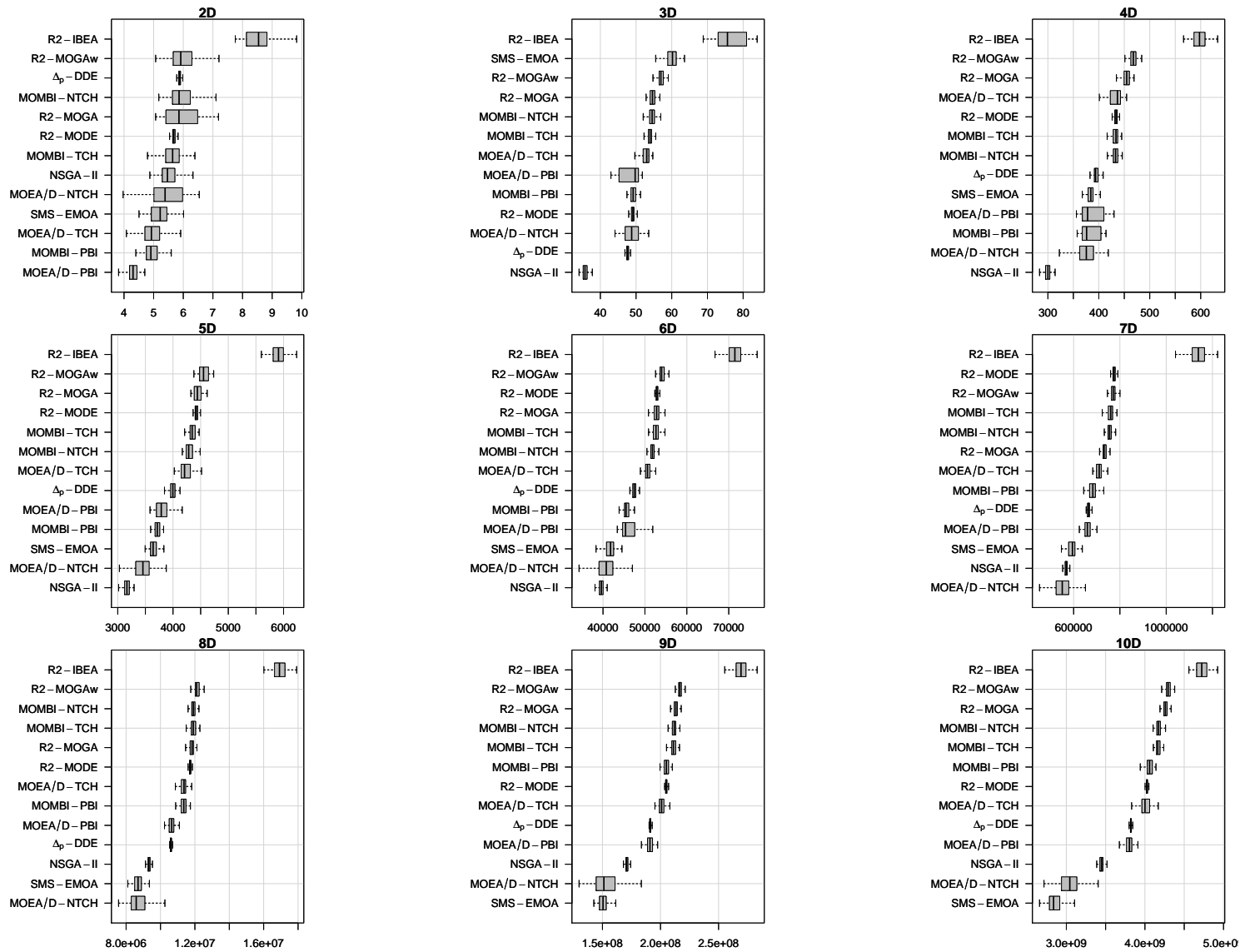


Figure A.113: Box-plot of hypervolume indicator values for different optimizers on the WFG1 test problem.

Table A.45: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG1 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D													
NSGA-II	—	$5.71e-15$	$> 0.05$	$1.28e-34$	$5.60e-09$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$2.77e-24$
MOEA/D-TCH	$> 0.05$	—	$> 0.05$	$7.58e-23$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-NTCH	$> 0.05$	$8.12e-09$	—	$1.06e-29$	$2.97e-04$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$2.26e-11$
MOEA/D-PBI	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
SMS-EMOA	$> 0.05$	$9.49e-05$	$> 0.05$	$4.92e-34$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.34e-07$
$\Delta_p$ -DDE	$2.28e-14$	$6.88e-23$	$1.95e-05$	$1.28e-34$	$2.99e-31$	—	$> 0.05$	$> 0.05$	$8.16e-34$	$> 0.05$	$9.91e-13$	$> 0.05$	$2.51e-33$
R2-MOGA	$4.96e-06$	$5.12e-22$	$1.86e-08$	$1.28e-34$	$6.01e-19$	$> 0.05$	—	$> 0.05$	$8.80e-03$	$> 0.05$	$1.35e-04$	$> 0.05$	$6.79e-30$
R2-MOGAw	$9.76e-09$	$9.65e-23$	$1.37e-08$	$1.28e-34$	$8.15e-23$	$> 0.05$	$> 0.05$	—	$2.97e-08$	$> 0.05$	$1.63e-07$	$> 0.05$	$8.77e-31$
R2-MODE	$1.11e-09$	$8.48e-16$	$> 0.05$	$1.28e-34$	$2.02e-26$	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$2.74e-33$
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$6.64e-03$	$5.06e-16$	$4.65e-02$	$1.32e-34$	$3.14e-15$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$	$2.75e-27$
MOMBI-NTCH	$3.05e-10$	$1.45e-22$	$2.19e-08$	$1.28e-34$	$7.15e-25$	$> 0.05$	$> 0.05$	$> 0.05$	$1.88e-06$	$> 0.05$	$1.53e-07$	—	$6.00e-32$
MOMBI-PBI	$> 0.05$	$> 0.05$	$> 0.05$	$4.14e-33$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—
3D													
NSGA-II	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-TCH	$1.28e-34$	—	$2.93e-20$	$2.24e-19$	$> 0.05$	$1.61e-33$	$> 0.05$	$> 0.05$	$1.84e-19$	$> 0.05$	$> 0.05$	$> 0.05$	$2.29e-19$
MOEA/D-NTCH	$1.28e-34$	$> 0.05$	—	$> 0.05$	$> 0.05$	$2.21e-02$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-PBI	$1.28e-34$	$> 0.05$	$> 0.05$	—	$> 0.05$	$6.04e-05$	$> 0.05$	$> 0.05$	$1.30e-02$	$> 0.05$	$> 0.05$	$> 0.05$	$8.98e-03$
SMS-EMOA	$1.28e-34$	$4.93e-33$	$1.44e-34$	$1.28e-34$	—	$1.28e-34$	$6.07e-27$	$2.61e-19$	$1.28e-34$	$> 0.05$	$3.29e-30$	$2.05e-27$	$1.28e-34$
$\Delta_p$ -DDE	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
R2-MOGA	$1.28e-34$	$5.28e-25$	$3.05e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	—	$> 0.05$	$1.28e-34$	$> 0.05$	$4.38e-09$	$> 0.05$	$1.28e-34$
R2-MOGAw	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$1.84e-29$	—	$1.28e-34$	$> 0.05$	$3.24e-34$	$6.61e-31$	$1.28e-34$
R2-MODE	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$3.05e-34$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$3.67e-15$	$5.39e-33$	$4.12e-34$	$> 0.05$	$1.28e-34$	$> 0.05$	$> 0.05$	$1.28e-34$	$> 0.05$	—	$> 0.05$	$1.49e-34$
MOMBI-NTCH	$1.28e-34$	$9.40e-21$	$1.41e-32$	$2.82e-33$	$> 0.05$	$1.28e-34$	$> 0.05$	$> 0.05$	$1.36e-34$	$> 0.05$	$9.52e-07$	—	$2.26e-34$
MOMBI-PBI	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$3.27e-15$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—
4D													
NSGA-II	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-TCH	$1.28e-34$	—	$9.39e-33$	$6.00e-29$	$4.93e-33$	$5.22e-34$	$> 0.05$	$> 0.05$	$3.71e-02$	$> 0.05$	$3.88e-02$	$4.26e-02$	$1.54e-32$
MOEA/D-NTCH	$1.28e-34$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-PBI	$1.28e-34$	$> 0.05$	$1.17e-03$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
SMS-EMOA	$1.28e-34$	$> 0.05$	$4.65e-05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$4.13e-02$
$\Delta_p$ -DDE	$1.28e-34$	$> 0.05$	$9.54e-16$	$2.66e-03$	$2.34e-20$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$3.37e-04$
R2-MOGA	$1.28e-34$	$1.47e-28$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$> 0.05$	$1.15e-32$	$> 0.05$	$2.38e-32$	$2.18e-32$	$1.28e-34$
R2-MOGAw	$1.28e-34$	$2.33e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$4.21e-20$	—	$1.28e-34$	$> 0.05$	$1.32e-34$	$1.32e-34$	$1.28e-34$
R2-MODE	$1.28e-34$	$> 0.05$	$1.00e-33$	$2.79e-34$	$1.89e-34$	$1.28e-34$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$> 0.05$	$1.66e-33$	$7.88e-33$	$8.16e-34$	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$	$1.32e-34$
MOMBI-NTCH	$1.28e-34$	$> 0.05$	$2.44e-33$	$1.54e-32$	$9.75e-34$	$1.32e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$1.68e-34$
MOMBI-PBI	$1.28e-34$	$> 0.05$	$6.46e-03$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—

Table A.46: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG1 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
5D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$1.28e-34$	—	$3.05e-34$	$3.06e-28$	$1.28e-34$	$3.68e-33$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.13e-33$
MOEA/D-NTCH	$2.30e-21$	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$1.28e-34$	> 0.05	$6.74e-27$	—	$1.28e-18$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$4.46e-05$
SMS-EMOA	$1.28e-34$	> 0.05	$6.56e-16$	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	$1.28e-34$	> 0.05	$4.01e-32$	$4.77e-12$	$1.53e-34$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$3.34e-22$
R2-MOGA	$1.28e-34$	$7.28e-24$	$1.28e-34$	$9.75e-34$	$1.28e-34$	$1.28e-34$	—	> 0.05	$2.70e-02$	> 0.05	$1.93e-16$	$1.00e-23$	$1.28e-34$
R2-MOGAw	$1.28e-34$	$2.18e-32$	$1.28e-34$	$3.44e-34$	$1.28e-34$	$1.28e-34$	$2.96e-14$	—	$2.46e-25$	> 0.05	$5.42e-31$	$1.83e-32$	$1.28e-34$
R2-MODE	$1.28e-34$	$1.04e-22$	$1.28e-34$	$2.44e-33$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	—	> 0.05	$2.11e-17$	$6.40e-23$	$1.28e-34$
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$7.65e-12$	$1.28e-34$	$2.91e-33$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	—	$1.01e-06$	$1.32e-34$
MOMBI-NTCH	$1.28e-34$	$4.48e-06$	$1.28e-34$	$9.39e-33$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	$1.40e-34$
MOMBI-PBI	$1.28e-34$	> 0.05	$4.33e-24$	> 0.05	$4.09e-12$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—
6D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$1.28e-34$	—	$1.28e-34$	$2.13e-26$	$1.28e-34$	$1.58e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$7.69e-34$
MOEA/D-NTCH	$1.13e-04$	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$1.28e-34$	> 0.05	$1.15e-28$	—	$7.92e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	$1.75e-27$	> 0.05	$4.59e-04$	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	$1.28e-34$	> 0.05	$2.01e-34$	$1.33e-09$	$1.28e-34$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$3.15e-28$
R2-MOGA	$1.28e-34$	$1.69e-26$	$1.28e-34$	$3.88e-34$	$1.28e-34$	$1.28e-34$	—	> 0.05	> 0.05	> 0.05	> 0.05	$1.41e-15$	$1.28e-34$
R2-MOGAw	$1.28e-34$	$2.17e-33$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$5.63e-19$	—	$1.08e-26$	> 0.05	$2.90e-22$	$6.36e-32$	$1.28e-34$
R2-MODE	$1.28e-34$	$5.84e-29$	$1.28e-34$	$1.32e-34$	$1.28e-34$	$1.28e-34$	$2.00e-02$	> 0.05	—	> 0.05	$1.01e-03$	$1.40e-26$	$1.28e-34$
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$1.03e-23$	$1.28e-34$	$2.10e-33$	$1.28e-34$	$1.32e-34$	> 0.05	> 0.05	> 0.05	> 0.05	—	$4.35e-10$	$1.40e-34$
MOMBI-NTCH	$1.28e-34$	$8.95e-18$	$1.28e-34$	$7.56e-32$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	$1.28e-34$
MOMBI-PBI	$1.28e-34$	> 0.05	$1.79e-29$	> 0.05	$2.71e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—
7D													
NSGA-II	—	> 0.05	$1.59e-05$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$1.28e-34$	—	$1.28e-34$	$8.11e-25$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$9.88e-23$
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$1.28e-34$	> 0.05	$6.83e-34$	—	$3.88e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	$8.15e-23$	> 0.05	$1.20e-17$	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	$1.28e-34$	> 0.05	$1.28e-34$	$2.20e-04$	$1.28e-34$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$1.28e-34$	$3.98e-27$	$1.28e-34$	$5.42e-31$	$1.28e-34$	$1.28e-34$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.45e-32$
R2-MOGAw	$1.28e-34$	$1.44e-34$	$1.28e-34$	$1.32e-34$	$1.28e-34$	$1.28e-34$	$2.75e-32$	—	> 0.05	> 0.05	$1.42e-09$	$1.01e-16$	$1.68e-34$
R2-MODE	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.68e-34$	—	> 0.05	> 0.05	$6.28e-19$	$3.39e-27$	$1.28e-34$
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$2.51e-33$	$1.28e-34$	$3.05e-34$	$1.28e-34$	$1.28e-34$	$9.55e-24$	> 0.05	> 0.05	> 0.05	—	$6.55e-03$	$4.78e-34$
MOMBI-NTCH	$1.28e-34$	$2.55e-34$	$1.28e-34$	$1.63e-34$	$1.28e-34$	$1.28e-34$	$1.41e-24$	> 0.05	> 0.05	> 0.05	> 0.05	—	$5.22e-34$
MOMBI-PBI	$1.28e-34$	> 0.05	$1.73e-34$	$1.62e-12$	$1.53e-34$	$2.85e-15$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—

Table A.47: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG1 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
8D													
NSGA-II	–	> 0.05	1.93e – 15	> 0.05	1.87e – 33	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e – 34	–	1.28e – 34	1.72e – 24	1.28e – 34	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e – 34	> 0.05	2.63e – 34	–	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e – 34	> 0.05	1.36e – 34	> 0.05	1.28e – 34	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e – 34	6.18e – 32	1.28e – 34	5.55e – 33	1.28e – 34	1.28e – 34	–	> 0.05	1.64e – 08	> 0.05	> 0.05	> 0.05	4.25e – 32
R2-MOGAw	1.28e – 34	1.32e – 34	1.28e – 34	1.40e – 34	1.28e – 34	1.28e – 34	1.12e – 26	–	7.69e – 34	> 0.05	1.12e – 13	1.51e – 17	1.53e – 34
R2-MODE	1.28e – 34	7.40e – 31	1.28e – 34	6.36e – 32	1.28e – 34	1.28e – 34	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	1.16e – 31
R2-IBEA	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	–	–	1.28e – 34	1.28e – 34	1.28e – 34
MOMBI-TCH	1.28e – 34	1.94e – 32	1.28e – 34	2.04e – 33	1.28e – 34	1.28e – 34	1.49e – 04	> 0.05	2.35e – 15	> 0.05	–	> 0.05	2.06e – 32
MOMBI-NTCH	1.28e – 34	5.88e – 34	1.28e – 34	5.22e – 34	1.28e – 34	1.28e – 34	3.84e – 06	> 0.05	1.95e – 23	> 0.05	> 0.05	–	7.92e – 34
MOMBI-PBI	1.28e – 34	> 0.05	1.28e – 34	9.20e – 25	1.28e – 34	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
9D													
NSGA-II	–	> 0.05	1.92e – 29	> 0.05	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e – 34	–	1.28e – 34	1.44e – 29	1.28e – 34	6.00e – 32	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e – 34	> 0.05	1.28e – 34	–	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e – 34	> 0.05	1.28e – 34	> 0.05	1.28e – 34	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e – 34	2.01e – 34	1.28e – 34	1.32e – 34	1.28e – 34	1.28e – 34	–	> 0.05	1.28e – 34	> 0.05	3.68e – 09	1.85e – 07	2.07e – 34
R2-MOGAw	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	2.20e – 23	–	1.28e – 34	> 0.05	1.13e – 30	1.90e – 30	1.28e – 34
R2-MODE	1.28e – 34	3.57e – 21	1.28e – 34	4.37e – 32	1.28e – 34	1.28e – 34	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	–	1.28e – 34	1.28e – 34	1.28e – 34
MOMBI-TCH	1.28e – 34	1.57e – 33	1.28e – 34	2.55e – 34	1.28e – 34	1.28e – 34	> 0.05	> 0.05	5.38e – 34	> 0.05	–	> 0.05	1.27e – 30
MOMBI-NTCH	1.28e – 34	7.46e – 34	1.28e – 34	1.95e – 34	1.28e – 34	1.28e – 34	> 0.05	> 0.05	4.37e – 34	> 0.05	> 0.05	–	3.09e – 32
MOMBI-PBI	1.28e – 34	7.44e – 17	1.28e – 34	2.31e – 32	1.28e – 34	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
10D													
NSGA-II	–	> 0.05	2.31e – 32	> 0.05	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e – 34	–	1.28e – 34	3.57e – 32	1.28e – 34	3.05e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	–	> 0.05	3.76e – 18	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e – 34	> 0.05	1.28e – 34	–	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e – 34	> 0.05	1.28e – 34	1.93e – 04	1.28e – 34	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	–	> 0.05	1.28e – 34	> 0.05	2.66e – 31	8.25e – 30	1.28e – 34
R2-MOGAw	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	8.50e – 14	–	1.28e – 34	> 0.05	4.52e – 33	1.50e – 32	1.28e – 34
R2-MODE	1.28e – 34	8.21e – 04	1.28e – 34	2.51e – 33	1.28e – 34	1.28e – 34	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	–	1.28e – 34	1.28e – 34	1.28e – 34
MOMBI-TCH	1.28e – 34	1.02e – 32	1.28e – 34	3.15e – 34	1.28e – 34	1.28e – 34	> 0.05	> 0.05	1.44e – 34	> 0.05	–	> 0.05	1.54e – 30
MOMBI-NTCH	1.28e – 34	1.35e – 33	1.28e – 34	2.48e – 34	1.28e – 34	1.28e – 34	> 0.05	> 0.05	1.28e – 34	> 0.05	> 0.05	–	1.89e – 32
MOMBI-PBI	1.28e – 34	1.82e – 11	1.28e – 34	2.51e – 33	1.28e – 34	1.28e – 34	> 0.05	> 0.05	6.73e – 13	> 0.05	> 0.05	> 0.05	–

Table A.48: Comparison of R2 indicator values for different optimizers on the WFG1 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	9.375938e-01	1.001959e+00	9.453425e-01	1.078839e+00	9.639018e-01	8.644034e-01	8.889231e-01	8.824912e-01	8.720700e-01	6.241693e-01	9.162205e-01	8.904245e-01	1.001597e+00
	avg.	9.206228e-01	9.937047e-01	9.391134e-01	1.080756e+00	9.648576e-01	8.647069e-01	8.777192e-01	8.744976e-01	8.711804e-01	6.126170e-01	9.183190e-01	8.772307e-01	9.953322e-01
	min.	7.521156e-01	8.398816e-01	7.984555e-01	1.018667e+00	7.257898e-01	8.526200e-01	7.442464e-01	7.143702e-01	8.554416e-01	4.690618e-01	8.316638e-01	7.206107e-01	8.321220e-01
	max.	1.007941e+00	1.202229e+00	1.116065e+00	1.157798e+00	1.046478e+00	8.780405e-01	9.833188e-01	9.815880e-01	8.834155e-01	7.025908e-01	1.012680e+00	9.698903e-01	1.063071e+00
	std.	5.765479e-02	6.818270e-02	6.923065e-02	2.417012e-02	4.433476e-02	5.237674e-03	6.384676e-02	6.130270e-02	5.729420e-03	6.782729e-02	3.407373e-02	5.834151e-02	3.444263e-02
3D	med.	6.982200e-01	5.169933e-01	5.518729e-01	5.627388e-01	4.334133e-01	5.936407e-01	5.033342e-01	4.786867e-01	5.685910e-01	2.766115e-01	5.093332e-01	5.040276e-01	5.629926e-01
	avg.	6.970274e-01	5.223310e-01	5.565061e-01	5.728818e-01	4.380822e-01	5.939669e-01	5.028565e-01	4.788257e-01	5.686093e-01	2.698930e-01	5.086048e-01	5.040589e-01	5.684508e-01
	min.	6.690018e-01	4.949183e-01	5.108864e-01	5.402806e-01	3.996621e-01	5.838042e-01	4.674609e-01	4.547335e-01	5.552333e-01	2.207328e-01	4.857090e-01	4.750310e-01	5.410157e-01
	max.	7.195362e-01	5.664455e-01	6.426093e-01	6.208716e-01	4.952644e-01	6.067498e-01	5.235946e-01	5.042245e-01	5.784413e-01	3.297555e-01	5.390771e-01	5.416539e-01	6.221182e-01
	std.	9.651503e-03	1.869015e-02	2.777290e-02	2.426884e-02	2.068376e-02	4.624771e-03	1.124926e-02	1.082814e-02	4.690906e-03	2.770422e-02	8.420053e-03	1.294843e-02	1.957390e-02
4D	med.	5.724620e-01	4.376317e-01	4.810720e-01	4.906216e-01	4.794287e-01	5.065817e-01	4.238137e-01	4.158074e-01	4.555764e-01	2.562766e-01	4.472545e-01	4.483474e-01	4.940165e-01
	avg.	5.728519e-01	4.399687e-01	4.816350e-01	4.867436e-01	4.783023e-01	5.047643e-01	4.242113e-01	4.151823e-01	4.555341e-01	2.563933e-01	4.472141e-01	4.490222e-01	4.896320e-01
	min.	5.598178e-01	4.211078e-01	4.296439e-01	4.453358e-01	4.468511e-01	4.852453e-01	4.016463e-01	3.972711e-01	4.449522e-01	2.095085e-01	4.335317e-01	4.358347e-01	4.654661e-01
	max.	5.885206e-01	4.645220e-01	5.249017e-01	5.159563e-01	4.921520e-01	4.447400e-01	4.351273e-01	4.640704e-01	5.351273e-01	2.842139e-01	4.679730e-01	4.673314e-01	5.128177e-01
	std.	5.862924e-03	1.047155e-02	1.823417e-02	1.739464e-02	8.612667e-03	6.453609e-03	8.245404e-03	7.498752e-03	3.572386e-03	1.429143e-02	5.773099e-03	6.862364e-03	1.400908e-02
5D	med.	4.838814e-01	3.984498e-01	4.341223e-01	4.296596e-01	4.320895e-01	4.344741e-01	3.850626e-01	3.789867e-01	3.992118e-01	2.471499e-01	3.980086e-01	4.017353e-01	4.353347e-01
	avg.	4.839917e-01	3.963852e-01	4.372385e-01	4.315231e-01	4.353278e-01	4.345220e-01	3.845250e-01	3.779467e-01	3.987194e-01	2.469624e-01	3.977963e-01	4.005765e-01	4.336544e-01
	min.	4.756780e-01	3.731850e-01	3.886353e-01	3.796119e-01	4.139161e-01	4.228529e-01	3.704050e-01	3.622225e-01	3.930370e-01	2.189486e-01	3.789474e-01	3.855370e-01	4.125591e-01
	max.	4.944801e-01	4.107037e-01	4.804266e-01	4.448171e-01	4.460884e-01	4.553896e-01	3.960995e-01	3.938074e-01	4.041584e-01	2.791740e-01	4.127562e-01	4.115559e-01	4.477210e-01
	std.	3.910366e-03	8.203757e-03	1.526648e-02	1.214040e-02	5.266946e-03	6.521104e-03	5.591867e-03	6.767793e-03	2.687618e-03	1.035316e-02	5.574593e-03	6.201348e-03	7.407742e-03
6D	med.	4.233236e-01	3.603604e-01	3.916802e-01	3.832574e-01	3.941113e-01	4.037581e-01	3.498972e-01	3.451223e-01	3.635502e-01	2.296166e-01	3.589274e-01	3.609115e-01	3.832327e-01
	avg.	4.235692e-01	3.599972e-01	3.921685e-01	3.811652e-01	3.942029e-01	4.032862e-01	3.497361e-01	3.449779e-01	3.633452e-01	2.296179e-01	3.593665e-01	3.605236e-01	3.829094e-01
	min.	4.176624e-01	3.476292e-01	3.686936e-01	3.567733e-01	3.844010e-01	3.903264e-01	3.367266e-01	3.299984e-01	3.587754e-01	2.078198e-01	3.441051e-01	3.463685e-01	3.688100e-01
	max.	4.298978e-01	3.679061e-01	4.178690e-01	4.022724e-01	4.120204e-01	4.119709e-01	3.604340e-01	3.540438e-01	3.695593e-01	2.513301e-01	3.716666e-01	3.718527e-01	4.000674e-01
	std.	2.404757e-03	4.290277e-03	1.186533e-02	1.005237e-02	5.170983e-03	3.594660e-03	4.702508e-03	4.583443e-03	1.839039e-03	9.446576e-03	6.044783e-03	4.073582e-03	5.246441e-03
7D	med.	3.780054e-01	3.256168e-01	3.577425e-01	3.377092e-01	3.535218e-01	3.622657e-01	3.183790e-01	3.091393e-01	3.166443e-01	1.745119e-01	3.204614e-01	3.191770e-01	3.268497e-01
	avg.	3.782177e-01	3.258339e-01	3.580357e-01	3.373452e-01	3.536061e-01	3.625836e-01	3.182107e-01	3.091556e-01	3.167153e-01	1.752679e-01	3.208470e-01	3.188522e-01	3.270348e-01
	min.	3.746325e-01	3.172456e-01	3.289772e-01	3.200885e-01	3.418541e-01	3.559486e-01	3.073123e-01	2.969896e-01	3.092164e-01	1.538562e-01	3.074697e-01	3.071607e-01	2.998293e-01
	max.	3.852651e-01	3.342231e-01	3.960538e-01	3.531128e-01	3.645642e-01	3.700852e-01	3.262807e-01	3.211904e-01	3.225786e-01	2.061821e-01	3.325448e-01	3.275359e-01	3.438094e-01
	std.	1.952373e-03	3.404692e-03	1.227046e-02	6.845202e-03	4.794228e-03	3.205269e-03	3.863126e-03	4.723830e-03	2.415179e-03	1.057856e-02	5.616160e-03	4.117416e-03	7.458102e-03
8D	med.	3.426277e-01	3.005447e-01	3.265068e-01	3.087363e-01	3.320614e-01	3.341478e-01	2.921592e-01	2.892886e-01	3.072715e-01	1.885813e-01	3.021638e-01	2.957865e-01	2.922283e-01
	avg.	3.426901e-01	3.004321e-01	3.265782e-01	3.086193e-01	3.327307e-01	3.342505e-01	2.920826e-01	2.887653e-01	3.072176e-01	1.896432e-01	3.013871e-01	2.954852e-01	2.926705e-01
	min.	3.389367e-01	2.909792e-01	3.082045e-01	2.928699e-01	3.233871e-01	3.317858e-01	2.861009e-01	2.731740e-01	3.027296e-01	1.754972e-01	2.882559e-01	2.880046e-01	2.807911e-01
	max.	3.453671e-01	3.065963e-01	3.495136e-01	3.238372e-01	3.430452e-01	3.374196e-01	2.989301e-01	2.955682e-01	3.107147e-01	2.152538e-01	3.091811e-01	3.025753e-01	3.020001e-01
	std.	1.066351e-03	3.295826e-03	9.394332e-03	4.816570e-03	3.853792e-03	1.147557e-03	2.566951e-03	4.092694e-03	1.503050e-03	6.502351e-03	4.311205e-03	2.835350e-03	4.395040e-03
9D	med.	3.151238e-01	2.806155e-01	3.033090e-01	2.872548e-01	3.114492e-01	3.086037e-01	2.717964e-01	2.700896e-01	2.922182e-01	2.088334e-01	2.839842e-01	2.765998e-01	2.728724e-01
	avg.	3.150826e-01	2.806278e-01	3.028522e-01	2.870639e-01	3.115670e-01	3.085650e-01	2.716991e-01	2.699367e-01	2.921204e-01	2.083879e-01	2.836583e-01	2.766219e-01	2.727982e-01
	min.	3.127294e-01	2.726813e-01	2.873707e-01	2.801852e-01	3.042988e-01	3.063019e-01	2.662458e-01	2.629433e-01	2.890872e-01	1.988335e-01	2.745953e-01	2.650028e-01	2.668476e-01
	max.	3.166645e-01	2.872720e-01	3.202524e-01	2.953008e-01	3.240042e-01	3.105977e-01	2.756843e-01	2.757335e-01	2.936406e-01	2.193079e-01	2.919473e-01	2.817376e-01	2.783510e-01
	std.	8.038419e-04	2.435131e-03	8.424958e-03	3.413711e-03	3.155836e-03	9.163021e-04	1.750431e-03	2.463660e-03	8.564671e-04	4.369115e-03	3.021166e-03	2.392057e-03	2.696270e-03
10D	med.	2.927267e-01	2.629897e-01	2.811774e-01	2.689725e-01	2.943448e-01	2.878844e-01	2.551280e-01	2.549565e-01	2.771826e-01	2.241873e-01	2.682160e-01	2.609384e-01	2.585667e-01
	avg.	2.927392e-01	2.629611e-01	2.811278e-01	2.685815e-01	2.943909e-01	2.877599e-01	2.551739e-01	2.549573e-01	2.771859e-01	2.237378e-01	2.681722e-01	2.608158e-01	2.584682e-01
	min.	2.914873e-01	2.577833e-01	2.705560e-01	2.629741e-01	2.874344e-01	2.839849e-01	2.521911e-01	2.490161e-01	2.753129e-01	2.147344e-01	2.599037e-01	2.523458e-01	2.547129e-01
	max.	2.942794e-01	2.660248e-01	2.964057e-01	2.786217e-01	3.021415e-01	2.891751e-01	2.584811e-01	2.605620e-01	2.784896e-01	2.300981e-01	2.734614e-01	2.649496e-01	2.639297e-01
	std.	5.658038e-04	2.023607e-03	5.441397e-03	2.481917e-03	2.898527e-03	7.683005e-04	1.274217e-03	1.861964e-03	5.643912e-04	3.125840e-03	2.193019e-03	1.855404e-03	1.780405e-03



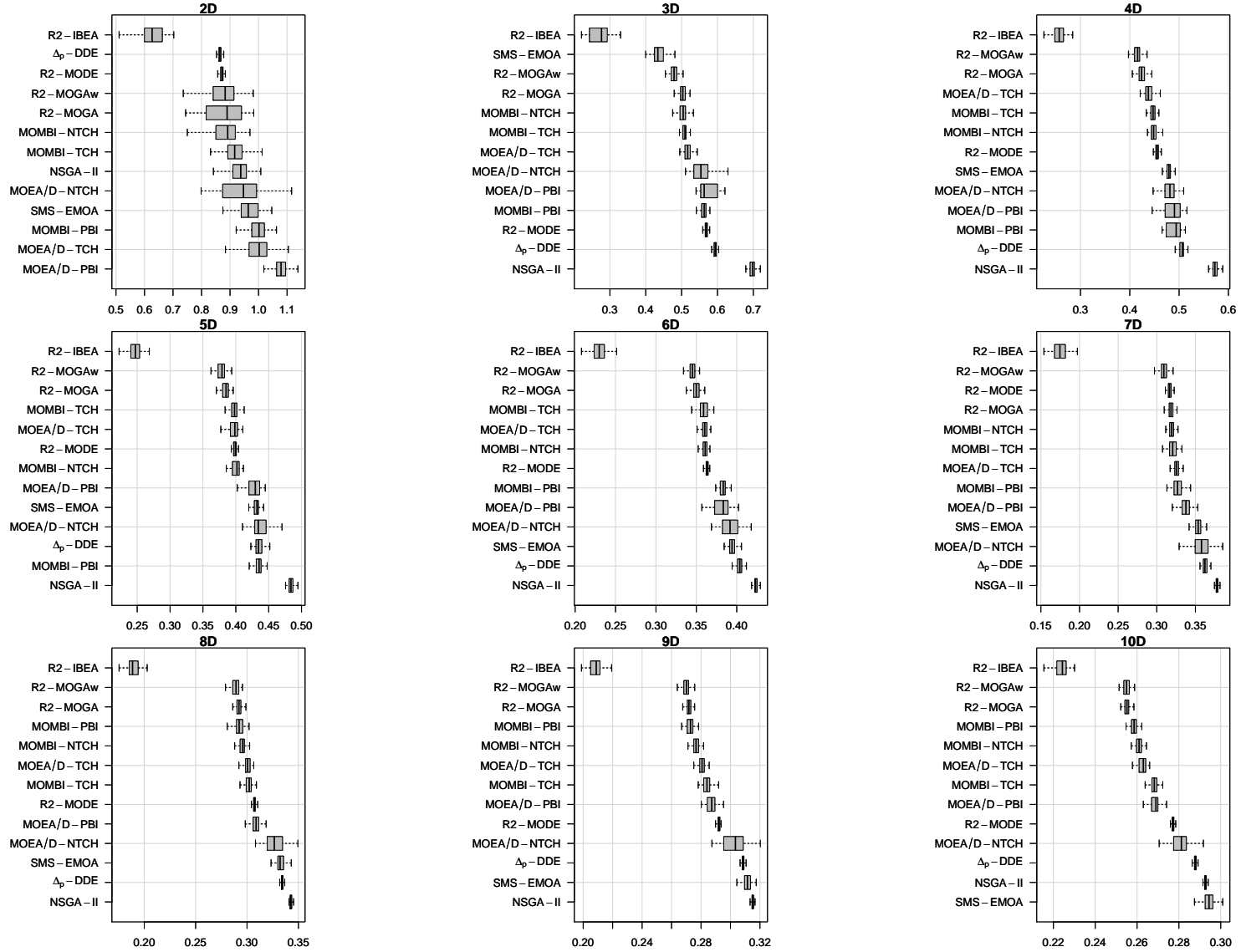


Figure A.114: Box-plot of R2 indicator values for different optimizers on the WFG1 test problem.

Table A.49: Comparison of runtime (in milliseconds) for different optimizers on the WFG1 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	4.599040e+02	7.978240e+02	1.369769e+03	1.327181e+03	1.650294e+04	1.775099e+03	3.514412e+03	2.827207e+03	2.126567e+03	5.503980e+05	2.250321e+03	2.250589e+03	3.222546e+03
	avg.	4.739781e+02	8.265625e+02	1.378408e+03	1.315896e+03	1.649517e+04	1.779253e+03	3.488344e+03	2.836140e+03	2.114776e+03	5.501848e+05	2.221327e+03	2.263188e+03	3.207090e+03
	min.	4.187740e+02	6.305080e+02	1.004605e+03	1.010038e+03	1.235569e+04	1.504107e+03	2.530867e+03	2.087164e+03	1.327770e+03	5.390870e+05	1.740869e+03	1.889415e+03	2.769560e+03
	max.	6.061510e+02	1.260769e+03	1.912506e+03	1.766484e+03	2.045478e+04	2.147919e+03	4.433733e+03	3.758985e+03	2.685267e+03	5.650890e+05	2.572196e+03	2.661327e+03	3.494649e+03
	std.	4.581459e+01	1.099977e+02	1.307240e+02	1.262288e+02	1.533464e+03	1.338528e+02	3.669880e+02	3.305442e+02	2.829088e+02	5.081207e+03	1.883581e+02	1.744874e+02	1.543343e+02
3D	med.	6.076170e+02	9.520120e+02	1.763442e+03	1.444471e+03	6.644996e+05	1.360474e+03	3.541519e+03	3.037424e+03	3.773168e+03	6.039000e+05	2.240893e+03	2.336409e+03	3.127573e+03
	avg.	6.209734e+02	9.592217e+02	1.749623e+03	1.425161e+03	6.594024e+05	1.359696e+03	3.566020e+03	3.093229e+03	3.710255e+03	5.916937e+05	2.248305e+03	2.333371e+03	3.103430e+03
	min.	5.318820e+02	7.914550e+02	1.378975e+03	1.100909e+03	4.656447e+05	1.034859e+03	2.752029e+03	2.440617e+03	2.590512e+03	4.834120e+05	1.812255e+03	1.977718e+03	2.735969e+03
	max.	7.926200e+02	1.240851e+03	2.030553e+03	1.703123e+03	7.961154e+05	1.760361e+03	4.291325e+03	3.873548e+03	4.153800e+03	6.431300e+05	2.629235e+03	2.778177e+03	3.327700e+03
	std.	6.025720e+01	9.160318e+01	1.161973e+02	1.051987e+02	6.295862e+04	1.571413e+02	3.220821e+02	3.241165e+02	3.096344e+02	4.283415e+04	1.719322e+02	1.618298e+02	1.113316e+02
4D	med.	9.498320e+02	1.285851e+03	2.039500e+03	1.767586e+03	3.429458e+06	1.230598e+03	4.624097e+03	4.019422e+03	4.633445e+03	5.957620e+05	2.462817e+03	2.470719e+03	3.664957e+03
	avg.	9.525834e+02	1.295106e+03	2.028154e+03	1.759789e+03	3.390848e+06	1.211811e+03	4.664628e+03	4.034988e+03	4.657835e+03	5.959166e+05	2.467582e+03	2.488592e+03	3.670486e+03
	min.	8.048870e+02	1.065013e+03	1.553539e+03	1.413909e+03	2.329436e+06	9.309440e+02	3.636943e+03	3.069285e+03	3.484192e+03	5.649310e+05	2.065222e+03	2.143500e+03	3.264199e+03
	max.	1.163868e+03	1.734268e+03	2.455969e+03	2.069860e+03	3.995100e+06	1.553362e+03	5.467875e+03	4.820219e+03	5.401554e+03	6.078050e+05	3.600358e+03	3.401949e+03	4.101949e+03
	std.	7.859461e+01	1.167252e+02	1.705322e+02	1.163566e+02	3.419715e+05	1.302088e+02	3.757812e+02	3.690568e+02	4.264976e+02	5.797291e+03	2.573669e+02	1.712435e+02	1.872381e+02
5D	med.	1.253192e+03	1.530218e+03	2.713814e+03	2.134973e+03	4.389455e+06	1.701240e+03	5.972455e+03	5.258757e+03	6.322868e+03	6.806940e+05	3.072684e+03	3.804125e+03	4.333667e+03
	avg.	1.257725e+03	1.551704e+03	2.659519e+03	2.081369e+03	4.255093e+06	1.680470e+03	6.041554e+03	5.325780e+03	6.261485e+03	6.590060e+05	3.175102e+03	3.824922e+03	4.307610e+03
	min.	1.046702e+03	1.297255e+03	2.014588e+03	1.625991e+03	3.002198e+06	1.236133e+03	4.934268e+03	4.279735e+03	4.925179e+03	5.876310e+05	2.431411e+03	2.555845e+03	3.723028e+03
	max.	1.474207e+03	2.319348e+03	3.157320e+03	2.524631e+03	5.053304e+06	2.037231e+03	6.991850e+03	6.321680e+03	7.093521e+03	6.951640e+05	5.933559e+03	3.299623e+03	4.651284e+03
	std.	9.800947e+01	1.511366e+02	2.307795e+02	1.865604e+02	4.497210e+05	1.489311e+02	4.620817e+02	4.185824e+02	4.409800e+02	3.358915e+04	5.660226e+02	1.545865e+02	1.571741e+02
6D	med.	1.536440e+03	1.839516e+03	3.035129e+03	2.230481e+03	4.165352e+06	1.941140e+03	6.977057e+03	6.330150e+03	6.920349e+03	7.280980e+05	3.076965e+03	3.140848e+03	4.825427e+03
	avg.	1.555396e+03	1.825059e+03	2.972861e+03	2.280829e+03	4.151630e+06	1.920996e+03	6.959557e+03	6.408783e+03	6.987390e+03	7.278189e+05	3.114626e+03	3.182022e+03	4.802029e+03
	min.	1.304924e+03	1.472856e+03	2.332659e+03	1.803894e+03	2.976527e+06	1.521238e+03	5.121685e+03	5.365825e+03	5.428440e+03	6.601850e+05	2.602175e+03	2.769607e+03	3.860246e+03
	max.	1.904859e+03	2.169081e+03	3.443573e+03	2.854826e+03	4.781250e+06	2.437058e+03	7.989170e+03	8.003711e+03	8.173662e+03	7.444390e+05	4.640308e+03	3.668181e+03	5.339691e+03
	std.	1.088754e+02	1.387797e+02	2.056680e+02	2.571295e+02	3.385643e+05	1.857568e+02	6.437981e+02	5.132212e+02	6.233922e+02	8.505124e+03	3.040182e+02	1.952434e+02	3.068488e+02
7D	med.	1.646457e+03	1.967075e+03	3.269459e+03	2.442194e+03	3.122449e+06	2.575790e+03	5.545903e+03	5.046137e+03	5.408164e+03	3.356720e+05	2.288448e+03	2.601552e+03	3.903844e+03
	avg.	1.646102e+03	1.972271e+03	3.223642e+03	2.482175e+03	3.054187e+06	2.565317e+03	5.547479e+03	5.050387e+03	5.375008e+03	3.346271e+05	2.318627e+03	2.642777e+03	3.906695e+03
	min.	1.359092e+03	1.548294e+03	2.346531e+03	1.966785e+03	1.965700e+06	1.706115e+03	4.454019e+03	4.037244e+03	4.205779e+03	2.770290e+05	1.972669e+03	2.362066e+03	3.339931e+03
	max.	1.982229e+03	2.337269e+03	3.861746e+03	3.252408e+03	3.954182e+06	3.292697e+03	6.326435e+03	6.707945e+03	6.064690e+03	3.427840e+05	2.815533e+03	3.146068e+03	4.325155e+03
	std.	1.231829e+02	1.630277e+02	2.388573e+02	2.849395e+02	3.500658e+05	2.903546e+02	4.184391e+02	4.310051e+02	4.381122e+02	8.024905e+03	1.777084e+02	1.796061e+02	1.808474e+02
8D	med.	2.167185e+03	2.193240e+03	3.704736e+03	2.981593e+03	3.802408e+06	3.848306e+03	9.183757e+03	7.737465e+03	9.474989e+03	1.378983e+06	3.410835e+03	3.924526e+03	5.595774e+03
	avg.	2.160264e+03	2.208894e+03	3.623606e+03	2.917505e+03	3.719679e+06	3.821784e+03	9.111259e+03	7.772869e+03	9.473871e+03	1.295356e+06	3.404087e+03	3.947010e+03	5.580078e+03
	min.	1.780232e+03	1.822037e+03	2.970917e+03	2.221145e+03	2.643867e+06	2.983892e+03	6.857448e+03	5.929261e+03	7.241233e+03	1.116745e+06	2.729734e+03	3.338571e+03	4.692790e+03
	max.	2.456592e+03	2.677876e+03	4.112148e+03	3.326631e+03	4.601654e+06	4.537304e+03	9.926848e+03	9.620658e+03	1.069685e+04	1.524315e+06	3.781260e+03	4.419158e+03	6.223474e+03
	std.	1.235362e+02	1.687387e+02	2.455968e+02	2.563985e+02	3.955045e+05	3.112879e+02	5.202473e+02	8.120775e+02	7.847562e+02	1.224899e+05	2.089795e+02	2.120098e+02	2.632207e+02
9D	med.	2.827020e+03	2.489225e+03	4.353911e+03	2.770541e+03	4.903879e+06	4.853571e+03	1.145640e+04	1.252190e+04	1.503425e+04	2.633324e+06	4.453848e+03	5.434557e+03	7.228155e+03
	avg.	2.796308e+03	2.494103e+03	4.272091e+03	2.848176e+03	4.897896e+06	4.887009e+03	1.134691e+04	1.242638e+04	1.470835e+04	2.632668e+06	4.469034e+03	5.441662e+03	7.240362e+03
	min.	2.339061e+03	2.100650e+03	3.203504e+03	2.364661e+03	3.261805e+06	3.207692e+03	8.963909e+03	9.048510e+03	1.162026e+04	2.561492e+06	3.759675e+03	4.638053e+03	6.124840e+03
	max.	3.145131e+03	2.930809e+03	4.647811e+03	3.528123e+03	7.123193e+06	6.929979e+03	1.503330e+04	1.518233e+04	1.629308e+04	2.655306e+06	5.100257e+03	5.999190e+03	7.794497e+03
	std.	1.642360e+02	1.825862e+02	2.644259e+02	2.836962e+02	7.437062e+05	8.024306e+02	1.896495e+03	1.054466e+03	1.091330e+03	1.038471e+04	2.844822e+02	2.989491e+02	3.858692e+02
10D	med.	3.474011e+03	2.745876e+03	4.760842e+03	3.080881e+03	6.443736e+06	5.120893e+03	1.671761e+04	1.878578e+04	1.532721e+04	4.972440e+06	5.170058e+03	6.331487e+03	9.729734e+03
	avg.	3.447380e+03	2.737106e+03	4.674965e+03	3.094739e+03	6.485918e+06	5.146723e+03	1.728806e+04	1.846988e+04	1.491937e+04	4.919373e+06	5.268072e+03	6.463623e+03	9.807793e+03
	min.	2.802310e+03	2.306067e+03	3.727558e+03	2.412886e+03	4.652599e+06	4.331631e+03	1.426415e+04	1.413369e+04	1.288849e+04	4.386301e+06	4.495852e+03	5.716639e+03	8.541828e+03
	max.	3.851660e+03	3.086076e+03	5.094434e+03	3.647505e+03	1.059404e+07	6.403836e+03	2.339781e+04	2.178000e+04	2.604795e+04	5.023077e+06	6.365499e+03	8.035940e+03	1.178021e+04
	std.	2.285265e+02	1.845542e+02	2.776199e+02	3.046095e+02	1.131184e+06	4.192698e+02	2.633963e+03	1.234027e+03	5.244051e+03	9.877083e+04	5.031954e+02	7.706463e+02	8.014541e+02

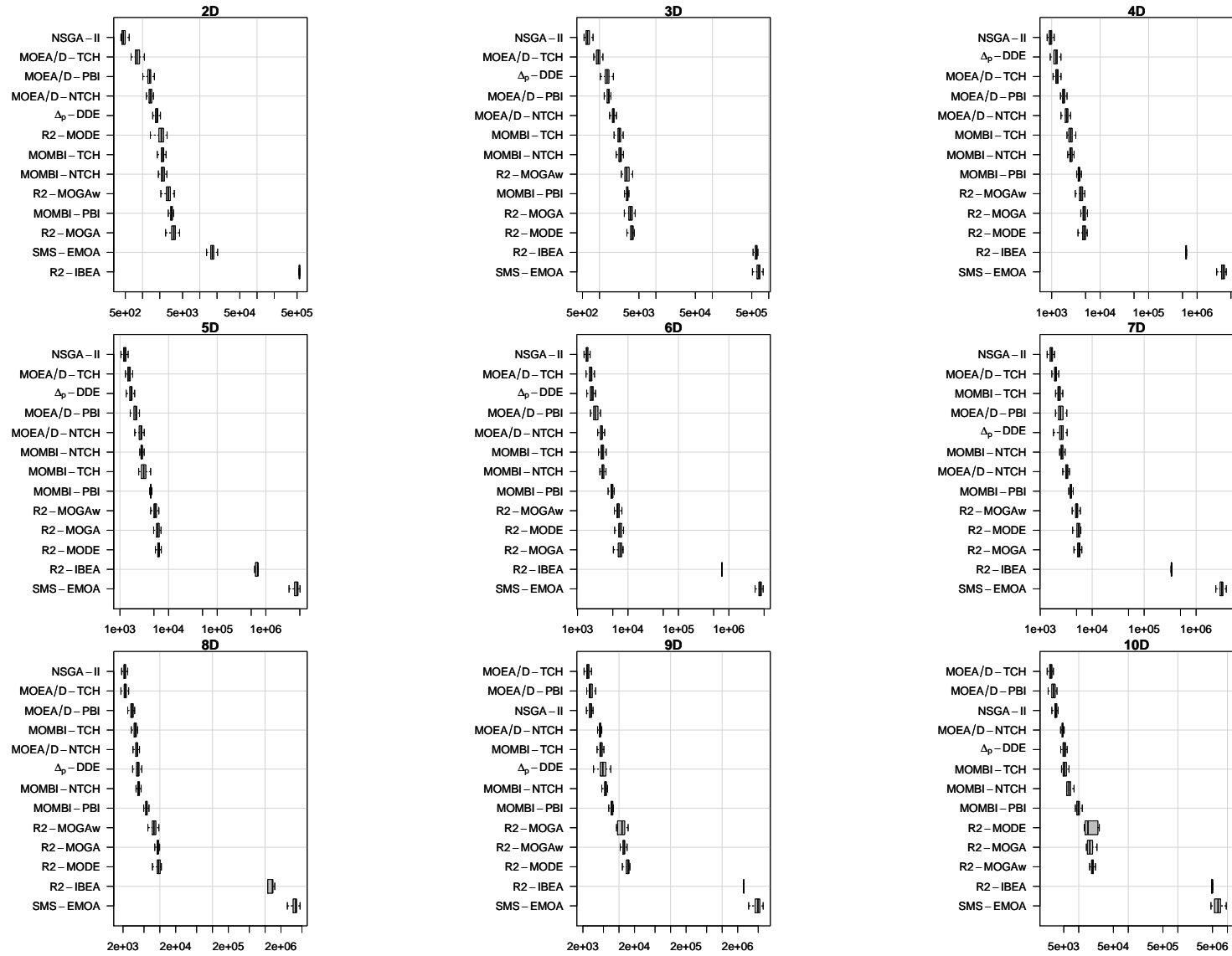


Figure A.115: Box-plot of runtime (in logarithmic scale) for different optimizers on the WFG1 test problem.

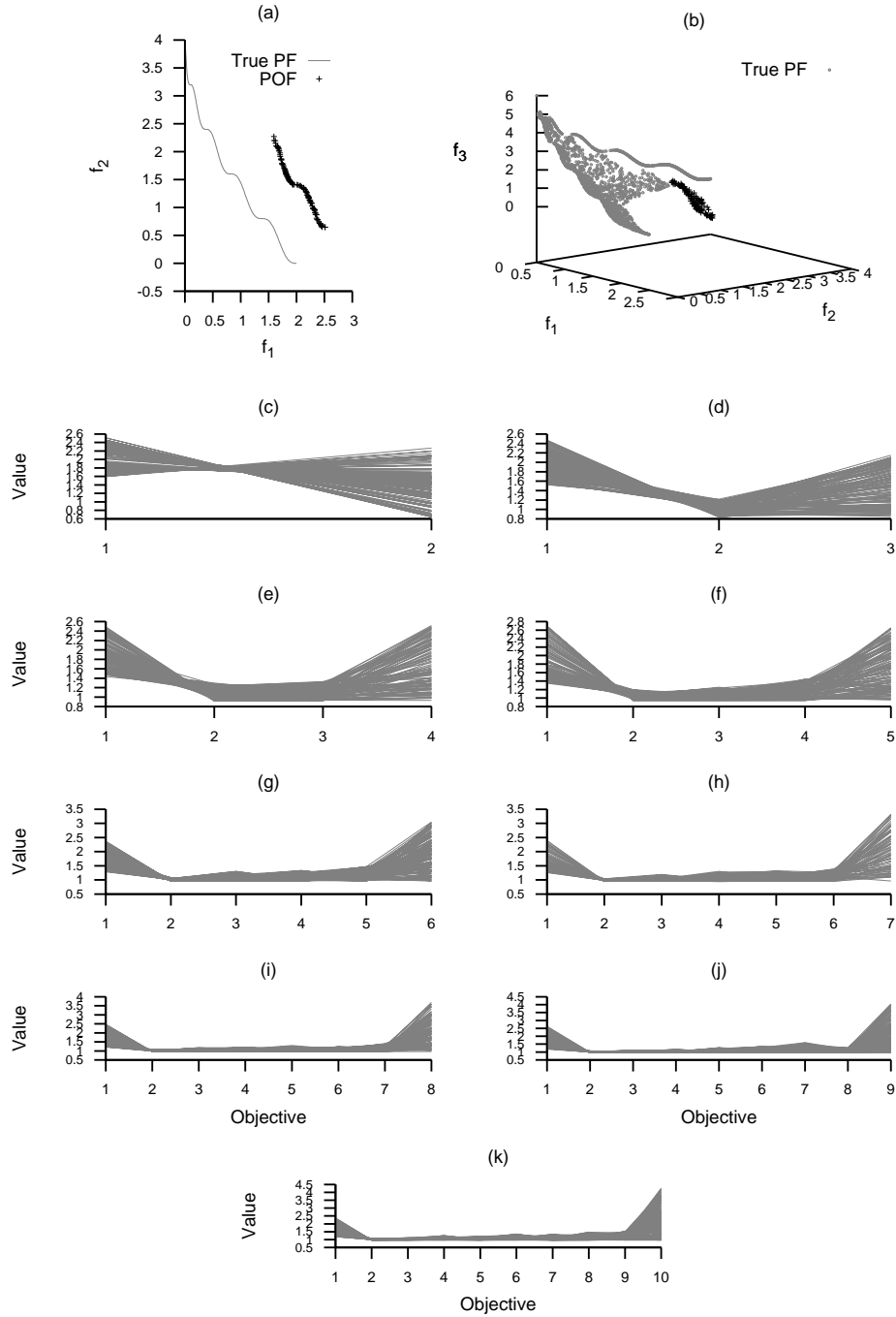


Figure A.116: Plots of the approximations obtained by NSGA-II from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

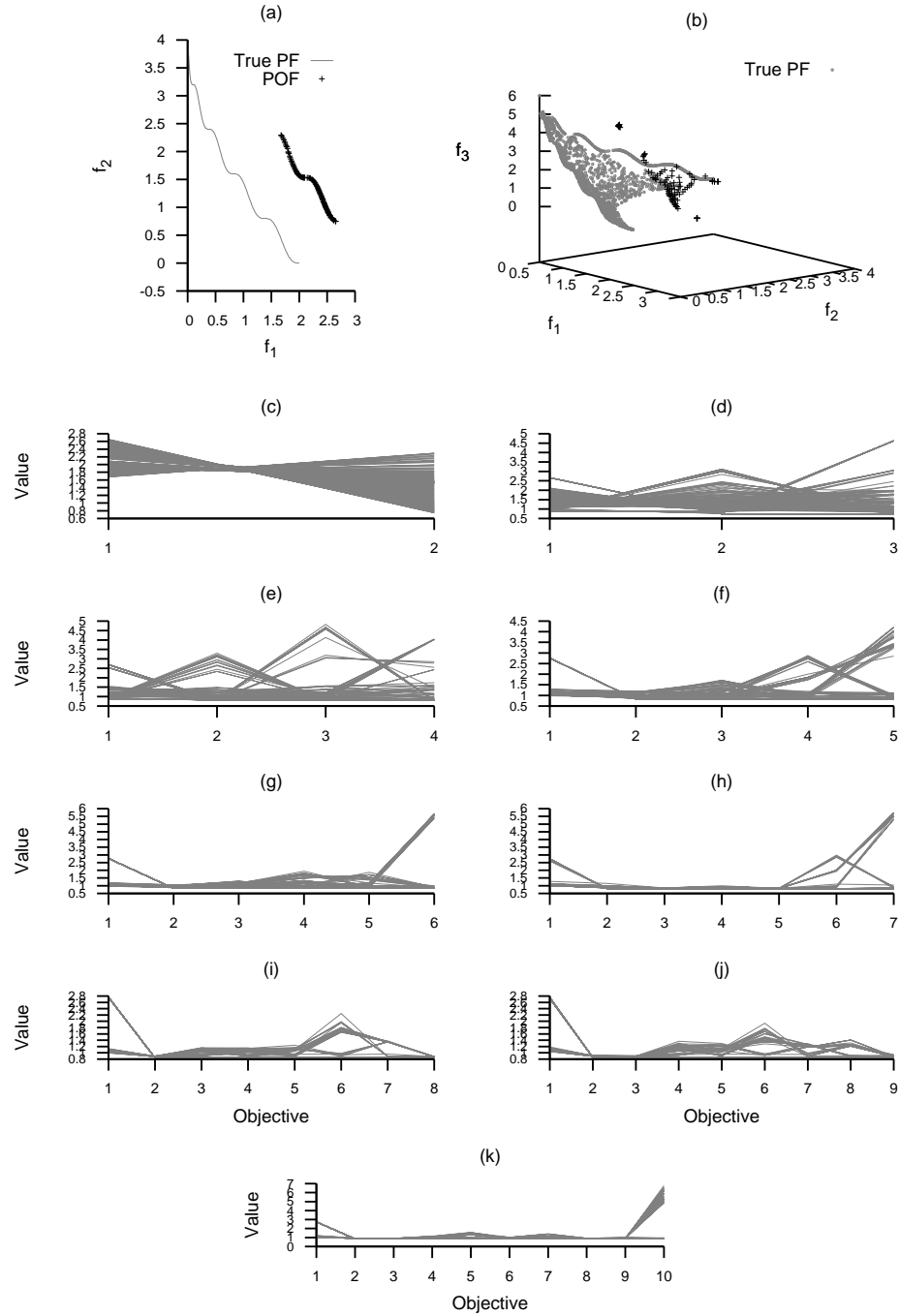


Figure A.117: Plots of the approximations obtained by MOEA/D-TCH from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

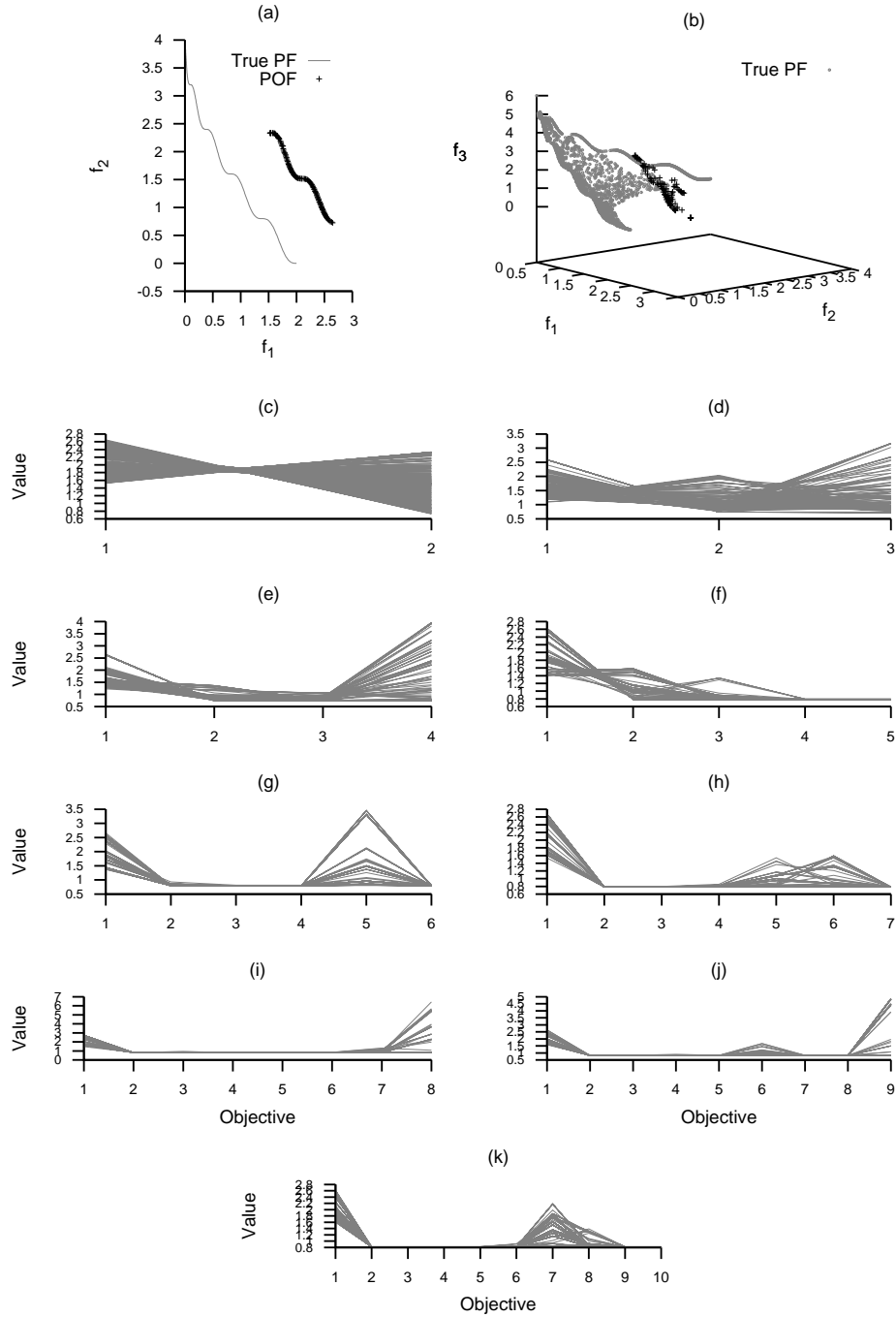


Figure A.118: Plots of the approximations obtained by MOEA/D-NTCH from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

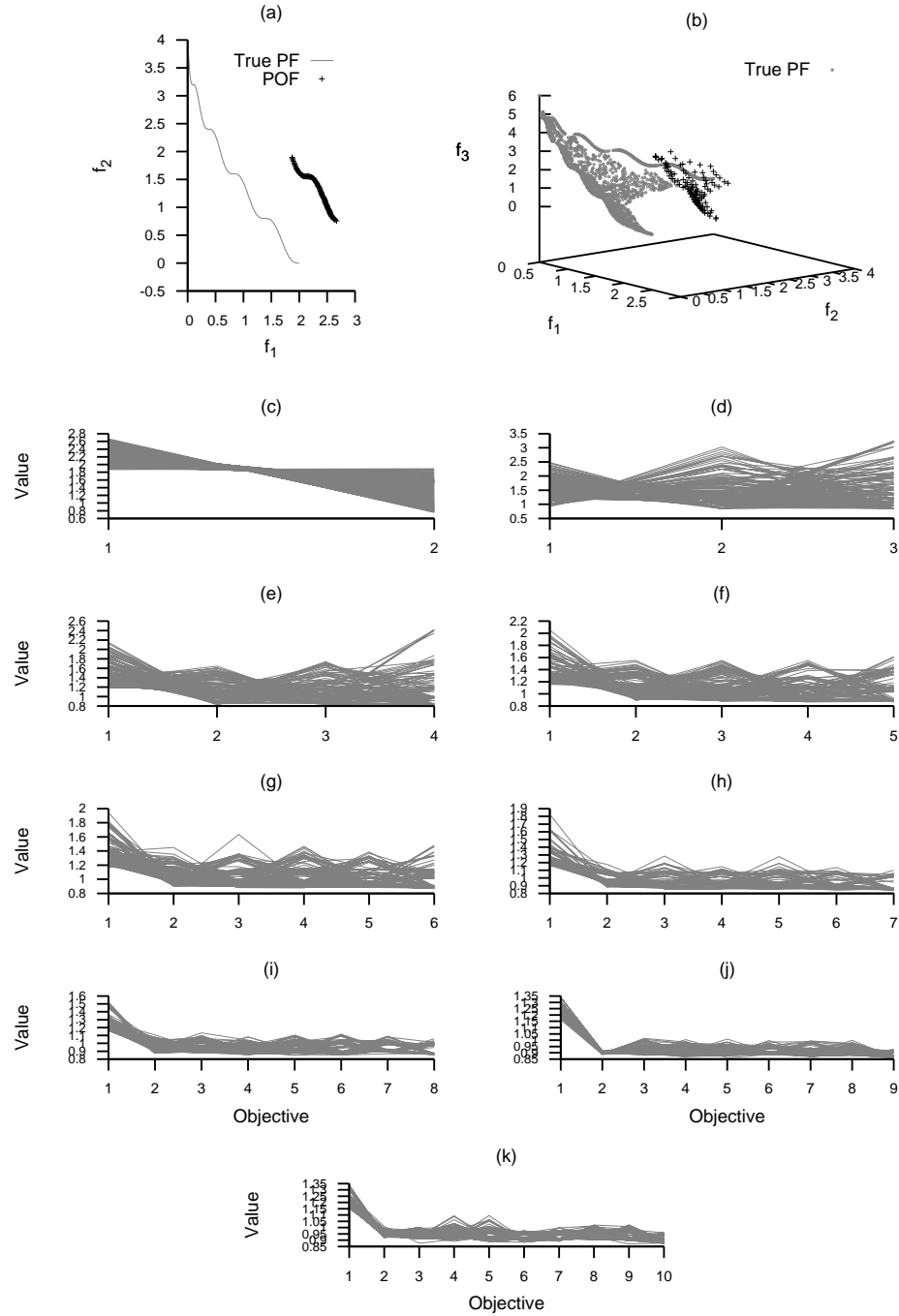


Figure A.119: Plots of the approximations obtained by MOEA/D-PBI from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

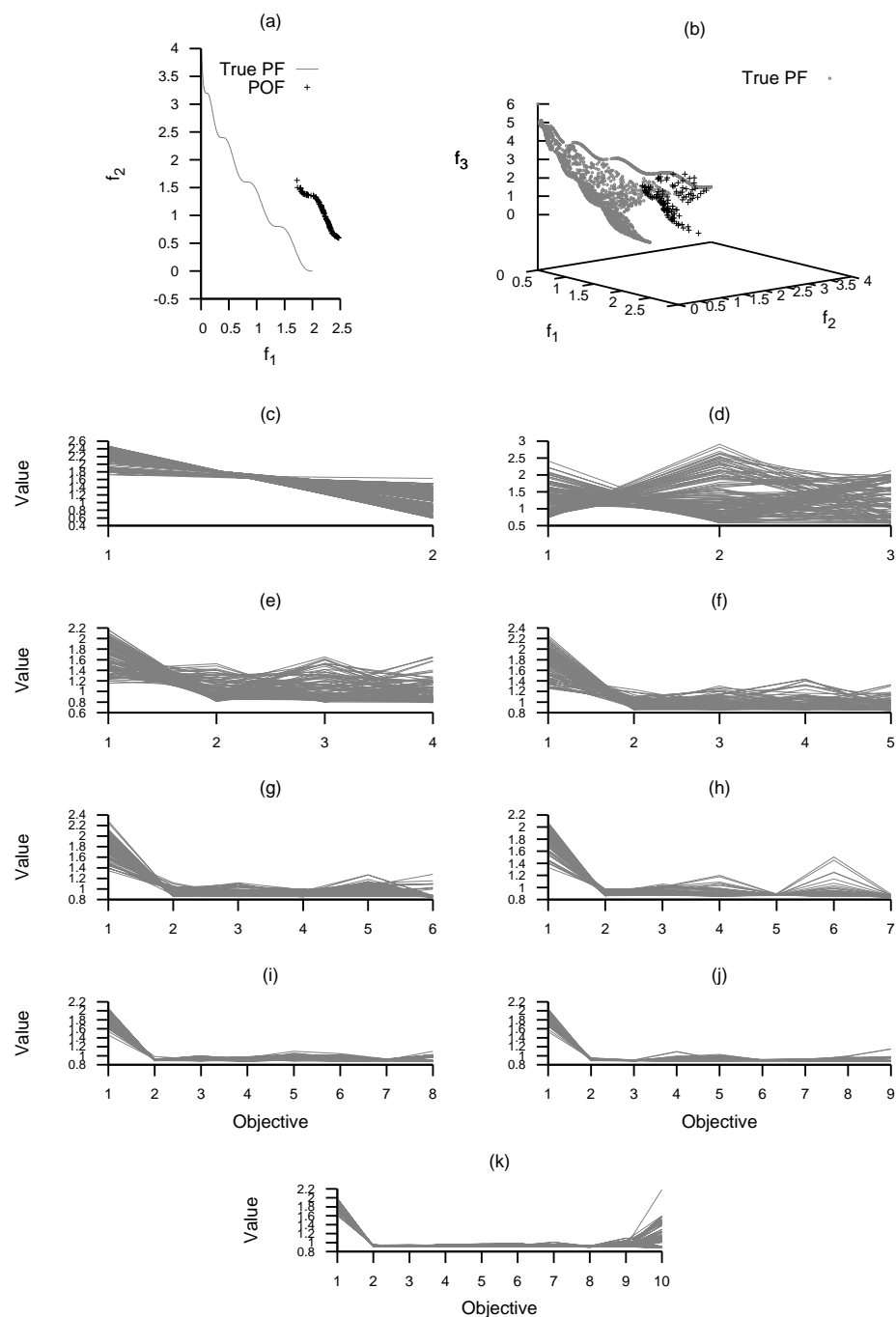


Figure A.120: Plots of the approximations obtained by SMS-EMOA from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



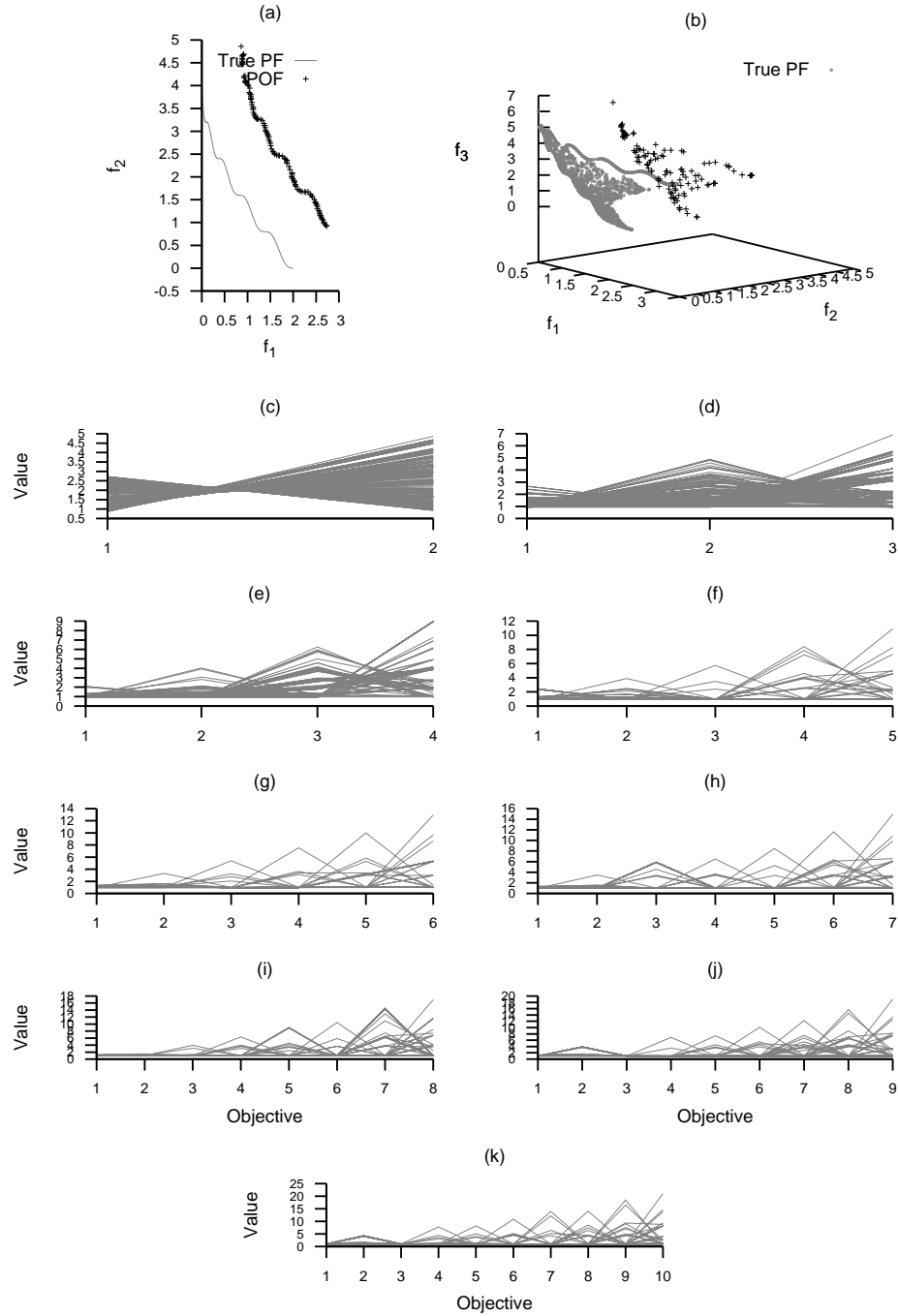


Figure A.121: Plots of the approximations obtained by  $\Delta_p$ -DDE from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

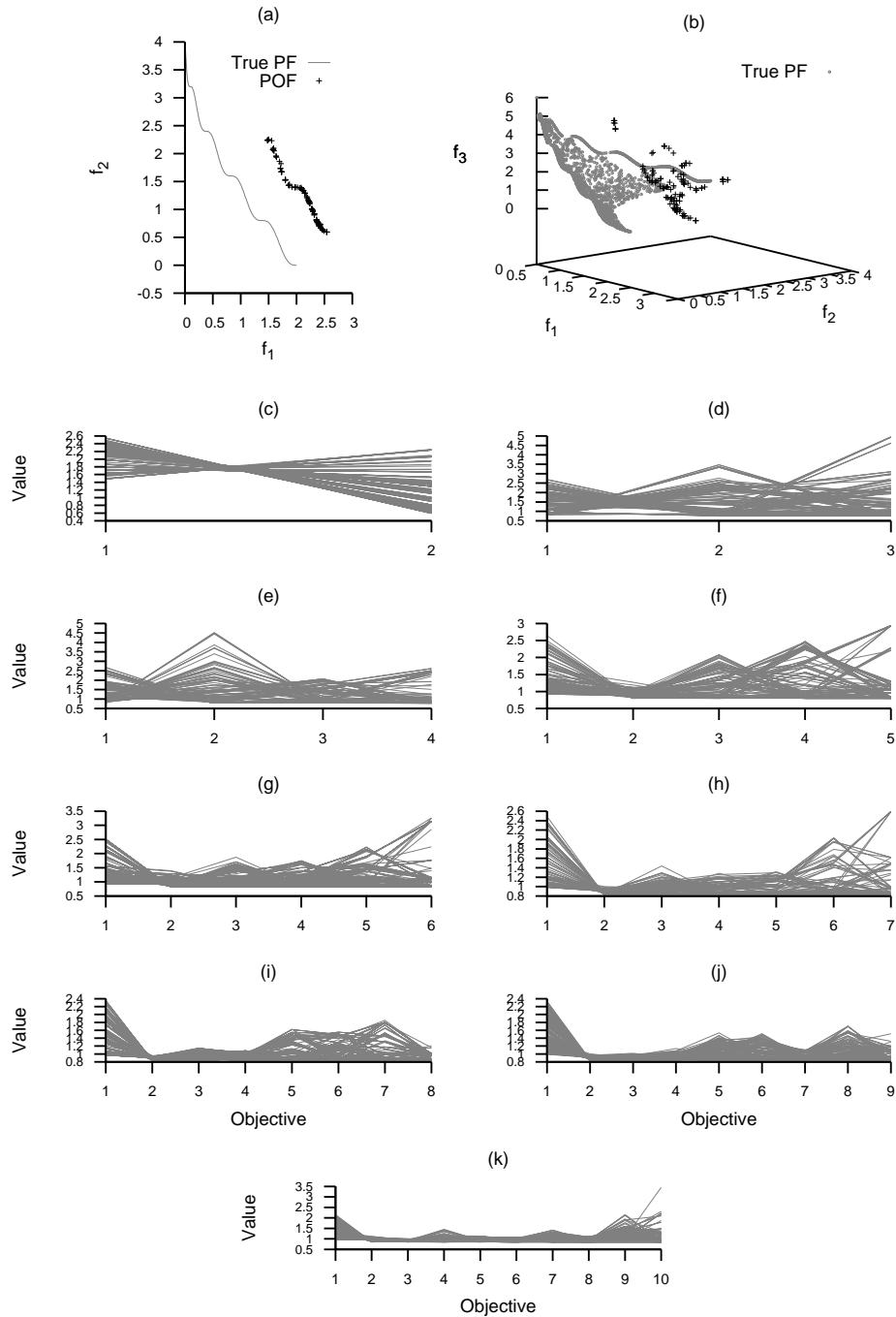


Figure A.122: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

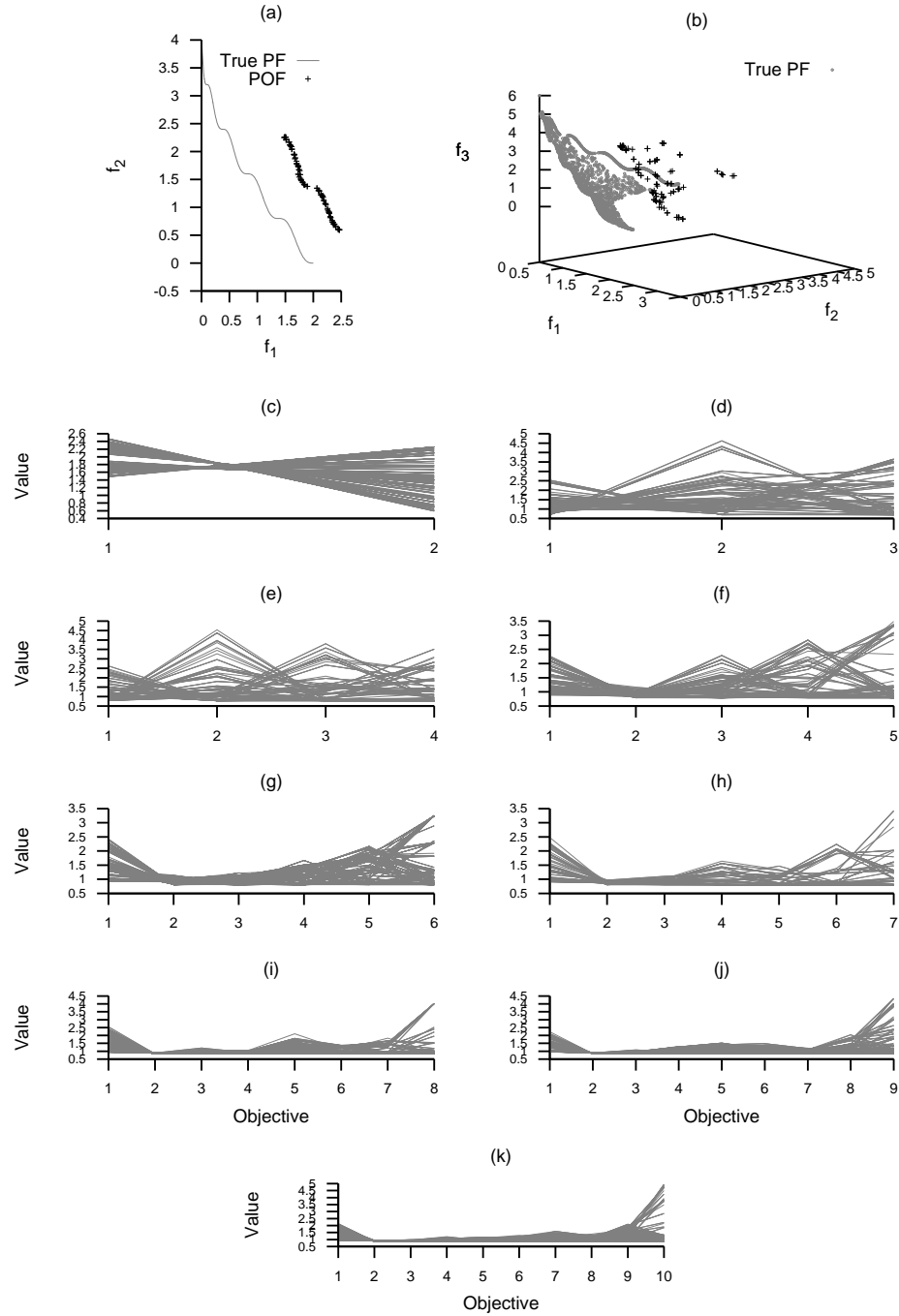


Figure A.123: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

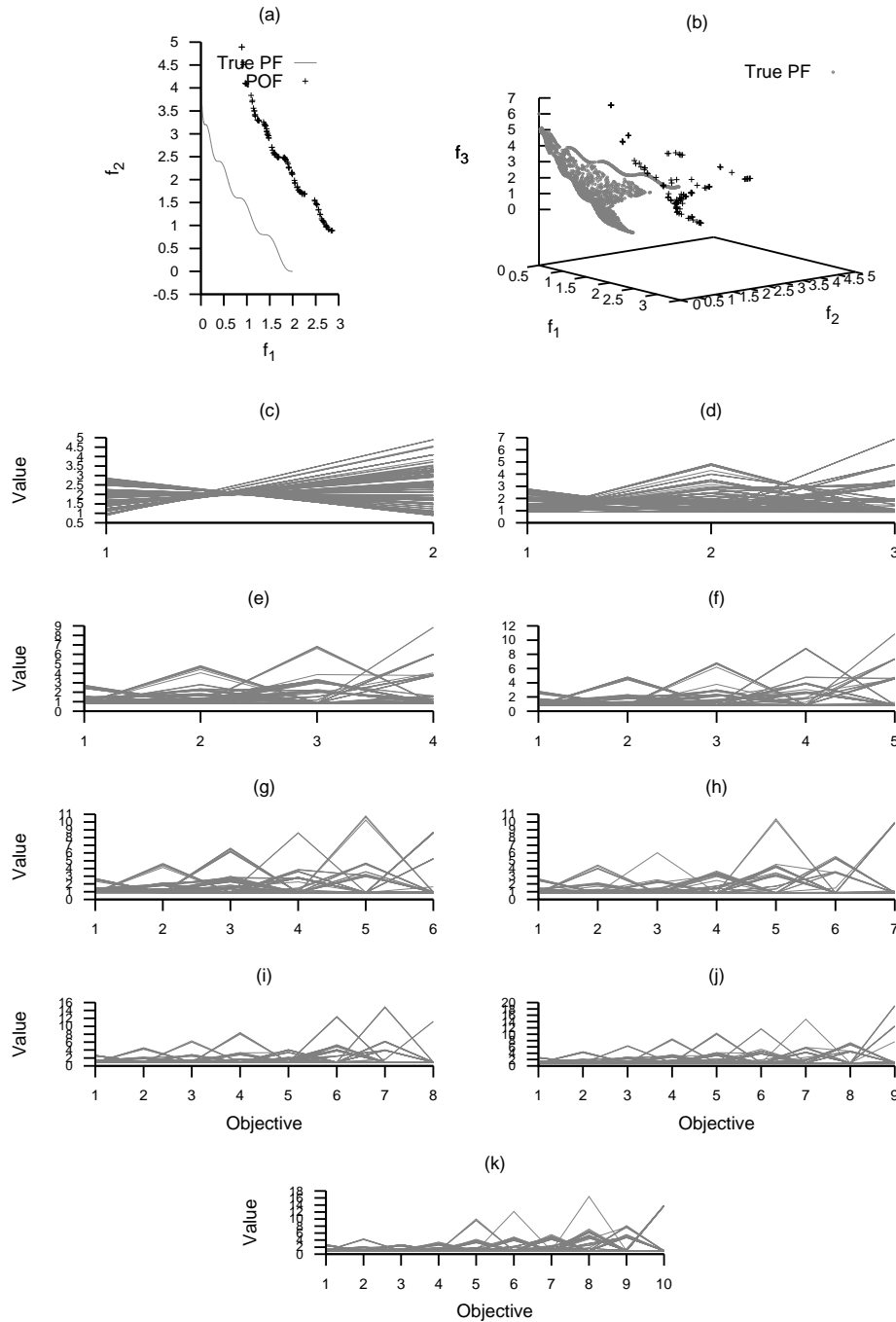


Figure A.124: Plots of the approximations obtained by *R2-MODE* from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

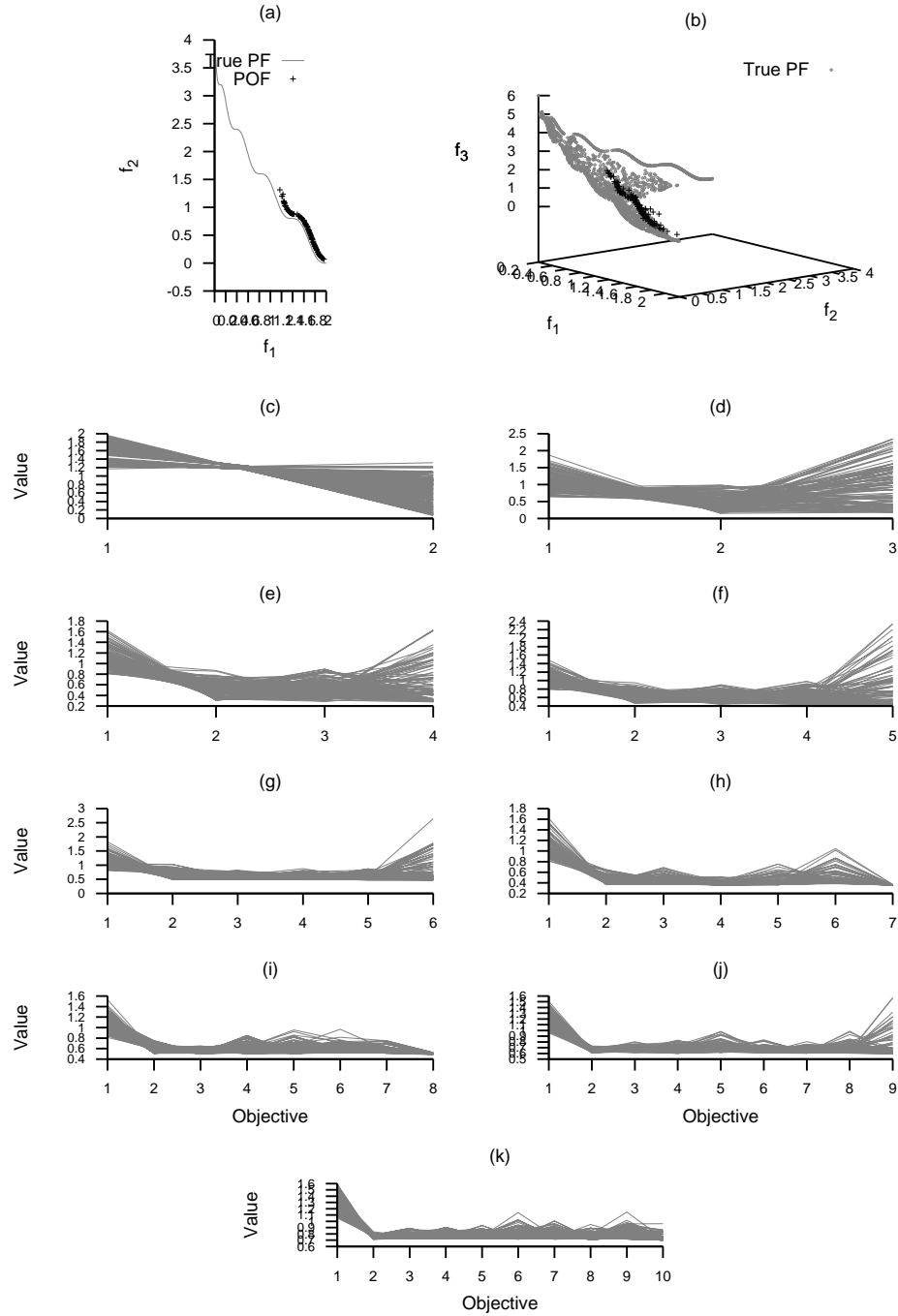


Figure A.125: Plots of the approximations obtained by *R2*-IBEA from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

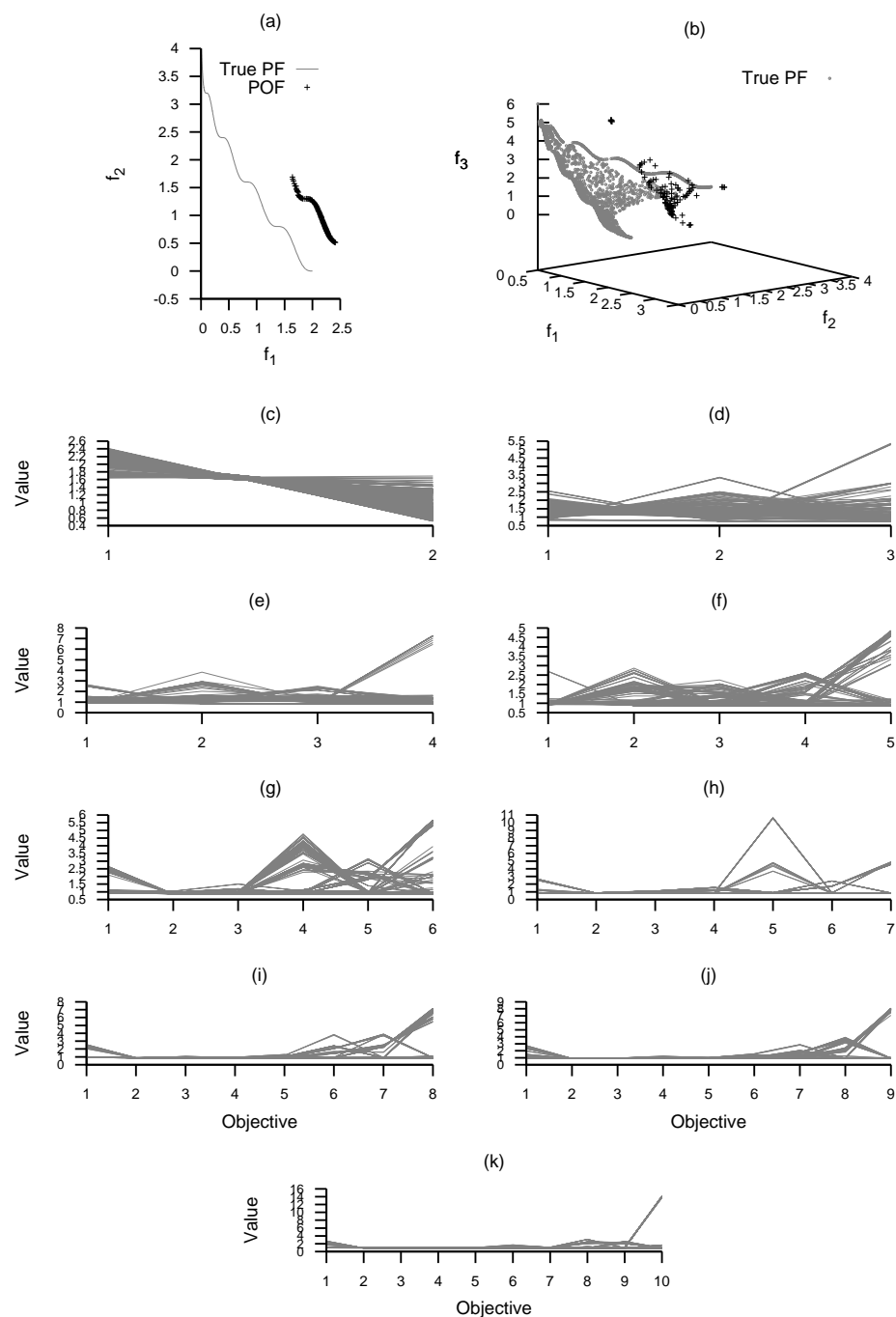


Figure A.126: Plots of the approximations obtained by MOMBI-TCH from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

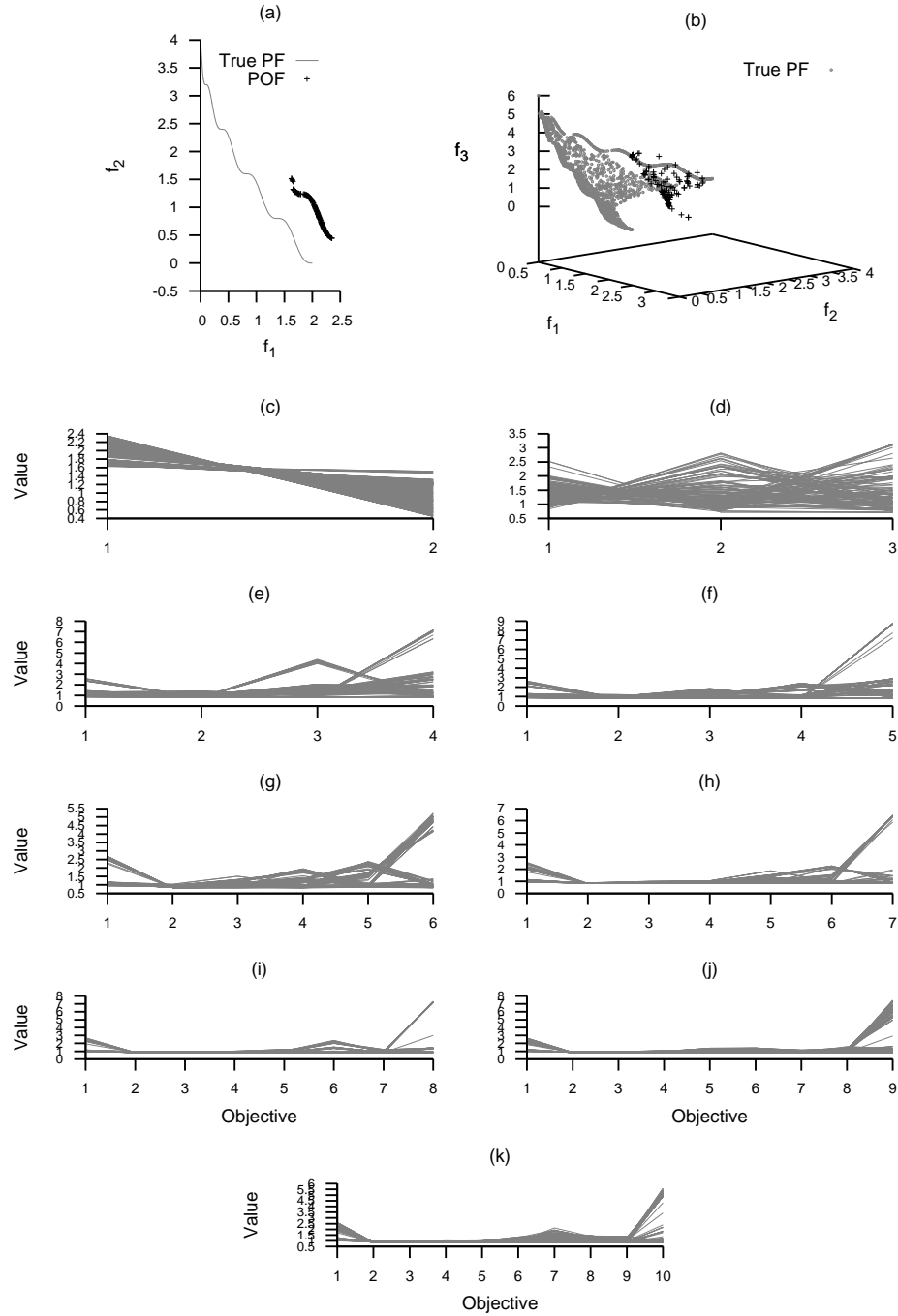


Figure A.127: Plots of the approximations obtained by MOMBI-NTCH from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

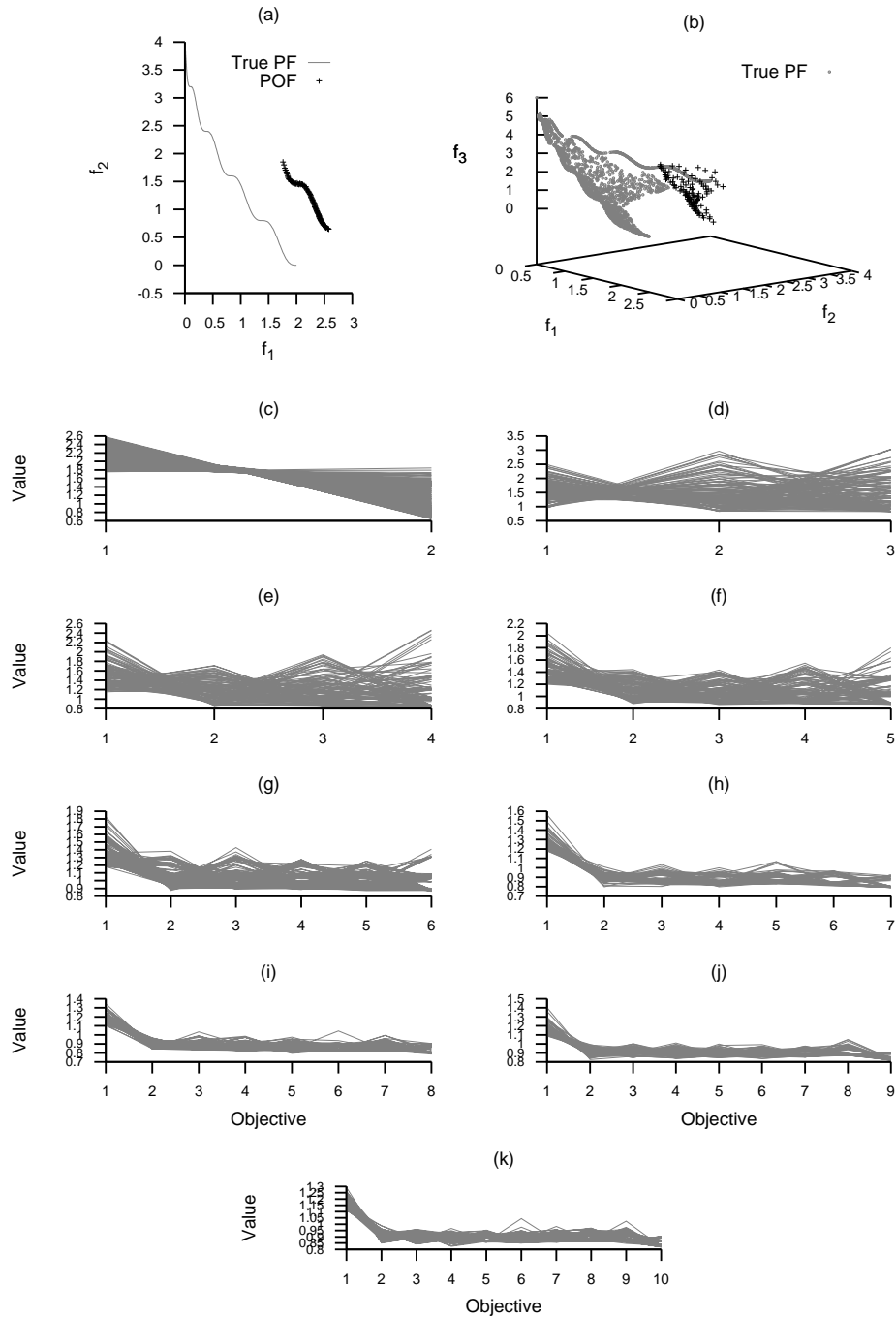


Figure A.128: Plots of the approximations obtained by MOMBI-PBI from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



Table A.50: Comparison of hypervolume indicator values for different optimizers on the WFG2 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_P$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	1.056834e+01	1.031620e+01	9.254194e+00	8.562011e+00	1.059374e+01	1.110396e+01	1.049714e+01	1.053975e+01	1.133364e+01	1.059186e+01	1.057496e+01	1.058443e+01	1.037753e+01
	avg.	1.059733e+01	9.879543e+00	9.822272e+00	9.038818e+00	1.058693e+01	1.110198e+01	1.048741e+01	1.053604e+01	1.132532e+01	1.058504e+01	1.055085e+01	1.062808e+01	1.028095e+01
	min.	1.049669e+01	8.973490e+00	8.802401e+00	7.919902e+00	1.038937e+01	1.103344e+01	9.186385e+00	1.040859e+01	1.122601e+01	1.046523e+01	9.241406e+00	1.051236e+01	9.796329e+00
	max.	1.137921e+01	1.056364e+01	1.059085e+01	1.035798e+01	1.061746e+01	1.117534e+01	1.130595e+01	1.129823e+01	1.136480e+01	1.060982e+01	1.137410e+01	1.143464e+01	1.121664e+01
	std.	1.555191e-01	6.595801e-01	6.758152e-01	7.352634e-01	2.935089e-02	2.935188e-02	1.569239e-01	8.551981e-02	2.838323e-02	2.477990e-02	2.047586e-01	1.974730e-01	2.523929e-01
3D	med.	9.676038e+01	8.348310e+01	7.438178e+01	7.930761e+01	1.007076e+02	9.724574e+01	9.773399e+01	9.848679e+01	9.807757e+01	9.976488e+01	9.726926e+01	9.861582e+01	9.680853e+01
	avg.	9.294482e+01	8.530760e+01	7.564650e+01	8.065022e+01	9.358952e+01	9.725256e+01	9.220152e+01	9.414204e+01	9.806217e+01	9.400347e+01	9.262112e+01	9.334010e+01	9.211069e+01
	min.	8.202849e+01	6.649721e+01	6.455183e+01	6.426446e+01	8.499928e+01	9.632287e+01	8.252900e+01	8.158626e+01	9.694848e+01	8.401966e+01	8.279672e+01	8.411528e+01	8.171586e+01
	max.	9.800972e+01	9.807069e+01	9.284340e+01	9.475782e+01	1.011362e+02	9.812582e+01	9.929144e+01	9.963081e+01	9.888000e+01	1.005083e+02	9.852800e+01	9.982419e+01	9.835445e+01
	std.	6.420325e+00	5.624846e+00	6.793015e+00	5.410482e+00	7.825166e+00	4.398087e-01	7.410519e+00	7.069748e+00	3.937220e-01	7.473167e+00	6.653694e+00	7.080228e+00	6.999452e+00
4D	med.	8.580452e+02	7.352646e+02	5.972418e+02	6.642789e+02	9.023297e+02	8.070014e+02	8.986689e+02	9.031349e+02	8.853085e+02	9.081908e+02	8.813632e+02	9.058905e+02	8.669970e+02
	avg.	8.367862e+02	7.407499e+02	6.228836e+02	6.718090e+02	8.442911e+02	8.056685e+02	8.376611e+02	8.530653e+02	8.854664e+02	8.637822e+02	8.446033e+02	8.692146e+02	8.311202e+02
	min.	7.143466e+02	5.159311e+02	5.189946e+02	5.345101e+02	7.381353e+02	7.647743e+02	7.298210e+02	7.393345e+02	8.748484e+02	7.514085e+02	7.484682e+02	7.516358e+02	7.089115e+02
	max.	8.803917e+02	8.964366e+02	8.455987e+02	8.243408e+02	9.237064e+02	8.492161e+02	9.194767e+02	9.167273e+02	8.940807e+02	9.185441e+02	9.159024e+02	9.222784e+02	8.909363e+02
	std.	5.197759e+01	6.834466e+01	4.417932e+01	1.364793e+01	7.890589e+01	7.352414e+01	7.410519e+01	7.351202e+01	3.291263e+00	7.080310e+01	6.477972e+01	6.475272e+01	6.475272e+01
5D	med.	8.929884e+03	7.829372e+03	6.044414e+03	6.910237e+03	9.699517e+03	8.234614e+03	9.922286e+03	9.883547e+03	9.727051e+03	9.927452e+03	9.945436e+03	1.004203e+04	9.240369e+03
	avg.	8.715990e+03	7.999154e+03	6.292882e+03	6.966263e+03	9.094139e+03	8.216906e+03	9.360747e+03	9.247260e+03	9.726322e+03	9.588148e+03	9.502540e+03	9.520055e+03	8.788251e+03
	min.	7.376337e+03	5.055141e+03	3.863645e+03	5.363118e+03	7.869579e+03	7.691002e+03	7.987850e+03	8.042248e+03	9.617339e+03	8.093220e+03	8.101632e+03	8.198840e+03	7.565369e+03
	max.	9.330339e+03	9.715336e+03	8.760137e+03	8.637415e+03	1.005313e+04	8.757164e+03	1.009657e+04	1.007388e+04	9.812766e+03	1.010269e+04	1.018583e+04	1.026931e+04	9.595323e+03
	std.	5.528894e+02	8.053158e+02	8.981644e+02	5.276409e+02	8.608010e+02	1.886424e+02	8.593869e+02	8.535358e+02	2.965877e+01	7.006414e+02	7.992169e+02	8.431929e+02	7.393262e+02
6D	med.	1.081027e+05	9.590283e+04	7.243951e+04	8.524195e+04	1.228404e+05	9.727521e+04	1.272516e+05	1.271174e+05	1.251269e+05	1.270968e+05	1.269257e+05	1.286095e+05	1.139028e+05
	avg.	1.074662e+05	9.703651e+04	7.408828e+04	8.645396e+04	1.171675e+05	9.741925e+04	1.190465e+05	1.208405e+05	1.251863e+05	1.211791e+05	1.196512e+05	1.225002e+05	1.091532e+05
	min.	8.905776e+04	6.894487e+04	5.163365e+04	7.116415e+04	9.896231e+04	9.098477e+04	1.011702e+05	1.027996e+05	1.241439e+05	1.033963e+05	1.037204e+05	1.042317e+05	9.506095e+04
	max.	1.149832e+05	1.185328e+05	9.915942e+04	1.087461e+05	1.284959e+05	1.043727e+05	1.302028e+05	1.309020e+05	1.266075e+05	1.298999e+05	1.322741e+05	1.324932e+05	1.231415e+05
	std.	4.732781e+03	8.149111e+03	9.445114e+03	6.322231e+03	1.071638e+04	3.179418e+03	1.191138e+04	1.074409e+04	4.614298e+02	1.015511e+04	1.112846e+04	1.074149e+04	9.773934e+03
7D	med.	1.537517e+06	1.360209e+06	1.015974e+06	1.230139e+06	1.817102e+06	1.447805e+06	1.537589e+06	1.803138e+06	1.858559e+06	1.821598e+06	1.550316e+06	1.572270e+06	1.544180e+06
	avg.	1.511360e+06	1.338831e+06	1.018542e+06	1.230795e+06	1.701311e+06	1.447141e+06	1.684223e+06	1.698826e+06	1.858580e+06	1.723082e+06	1.651073e+06	1.677069e+06	1.528445e+06
	min.	1.232113e+06	8.811574e+05	6.816297e+05	9.352757e+05	1.442884e+06	1.346795e+06	1.446620e+06	1.451882e+06	1.832253e+06	1.502653e+06	1.468409e+06	1.441492e+06	1.307295e+06
	max.	1.693106e+06	1.686997e+06	1.423734e+06	1.497698e+06	1.919818e+06	1.565410e+06	1.927773e+06	1.921562e+06	1.875611e+06	1.919701e+06	1.936536e+06	1.911159e+06	1.765878e+06
	std.	9.043842e+04	1.574924e+05	1.534324e+05	9.873037e+04	1.743276e+05	4.709328e+04	1.917199e+05	1.783423e+05	8.485134e+03	1.618875e+05	1.583995e+05	1.596484e+05	1.328284e+05
8D	med.	2.446881e+07	2.253557e+07	1.686780e+07	2.053183e+07	2.998301e+07	2.148810e+07	3.176784e+07	3.147606e+07	3.109770e+07	3.165479e+07	2.681464e+07	3.099733e+07	2.619871e+07
	avg.	2.434648e+07	2.222245e+07	1.680252e+07	2.068263e+07	2.837180e+07	2.154779e+07	2.977179e+07	2.967087e+07	3.110851e+07	3.008904e+07	2.865954e+07	2.959286e+07	2.566377e+07
	min.	2.025755e+07	1.539614e+07	1.131650e+07	1.575147e+07	2.386410e+07	2.027845e+07	2.522370e+07	2.472044e+07	3.079214e+07	2.585079e+07	2.504881e+07	2.463296e+07	2.255003e+07
	max.	2.633585e+07	2.786271e+07	2.321270e+07	2.486662e+07	3.142348e+07	2.278880e+07	3.269185e+07	3.279636e+07	3.144480e+07	3.265354e+07	3.262059e+07	3.258992e+07	2.906286e+07
	std.	1.085017e+06	2.291467e+06	1.895094e+06	1.250451e+06	2.732431e+06	5.455143e+05	3.083728e+06	2.857286e+06	1.391374e+05	2.649888e+06	2.730782e+06	2.678507e+06	2.148209e+06
9D	med.	4.391613e+08	4.233744e+08	3.156705e+08	3.819245e+08	5.593446e+08	3.685165e+08	6.075989e+08	6.025596e+08	5.592964e+08	6.078008e+08	5.911064e+08	5.965633e+08	5.210862e+08
	avg.	4.392415e+08	4.212582e+08	3.195349e+08	3.875035e+08	5.471798e+08	3.693754e+08	5.834038e+08	5.811032e+08	5.591493e+08	5.899363e+08	5.639176e+08	5.789271e+08	5.069613e+08
	min.	4.002431e+08	2.930129e+08	2.111415e+08	3.508651e+08	4.462704e+08	3.550719e+08	4.841421e+08	4.829563e+08	5.448589e+08	4.930676e+08	4.824572e+08	4.867659e+08	4.270662e+08
	max.	4.694057e+08	5.229989e+08	4.424426e+08	4.667665e+08	5.793282e+08	3.930594e+08	6.218282e+08	6.173709e+08	5.737261e+08	6.211857e+08	6.232942e+08	6.187652e+08	5.588184e+08
	std.	1.231910e+07	3.965819e+07	4.762269e+07	2.343985e+07	3.501682e+07	8.020867e+06	4.947679e+07	4.682574e+07	5.045957e+06	4.196664e+07	5.146583e+07	4.131862e+07	3.942438e+07
10D	med.	8.756451e+09	8.737562e+09	6.642250e+09	8.065695e+09	1.116544e+10	7.208815e+09	1.277258e+10	1.260315e+10	1.054056e+10	1.274692e+10	1.253637e+10	1.255996e+10	1.102896e+10
	avg.	8.781409e+09	8.843681e+09	6.871359e+09	8.204751e+09	1.102356e+10	7.219050e+09	1.237788e+10	1.216290e+10	1.053285e+10	1.250636e+10	1.217893e+10	1.220692e+10	1.080559e+10
	min.	7.718258e+09	7.344721e+09	4.744775e+09	7.471289e+09	8.957500e+09	6.855479e+09	1.008451e+10	1.000153e+10	1.023637e+10	1.031943e+10	1.022977e+10	1.022977e+10	9.211370e+09
	max.	9.320955e+09	1.094106e+10	9.153010e+09	1.024960e+10	1.177153e+10	7.791966e+09	1.298303e+10	1.290395e+10	1.081126e+10	1.300051e+10	1.301250e+10	1.303710e+10	1.168455e+10
	std.	1.986274e+08	6.889871e+08	1.116406e+09	5.906145e+08	5.690336e+08	1.454627e+08	9.44569e+08	9.682822e+08	1.256696e+08	7.374835e+08	9.042752e+08	8.689123e+08	6.336224e+08

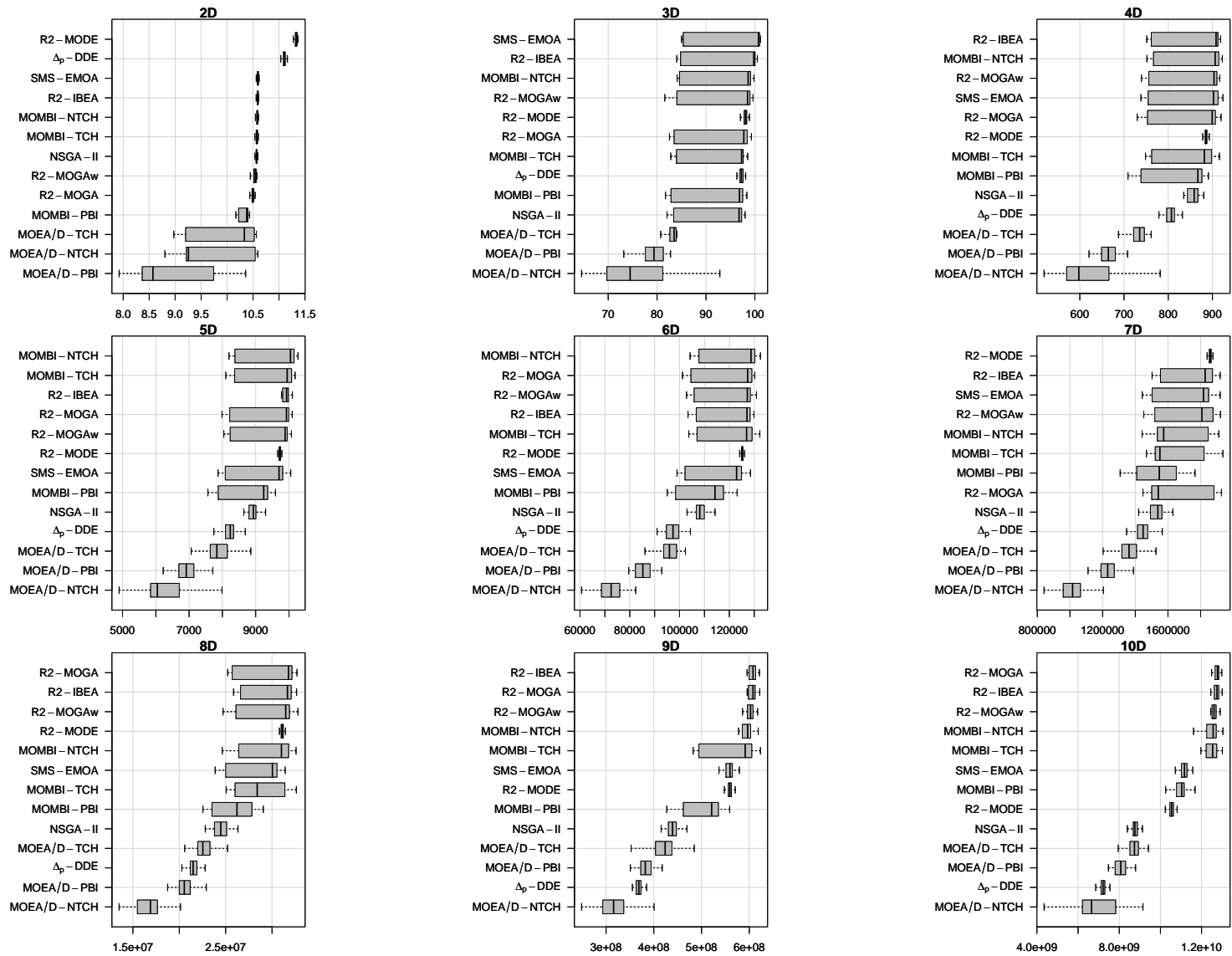


Figure A.129: Box-plot of hypervolume indicator values for different optimizers on the WFG2 test problem.

Table A.51: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG2 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D													
NSGA-II	—	1.84e-31	4.30e-21	1.28e-34	> 0.05	> 0.05	1.37e-32	2.58e-18	> 0.05	> 0.05	> 0.05	> 0.05	3.68e-32
MOEA/D-TCH	> 0.05	—	> 0.05	5.74e-12	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	1.08e-09	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	9.32e-17	1.19e-32	7.28e-30	1.28e-34	—	> 0.05	3.11e-30	1.25e-27	> 0.05	> 0.05	4.17e-13	2.67e-05	1.42e-31
$\Delta_p$ -DDE	1.32e-29	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	2.51e-33	2.51e-33	> 0.05	1.28e-34	2.51e-33	2.97e-27	4.63e-32
R2-MOGA	> 0.05	1.36e-05	5.70e-03	5.38e-34	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	3.78e-30
R2-MOGAw	> 0.05	1.01e-16	7.46e-09	1.28e-34	> 0.05	> 0.05	1.55e-13	—	> 0.05	> 0.05	> 0.05	> 0.05	6.93e-32
R2-MODE	1.13e-30	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	2.26e-34	2.01e-34	—	1.28e-34	2.51e-33	5.39e-28	1.28e-34
R2-IBEA	1.98e-14	2.92e-32	6.97e-29	1.28e-34	> 0.05	> 0.05	1.42e-30	3.87e-26	> 0.05	—	1.03e-10	8.92e-04	4.63e-32
MOMBI-TCH	2.04e-02	4.14e-29	1.14e-21	2.23e-33	> 0.05	> 0.05	2.11e-27	3.01e-17	> 0.05	> 0.05	—	> 0.05	1.12e-29
MOMBI-NTCH	2.48e-09	1.31e-32	6.92e-27	1.28e-34	> 0.05	> 0.05	7.01e-33	2.53e-25	> 0.05	> 0.05	5.14e-05	—	3.27e-32
MOMBI-PBI	> 0.05	> 0.05	> 0.05	6.36e-32	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—
3D													
NSGA-II	—	1.19e-10	9.21e-30	2.42e-29	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	> 0.05	—	1.91e-20	4.94e-19	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	> 0.05	4.07e-07	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	2.43e-05	1.68e-24	1.27e-31	1.37e-26	—	> 0.05	2.66e-08	1.48e-05	> 0.05	1.41e-07	5.69e-07	9.42e-08	1.01e-06
$\Delta_p$ -DDE	9.10e-09	1.99e-28	1.28e-34	1.28e-34	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	8.29e-06
R2-MOGA	2.56e-05	5.19e-12	1.12e-28	1.61e-27	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	4.08e-02	> 0.05	7.28e-07
R2-MOGAw	9.53e-11	2.11e-18	4.86e-30	3.56e-29	> 0.05	1.67e-06	4.03e-05	—	4.58e-03	> 0.05	7.83e-08	> 0.05	2.81e-12
R2-MODE	6.60e-30	1.16e-33	1.28e-34	1.28e-34	> 0.05	2.38e-24	8.63e-03	> 0.05	—	> 0.05	3.14e-20	> 0.05	2.40e-27
R2-IBEA	2.52e-08	4.64e-26	8.01e-32	7.05e-28	> 0.05	3.61e-03	2.98e-11	3.69e-08	3.69e-03	—	5.55e-10	5.04e-08	8.63e-10
MOMBI-TCH	2.46e-04	2.10e-19	1.18e-29	2.27e-28	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	7.55e-04
MOMBI-NTCH	2.62e-08	4.41e-26	7.56e-32	7.05e-28	> 0.05	3.77e-03	1.13e-07	3.71e-02	3.52e-02	> 0.05	1.20e-09	—	1.03e-09
MOMBI-PBI	> 0.05	3.68e-07	3.23e-28	4.97e-28	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—
4D													
NSGA-II	—	1.31e-17	3.68e-32	1.89e-32	> 0.05	3.53e-15	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	> 0.05	—	3.34e-22	4.77e-22	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	> 0.05	7.30e-10	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	7.21e-05	1.45e-24	5.05e-32	5.91e-30	—	2.52e-02	1.23e-02	> 0.05	2.73e-02	> 0.05	> 0.05	> 0.05	4.97e-07
$\Delta_p$ -DDE	> 0.05	1.38e-21	2.66e-33	7.14e-32	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	3.65e-04	1.11e-23	9.51e-32	1.18e-29	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	3.25e-06
R2-MOGAw	9.79e-08	1.52e-26	1.78e-32	1.13e-30	> 0.05	1.24e-04	1.04e-02	—	1.42e-04	> 0.05	4.61e-03	> 0.05	4.48e-10
R2-MODE	1.78e-34	4.79e-33	1.28e-34	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	1.94e-27
R2-IBEA	9.59e-10	4.69e-29	3.27e-33	2.59e-31	1.46e-02	1.73e-06	5.93e-09	1.65e-04	1.73e-06	—	2.72e-07	> 0.05	4.02e-12
MOMBI-TCH	4.14e-06	1.61e-27	4.52e-33	6.25e-31	> 0.05	3.00e-04	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	1.61e-06
MOMBI-NTCH	5.28e-12	4.35e-30	1.24e-33	5.67e-32	7.91e-03	8.47e-09	5.35e-07	9.54e-04	7.04e-08	> 0.05	1.76e-08	—	4.54e-14
MOMBI-PBI	1.73e-03	1.11e-14	3.54e-31	3.16e-31	> 0.05	1.51e-07	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—

Table A.52: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG2 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
5D													
NSGA-II	–	$7.04e-08$	$1.20e-31$	$2.25e-32$	$> 0.05$	$2.69e-15$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-TCH	$> 0.05$	–	$2.25e-23$	$7.22e-23$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-NTCH	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-PBI	$> 0.05$	$> 0.05$	$1.82e-11$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
SMS-EMOA	$1.33e-05$	$4.82e-21$	$5.39e-33$	$4.57e-31$	–	$5.74e-06$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$3.91e-09$
$\Delta_p$ -DDE	$> 0.05$	$8.63e-10$	$2.12e-31$	$2.15e-28$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
R2-MOGA	$3.27e-08$	$8.53e-25$	$1.07e-33$	$6.74e-32$	$1.14e-10$	$3.08e-13$	–	$2.51e-02$	$4.65e-05$	$> 0.05$	$> 0.05$	$> 0.05$	$2.86e-12$
R2-MOGAw	$1.33e-05$	$1.25e-23$	$1.39e-33$	$1.07e-31$	$7.32e-08$	$1.18e-12$	$> 0.05$	–	$8.13e-03$	$> 0.05$	$> 0.05$	$> 0.05$	$1.26e-09$
R2-MODE	$1.28e-34$	$4.64e-34$	$1.28e-34$	$1.28e-34$	$2.60e-02$	$1.28e-34$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$
R2-IBEA	$5.82e-16$	$7.67e-29$	$4.50e-34$	$2.58e-33$	$1.01e-15$	$5.67e-23$	$> 0.05$	$5.95e-04$	$7.89e-13$	–	$> 0.05$	$> 0.05$	$2.10e-19$
MOMBI-TCH	$2.74e-10$	$1.14e-26$	$7.69e-34$	$4.39e-33$	$5.95e-13$	$1.14e-22$	$8.35e-04$	$3.22e-06$	$6.60e-07$	$1.43e-02$	–	$> 0.05$	$2.65e-14$
MOMBI-NTCH	$1.05e-08$	$3.78e-26$	$9.19e-34$	$3.57e-33$	$2.44e-13$	$6.93e-24$	$1.73e-08$	$1.51e-10$	$1.64e-05$	$4.85e-06$	$6.59e-03$	–	$8.61e-13$
MOMBI-PBI	$6.64e-06$	$5.47e-10$	$6.61e-31$	$2.45e-31$	$> 0.05$	$8.12e-05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–
6D													
NSGA-II	–	$1.39e-17$	$6.43e-34$	$1.12e-32$	$> 0.05$	$1.56e-26$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-TCH	$> 0.05$	–	$4.23e-28$	$9.84e-21$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-NTCH	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-PBI	$> 0.05$	$> 0.05$	$2.56e-20$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
SMS-EMOA	$2.14e-08$	$1.74e-28$	$1.40e-34$	$8.24e-32$	–	$1.29e-28$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.66e-14$
$\Delta_p$ -DDE	$> 0.05$	$2.30e-02$	$9.28e-31$	$1.56e-24$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
R2-MOGA	$5.30e-05$	$3.77e-29$	$1.28e-34$	$3.57e-33$	$6.00e-08$	$1.35e-33$	–	$> 0.05$	$3.88e-03$	$> 0.05$	$> 0.05$	$> 0.05$	$1.13e-11$
R2-MOGAw	$2.83e-09$	$2.55e-30$	$1.28e-34$	$9.75e-34$	$2.22e-11$	$2.79e-34$	$> 0.05$	–	$2.67e-06$	$> 0.05$	$> 0.05$	$> 0.05$	$2.59e-15$
R2-MODE	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.64e-12$	$1.28e-34$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$
R2-IBEA	$1.15e-10$	$1.88e-30$	$1.28e-34$	$4.00e-34$	$2.45e-11$	$1.49e-34$	$> 0.05$	$> 0.05$	$6.28e-06$	–	$> 0.05$	$> 0.05$	$8.31e-16$
MOMBI-TCH	$3.17e-07$	$5.38e-29$	$1.28e-34$	$5.38e-34$	$3.09e-07$	$1.49e-34$	$2.64e-02$	$> 0.05$	$2.08e-02$	$> 0.05$	–	$> 0.05$	$7.50e-11$
MOMBI-NTCH	$1.82e-12$	$2.55e-30$	$1.28e-34$	$3.55e-34$	$6.58e-12$	$1.36e-34$	$1.58e-06$	$4.18e-06$	$4.09e-06$	$8.57e-06$	$2.17e-03$	–	$2.59e-15$
MOMBI-PBI	$4.17e-03$	$1.19e-16$	$1.07e-33$	$5.28e-30$	$> 0.05$	$1.05e-15$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–
7D													
NSGA-II	–	$2.77e-17$	$2.91e-33$	$1.27e-31$	$> 0.05$	$7.48e-13$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-TCH	$> 0.05$	–	$4.66e-22$	$3.64e-14$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-NTCH	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-PBI	$> 0.05$	$> 0.05$	$6.16e-20$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
SMS-EMOA	$1.90e-07$	$1.09e-29$	$1.28e-34$	$7.92e-34$	–	$4.30e-26$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$2.95e-12$
$\Delta_p$ -DDE	$> 0.05$	$4.28e-15$	$1.39e-33$	$1.28e-29$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
R2-MOGA	$3.11e-04$	$1.43e-28$	$1.28e-34$	$5.22e-34$	$> 0.05$	$8.63e-26$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$2.02e-08$
R2-MOGAw	$1.57e-07$	$1.15e-29$	$1.28e-34$	$2.26e-34$	$8.46e-03$	$2.49e-29$	$> 0.05$	–	$> 0.05$	$> 0.05$	$4.06e-02$	$> 0.05$	$1.02e-09$
R2-MODE	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.31e-15$	$1.28e-34$	$1.00e-02$	$7.42e-04$	–	$2.85e-03$	$7.64e-19$	$5.25e-17$	$1.28e-34$
R2-IBEA	$9.36e-16$	$8.77e-31$	$1.28e-34$	$1.28e-34$	$1.20e-04$	$1.12e-32$	$7.01e-03$	$1.74e-02$	$> 0.05$	–	$2.35e-06$	$3.11e-04$	$1.20e-11$
MOMBI-TCH	$9.06e-06$	$8.55e-29$	$1.28e-34$	$2.48e-34$	$> 0.05$	$4.56e-29$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$2.22e-06$
MOMBI-NTCH	$2.83e-09$	$1.40e-29$	$1.28e-34$	$2.40e-34$	$> 0.05$	$1.69e-29$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$1.73e-08$
MOMBI-PBI	$> 0.05$	$3.75e-16$	$4.93e-33$	$4.20e-31$	$> 0.05$	$7.49e-04$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–

Table A.53: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG2 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
8D													
NSGA-II	–	5.27e – 19	3.15e – 34	2.42e – 30	> 0.05	3.20e – 30	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	> 0.05	–	6.50e – 26	8.82e – 16	> 0.05	1.08e – 12	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	> 0.05	1.61e – 27	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	2.45e – 19	1.07e – 31	1.28e – 34	4.00e – 34	–	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	8.86e – 15
$\Delta_p$ -DDE	> 0.05	> 0.05	3.48e – 30	1.48e – 14	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	4.20e – 31	2.91e – 33	1.28e – 34	1.28e – 34	1.04e – 11	1.28e – 34	–	> 0.05	1.38e – 04	> 0.05	5.51e – 04	2.91e – 02	2.01e – 15
R2-MOGAw	3.09e – 32	3.57e – 33	1.28e – 34	1.36e – 34	5.90e – 11	1.28e – 34	> 0.05	–	1.28e – 03	> 0.05	3.68e – 04	> 0.05	3.08e – 15
R2-MODE	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.29e – 28	1.28e – 34	> 0.05	> 0.05	–	> 0.05	5.68e – 05	> 0.05	1.28e – 34
R2-IBEA	2.55e – 34	1.81e – 33	1.28e – 34	1.28e – 34	1.20e – 12	1.28e – 34	> 0.05	6.70e – 03	4.74e – 05	–	4.32e – 09	9.38e – 04	4.27e – 17
MOMBI-TCH	3.27e – 32	1.12e – 32	1.28e – 34	1.28e – 34	2.04e – 04	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	6.18e – 10
MOMBI-NTCH	2.31e – 32	3.57e – 33	1.28e – 34	1.40e – 34	1.17e – 09	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	2.87e – 03	–	6.82e – 16
MOMBI-PBI	3.66e – 03	1.42e – 22	1.68e – 34	7.78e – 32	> 0.05	2.01e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
9D													
NSGA-II	–	8.36e – 09	1.87e – 33	7.89e – 27	> 0.05	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	> 0.05	–	7.29e – 27	1.65e – 15	> 0.05	2.66e – 25	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	> 0.05	4.82e – 21	–	> 0.05	8.05e – 14	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	1.92e – 33	8.35e – 33	1.28e – 34	3.05e – 34	–	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.17e – 21
$\Delta_p$ -DDE	> 0.05	> 0.05	1.86e – 17	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e – 34	3.79e – 33	1.28e – 34	1.28e – 34	2.37e – 14	1.28e – 34	–	1.20e – 03	3.95e – 12	> 0.05	3.97e – 07	3.82e – 07	8.76e – 17
R2-MOGAw	1.28e – 34	3.18e – 33	1.28e – 34	1.28e – 34	4.54e – 15	1.28e – 34	> 0.05	–	6.97e – 13	> 0.05	3.31e – 04	7.24e – 04	1.90e – 17
R2-MODE	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	1.28e – 34	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	2.23e – 33
R2-IBEA	1.28e – 34	9.75e – 34	1.28e – 34	1.28e – 34	2.44e – 17	1.28e – 34	> 0.05	5.74e – 05	2.69e – 15	–	6.00e – 10	7.88e – 10	1.08e – 19
MOMBI-TCH	1.28e – 34	1.37e – 32	1.28e – 34	1.28e – 34	1.61e – 07	1.28e – 34	> 0.05	> 0.05	2.05e – 05	> 0.05	–	> 0.05	1.88e – 10
MOMBI-NTCH	1.28e – 34	1.57e – 33	1.28e – 34	1.28e – 34	4.49e – 16	1.28e – 34	> 0.05	> 0.05	5.99e – 14	> 0.05	> 0.05	–	7.98e – 19
MOMBI-PBI	4.91e – 23	1.84e – 27	1.84e – 34	1.05e – 32	> 0.05	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
10D													
NSGA-II	–	> 0.05	8.24e – 32	8.76e – 18	> 0.05	1.32e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	> 0.05	–	9.46e – 28	4.63e – 14	> 0.05	3.15e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	> 0.05	8.86e – 15	–	> 0.05	3.44e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	3.05e – 34	1.19e – 32	1.40e – 34	1.66e – 33	–	1.28e – 34	> 0.05	> 0.05	7.15e – 25	> 0.05	> 0.05	> 0.05	4.70e – 05
$\Delta_p$ -DDE	> 0.05	> 0.05	1.62e – 04	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e – 34	3.08e – 33	1.28e – 34	1.68e – 34	3.38e – 18	1.28e – 34	–	8.47e – 09	1.31e – 17	> 0.05	3.17e – 07	7.93e – 07	2.56e – 19
R2-MOGAw	1.28e – 34	4.39e – 33	1.28e – 34	1.53e – 34	3.31e – 14	1.28e – 34	> 0.05	–	7.19e – 15	> 0.05	> 0.05	> 0.05	1.41e – 15
R2-MODE	1.28e – 34	6.07e – 31	1.28e – 34	1.32e – 34	> 0.05	1.28e – 34	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	1.28e – 34	3.76e – 34	1.28e – 34	1.28e – 34	9.88e – 23	1.28e – 34	> 0.05	6.94e – 09	6.80e – 25	–	5.46e – 08	1.04e – 07	1.00e – 23
MOMBI-TCH	1.28e – 34	2.10e – 33	1.28e – 34	1.49e – 34	1.55e – 16	1.28e – 34	> 0.05	> 0.05	5.94e – 17	> 0.05	–	> 0.05	8.58e – 18
MOMBI-NTCH	1.28e – 34	1.24e – 33	1.28e – 34	1.40e – 34	2.39e – 17	1.28e – 34	> 0.05	> 0.05	1.74e – 18	> 0.05	> 0.05	–	1.37e – 18
MOMBI-PBI	1.58e – 34	3.25e – 31	1.28e – 34	4.65e – 33	> 0.05	1.28e – 34	> 0.05	> 0.05	2.30e – 15	> 0.05	> 0.05	> 0.05	–

Table A.54: Comparison of R2 indicator values for different optimizers on the WFG2 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	5.509067e-01	5.629176e-01	8.355440e-01	8.619363e-01	5.501254e-01	4.482076e-01	5.523035e-01	5.524720e-01	4.368853e-01	5.500633e-01	5.509380e-01	5.507360e-01	5.576963e-01
	avg.	5.465477e-01	6.867561e-01	7.058292e-01	7.420888e-01	5.506856e-01	4.481924e-01	5.543338e-01	5.514873e-01	4.369824e-01	5.506429e-01	5.558395e-01	5.441788e-01	5.595259e-01
	min.	4.343521e-01	5.502188e-01	5.496836e-01	5.615269e-01	5.484716e-01	4.446230e-01	4.358740e-01	4.367708e-01	4.360233e-01	5.488026e-01	4.353584e-01	4.332272e-01	4.389673e-01
	max.	5.561210e-01	8.614491e-01	8.758045e-01	9.274068e-01	5.658236e-01	4.511414e-01	8.364528e-01	5.561383e-01	4.384455e-01	5.598144e-01	8.371610e-01	5.561638e-01	5.834493e-01
	std.	2.283044e-02	1.429949e-01	1.438358e-01	1.467529e-01	2.211862e-03	1.349828e-03	3.083486e-02	1.167892e-02	4.932772e-04	1.899230e-03	4.195854e-02	2.785883e-02	1.911179e-02
3D	med.	1.771995e-01	4.210552e-01	4.820116e-01	4.447641e-01	1.521569e-01	1.669598e-01	1.568849e-01	1.569770e-01	1.585558e-01	1.527429e-01	1.589030e-01	1.617875e-01	1.671438e-01
	avg.	2.483247e-01	3.808227e-01	5.219057e-01	4.139394e-01	2.758840e-01	1.676346e-01	2.632438e-01	2.351096e-01	1.585407e-01	2.551656e-01	2.523957e-01	2.613419e-01	2.561958e-01
	min.	1.681775e-01	1.578985e-01	2.135812e-01	1.744948e-01	1.507022e-01	1.630439e-01	1.537161e-01	1.539833e-01	1.566601e-01	1.513408e-01	1.566470e-01	1.560086e-01	1.606167e-01
	max.	4.318722e-01	8.077628e-01	8.268548e-01	8.214414e-01	4.176139e-01	1.744769e-01	4.200329e-01	4.223075e-01	1.610652e-01	4.180173e-01	4.254224e-01	4.209715e-01	4.302427e-01
	std.	1.149695e-01	1.100546e-01	1.467926e-01	1.007801e-01	1.326135e-01	2.542708e-03	1.293812e-01	1.208999e-01	9.545168e-04	1.291849e-01	1.261518e-01	1.265752e-01	1.241779e-01
4D	med.	1.345607e-01	4.193535e-01	6.118082e-01	4.483096e-01	9.125584e-02	1.459093e-01	8.664505e-02	8.675544e-02	9.391793e-02	8.143650e-02	1.048407e-01	1.068416e-01	9.758192e-02
	avg.	1.858891e-01	3.934487e-01	6.188325e-01	4.263314e-01	2.114715e-01	1.468750e-01	2.266752e-01	1.991046e-01	9.388000e-02	1.814345e-01	2.134353e-01	1.876175e-01	1.889688e-01
	min.	1.174577e-01	1.043496e-01	1.467629e-01	1.151953e-01	8.363930e-02	1.190950e-01	8.160444e-02	8.172117e-02	9.116865e-02	7.817746e-02	9.766234e-02	9.840450e-02	9.030989e-02
	max.	4.345922e-01	8.226873e-01	8.311635e-01	8.422618e-01	4.205579e-01	1.685479e-01	4.158948e-01	9.645180e-02	9.645180e-02	4.080947e-01	4.158033e-01	4.138520e-01	4.254100e-01
	std.	1.139998e-01	1.178221e-01	1.654714e-01	1.000689e-01	1.559038e-01	9.795660e-03	1.609306e-01	1.548798e-01	1.134957e-03	1.509435e-01	1.480938e-01	1.357728e-01	1.454700e-01
5D	med.	1.374312e-01	4.527316e-01	6.891412e-01	4.793933e-01	7.396946e-02	1.433871e-01	5.882865e-02	6.201454e-02	6.706618e-02	5.720997e-02	8.452817e-02	8.587330e-02	7.654083e-02
	avg.	1.956440e-01	3.871248e-01	7.178507e-01	4.645121e-01	2.195634e-01	1.443067e-01	1.861176e-01	2.104947e-01	6.719654e-02	1.357506e-01	1.888032e-01	2.005795e-01	2.073330e-01
	min.	1.145644e-01	9.106638e-02	1.890890e-01	9.934767e-02	6.180008e-02	1.223366e-01	5.458669e-02	5.582054e-02	6.516124e-02	5.316834e-02	7.486381e-02	7.741748e-02	6.562094e-02
	max.	4.746590e-01	9.021157e-01	1.321585e+00	9.046885e-01	4.492000e-01	1.709116e-01	4.398565e-01	4.403313e-01	6.981525e-02	4.377043e-01	4.443228e-01	4.414410e-01	4.513151e-01
	std.	1.277808e-01	1.630229e-01	2.089413e-01	1.258416e-01	1.833244e-01	9.717204e-03	1.795293e-01	1.850975e-01	8.986316e-04	1.543681e-01	1.637922e-01	1.669663e-01	1.788851e-01
6D	med.	1.410658e-01	4.316035e-01	6.952437e-01	4.485727e-01	6.881225e-02	1.569663e-01	4.714725e-02	4.851526e-02	5.319667e-02	4.645687e-02	9.015568e-02	8.689654e-02	7.385573e-02
	avg.	1.638427e-01	4.020969e-01	7.125288e-01	4.227478e-01	1.712002e-01	1.576758e-01	1.868635e-01	1.593632e-01	5.318975e-02	1.537558e-01	2.155901e-01	1.859929e-01	2.141730e-01
	min.	1.129298e-01	9.668514e-02	4.272225e-01	8.928364e-02	5.668769e-02	1.279254e-01	4.126799e-02	4.164990e-02	5.115244e-02	4.080762e-02	6.145387e-02	7.142838e-02	5.449554e-02
	max.	4.527833e-01	8.320644e-01	1.225356e+00	8.366271e-01	4.221700e-01	1.917857e-01	4.166554e-01	4.138203e-01	5.532720e-02	4.110923e-01	4.156557e-01	4.156110e-01	4.246993e-01
	std.	8.248402e-02	1.094756e-01	1.572984e-01	1.071461e-01	1.624045e-01	1.252243e-02	1.785527e-01	1.686337e-01	9.270829e-04	1.663811e-01	1.640949e-01	1.518217e-01	1.744781e-01
7D	med.	1.562814e-01	4.400355e-01	8.146676e-01	4.389581e-01	6.297648e-02	1.362983e-01	4.036432e-01	5.351637e-02	4.674832e-02	4.683818e-02	4.112465e-01	4.093871e-01	9.020946e-02
	avg.	1.953486e-01	4.566804e-01	7.594528e-01	4.430590e-01	2.034133e-01	1.373140e-01	2.325649e-01	2.232991e-01	4.670662e-02	2.091386e-01	2.993494e-01	2.874520e-01	2.338114e-01
	min.	1.189942e-01	1.130773e-01	4.369829e-01	1.009588e-01	4.665026e-02	1.087062e-01	3.707693e-02	3.890046e-02	4.358165e-02	3.548562e-02	6.989330e-02	8.044164e-02	5.299963e-02
	max.	4.508722e-01	8.462603e-01	1.226444e+00	8.398827e-01	4.182436e-01	1.671900e-01	4.152054e-01	4.154090e-01	4.922280e-02	4.084389e-01	4.341518e-01	4.331075e-01	4.281357e-01
	std.	1.042808e-01	1.556342e-01	1.811747e-01	1.267957e-01	1.750827e-01	1.188941e-02	1.831205e-01	1.824584e-01	1.073550e-03	1.806120e-01	1.487381e-01	1.437185e-01	1.726786e-01
8D	med.	1.544106e-01	4.394001e-01	8.115868e-01	4.306677e-01	6.342260e-02	1.564076e-01	3.620179e-02	3.928097e-02	4.218946e-02	3.783475e-02	1.728861e-01	1.635663e-01	7.753328e-02
	avg.	1.694346e-01	4.410959e-01	7.496300e-01	4.162375e-01	1.859821e-01	1.563182e-01	1.636490e-01	1.690842e-01	4.215833e-02	1.563350e-01	2.639585e-01	2.337161e-01	2.339625e-01
	min.	1.335248e-01	1.402869e-01	4.386568e-01	9.423140e-02	4.973787e-02	1.403169e-01	3.008823e-02	2.991931e-02	3.918805e-02	3.215839e-02	5.495201e-02	7.071847e-02	5.185509e-02
	max.	4.498842e-01	8.312885e-01	1.213896e+00	8.288544e-01	4.161113e-01	1.751489e-01	4.057496e-01	4.108454e-01	4.448519e-02	4.026665e-01	4.291625e-01	4.317594e-01	4.170581e-01
	std.	6.365514e-02	1.360627e-01	1.186852e-01	8.919679e-02	1.689897e-01	7.528331e-03	1.767426e-01	1.768636e-01	1.058334e-03	1.720417e-01	1.498550e-01	1.362628e-01	1.739441e-01
9D	med.	1.514557e-01	4.461635e-01	7.374105e-01	4.340445e-01	6.074353e-02	1.646452e-01	2.917378e-02	3.158399e-02	4.906031e-02	3.073530e-02	1.398884e-01	1.601060e-01	5.745264e-02
	avg.	1.538512e-01	4.405183e-01	7.390677e-01	4.069902e-01	1.070650e-01	1.642462e-01	1.119009e-01	1.102161e-01	4.922734e-02	9.779601e-02	2.172328e-01	2.016218e-01	1.481019e-01
	min.	1.298367e-01	1.304002e-01	4.314007e-01	8.490915e-02	5.096911e-02	1.503817e-01	2.487170e-02	2.714835e-02	4.374777e-02	2.646511e-02	4.376930e-02	7.189553e-02	4.148218e-02
	max.	3.402912e-01	8.429496e-01	1.230104e+00	4.453905e-01	4.198136e-01	1.759555e-01	4.093764e-01	4.103918e-01	5.518370e-02	4.076185e-01	4.380452e-01	4.309806e-01	4.181883e-01
	std.	2.079300e-02	1.209584e-01	1.733800e-01	9.327203e-02	1.202528e-01	5.096153e-03	1.577052e-01	1.546213e-01	2.117599e-03	1.449528e-01	1.448544e-01	1.093987e-01	1.585843e-01
10D	med.	1.478555e-01	4.455642e-01	7.324044e-01	4.308923e-01	6.735296e-02	1.646770e-01	2.478410e-02	2.788835e-02	6.910182e-02	2.691013e-02	1.201930e-01	1.495215e-01	4.975581e-02
	avg.	1.504131e-01	4.132028e-01	7.037796e-01	3.896623e-01	9.559005e-02	1.642697e-01	8.613055e-02	1.074136e-01	6.919768e-02	6.866154e-02	1.712366e-01	1.933890e-01	1.039354e-01
	min.	1.318261e-01	1.697752e-01	3.648702e-01	6.443212e-02	5.039451e-02	1.484991e-01	2.170980e-02	2.351178e-02	6.342271e-02	2.318741e-02	5.041146e-02	7.698682e-02	4.023777e-02
	max.	4.489975e-01	5.199393e-01	1.233707e+00	4.402953e-01	4.232086e-01	1.735750e-01	4.110681e-01	4.120242e-01	7.502490e-02	4.086536e-01	4.375043e-01	4.321308e-01	4.155099e-01
	std.	3.068707e-02	9.134868e-02	1.960286e-01	1.141791e-01	9.650466e-02	3.905194e-03	1.416760e-01	1.563131e-01	2.659615e-03	1.196968e-01	1.200487e-01	1.080923e-01	1.304803e-01

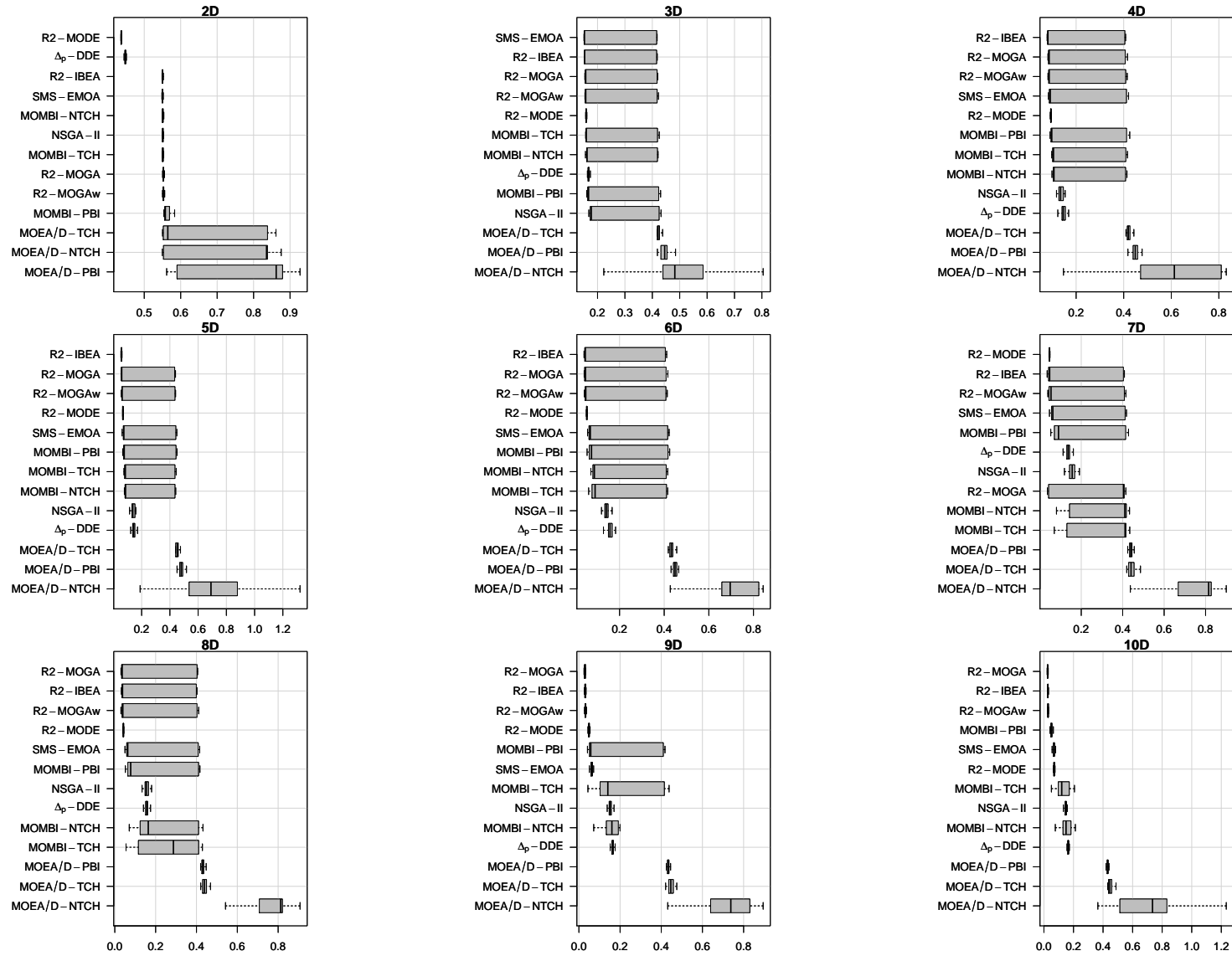


Figure A.130: Box-plot of R2 indicator values for different optimizers on the WFG2 test problem.

Table A.55: Comparison of runtime (in milliseconds) for different optimizers on the WFG2 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	3.345390e+02	7.262890e+02	1.278967e+03	1.212585e+03	2.425863e+04	1.625810e+03	2.856282e+03	2.133275e+03	2.525457e+03	5.506860e+05	2.091109e+03	2.214273e+03	3.074369e+03
	avg.	3.453978e+02	7.464595e+02	1.269492e+03	1.206820e+03	2.409929e+04	1.625405e+03	2.837474e+03	2.192003e+03	2.516318e+03	5.499178e+05	2.078353e+03	2.187710e+03	3.068510e+03
	min.	2.882800e+02	5.035370e+02	9.172500e+02	9.512090e+02	1.632564e+04	1.250334e+03	2.024914e+03	1.686591e+03	1.612872e+03	5.340020e+05	1.649449e+03	1.834786e+03	2.596267e+03
	max.	4.977700e+02	1.127619e+03	1.567379e+03	1.750800e+03	2.795851e+04	2.024486e+03	4.350478e+03	2.961091e+03	3.151373e+03	5.626510e+05	2.431046e+03	2.572266e+03	3.358851e+03
	std.	4.344720e+01	1.086612e+02	1.112462e+02	1.276765e+02	2.328075e+03	1.476138e+02	3.648817e+02	2.600769e+02	3.432491e+02	5.131495e+03	1.577735e+02	1.588372e+02	1.425401e+02
3D	med.	4.886040e+02	8.793350e+02	1.535865e+03	1.299931e+03	7.889835e+05	1.367830e+03	2.361714e+03	1.991024e+03	2.143478e+03	5.666400e+05	2.064188e+03	2.202048e+03	3.195490e+03
	avg.	4.928136e+02	8.862223e+02	1.520334e+03	1.274527e+03	7.668611e+05	1.376875e+03	2.780599e+03	2.211664e+03	2.104085e+03	5.622009e+05	2.064834e+03	2.205092e+03	3.183629e+03
	min.	4.080990e+02	6.867960e+02	1.135027e+03	9.001140e+02	5.821441e+05	1.037329e+03	1.682535e+03	1.409427e+03	1.499758e+03	5.398660e+05	1.667095e+03	1.858030e+03	2.822778e+03
	max.	7.062350e+02	1.271220e+03	1.786548e+03	1.552127e+03	8.707983e+05	1.751289e+03	4.234843e+03	3.542866e+03	2.530722e+03	5.839080e+05	2.342884e+03	2.658438e+03	3.477891e+03
	std.	5.979428e+01	1.036167e+02	1.487559e+02	1.190103e+02	7.293934e+04	1.324462e+02	8.182645e+02	5.534540e+02	2.365818e+02	1.462410e+04	1.480510e+02	1.383716e+02	1.559144e+02
4D	med.	7.228740e+02	1.015215e+03	1.785109e+03	1.559103e+03	1.253893e+06	1.019861e+03	2.838986e+03	2.571302e+03	2.515195e+03	5.999010e+05	2.218814e+03	2.258023e+03	3.512437e+03
	avg.	7.203291e+02	1.029875e+03	1.784767e+03	1.545096e+03	1.559487e+06	1.026775e+03	3.688210e+03	3.182818e+03	2.513503e+03	5.993929e+05	2.270112e+03	2.282780e+03	3.470437e+03
	min.	5.907920e+02	7.337250e+02	1.442295e+03	1.150044e+03	5.700336e+05	7.735330e+02	1.853592e+03	1.908225e+03	1.907659e+03	5.491860e+05	1.811688e+03	2.018826e+03	2.875574e+03
	max.	8.834120e+02	1.424844e+03	2.247302e+03	1.820086e+03	3.515123e+06	1.416504e+03	6.027580e+03	5.956195e+03	2.978058e+03	6.136830e+05	3.483516e+03	3.625339e+03	3.153884e+03
	std.	6.465154e+01	1.169037e+02	1.528860e+02	1.199790e+02	7.108942e+05	1.341946e+02	1.388037e+03	1.161985e+03	2.462760e+02	7.791764e+03	2.462315e+02	1.478280e+02	2.053388e+02
5D	med.	9.590920e+02	1.219107e+03	2.415639e+03	1.850919e+03	1.571026e+06	1.332325e+03	3.399286e+03	3.289606e+03	3.450026e+03	6.771770e+05	2.916274e+03	2.558849e+03	3.985927e+03
	avg.	9.547543e+02	1.221172e+03	2.384346e+03	1.811323e+03	2.271416e+06	1.316353e+03	4.420130e+03	4.523496e+03	3.382494e+03	6.559675e+05	2.958747e+03	2.606459e+03	3.960668e+03
	min.	7.757740e+02	9.015620e+02	1.997423e+03	1.389064e+03	9.308643e+05	9.086330e+02	2.438963e+03	2.364076e+03	2.343319e+03	6.029350e+05	2.198754e+03	2.301083e+03	3.153884e+03
	max.	1.136902e+03	1.644935e+03	2.965298e+03	2.299868e+03	5.718994e+06	1.711740e+03	8.131545e+03	9.962987e+03	4.097011e+03	6.896380e+05	4.883646e+03	3.073614e+03	4.563969e+03
	std.	7.870008e+01	1.207784e+02	1.641476e+02	1.785602e+02	1.118436e+06	1.274145e+02	1.820895e+03	1.936454e+03	3.736615e+02	3.302329e+04	4.598872e+02	1.766596e+02	1.978033e+02
6D	med.	1.167414e+03	1.277660e+03	2.580283e+03	2.105379e+03	1.683542e+06	1.391962e+03	4.181806e+03	3.732243e+03	4.113550e+03	7.220700e+05	2.650306e+03	3.118024e+03	4.350964e+03
	avg.	1.158408e+03	1.278111e+03	2.533764e+03	2.048239e+03	2.164247e+06	1.378084e+03	5.631318e+03	4.942637e+03	4.106547e+03	7.220976e+05	2.724786e+03	3.116467e+03	4.328567e+03
	min.	9.679390e+02	9.376810e+02	1.880318e+03	1.536027e+03	9.983093e+05	9.767480e+02	3.173710e+03	2.754456e+03	3.050836e+03	7.089770e+05	2.207446e+03	2.665003e+03	3.601414e+03
	max.	1.420225e+03	1.727701e+03	3.078477e+03	2.498446e+03	4.987909e+06	1.713388e+03	9.811095e+03	1.011324e+04	4.783695e+03	7.347340e+05	4.062648e+03	3.986056e+03	4.893044e+03
	std.	1.031721e+02	1.378447e+02	2.274226e+02	2.464616e+02	1.023918e+06	1.409806e+02	2.270784e+03	2.130905e+03	3.526342e+02	4.796506e+03	3.332058e+03	2.656289e+02	1.421963e+02
7D	med.	1.182025e+03	1.432632e+03	2.809703e+03	2.313827e+03	1.705061e+06	2.012074e+03	5.651201e+03	3.584327e+03	3.023862e+03	3.293860e+05	2.057256e+03	2.418306e+03	3.505333e+03
	avg.	1.180805e+03	1.426743e+03	2.740331e+03	2.235343e+03	2.249612e+06	2.008265e+03	5.346074e+03	4.998013e+03	3.045485e+03	3.295499e+05	2.064541e+03	2.441466e+03	3.496266e+03
	min.	9.436300e+02	1.071078e+03	1.940729e+03	1.529134e+03	9.650080e+05	1.308041e+03	2.619420e+03	2.675955e+03	2.220059e+03	3.233350e+05	1.627937e+03	2.065339e+03	2.802935e+03
	max.	1.452697e+03	1.796895e+03	2.989056e+03	2.599224e+03	4.305614e+06	2.578258e+03	8.674916e+03	1.001558e+04	3.497580e+03	3.364170e+05	2.689714e+03	3.039056e+03	3.845336e+03
	std.	9.397241e+01	1.460613e+02	2.100911e+02	2.464616e+02	9.420113e+05	2.489434e+02	2.063020e+03	2.007164e+03	3.278459e+02	3.102705e+03	1.847331e+02	1.670864e+02	1.421963e+02
8D	med.	1.527856e+03	1.558790e+03	3.014945e+03	2.350864e+03	2.542457e+06	3.146104e+03	5.235971e+03	4.847515e+03	5.580676e+03	1.369324e+06	2.819710e+03	3.181451e+03	5.004631e+03
	avg.	1.533274e+03	1.564874e+03	2.966271e+03	2.395061e+03	3.257996e+06	3.142192e+03	7.461905e+03	6.879745e+03	5.488597e+03	1.280719e+06	2.851148e+03	3.210929e+03	4.978967e+03
	min.	1.264971e+03	1.180236e+03	2.176551e+03	1.506348e+03	1.355174e+06	2.388719e+03	3.421225e+03	3.481510e+03	3.869046e+03	1.103936e+06	2.359954e+03	2.829605e+03	4.062168e+03
	max.	1.787993e+03	1.861518e+03	3.464294e+03	3.231683e+03	5.942218e+06	3.904892e+03	1.387438e+04	1.629509e+04	6.158536e+03	1.512955e+06	3.238820e+03	3.737140e+03	5.507396e+03
	std.	1.144172e+02	1.296998e+02	2.276808e+02	2.803279e+02	1.307805e+06	3.648663e+02	3.647803e+03	3.459804e+03	5.155829e+02	1.239000e+05	1.923503e+02	2.199042e+02	1.212746e+02
9D	med.	1.958907e+03	1.789000e+03	3.531822e+03	2.745570e+03	2.585848e+06	4.696989e+03	7.058205e+03	6.752148e+03	8.630331e+03	2.608950e+06	3.515312e+03	4.482409e+03	6.914163e+03
	avg.	1.947209e+03	1.780612e+03	3.494595e+03	2.640661e+03	2.820154e+06	4.628156e+03	8.912568e+03	9.204954e+03	8.631767e+03	2.609242e+06	3.569566e+03	4.548252e+03	6.916195e+03
	min.	1.576000e+03	1.270533e+03	2.615962e+03	1.881765e+03	1.489870e+06	3.182092e+03	4.700333e+03	4.970201e+03	6.064239e+03	2.587324e+06	3.062480e+03	3.964786e+03	5.909351e+03
	max.	2.282691e+03	2.169989e+03	3.852501e+03	3.006349e+03	7.049797e+06	6.088240e+03	2.272820e+04	2.433915e+04	9.593284e+03	2.625854e+06	4.249619e+03	5.304599e+03	7.737455e+03
	std.	1.474649e+02	1.666589e+02	2.444202e+02	2.736947e+02	1.351381e+06	6.699057e+02	4.816613e+03	5.363517e+03	7.640678e+02	7.227164e+03	3.060543e+02	3.558426e+02	3.379968e+02
10D	med.	2.525374e+03	1.936406e+03	3.772370e+03	2.747205e+03	3.156089e+06	4.738551e+03	8.703010e+03	9.701387e+03	1.328070e+04	4.936533e+06	5.056696e+03	6.428666e+03	9.748401e+03
	avg.	2.524337e+03	1.936195e+03	3.705489e+03	2.734651e+03	3.594929e+06	4.734469e+03	1.120107e+04	1.297954e+04	4.888284e+03	4.986671e+06	5.150000e+03	6.351500e+03	9.670656e+03
	min.	2.131581e+03	1.562848e+03	2.728531e+03	1.929886e+03	2.074289e+06	4.149600e+03	7.020993e+03	7.645522e+03	9.486697e+03	4.712145e+06	3.965217e+03	5.183185e+03	7.750602e+03
	max.	2.887393e+03	2.286454e+03	4.067045e+03	3.464912e+03	8.795680e+06	5.334465e+03	3.180857e+04	3.814526e+04	1.467042e+04	4.982956e+06	5.831425e+03	7.455145e+03	1.130453e+04
	std.	1.494189e+02	1.646869e+02	2.658679e+02	2.645116e+02	1.226261e+06	2.518822e+02	6.457870e+03	7.936954e+03	1.247815e+03	8.512605e+04	5.696957e+02	5.994707e+02	9.566865e+02



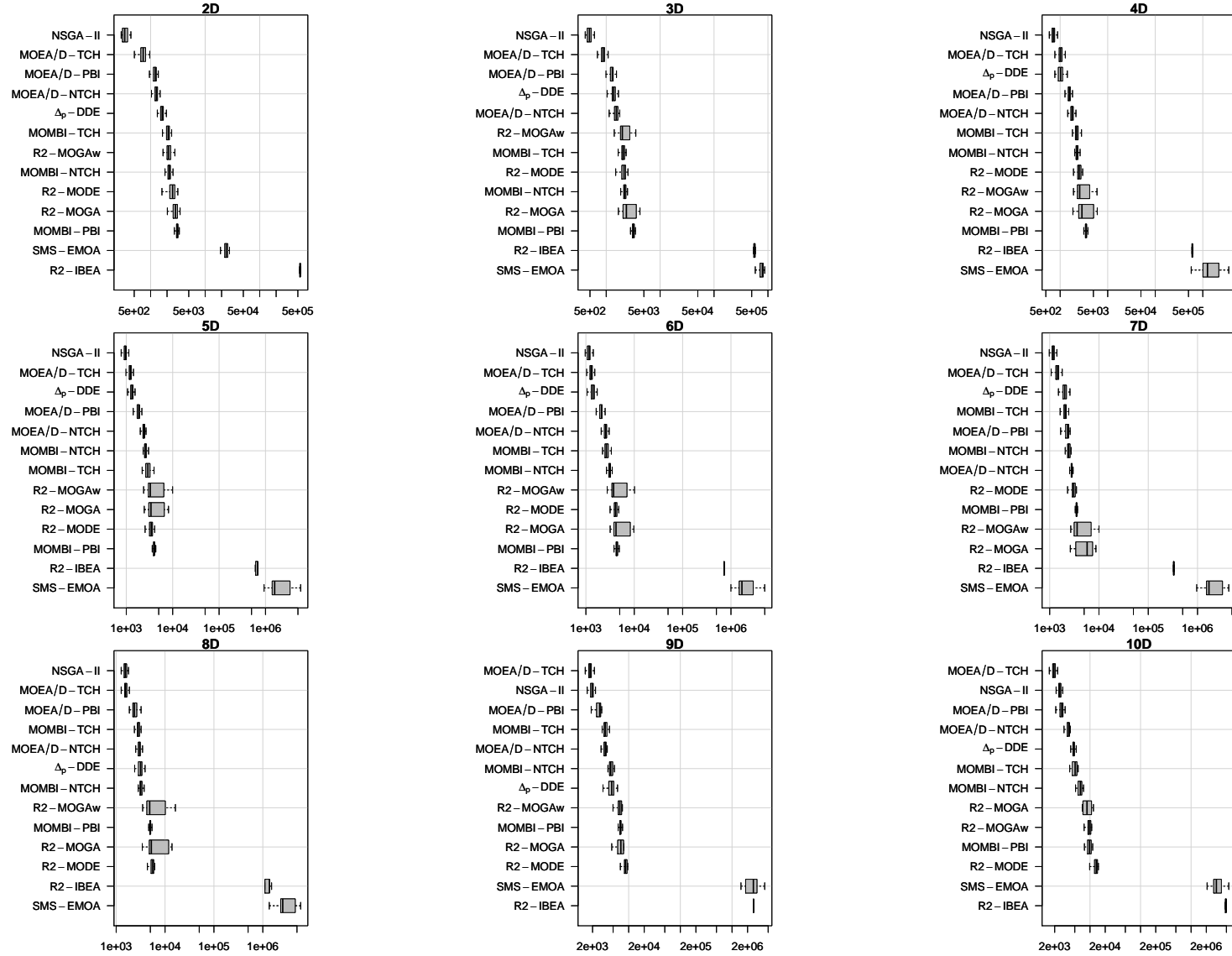


Figure A.131: Box-plot of runtime (in logarithmic scale) for different optimizers on the WFG2 test problem.

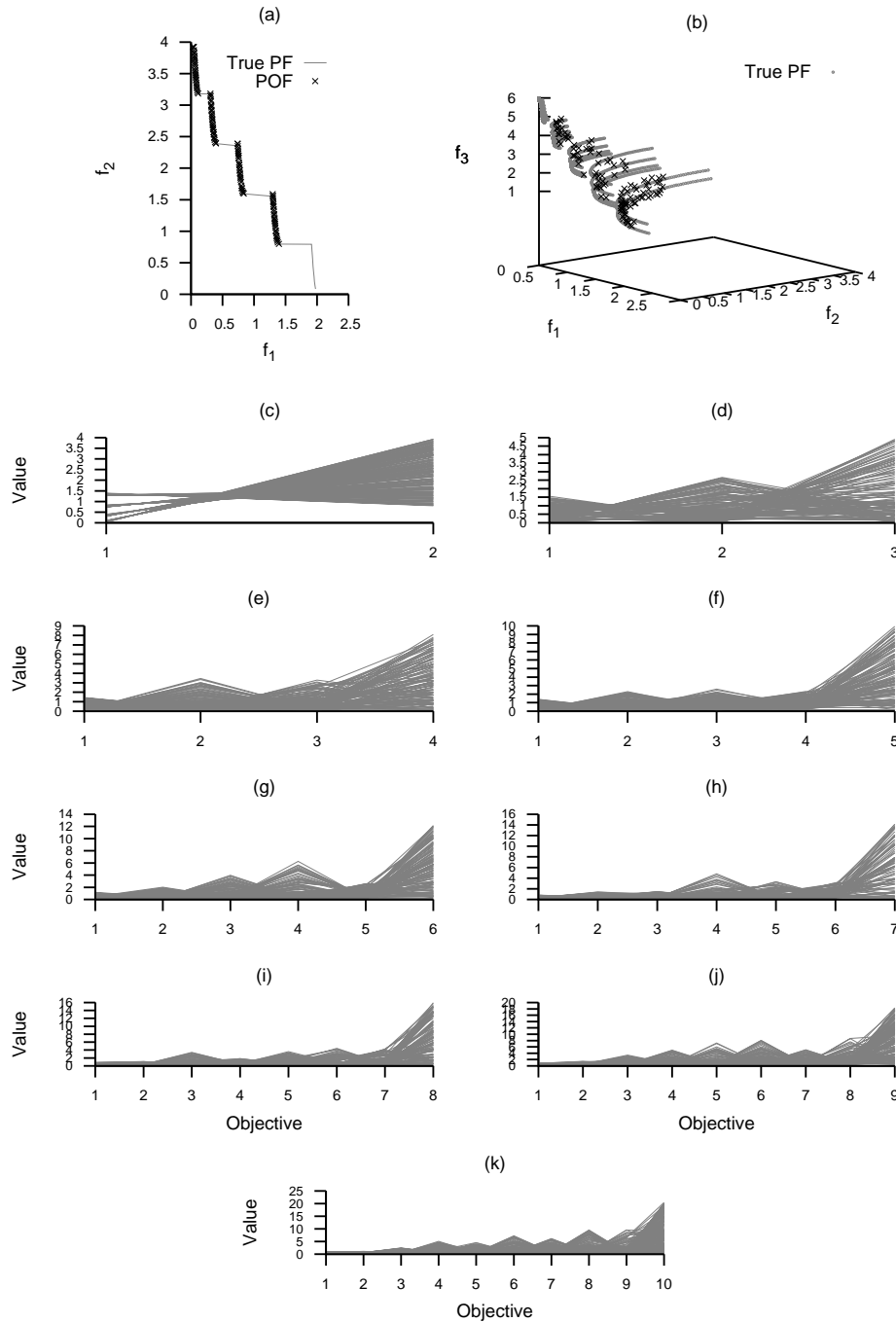


Figure A.132: Plots of the approximations obtained by NSGA-II from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

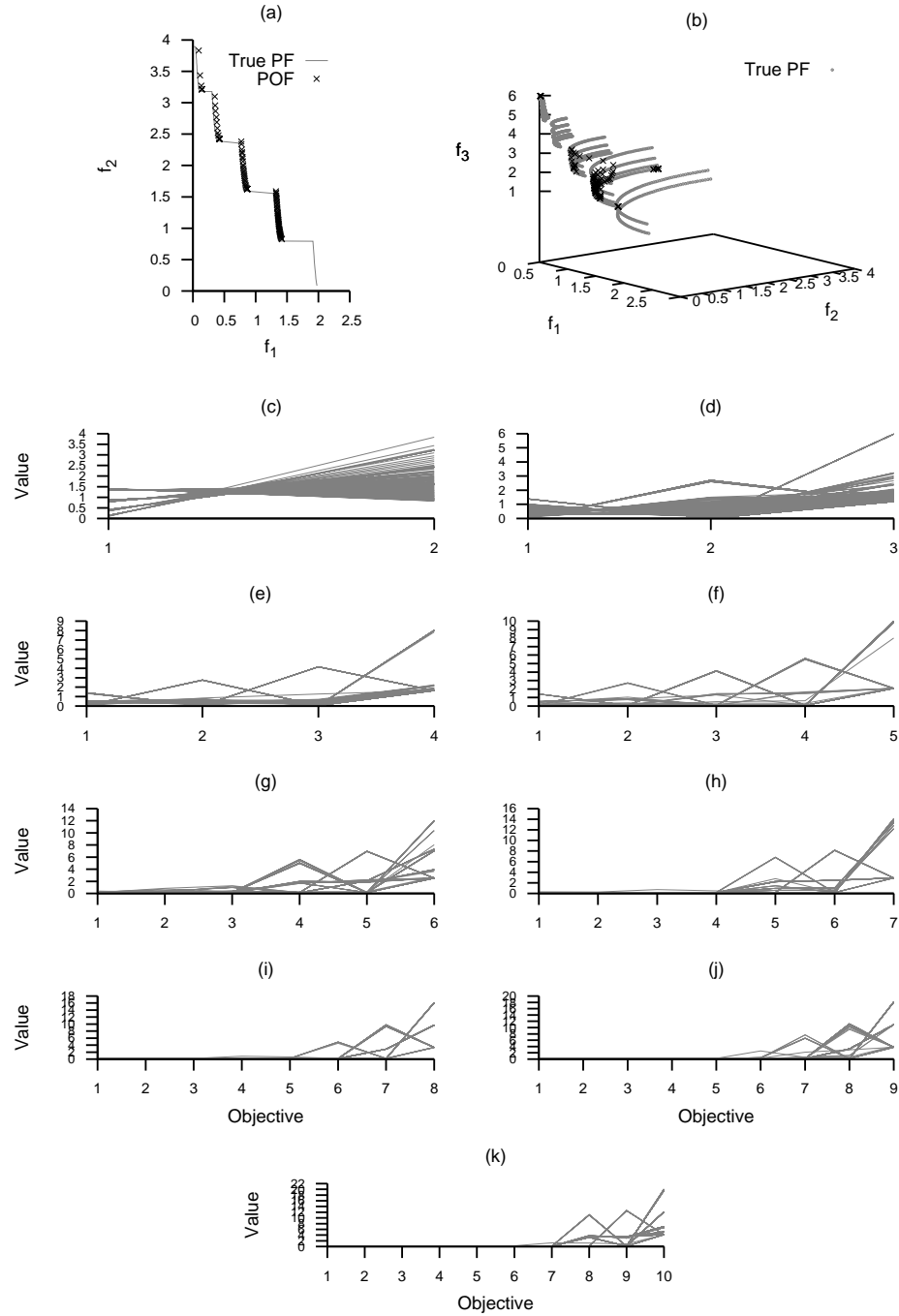


Figure A.133: Plots of the approximations obtained by MOEA/D-TCH from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

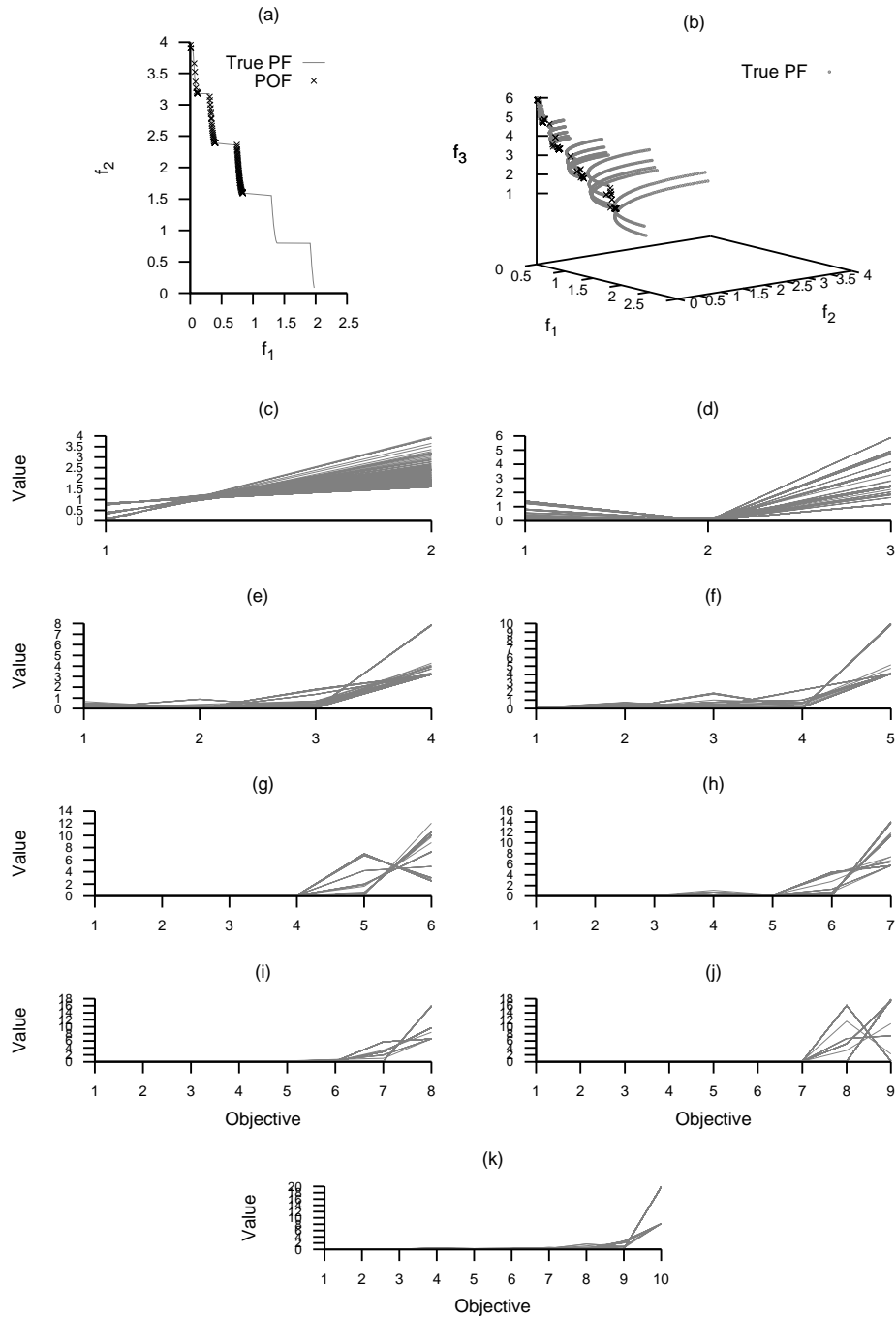


Figure A.134: Plots of the approximations obtained by MOEA/D-NTCH from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

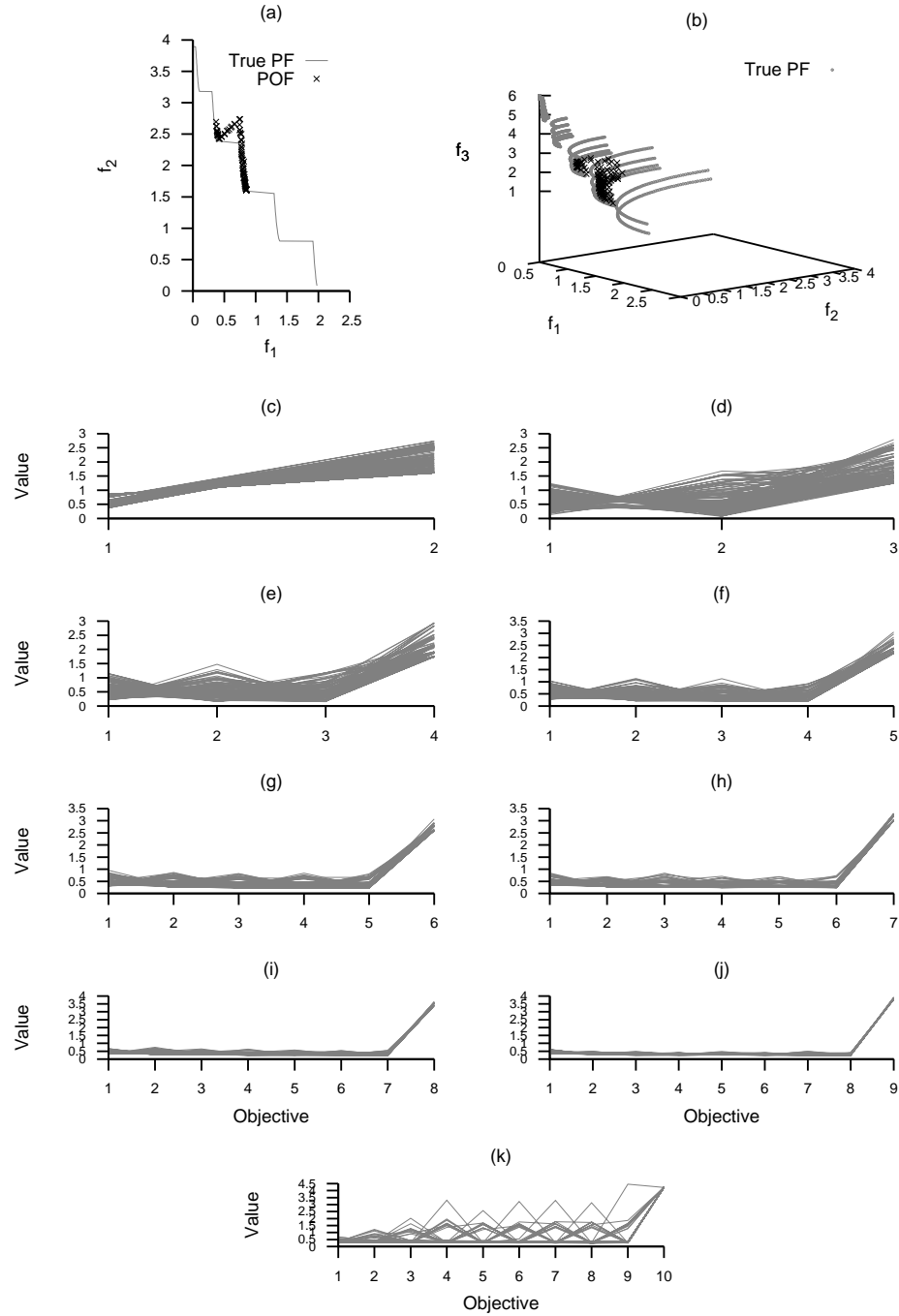


Figure A.135: Plots of the approximations obtained by MOEA/D-PBI from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

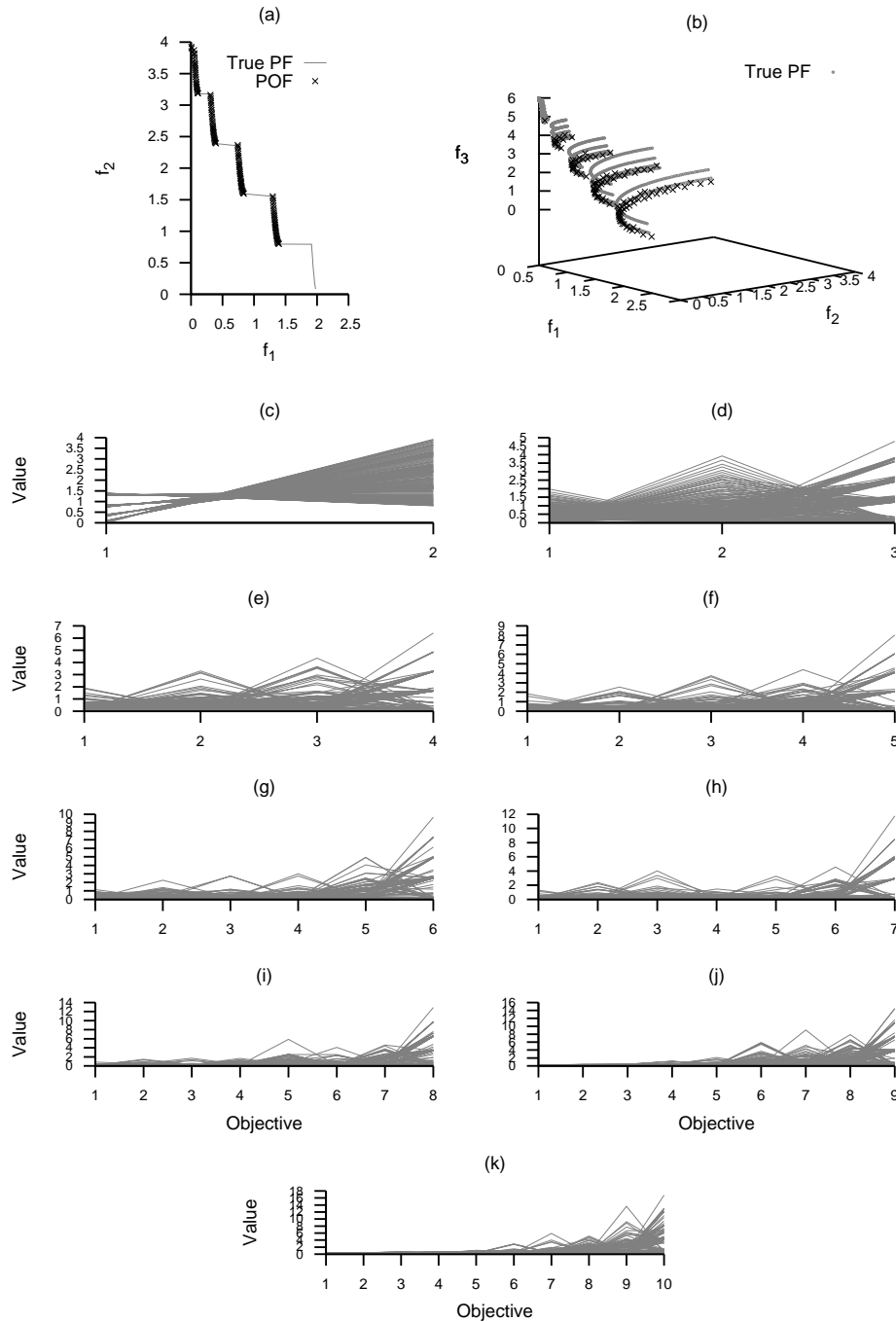


Figure A.136: Plots of the approximations obtained by SMS-EMOA from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

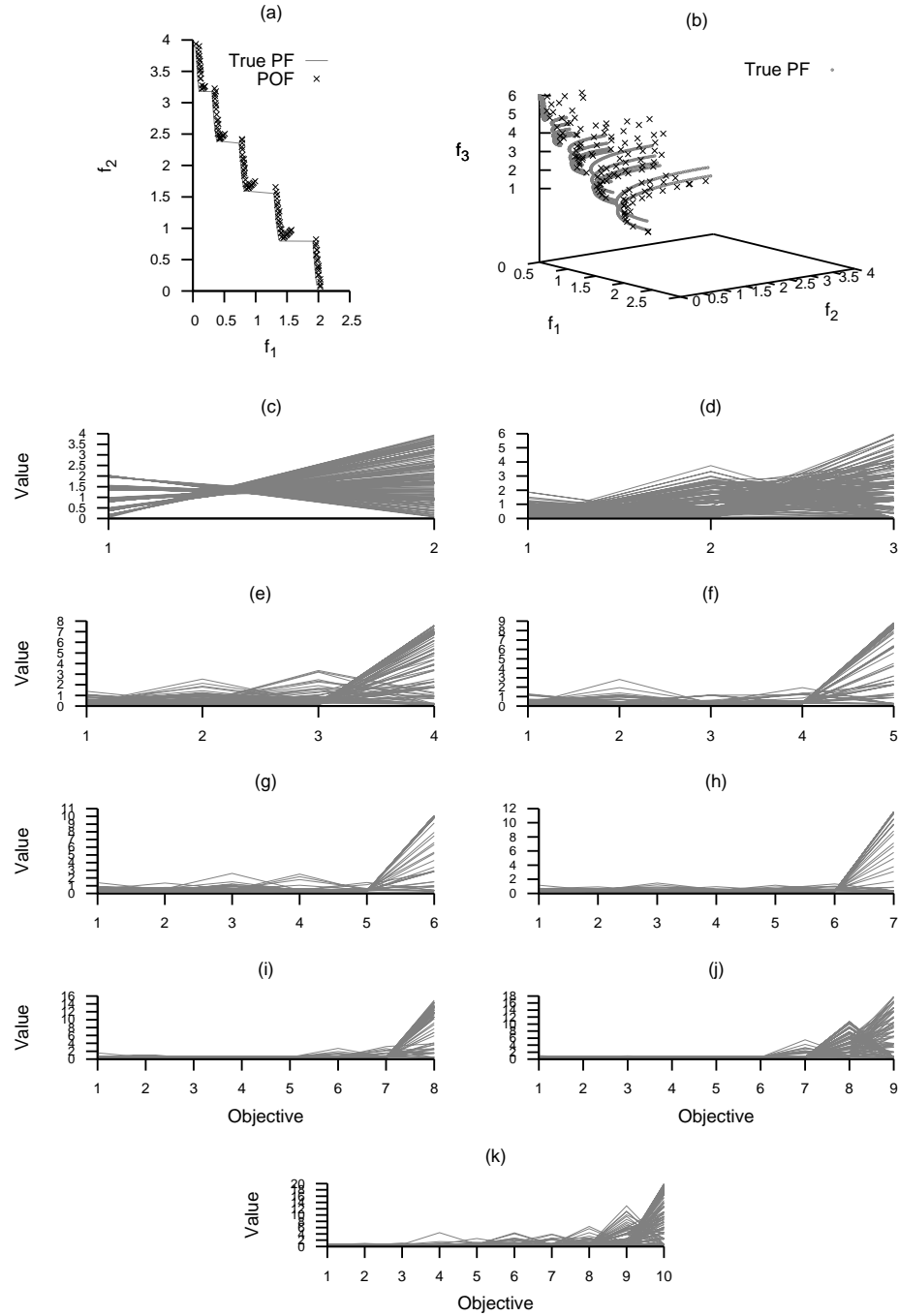


Figure A.137: Plots of the approximations obtained by  $\Delta_p$ -DDE from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

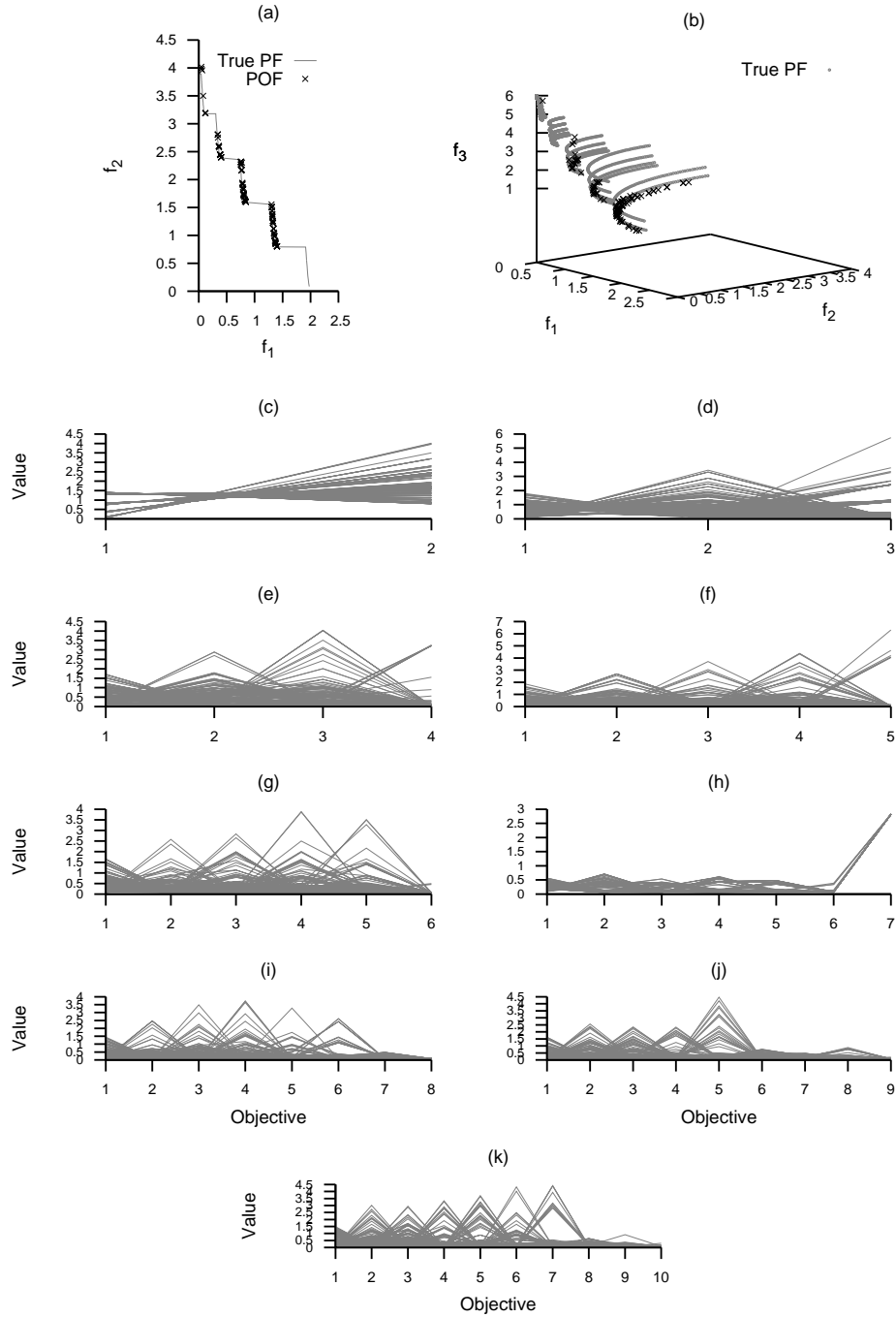


Figure A.138: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



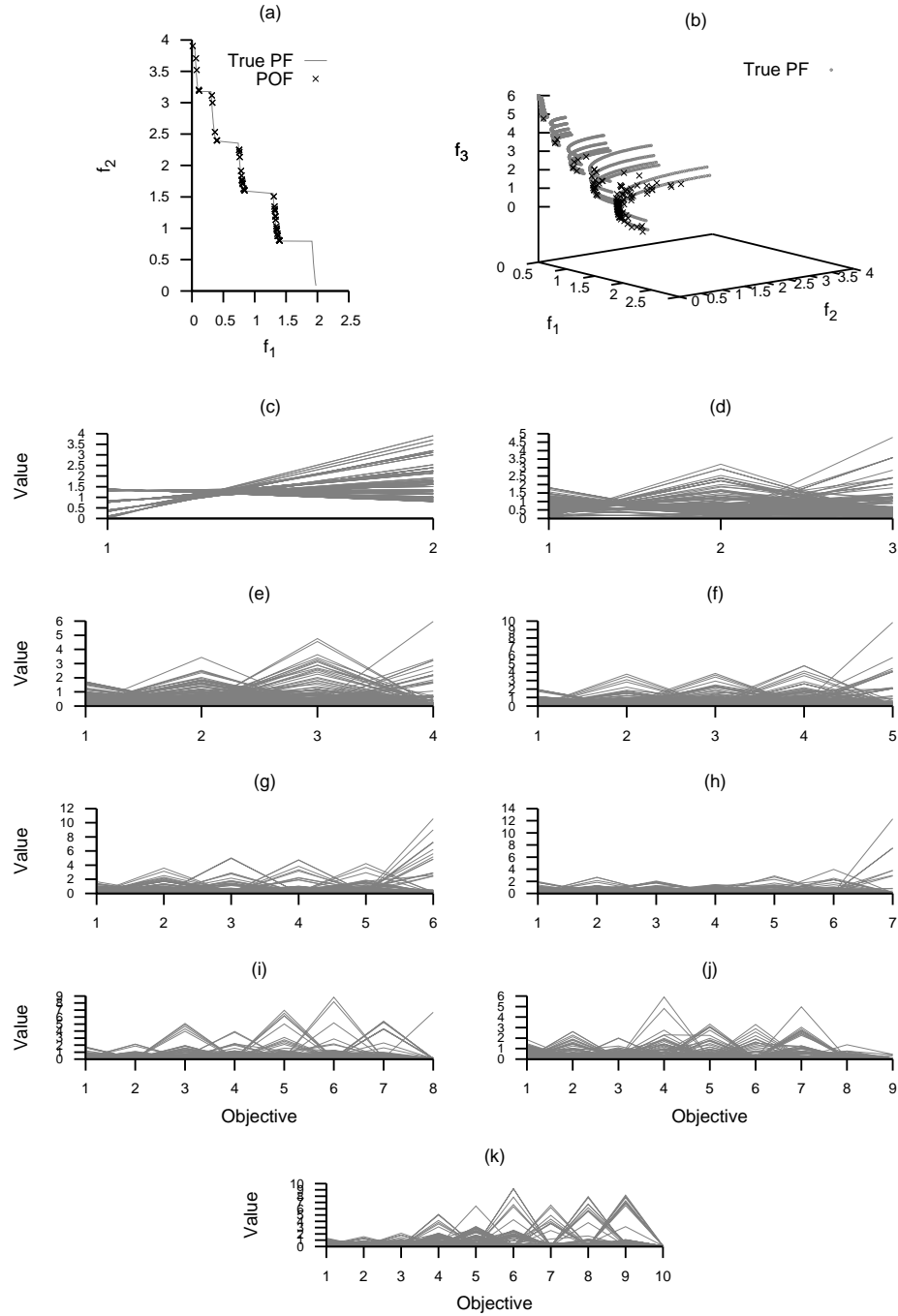


Figure A.139: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

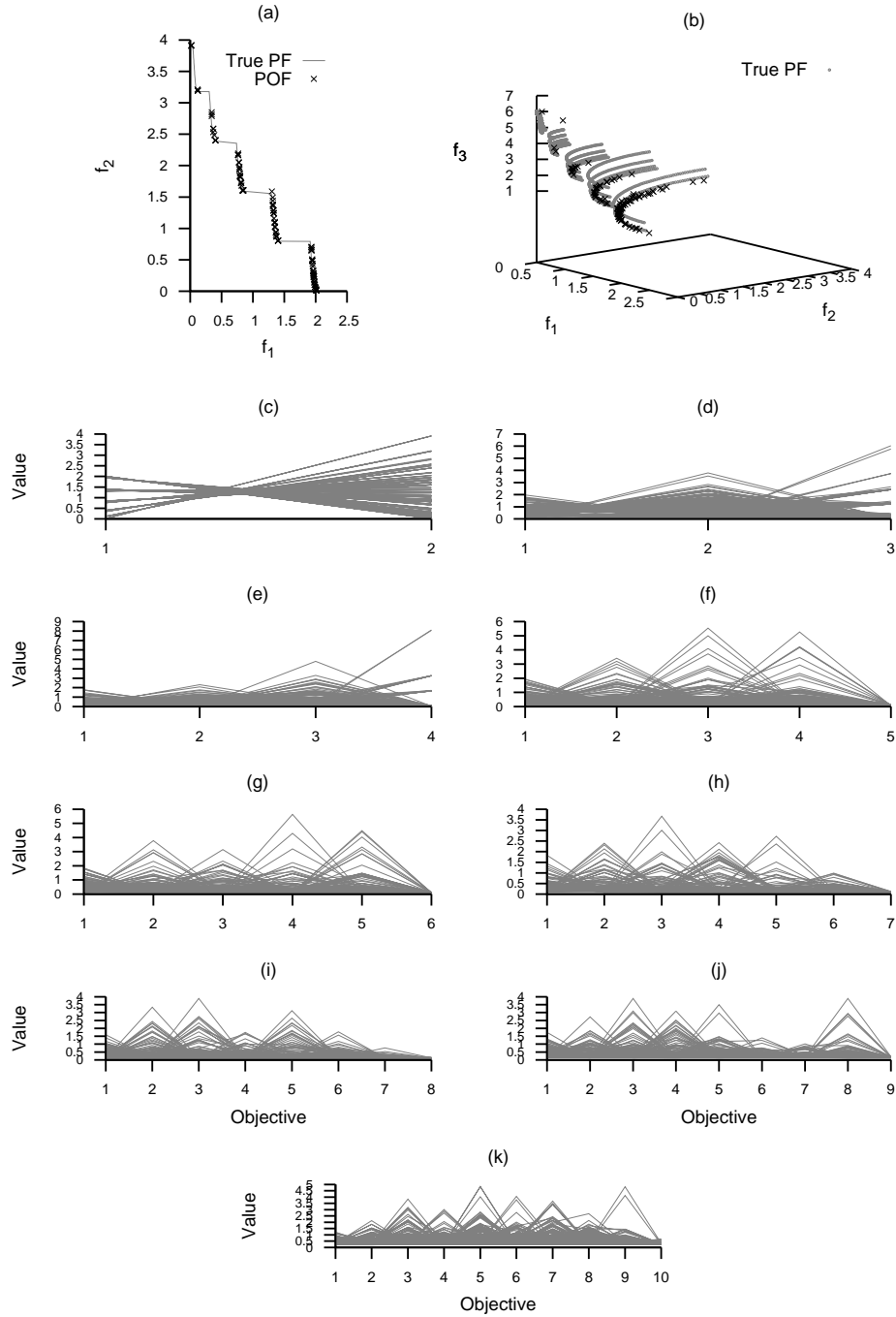


Figure A.140: Plots of the approximations obtained by *R2*-MODE from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

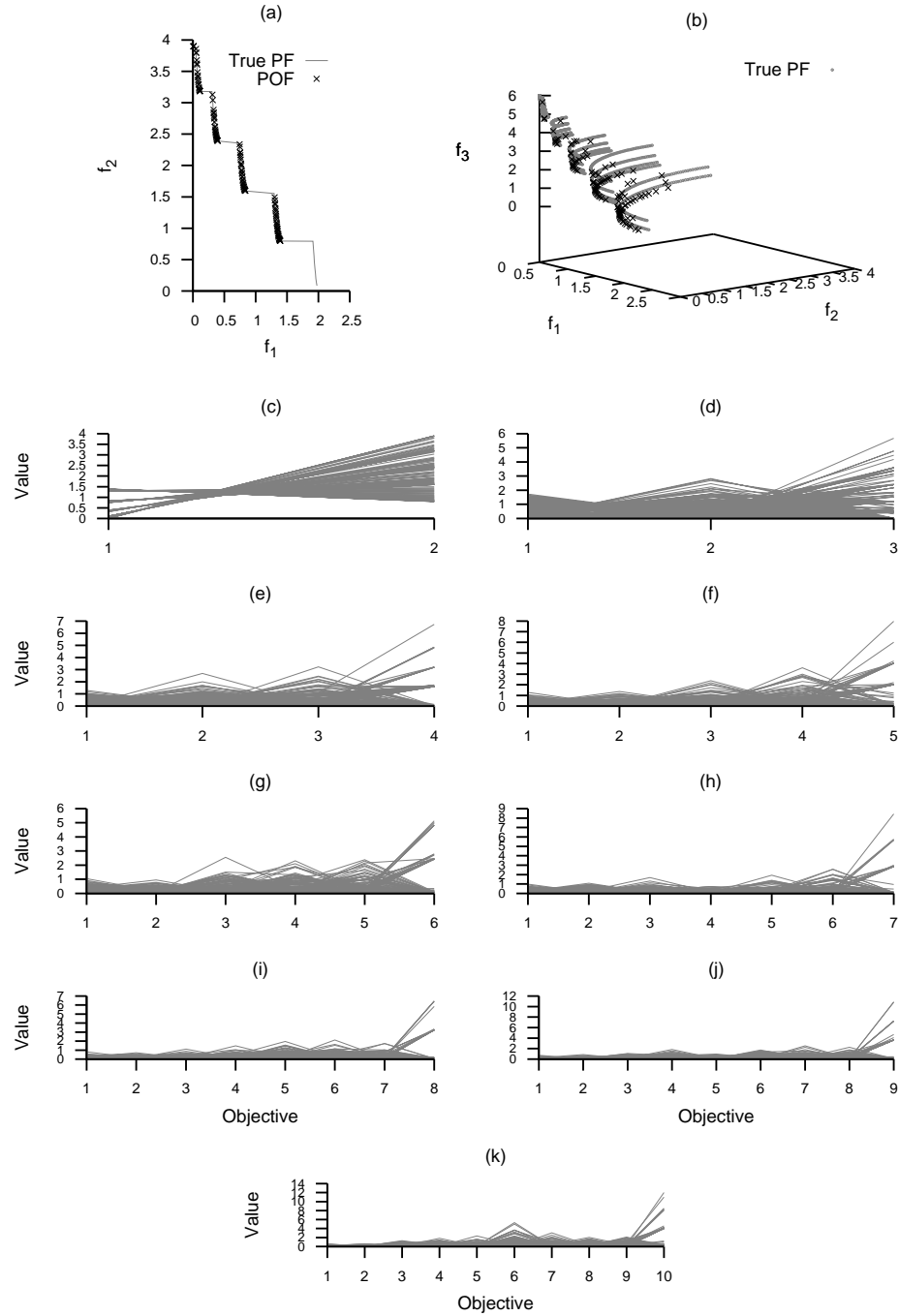


Figure A.141: Plots of the approximations obtained by *R2*-IBEA from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

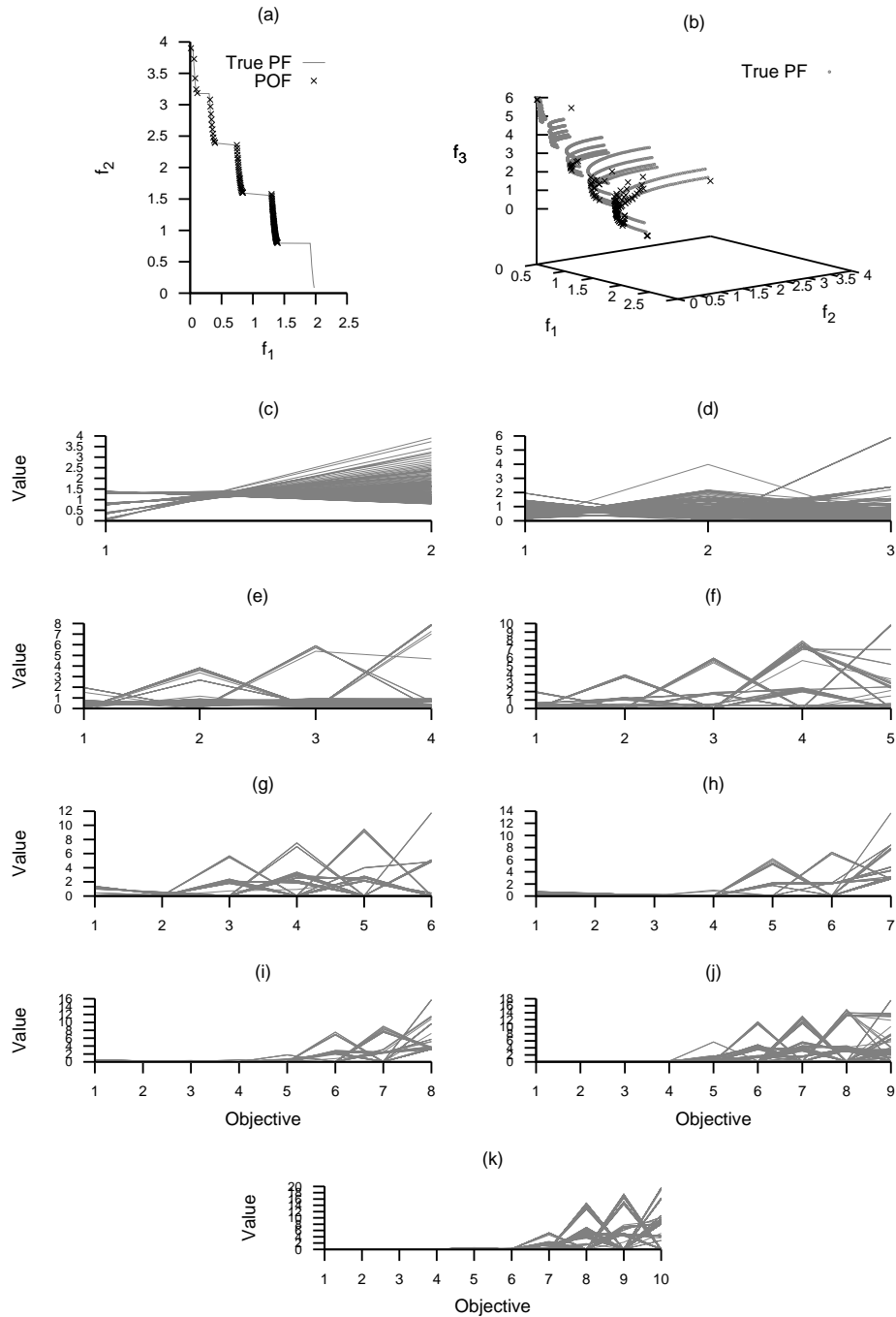


Figure A.142: Plots of the approximations obtained by MOMBI-TCH from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

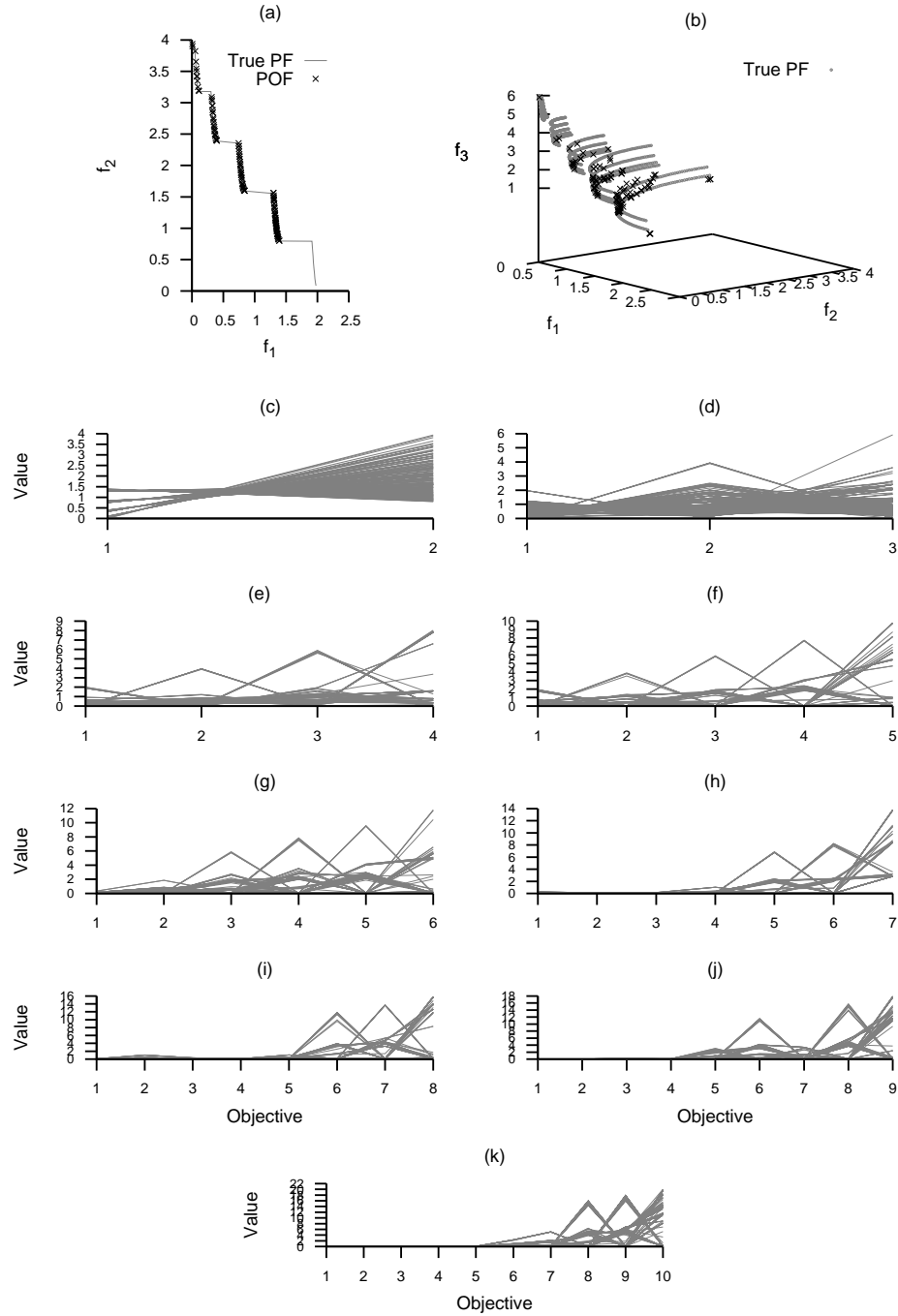


Figure A.143: Plots of the approximations obtained by MOMBI-NTCH from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

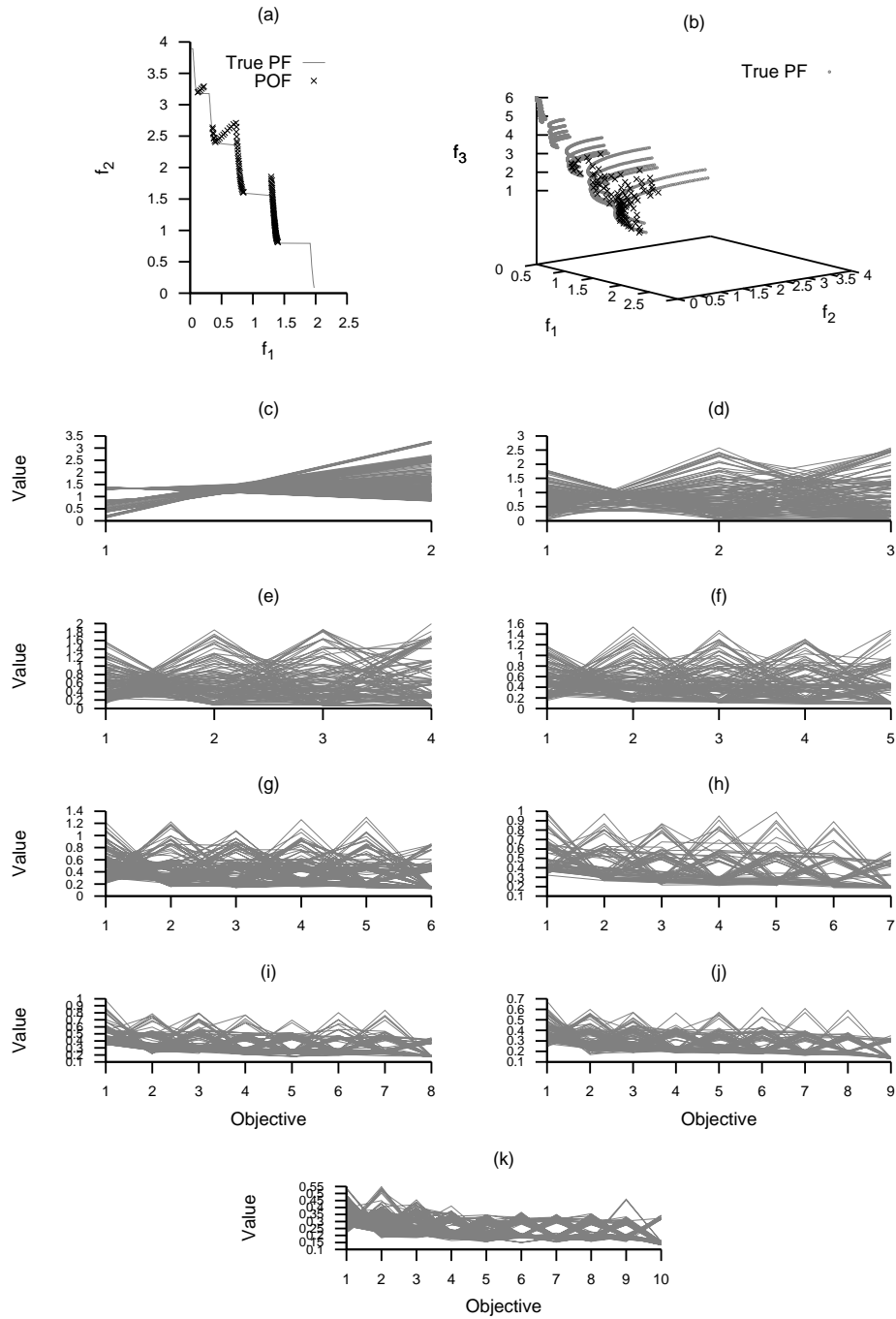


Figure A.144: Plots of the approximations obtained by MOMBI-PBI from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.56: Comparison of hypervolume indicator values for different optimizers on the WFG3 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_P$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	1.080929e+01	1.082073e+01	1.084268e+01	1.062089e+01	1.091293e+01	1.088348e+01	1.075011e+01	1.083164e+01	1.073261e+01	1.088766e+01	1.088683e+01	1.089172e+01	1.081771e+01
	avg.	1.080367e+01	1.079727e+01	1.082819e+01	1.059923e+01	1.091125e+01	1.088337e+01	1.074028e+01	1.082056e+01	1.072604e+01	1.088460e+01	1.088513e+01	1.088926e+01	1.080348e+01
	min.	1.072798e+01	1.057135e+01	1.058798e+01	9.929915e+00	1.081503e+01	1.081213e+01	1.063483e+01	1.067673e+01	1.060252e+01	1.083557e+01	1.076554e+01	1.083578e+01	1.045460e+01
	max.	1.085037e+01	1.089914e+01	1.090870e+01	1.080497e+01	1.094099e+01	1.092627e+01	1.080885e+01	1.087582e+01	1.078188e+01	1.091535e+01	1.091715e+01	1.092592e+01	1.087375e+01
	std.	2.689809e-02	7.401296e-02	5.845298e-02	1.385856e-01	1.705812e-02	1.976086e-02	3.588083e-02	3.767611e-02	3.311820e-02	1.842918e-02	1.851391e-02	1.689314e-02	6.641551e-02
3D	med.	7.396661e+01	7.358340e+01	7.466708e+01	6.768157e+01	7.605552e+01	6.598147e+01	7.397549e+01	7.437005e+01	7.062335e+01	7.525225e+01	7.413735e+01	7.525643e+01	7.291207e+01
	avg.	7.393729e+01	7.347770e+01	7.446448e+01	6.783458e+01	7.602105e+01	6.598086e+01	7.395483e+01	7.434212e+01	7.064316e+01	7.524283e+01	7.414499e+01	7.522102e+01	7.281491e+01
	min.	7.326206e+01	7.064458e+01	7.156235e+01	6.270223e+01	7.537755e+01	6.314836e+01	7.303794e+01	7.284886e+01	6.970303e+01	7.445464e+01	7.297164e+01	7.457805e+01	6.998879e+01
	max.	7.445144e+01	7.465372e+01	7.539304e+01	7.239273e+01	7.623791e+01	6.756226e+01	7.473092e+01	7.545268e+01	7.162136e+01	7.564052e+01	7.475672e+01	7.563180e+01	7.355500e+01
	std.	2.521584e-01	8.284694e-01	6.703682e-01	2.170551e+00	1.509563e-01	7.506648e-01	3.662618e-01	4.832915e-01	4.138130e-01	2.183356e-01	2.908267e-01	2.169986e-01	6.180284e-01
4D	med.	6.042265e+02	5.654308e+02	6.205386e+02	5.319824e+02	4.114227e+02	5.181768e+02	6.382255e+02	6.406672e+02	5.582757e+02	6.252561e+02	5.710341e+02	6.352570e+02	5.998385e+02
	avg.	6.043845e+02	5.714357e+02	6.201350e+02	5.365397e+02	4.114308e+02	5.193379e+02	6.380942e+02	6.402401e+02	5.580652e+02	6.230327e+02	5.749988e+02	6.356407e+02	5.991985e+02
	min.	5.834848e+02	5.409257e+02	5.908843e+02	4.938397e+02	3.963935e+02	4.885361e+02	6.239606e+02	6.203449e+02	5.398772e+02	5.953855e+02	5.534377e+02	6.219857e+02	5.739971e+02
	max.	6.239929e+02	6.136477e+02	6.408495e+02	5.861216e+02	4.333750e+02	5.477353e+02	6.494884e+02	6.547134e+02	5.793513e+02	6.410710e+02	6.124131e+02	6.532811e+02	6.119626e+02
	std.	7.684829e+00	1.788037e+01	9.858316e+00	1.918334e+01	7.080513e+00	1.149648e+01	5.686265e+00	6.453276e+00	7.170581e+00	1.019094e+01	1.299633e+01	5.479134e+00	7.264621e+00
5D	med.	5.968174e+03	5.442422e+03	6.187873e+03	5.013715e+03	4.076214e+03	4.888444e+03	6.691556e+03	6.728691e+03	5.561652e+03	6.443108e+03	5.584336e+03	6.313007e+03	5.836843e+03
	avg.	5.975131e+03	5.444623e+03	6.169376e+03	5.033589e+03	4.080444e+03	4.876523e+03	6.694540e+03	6.723385e+03	5.571339e+03	6.422983e+03	5.565168e+03	6.314265e+03	5.849068e+03
	min.	5.626657e+03	5.090863e+03	5.690648e+03	4.714730e+03	3.892292e+03	4.336813e+03	6.505695e+03	6.394385e+03	5.394724e+03	6.001860e+03	5.202548e+03	6.176961e+03	5.606006e+03
	max.	6.285037e+03	5.903073e+03	6.556340e+03	5.503081e+03	4.335865e+03	5.270422e+03	6.911828e+03	6.925501e+03	5.819613e+03	6.707774e+03	5.795084e+03	6.551263e+03	6.161033e+03
	std.	1.316193e+02	1.462548e+02	1.579990e+02	1.771060e+02	9.123342e+01	1.980250e+02	8.428345e+01	1.022821e+02	8.753428e+01	1.478931e+02	9.583778e+01	7.104411e+01	1.218805e+02
6D	med.	6.947715e+04	6.118550e+04	7.409561e+04	5.550813e+04	5.110492e+04	5.690649e+04	8.165422e+04	8.285271e+04	6.666537e+04	8.472536e+04	6.503620e+04	7.791069e+04	6.785660e+04
	avg.	6.983225e+04	6.137359e+04	7.383787e+04	5.544411e+04	5.108420e+04	5.639551e+04	8.157367e+04	8.291366e+04	6.675284e+04	8.461369e+04	6.511463e+04	7.792368e+04	6.778879e+04
	min.	6.552391e+04	5.804626e+04	6.798345e+04	4.977190e+04	4.779721e+04	5.036483e+04	7.848407e+04	7.889210e+04	6.455816e+04	8.011002e+04	6.148506e+04	7.550737e+04	6.484860e+04
	max.	7.387094e+04	6.809099e+04	8.031128e+04	6.241685e+04	5.367779e+04	6.415447e+04	8.487010e+04	8.672255e+04	6.962332e+04	8.757251e+04	7.206598e+04	8.072205e+04	7.080093e+04
	std.	1.733108e+03	1.912985e+03	2.136428e+03	2.291702e+03	1.251632e+03	2.992801e+03	1.442275e+03	1.406138e+03	1.010859e+03	1.459085e+03	1.500636e+03	9.474970e+02	1.390665e+03
7D	med.	9.691010e+05	8.122205e+05	1.000734e+06	6.969254e+05	6.265543e+05	7.238729e+05	1.055682e+06	1.157163e+06	8.958505e+05	1.266649e+06	8.939361e+05	1.063537e+06	9.278825e+05
	avg.	9.713403e+05	8.145525e+05	1.005046e+06	6.996527e+05	6.264764e+05	7.234293e+05	1.059337e+06	1.154533e+06	8.991558e+05	1.266179e+06	8.933735e+05	1.061432e+06	9.040661e+05
	min.	9.119627e+05	7.538223e+05	9.292635e+05	5.933325e+05	5.684170e+05	6.888311e+05	9.988626e+05	1.035376e+06	8.635073e+05	1.236996e+06	8.240472e+05	9.999701e+05	7.244090e+05
	max.	1.052841e+06	9.011238e+05	1.111506e+06	7.843637e+05	6.777921e+05	7.558265e+05	1.123123e+06	1.217087e+06	9.430463e+05	1.289550e+06	9.464357e+05	1.135122e+06	9.788843e+05
	std.	2.895581e+04	2.892285e+04	3.084000e+04	4.503282e+04	2.346811e+04	1.388153e+04	2.696504e+04	3.132954e+04	1.840501e+04	1.120998e+04	2.476407e+04	2.160473e+04	6.439099e+04
8D	med.	1.573432e+07	1.349834e+07	1.666934e+07	9.277903e+06	1.239844e+07	1.209251e+07	1.757073e+07	1.882158e+07	1.491623e+07	2.135198e+07	1.496593e+07	1.767501e+07	1.186300e+07
	avg.	1.575243e+07	1.361386e+07	1.664953e+07	9.503001e+06	1.237393e+07	1.209596e+07	1.759909e+07	1.883578e+07	1.494786e+07	2.131521e+07	1.501481e+07	1.770375e+07	1.174053e+07
	min.	1.440350e+07	1.249661e+07	1.532744e+07	7.614940e+06	1.160241e+07	1.162767e+07	1.650301e+07	1.764494e+07	1.442826e+07	1.987108e+07	1.409654e+07	1.701365e+07	7.658874e+06
	max.	1.681993e+07	1.525898e+07	1.787993e+07	1.151703e+07	1.323281e+07	1.251061e+07	1.884562e+07	2.005961e+07	1.564438e+07	2.172340e+07	1.590545e+07	1.869919e+07	1.417287e+07
	std.	4.362249e+05	5.697359e+05	4.689376e+05	1.019703e+06	3.299865e+05	1.862970e+05	4.400014e+05	5.373520e+05	2.619275e+05	2.685813e+05	3.596242e+05	3.377660e+05	1.271330e+06
9D	med.	2.945376e+08	2.494244e+08	3.092004e+08	1.334286e+08	2.627673e+08	2.258205e+08	3.274848e+08	3.434477e+08	2.789685e+08	3.996065e+08	2.839948e+08	3.356152e+08	2.099746e+08
	avg.	2.951807e+08	2.500507e+08	3.089239e+08	1.424420e+08	2.629889e+08	2.255183e+08	3.278326e+08	3.433815e+08	2.792802e+08	3.982823e+08	2.845720e+08	3.355348e+08	2.064797e+08
	min.	2.802249e+08	2.300386e+08	2.843121e+08	1.227793e+08	2.539128e+08	2.125701e+08	3.096017e+08	3.212501e+08	2.706198e+08	3.769503e+08	2.613939e+08	3.167355e+08	1.239402e+08
	max.	3.182473e+08	2.760080e+08	3.346849e+08	1.952890e+08	7.225135e+08	3.359073e+08	3.482528e+08	3.657256e+08	2.889755e+08	4.050716e+08	3.087636e+08	3.503008e+08	2.516549e+08
	std.	7.066818e+06	1.034951e+07	9.743760e+06	1.864563e+07	4.183469e+06	4.008788e+06	8.001253e+06	9.617603e+06	3.753466e+06	4.545478e+06	8.801272e+06	2.675900e+06	2.008803e+07
10D	med.	6.143779e+09	5.263896e+09	6.410566e+09	2.425768e+09	7.722317e+09	4.676175e+09	6.654170e+09	6.944757e+09	5.765740e+09	8.147315e+09	5.945838e+09	7.021610e+09	3.909088e+09
	avg.	6.130052e+09	5.279773e+09	6.400860e+09	2.428676e+09	7.443058e+09	4.671773e+09	6.648185e+09	6.937659e+09	5.769412e+09	8.136016e+09	6.098895e+09	7.065997e+09	3.750224e+09
	min.	5.827035e+09	4.824468e+09	5.949479e+09	2.236274e+09	5.892548e+09	4.528799e+09	6.338627e+09	6.537097e+09	5.620850e+09	7.651215e+09	5.575701e+09	6.651744e+09	2.065892e+09
	max.	6.395603e+09	6.091023e+09	6.936544e+09	2.657650e+09	7.962092e+09	4.882896e+09	7.018640e+09	7.613115e+09	5.951529e+09	8.341356e+09	7.085086e+09	7.662851e+09	4.564315e+09
	std.	1.250371e+08	2.428456e+08	2.338107e+08	7.984825e+07	6.327740e+08	7.032866e+07	1.345885e+08	1.727941e+08	6.603764e+07	1.175515e+08	4.031995e+08	2.186951e+08	6.198866e+08

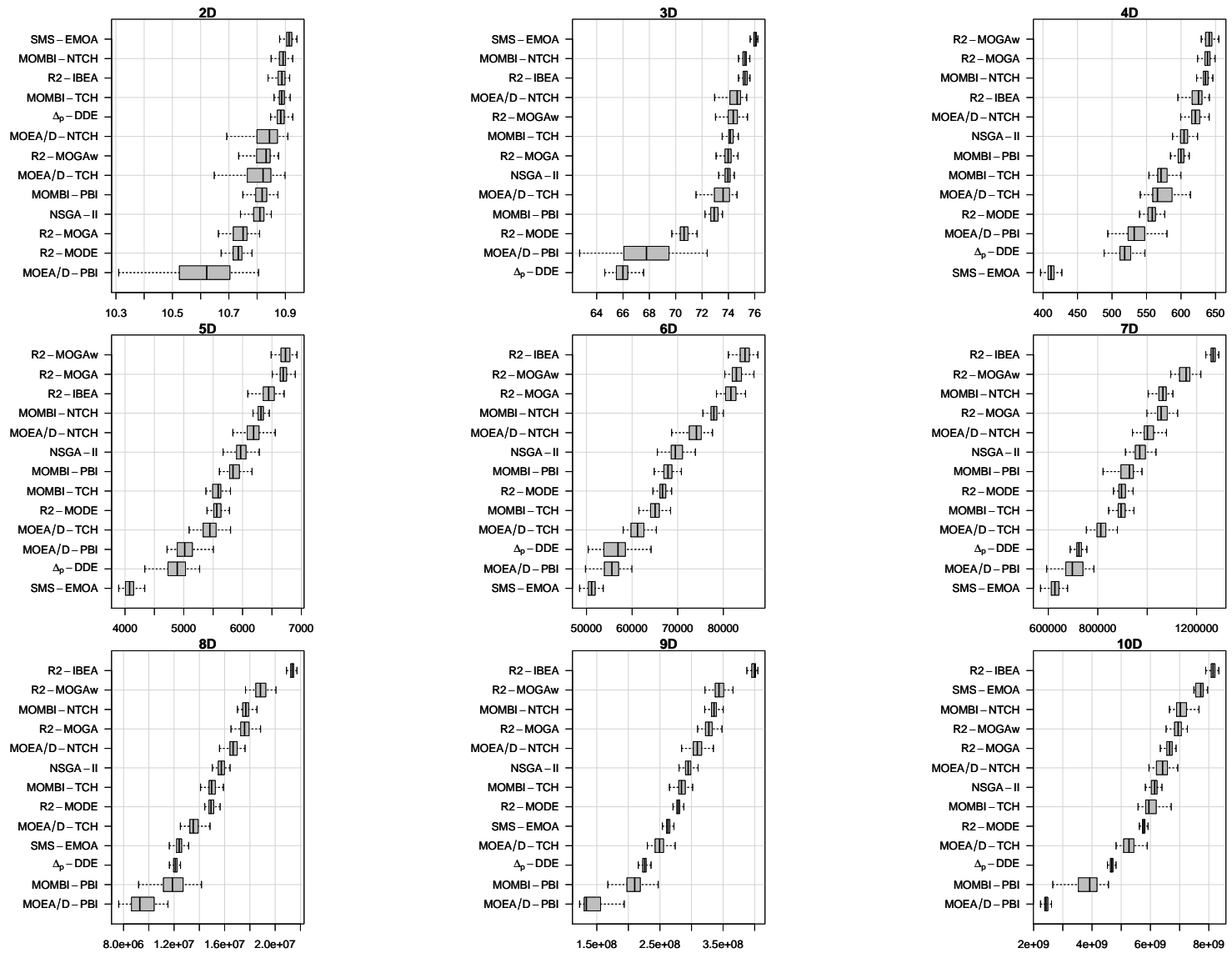


Figure A.145: Box-plot of hypervolume indicator values for different optimizers on the WFG3 test problem.



Table A.57: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG3 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D													
NSGA-II	—	> 0.05	> 0.05	6.18e-32	> 0.05	> 0.05	5.86e-26	> 0.05	3.44e-31	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	2.88e-02	—	> 0.05	1.44e-25	> 0.05	> 0.05	7.41e-14	> 0.05	7.29e-17	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	4.36e-07	3.62e-04	—	2.16e-30	> 0.05	> 0.05	1.91e-22	6.84e-03	4.89e-26	> 0.05	> 0.05	> 0.05	8.82e-05
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	3.88e-34	2.51e-33	2.48e-31	1.28e-34	—	8.19e-21	1.28e-34	1.27e-33	1.28e-34	3.26e-21	1.47e-23	4.83e-19	6.83e-34
$\Delta_p$ -DDE	1.07e-33	2.22e-27	4.94e-17	1.28e-34	> 0.05	—	1.28e-34	6.60e-30	1.28e-34	> 0.05	> 0.05	> 0.05	7.67e-32
R2-MOGA	> 0.05	> 0.05	> 0.05	2.02e-18	> 0.05	> 0.05	—	> 0.05	2.62e-04	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	2.49e-06	> 0.05	> 0.05	3.78e-32	> 0.05	> 0.05	4.08e-27	—	3.38e-30	> 0.05	> 0.05	> 0.05	1.82e-02
R2-MODE	> 0.05	> 0.05	> 0.05	3.74e-15	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	2.13e-34	4.41e-28	7.72e-18	1.28e-34	> 0.05	> 0.05	1.28e-34	1.23e-30	1.28e-34	—	> 0.05	> 0.05	2.67e-32
MOMBI-TCH	2.82e-33	3.98e-29	1.57e-19	1.63e-34	> 0.05	> 0.05	2.63e-34	2.86e-31	1.49e-34	> 0.05	—	> 0.05	2.92e-32
MOMBI-NTCH	1.95e-34	5.21e-30	3.74e-21	1.28e-34	> 0.05	6.22e-03	1.28e-34	2.60e-32	1.28e-34	3.58e-02	3.05e-02	—	2.30e-33
MOMBI-PBI	6.61e-03	> 0.05	> 0.05	6.34e-29	> 0.05	> 0.05	3.67e-23	> 0.05	2.23e-25	> 0.05	> 0.05	> 0.05	—
3D													
NSGA-II	—	3.87e-05	> 0.05	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	5.88e-34
MOEA/D-TCH	> 0.05	—	> 0.05	3.05e-34	> 0.05	1.28e-34	> 0.05	> 0.05	1.71e-33	> 0.05	> 0.05	> 0.05	6.92e-11
MOEA/D-NTCH	6.61e-13	1.51e-17	—	1.40e-34	> 0.05	1.28e-34	5.37e-12	3.97e-03	1.36e-34	> 0.05	1.78e-08	> 0.05	2.16e-30
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	> 0.05	1.54e-11	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	1.28e-34	1.28e-34	1.32e-34	1.28e-34	—	1.28e-34	1.28e-34	1.32e-34	1.28e-34	4.00e-34	1.28e-34	2.96e-34	1.28e-34
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	> 0.05	1.23e-05	> 0.05	1.28e-34	> 0.05	1.28e-34	—	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	1.89e-31
R2-MOGAw	5.94e-12	7.83e-16	> 0.05	1.28e-34	> 0.05	1.28e-34	1.03e-09	—	1.28e-34	> 0.05	9.96e-05	> 0.05	7.01e-33
R2-MODE	> 0.05	> 0.05	> 0.05	4.34e-22	> 0.05	1.28e-34	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	1.28e-34	1.68e-34	5.84e-25	1.28e-34	> 0.05	1.28e-34	1.58e-34	9.55e-31	1.28e-34	—	1.84e-34	> 0.05	1.28e-34
MOMBI-TCH	5.32e-08	2.84e-11	> 0.05	1.28e-34	> 0.05	1.28e-34	7.81e-05	> 0.05	1.28e-34	> 0.05	—	> 0.05	5.54e-34
MOMBI-NTCH	1.28e-34	1.84e-34	2.57e-24	1.28e-34	> 0.05	1.28e-34	1.49e-34	6.42e-30	1.28e-34	> 0.05	1.84e-34	—	1.28e-34
MOMBI-PBI	> 0.05	> 0.05	> 0.05	9.75e-34	> 0.05	1.28e-34	> 0.05	> 0.05	6.36e-32	> 0.05	> 0.05	> 0.05	—
4D													
NSGA-II	—	7.78e-26	> 0.05	1.36e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	1.28e-34	> 0.05	4.35e-28	> 0.05	3.33e-06
MOEA/D-TCH	> 0.05	—	> 0.05	2.42e-23	1.28e-34	1.44e-34	> 0.05	> 0.05	1.86e-08	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	7.30e-22	4.26e-33	—	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	1.28e-34	> 0.05	1.57e-33	> 0.05	2.40e-28
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	1.28e-34	1.85e-11	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	1.28e-34	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.32e-34	1.28e-34	1.09e-28	1.28e-34	1.28e-34	1.28e-34	—	> 0.05	1.28e-34	1.72e-24	1.28e-34	3.34e-04	1.28e-34
R2-MOGAw	1.49e-34	1.28e-34	2.10e-30	1.28e-34	1.28e-34	1.28e-34	1.39e-02	—	1.28e-34	1.03e-27	1.28e-34	9.05e-08	1.28e-34
R2-MODE	> 0.05	> 0.05	> 0.05	6.19e-17	1.28e-34	1.73e-34	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	2.70e-24	1.61e-33	1.16e-02	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	1.28e-34	—	7.03e-34	> 0.05	7.18e-30
MOMBI-TCH	> 0.05	1.18e-03	> 0.05	6.51e-28	1.28e-34	1.28e-34	> 0.05	> 0.05	6.30e-25	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.36e-34	1.28e-34	5.02e-26	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	1.28e-34	1.70e-20	1.28e-34	—	1.28e-34
MOMBI-PBI	> 0.05	2.64e-21	> 0.05	1.89e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	1.36e-34	> 0.05	5.98e-24	> 0.05	—

Table A.58: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG3 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
5D													
NSGA-II	—	$6.24e-34$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$2.58e-33$	> 0.05	$1.81e-33$	> 0.05	$2.75e-11$
MOEA/D-TCH	> 0.05	—	> 0.05	$3.87e-29$	$1.28e-34$	$5.07e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	$3.99e-16$	$1.73e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$1.63e-34$	> 0.05	$1.49e-34$	> 0.05	$4.66e-27$
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	$1.28e-34$	$2.55e-07$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$1.28e-34$	$1.28e-34$	$1.49e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	> 0.05	$1.28e-34$	$6.07e-30$	$1.28e-34$	$1.73e-34$	$1.28e-34$
R2-MOGAw	$1.28e-34$	$1.28e-34$	$1.63e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$9.10e-03$	—	$1.28e-34$	$2.94e-30$	$1.28e-34$	$2.13e-34$	$1.28e-34$
R2-MODE	> 0.05	$1.65e-11$	> 0.05	$2.55e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	$1.50e-32$	$1.28e-34$	$1.10e-20$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$1.28e-34$	—	$1.28e-34$	$1.98e-11$	$2.79e-34$
MOMBI-TCH	> 0.05	$8.00e-11$	> 0.05	$5.88e-34$	$1.28e-34$	$1.36e-34$	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	$2.66e-33$	$1.28e-34$	$8.98e-14$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	—	$1.28e-34$
MOMBI-PBI	> 0.05	$1.22e-32$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$2.16e-30$	> 0.05	$3.07e-31$	> 0.05	—
6D													
NSGA-II	—	$2.01e-34$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$9.22e-28$	> 0.05	$2.38e-32$	> 0.05	$5.71e-15$
MOEA/D-TCH	> 0.05	—	> 0.05	$2.52e-32$	$1.28e-34$	$3.90e-25$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	$6.46e-25$	$1.32e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$2.63e-34$	> 0.05	$2.33e-34$	> 0.05	$8.35e-33$
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	$8.79e-29$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	$8.92e-03$	$2.11e-29$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$1.28e-34$	$1.28e-34$	$2.33e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	$7.22e-33$	$1.28e-34$
R2-MOGAw	$1.28e-34$	$1.28e-34$	$1.36e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-09$	—	$1.28e-34$	> 0.05	$1.28e-34$	$3.34e-34$	$1.28e-34$
R2-MODE	> 0.05	$2.91e-33$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	—	> 0.05	$4.23e-16$	> 0.05	> 0.05
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.32e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$5.15e-26$	$2.22e-14$	$1.28e-34$	—	$1.28e-34$	$1.36e-34$	$1.28e-34$
MOMBI-TCH	> 0.05	$2.16e-27$	> 0.05	$1.32e-34$	$1.28e-34$	$3.15e-34$	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$2.24e-31$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	—	$1.28e-34$
MOMBI-PBI	> 0.05	$1.07e-33$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$1.39e-08$	> 0.05	$6.44e-24$	> 0.05	—
7D													
NSGA-II	—	$1.28e-34$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$7.43e-33$	> 0.05	$6.23e-33$	> 0.05	$1.29e-20$
MOEA/D-TCH	> 0.05	—	> 0.05	$8.92e-34$	$1.28e-34$	$1.32e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	$6.50e-13$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$1.73e-34$	> 0.05	$1.68e-34$	> 0.05	$5.35e-32$
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	$1.60e-24$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	$3.39e-05$	$1.28e-34$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$7.88e-33$	$1.28e-34$	$5.55e-24$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$
R2-MOGAw	$1.36e-34$	$1.28e-34$	$2.48e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$4.79e-33$	—	$1.28e-34$	> 0.05	$1.28e-34$	$5.39e-33$	$1.28e-34$
R2-MODE	> 0.05	$1.48e-33$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	> 0.05	$1.50e-31$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	$1.71e-33$	$1.28e-34$	$1.20e-26$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	—	$1.28e-34$
MOMBI-PBI	> 0.05	$1.81e-18$	> 0.05	$6.61e-33$	$1.28e-34$	$1.81e-33$	> 0.05	> 0.05	$8.96e-07$	> 0.05	$2.33e-07$	> 0.05	—

Table A.59: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG3 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
8D													
NSGA-II	–	3.34e – 34	> 0.05	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	> 0.05	5.25e – 28	> 0.05	6.76e – 24	> 0.05	1.28e – 34
MOEA/D-TCH	> 0.05	–	> 0.05	1.28e – 34	8.35e – 33	1.32e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	5.69e – 28
MOEA/D-NTCH	1.48e – 24	1.28e – 34	–	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	> 0.05	1.73e – 34	> 0.05	3.65e – 34	> 0.05	1.28e – 34
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	1.28e – 34	–	3.39e – 12	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	2.67e – 04
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	1.28e – 34	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	2.07e – 34	1.28e – 34	5.60e – 27	1.28e – 34	1.28e – 34	1.28e – 34	–	> 0.05	1.28e – 34	> 0.05	1.28e – 34	> 0.05	1.28e – 34
R2-MOGAw	1.28e – 34	1.28e – 34	1.49e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.29e – 29	–	1.28e – 34	> 0.05	1.28e – 34	2.00e – 29	1.28e – 34
R2-MODE	> 0.05	1.30e – 30	> 0.05	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	1.28e – 34
R2-IBEA	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.36e – 34	1.28e – 34	–	1.28e – 34	1.28e – 34	1.28e – 34
MOMBI-TCH	> 0.05	1.54e – 30	> 0.05	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	1.36e – 34
MOMBI-NTCH	1.28e – 34	1.28e – 34	1.84e – 31	1.28e – 34	1.28e – 34	1.28e – 34	2.73e – 02	> 0.05	1.28e – 34	> 0.05	1.28e – 34	–	1.28e – 34
MOMBI-PBI	> 0.05	> 0.05	> 0.05	1.76e – 23	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
9D													
NSGA-II	–	1.28e – 34	> 0.05	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	> 0.05	1.19e – 32	> 0.05	8.59e – 17	> 0.05	1.28e – 34
MOEA/D-TCH	> 0.05	–	> 0.05	1.28e – 34	> 0.05	3.88e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	7.01e – 33
MOEA/D-NTCH	2.92e – 19	1.28e – 34	–	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	> 0.05	1.95e – 34	> 0.05	2.94e – 30	> 0.05	1.28e – 34
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	2.58e – 21	> 0.05	1.28e – 34	–	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.28e – 34
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	1.28e – 34	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.47e – 19
R2-MOGA	2.26e – 34	1.28e – 34	1.89e – 27	1.28e – 34	1.28e – 34	1.28e – 34	–	> 0.05	1.28e – 34	> 0.05	1.28e – 34	> 0.05	1.28e – 34
R2-MOGAw	1.28e – 34	1.28e – 34	5.71e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.09e – 21	–	1.28e – 34	> 0.05	1.28e – 34	3.32e – 10	1.28e – 34
R2-MODE	> 0.05	3.88e – 34	> 0.05	1.28e – 34	1.89e – 34	1.28e – 34	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	1.28e – 34
R2-IBEA	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	–	1.28e – 34	1.28e – 34	1.28e – 34
MOMBI-TCH	> 0.05	1.10e – 33	> 0.05	1.28e – 34	7.64e – 33	1.28e – 34	> 0.05	> 0.05	1.47e – 08	> 0.05	–	> 0.05	1.28e – 34
MOMBI-NTCH	1.32e – 34	1.28e – 34	1.48e – 33	1.28e – 34	1.28e – 34	1.28e – 34	5.94e – 12	> 0.05	1.28e – 34	> 0.05	1.28e – 34	–	1.28e – 34
MOMBI-PBI	> 0.05	> 0.05	> 0.05	2.00e – 31	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
10D													
NSGA-II	–	4.64e – 34	> 0.05	1.28e – 34	> 0.05	1.28e – 34	> 0.05	> 0.05	8.16e – 34	> 0.05	8.29e – 06	> 0.05	1.28e – 34
MOEA/D-TCH	> 0.05	–	> 0.05	1.28e – 34	> 0.05	1.40e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.28e – 34
MOEA/D-NTCH	3.07e – 16	1.89e – 34	–	1.28e – 34	> 0.05	1.28e – 34	> 0.05	> 0.05	1.32e – 34	> 0.05	2.18e – 11	> 0.05	1.28e – 34
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	2.87e – 25	1.53e – 34	3.83e – 18	1.28e – 34	–	1.28e – 34	1.56e – 14	2.28e – 14	1.63e – 34	> 0.05	2.97e – 27	4.14e – 14	1.28e – 34
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	1.28e – 34	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.68e – 34
R2-MOGA	1.49e – 34	1.28e – 34	2.08e – 14	1.28e – 34	> 0.05	1.28e – 34	–	> 0.05	1.28e – 34	> 0.05	8.24e – 17	> 0.05	1.28e – 34
R2-MOGAw	1.28e – 34	1.28e – 34	9.28e – 31	1.28e – 34	> 0.05	1.28e – 34	2.09e – 23	–	1.28e – 34	> 0.05	2.75e – 27	> 0.05	1.28e – 34
R2-MODE	> 0.05	6.07e – 30	> 0.05	1.28e – 34	> 0.05	1.28e – 34	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	1.28e – 34
R2-IBEA	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	5.87e – 33	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	–	1.28e – 34	1.36e – 34	1.28e – 34
MOMBI-TCH	> 0.05	2.06e – 32	> 0.05	1.28e – 34	> 0.05	1.28e – 34	> 0.05	> 0.05	2.84e – 16	> 0.05	–	> 0.05	1.28e – 34
MOMBI-NTCH	1.28e – 34	1.28e – 34	2.58e – 33	1.28e – 34	> 0.05	1.28e – 34	2.55e – 30	1.10e – 04	1.28e – 34	> 0.05	5.00e – 30	–	1.28e – 34
MOMBI-PBI	> 0.05	> 0.05	> 0.05	7.02e – 26	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–

Table A.60: Comparison of R2 indicator values for different optimizers on the WFG3 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	4.612197e-01	4.605219e-01	4.603354e-01	4.680769e-01	4.582305e-01	4.593487e-01	4.630775e-01	4.613851e-01	4.660209e-01	4.582911e-01	4.591221e-01	4.593078e-01	4.617658e-01
	avg.	4.615176e-01	4.614054e-01	4.610143e-01	4.695080e-01	4.583354e-01	4.593760e-01	4.632266e-01	4.616931e-01	4.661902e-01	4.583564e-01	4.592457e-01	4.594270e-01	4.623027e-01
	min.	4.599688e-01	4.583550e-01	4.583449e-01	4.616423e-01	4.574407e-01	4.583134e-01	4.611230e-01	4.596374e-01	4.646008e-01	4.575759e-01	4.582081e-01	4.580598e-01	4.599994e-01
	max.	4.645379e-01	4.716125e-01	4.718628e-01	5.051428e-01	4.617629e-01	4.612760e-01	4.666792e-01	4.666360e-01	4.694720e-01	4.603585e-01	4.642735e-01	4.617361e-01	4.750471e-01
	std.	9.253650e-04	2.587364e-03	2.231055e-03	6.048154e-03	6.410846e-04	4.589613e-04	1.013787e-03	1.218745e-03	9.070707e-04	4.769647e-04	7.630053e-04	6.933513e-04	2.256776e-03
3D	med.	4.179902e-01	3.880501e-01	4.032716e-01	4.192083e-01	3.860173e-01	4.249437e-01	3.916052e-01	3.905965e-01	4.035464e-01	3.962952e-01	3.881609e-01	3.899442e-01	3.920990e-01
	avg.	4.180187e-01	3.884251e-01	4.031574e-01	4.202497e-01	3.860429e-01	4.252182e-01	3.917934e-01	3.906672e-01	4.036386e-01	3.967842e-01	3.882237e-01	3.901809e-01	3.926494e-01
	min.	4.156706e-01	3.853189e-01	3.907974e-01	3.929138e-01	3.839643e-01	4.169163e-01	3.889234e-01	3.875899e-01	3.995882e-01	3.922586e-01	3.864066e-01	3.875627e-01	3.896698e-01
	max.	4.211770e-01	3.969289e-01	4.156422e-01	4.547732e-01	3.890892e-01	4.404293e-01	3.957388e-01	3.943776e-01	4.073084e-01	4.037957e-01	3.904700e-01	3.949699e-01	4.086341e-01
	std.	1.155574e-03	2.272345e-03	4.970261e-03	1.500631e-02	1.028553e-03	4.301334e-03	1.295214e-03	1.343450e-03	1.684587e-03	2.333724e-03	8.325749e-04	1.432918e-03	2.866844e-03
4D	med.	4.270996e-01	3.660524e-01	4.291742e-01	4.080201e-01	5.314047e-01	4.057405e-01	3.546368e-01	3.555197e-01	3.840794e-01	3.949858e-01	3.666574e-01	3.575585e-01	3.622716e-01
	avg.	4.267088e-01	3.657722e-01	4.276561e-01	4.032646e-01	5.319328e-01	4.062117e-01	3.548492e-01	3.555803e-01	3.845025e-01	3.955433e-01	3.663411e-01	3.578508e-01	3.630003e-01
	min.	4.145633e-01	3.564930e-01	4.001087e-01	3.636943e-01	5.218254e-01	3.909849e-01	3.506640e-01	3.506058e-01	3.747023e-01	3.802792e-01	3.584034e-01	3.531777e-01	3.587272e-01
	max.	4.358169e-01	3.755294e-01	4.549059e-01	4.274199e-01	5.449370e-01	4.300945e-01	3.617757e-01	3.603530e-01	3.928269e-01	4.127321e-01	3.723043e-01	3.630313e-01	3.770066e-01
	std.	4.046783e-03	4.151395e-03	9.041382e-03	1.547012e-02	4.893718e-03	8.180719e-03	1.925540e-03	2.123009e-03	3.058994e-03	6.448358e-03	3.026243e-03	2.186062e-03	3.139481e-03
5D	med.	4.103190e-01	3.520836e-01	4.469863e-01	3.820860e-01	4.595428e-01	3.963724e-01	3.244205e-01	3.286043e-01	3.669833e-01	3.889698e-01	3.431153e-01	3.461458e-01	3.400813e-01
	avg.	4.107355e-01	3.517574e-01	4.454263e-01	3.805391e-01	4.599716e-01	3.969227e-01	3.244564e-01	3.281609e-01	3.673056e-01	3.894949e-01	3.433896e-01	3.458743e-01	3.403910e-01
	min.	3.982834e-01	3.399244e-01	4.095922e-01	3.508607e-01	4.500382e-01	3.767246e-01	3.164942e-01	3.201057e-01	3.576977e-01	3.724162e-01	3.377566e-01	3.33951e-01	3.299311e-01
	max.	4.331571e-01	3.687656e-01	5.686697e-01	4.010689e-01	4.725836e-01	4.274265e-01	3.353728e-01	3.389225e-01	3.765863e-01	4.129629e-01	3.530514e-01	3.610771e-01	3.541982e-01
	std.	6.234382e-03	5.648539e-03	2.515039e-02	1.081503e-02	4.176405e-03	1.228887e-02	3.644914e-03	4.043953e-03	3.751013e-03	8.554031e-03	2.926728e-03	6.205007e-03	4.759128e-03
6D	med.	4.087125e-01	3.866164e-01	5.039626e-01	3.847952e-01	4.307170e-01	3.933080e-01	3.227788e-01	3.294424e-01	3.596170e-01	4.146512e-01	3.576900e-01	3.732178e-01	3.390144e-01
	avg.	4.080410e-01	3.860765e-01	5.024216e-01	3.852560e-01	4.307267e-01	3.966052e-01	3.228678e-01	3.292630e-01	3.594000e-01	4.148944e-01	3.577667e-01	3.722788e-01	3.389918e-01
	min.	3.865337e-01	3.688220e-01	4.542633e-01	3.656774e-01	4.219686e-01	3.684600e-01	3.141817e-01	3.189529e-01	3.496006e-01	3.936451e-01	3.450315e-01	3.550024e-01	3.304915e-01
	max.	4.301586e-01	4.110336e-01	5.686231e-01	3.987124e-01	4.411558e-01	4.231469e-01	3.286851e-01	3.389443e-01	3.651212e-01	4.16282e-01	3.681063e-01	3.901434e-01	3.516031e-01
	std.	7.457740e-03	8.037557e-03	1.802507e-02	6.725483e-03	3.880452e-03	1.353722e-02	2.688947e-03	3.611149e-03	2.785121e-03	1.070786e-02	4.283835e-03	6.925226e-03	3.961126e-03
7D	med.	4.325679e-01	4.333228e-01	5.496249e-01	3.693836e-01	4.145850e-01	4.041218e-01	3.257859e-01	3.274996e-01	3.522845e-01	4.710821e-01	3.807832e-01	3.983718e-01	3.229364e-01
	avg.	4.326264e-01	4.367546e-01	5.477011e-01	3.704788e-01	4.148371e-01	4.056252e-01	3.265942e-01	3.275599e-01	3.518672e-01	4.711455e-01	3.816704e-01	3.998820e-01	3.331128e-01
	min.	4.075983e-01	3.925519e-01	5.059061e-01	3.568451e-01	4.012688e-01	3.935747e-01	3.162338e-01	3.102823e-01	3.425512e-01	4.679335e-01	3.599658e-01	3.717167e-01	3.152339e-01
	max.	4.571532e-01	5.243250e-01	5.819005e-01	3.857220e-01	4.276428e-01	4.223161e-01	3.408625e-01	3.435127e-01	3.605850e-01	4.733879e-01	4.107316e-01	4.458222e-01	3.928273e-01
	std.	9.709373e-03	2.264459e-02	1.569868e-02	6.204098e-03	4.902374e-03	6.011013e-03	5.317765e-03	6.365674e-03	4.210672e-03	1.207717e-03	8.851462e-03	1.437215e-02	2.141803e-02
8D	med.	3.906126e-01	4.572602e-01	5.604370e-01	3.620459e-01	3.752678e-01	3.856622e-01	3.037428e-01	3.156118e-01	3.397301e-01	4.811961e-01	3.780232e-01	3.979867e-01	3.694042e-01
	avg.	3.912093e-01	4.596658e-01	5.593063e-01	3.626933e-01	3.765551e-01	3.857529e-01	3.047406e-01	3.164091e-01	3.395201e-01	4.813007e-01	3.795381e-01	3.975012e-01	3.768726e-01
	min.	3.743942e-01	4.176043e-01	5.205348e-01	3.463858e-01	3.644160e-01	3.640242e-01	2.940302e-01	3.032738e-01	3.326411e-01	4.648234e-01	3.550445e-01	3.652450e-01	3.418659e-01
	max.	4.074602e-01	5.347128e-01	6.016717e-01	3.783579e-01	3.976072e-01	3.964025e-01	3.06188e-01	3.331650e-01	3.45914e-01	5.092019e-01	4.079497e-01	4.307691e-01	4.301966e-01
	std.	6.941745e-03	2.197173e-02	1.477285e-02	6.757765e-03	5.936210e-03	5.147112e-03	4.995251e-03	6.474351e-03	2.692015e-03	4.500046e-03	1.010047e-02	1.306333e-02	1.971637e-02
9D	med.	3.649738e-01	4.653689e-01	5.524433e-01	3.502885e-01	3.659200e-01	3.636003e-01	2.912230e-01	3.096731e-01	3.286262e-01	4.642370e-01	3.750937e-01	3.983879e-01	3.679994e-01
	avg.	3.651997e-01	4.690330e-01	5.527957e-01	3.503591e-01	3.660385e-01	3.640391e-01	2.915361e-01	3.090116e-01	3.285626e-01	4.646339e-01	3.753287e-01	3.976985e-01	3.677983e-01
	min.	3.503232e-01	4.225776e-01	5.182622e-01	3.395554e-01	3.507874e-01	3.491620e-01	2.822465e-01	2.924928e-01	3.238343e-01	4.411384e-01	3.427893e-01	3.664810e-01	3.336026e-01
	max.	3.782440e-01	5.297503e-01	5.791488e-01	3.638500e-01	3.790778e-01	3.809563e-01	3.046226e-01	3.198159e-01	3.323438e-01	4.829591e-01	3.971600e-01	4.232838e-01	4.057511e-01
	std.	6.067857e-03	2.367220e-02	1.403845e-02	5.428803e-03	6.827398e-03	5.451216e-03	4.355132e-03	5.727185e-03	1.681350e-03	8.501402e-03	9.853630e-03	1.190948e-02	1.507821e-02
10D	med.	3.540019e-01	4.855494e-01	5.518684e-01	3.320301e-01	3.253579e-01	3.503110e-01	2.828682e-01	3.047343e-01	3.194492e-01	4.536001e-01	3.718080e-01	3.923206e-01	3.631682e-01
	avg.	3.549433e-01	4.887552e-01	5.521902e-01	3.322658e-01	3.267640e-01	3.505597e-01	2.838792e-01	3.047835e-01	3.193607e-01	4.523395e-01	3.728347e-01	3.938243e-01	3.660309e-01
	min.	3.454098e-01	4.209742e-01	5.126420e-01	3.233361e-01	3.184102e-01	3.386371e-01	2.765473e-01	2.949211e-01	3.156696e-01	4.312340e-01	3.502977e-01	3.669533e-01	3.422870e-01
	max.	3.800443e-01	5.530462e-01	5.791394e-01	3.394291e-01	4.043082e-01	3.634022e-01	2.964757e-01	3.179083e-01	3.223611e-01	4.751695e-01	4.073446e-01	4.281730e-01	4.027468e-01
	std.	5.987872e-03	2.775273e-02	1.491555e-02	3.184189e-03	2.583129e-02	4.101584e-03	3.926204e-03	5.026097e-03	1.283576e-03	9.177830e-03	1.135094e-02	1.273223e-02	1.481942e-02

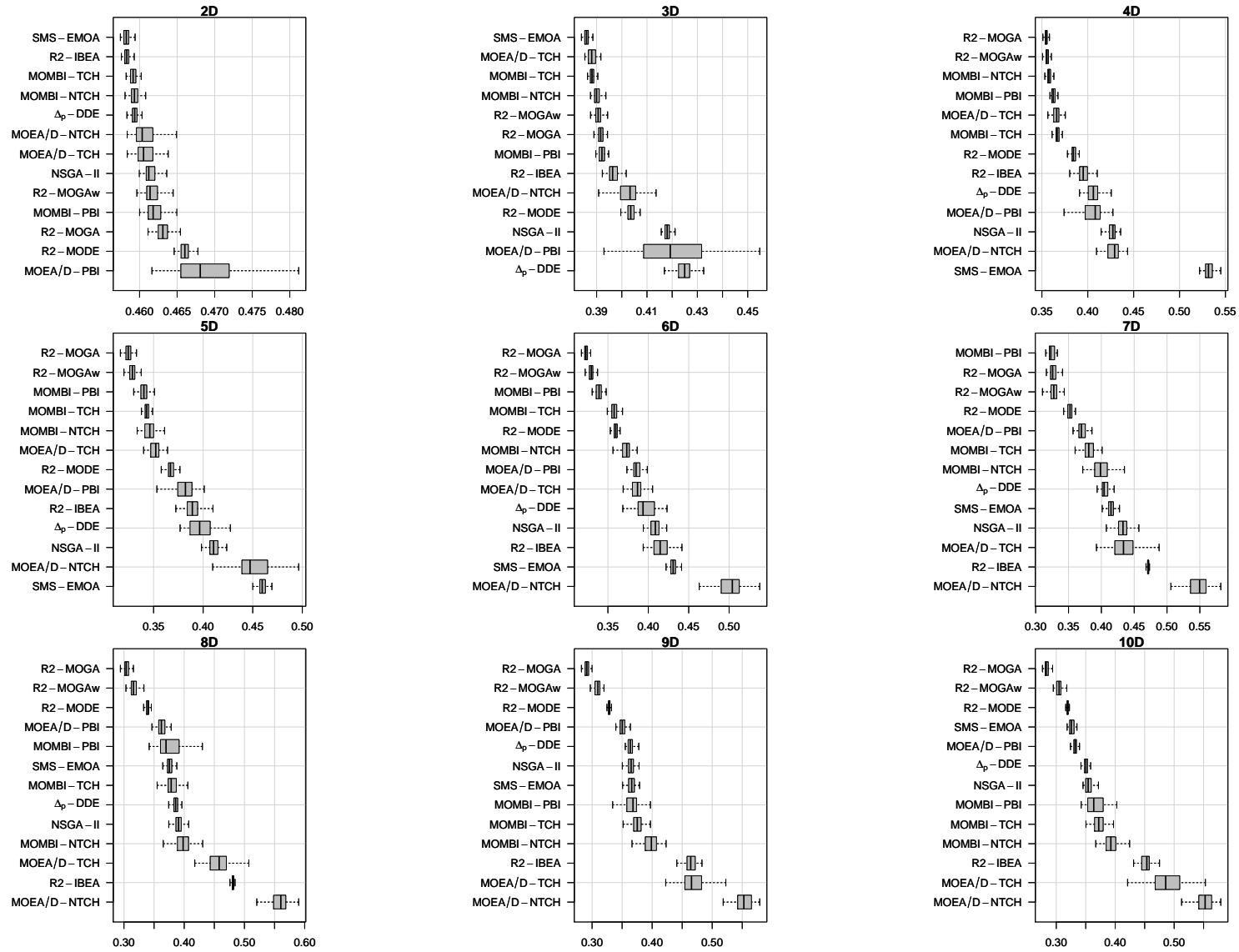


Figure A.146: Box-plot of R2 indicator values for different optimizers on the WFG3 test problem.

Table A.61: Comparison of runtime (in milliseconds) for different optimizers on the WFG3 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	3.639540e+02	8.128330e+02	1.343471e+03	1.233565e+03	3.601478e+04	1.647903e+03	1.848121e+03	1.510518e+03	1.596650e+03	5.534500e+05	2.042484e+03	2.164002e+03	2.905812e+03
	avg.	3.745948e+02	8.077670e+02	1.345401e+03	1.223695e+03	3.543423e+04	1.642275e+03	1.808190e+03	1.499734e+03	1.538651e+03	5.535951e+05	2.059582e+03	2.147879e+03	2.879580e+03
	min.	3.099510e+02	6.242140e+02	9.841470e+02	8.271550e+02	2.601571e+04	1.319180e+03	1.234048e+03	1.115257e+03	9.565480e+02	4.773560e+05	1.700439e+03	1.717252e+03	2.354943e+03
	max.	5.257020e+02	1.084460e+03	1.885631e+03	1.989296e+03	3.912032e+04	1.978673e+03	2.294056e+03	1.894249e+03	2.023683e+03	5.692920e+05	2.497995e+03	2.509498e+03	3.160925e+03
	std.	4.904388e+01	9.079911e+01	1.352225e+02	1.317212e+02	3.198988e+03	1.332590e+02	2.094259e+02	1.781301e+02	2.329706e+02	9.088279e+03	1.629998e+02	1.605982e+02	1.687873e+02
3D	med.	4.415060e+02	8.484170e+02	1.666888e+03	1.292427e+03	8.598492e+05	1.589462e+03	2.282210e+03	1.830632e+03	2.636120e+03	5.675220e+05	2.058003e+03	2.008265e+03	3.676184e+03
	avg.	4.589927e+02	8.496150e+02	1.647855e+03	1.291126e+03	8.480990e+05	1.568076e+03	2.252342e+03	1.847842e+03	2.530709e+03	5.605014e+05	2.076645e+03	2.031840e+03	3.643434e+03
	min.	3.734340e+02	6.746360e+02	1.161225e+03	1.004446e+03	6.125424e+05	1.088746e+03	1.565898e+03	1.452920e+03	1.561716e+03	5.376890e+05	1.714317e+03	1.763652e+03	3.092799e+03
	max.	6.212150e+02	1.214705e+03	1.896434e+03	1.961382e+03	9.678489e+05	2.197755e+03	2.805096e+03	2.247909e+03	3.136261e+03	5.810230e+05	2.461806e+03	2.478521e+03	4.043284e+03
	std.	6.564229e+01	9.350345e+01	1.340387e+02	1.330987e+02	7.900921e+04	1.796226e+02	2.448071e+02	1.680803e+02	2.912640e+02	1.474053e+04	1.587783e+02	1.424083e+02	1.886212e+02
4D	med.	6.677380e+02	1.058852e+03	1.880588e+03	1.555919e+03	6.777314e+06	1.722191e+03	2.911003e+03	2.455980e+03	3.013816e+03	5.994850e+05	2.189616e+03	2.304286e+03	3.517766e+03
	avg.	6.728811e+02	1.067853e+03	1.855600e+03	1.535952e+03	6.683073e+06	1.752749e+03	2.843983e+03	2.433014e+03	2.972066e+03	5.990224e+05	2.251171e+03	2.305781e+03	3.466700e+03
	min.	5.477160e+02	8.487820e+02	1.344875e+03	1.247105e+03	5.411974e+06	1.301225e+03	2.001527e+03	1.790933e+03	2.366614e+03	5.499380e+05	1.819815e+03	1.971535e+03	2.701233e+03
	max.	8.330450e+02	1.371375e+03	2.183257e+03	1.817624e+03	7.357131e+06	2.235947e+03	3.329736e+03	2.815886e+03	3.411975e+03	6.109890e+05	3.715051e+03	2.798832e+03	3.752596e+03
	std.	6.546107e+01	9.523738e+01	1.472504e+02	1.068083e+02	4.501248e+05	2.015918e+02	2.476566e+02	2.109228e+02	2.706714e+02	7.566159e+03	3.051023e+02	1.541542e+02	2.086724e+02
5D	med.	8.847470e+02	1.272841e+03	2.529114e+03	1.684309e+03	9.607140e+06	3.059612e+03	3.471906e+03	3.1144577e+03	3.608365e+03	6.782920e+05	2.861447e+03	2.754171e+03	3.959581e+03
	avg.	8.862473e+02	1.271560e+03	2.473587e+03	1.700181e+03	9.428853e+06	3.047938e+03	3.470732e+03	3.185374e+03	3.560749e+03	6.567699e+05	2.866132e+03	2.776843e+03	3.959531e+03
	min.	7.064750e+02	9.956970e+02	1.911532e+03	1.457112e+03	7.824987e+06	2.016168e+03	2.733100e+03	2.597853e+03	2.569675e+03	6.026270e+05	2.199480e+03	2.386638e+03	3.344283e+03
	max.	1.122425e+03	1.637701e+03	2.850978e+03	2.040626e+03	1.052222e+07	3.791263e+03	4.000740e+03	3.612491e+03	4.094862e+03	6.912620e+05	4.112776e+03	3.435169e+03	4.441694e+03
	std.	8.007042e+01	1.199572e+02	1.885933e+02	1.603996e+02	6.141545e+05	3.434217e+02	3.011803e+02	2.213211e+02	3.072302e+02	3.296805e+04	3.909242e+02	1.726418e+02	2.418511e+02
6D	med.	1.078231e+03	1.378868e+03	2.708700e+03	1.997326e+03	1.026766e+07	3.306348e+03	4.243052e+03	3.886073e+03	4.050697e+03	7.221600e+05	2.726423e+03	3.046625e+03	4.466956e+03
	avg.	1.080613e+03	1.369736e+03	2.661759e+03	1.960415e+03	1.013562e+07	3.382420e+03	4.132820e+03	3.885756e+03	4.073476e+03	7.224010e+05	2.748774e+03	3.038879e+03	4.475314e+03
	min.	8.802750e+02	1.071921e+03	1.909066e+03	1.457112e+03	8.631516e+06	2.284746e+03	3.208554e+03	2.996444e+03	3.155666e+03	7.127540e+05	2.185022e+03	2.507434e+03	3.583193e+03
	max.	1.416119e+03	1.795859e+03	3.137706e+03	2.373925e+03	1.113459e+07	4.741440e+03	4.616403e+03	4.434317e+03	4.584614e+03	7.348640e+05	4.370556e+03	3.470739e+03	4.977068e+03
	std.	1.079460e+02	1.187907e+02	1.714520e+02	2.079984e+02	6.691795e+05	5.563704e+02	3.728149e+02	2.680135e+02	2.983608e+02	5.068818e+03	3.086861e+02	1.780499e+02	2.462165e+02
7D	med.	1.084323e+03	1.459247e+03	2.859018e+03	2.138166e+03	6.500561e+06	9.506986e+03	3.610147e+03	3.204483e+03	3.150643e+03	3.298290e+05	1.811446e+03	2.208010e+03	3.967106e+03
	avg.	1.072598e+03	1.470922e+03	2.814806e+03	2.142910e+03	6.404473e+06	9.336056e+03	3.598337e+03	3.232838e+03	3.157961e+03	3.301447e+05	1.834435e+03	2.230081e+03	3.962137e+03
	min.	8.763300e+02	1.182377e+03	2.000824e+03	1.695419e+03	5.198053e+06	5.510551e+03	2.735189e+03	2.589037e+03	2.570506e+03	3.212220e+05	1.564211e+03	1.907372e+03	3.326650e+03
	max.	1.425651e+03	2.663294e+03	3.170924e+03	3.001476e+03	7.376888e+06	1.263098e+04	4.156311e+03	3.808819e+03	3.657814e+03	3.372740e+05	2.480830e+03	2.590865e+03	4.402167e+03
	std.	1.007623e+02	1.617367e+02	2.001496e+02	2.392740e+02	5.771116e+05	1.395223e+03	2.677807e+02	2.501122e+02	2.441667e+02	3.295206e+03	1.719455e+02	1.780406e+02	1.642788e+02
8D	med.	1.475340e+03	1.619779e+03	3.101515e+03	2.422101e+03	1.101764e+07	2.005427e+04	5.193143e+03	4.769473e+03	4.889361e+03	1.262239e+06	2.522081e+03	3.363008e+03	5.670626e+03
	avg.	1.474771e+03	1.609320e+03	3.066358e+03	2.440897e+03	1.086434e+07	2.008281e+04	5.064539e+03	4.776171e+03	4.853834e+03	1.232551e+06	2.578801e+03	3.385379e+03	5.629490e+03
	min.	1.125689e+03	1.185802e+03	2.371786e+03	1.925047e+03	8.407887e+06	1.151908e+04	3.934235e+03	3.888168e+03	3.689226e+03	1.154687e+06	2.157686e+03	2.833578e+03	4.686531e+03
	max.	1.849315e+03	2.209513e+03	3.304750e+03	3.018727e+03	1.220856e+07	2.436363e+04	5.731098e+03	5.431620e+03	5.442720e+03	1.307899e+06	3.406524e+03	3.850951e+03	6.205033e+03
	std.	1.196513e+02	1.420646e+02	1.753435e+02	2.458554e+02	9.643918e+05	2.208192e+03	4.625600e+02	3.321027e+02	3.351533e+02	5.190796e+04	2.423678e+02	2.198717e+02	2.689888e+02
9D	med.	1.933228e+03	1.782492e+03	3.374654e+03	2.639397e+03	1.594676e+07	2.847269e+04	7.070153e+03	6.779913e+03	7.680282e+03	2.667279e+06	3.825491e+03	4.388868e+03	6.920657e+03
	avg.	1.915834e+03	1.782187e+03	3.373708e+03	2.644390e+03	1.553277e+07	3.047572e+04	7.052933e+03	6.796865e+03	7.646587e+03	2.609546e+06	3.835234e+03	4.450363e+03	6.940366e+03
	min.	1.631921e+03	1.384100e+03	2.657982e+03	2.065482e+03	1.201814e+07	2.023019e+04	5.303483e+03	5.536360e+03	5.653752e+03	2.223086e+06	3.204111e+03	3.866779e+03	5.976809e+03
	max.	2.224529e+03	2.130222e+03	3.840133e+03	3.022273e+03	1.797442e+07	4.608926e+04	8.118932e+03	7.727698e+03	8.367524e+03	2.788076e+06	4.266693e+03	5.260434e+03	7.649396e+03
	std.	1.360266e+02	1.294710e+02	2.676059e+02	2.359226e+02	1.780053e+06	6.614528e+03	6.395502e+02	4.682507e+02	5.943387e+02	1.808275e+05	2.252953e+02	3.130499e+02	3.134191e+02
10D	med.	2.407036e+03	1.915854e+03	3.737363e+03	2.817801e+03	1.368565e+07	5.015673e+04	9.534846e+03	9.574109e+03	1.053063e+04	4.924168e+06	4.349289e+03	5.790585e+03	9.306603e+03
	avg.	2.424973e+03	1.913517e+03	3.674626e+03	2.808259e+03	1.383092e+07	5.038180e+04	9.436301e+03	9.497037e+03	1.023396e+04	4.875216e+06	4.411740e+03	5.821246e+03	9.186976e+03
	min.	1.965079e+03	1.468075e+03	2.840700e+03	2.185537e+03	8.122166e+06	4.327624e+04	7.547313e+03	7.615345e+03	7.200540e+03	4.728646e+06	3.788357e+03	5.040558e+03	7.786326e+03
	max.	2.792008e+03	2.479594e+03	4.036750e+03	2.540593e+03	2.374272e+07	6.084224e+04	1.143007e+04	1.049953e+04	1.182326e+04	4.974642e+06	5.394815e+03	7.134298e+03	1.132928e+04
	std.	1.696803e+02	1.547371e+02	2.725715e+02	2.445429e+02	3.250561e+06	3.569957e+03	1.332044e+03	5.883422e+02	1.363922e+03	8.516141e+04	4.048990e+02	7.363721e+02	5.633807e+02

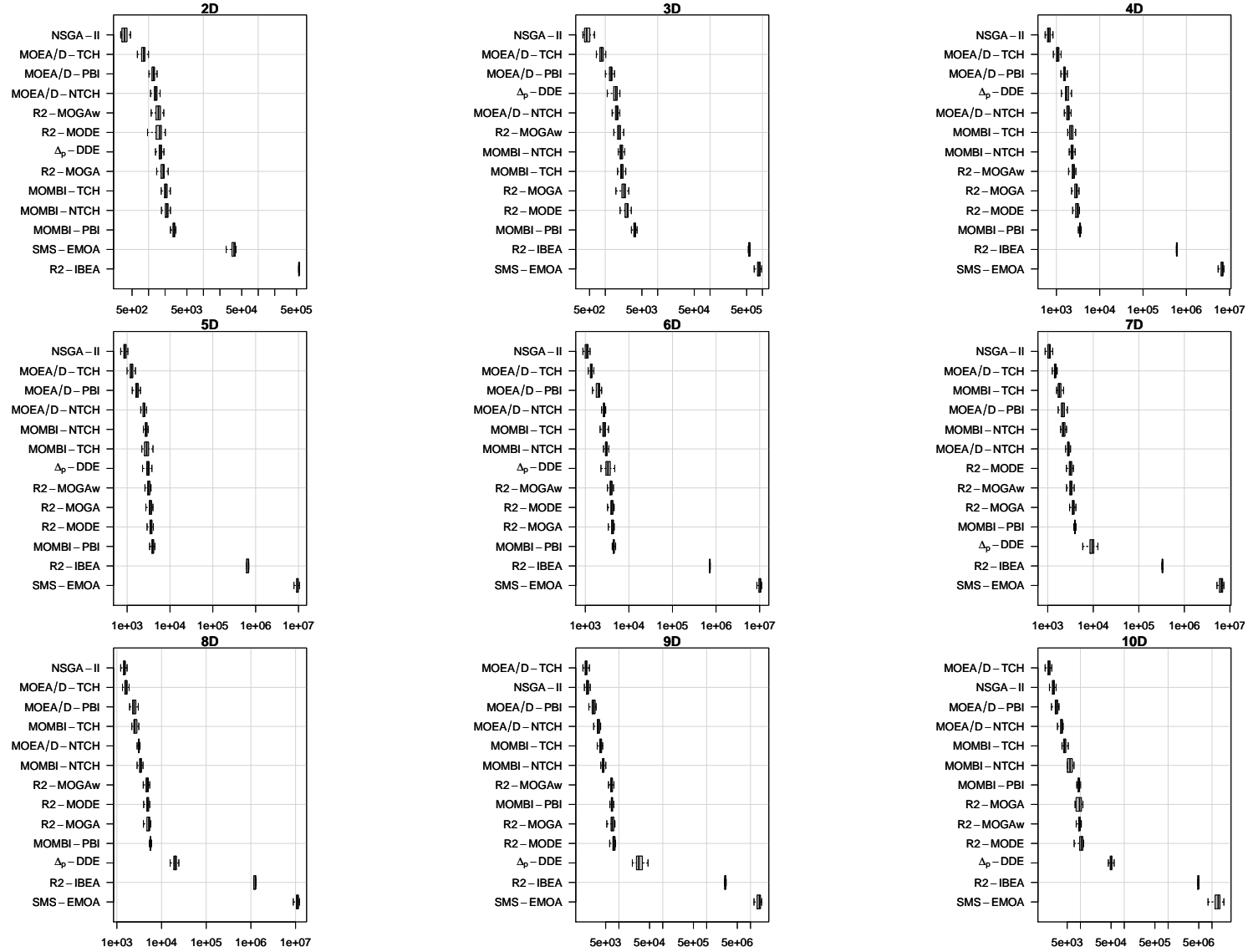


Figure A.147: Box-plot of runtime (in logarithmic scale) for different optimizers on the WFG3 test problem.

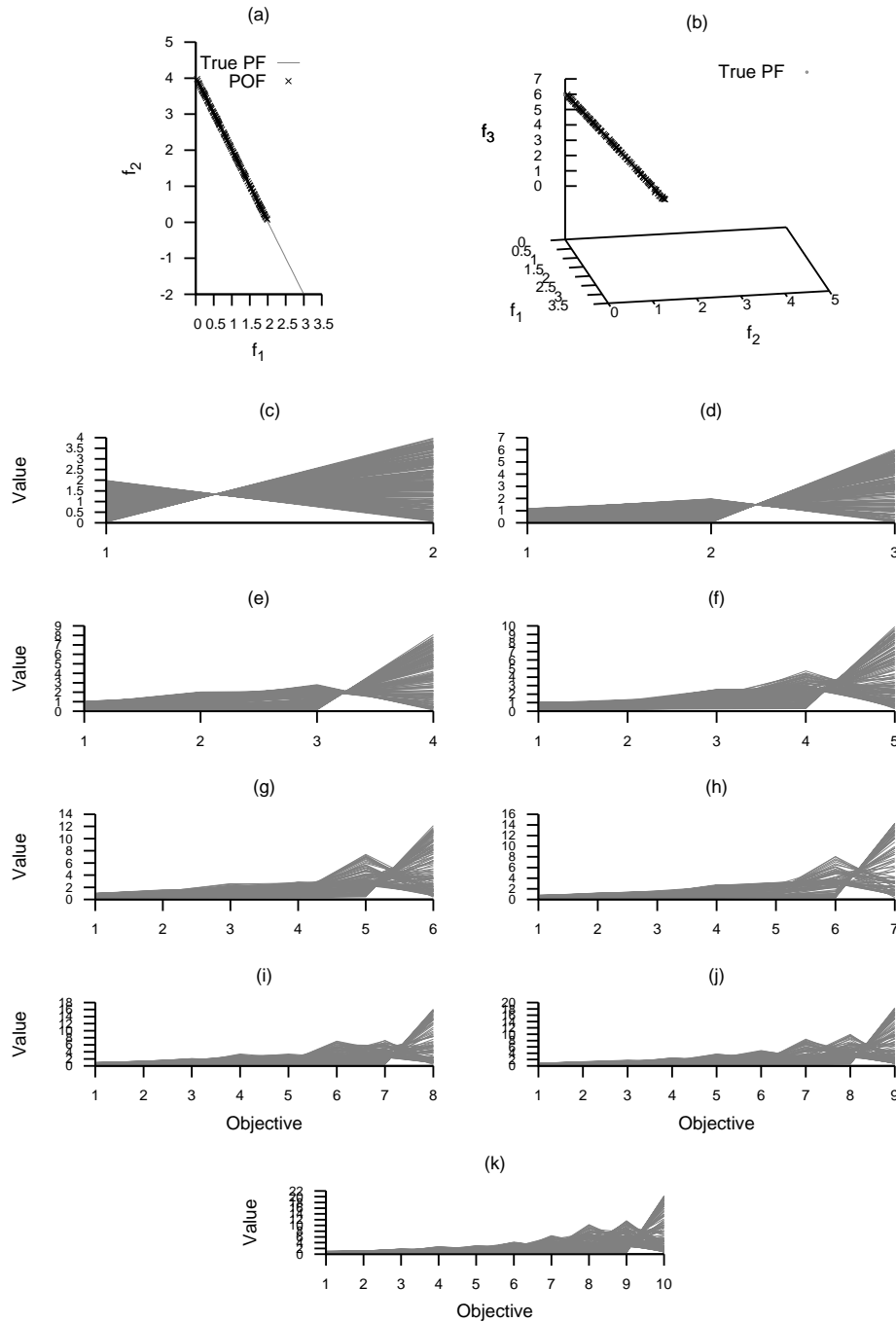


Figure A.148: Plots of the approximations obtained by NSGA-II from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



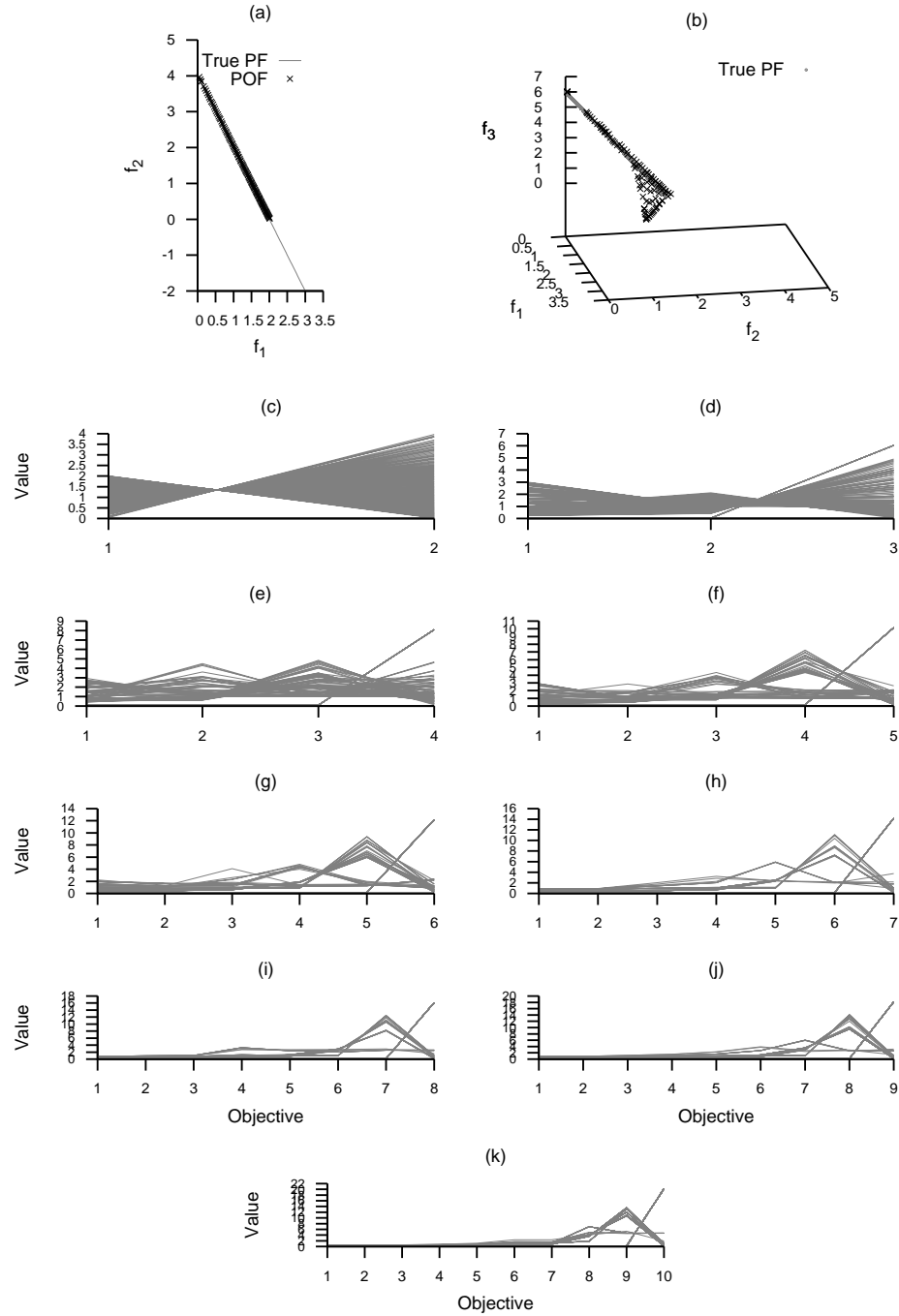


Figure A.149: Plots of the approximations obtained by MOEA/D-TCH from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

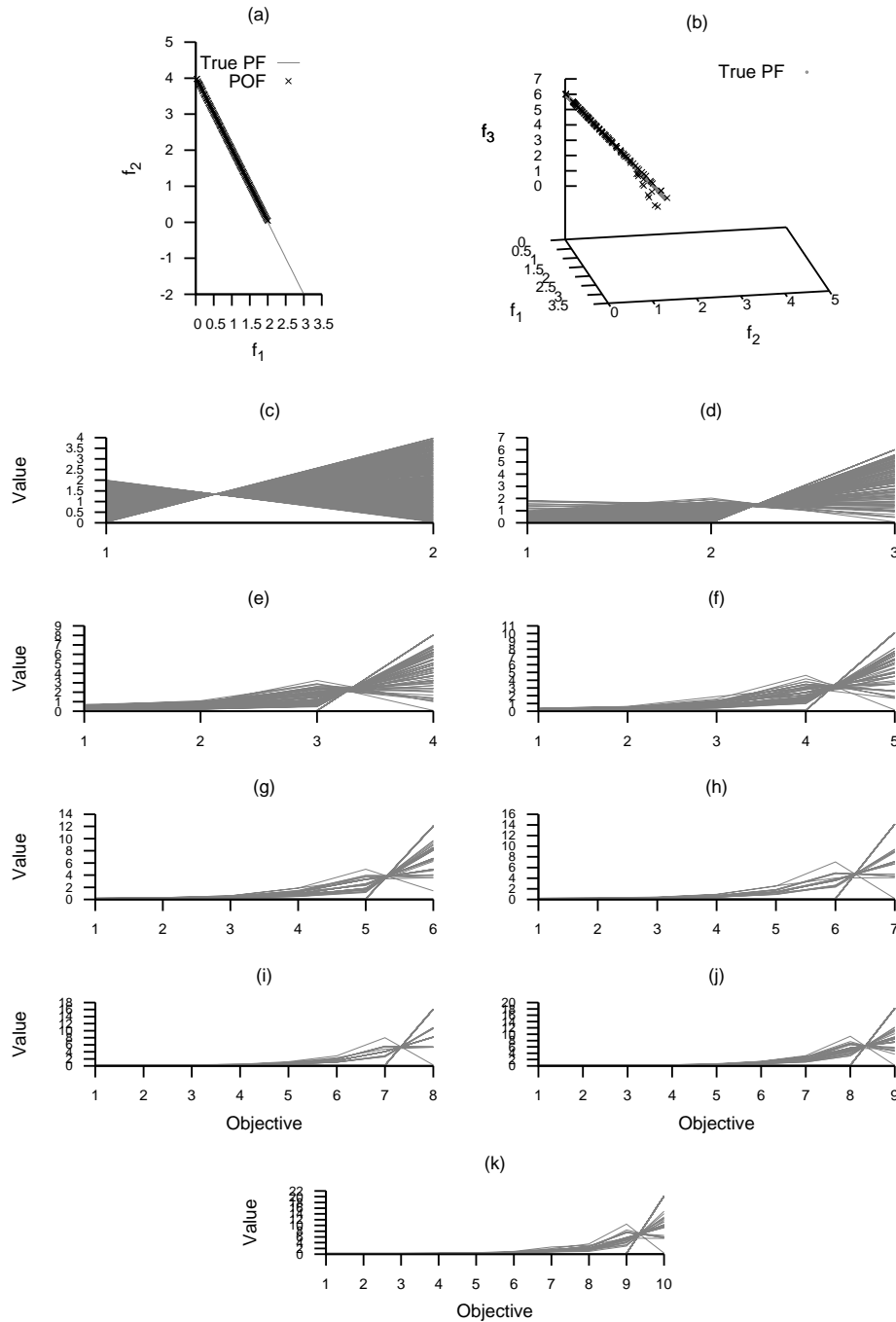


Figure A.150: Plots of the approximations obtained by MOEA/D-NTCH from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

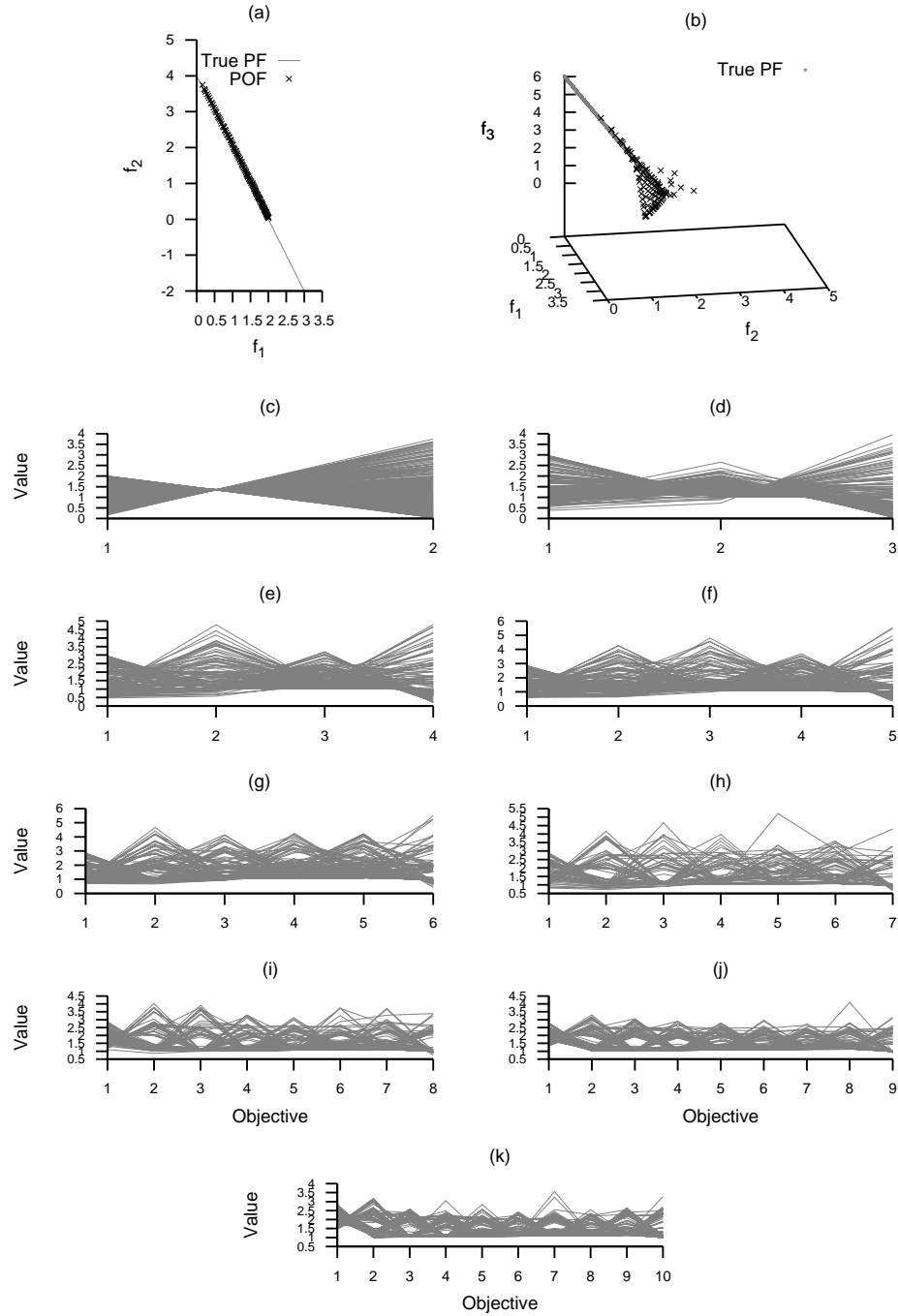


Figure A.151: Plots of the approximations obtained by MOEA/D-PBI from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

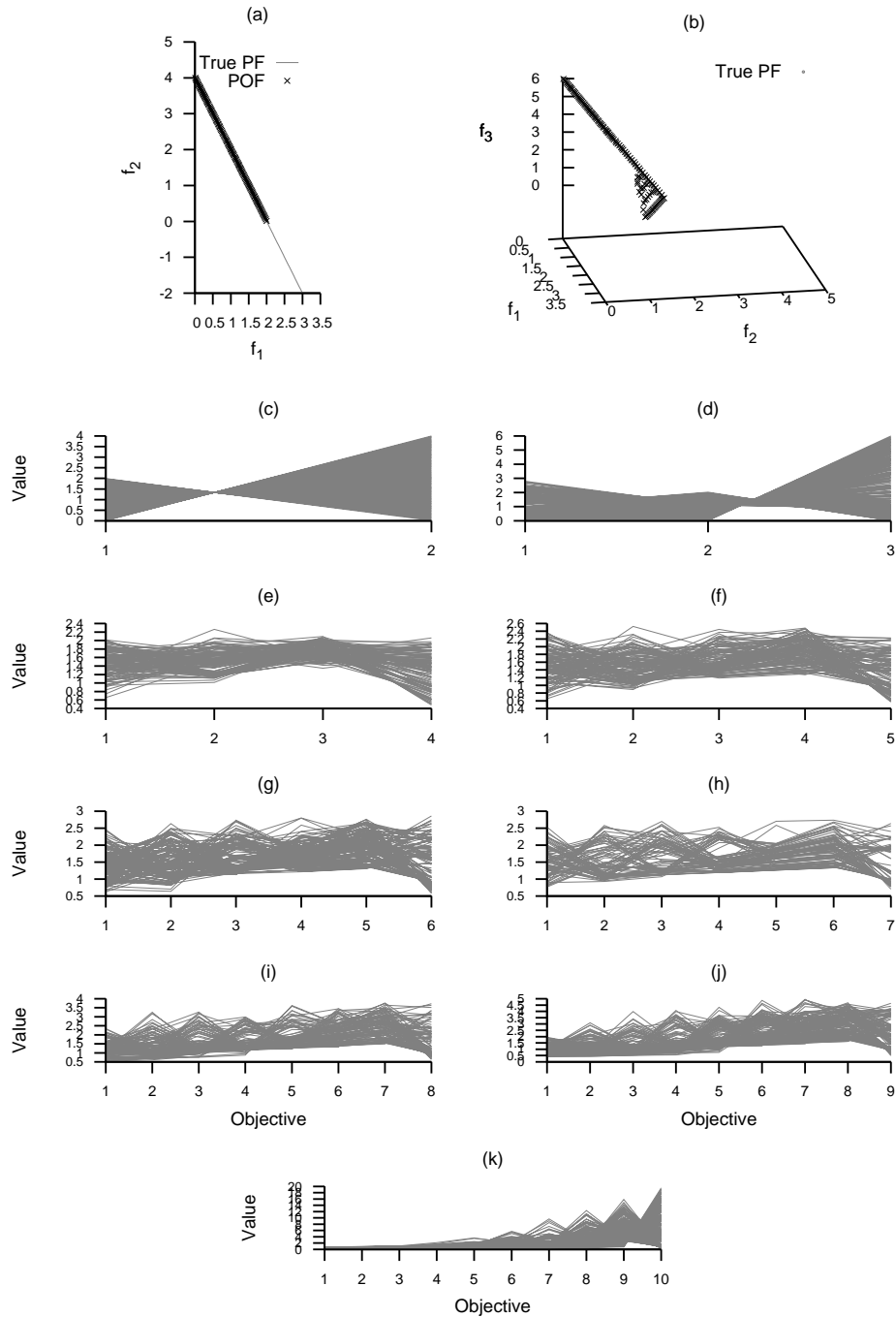


Figure A.152: Plots of the approximations obtained by SMS-EMOA from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

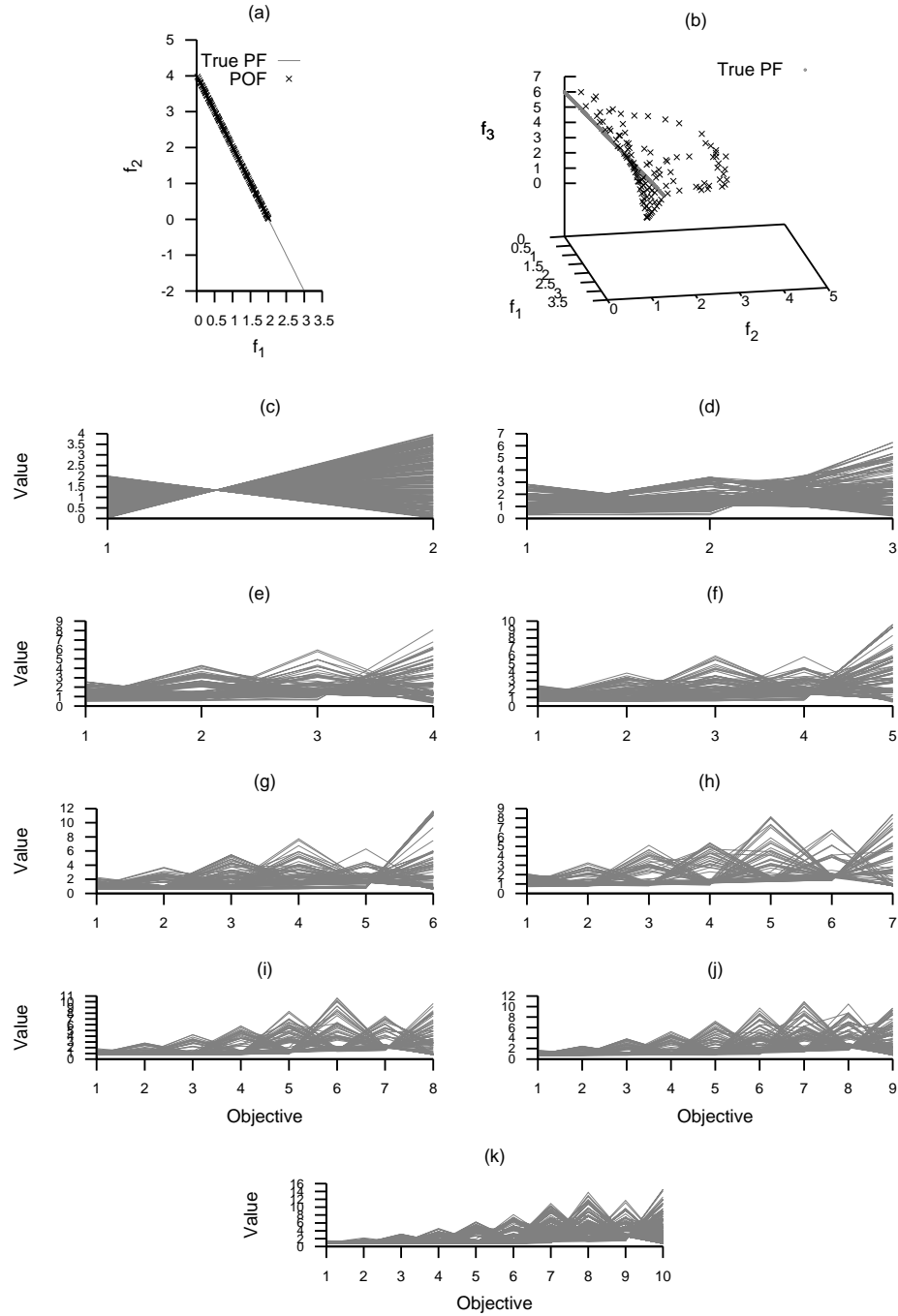


Figure A.153: Plots of the approximations obtained by  $\Delta_p$ -DDE from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

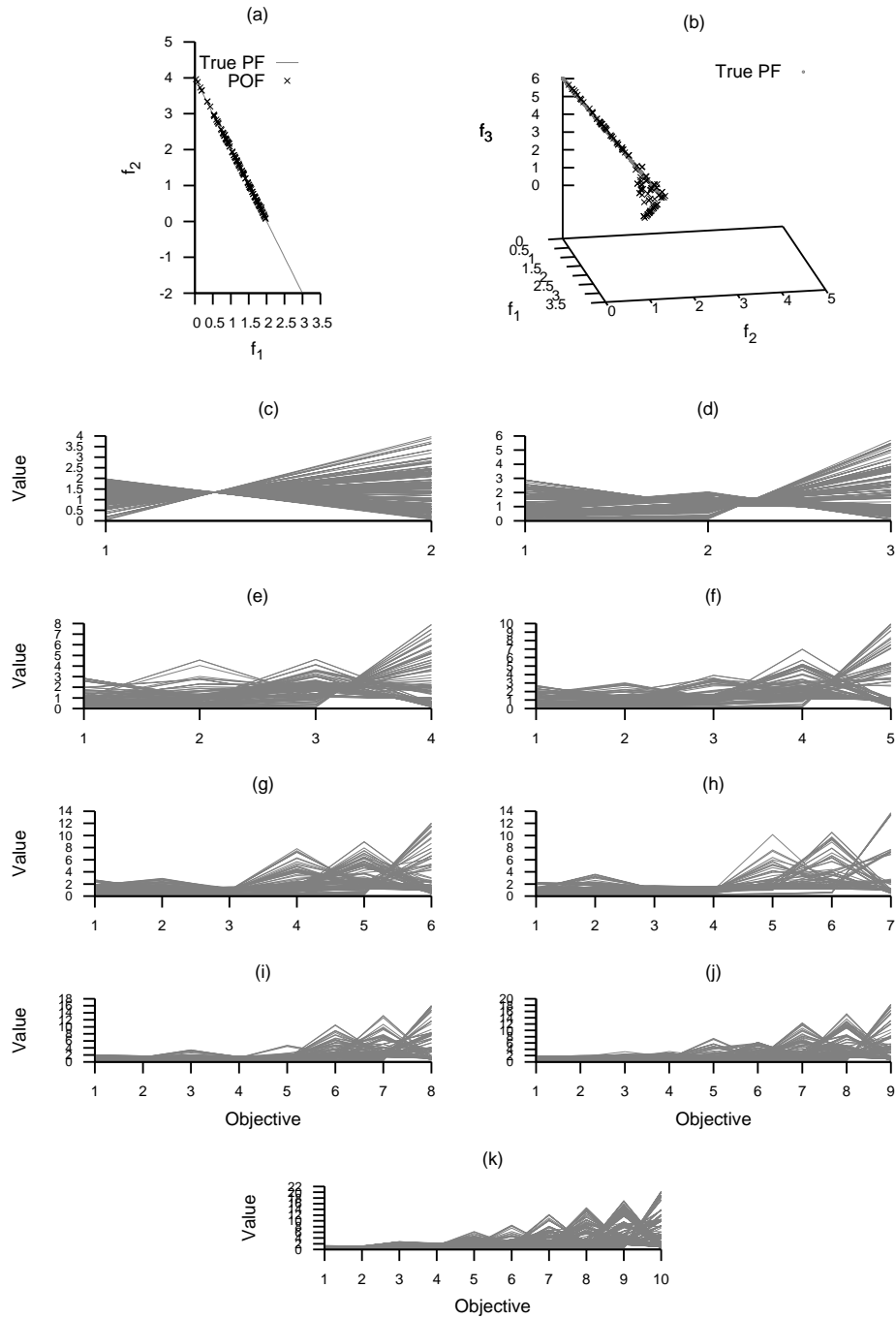


Figure A.154: Plots of the approximations obtained by R2-MOGA from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

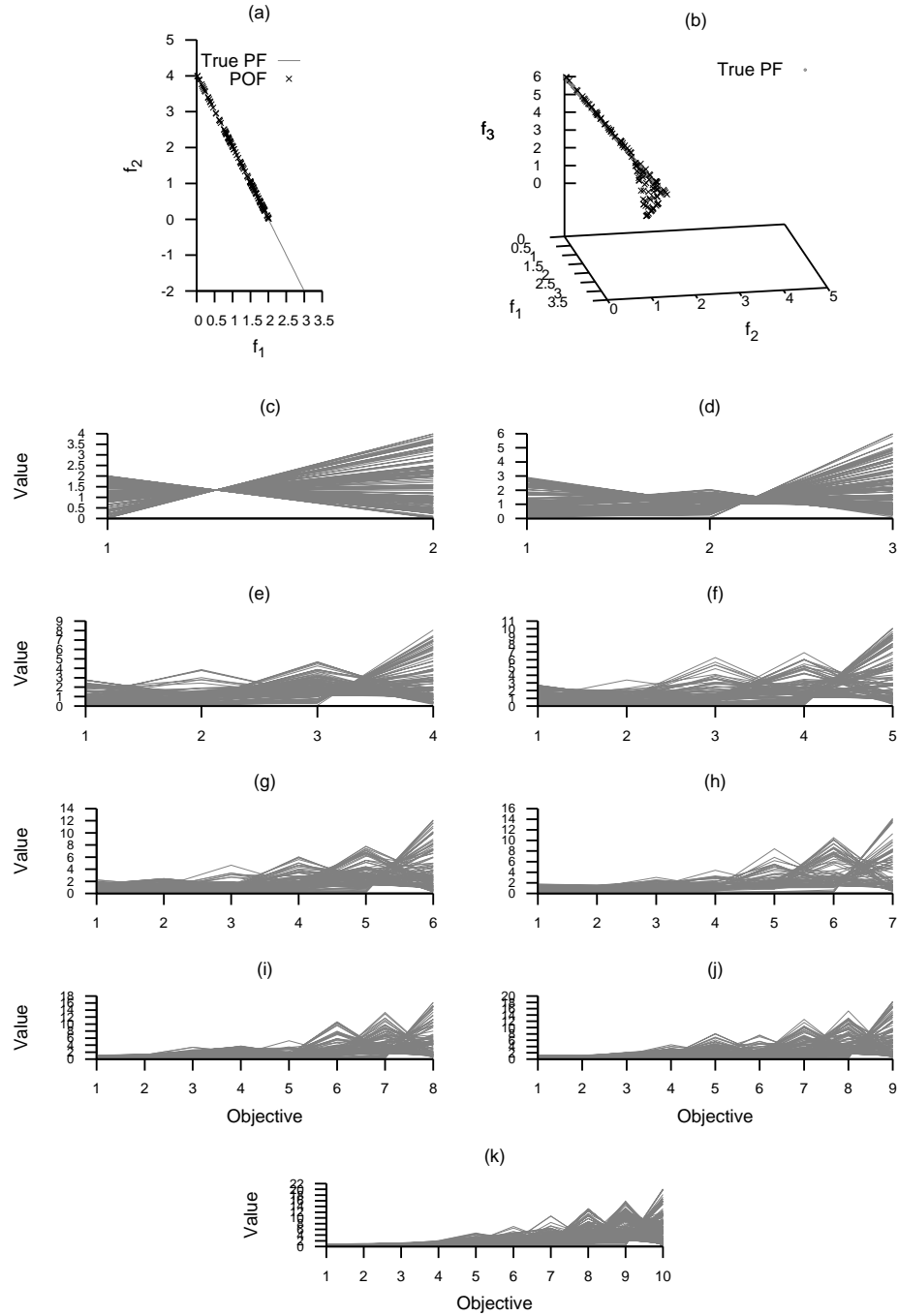


Figure A.155: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

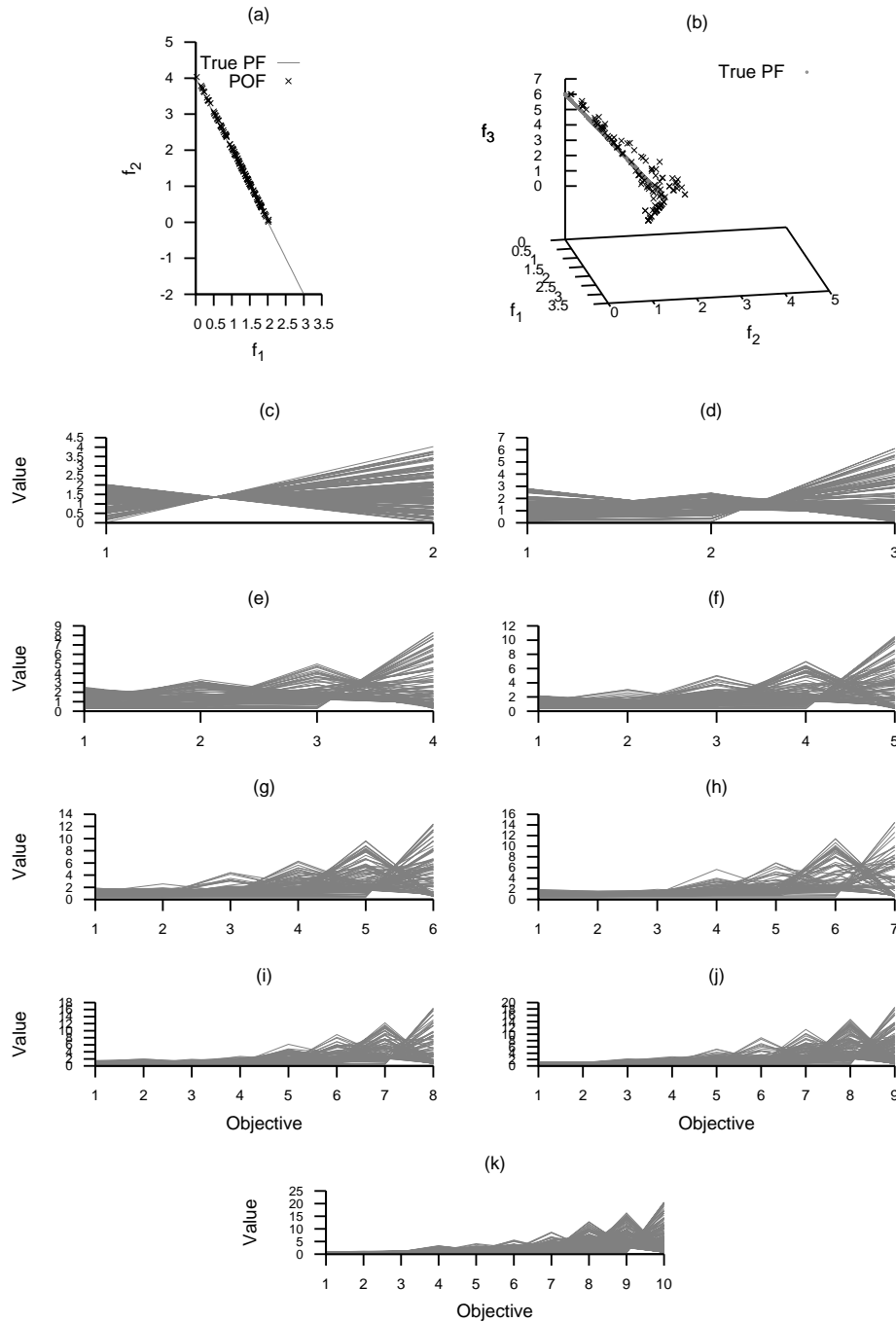


Figure A.156: Plots of the approximations obtained by *R2-MODE* from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



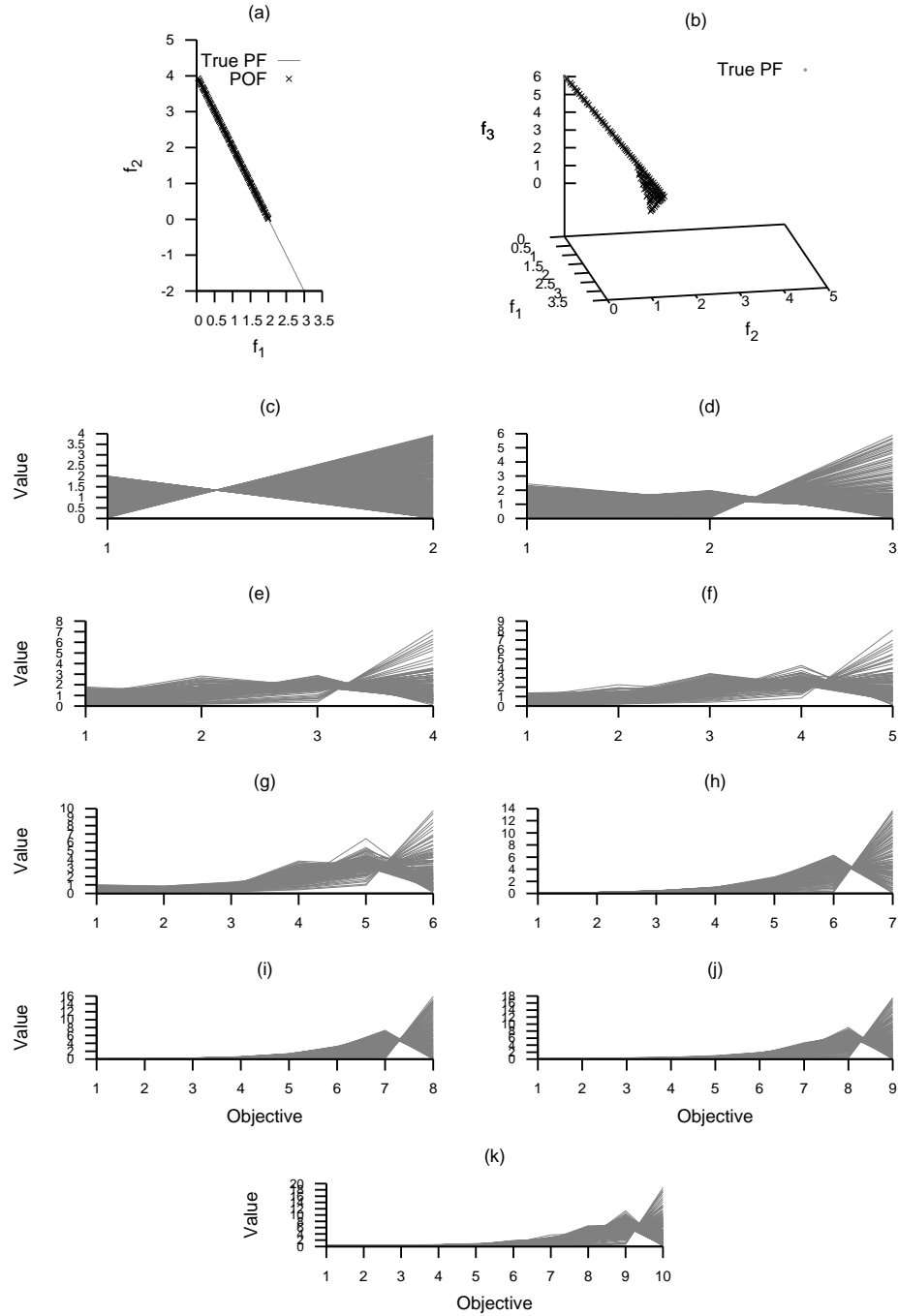


Figure A.157: Plots of the approximations obtained by *R2*-IBEA from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

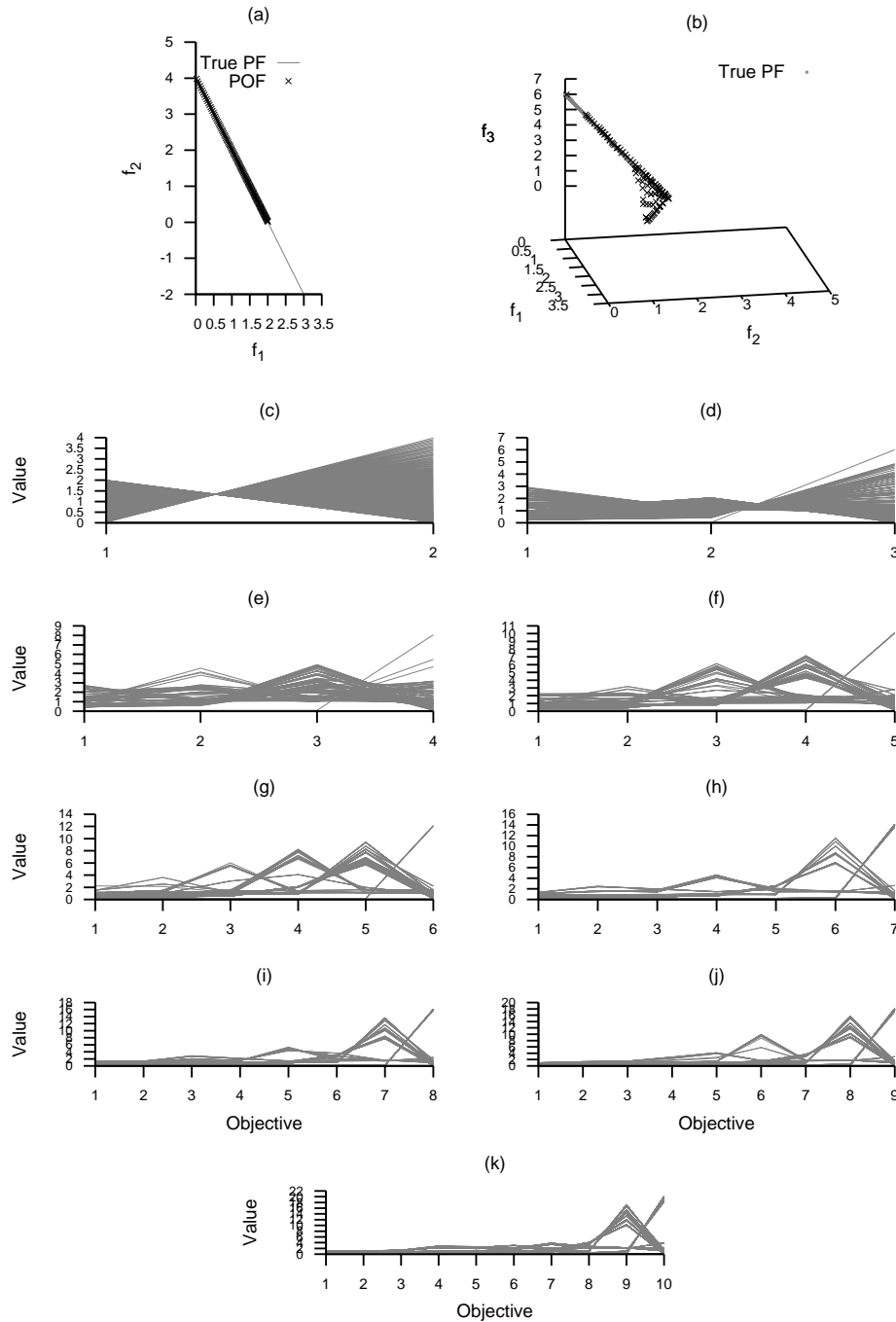


Figure A.158: Plots of the approximations obtained by MOMBI-TCH from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

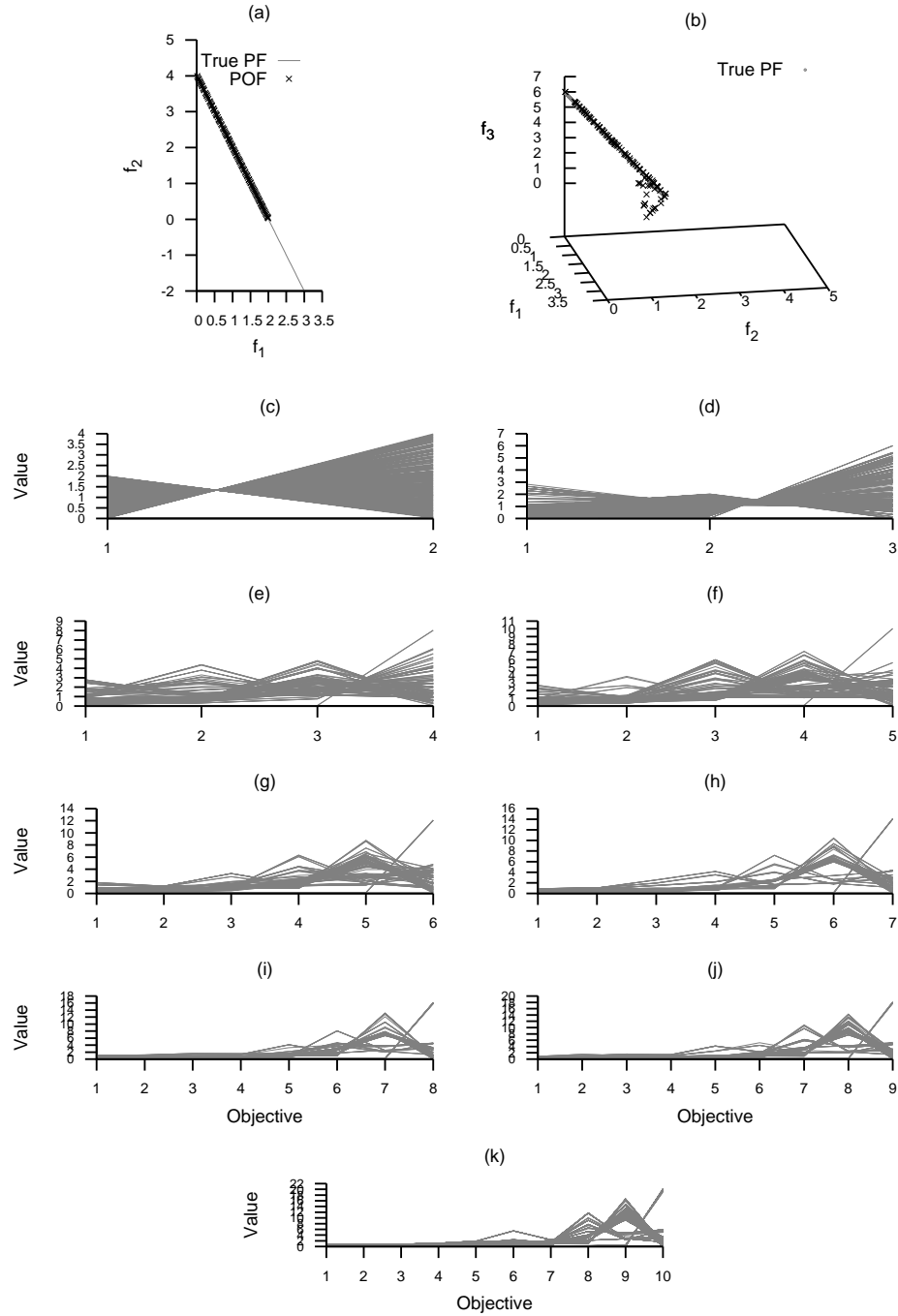


Figure A.159: Plots of the approximations obtained by MOMBI-NTCH from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

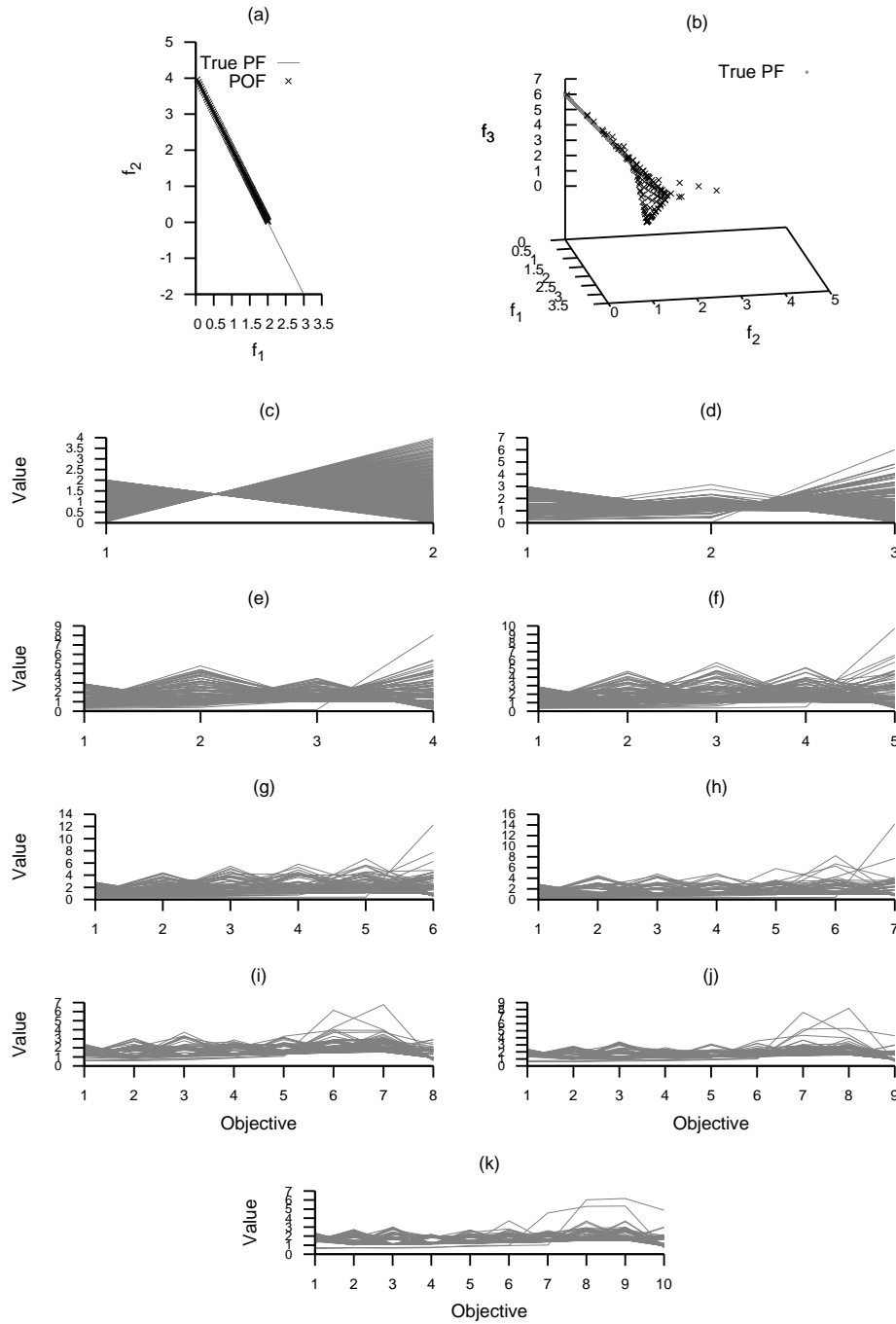


Figure A.160: Plots of the approximations obtained by MOMBI-PBI from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.62: Comparison of hypervolume indicator values for different optimizers on the WFG4 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_P$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	8.402261e+00	8.607865e+00	8.621991e+00	8.435212e+00	8.624041e+00	8.217368e+00	8.515415e+00	8.569914e+00	8.172909e+00	8.639795e+00	8.627164e+00	8.626990e+00	8.509979e+00
	avg.	8.402740e+00	8.600385e+00	8.616291e+00	8.431820e+00	8.615366e+00	8.216373e+00	8.512404e+00	8.563387e+00	8.164967e+00	8.628237e+00	8.622771e+00	8.623034e+00	8.509093e+00
	min.	8.203686e+00	8.453109e+00	8.521426e+00	8.268336e+00	8.488859e+00	8.144535e+00	8.427318e+00	8.411010e+00	8.054434e+00	8.533454e+00	8.564456e+00	8.537375e+00	8.420897e+00
	max.	8.535149e+00	8.641490e+00	8.645153e+00	8.516000e+00	8.672459e+00	8.291247e+00	8.572662e+00	8.608518e+00	8.229426e+00	8.671199e+00	8.651225e+00	8.650488e+00	8.561596e+00
	std.	7.233192e-02	3.054213e-02	2.226713e-02	4.591062e-02	3.747269e-02	2.789951e-02	3.140398e-02	3.379753e-02	3.368487e-02	3.233533e-02	1.664951e-02	2.015437e-02	2.641076e-02
3D	med.	6.624712e+01	7.371026e+01	6.829832e+01	7.255097e+01	7.722033e+01	7.020343e+01	7.297241e+01	7.323581e+01	6.926202e+01	7.602284e+01	7.333068e+01	7.421111e+01	7.332515e+01
	avg.	6.632662e+01	7.375399e+01	6.796094e+01	7.253488e+01	7.722187e+01	7.019103e+01	7.294510e+01	7.320697e+01	6.927881e+01	7.601575e+01	7.339666e+01	7.421154e+01	7.331585e+01
	min.	6.285120e+01	7.337854e+01	5.064696e+01	7.120644e+01	7.702278e+01	6.903319e+01	7.158659e+01	7.161580e+01	6.801149e+01	7.562070e+01	7.298402e+01	7.354928e+01	7.218659e+01
	max.	6.840041e+01	7.454147e+01	7.536008e+01	7.355646e+01	7.736688e+01	7.098861e+01	7.399205e+01	7.416222e+01	7.047653e+01	7.639395e+01	7.465349e+01	7.478859e+01	7.405878e+01
	std.	1.049171e+00	2.190785e-01	3.962500e+00	5.120868e-01	8.649989e-02	3.620598e-01	5.026532e-01	5.566442e-01	5.010688e-01	1.543570e-01	2.893348e-01	2.666076e-01	2.888702e-01
4D	med.	5.436381e+02	6.566333e+02	4.794876e+02	6.959348e+02	3.930478e+02	6.613466e+02	7.058745e+02	7.123503e+02	6.609910e+02	7.348977e+02	6.593933e+02	6.749665e+02	7.119989e+02
	avg.	5.440889e+02	6.578855e+02	4.757090e+02	6.970242e+02	3.950572e+02	6.583558e+02	7.053418e+02	7.109780e+02	6.612472e+02	7.348269e+02	6.593987e+02	6.753218e+02	7.119021e+02
	min.	5.106963e+02	6.402526e+02	3.511337e+02	6.700559e+02	3.833243e+02	6.208600e+02	6.853580e+02	6.969828e+02	6.419444e+02	7.251673e+02	6.344163e+02	6.500308e+02	6.980444e+02
	max.	5.820597e+02	6.878323e+02	6.796938e+02	7.209944e+02	4.350171e+02	6.842162e+02	7.214957e+02	7.261831e+02	6.873816e+02	7.450069e+02	7.054941e+02	6.931214e+02	7.202057e+02
	std.	1.456814e+01	1.068798e+01	7.502973e+01	9.625091e+00	8.116507e+00	1.361310e+01	7.176458e+00	6.258527e+00	3.797056e+00	1.477584e+01	8.751584e+00	4.381689e+00	4.381689e+00
5D	med.	5.173619e+03	7.014378e+03	4.197752e+03	7.183985e+03	4.294043e+03	5.785981e+03	8.064670e+03	8.118096e+03	7.449736e+03	8.225646e+03	7.826277e+03	7.611915e+03	7.758123e+03
	avg.	5.178641e+03	6.995600e+03	4.078744e+03	7.160882e+03	4.341308e+03	5.770549e+03	8.058394e+03	8.112544e+03	7.439987e+03	8.222874e+03	7.826969e+03	7.628931e+03	7.753989e+03
	min.	4.752303e+03	6.362235e+03	3.247571e+03	6.236177e+03	4.202333e+03	5.350865e+03	7.702646e+03	7.894278e+03	7.227092e+03	8.092201e+03	7.095634e+03	6.942698e+03	7.375978e+03
	max.	5.609634e+03	7.713655e+03	4.871508e+03	7.643359e+03	8.534824e+03	6.086980e+03	8.301093e+03	8.295196e+03	7.647066e+03	8.358094e+03	8.191277e+03	8.124530e+03	7.996644e+03
	std.	1.946625e+02	2.353202e+02	4.699194e+02	2.884082e+02	4.258081e+02	1.511573e+02	1.004235e+02	7.710411e+01	7.713842e+01	5.461149e+01	2.072889e+02	2.839131e+02	1.026923e+02
6D	med.	5.968287e+04	6.813388e+04	4.405267e+04	8.272420e+04	5.551979e+04	6.039282e+04	1.064885e+05	1.069683e+05	9.653418e+04	1.054024e+05	8.863893e+04	8.522044e+04	9.294528e+04
	avg.	5.966841e+04	6.832264e+04	4.551140e+04	8.253695e+04	5.585827e+04	6.081722e+04	1.063186e+05	1.067909e+05	9.644725e+04	1.053236e+05	8.849589e+04	8.531579e+04	9.228950e+04
	min.	5.505789e+04	5.053486e+04	3.716622e+04	6.665281e+04	5.411255e+04	5.347425e+04	1.023245e+05	1.036746e+05	9.310056e+04	1.030777e+05	7.167535e+04	6.174908e+04	4.450593e+04
	max.	6.513949e+04	8.785432e+04	5.819591e+04	9.788082e+04	6.227946e+04	6.949551e+04	1.097182e+05	1.093415e+05	9.930925e+04	1.077189e+05	9.910423e+04	9.868637e+04	9.998659e+04
	std.	2.170738e+03	9.047219e+03	5.748510e+03	5.877294e+03	1.237856e+03	2.937652e+03	1.536552e+03	1.252760e+03	1.295710e+03	9.748776e+02	5.016455e+03	7.337352e+03	6.288478e+03
7D	med.	7.626824e+05	7.039987e+05	5.992648e+05	1.049779e+06	7.721466e+05	7.837904e+05	1.473203e+06	1.575570e+06	1.359873e+06	1.552771e+06	1.065118e+06	9.982085e+05	1.266178e+06
	avg.	7.681575e+05	7.133513e+05	6.012694e+05	1.046112e+06	7.674721e+05	7.849812e+05	1.472897e+06	1.573748e+06	1.358055e+06	1.550362e+06	1.064619e+06	9.946800e+05	1.149893e+06
	min.	6.935230e+05	5.254554e+05	4.970695e+05	8.813301e+05	7.007245e+05	6.823138e+05	1.387596e+06	1.510828e+06	1.289778e+06	1.477144e+06	9.048193e+05	8.007528e+05	6.625547e+05
	max.	8.950067e+05	9.484654e+05	7.124490e+05	1.425434e+06	8.064882e+05	8.866423e+05	1.552191e+06	1.615588e+06	1.445082e+06	1.595986e+06	1.231470e+06	1.136421e+06	1.445869e+06
	std.	4.043502e+04	8.606977e+04	5.994848e+04	1.215214e+05	2.026713e+04	3.772201e+04	3.212789e+04	2.290530e+04	3.743394e+04	2.704500e+04	6.082803e+04	6.542364e+04	2.760156e+05
8D	med.	1.298781e+07	1.082788e+07	9.412132e+06	1.678641e+07	1.380735e+07	1.384845e+07	2.648057e+07	2.696762e+07	2.367411e+07	2.669825e+07	1.710310e+07	1.629466e+07	2.031159e+07
	avg.	1.297446e+07	1.076820e+07	9.423581e+06	1.688463e+07	1.387526e+07	1.380412e+07	2.651485e+07	2.686409e+07	2.369309e+07	2.669547e+07	1.728115e+07	1.630008e+07	1.852220e+07
	min.	1.124428e+07	7.911378e+06	6.791281e+06	1.450647e+07	1.345856e+07	1.149445e+07	2.473918e+07	2.296455e+07	2.260423e+07	2.567236e+07	1.488189e+07	1.174214e+07	1.091310e+07
	max.	1.417993e+07	1.378625e+07	1.105532e+07	2.147867e+07	1.614675e+07	1.540554e+07	2.764875e+07	2.774171e+07	2.461509e+07	2.761079e+07	2.022809e+07	1.866129e+07	2.346539e+07
	std.	5.548191e+05	1.323678e+06	9.424407e+05	1.622176e+06	3.185371e+05	7.627866e+05	4.627726e+05	6.424710e+05	3.967887e+05	3.585358e+05	9.690983e+05	1.091205e+06	4.572484e+06
9D	med.	2.483190e+08	1.881403e+08	1.739954e+08	2.892747e+08	3.061929e+08	2.568861e+08	5.178957e+08	4.994474e+08	4.356138e+08	4.936697e+08	3.104687e+08	3.016706e+08	3.524195e+08
	avg.	2.479821e+08	1.864362e+08	1.741570e+08	3.035038e+08	3.069210e+08	2.588273e+08	5.156018e+08	4.972857e+08	4.352006e+08	4.936749e+08	3.091686e+08	2.998064e+08	3.194467e+08
	min.	2.289088e+08	1.319176e+08	1.463208e+08	2.591960e+08	2.928161e+08	2.152132e+08	4.812925e+08	4.371322e+08	4.099766e+08	4.730888e+08	2.390237e+08	2.474819e+08	2.007298e+08
	max.	2.648017e+08	2.504104e+08	2.084345e+08	3.818363e+08	3.419635e+08	3.039391e+08	5.404442e+08	5.250443e+08	4.502305e+08	5.080616e+08	3.421019e+08	3.657954e+08	4.320006e+08
	std.	9.024546e+06	2.571084e+07	1.559876e+07	2.721270e+07	6.733861e+06	2.010524e+07	1.479938e+07	7.114416e+06	6.696077e+06	7.114416e+06	1.730878e+07	2.012126e+07	8.346708e+07
10D	med.	5.314407e+09	3.597912e+09	3.459685e+09	6.030432e+09	8.045404e+09	4.971920e+09	1.058143e+10	9.641915e+09	8.727140e+09	9.936323e+09	6.321575e+09	6.016075e+09	6.746440e+09
	avg.	5.318515e+09	3.553657e+09	3.495181e+09	6.243653e+09	8.034174e+09	5.128531e+09	1.053549e+10	9.641559e+09	8.719371e+09	9.935956e+09	6.299927e+09	5.981243e+09	6.234957e+09
	min.	4.896400e+09	2.689995e+09	2.771260e+09	5.476586e+09	7.713205e+09	4.189190e+09	9.516036e+09	8.860524e+09	8.417787e+09	9.534824e+09	5.027208e+09	4.755706e+09	3.958771e+09
	max.	5.691618e+09	4.876270e+09	4.065683e+09	8.114881e+09	8.572660e+09	6.531381e+09	1.112250e+10	1.044798e+10	9.125954e+09	1.026240e+10	7.107583e+09	7.018982e+09	8.336053e+09
	std.	1.520509e+08	4.877549e+08	2.486847e+08	6.002466e+08	1.762863e+08	5.355756e+08	3.338487e+08	3.440622e+08	1.336270e+08	1.458795e+08	4.159559e+08	4.721691e+08	1.549937e+09

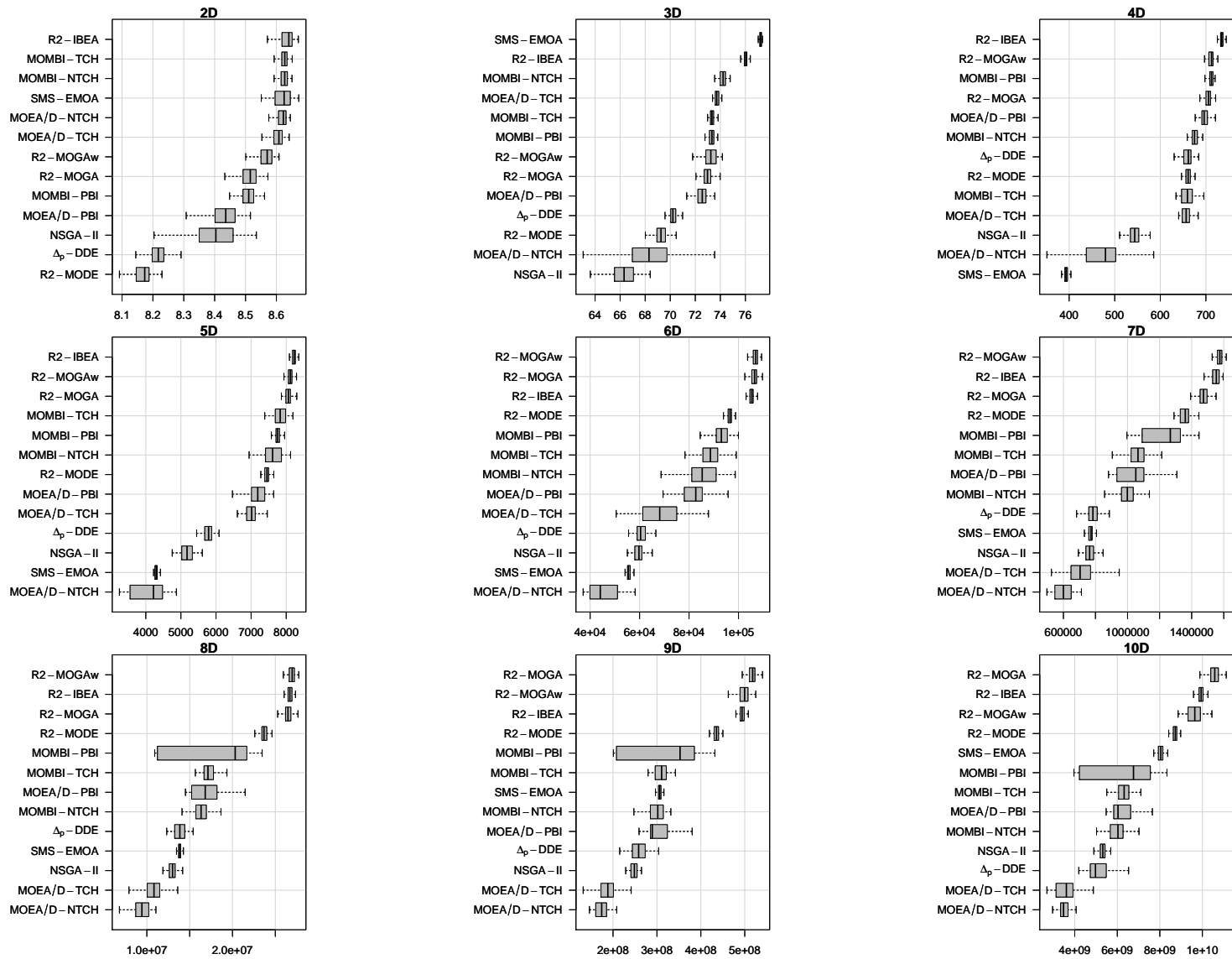


Figure A.161: Box-plot of hypervolume indicator values for different optimizers on the WFG4 test problem.

Table A.63: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG4 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	1.33e-32	> 0.05	> 0.05	2.07e-34	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	5.38e-34	—	> 0.05	4.78e-34	> 0.05	1.28e-34	2.28e-30	1.74e-18	1.28e-34	> 0.05	> 0.05	> 0.05	7.83e-31
MOEA/D-NTCH	1.49e-34	1.45e-07	—	1.28e-34	> 0.05	1.28e-34	1.52e-33	5.04e-27	1.28e-34	> 0.05	> 0.05	> 0.05	6.83e-34
MOEA/D-PBI	1.12e-03	> 0.05	> 0.05	—	> 0.05	1.32e-34	> 0.05	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	3.15e-34	2.62e-05	> 0.05	1.84e-34	—	1.28e-34	1.20e-30	8.06e-20	1.28e-34	> 0.05	> 0.05	> 0.05	2.59e-31
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	5.56e-22	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	6.33e-26	> 0.05	> 0.05	1.38e-27	> 0.05	1.28e-34	—	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	1.50e-32	> 0.05	> 0.05	1.83e-32	> 0.05	1.28e-34	2.25e-21	—	1.28e-34	> 0.05	> 0.05	> 0.05	8.87e-24
R2-MODE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	1.36e-34	2.00e-13	1.15e-07	1.28e-34	2.51e-03	1.28e-34	1.76e-33	1.76e-23	1.28e-34	—	3.22e-05	7.65e-05	4.64e-34
MOMBI-TCH	1.28e-34	2.67e-12	1.55e-02	1.28e-34	> 0.05	1.28e-34	1.58e-34	3.07e-31	1.28e-34	> 0.05	—	> 0.05	1.28e-34
MOMBI-NTCH	1.28e-34	1.79e-13	3.74e-03	1.28e-34	> 0.05	1.28e-34	6.06e-34	8.72e-30	1.28e-34	> 0.05	> 0.05	—	2.96e-34
MOMBI-PBI	7.78e-26	> 0.05	> 0.05	2.40e-28	> 0.05	1.28e-34	> 0.05	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	—
3D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	3.77e-27	3.15e-34	> 0.05	1.28e-34	3.77e-27	4.89e-14	1.28e-34	> 0.05	3.93e-20	> 0.05	4.50e-24
MOEA/D-NTCH	7.90e-14	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	> 0.05	7.02e-26	—	> 0.05	1.28e-34	> 0.05	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34
$\Delta_p$ -DDE	1.28e-34	> 0.05	1.14e-11	> 0.05	> 0.05	—	> 0.05	> 0.05	1.01e-25	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	> 0.05	3.58e-26	2.42e-08	> 0.05	1.28e-34	—	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	1.28e-34	> 0.05	2.13e-26	8.21e-15	> 0.05	1.28e-34	1.45e-04	—	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	1.89e-34	> 0.05	7.94e-06	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	1.28e-34	> 0.05	2.30e-26	6.48e-27	> 0.05	1.28e-34	2.86e-13	1.36e-02	1.28e-34	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.28e-34	1.19e-23	5.54e-28	1.32e-34	> 0.05	1.28e-34	1.07e-33	1.98e-30	1.28e-34	> 0.05	2.22e-30	—	2.30e-33
MOMBI-PBI	1.28e-34	> 0.05	2.49e-26	4.78e-24	> 0.05	1.28e-34	8.76e-10	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	—
4D													
NSGA-II	—	> 0.05	5.39e-20	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	1.16e-30	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	8.76e-13	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	6.63e-34	1.49e-34	—	1.28e-34	4.00e-34	> 0.05	> 0.05	2.63e-34	> 0.05	9.24e-32	1.32e-28	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e-34	> 0.05	9.55e-31	> 0.05	1.28e-34	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.36e-34	1.28e-34	3.73e-10	1.28e-34	1.28e-34	—	> 0.05	1.36e-34	> 0.05	8.92e-34	5.38e-34	> 0.05
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	2.24e-21	1.28e-34	1.28e-34	1.53e-07	—	1.28e-34	> 0.05	2.26e-34	1.28e-34	> 0.05
R2-MODE	1.28e-34	1.76e-03	7.83e-31	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.40e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	1.28e-34	> 0.05	7.61e-31	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.28e-34	7.13e-21	3.90e-33	> 0.05	1.28e-34	3.93e-20	> 0.05	> 0.05	2.76e-22	> 0.05	8.48e-16	—	> 0.05
MOMBI-PBI	1.28e-34	1.28e-34	1.28e-34	7.71e-25	1.28e-34	1.28e-34	3.18e-11	> 0.05	1.28e-34	> 0.05	1.49e-34	1.28e-34	—

Table A.64: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG4 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
5D													
NSGA-II	—	> 0.05	1.89e-34	> 0.05	2.51e-33	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	1.28e-34	> 0.05	2.51e-33	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	7.65e-07	1.28e-34	—	2.51e-33	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	4.58e-02	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.07e-33	> 0.05	1.28e-34	> 0.05	2.51e-33	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.32e-34	1.28e-34	1.28e-34	2.51e-33	1.28e-34	—	> 0.05	1.28e-34	> 0.05	7.14e-17	1.92e-26	4.01e-32
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.28e-34	2.51e-33	1.28e-34	1.87e-05	—	1.28e-34	> 0.05	2.51e-24	4.20e-31	2.48e-34
R2-MODE	1.28e-34	4.73e-30	1.28e-34	4.07e-16	2.51e-33	1.28e-34	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	2.51e-33	1.28e-34	2.99e-26	3.57e-21	1.28e-34	—	8.40e-34	1.63e-34	1.28e-34
MOMBI-TCH	1.28e-34	9.75e-34	1.28e-34	3.18e-32	2.51e-33	1.28e-34	> 0.05	> 0.05	3.66e-29	> 0.05	—	3.09e-07	8.14e-04
MOMBI-NTCH	1.28e-34	7.07e-29	1.28e-34	1.55e-20	2.51e-33	1.28e-34	> 0.05	> 0.05	1.41e-08	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	1.28e-34	3.44e-34	1.28e-34	1.10e-33	2.51e-33	1.28e-34	> 0.05	> 0.05	4.37e-32	> 0.05	> 0.05	8.85e-04	—
6D													
NSGA-II	—	> 0.05	4.24e-34	> 0.05	6.23e-27	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	6.95e-14	—	8.35e-33	> 0.05	7.54e-20	3.29e-11	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	1.68e-23	1.28e-34	—	1.28e-34	1.44e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	5.90e-31	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	2.08e-03	> 0.05	3.34e-34	> 0.05	1.12e-28	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	> 0.05	1.28e-34	2.16e-08	1.28e-34	1.28e-34	1.28e-34
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.73e-02	—	1.28e-34	8.86e-15	1.28e-34	1.28e-34	1.28e-34
R2-MODE	1.28e-34	1.28e-34	1.28e-34	2.82e-31	1.28e-34	1.28e-34	> 0.05	> 0.05	—	> 0.05	2.56e-26	1.10e-30	8.59e-17
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	1.28e-34	6.93e-32	1.28e-34	1.64e-13	1.28e-34	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	—	1.39e-03	> 0.05
MOMBI-NTCH	3.44e-34	9.81e-26	1.28e-34	3.31e-04	1.32e-34	9.19e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	2.51e-33	1.33e-32	5.38e-34	4.35e-23	2.51e-33	2.51e-33	> 0.05	> 0.05	> 0.05	> 0.05	1.02e-10	4.99e-15	—
7D													
NSGA-II	—	2.38e-08	2.40e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	> 0.05	—	4.18e-18	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.89e-34	1.27e-33	1.28e-34	—	1.28e-34	1.40e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	4.21e-04	> 0.05
SMS-EMOA	> 0.05	3.85e-09	1.53e-34	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	3.28e-04	1.26e-11	2.55e-34	> 0.05	5.68e-05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	1.53e-34	1.28e-34	1.28e-34	—	> 0.05	2.04e-33	> 0.05	1.28e-34	1.28e-34	2.17e-33
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	6.63e-34	—	1.28e-34	1.62e-09	1.28e-34	1.28e-34	1.28e-34
R2-MODE	1.28e-34	1.28e-34	1.28e-34	5.35e-32	1.28e-34	1.28e-34	> 0.05	> 0.05	—	> 0.05	1.28e-34	1.28e-34	1.03e-14
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	2.86e-30	> 0.05	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	1.28e-34	1.44e-34	1.28e-34	> 0.05	1.28e-34	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	—	7.48e-13	> 0.05
MOMBI-NTCH	5.88e-34	1.39e-33	1.28e-34	> 0.05	1.40e-34	1.31e-33	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	2.11e-11	9.14e-18	4.09e-29	1.83e-08	2.11e-11	2.11e-11	> 0.05	> 0.05	> 0.05	> 0.05	3.90e-10	5.80e-11	—



Table A.65: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG4 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
8D													
NSGA-II	—	$2.25e-26$	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-TCH	$> 0.05$	—	$1.93e-13$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-NTCH	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$5.07e-34$	$4.65e-33$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$2.91e-02$	$> 0.05$
SMS-EMOA	$8.75e-25$	$6.43e-34$	$1.28e-34$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
$\Delta_p$ -DDE	$2.85e-14$	$2.06e-31$	$1.28e-34$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
R2-MOGA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$> 0.05$	$1.28e-34$	$> 0.05$	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MOGAw	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.21e-09$	—	$2.23e-33$	$2.21e-05$	$1.28e-34$	$1.28e-34$	$1.58e-34$
R2-MODE	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$> 0.05$	—	$> 0.05$	$1.28e-34$	$1.28e-34$	$5.71e-33$
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.20e-03$	$> 0.05$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.20e-02$	$1.68e-34$	$1.95e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$1.32e-10$	$> 0.05$
MOMBI-NTCH	$3.00e-32$	$3.05e-34$	$1.28e-34$	$> 0.05$	$2.31e-31$	$2.10e-30$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$
MOMBI-PBI	$2.24e-09$	$8.40e-22$	$1.68e-34$	$4.84e-08$	$2.27e-09$	$2.27e-09$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.39e-08$	$3.09e-09$	—
9D													
NSGA-II	—	$3.57e-32$	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-TCH	$> 0.05$	—	$2.12e-05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-NTCH	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-PBI	$1.89e-34$	$1.28e-34$	$1.28e-34$	—	$> 0.05$	$3.18e-25$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
SMS-EMOA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.15e-03$	—	$5.54e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$7.86e-03$	$> 0.05$
$\Delta_p$ -DDE	$1.93e-05$	$1.59e-32$	$1.28e-34$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
R2-MOGA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$8.70e-19$	$1.28e-34$	$3.80e-28$	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MOGAw	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	—	$4.78e-34$	$6.79e-04$	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MODE	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$> 0.05$	—	$> 0.05$	$1.28e-34$	$1.28e-34$	$6.06e-34$
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$> 0.05$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$1.39e-33$	$1.49e-34$	$1.28e-34$	$1.10e-03$	$> 0.05$	$5.12e-31$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$4.97e-04$	$> 0.05$
MOMBI-NTCH	$4.52e-33$	$1.36e-34$	$1.28e-34$	$> 0.05$	$> 0.05$	$4.00e-25$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$
MOMBI-PBI	$1.64e-05$	$2.27e-28$	$2.23e-33$	$3.82e-04$	$2.46e-05$	$1.64e-05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.09e-04$	$4.79e-05$	—
10D													
NSGA-II	—	$1.28e-34$	$1.28e-34$	$> 0.05$	$> 0.05$	$8.76e-06$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-TCH	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-NTCH	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOEA/D-PBI	$1.43e-33$	$1.28e-34$	$1.28e-34$	—	$> 0.05$	$1.77e-24$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.83e-02$	$> 0.05$
SMS-EMOA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$4.25e-33$	—	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$	$1.28e-34$	$5.54e-24$
$\Delta_p$ -DDE	$> 0.05$	$2.58e-33$	$1.28e-34$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
R2-MOGA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$3.77e-29$	$1.28e-34$	$1.06e-25$	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MOGAw	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	—	$2.40e-34$	$> 0.05$	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MODE	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.70e-34$	$1.28e-34$	$> 0.05$	$> 0.05$	—	$> 0.05$	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.44e-11$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$9.55e-31$	$1.28e-34$	$1.28e-34$	$8.13e-03$	$> 0.05$	$1.17e-27$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$6.68e-07$	$> 0.05$
MOMBI-NTCH	$4.14e-22$	$1.36e-34$	$1.28e-34$	$> 0.05$	$> 0.05$	$4.60e-20$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$
MOMBI-PBI	$1.64e-05$	$2.12e-31$	$1.53e-34$	$1.66e-02$	$> 0.05$	$1.97e-05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$8.57e-03$	$4.28e-04$	—

Table A.66: Comparison of R2 indicator values for different optimizers on the WFG4 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	5.968954e-01	5.893831e-01	5.894567e-01	5.958044e-01	5.883128e-01	6.050051e-01	5.923528e-01	5.909439e-01	6.077013e-01	5.885312e-01	5.890318e-01	5.891732e-01	5.934459e-01
	avg.	5.982651e-01	5.895547e-01	5.895336e-01	5.961134e-01	5.886842e-01	6.048998e-01	5.923448e-01	5.910764e-01	6.078643e-01	5.888050e-01	5.890732e-01	5.892798e-01	5.933976e-01
	min.	5.909987e-01	5.889123e-01	5.888166e-01	5.931299e-01	5.878582e-01	6.028483e-01	5.906437e-01	5.896780e-01	6.052600e-01	5.881121e-01	5.885902e-01	5.886637e-01	5.915561e-01
	max.	6.127559e-01	5.939254e-01	5.917093e-01	6.028222e-01	5.948928e-01	6.079536e-01	5.948347e-01	5.942533e-01	6.128262e-01	5.914664e-01	5.898895e-01	5.920256e-01	5.958683e-01
	std.	5.093307e-03	6.720050e-04	4.643018e-04	1.587771e-03	1.009018e-03	1.100726e-03	7.947335e-04	7.707331e-04	1.372570e-03	7.664315e-04	2.480318e-04	5.152384e-04	7.828592e-04
3D	med.	4.019292e-01	3.166663e-01	3.395762e-01	3.235791e-01	3.118103e-01	3.378711e-01	3.187235e-01	3.176608e-01	3.304426e-01	3.160793e-01	3.169658e-01	3.175645e-01	3.211940e-01
	avg.	4.029347e-01	3.166564e-01	3.543529e-01	3.237085e-01	3.118827e-01	3.402559e-01	3.188127e-01	3.180416e-01	3.305959e-01	3.161398e-01	3.169593e-01	3.176309e-01	3.212364e-01
	min.	3.579314e-01	3.158661e-01	3.161999e-01	3.204776e-01	3.109073e-01	3.307842e-01	3.167264e-01	3.156057e-01	3.278592e-01	3.149871e-01	3.162730e-01	3.165554e-01	3.193212e-01
	max.	4.506043e-01	3.172020e-01	6.687949e-01	3.288746e-01	3.134401e-01	3.670400e-01	3.211149e-01	3.304712e-01	3.348022e-01	3.180183e-01	3.178425e-01	3.193623e-01	3.249576e-01
	std.	1.859683e-02	2.733878e-04	4.333286e-02	1.719673e-03	4.754340e-04	7.616963e-03	9.339664e-04	1.844135e-03	1.211684e-03	6.061344e-04	3.105725e-04	5.171421e-04	8.347328e-04
4D	med.	3.954465e-01	2.244233e-01	4.452893e-01	2.223376e-01	4.382988e-01	2.442731e-01	2.144612e-01	2.148248e-01	2.259865e-01	2.185133e-01	2.238863e-01	2.243878e-01	2.179157e-01
	avg.	3.976841e-01	2.242709e-01	5.016931e-01	2.231290e-01	4.367348e-01	2.480040e-01	2.144800e-01	2.150598e-01	2.261251e-01	2.186068e-01	2.238529e-01	2.245208e-01	2.181252e-01
	min.	3.576598e-01	2.202324e-01	2.320171e-01	2.159667e-01	3.891023e-01	2.295732e-01	2.116759e-01	2.116276e-01	2.222439e-01	2.156899e-01	2.183629e-01	2.187600e-01	2.160232e-01
	max.	4.484759e-01	2.264360e-01	7.637978e-01	2.379403e-01	4.53752e-01	2.852450e-01	2.177450e-01	2.218147e-01	2.292809e-01	2.227790e-01	2.271579e-01	2.295034e-01	2.218255e-01
	std.	1.795259e-02	1.451004e-03	1.528863e-01	4.301968e-03	9.983492e-03	1.240364e-02	1.278651e-03	1.910900e-03	1.348747e-03	1.557659e-03	1.875967e-03	2.108635e-03	1.024092e-03
5D	med.	4.080081e-01	1.777476e-01	6.605617e-01	2.052628e-01	3.514323e-01	2.765545e-01	1.642713e-01	1.661569e-01	1.767401e-01	1.760500e-01	1.716111e-01	1.717490e-01	1.84289e-01
	avg.	4.049802e-01	1.796245e-01	6.540062e-01	2.041883e-01	3.492964e-01	2.760173e-01	1.645925e-01	1.666009e-01	1.766417e-01	1.768236e-01	1.716182e-01	1.723277e-01	1.848132e-01
	min.	3.314935e-01	1.714943e-01	4.093493e-01	1.872953e-01	1.687110e-01	2.494291e-01	1.615791e-01	1.629170e-01	1.733691e-01	1.720377e-01	1.673401e-01	1.655063e-01	1.801272e-01
	max.	4.519577e-01	2.605914e-01	8.162317e-01	2.357386e-01	3.611213e-01	2.958681e-01	1.701479e-01	1.740671e-01	1.797916e-01	1.895969e-01	1.781067e-01	1.795366e-01	1.917024e-01
	std.	2.079681e-02	1.029896e-02	1.278031e-01	1.089279e-02	1.909467e-02	1.024827e-02	1.505356e-03	1.989984e-03	1.154139e-03	3.315044e-03	2.202242e-03	3.099583e-03	2.093283e-03
6D	med.	4.204408e-01	2.483095e-01	7.642474e-01	1.894647e-01	2.911457e-01	2.776565e-01	1.243788e-01	1.275847e-01	1.387500e-01	1.495548e-01	2.228708e-01	2.234637e-01	1.630586e-01
	avg.	4.187795e-01	2.796029e-01	7.095514e-01	1.921291e-01	2.902664e-01	2.771404e-01	1.249682e-01	1.280482e-01	1.389902e-01	1.496635e-01	2.226328e-01	2.334981e-01	1.631902e-01
	min.	3.775654e-01	1.743787e-01	5.008364e-01	1.553191e-01	2.535070e-01	2.411172e-01	1.214262e-01	1.232158e-01	1.352957e-01	1.382579e-01	1.610581e-01	1.679484e-01	1.450111e-01
	max.	4.577181e-01	4.638270e-01	8.273432e-01	2.504130e-01	3.020770e-01	3.062359e-01	1.312055e-01	1.369571e-01	1.463630e-01	1.667445e-01	3.133865e-01	3.351607e-01	3.341968e-01
	std.	1.598259e-02	6.555256e-02	1.175071e-01	1.539284e-02	7.346647e-03	1.280960e-02	1.996897e-03	2.322095e-03	1.845941e-03	5.069189e-03	2.488280e-02	3.549380e-02	2.054828e-02
7D	med.	4.506332e-01	4.781602e-01	7.337044e-01	1.873031e-01	2.729388e-01	2.975271e-01	1.214702e-01	1.134576e-01	1.303561e-01	1.352832e-01	3.488306e-01	3.528780e-01	1.625708e-01
	avg.	4.498892e-01	4.597384e-01	7.195966e-01	1.958647e-01	2.729795e-01	2.980508e-01	1.216200e-01	1.141198e-01	1.309732e-01	1.375988e-01	3.493532e-01	3.630920e-01	1.823456e-01
	min.	3.974927e-01	3.259752e-01	5.338801e-01	1.403322e-01	2.592170e-01	2.665551e-01	1.107377e-01	1.079232e-01	1.211006e-01	1.242274e-01	2.894336e-01	3.390267e-01	1.290744e-01
	max.	5.059859e-01	7.120207e-01	8.531602e-01	2.399798e-01	2.878332e-01	3.218617e-01	1.313045e-01	1.284035e-01	1.394943e-01	1.639021e-01	4.528440e-01	5.168260e-01	2.873341e-01
	std.	2.240903e-02	7.201076e-02	1.151521e-01	2.802145e-02	5.471006e-03	1.093695e-02	4.525780e-03	4.114910e-03	4.943084e-03	9.214860e-03	1.936500e-02	3.487161e-02	5.862849e-02
8D	med.	4.225262e-01	5.021052e-01	7.994816e-01	1.822726e-01	2.205575e-01	2.604143e-01	9.454180e-02	9.638185e-02	1.091563e-01	1.300090e-01	3.663728e-01	3.716268e-01	1.453747e-01
	avg.	4.212077e-01	4.855716e-01	7.585284e-01	1.814360e-01	2.199518e-01	2.606644e-01	9.513798e-02	9.824595e-02	1.090098e-01	1.303356e-01	3.700466e-01	3.736658e-01	1.647635e-01
	min.	3.854980e-01	3.778778e-01	5.476061e-01	1.414183e-01	1.852298e-01	2.324586e-01	8.916629e-02	9.137920e-02	1.031697e-01	1.131872e-01	3.024376e-01	3.507474e-01	1.152551e-01
	max.	4.557762e-01	7.127149e-01	8.895749e-01	2.161125e-01	2.276117e-01	2.834481e-01	1.264828e-01	1.734308e-01	1.142473e-01	1.567423e-01	4.775364e-01	4.033707e-01	2.537164e-01
	std.	1.668772e-02	6.427075e-02	1.051457e-01	2.286754e-02	5.283908e-03	8.980835e-03	4.208741e-03	9.688457e-03	2.323874e-03	7.809309e-03	2.211776e-02	9.197763e-03	5.314178e-02
9D	med.	3.971328e-01	5.175216e-01	7.569856e-01	1.830838e-01	1.682536e-01	2.304216e-01	7.879063e-02	8.700394e-02	1.025940e-01	1.354735e-01	3.841582e-01	3.870983e-01	1.363416e-01
	avg.	3.981420e-01	5.076798e-01	7.455422e-01	1.727872e-01	1.681257e-01	2.301317e-01	7.975630e-02	8.993152e-02	1.025686e-01	1.359862e-01	3.868156e-01	3.888239e-01	1.586177e-01
	min.	3.688046e-01	3.934118e-01	5.489295e-01	1.313375e-01	1.522735e-01	2.117492e-01	7.576132e-02	8.037628e-02	9.879080e-02	1.167987e-01	3.684911e-01	3.560198e-01	1.044866e-01
	max.	4.321571e-01	7.329847e-01	8.589309e-01	2.007999e-01	1.767434e-01	2.529415e-01	1.076408e-01	1.608156e-01	1.070418e-01	1.766611e-01	4.301330e-01	4.238933e-01	2.277035e-01
	std.	1.390041e-02	6.044621e-02	9.699103e-02	1.810575e-02	3.663791e-03	8.259913e-03	4.576690e-03	1.079390e-02	1.820823e-03	9.018436e-03	1.216835e-02	1.142085e-02	4.833941e-02
10D	med.	3.732502e-01	5.638393e-01	7.405062e-01	1.651725e-01	1.233657e-01	2.158933e-01	7.267322e-02	1.009839e-01	1.034091e-01	1.458816e-01	3.931289e-01	3.950456e-01	1.295123e-01
	avg.	3.745835e-01	5.811077e-01	7.334680e-01	1.588944e-01	1.231290e-01	2.149472e-01	7.589373e-02	1.051466e-01	1.032741e-01	1.465696e-01	3.966849e-01	3.965357e-01	1.481186e-01
	min.	3.404570e-01	4.249360e-01	5.654177e-01	1.184564e-01	1.126060e-01	1.917817e-01	6.897552e-02	7.844036e-02	9.858044e-02	1.335942e-01	3.685607e-01	3.712444e-01	1.020942e-01
	max.	4.112895e-01	8.363303e-01	8.625790e-01	1.806343e-01	1.299836e-01	2.427686e-01	1.555499e-01	1.488917e-01	1.081102e-01	1.649046e-01	4.381634e-01	4.382189e-01	2.089573e-01
	std.	1.332675e-02	8.492038e-02	9.434916e-02	1.688159e-02	2.968290e-03	1.050087e-02	1.202613e-02	1.825495e-02	1.988521e-03	6.479387e-03	1.303182e-02	1.018004e-02	4.138786e-02

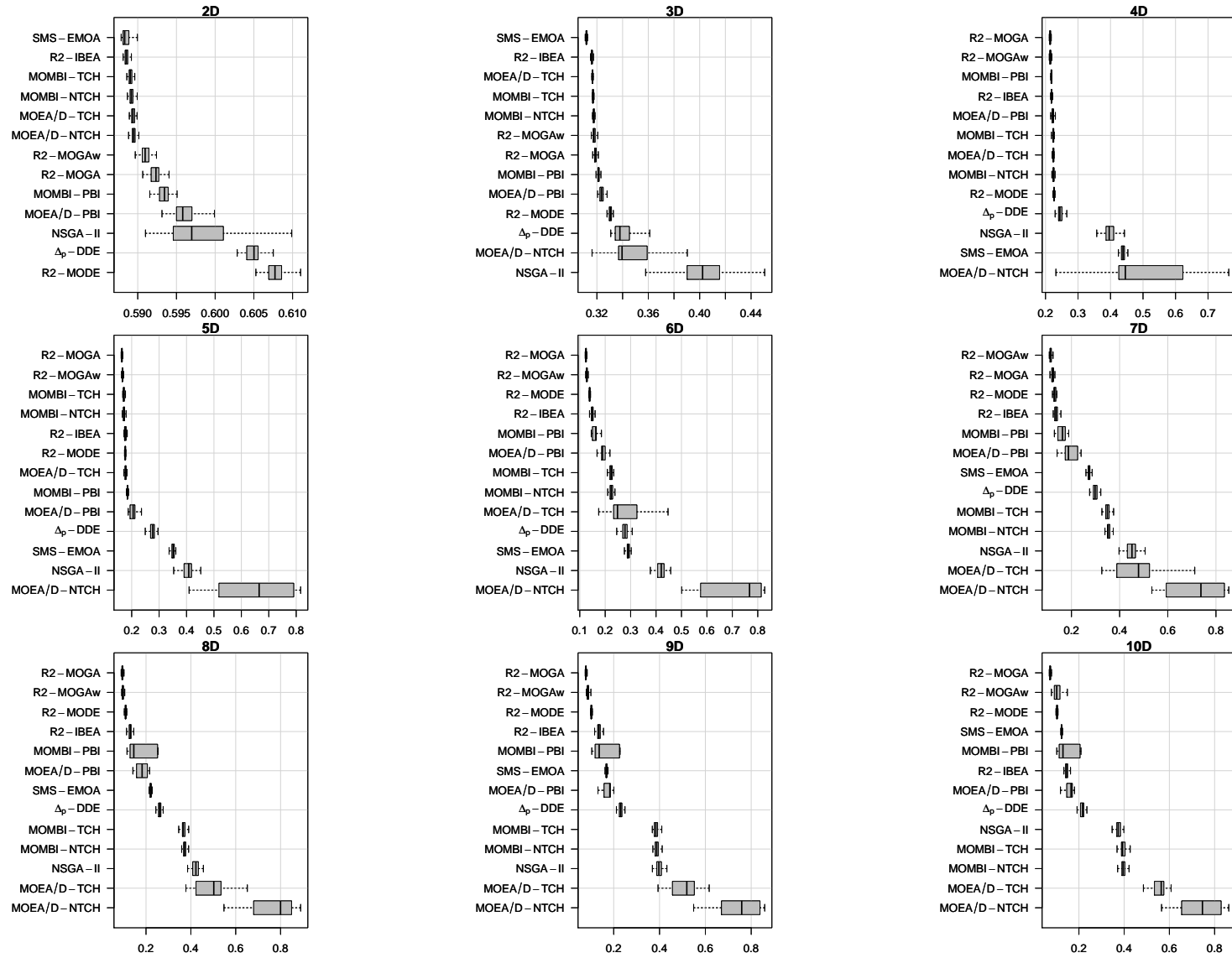


Figure A.162: Box-plot of R2 indicator values for different optimizers on the WFG4 test problem.

Table A.67: Comparison of runtime (in milliseconds) for different optimizers on the WFG4 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_\rho$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	4.470710e + 02	8.779480e + 02	1.443400e + 03	1.338384e + 03	3.344709e + 04	1.585969e + 03	2.046265e + 03	1.583489e + 03	2.111055e + 03	5.491690e + 05	2.153068e + 03	2.252051e + 03	2.982590e + 03
	avg.	4.490646e + 02	8.937412e + 02	1.428392e + 03	1.323693e + 03	3.354714e + 04	1.604684e + 03	2.043014e + 03	1.594237e + 03	2.088393e + 03	5.494535e + 05	2.154816e + 03	2.243683e + 03	2.980194e + 03
	min.	3.815790e + 02	6.948240e + 02	1.147738e + 03	9.501890e + 02	2.508221e + 04	1.269665e + 03	1.482542e + 03	1.287528e + 03	1.249909e + 03	5.349290e + 05	1.721858e + 03	1.940390e + 03	2.445097e + 03
	max.	6.056180e + 02	1.277792e + 03	1.774264e + 03	2.071201e + 03	3.974074e + 04	2.053580e + 03	2.539068e + 03	1.947884e + 03	2.553276e + 03	5.631660e + 05	2.510240e + 03	2.731941e + 03	3.325227e + 03
	std.	4.713518e + 01	1.019050e + 02	9.934659e + 01	1.449902e + 02	3.108898e + 03	1.407294e + 02	2.153957e + 02	1.554868e + 02	2.718630e + 02	4.964297e + 03	1.520488e + 02	1.374778e + 02	1.359120e + 02
3D	med.	6.159790e + 02	9.353280e + 02	1.689499e + 03	1.408689e + 03	1.060722e + 06	1.872357e + 03	2.222581e + 03	1.880057e + 03	2.480011e + 03	5.675860e + 05	2.164039e + 03	2.103516e + 03	3.731280e + 03
	avg.	6.154608e + 02	9.369093e + 02	1.697762e + 03	1.402879e + 03	1.037376e + 06	1.859479e + 03	2.183010e + 03	1.903421e + 03	2.454535e + 03	5.648722e + 05	2.151271e + 03	2.137436e + 03	3.723747e + 03
	min.	5.235670e + 02	7.191640e + 02	1.218761e + 03	1.142208e + 03	7.842698e + 05	1.336200e + 03	1.652988e + 03	1.428577e + 03	1.992439e + 03	4.990470e + 05	1.805905e + 03	1.854010e + 03	3.159822e + 03
	max.	7.743760e + 02	1.256167e + 03	2.218254e + 03	1.688329e + 03	1.123962e + 06	2.317819e + 03	2.705093e + 03	2.392700e + 03	2.902496e + 03	5.867250e + 05	2.528979e + 03	2.471292e + 03	4.241841e + 03
	std.	5.645184e + 01	1.009847e + 02	1.449953e + 02	1.084227e + 02	6.884624e + 04	1.810174e + 02	2.591413e + 02	2.028276e + 02	2.097023e + 02	1.634693e + 04	1.459957e + 02	1.436304e + 02	2.025789e + 02
4D	med.	9.022460e + 02	1.168411e + 03	1.845362e + 03	1.676547e + 03	3.207586e + 06	2.429413e + 03	2.978977e + 03	2.502901e + 03	3.307993e + 03	5.937710e + 05	2.323766e + 03	2.400494e + 03	3.666117e + 03
	avg.	8.946101e + 02	1.168337e + 03	1.838336e + 03	1.664396e + 03	3.100718e + 06	2.404085e + 03	2.911856e + 03	2.514263e + 03	3.264043e + 03	5.933662e + 05	2.343869e + 03	2.414335e + 03	3.668313e + 03
	min.	7.328420e + 02	9.712760e + 02	1.405948e + 03	1.200825e + 03	1.961072e + 06	1.627919e + 03	1.953828e + 03	1.844089e + 03	2.338973e + 03	5.689710e + 05	1.922981e + 03	2.123785e + 03	3.267399e + 03
	max.	1.057424e + 03	1.570020e + 03	2.252906e + 03	2.009515e + 03	3.622133e + 06	3.085173e + 03	3.261330e + 03	2.914588e + 03	3.882679e + 03	5.999540e + 05	3.392436e + 03	2.751977e + 03	4.058873e + 03
	std.	7.839933e + 01	9.552902e + 01	1.534459e + 02	1.311175e + 02	4.150029e + 05	2.663618e + 02	2.819879e + 02	2.155398e + 02	3.737301e + 02	4.211105e + 03	2.732199e + 02	1.630519e + 02	1.539831e + 02
5D	med.	1.122286e + 03	1.370466e + 03	2.558837e + 03	1.819406e + 03	4.368952e + 06	5.019039e + 03	3.413276e + 03	3.174881e + 03	4.388150e + 03	6.789070e + 05	2.978639e + 03	2.916297e + 03	4.238542e + 03
	avg.	1.123662e + 03	1.374162e + 03	2.531228e + 03	1.766973e + 03	4.296553e + 06	5.059038e + 03	3.428241e + 03	3.190649e + 03	4.303728e + 03	6.583716e + 05	3.039215e + 03	2.921011e + 03	4.201601e + 03
	min.	9.355770e + 02	1.112165e + 03	1.802396e + 03	1.392572e + 03	2.655180e + 06	2.843021e + 03	2.596067e + 03	2.361170e + 03	3.130188e + 03	5.915920e + 05	2.377017e + 03	2.400902e + 03	3.423149e + 03
	max.	1.358381e + 03	1.608937e + 03	2.885617e + 03	2.256026e + 03	4.873259e + 06	6.835072e + 03	3.989273e + 03	3.684806e + 03	4.901942e + 03	6.905980e + 05	4.805607e + 03	3.271403e + 03	4.438628e + 03
	std.	8.803051e + 01	1.015186e + 02	1.869439e + 02	2.050763e + 02	3.905004e + 05	6.792380e + 02	3.420929e + 02	2.901785e + 02	3.355187e + 02	3.211782e + 04	4.735484e + 02	1.848638e + 02	1.563084e + 02
6D	med.	1.402352e + 03	1.560271e + 03	2.741399e + 03	2.029773e + 03	4.143657e + 06	5.307773e + 03	4.198023e + 03	3.802259e + 03	4.663156e + 03	7.284070e + 05	2.853231e + 03	2.930359e + 03	4.690120e + 03
	avg.	1.386050e + 03	1.564623e + 03	2.708585e + 03	1.995657e + 03	4.093750e + 06	5.341292e + 03	4.114155e + 03	3.800246e + 03	4.656999e + 03	7.273937e + 05	2.865561e + 03	2.962681e + 03	4.675535e + 03
	min.	1.141684e + 03	1.273515e + 03	1.914693e + 03	1.574645e + 03	2.977935e + 06	3.299497e + 03	2.887571e + 03	3.189789e + 03	3.543513e + 03	6.627550e + 05	2.292569e + 03	2.651007e + 03	3.829742e + 03
	max.	1.641005e + 03	2.169128e + 03	3.057452e + 03	2.649490e + 03	4.655950e + 06	7.500763e + 03	4.806647e + 03	4.280226e + 03	5.292396e + 03	7.379210e + 05	4.128782e + 03	3.674156e + 03	5.141707e + 03
	std.	1.149490e + 02	1.405484e + 02	1.769777e + 02	1.908198e + 02	3.777327e + 05	8.459911e + 02	4.162639e + 02	2.514362e + 02	4.283213e + 02	8.225047e + 03	2.870413e + 02	1.762693e + 02	2.178017e + 02
7D	med.	1.403137e + 03	1.664914e + 03	2.935865e + 03	2.021322e + 03	3.000903e + 06	1.821441e + 04	3.685950e + 03	3.324000e + 03	4.303305e + 03	3.322430e + 05	2.328719e + 03	2.586422e + 03	3.627201e + 03
	avg.	1.400228e + 03	1.662465e + 03	2.862228e + 03	1.994123e + 03	2.921112e + 06	1.813389e + 04	3.643172e + 03	3.340823e + 03	4.183727e + 03	3.317201e + 05	2.313016e + 03	2.593766e + 03	3.600246e + 03
	min.	1.148069e + 03	1.302397e + 03	2.171432e + 03	1.529486e + 03	2.005352e + 06	9.693703e + 03	2.558663e + 03	2.553672e + 03	3.068701e + 03	2.925060e + 05	1.833320e + 03	2.281792e + 03	3.064173e + 03
	max.	1.638248e + 03	2.092671e + 03	3.550282e + 03	2.478306e + 03	3.503640e + 06	2.741478e + 04	4.179493e + 03	3.783508e + 03	4.821806e + 03	3.376640e + 05	2.666992e + 03	3.087327e + 03	4.047016e + 03
	std.	9.762352e + 01	1.333312e + 02	2.425762e + 02	1.876982e + 02	3.932331e + 05	3.525423e + 03	3.080592e + 02	2.494834e + 02	3.965065e + 02	4.879863e + 03	1.555051e + 02	1.586550e + 02	1.664791e + 02
8D	med.	1.826294e + 03	1.887402e + 03	3.238599e + 03	2.220266e + 03	5.104427e + 06	3.815153e + 04	5.109942e + 03	4.776669e + 03	6.302960e + 03	1.274785e + 06	3.188455e + 03	3.652244e + 03	5.213327e + 03
	avg.	1.830351e + 03	1.898479e + 03	3.187806e + 03	2.177865e + 03	5.097469e + 06	3.971690e + 04	5.097669e + 03	4.760796e + 03	6.070609e + 03	1.244761e + 06	3.157945e + 03	3.665927e + 03	5.226143e + 03
	min.	1.520249e + 03	1.612979e + 03	2.320843e + 03	1.667061e + 03	3.593208e + 06	2.442367e + 04	3.768681e + 03	3.584870e + 03	4.500322e + 03	1.173644e + 06	2.451902e + 03	3.148821e + 03	4.396995e + 03
	max.	2.069970e + 03	2.283731e + 03	3.588546e + 03	2.640675e + 03	6.207530e + 06	6.107745e + 04	5.966140e + 03	5.471446e + 03	6.774766e + 03	1.314622e + 06	3.476109e + 03	4.134715e + 03	5.855942e + 03
	std.	1.218036e + 02	1.320204e + 02	2.207019e + 02	2.069576e + 02	6.201526e + 05	8.448882e + 03	4.794167e + 02	4.070813e + 02	5.486962e + 02	5.011233e + 04	1.832279e + 02	1.906852e + 02	3.509280e + 02
9D	med.	2.414922e + 03	2.068005e + 03	3.841181e + 03	2.445100e + 03	7.359190e + 06	6.212938e + 04	7.213361e + 03	6.906769e + 03	9.515049e + 03	2.657788e + 06	3.749573e + 03	4.796865e + 03	7.324264e + 03
	avg.	2.403063e + 03	2.071145e + 03	3.820838e + 03	2.348936e + 03	7.240538e + 06	6.307376e + 04	7.119695e + 03	6.863815e + 03	9.524792e + 03	2.626167e + 06	3.831297e + 03	4.869909e + 03	7.371141e + 03
	min.	2.055357e + 03	1.617044e + 03	3.285957e + 03	1.808675e + 03	5.385609e + 06	3.749120e + 04	5.482280e + 03	5.138983e + 03	7.834676e + 03	2.241403e + 06	3.213881e + 03	4.114570e + 03	6.405420e + 03
	max.	2.794334e + 03	2.529750e + 03	4.200705e + 03	2.709230e + 03	8.671254e + 06	1.055171e + 05	8.221760e + 03	7.678344e + 03	1.058651e + 04	2.804116e + 06	4.500526e + 03	5.571773e + 03	8.299194e + 03
	std.	1.416105e + 02	1.701537e + 02	1.709079e + 02	2.352856e + 02	8.892716e + 05	1.438960e + 04	8.059654e + 02	5.047324e + 02	7.217262e + 02	1.834327e + 05	3.006742e + 02	3.994675e + 02	4.375044e + 02
10D	med.	3.004176e + 03	2.214982e + 03	4.096950e + 03	2.585131e + 03	1.416796e + 07	1.310403e + 05	8.865364e + 03	9.635656e + 03	1.252078e + 04	4.944454e + 06	4.815857e + 03	5.476672e + 03	1.108051e + 04
	avg.	2.973208e + 03	2.204789e + 03	4.033173e + 03	2.556056e + 03	1.385822e + 07	1.351637e + 05	9.249340e + 03	9.669426e + 03	1.215139e + 04	4.893383e + 06	4.956730e + 03	5.931475e + 03	1.089621e + 04
	min.	2.466975e + 03	1.885005e + 03	2.783800e + 03	1.991280e + 03	1.040234e + 07	8.646778e + 04	7.545313e + 03	8.069157e + 03	8.758823e + 03	4.659417e + 06	4.121541e + 03	5.408001e + 03	9.552024e + 03
	max.	3.487000e + 03	2.558413e + 03	4.366949e + 03	3.022088e + 03	1.694847e + 07	1.966351e + 05	1.199533e + 04	1.108623e + 04	1.427062e + 04	4.997669e + 06	6.124023e + 03	7.610472e + 03	1.242470e + 04
	std.	2.006998e + 02	1.551638e + 02	2.469317e + 02	2.471004e + 02	1.850395e + 06	2.403720e + 04	1.192640e + 03	6.170717e + 02	1.564926e + 03	8.820352e + 04	6.832559e + 02	6.616012e + 02	6.945841e + 02

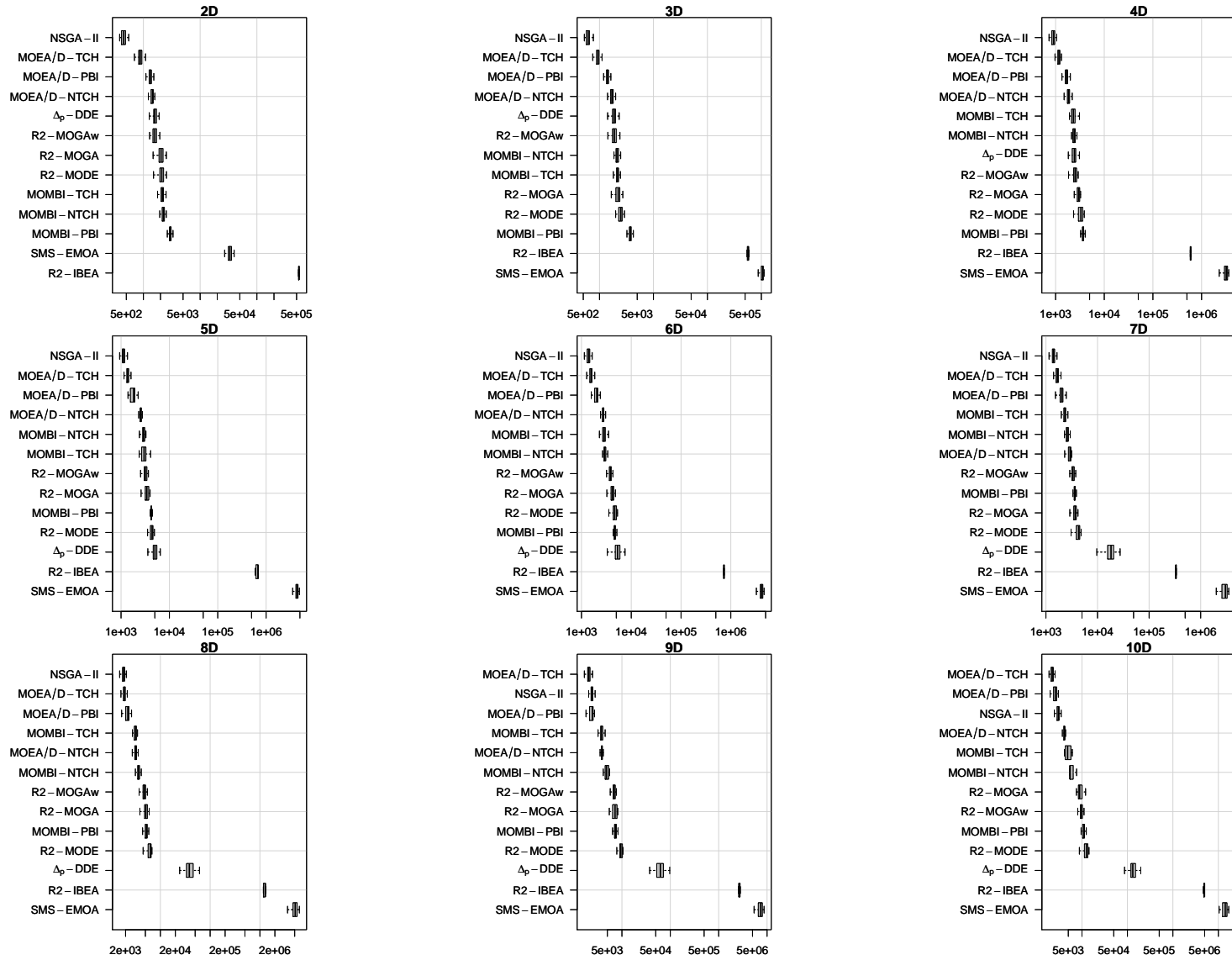


Figure A.163: Box-plot of runtime (in logarithmic scale) for different optimizers on the WFG4 test problem.

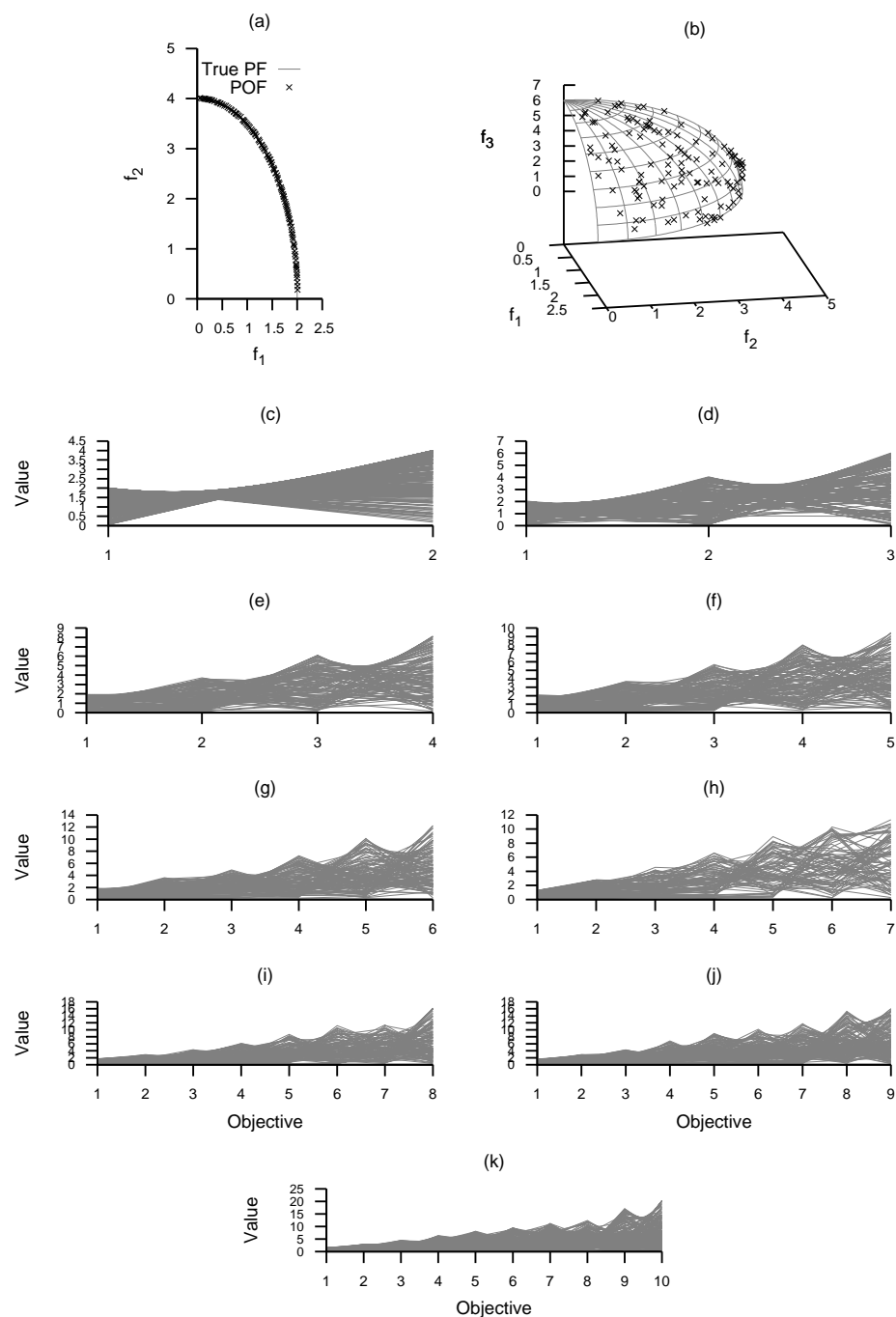


Figure A.164: Plots of the approximations obtained by NSGA-II from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

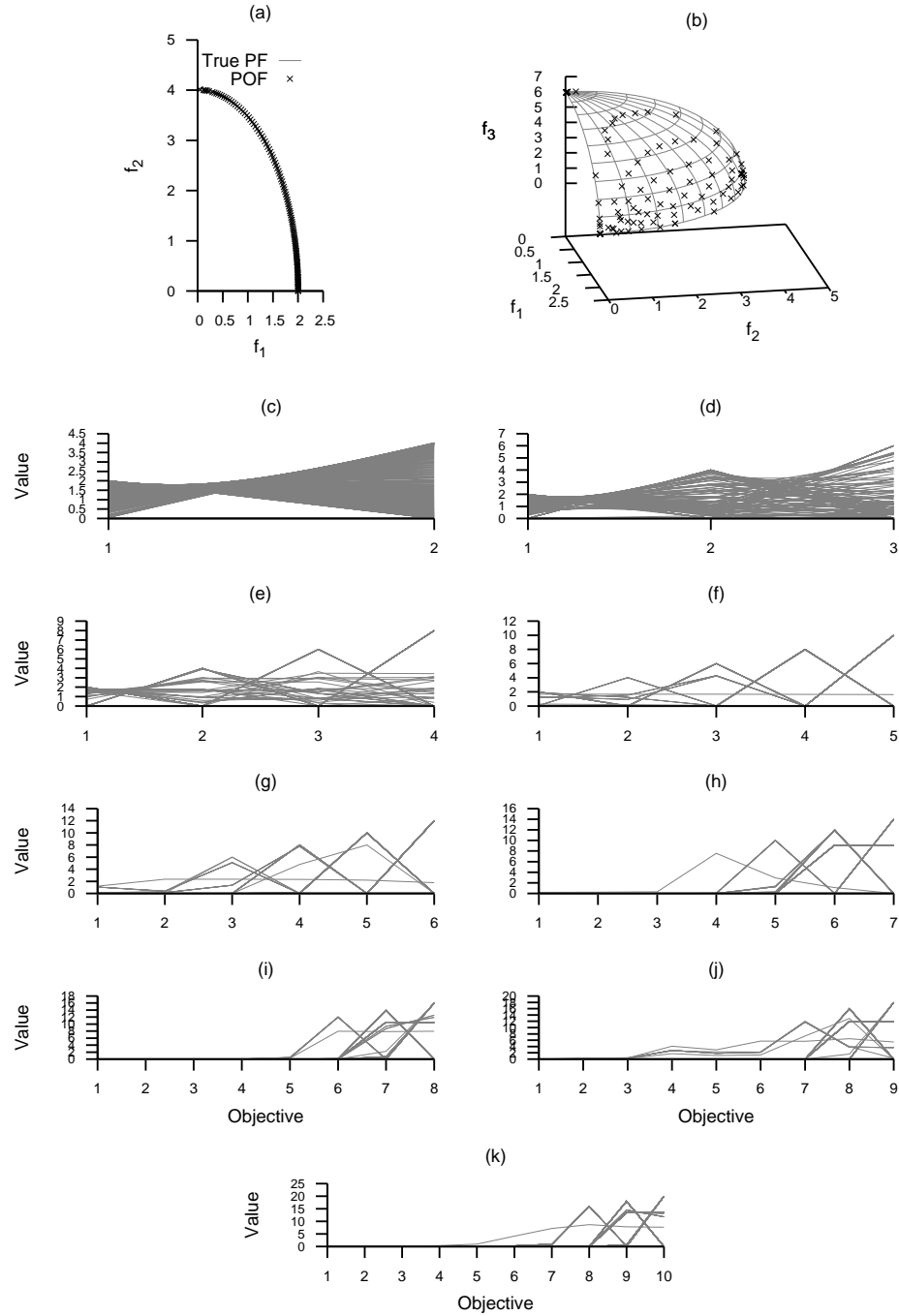


Figure A.165: Plots of the approximations obtained by MOEA/D-TCH from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

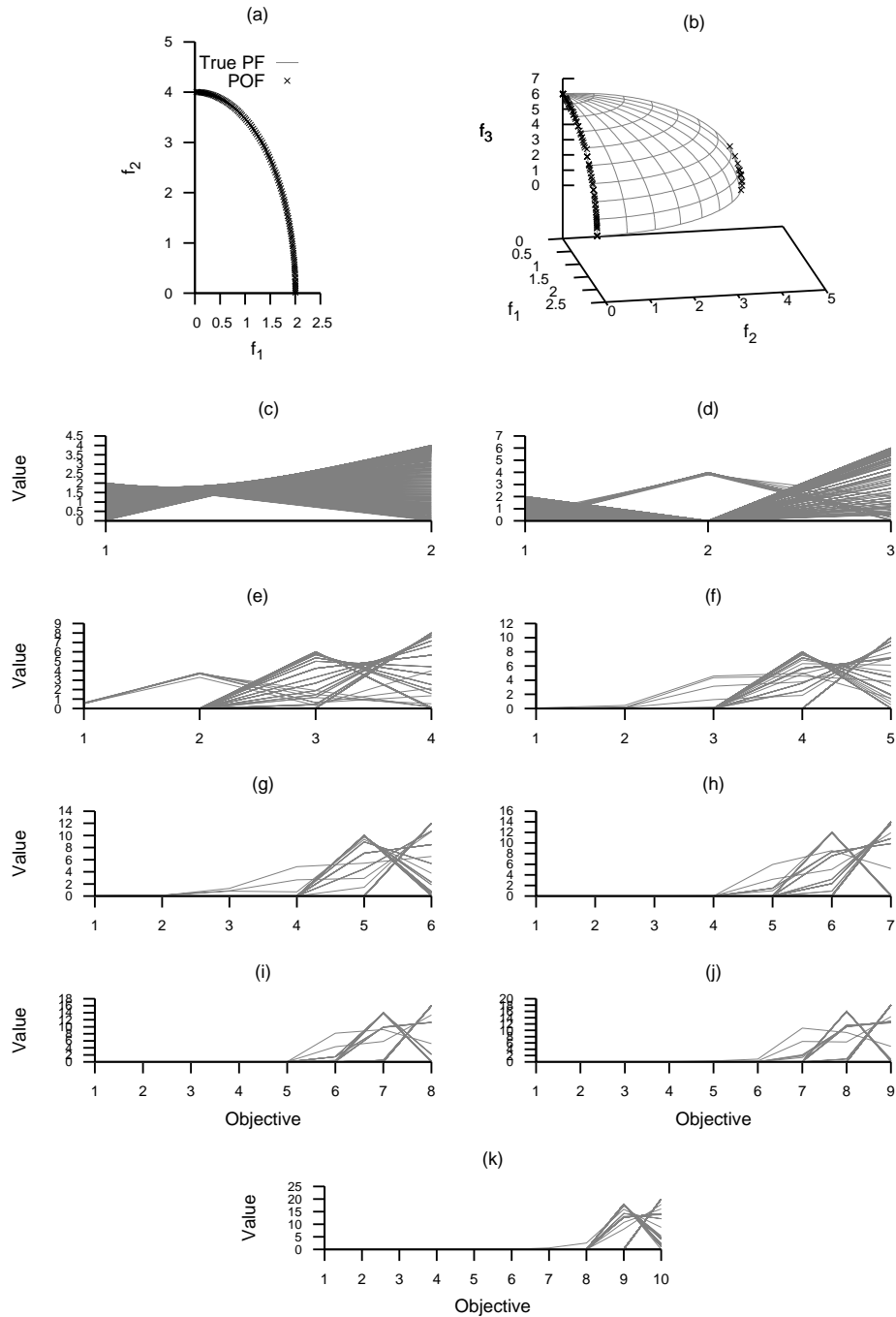


Figure A.166: Plots of the approximations obtained by MOEA/D-NTCH from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



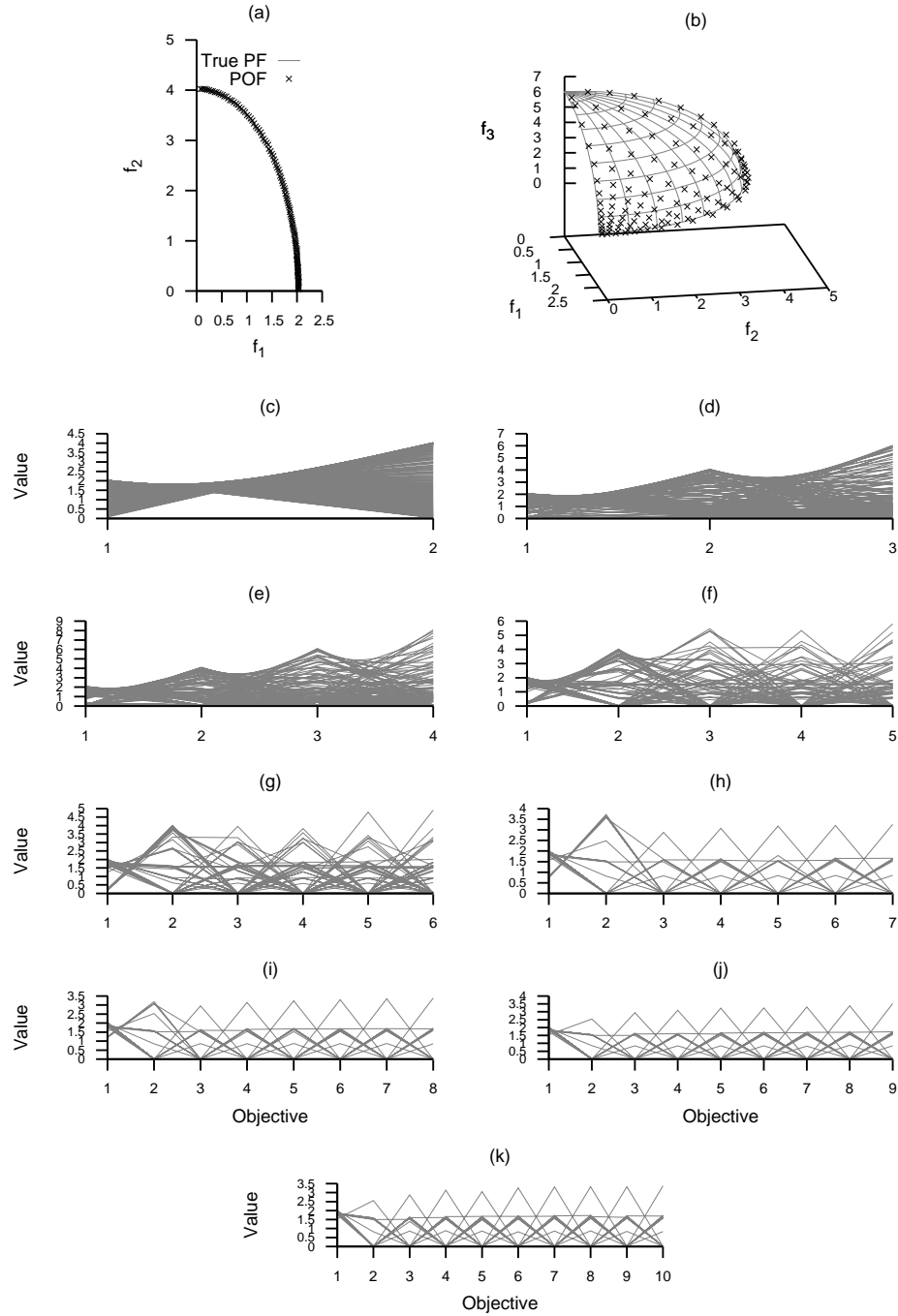


Figure A.167: Plots of the approximations obtained by MOEA/D-PBI from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

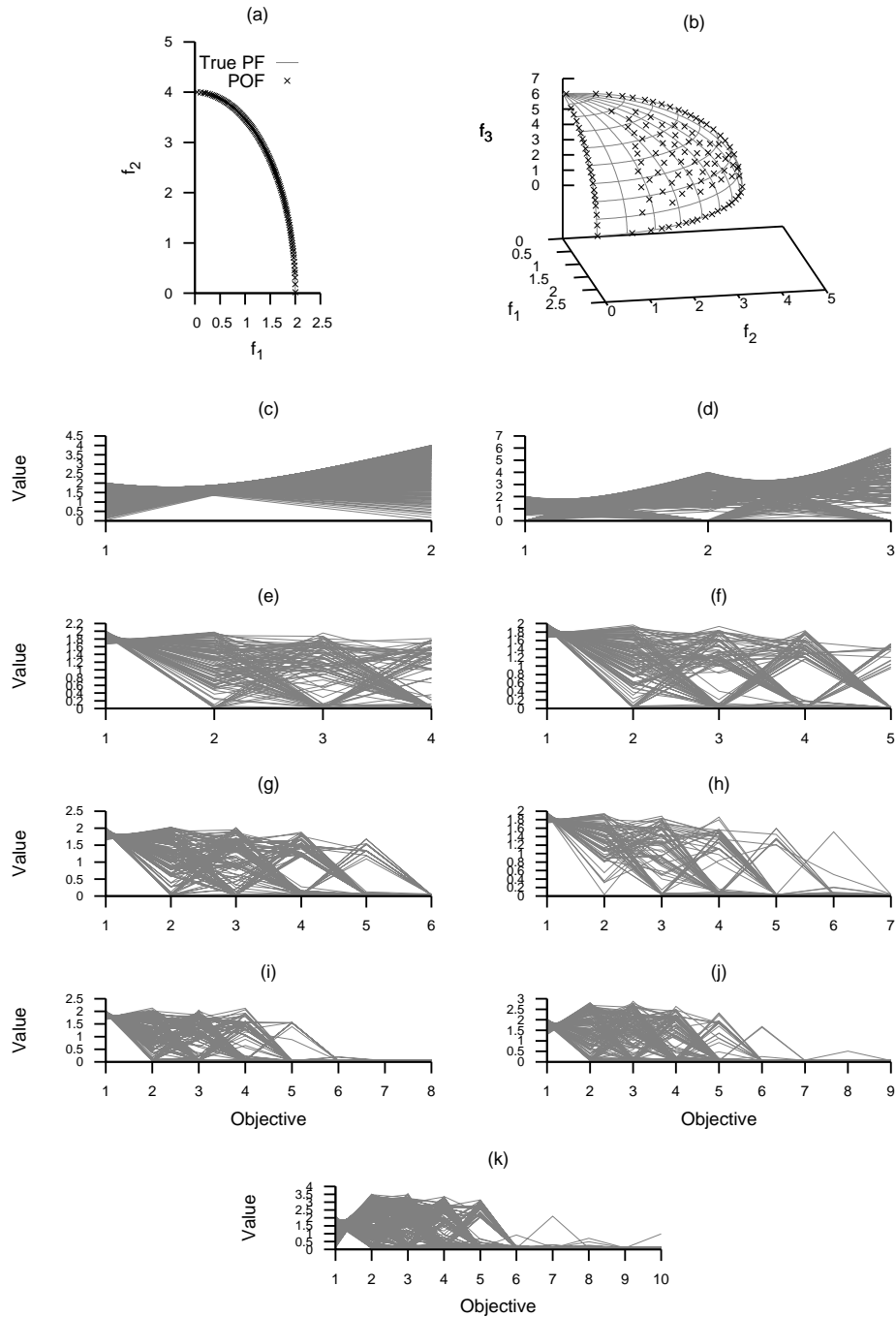


Figure A.168: Plots of the approximations obtained by SMS-EMOA from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

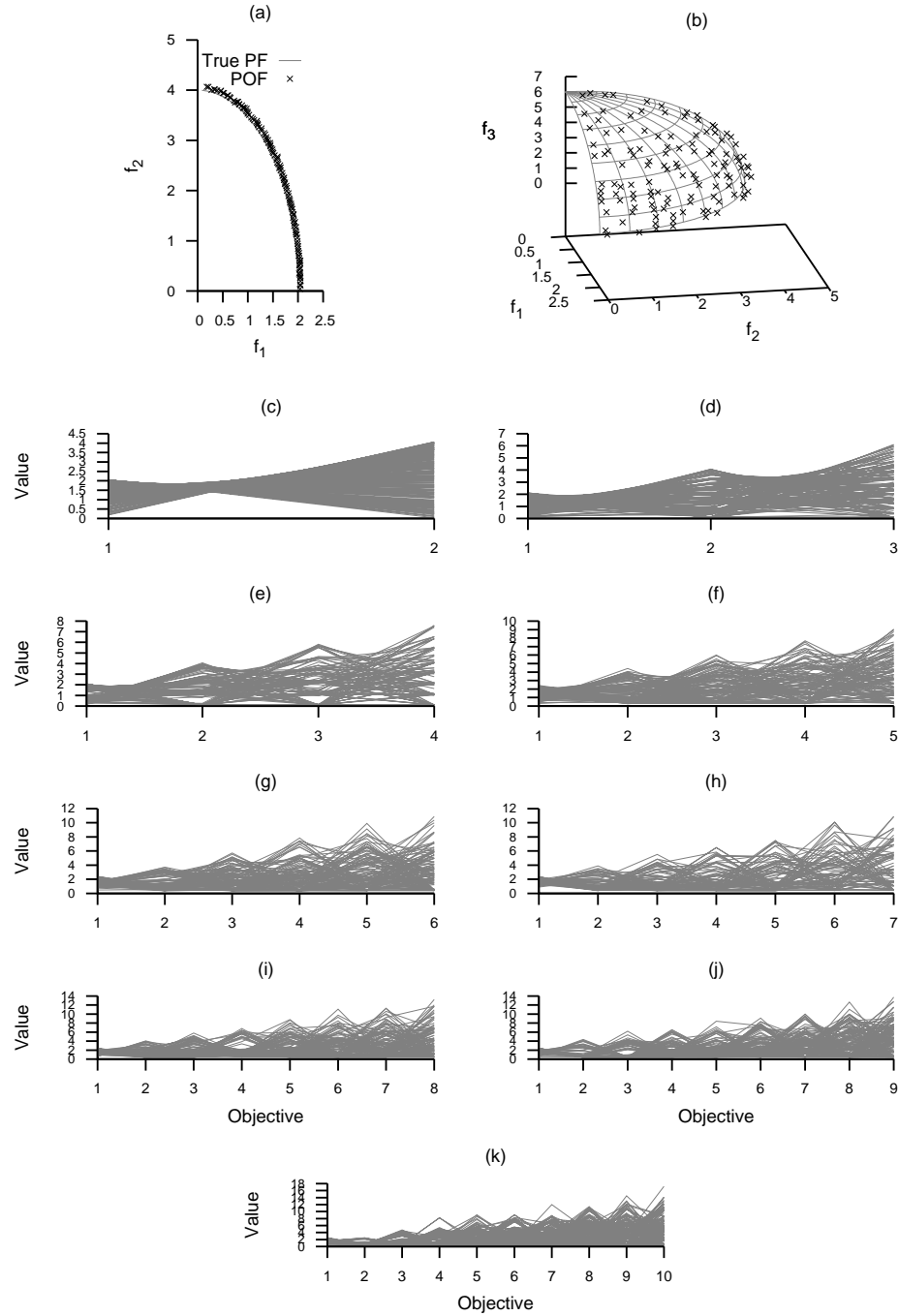


Figure A.169: Plots of the approximations obtained by  $\Delta_p$ -DDE from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

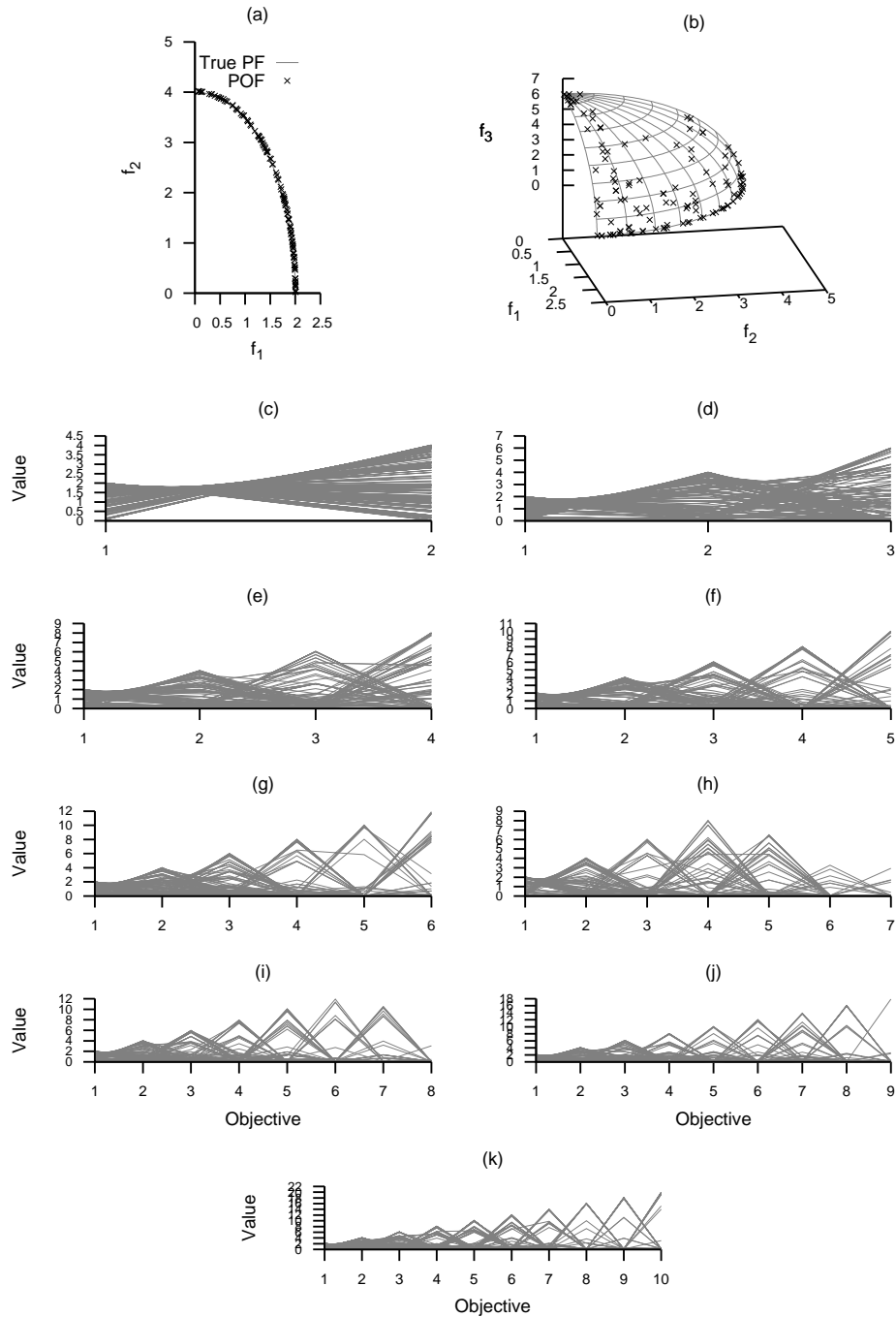


Figure A.170: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

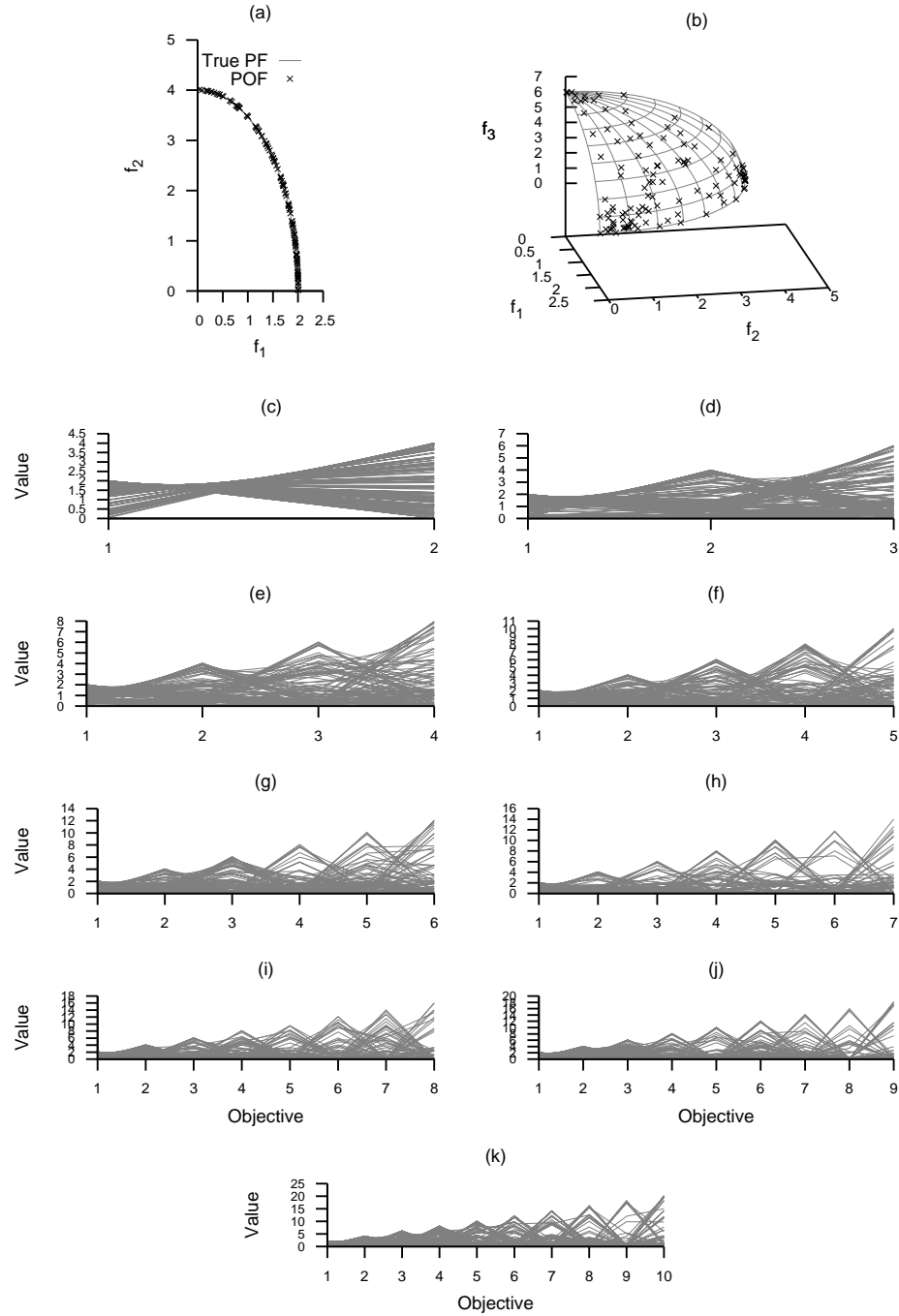


Figure A.171: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

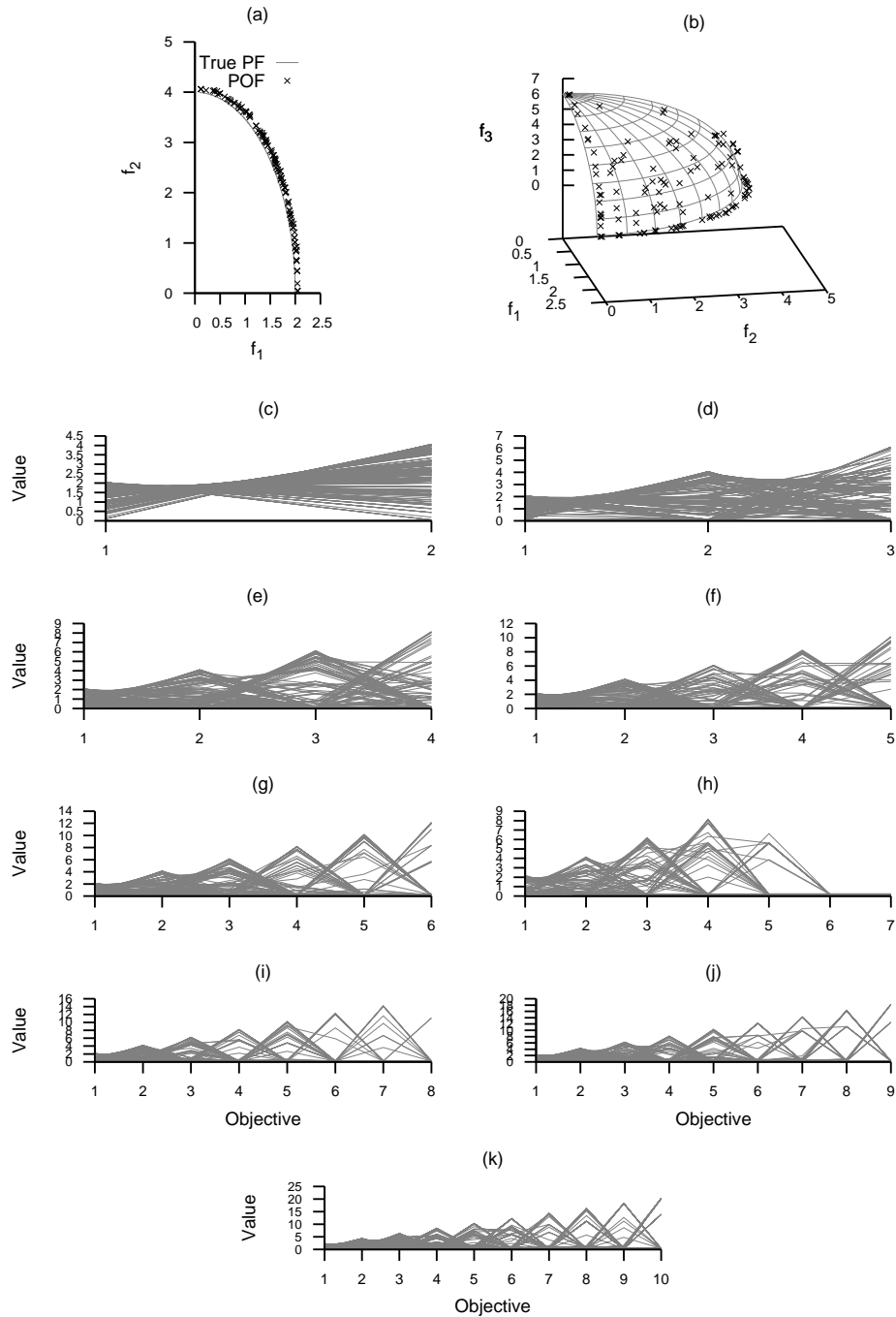


Figure A.172: Plots of the approximations obtained by *R2-MODE* from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

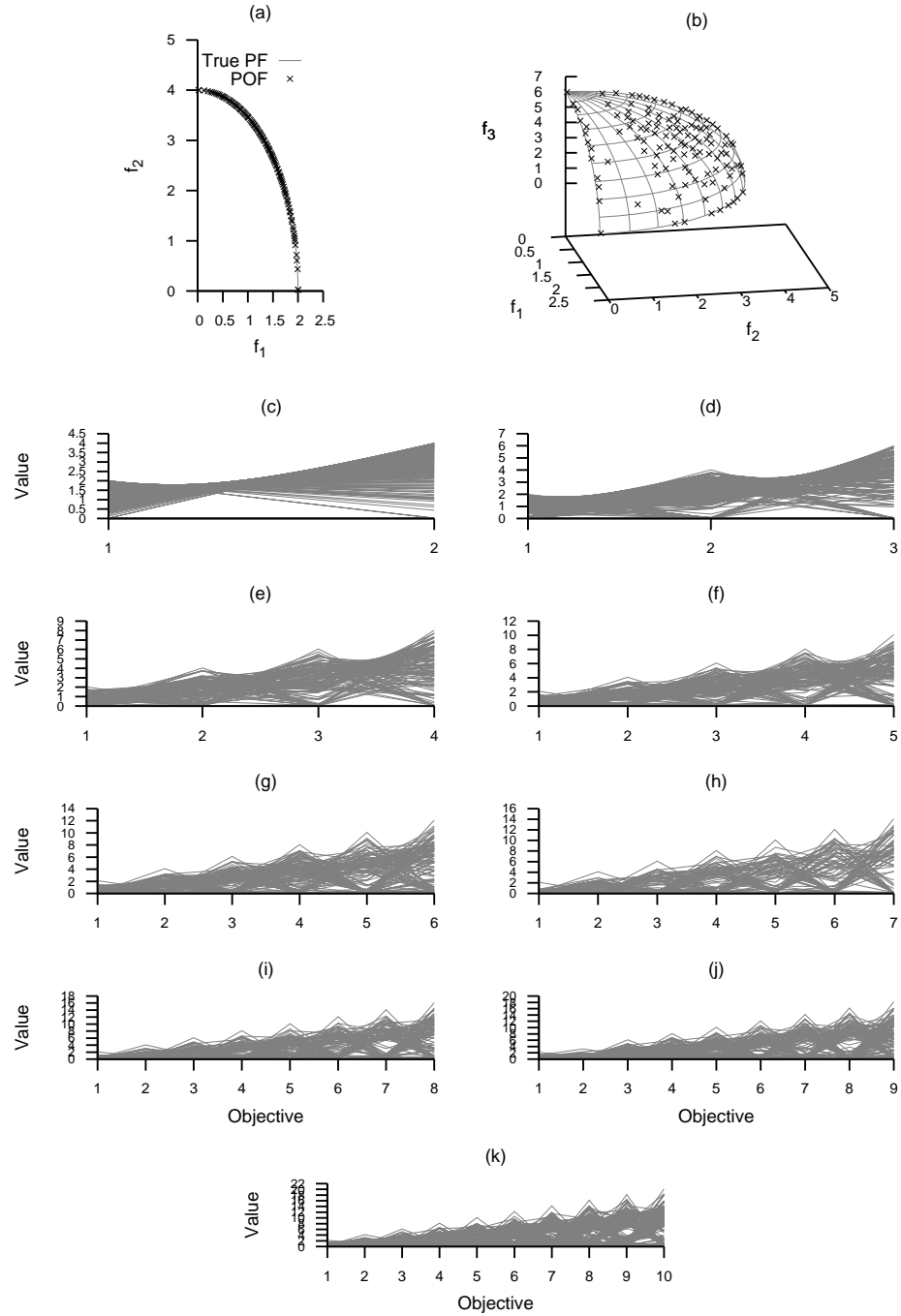


Figure A.173: Plots of the approximations obtained by *R2-IBEA* from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

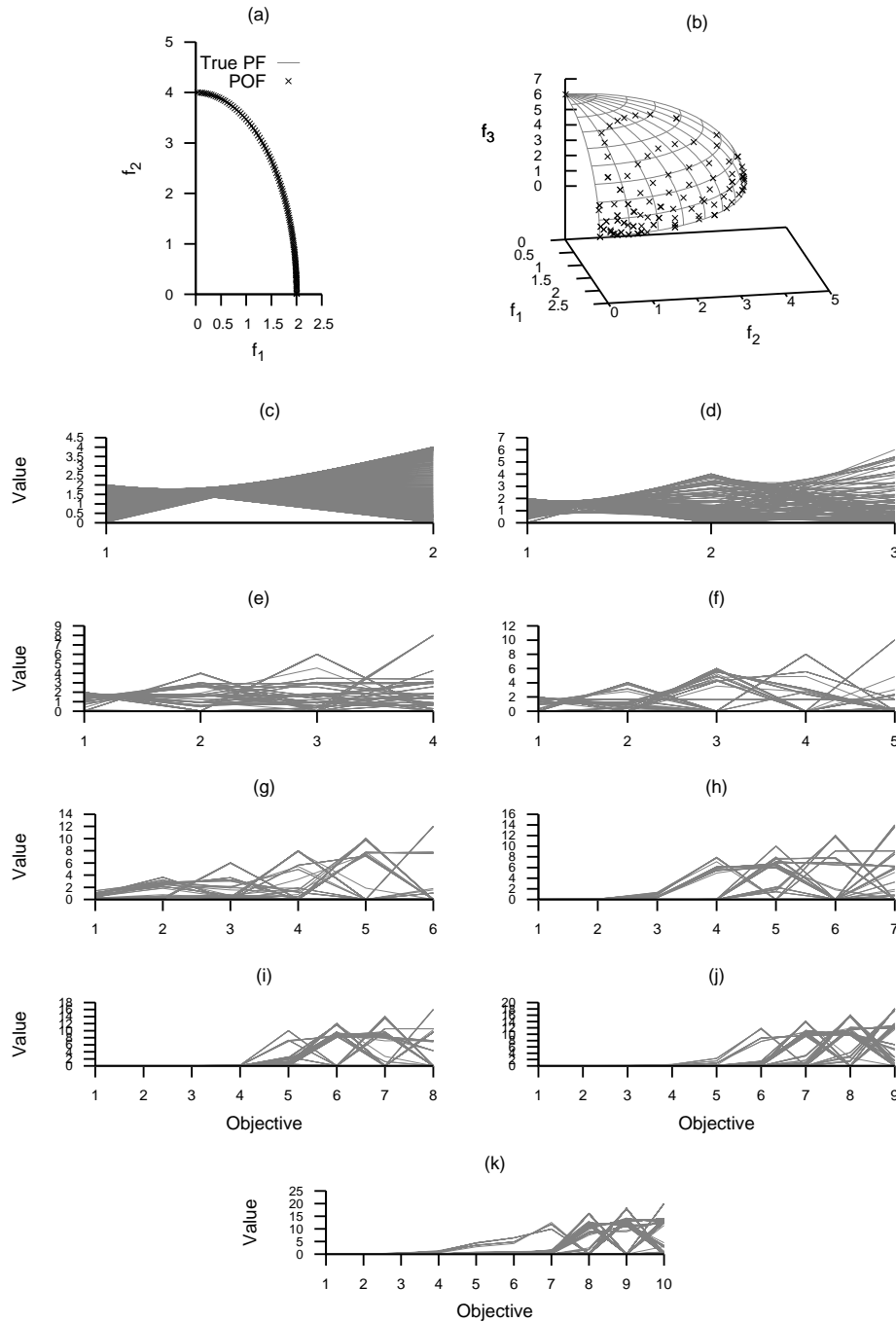


Figure A.174: Plots of the approximations obtained by MOMBI-TCH from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



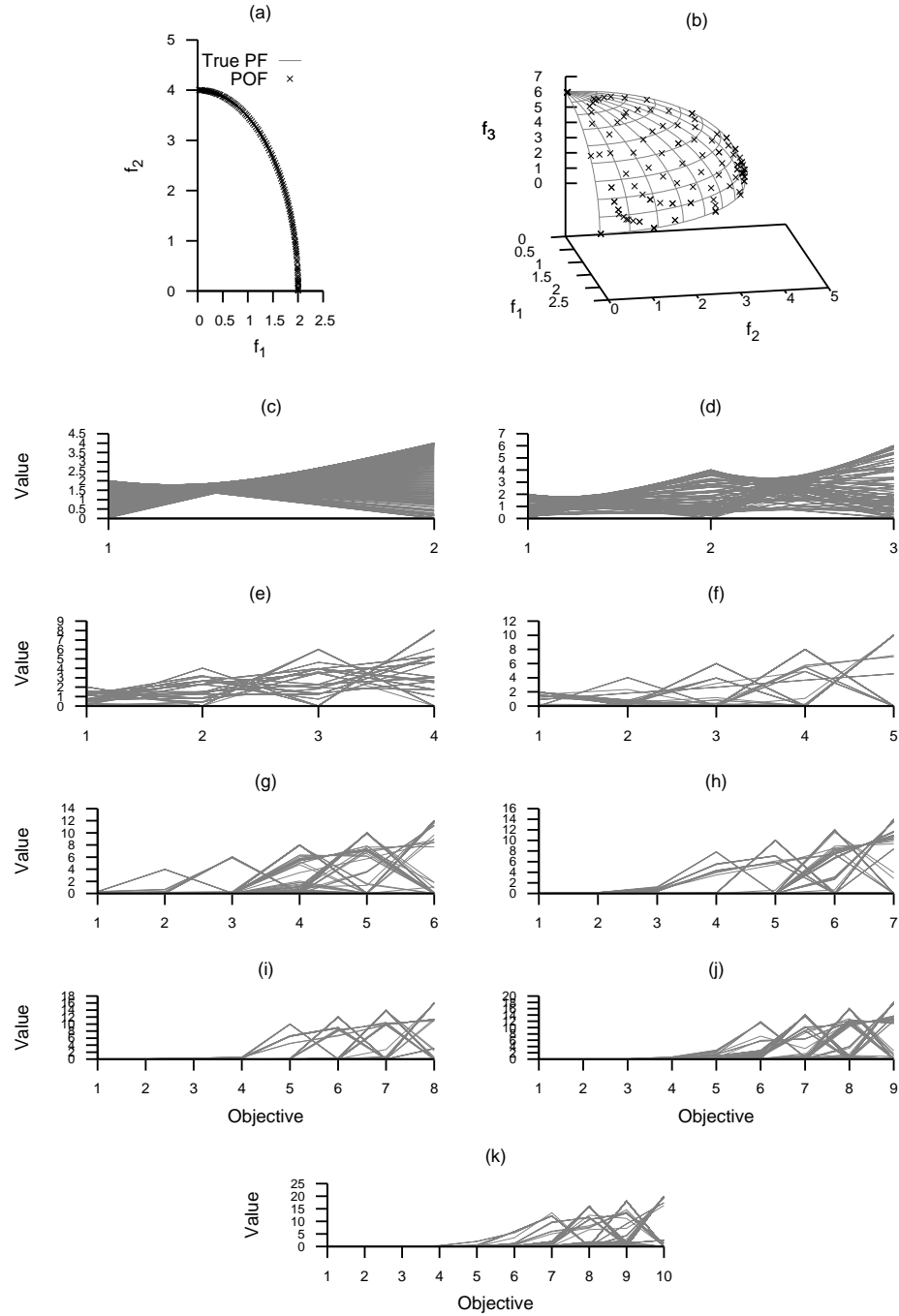


Figure A.175: Plots of the approximations obtained by MOMBI-NTCH from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

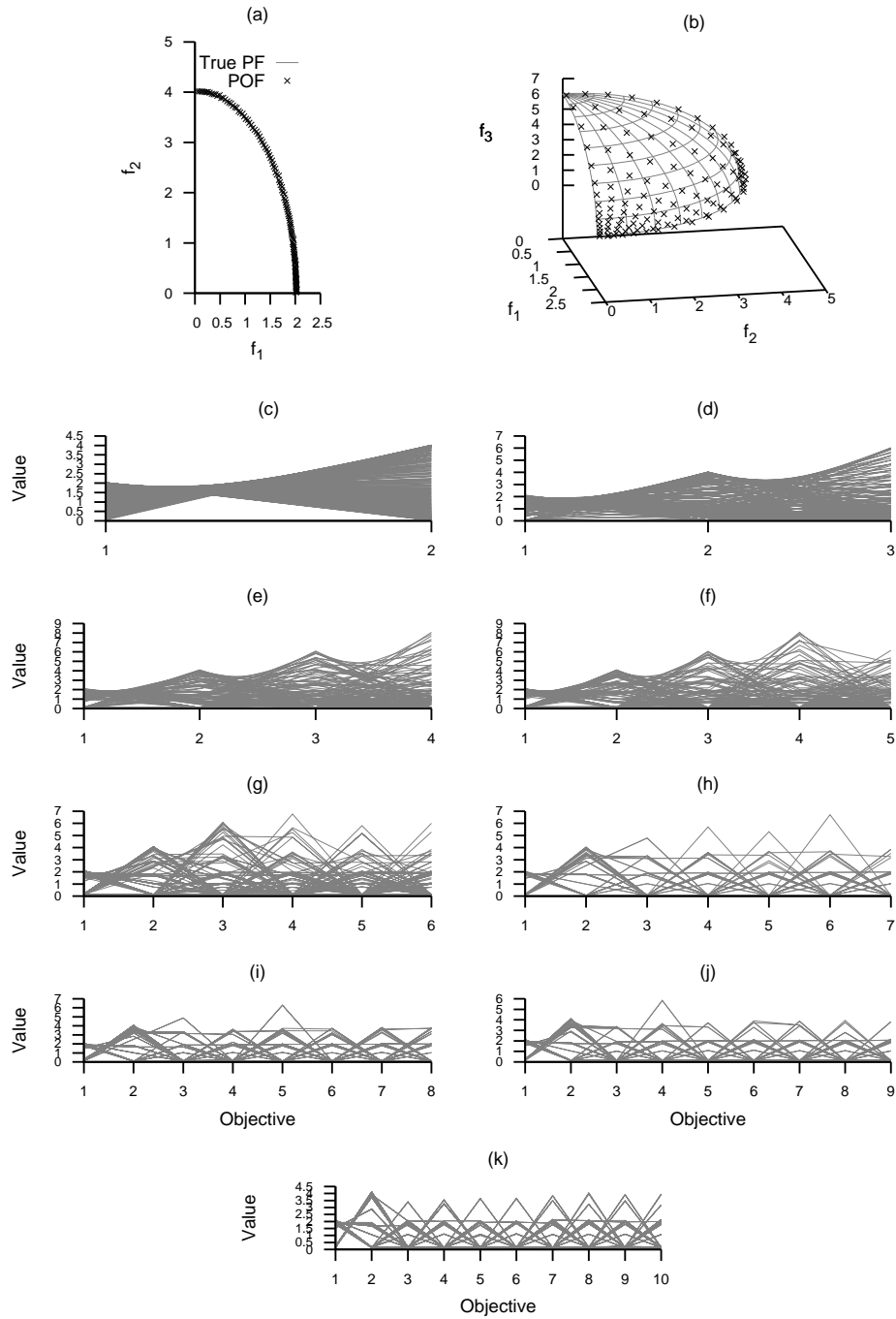


Figure A.176: Plots of the approximations obtained by MOMBI-PBI from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.68: Comparison of hypervolume indicator values for different optimizers on the WFG5 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	8.043419e+00	8.116876e+00	8.122947e+00	8.074498e+00	8.129609e+00	8.017967e+00	8.090092e+00	8.095556e+00	8.041423e+00	8.134081e+00	8.126378e+00	8.128996e+00	8.095134e+00
	avg.	8.039761e+00	8.116617e+00	8.121583e+00	8.070394e+00	8.129865e+00	8.009467e+00	8.090813e+00	8.097865e+00	8.040608e+00	8.136253e+00	8.131573e+00	8.132780e+00	8.099139e+00
	min.	7.907299e+00	8.081123e+00	8.088496e+00	7.999538e+00	8.070608e+00	7.913530e+00	8.045824e+00	8.053872e+00	7.950874e+00	8.122114e+00	8.110351e+00	8.106734e+00	8.074029e+00
	max.	8.091154e+00	8.159794e+00	8.146367e+00	8.104901e+00	8.166340e+00	8.094995e+00	8.143790e+00	8.134816e+00	8.096239e+00	8.171018e+00	8.195226e+00	8.181808e+00	8.141177e+00
	std.	2.968168e-02	1.027119e-02	9.931583e-03	1.957614e-02	1.262214e-02	3.895232e-02	1.884348e-02	1.615282e-02	2.955215e-02	1.035994e-02	1.657966e-02	1.372834e-02	1.664093e-02
3D	med.	6.598639e+01	6.973968e+01	6.495133e+01	7.075213e+01	7.399014e+01	6.921137e+01	7.023336e+01	7.001682e+01	6.923947e+01	7.263338e+01	6.972734e+01	7.099176e+01	7.082050e+01
	avg.	6.605882e+01	6.976597e+01	6.506149e+01	7.074854e+01	7.380784e+01	6.918929e+01	7.019862e+01	6.999604e+01	6.923435e+01	7.259704e+01	6.973229e+01	7.098923e+01	7.081965e+01
	min.	6.453793e+01	6.936479e+01	5.311561e+01	7.008848e+01	7.340703e+01	6.732695e+01	6.839176e+01	6.776808e+01	6.729063e+01	7.191027e+01	6.929005e+01	7.028204e+01	6.993921e+01
	max.	6.761195e+01	7.105328e+01	7.206676e+01	7.136500e+01	7.415203e+01	7.044328e+01	7.141937e+01	7.098520e+01	7.042415e+01	7.336819e+01	7.031182e+01	7.172502e+01	7.179174e+01
	std.	7.305982e-01	3.002604e-01	3.938233e+00	2.606644e-01	2.826087e-01	5.348797e-01	6.326657e-01	5.296783e-01	6.113510e-01	3.060789e-01	2.112301e-01	2.868983e-01	3.503949e-01
4D	med.	5.514359e+02	6.446846e+02	5.964827e+02	6.948729e+02	3.752183e+02	6.284322e+02	6.852148e+02	6.856321e+02	6.499659e+02	7.104471e+02	6.324102e+02	6.570696e+02	6.896220e+02
	avg.	5.510702e+02	6.413575e+02	5.914301e+02	6.942713e+02	3.810921e+02	6.274234e+02	6.841278e+02	6.847188e+02	6.509815e+02	7.108075e+02	6.331595e+02	6.573064e+02	6.896403e+02
	min.	5.230625e+02	6.088849e+02	3.563687e+02	6.647383e+02	3.674253e+02	5.949814e+02	6.546677e+02	6.723217e+02	6.309218e+02	7.051284e+02	6.055725e+02	6.371450e+02	6.789653e+02
	max.	5.761791e+02	6.794789e+02	6.775783e+02	7.158465e+02	4.400446e+02	6.487402e+02	7.013197e+02	7.016604e+02	6.697088e+02	7.208938e+02	6.631522e+02	6.815279e+02	6.973531e+02
	std.	1.122574e+01	1.641960e+01	5.692961e+01	9.502580e+00	1.355341e+01	1.130325e+01	7.893271e+00	5.398831e+00	8.643595e+00	3.044489e+00	1.451067e+01	9.302632e+00	3.581828e+00
5D	med.	5.000345e+03	7.443855e+03	6.332454e+03	7.487174e+03	4.322375e+03	5.792766e+03	7.841457e+03	7.862925e+03	7.058632e+03	8.046322e+03	7.294655e+03	7.710059e+03	7.515820e+03
	avg.	5.017541e+03	7.410630e+03	6.025836e+03	7.483201e+03	5.914798e+03	5.796672e+03	7.844224e+03	7.858257e+03	7.056976e+03	8.040320e+03	7.263320e+03	7.669490e+03	7.522794e+03
	min.	4.505780e+03	6.686180e+03	3.273020e+03	7.064514e+03	5.101339e+03	5.418729e+03	7.626487e+03	7.707365e+03	6.802798e+03	7.896554e+03	6.704001e+03	6.867470e+03	7.208881e+03
	max.	5.381611e+03	7.940270e+03	7.436792e+03	7.901973e+03	8.430234e+03	6.141593e+03	8.055412e+03	8.002696e+03	7.295184e+03	8.163834e+03	7.836754e+03	8.058533e+03	7.927130e+03
	std.	1.804037e+02	2.797719e+02	1.038416e+03	1.703072e+02	2.075138e+03	1.343242e+02	9.094609e+01	6.357653e+01	1.187889e+02	4.804328e+01	2.954439e+02	1.960012e+02	1.213560e+02
6D	med.	5.486112e+04	8.693052e+04	5.810585e+04	9.160808e+04	5.326937e+04	6.249133e+04	1.037413e+05	1.038268e+05	8.825585e+04	1.033310e+05	9.405506e+04	8.915694e+04	9.342507e+04
	avg.	5.489732e+04	8.607212e+04	6.098373e+04	9.123307e+04	5.384990e+04	6.262522e+04	1.038283e+05	1.038731e+05	8.826025e+04	1.033612e+05	9.420949e+04	8.883596e+04	9.336512e+04
	min.	4.980051e+04	6.873202e+04	4.053197e+04	8.125284e+04	5.160031e+04	5.421739e+04	9.945873e+04	1.012646e+05	8.325084e+04	1.014139e+05	8.055685e+04	7.041762e+04	7.061912e+04
	max.	6.108935e+04	9.715025e+04	8.954424e+04	9.760232e+04	6.949188e+04	6.924790e+04	1.083139e+05	1.063309e+05	9.448103e+04	1.052635e+05	1.040347e+05	1.023627e+05	1.023013e+05
	std.	2.209785e+03	5.772381e+03	1.169935e+04	3.356762e+03	2.228292e+03	3.156587e+03	1.444462e+03	1.018763e+03	1.745391e+03	7.506298e+02	4.728157e+03	5.377407e+03	3.977566e+03
7D	med.	7.185489e+05	9.468577e+05	6.354231e+05	1.195745e+06	7.470079e+05	8.534405e+05	1.488831e+06	1.542226e+06	1.272135e+06	1.509402e+06	1.231151e+06	1.025460e+06	1.335266e+06
	avg.	7.206783e+05	9.541403e+05	6.540940e+05	1.173492e+06	7.496225e+05	8.512788e+05	1.492174e+06	1.538578e+06	1.276933e+06	1.501400e+06	1.244622e+06	1.024896e+06	1.312338e+06
	min.	6.411052e+05	7.076598e+05	5.170595e+05	8.050368e+05	7.015238e+05	7.259144e+05	1.429273e+06	1.474994e+06	1.191836e+06	1.403983e+06	9.854822e+05	8.500892e+05	8.028283e+05
	max.	8.018880e+05	1.308244e+06	8.685789e+05	1.357848e+06	8.522936e+05	9.394218e+05	1.567635e+06	1.579821e+06	1.374304e+06	1.570955e+06	1.502944e+06	1.277734e+06	1.456584e+06
	std.	3.482546e+04	1.195674e+05	8.043351e+04	1.203076e+05	2.029143e+04	3.868575e+04	2.946807e+04	1.974325e+04	3.667736e+04	3.513889e+04	1.237238e+05	7.432176e+04	1.066999e+05
8D	med.	1.159778e+07	1.368180e+07	1.010988e+07	1.872563e+07	1.351215e+07	1.439280e+07	2.645510e+07	2.650334e+07	1.993626e+07	2.606022e+07	1.750639e+07	1.514216e+07	2.050383e+07
	avg.	1.162239e+07	1.381954e+07	1.022745e+07	1.859733e+07	1.406756e+07	1.432979e+07	2.640773e+07	2.650940e+07	1.995146e+07	2.593360e+07	1.775193e+07	1.512703e+07	1.864678e+07
	min.	1.059010e+07	1.056371e+07	8.145420e+06	1.369752e+07	1.289171e+07	1.284872e+07	2.538709e+07	2.583190e+07	1.879140e+07	2.475067e+07	1.452988e+07	1.215464e+07	1.054172e+07
	max.	1.257622e+07	1.862453e+07	1.318537e+07	2.181391e+07	2.833489e+07	1.546386e+07	2.752824e+07	2.714512e+07	2.112294e+07	2.691596e+07	2.285165e+07	1.842073e+07	2.313523e+07
	std.	4.471479e+05	1.463352e+06	9.840168e+05	1.723438e+06	2.543940e+06	6.049557e+05	4.131680e+05	2.944521e+05	4.501463e+05	5.163159e+05	1.626468e+06	1.338612e+06	4.337768e+06
9D	med.	2.174094e+08	2.305922e+08	1.802393e+08	3.324307e+08	3.068460e+08	2.676542e+08	5.103371e+08	5.025082e+08	3.549482e+08	4.798160e+08	2.913755e+08	2.656694e+08	2.055817e+08
	avg.	2.177236e+08	2.333834e+08	1.847879e+08	3.279235e+08	3.220910e+08	2.659915e+08	5.106258e+08	5.025733e+08	3.555012e+08	4.794096e+08	2.939024e+08	2.638298e+08	2.745134e+08
	min.	1.997491e+08	1.854080e+08	1.472944e+08	2.592417e+08	2.917401e+08	2.349126e+08	4.974271e+08	4.868510e+08	3.397012e+08	4.529265e+08	2.504742e+08	1.916171e+08	1.928997e+08
	max.	2.404990e+08	3.060646e+08	2.328872e+08	3.917096e+08	5.261185e+08	2.886417e+08	5.225329e+08	5.153205e+08	3.740309e+08	4.930127e+08	3.630712e+08	3.203371e+08	4.420259e+08
	std.	7.456366e+06	2.781763e+07	1.861265e+07	3.137790e+07	5.360345e+07	1.243438e+07	5.854970e+06	6.799860e+06	7.344428e+06	2.286232e+07	2.605688e+07	8.264773e+06	8.242773e+06
10D	med.	4.584477e+09	4.574586e+09	3.596432e+09	6.534129e+09	9.654504e+09	5.448771e+09	1.068818e+10	1.035402e+10	7.137522e+09	9.710447e+09	5.747460e+09	5.303109e+09	4.070623e+09
	avg.	4.572699e+09	4.537539e+09	3.616649e+09	6.445685e+09	9.194611e+09	5.455152e+09	1.066867e+10	1.034157e+10	7.137858e+09	9.698056e+09	5.663694e+09	5.303984e+09	4.923299e+09
	min.	4.056620e+09	3.546419e+09	2.737534e+09	5.396896e+09	7.589661e+09	4.650902e+09	1.024830e+10	1.007474e+10	6.814237e+09	9.366984e+09	3.893538e+09	3.471840e+09	3.773702e+09
	max.	4.848850e+09	5.691787e+09	4.509543e+09	7.489958e+09	1.028863e+10	5.929412e+09	1.098133e+10	1.057486e+10	7.448482e+09	9.984666e+09	6.824897e+09	6.426266e+09	7.849462e+09
	std.	1.264785e+08	5.034787e+08	3.894553e+08	6.089859e+08	8.606275e+08	2.369476e+08	1.06703e+08	1.205180e+08	1.20378e+08	1.288354e+08	5.336725e+08	5.571961e+08	1.344173e+09

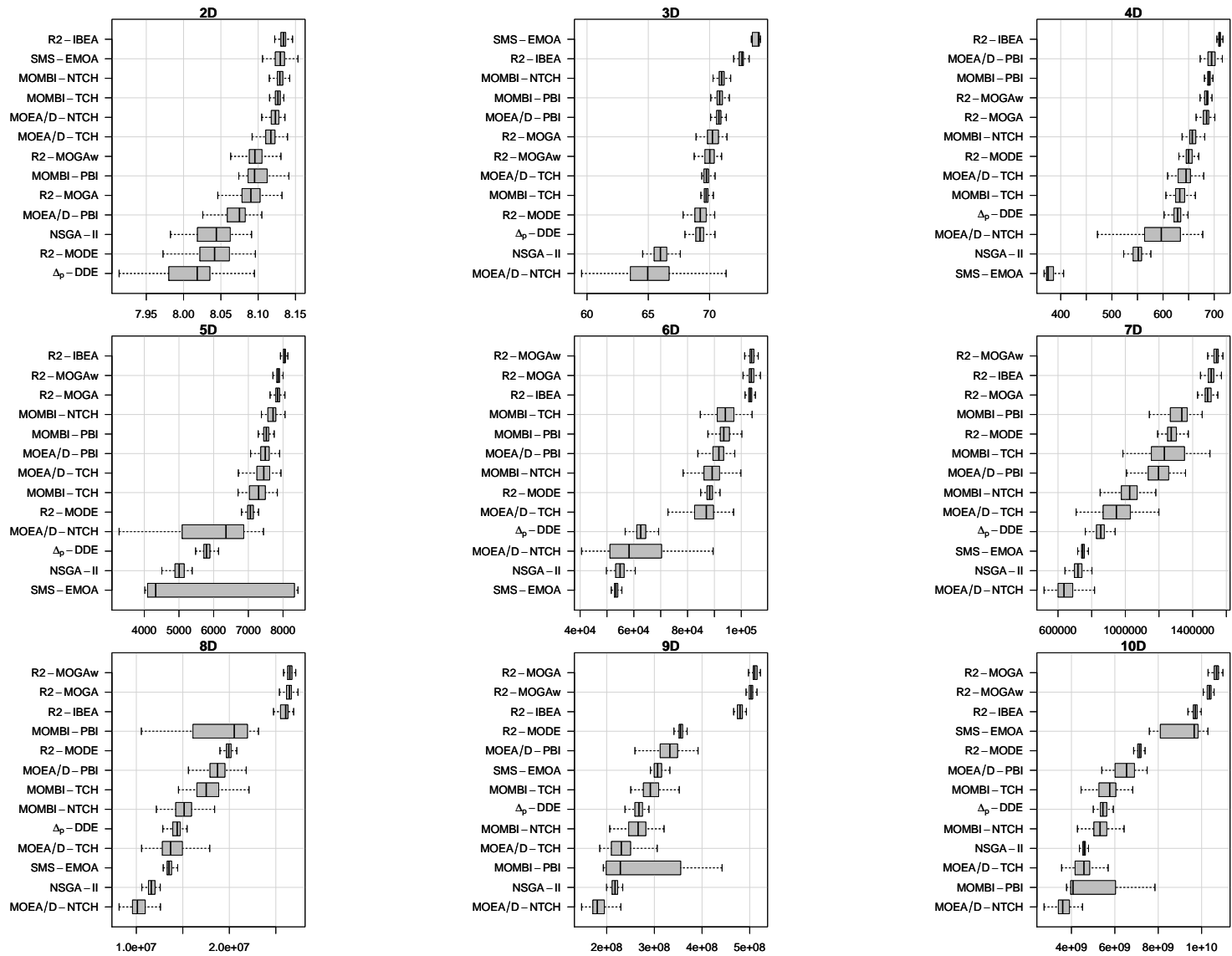


Figure A.177: Box-plot of hypervolume indicator values for different optimizers on the WFG5 test problem.

Table A.69: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG5 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	$1.76e-08$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$1.58e-34$	—	> 0.05	$8.66e-34$	> 0.05	$1.40e-34$	$7.82e-21$	$2.64e-16$	$1.73e-34$	> 0.05	> 0.05	> 0.05	$2.19e-13$
MOEA/D-NTCH	$1.53e-34$	$9.16e-06$	—	$5.71e-34$	> 0.05	$1.40e-34$	$1.86e-24$	$1.47e-19$	$1.49e-34$	> 0.05	> 0.05	> 0.05	$2.06e-18$
MOEA/D-PBI	$1.48e-14$	> 0.05	> 0.05	—	> 0.05	$8.41e-26$	> 0.05	> 0.05	$9.84e-14$	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	$1.89e-34$	$5.25e-17$	$3.87e-08$	$6.63e-34$	—	$1.49e-34$	$1.01e-28$	$9.32e-26$	$2.07e-34$	> 0.05	> 0.05	> 0.05	$4.42e-25$
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$1.45e-27$	> 0.05	> 0.05	$1.70e-11$	> 0.05	$3.37e-32$	—	> 0.05	$1.40e-26$	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	$8.01e-32$	> 0.05	> 0.05	$3.59e-20$	> 0.05	$1.66e-33$	$2.68e-03$	—	$7.20e-31$	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$7.40e-09$	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	$1.28e-34$	$1.50e-29$	$1.17e-22$	$1.28e-34$	$9.78e-06$	$1.28e-34$	$1.73e-32$	$1.59e-31$	$1.28e-34$	—	$1.29e-10$	$1.25e-06$	$2.66e-31$
MOMBI-TCH	$1.28e-34$	$2.54e-15$	$2.90e-05$	$1.28e-34$	> 0.05	$1.28e-34$	$3.28e-29$	$1.48e-24$	$1.28e-34$	> 0.05	—	> 0.05	$1.38e-24$
MOMBI-NTCH	$1.28e-34$	$7.13e-20$	$2.85e-09$	$1.28e-34$	> 0.05	$1.28e-34$	$4.00e-30$	$1.14e-26$	$1.28e-34$	> 0.05	$7.78e-03$	—	$1.03e-26$
MOMBI-PBI	$5.20e-32$	> 0.05	> 0.05	$1.39e-20$	> 0.05	$1.31e-33$	$1.35e-03$	> 0.05	$3.54e-31$	> 0.05	> 0.05	> 0.05	—
3D													
NSGA-II	—	> 0.05	$9.26e-05$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$1.28e-34$	—	$2.02e-18$	> 0.05	> 0.05	$4.94e-17$	> 0.05	> 0.05	$1.14e-10$	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$1.28e-34$	$2.12e-32$	$7.99e-26$	—	> 0.05	$2.07e-34$	$7.34e-12$	$2.55e-23$	$3.34e-34$	> 0.05	$2.20e-34$	> 0.05	> 0.05
SMS-EMOA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$
$\Delta_p$ -DDE	$1.49e-34$	> 0.05	$1.90e-17$	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$1.28e-34$	$6.47e-10$	$1.58e-21$	> 0.05	> 0.05	$1.17e-21$	—	$4.23e-03$	$3.26e-19$	> 0.05	$1.73e-11$	> 0.05	> 0.05
R2-MOGAw	$1.28e-34$	$3.39e-05$	$6.89e-20$	> 0.05	> 0.05	$5.27e-20$	> 0.05	—	$7.53e-16$	> 0.05	$2.73e-06$	> 0.05	> 0.05
R2-MODE	$1.49e-34$	> 0.05	$2.11e-17$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.68e-34$	$1.28e-34$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	> 0.05	$4.45e-18$	> 0.05	> 0.05	$4.74e-17$	> 0.05	> 0.05	$2.49e-10$	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.16e-33$	$8.56e-29$	$6.75e-09$	> 0.05	$1.40e-34$	$6.97e-21$	$1.65e-29$	$1.40e-34$	> 0.05	$1.36e-34$	—	$1.23e-04$
MOMBI-PBI	$1.28e-34$	$3.68e-32$	$1.44e-26$	> 0.05	> 0.05	$3.24e-34$	$7.48e-14$	$1.11e-23$	$5.88e-34$	> 0.05	$3.44e-34$	> 0.05	—
4D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$1.28e-34$	—	$3.81e-15$	> 0.05	$1.28e-34$	$5.07e-10$	> 0.05	> 0.05	> 0.05	> 0.05	$1.10e-04$	> 0.05	> 0.05
MOEA/D-NTCH	$1.39e-13$	> 0.05	—	> 0.05	$6.61e-33$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$1.28e-34$	$2.01e-34$	$1.68e-34$	—	$1.28e-34$	$1.28e-34$	$1.22e-13$	$6.41e-15$	$1.68e-34$	> 0.05	$1.28e-34$	$4.92e-34$	$2.32e-06$
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	$1.28e-34$	> 0.05	$2.52e-07$	> 0.05	$1.28e-34$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$1.28e-34$	$9.19e-34$	$4.00e-34$	> 0.05	$1.28e-34$	$1.28e-34$	—	> 0.05	$6.83e-34$	> 0.05	$1.78e-34$	$5.83e-32$	> 0.05
R2-MOGAw	$1.28e-34$	$2.40e-34$	$1.73e-34$	> 0.05	$1.28e-34$	$1.28e-34$	> 0.05	—	$1.28e-34$	> 0.05	$1.28e-34$	$1.52e-33$	> 0.05
R2-MODE	$1.28e-34$	$1.08e-05$	$1.06e-22$	> 0.05	$1.28e-34$	$6.60e-30$	> 0.05	> 0.05	—	> 0.05	$1.18e-17$	> 0.05	> 0.05
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$4.86e-30$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	> 0.05	$3.67e-10$	> 0.05	$1.28e-34$	$3.33e-03$	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	$1.28e-34$	$2.57e-13$	$2.63e-26$	> 0.05	$1.28e-34$	$6.23e-33$	> 0.05	> 0.05	$2.52e-06$	> 0.05	$4.12e-24$	—	> 0.05
MOMBI-PBI	$1.28e-34$	$1.32e-34$	$1.28e-34$	> 0.05	$1.28e-34$	$1.28e-34$	$1.59e-09$	$4.64e-13$	$1.28e-34$	> 0.05	$1.28e-34$	$1.63e-34$	—

Table A.70: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG5 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
5D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	2.55e-02	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	2.90e-28	> 0.05	2.54e-02	1.28e-34	> 0.05	> 0.05	7.88e-20	> 0.05	2.28e-04	> 0.05	> 0.05
MOEA/D-NTCH	3.69e-11	> 0.05	—	> 0.05	> 0.05	4.97e-04	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	> 0.05	2.67e-32	—	2.54e-02	1.28e-34	> 0.05	> 0.05	1.29e-32	> 0.05	2.81e-08	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e-34	> 0.05	> 0.05	> 0.05	2.54e-02	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.43e-28	1.28e-34	1.13e-30	2.54e-02	1.28e-34	—	> 0.05	1.28e-34	> 0.05	7.88e-33	6.53e-15	1.59e-31
R2-MOGAw	1.28e-34	3.29e-30	1.28e-34	3.00e-32	2.54e-02	1.28e-34	> 0.05	—	1.28e-34	> 0.05	1.20e-33	9.28e-19	1.63e-32
R2-MODE	1.28e-34	> 0.05	3.93e-17	> 0.05	2.54e-02	1.28e-34	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	1.28e-34	1.44e-34	1.28e-34	1.32e-34	2.54e-02	1.28e-34	3.86e-31	2.23e-33	1.28e-34	—	1.28e-34	5.55e-33	1.36e-34
MOMBI-TCH	1.28e-34	> 0.05	1.68e-23	> 0.05	2.54e-02	1.28e-34	> 0.05	> 0.05	3.43e-09	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.28e-34	7.22e-13	1.10e-33	4.24e-13	2.54e-02	1.28e-34	> 0.05	> 0.05	1.10e-31	> 0.05	2.58e-21	—	1.41e-12
MOMBI-PBI	1.28e-34	2.76e-03	6.83e-34	3.65e-02	2.54e-02	1.28e-34	> 0.05	> 0.05	1.89e-34	> 0.05	6.25e-12	> 0.05	—
6D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	2.00e-06	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	3.11e-30	> 0.05	1.32e-34	1.40e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	7.30e-04	> 0.05	—	> 0.05	1.06e-05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	1.06e-12	4.78e-34	—	1.28e-34	1.28e-34	> 0.05	> 0.05	1.35e-14	> 0.05	> 0.05	1.37e-05	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	3.07e-31	> 0.05	3.51e-03	> 0.05	5.05e-32	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	> 0.05	1.28e-34	3.40e-03	1.50e-31	2.20e-34	1.95e-34
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	—	1.28e-34	5.27e-05	3.27e-32	1.63e-34	1.58e-34
R2-MODE	1.28e-34	3.19e-03	2.04e-33	> 0.05	1.28e-34	1.28e-34	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	1.28e-34	—	2.90e-31	1.63e-34	1.63e-34
MOMBI-TCH	1.28e-34	8.62e-20	2.55e-34	3.33e-06	1.28e-34	1.28e-34	> 0.05	> 0.05	2.05e-21	> 0.05	—	3.39e-12	> 0.05
MOMBI-NTCH	1.28e-34	6.54e-04	1.83e-32	> 0.05	1.28e-34	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	1.28e-34	1.11e-21	5.07e-34	2.97e-07	1.28e-34	1.28e-34	> 0.05	> 0.05	1.87e-26	> 0.05	> 0.05	4.89e-13	—
7D													
NSGA-II	—	> 0.05	1.47e-13	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	4.13e-32	—	2.31e-32	> 0.05	6.42e-30	1.93e-13	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	4.04e-22	3.05e-34	—	1.49e-34	2.42e-30	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	9.01e-20	> 0.05
SMS-EMOA	1.37e-11	> 0.05	5.27e-19	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	2.44e-33	> 0.05	1.04e-30	> 0.05	6.74e-32	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	> 0.05	1.28e-34	> 0.05	4.39e-33	1.28e-34	4.00e-34
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	4.67e-23	—	1.28e-34	9.74e-18	1.53e-34	1.28e-34	1.28e-34
R2-MODE	1.28e-34	1.68e-32	1.28e-34	2.85e-14	1.28e-34	1.28e-34	> 0.05	> 0.05	—	> 0.05	3.45e-03	6.06e-34	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.64e-03	> 0.05	1.28e-34	—	9.66e-33	1.28e-34	1.76e-33
MOMBI-TCH	1.28e-34	8.33e-29	1.28e-34	5.32e-04	1.28e-34	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	—	3.98e-27	> 0.05
MOMBI-NTCH	1.28e-34	1.55e-07	1.40e-34	> 0.05	1.32e-34	1.59e-32	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	1.28e-34	1.50e-30	1.89e-34	1.84e-19	1.44e-34	1.22e-32	> 0.05	> 0.05	2.81e-08	> 0.05	2.46e-06	1.68e-30	—

Table A.71: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG5 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
8D													
NSGA-II	—	> 0.05	$2.70e-22$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$2.22e-27$	—	$1.33e-32$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$1.28e-34$	$2.42e-30$	$1.28e-34$	—	$1.89e-29$	$2.00e-29$	> 0.05	> 0.05	> 0.05	> 0.05	$8.86e-06$	$2.99e-26$	> 0.05
SMS-EMOA	$1.28e-34$	> 0.05	$1.53e-34$	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	$1.28e-34$	$2.37e-04$	$1.49e-34$	> 0.05	$2.92e-13$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$8.06e-31$	$1.28e-34$	—	> 0.05	$1.28e-34$	$2.07e-10$	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MOGAw	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$8.06e-31$	$1.28e-34$	$4.37e-02$	—	$1.28e-34$	$3.83e-16$	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MODE	$1.28e-34$	$1.28e-34$	$1.28e-34$	$4.14e-14$	$8.06e-31$	$1.28e-34$	> 0.05	> 0.05	—	> 0.05	$1.81e-23$	$1.28e-34$	> 0.05
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$8.06e-31$	$1.28e-34$	> 0.05	> 0.05	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$8.53e-31$	$1.28e-34$	> 0.05	$1.23e-30$	$1.07e-33$	> 0.05	> 0.05	> 0.05	> 0.05	—	$5.54e-23$	> 0.05
MOMBI-NTCH	$2.71e-34$	$3.10e-10$	$1.73e-34$	> 0.05	$4.49e-16$	$7.55e-07$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	$7.38e-16$	$4.56e-13$	$4.12e-28$	$2.20e-04$	$1.37e-11$	$4.45e-12$	> 0.05	> 0.05	$1.29e-02$	> 0.05	$7.68e-06$	$7.15e-11$	—
9D													
NSGA-II	—	> 0.05	$2.80e-25$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$2.33e-05$	—	$1.14e-26$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$1.28e-34$	$3.37e-32$	$1.28e-34$	—	$4.34e-08$	$2.56e-26$	> 0.05	> 0.05	> 0.05	> 0.05	$1.16e-14$	$3.68e-26$	$5.92e-05$
SMS-EMOA	$1.28e-34$	$4.92e-34$	$1.28e-34$	> 0.05	—	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	$1.33e-09$	$7.67e-29$	$5.75e-04$
$\Delta_p$ -DDE	$1.53e-34$	$4.55e-17$	$1.28e-34$	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.66e-31$	$1.28e-34$	—	$4.55e-17$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MOGAw	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$7.46e-29$	$1.28e-34$	> 0.05	—	$1.28e-34$	$2.33e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MODE	$1.28e-34$	$1.28e-34$	$1.28e-34$	$9.90e-17$	$5.28e-25$	$1.28e-34$	> 0.05	> 0.05	—	> 0.05	$7.01e-33$	$1.28e-34$	$2.11e-09$
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$4.08e-26$	$1.28e-34$	> 0.05	> 0.05	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$3.41e-28$	$1.28e-34$	> 0.05	> 0.05	$2.34e-19$	> 0.05	> 0.05	> 0.05	> 0.05	—	$4.06e-14$	$1.66e-02$
MOMBI-NTCH	$9.72e-28$	$1.08e-12$	$3.08e-33$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	> 0.05	> 0.05	$6.10e-23$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—
10D													
NSGA-II	—	> 0.05	$7.24e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$3.72e-04$
MOEA/D-TCH	> 0.05	—	$8.85e-26$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$1.28e-34$	$1.16e-33$	$1.28e-34$	—	> 0.05	$1.14e-22$	> 0.05	> 0.05	> 0.05	> 0.05	$1.59e-15$	$5.69e-24$	$6.61e-13$
SMS-EMOA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	> 0.05	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	$1.28e-34$	$1.78e-34$
$\Delta_p$ -DDE	$2.55e-34$	$8.06e-28$	$1.28e-34$	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.19e-02$	$8.43e-07$
R2-MOGA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.32e-34$	$1.28e-34$	—	$4.20e-29$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MOGAw	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$5.07e-34$	$1.28e-34$	> 0.05	—	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MODE	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.11e-21$	> 0.05	$1.28e-34$	> 0.05	> 0.05	—	> 0.05	$1.32e-34$	$1.28e-34$	$1.06e-21$
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$4.58e-02$	$1.28e-34$	> 0.05	> 0.05	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$7.61e-31$	$1.92e-26$	$2.88e-34$	> 0.05	> 0.05	$4.33e-05$	> 0.05	> 0.05	> 0.05	> 0.05	—	$3.25e-06$	$1.88e-07$
MOMBI-NTCH	$9.79e-24$	$2.30e-18$	$3.47e-32$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	$1.75e-05$
MOMBI-PBI	> 0.05	> 0.05	$9.23e-22$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—

Table A.72: Comparison of R2 indicator values for different optimizers on the WFG5 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	6.129801e-01	6.094383e-01	6.091842e-01	6.114424e-01	6.088057e-01	6.148179e-01	6.106243e-01	6.104990e-01	6.126571e-01	6.088160e-01	6.090566e-01	6.090126e-01	6.105583e-01
	avg.	6.132235e-01	6.094274e-01	6.092743e-01	6.116773e-01	6.088118e-01	6.155523e-01	6.105962e-01	6.103912e-01	6.128751e-01	6.087185e-01	6.087992e-01	6.088295e-01	6.103057e-01
	min.	6.104445e-01	6.075235e-01	6.080526e-01	6.099180e-01	6.071712e-01	6.100771e-01	6.082495e-01	6.086947e-01	6.099675e-01	6.071061e-01	6.061245e-01	6.067529e-01	6.084255e-01
	max.	6.242919e-01	6.112380e-01	6.105729e-01	6.153443e-01	6.122352e-01	6.256286e-01	6.126237e-01	6.124059e-01	6.183725e-01	6.092748e-01	6.097715e-01	6.097732e-01	6.115826e-01
	std.	1.851563e-03	4.382974e-04	3.672774e-04	9.993326e-04	5.926162e-04	3.022231e-03	8.256883e-04	7.372125e-04	1.684051e-03	4.752743e-04	7.324575e-04	6.224743e-04	7.874685e-04
3D	med.	3.873929e-01	3.262691e-01	3.460699e-01	3.294528e-01	3.210938e-01	3.426550e-01	3.268279e-01	3.274450e-01	3.310353e-01	3.248046e-01	3.260644e-01	3.259255e-01	3.284194e-01
	avg.	3.865726e-01	3.264746e-01	3.551661e-01	3.295019e-01	3.211329e-01	3.439660e-01	3.271222e-01	3.276071e-01	3.311706e-01	3.249634e-01	3.263135e-01	3.261221e-01	3.286178e-01
	min.	3.520862e-01	3.240493e-01	3.264074e-01	3.276121e-01	3.194972e-01	3.343746e-01	3.244743e-01	3.242253e-01	3.288127e-01	3.233421e-01	3.247842e-01	3.242080e-01	3.258346e-01
	max.	4.332041e-01	3.279356e-01	4.626739e-01	3.318158e-01	3.229070e-01	3.607887e-01	3.321254e-01	3.332709e-01	3.336130e-01	3.273616e-01	3.279935e-01	3.285045e-01	3.319009e-01
	std.	1.826686e-02	9.603140e-04	3.302418e-02	8.857138e-04	8.878127e-04	4.884062e-03	1.477465e-03	1.568581e-03	1.277731e-03	9.975522e-04	7.873311e-04	9.957230e-04	1.255120e-03
4D	med.	3.745922e-01	2.282042e-01	2.569971e-01	2.270069e-01	4.488627e-01	2.708626e-01	2.192662e-01	2.216852e-01	2.304995e-01	2.235527e-01	2.292960e-01	2.276434e-01	2.241360e-01
	avg.	3.719809e-01	2.282091e-01	2.816434e-01	2.262097e-01	4.427646e-01	2.720820e-01	2.193751e-01	2.218779e-01	2.305924e-01	2.234631e-01	2.290145e-01	2.277788e-01	2.244203e-01
	min.	3.372072e-01	2.236446e-01	2.259789e-01	2.168031e-01	3.880990e-01	2.494513e-01	2.157243e-01	2.174892e-01	2.259911e-01	2.200766e-01	2.234467e-01	2.240025e-01	2.214675e-01
	max.	3.963206e-01	2.324167e-01	7.425204e-01	2.376786e-01	4.611838e-01	3.068142e-01	2.231616e-01	2.304269e-01	2.372743e-01	2.256971e-01	2.329188e-01	2.331848e-01	2.330836e-01
	std.	1.472888e-02	1.808363e-03	8.259940e-02	5.187325e-03	1.536653e-02	1.183049e-02	1.355656e-03	1.796412e-03	2.131976e-03	1.075676e-03	2.083082e-03	1.998276e-03	1.846825e-03
5D	med.	3.904668e-01	1.765729e-01	2.297627e-01	1.892812e-01	3.372553e-01	2.902928e-01	1.675818e-01	1.719776e-01	1.876331e-01	1.788905e-01	1.773152e-01	1.740373e-01	1.885907e-01
	avg.	3.912177e-01	1.767368e-01	2.296113e-01	1.886846e-01	3.276807e-01	2.908354e-01	1.676906e-01	1.723962e-01	1.879122e-01	1.790051e-01	1.774900e-01	1.743091e-01	1.887024e-01
	min.	3.519754e-01	1.713363e-01	1.741110e-01	1.792903e-01	1.686763e-01	2.698621e-01	1.650745e-01	1.682136e-01	1.837288e-01	1.750477e-01	1.712447e-01	1.697822e-01	1.809842e-01
	max.	4.285520e-01	1.838509e-01	8.022982e-01	2.047705e-01	3.709820e-01	3.110726e-01	1.722978e-01	1.814613e-01	1.950060e-01	1.849082e-01	1.829445e-01	1.799850e-01	1.962126e-01
	std.	1.781508e-02	2.516324e-03	1.311533e-01	4.114834e-03	8.956854e-02	8.452565e-03	1.384345e-03	2.294958e-03	2.491182e-03	1.668309e-03	2.270933e-03	2.223012e-03	3.178489e-03
6D	med.	4.071846e-01	1.735603e-01	3.643581e-01	1.632124e-01	2.935947e-01	2.925904e-01	1.273957e-01	1.332287e-01	1.568307e-01	1.498016e-01	1.676534e-01	1.872721e-01	1.621531e-01
	avg.	4.080541e-01	1.841137e-01	3.888861e-01	1.655287e-01	2.917127e-01	2.942593e-01	1.276136e-01	1.334783e-01	1.566370e-01	1.502957e-01	1.786126e-01	1.909140e-01	1.620295e-01
	min.	3.699138e-01	1.397857e-01	1.724326e-01	1.465593e-01	2.294386e-01	2.738787e-01	1.245940e-01	1.287981e-01	1.491266e-01	1.445719e-01	1.341597e-01	1.383709e-01	1.434145e-01
	max.	4.613218e-01	2.392842e-01	7.125813e-01	1.921677e-01	3.044007e-01	3.176622e-01	1.323185e-01	1.407369e-01	1.632520e-01	1.582358e-01	2.305675e-01	2.389885e-01	2.103022e-01
	std.	1.698823e-02	2.666910e-02	1.234971e-01	8.520501e-03	1.042775e-02	9.532727e-03	1.631930e-03	2.361387e-03	2.687967e-03	2.911005e-03	2.759199e-02	2.877491e-02	8.672321e-03
7D	med.	4.311900e-01	3.436866e-01	5.400830e-01	1.674549e-01	2.761365e-01	3.065213e-01	1.163957e-01	1.192244e-01	1.404086e-01	1.348180e-01	2.615352e-01	3.480395e-01	1.437504e-01
	avg.	4.322482e-01	3.293047e-01	4.876964e-01	1.686609e-01	2.744234e-01	3.069907e-01	1.169155e-01	1.194252e-01	1.406185e-01	1.391168e-01	2.633503e-01	3.398768e-01	1.497197e-01
	min.	3.643039e-01	1.802707e-01	2.782727e-01	1.418015e-01	2.365794e-01	2.892002e-01	1.089308e-01	1.116364e-01	1.266908e-01	1.255747e-01	1.529215e-01	1.958846e-01	1.286979e-01
	max.	4.821528e-01	3.933334e-01	6.717227e-01	2.502530e-01	2.885165e-01	3.265117e-01	1.262873e-01	1.312937e-01	1.501919e-01	1.719667e-01	3.662005e-01	3.894961e-01	2.398240e-01
	std.	2.277853e-02	5.103514e-02	8.797836e-02	2.199934e-02	7.192739e-03	8.113799e-03	3.057579e-03	4.055165e-03	4.425764e-03	1.155841e-02	6.412527e-02	7.344317e-02	1.741583e-02
8D	med.	4.055688e-01	3.807146e-01	5.491331e-01	1.502936e-01	2.190912e-01	2.781773e-01	9.490542e-02	1.004230e-01	1.418803e-01	1.218461e-01	3.590150e-01	3.727925e-01	1.438922e-01
	avg.	4.055723e-01	3.781128e-01	5.255718e-01	1.529327e-01	2.139923e-01	2.789730e-01	9.516940e-02	1.007258e-01	1.418556e-01	1.255117e-01	3.484016e-01	3.662361e-01	1.619512e-01
	min.	3.703517e-01	1.970645e-01	3.875078e-01	1.269918e-01	1.017795e-01	2.667933e-01	9.009654e-02	9.548651e-02	1.352259e-01	1.131687e-01	1.611559e-01	2.091723e-01	1.164122e-01
	max.	4.437429e-01	4.196712e-01	8.168470e-01	2.180965e-01	2.279828e-01	2.946375e-01	1.025584e-01	1.076315e-01	1.509424e-01	1.459564e-01	3.996796e-01	4.127983e-01	2.555770e-01
	std.	1.559243e-02	2.635675e-02	8.120260e-02	1.943908e-02	2.080329e-02	5.714701e-03	2.353078e-03	2.690660e-03	3.192649e-03	8.841173e-03	4.353196e-02	3.077783e-02	4.678368e-02
9D	med.	3.823720e-01	4.015859e-01	5.648528e-01	1.393387e-01	1.640982e-01	2.588504e-01	8.057591e-02	8.784878e-02	1.568827e-01	1.262140e-01	3.906881e-01	3.921716e-01	1.830306e-01
	avg.	3.825138e-01	4.039606e-01	5.520656e-01	1.462966e-01	1.592800e-01	2.587326e-01	8.077990e-02	8.793883e-02	1.578280e-01	1.270658e-01	3.829275e-01	3.863281e-01	1.818004e-01
	min.	3.563949e-01	3.581493e-01	4.047917e-01	1.149476e-01	9.419864e-02	2.451614e-01	7.736624e-02	8.459929e-02	1.461297e-01	1.145501e-01	2.341816e-01	2.873159e-01	1.022805e-01
	max.	4.151223e-01	4.468325e-01	6.573674e-01	1.942442e-01	1.737280e-01	2.701830e-01	8.495684e-02	9.387182e-02	1.726097e-01	1.436561e-01	4.352021e-01	4.466810e-01	2.299997e-01
	std.	1.326103e-02	1.685922e-02	5.064260e-02	1.977873e-02	1.862336e-02	5.536620e-03	1.519292e-03	1.823484e-03	5.524897e-03	7.489167e-03	3.733162e-02	3.423981e-02	4.718757e-02
10D	med.	3.598774e-01	4.111331e-01	5.749714e-01	1.352614e-01	9.951978e-02	2.418517e-01	7.246939e-02	8.105250e-02	1.858636e-01	1.322519e-01	3.966950e-01	3.952945e-01	2.054091e-01
	avg.	3.601166e-01	4.157882e-01	5.859639e-01	1.421275e-01	1.052028e-01	2.414368e-01	7.271067e-02	8.116903e-02	1.856034e-01	1.328139e-01	3.911114e-01	3.912242e-01	1.812021e-01
	min.	3.264709e-01	3.918841e-01	4.223792e-01	1.165218e-01	9.112540e-02	2.304454e-01	7.040080e-02	7.777070e-02	1.662447e-01	1.200449e-01	2.814313e-01	3.058505e-01	1.102698e-01
	max.	3.880347e-01	5.675405e-01	7.700151e-01	1.757237e-01	1.254980e-01	2.536017e-01	7.660633e-02	8.703865e-02	1.979424e-01	1.491394e-01	4.527857e-01	4.547056e-01	2.112917e-01
	std.	1.068365e-02	1.991735e-02	4.670492e-02	1.858112e-02	1.068853e-02	4.567378e-03	1.231639e-03	1.785766e-03	5.653419e-03	6.082294e-03	3.443492e-02	3.266218e-02	3.614985e-02



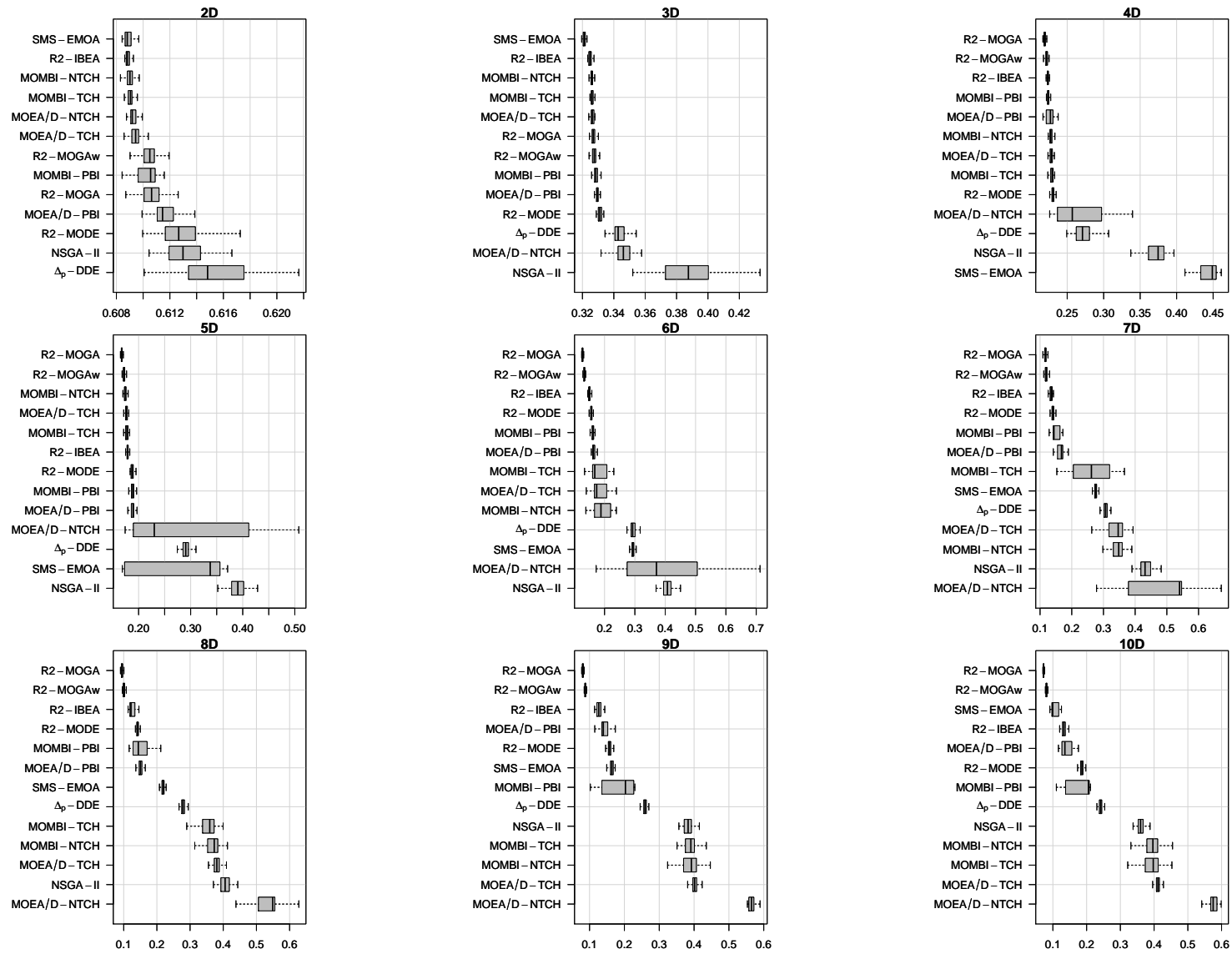


Figure A.178: Box-plot of R2 indicator values for different optimizers on the WFG5 test problem.

Table A.73: Comparison of runtime (in milliseconds) for different optimizers on the WFG5 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	4.109000e+02	8.479120e+02	1.388858e+03	1.294806e+03	4.337249e+04	1.515472e+03	1.894717e+03	1.502803e+03	1.983822e+03	4.563170e+05	2.115274e+03	2.125541e+03	2.910925e+03
	avg.	4.231354e+02	8.586184e+02	1.380692e+03	1.274029e+03	4.356078e+04	1.522022e+03	1.879597e+03	1.517391e+03	1.946516e+03	4.775948e+05	2.097574e+03	2.135635e+03	2.911662e+03
	min.	3.523730e+02	6.624110e+02	9.794850e+02	8.899430e+02	3.180675e+04	1.215047e+03	1.333013e+03	1.123345e+03	1.185192e+03	4.548160e+05	1.658717e+03	1.779294e+03	2.519499e+03
	max.	6.092890e+02	1.242424e+03	1.893251e+03	1.770062e+03	4.936930e+04	1.847642e+03	2.471699e+03	1.913784e+03	2.501248e+03	5.599660e+05	2.420928e+03	2.576211e+03	3.590104e+03
	std.	5.881379e+01	1.071197e+02	1.320416e+02	1.205661e+02	3.637953e+03	1.288999e+02	2.398662e+02	1.581340e+02	2.654378e+02	3.920277e+04	1.491964e+02	1.651321e+02	1.511291e+02
3D	med.	5.600090e+02	8.969970e+02	1.689476e+03	1.350645e+03	1.059802e+06	1.824314e+03	2.135677e+03	1.821885e+03	2.252421e+03	5.636440e+05	2.089912e+03	2.040209e+03	3.164945e+03
	avg.	5.633178e+02	8.979366e+02	1.666415e+03	1.330146e+03	1.028857e+06	1.801382e+03	2.111093e+03	1.837459e+03	2.247922e+03	5.647654e+05	2.094811e+03	2.061463e+03	3.151850e+03
	min.	4.719570e+02	6.691480e+02	1.220282e+03	1.030182e+03	7.772835e+05	1.362829e+03	1.582746e+03	1.451674e+03	1.718244e+03	5.085320e+05	1.734044e+03	1.805223e+03	2.470308e+03
	max.	7.271870e+02	1.272900e+03	1.982834e+03	1.985322e+03	1.133452e+06	2.430326e+03	2.737990e+03	2.278604e+03	2.689814e+03	5.852520e+05	2.527762e+03	2.390072e+03	3.493531e+03
	std.	6.244828e+01	1.067823e+02	1.417956e+02	1.412509e+02	7.984561e+04	1.957397e+02	2.334830e+02	1.805194e+02	2.273170e+02	1.533384e+04	1.588699e+02	1.384713e+02	1.749239e+02
4D	med.	8.098400e+02	1.108751e+03	1.966265e+03	1.627312e+03	3.391850e+06	2.392941e+03	2.732711e+03	2.328650e+03	2.747186e+03	5.953190e+05	2.234220e+03	2.346193e+03	3.309490e+03
	avg.	8.031400e+02	1.127968e+03	1.931428e+03	1.607364e+03	3.261264e+06	2.357776e+03	2.673173e+03	2.331507e+03	2.753236e+03	5.948572e+05	2.274330e+03	2.342211e+03	3.297211e+03
	min.	6.530590e+02	8.904760e+02	1.320584e+03	1.226955e+03	2.181628e+06	1.592605e+03	1.894267e+03	1.717934e+03	2.103624e+03	5.686100e+05	1.833953e+03	2.053699e+03	2.905092e+03
	max.	9.333890e+02	1.419729e+03	2.173494e+03	2.088201e+03	3.642182e+06	2.913865e+03	3.239295e+03	2.717251e+03	3.341715e+03	6.022970e+05	3.189343e+03	2.653434e+03	3.871599e+03
	std.	7.223322e+01	9.636594e+01	1.727320e+02	1.170157e+02	3.340293e+05	2.757062e+02	2.854164e+02	2.142397e+02	2.707986e+02	5.201505e+03	2.724060e+02	1.604761e+02	1.465827e+02
5D	med.	1.041276e+03	1.349634e+03	2.688201e+03	1.678737e+03	3.481514e+06	5.412628e+03	3.151608e+03	3.061088e+03	3.760539e+03	6.209950e+05	2.597786e+03	2.811571e+03	4.148384e+03
	avg.	1.032180e+03	1.343158e+03	2.634406e+03	1.684764e+03	3.496372e+06	5.389269e+03	3.157572e+03	3.027827e+03	3.711266e+03	6.308397e+05	2.655031e+03	2.811541e+03	4.161027e+03
	min.	8.427680e+02	1.081730e+03	2.033975e+03	1.346525e+03	2.131540e+06	3.329793e+03	2.213547e+03	2.536072e+03	3.579301e+03	5.986700e+05	2.132549e+03	2.308706e+03	3.886231e+03
	max.	1.308409e+03	1.559898e+03	3.201336e+03	2.084553e+03	4.745110e+06	6.578967e+03	3.811917e+03	3.461541e+03	4.335275e+03	7.211750e+05	4.172605e+03	3.133084e+03	4.510790e+03
	std.	9.754240e+01	1.116933e+02	1.970735e+02	1.684995e+02	8.930252e+05	5.699619e+02	2.871260e+02	2.200647e+02	3.815381e+02	2.998532e+04	3.193461e+02	1.620752e+02	1.059245e+02
6D	med.	1.289744e+03	1.649672e+03	2.921219e+03	1.756430e+03	4.880370e+06	5.868157e+03	3.793999e+03	3.565886e+03	4.168428e+03	7.281160e+05	2.490651e+03	3.204566e+03	3.509541e+03
	avg.	1.288052e+03	1.669687e+03	2.841159e+03	1.789822e+03	4.791138e+06	5.777951e+03	3.740166e+03	3.526714e+03	4.107668e+03	7.278370e+05	2.653451e+03	3.156682e+03	3.562536e+03
	min.	1.085438e+03	1.367126e+03	2.009127e+03	1.360956e+03	3.658297e+06	3.238234e+03	2.862704e+03	2.601145e+03	2.967225e+03	6.625070e+05	2.114617e+03	2.597622e+03	3.472335e+03
	max.	1.553827e+03	2.058384e+03	3.397169e+03	2.317797e+03	5.501579e+06	7.482078e+03	4.271885e+03	4.079667e+03	4.755734e+03	7.414060e+05	4.982885e+03	3.631883e+03	4.920554e+03
	std.	9.514429e+01	1.391559e+02	2.430976e+02	2.230134e+02	4.412188e+05	7.704886e+02	3.466475e+02	2.765202e+02	3.511941e+02	8.499423e+03	4.861023e+02	1.621420e+02	1.891823e+02
7D	med.	1.245965e+03	1.880354e+03	3.042309e+03	1.896697e+03	2.912475e+06	1.662676e+04	3.235032e+03	3.066522e+03	3.578081e+03	3.310410e+05	1.945981e+03	2.280858e+03	3.315928e+03
	avg.	1.244956e+03	1.888962e+03	2.997985e+03	1.859311e+03	2.867842e+06	1.697819e+04	3.236043e+03	3.041490e+03	3.493324e+03	3.303956e+05	1.976193e+03	2.330459e+03	3.299954e+03
	min.	1.012615e+03	1.571808e+03	2.158565e+03	1.326726e+03	1.982503e+06	1.019036e+04	2.271150e+03	2.217724e+03	2.750989e+03	3.151060e+05	1.657306e+03	2.041519e+03	2.971655e+03
	max.	1.449947e+03	2.287097e+03	3.426093e+03	2.282224e+03	3.341177e+06	2.486273e+04	3.738054e+03	3.499002e+03	3.947337e+03	3.365390e+05	2.508486e+03	3.725337e+03	3.550189e+03
	std.	9.898959e+01	1.459850e+02	2.682798e+02	1.798027e+02	3.636958e+05	2.868011e+03	3.065833e+02	2.638655e+02	3.043766e+02	3.458418e+03	1.556013e+02	1.619382e+02	1.238263e+02
8D	med.	1.710569e+03	2.143003e+03	3.345920e+03	1.988361e+03	4.894641e+06	4.005914e+04	5.073393e+03	4.350036e+03	4.937472e+03	1.271566e+06	2.617424e+03	3.321277e+03	5.168623e+03
	avg.	1.702938e+03	2.143004e+03	3.320513e+03	1.986080e+03	4.811242e+06	4.049775e+04	4.957911e+03	4.346879e+03	4.927145e+03	1.242501e+06	2.649709e+03	3.373677e+03	5.203682e+03
	min.	1.405317e+03	1.761754e+03	2.557215e+03	1.544740e+03	3.415879e+06	2.500180e+04	3.546940e+03	3.272274e+03	3.584943e+03	1.168679e+06	2.296923e+03	2.848024e+03	4.313324e+03
	max.	1.958539e+03	2.465618e+03	3.722241e+03	2.367323e+03	5.657337e+06	6.307659e+04	5.328079e+03	4.979307e+03	5.573192e+03	1.331182e+06	3.155141e+03	3.900888e+03	5.849171e+03
	std.	1.212051e+02	1.496792e+02	2.533240e+02	2.091432e+02	4.676210e+05	7.622986e+03	3.357945e+02	3.808443e+02	4.649243e+02	5.177284e+04	2.112706e+02	2.597542e+02	2.805457e+02
9D	med.	2.232634e+03	2.421932e+03	3.746654e+03	2.105793e+03	7.576993e+06	8.431282e+04	6.691788e+03	6.044243e+03	7.574225e+03	3.222520e+06	3.501454e+03	5.006306e+03	6.936856e+03
	avg.	2.218320e+03	2.408760e+03	3.734583e+03	2.157113e+03	7.456120e+06	8.633587e+04	6.531970e+03	6.083248e+03	7.488123e+03	3.003484e+06	3.598701e+03	4.963433e+03	6.973659e+03
	min.	1.866896e+03	2.050403e+03	2.946744e+03	1.659887e+03	4.829151e+06	5.491021e+04	4.906592e+03	4.923233e+03	5.435873e+03	2.246799e+06	3.040239e+03	3.974650e+03	5.740991e+03
	max.	2.473589e+03	2.806489e+03	4.254884e+03	2.562849e+03	8.561198e+06	1.198186e+05	7.437315e+03	7.053918e+03	8.196487e+03	3.273871e+06	4.296059e+03	5.369191e+03	8.393087e+03
	std.	1.450857e+02	1.629426e+02	2.835814e+02	2.388246e+02	7.040015e+05	1.329627e+04	7.013194e+02	5.260451e+02	5.449352e+02	3.517821e+05	3.282091e+02	3.025287e+02	4.063011e+02
10D	med.	2.852048e+03	2.508774e+03	4.269482e+03	2.410079e+03	9.115800e+06	2.323947e+05	9.290334e+03	8.759011e+03	1.089795e+04	4.959763e+06	4.450578e+03	5.919489e+03	9.569731e+03
	avg.	2.842408e+03	2.516797e+03	4.197346e+03	2.404021e+03	9.202950e+06	2.266936e+05	8.997345e+03	8.690314e+03	1.061625e+04	4.804007e+06	4.576815e+03	6.049272e+03	9.520805e+03
	min.	2.254799e+03	2.111383e+03	3.103948e+03	1.841148e+03	5.999163e+06	1.541557e+05	6.684437e+03	6.723526e+03	8.401421e+03	4.273221e+06	3.952563e+03	5.204385e+03	8.263868e+03
	max.	3.130882e+03	2.869052e+03	4.848976e+03	2.783534e+03	1.196085e+07	3.766797e+05	1.075673e+04	9.996179e+03	1.158336e+04	5.306902e+06	5.742006e+03	7.574863e+03	1.090207e+04
	std.	1.997543e+02	1.787561e+02	2.870990e+02	2.568925e+02	1.529639e+06	4.744690e+04	1.227220e+03	6.883439e+02	7.786266e+02	2.986241e+05	5.234335e+02	7.455630e+02	5.231241e+02

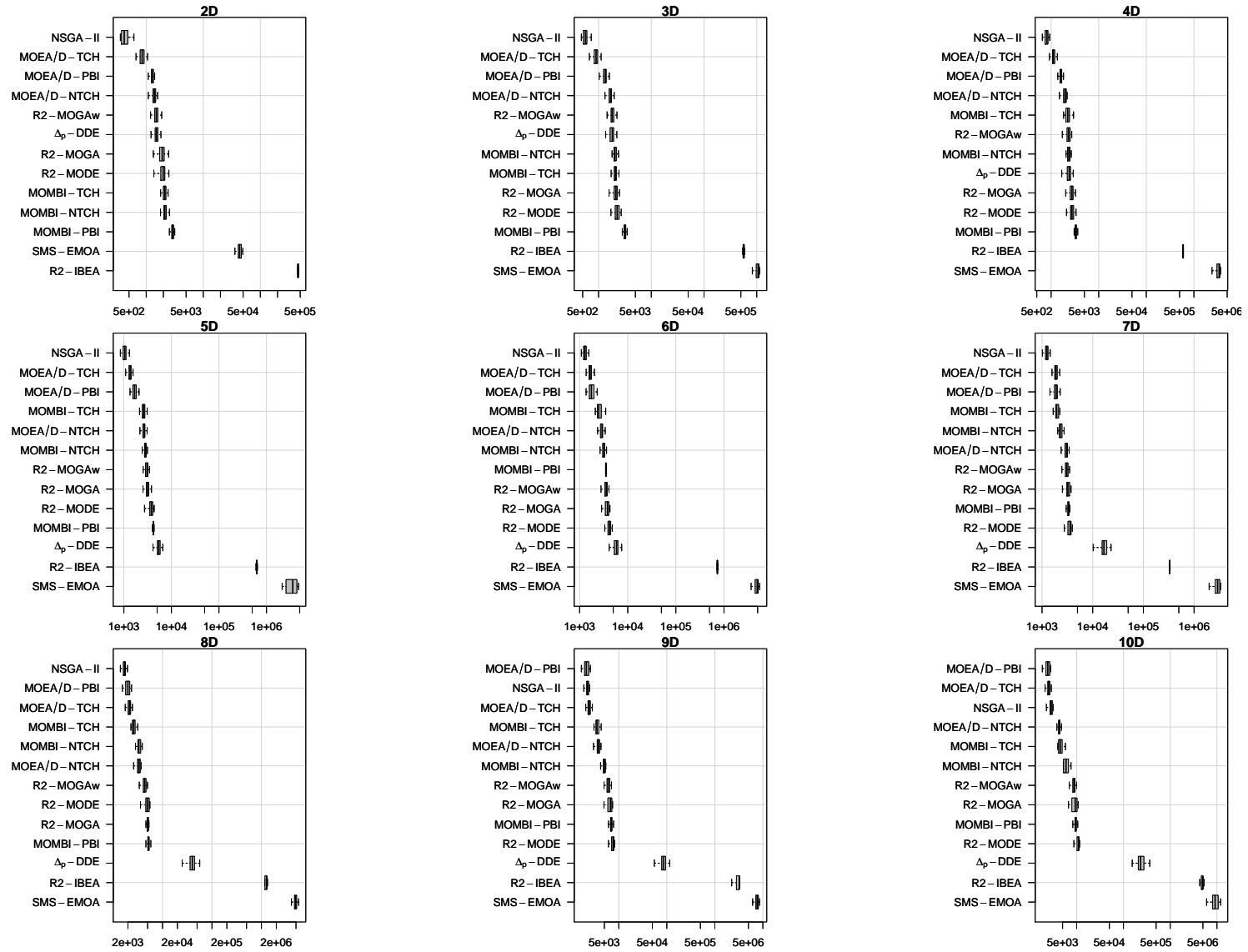


Figure A.179: Box-plot of runtime (in logarithmic scale) for different optimizers on the WFG5 test problem.

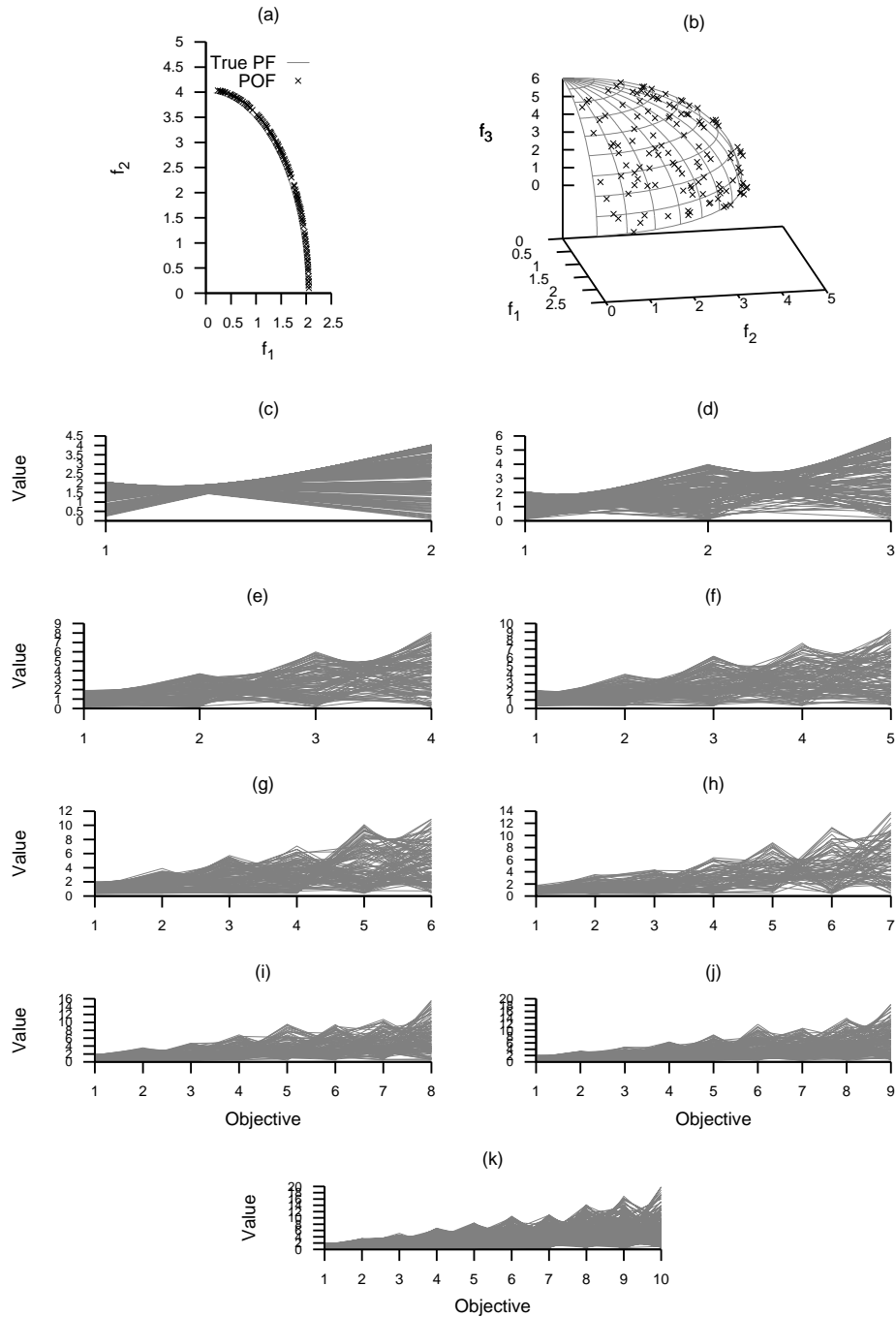


Figure A.180: Plots of the approximations obtained by NSGA-II from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

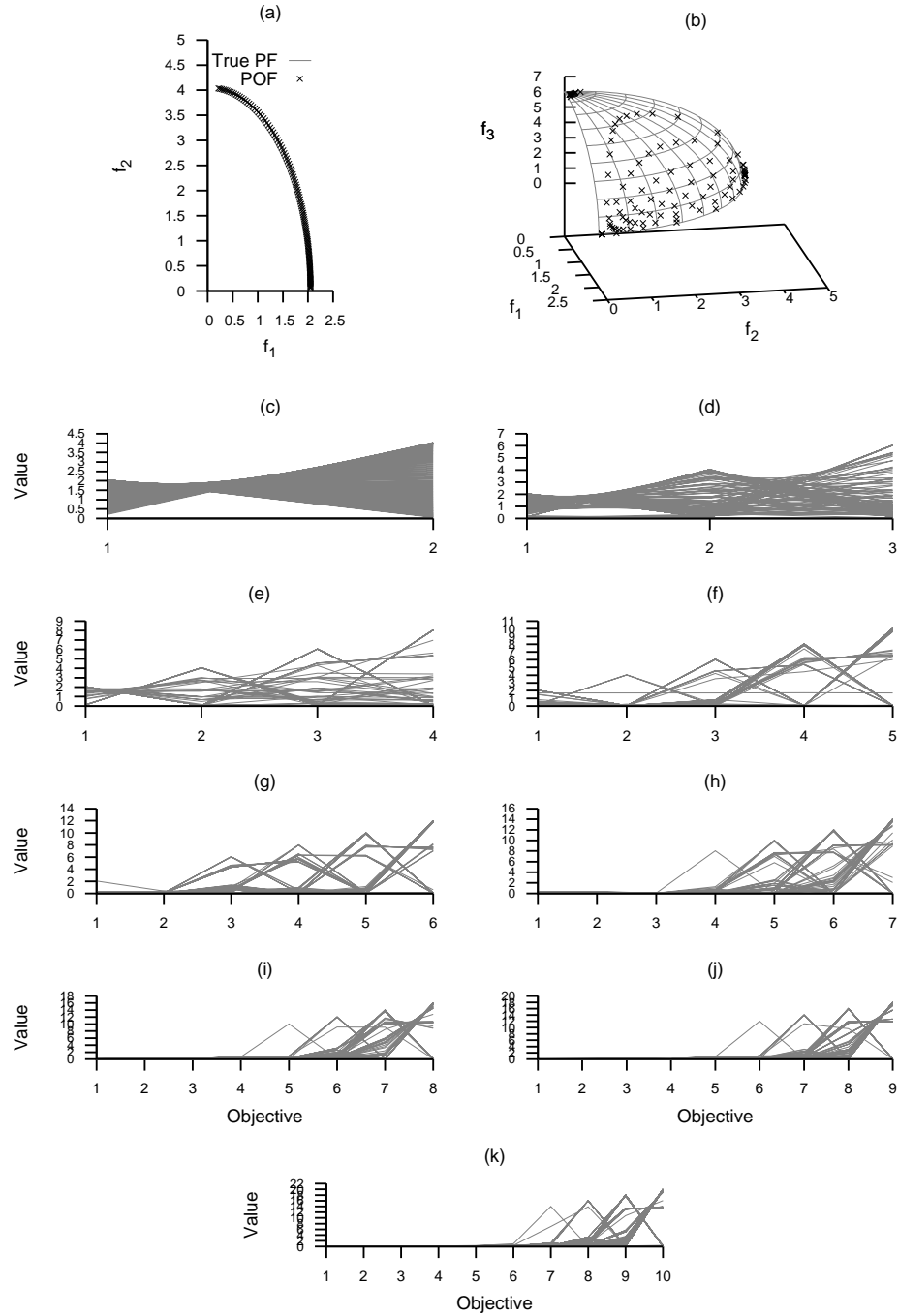


Figure A.181: Plots of the approximations obtained by MOEA/D-TCH from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

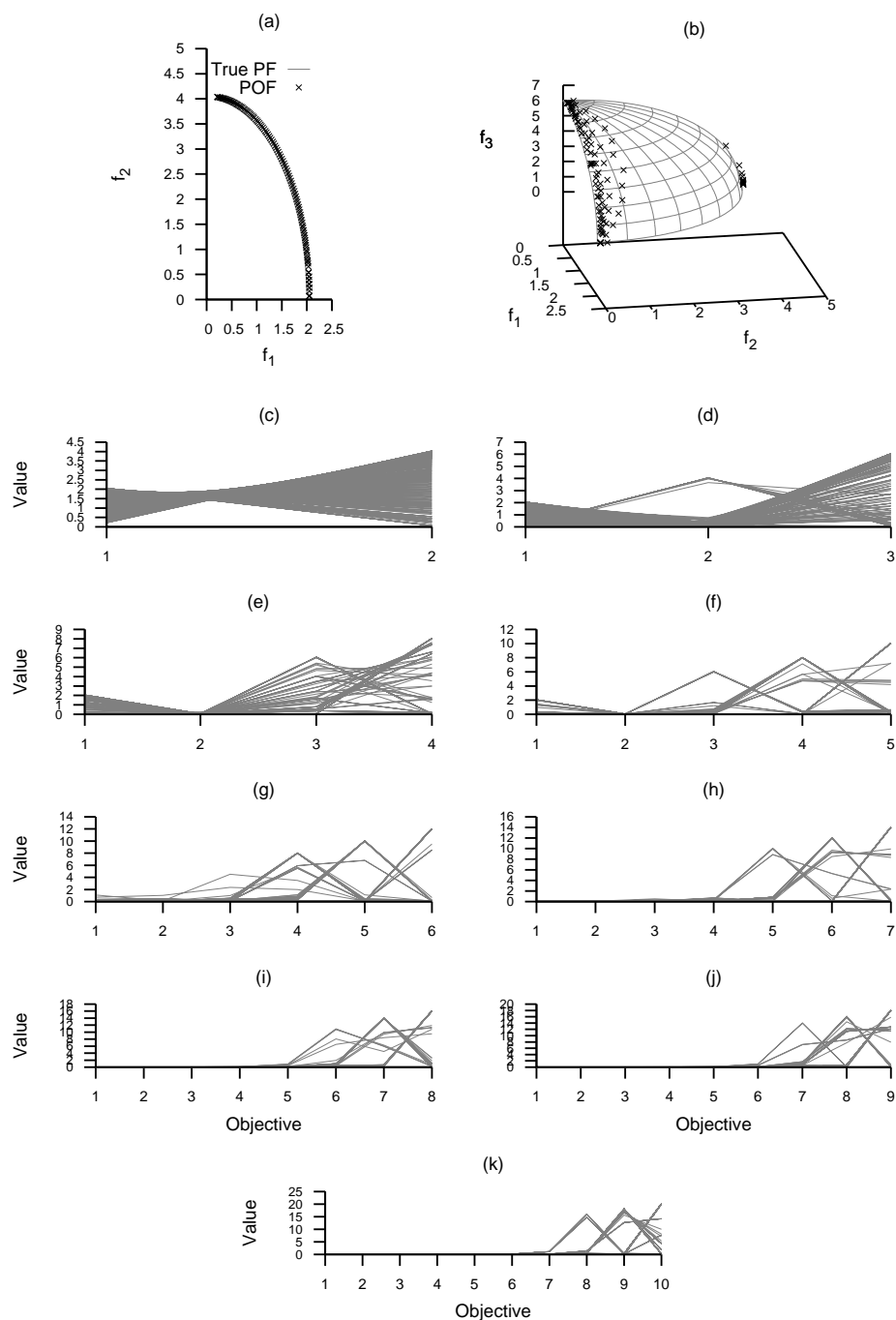


Figure A.182: Plots of the approximations obtained by MOEA/D-NTCH from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

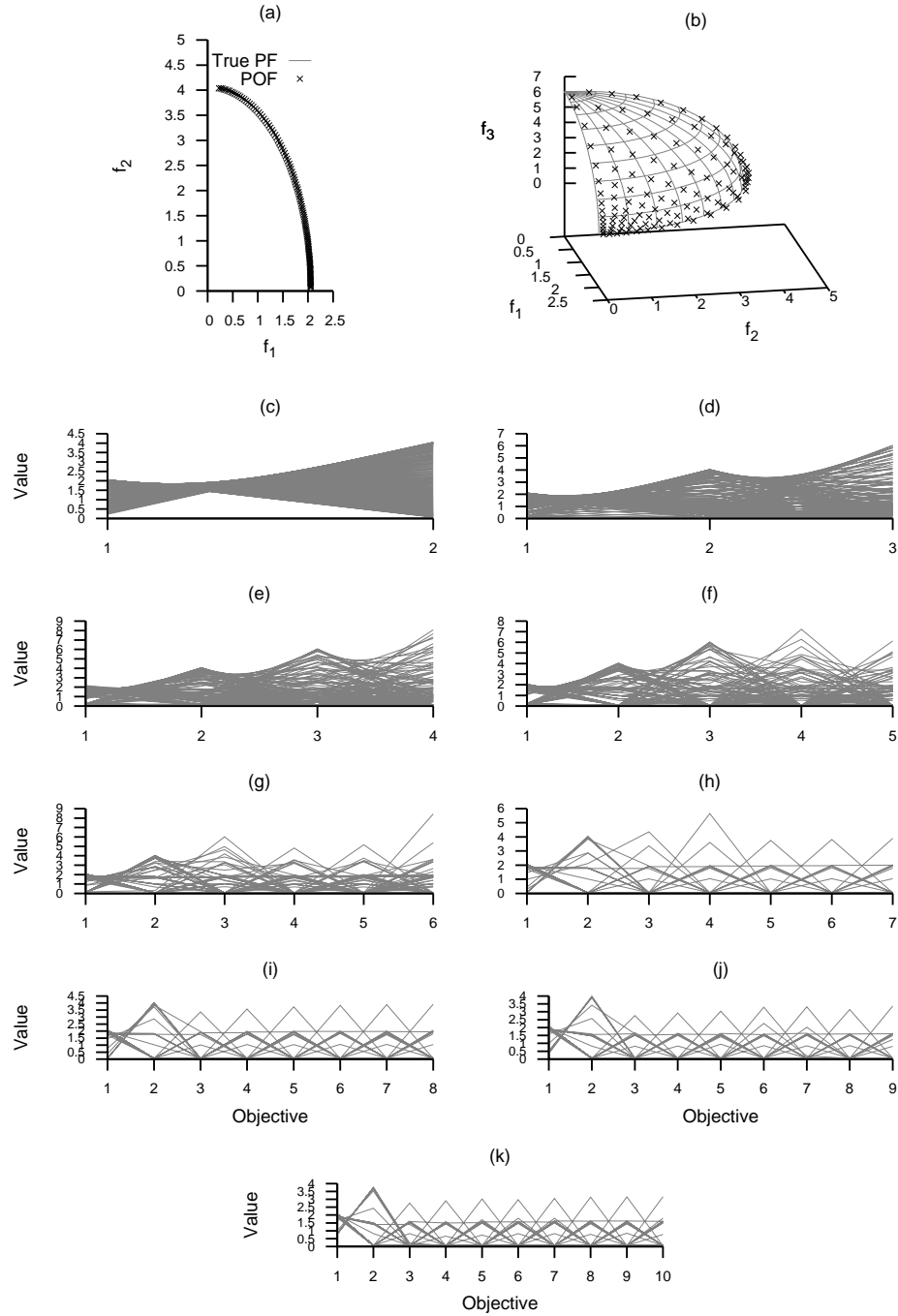


Figure A.183: Plots of the approximations obtained by MOEA/D-PBI from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

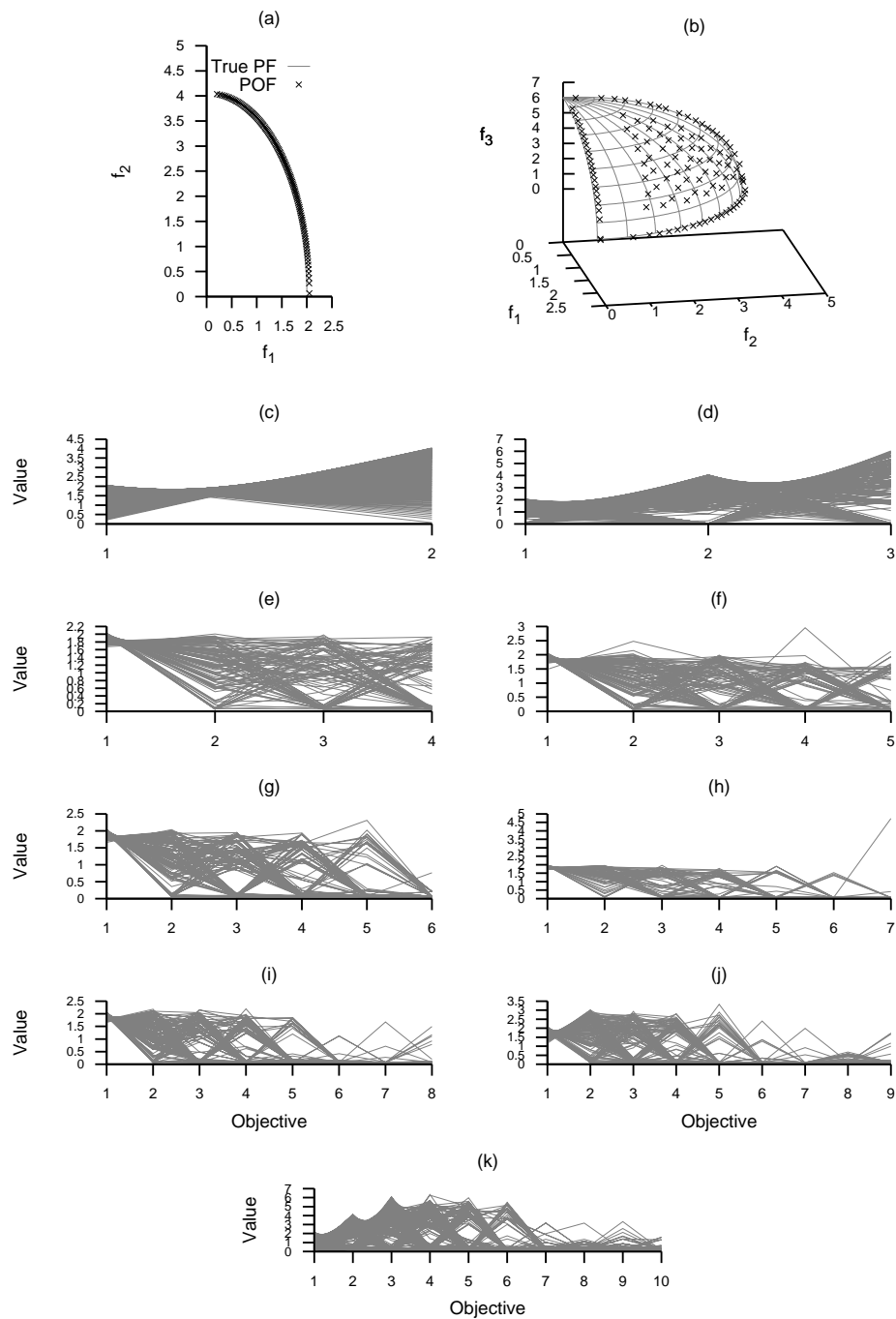


Figure A.184: Plots of the approximations obtained by SMS-EMOA from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



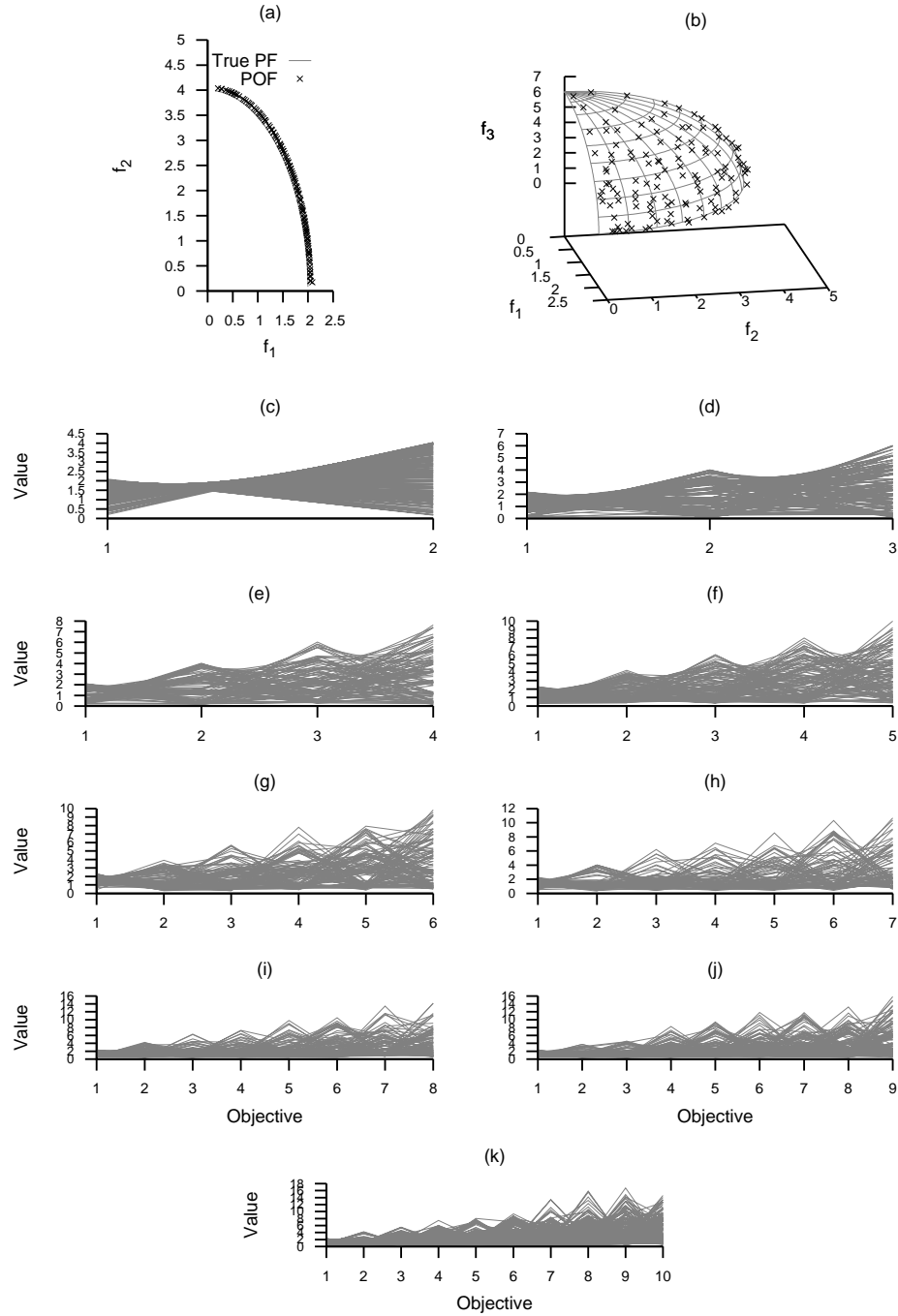


Figure A.185: Plots of the approximations obtained by  $\Delta_p$ -DDE from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

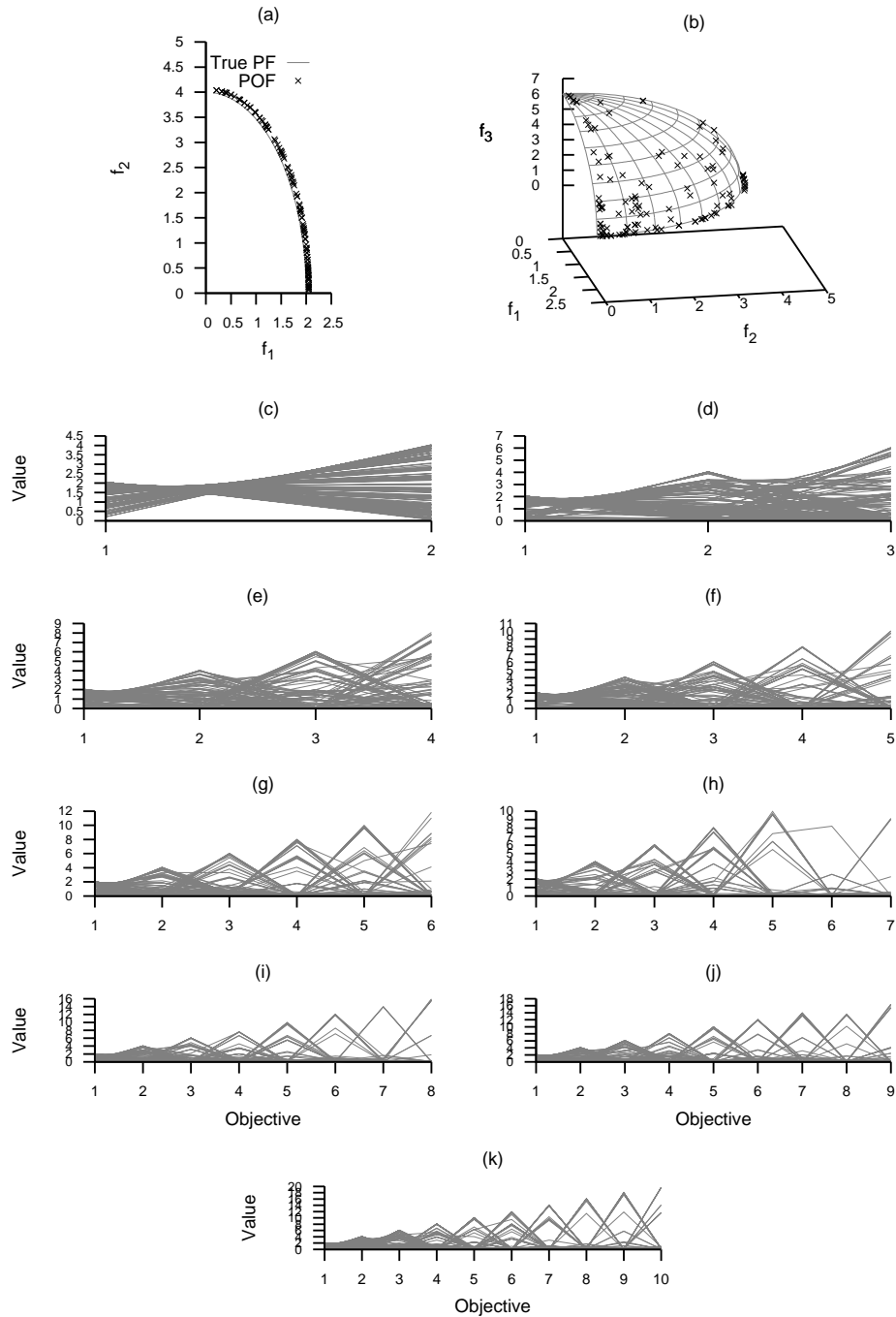


Figure A.186: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

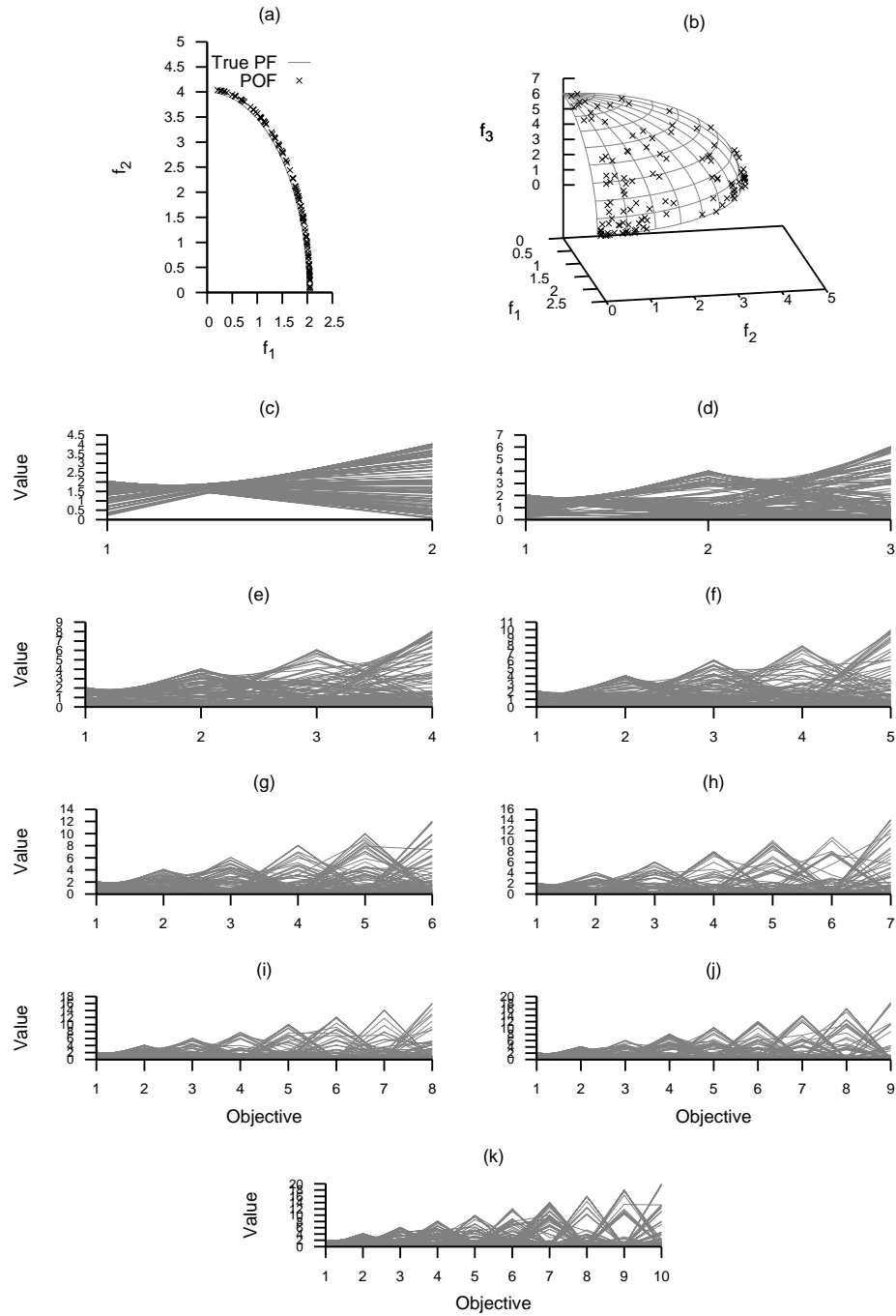


Figure A.187: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

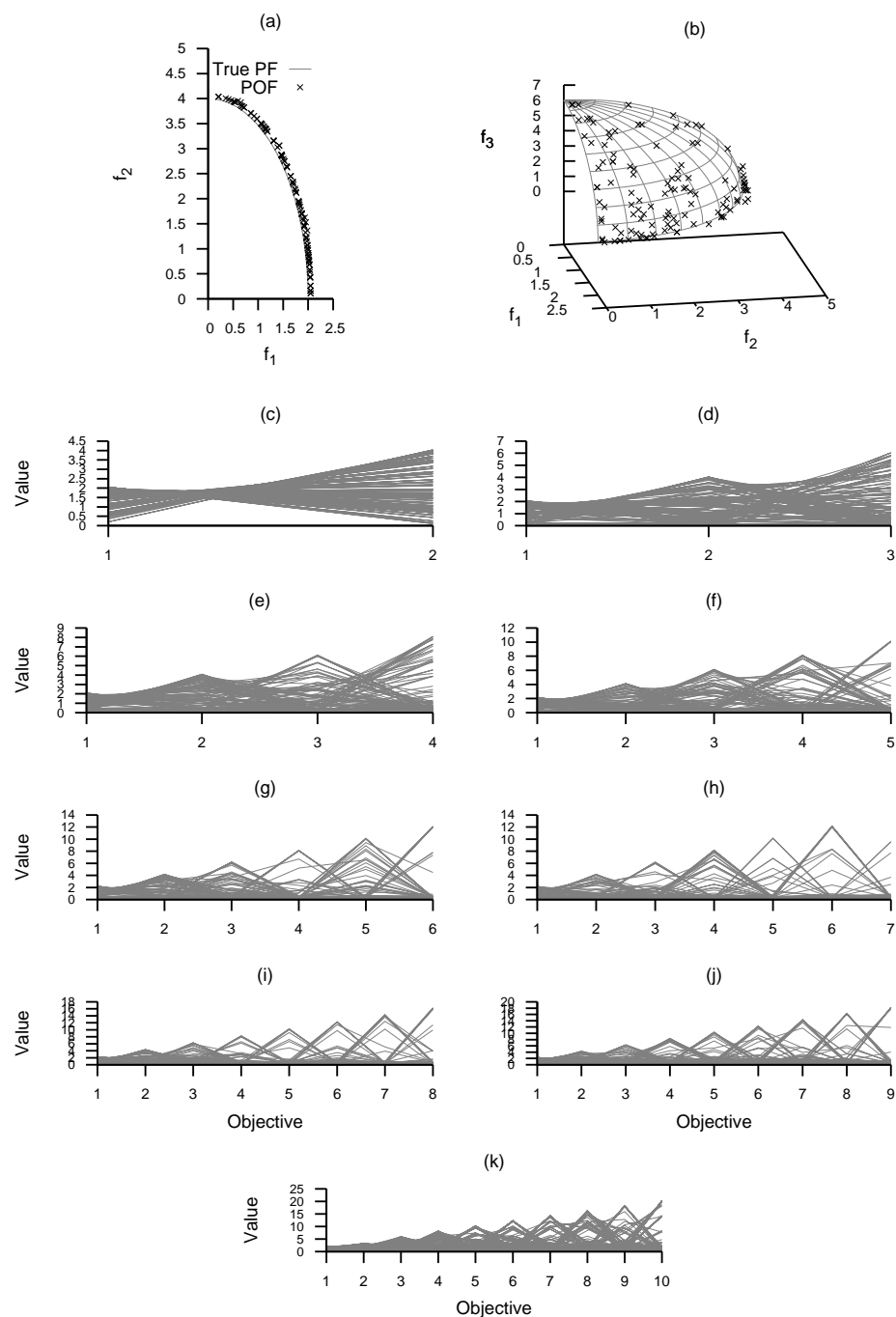


Figure A.188: Plots of the approximations obtained by *R2-MODE* from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

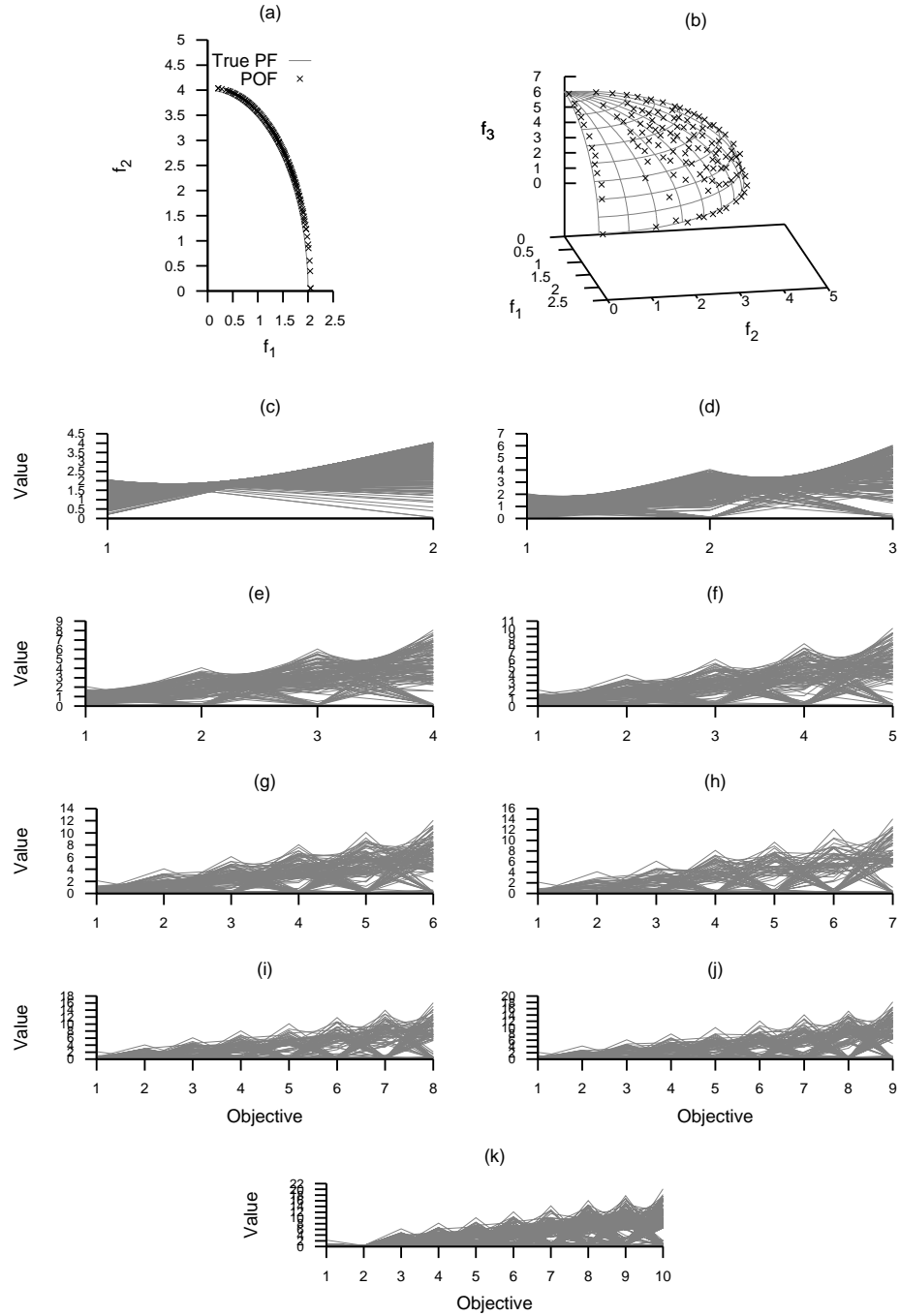


Figure A.189: Plots of the approximations obtained by *R2-IBEA* from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

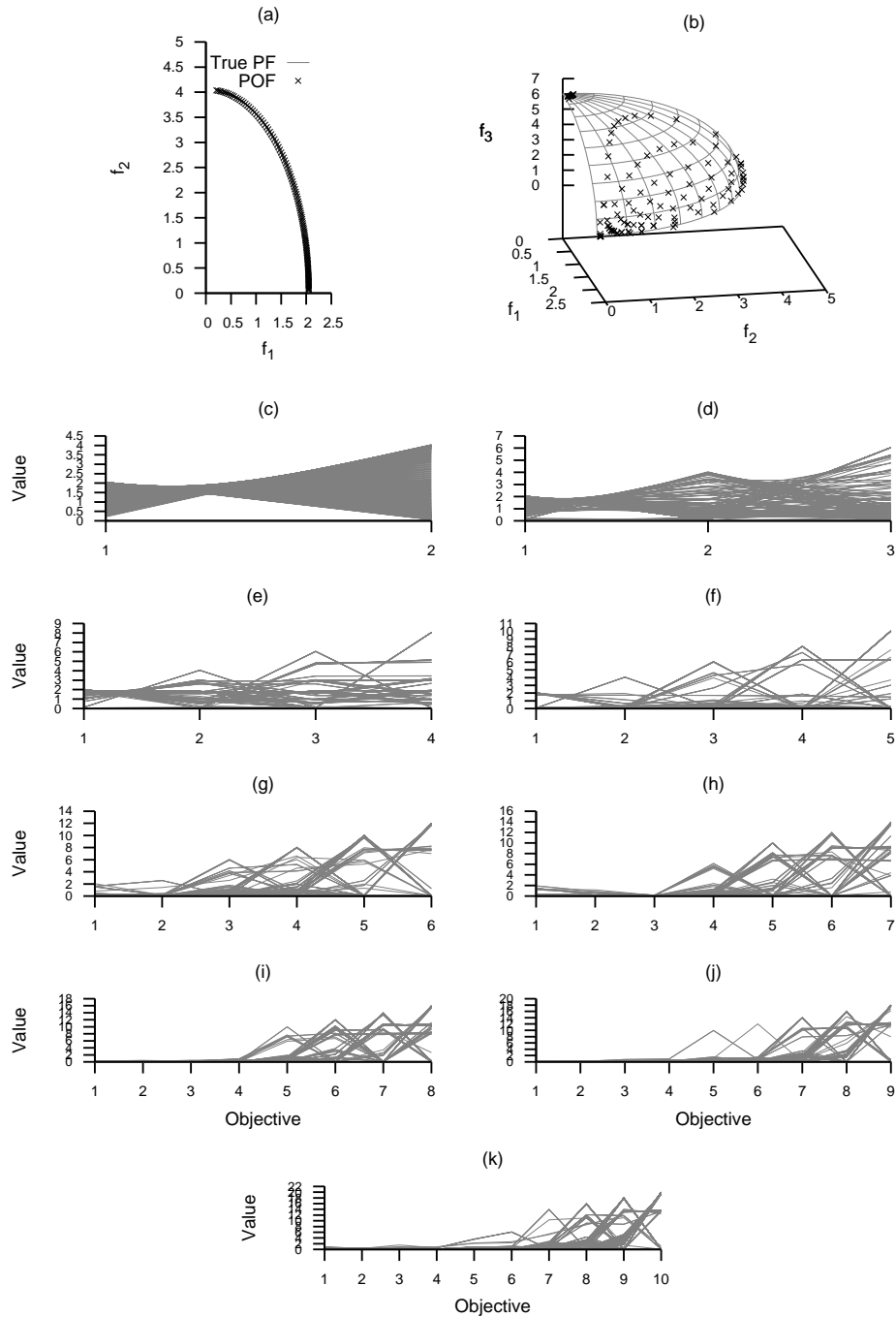


Figure A.190: Plots of the approximations obtained by MOMBI-TCH from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

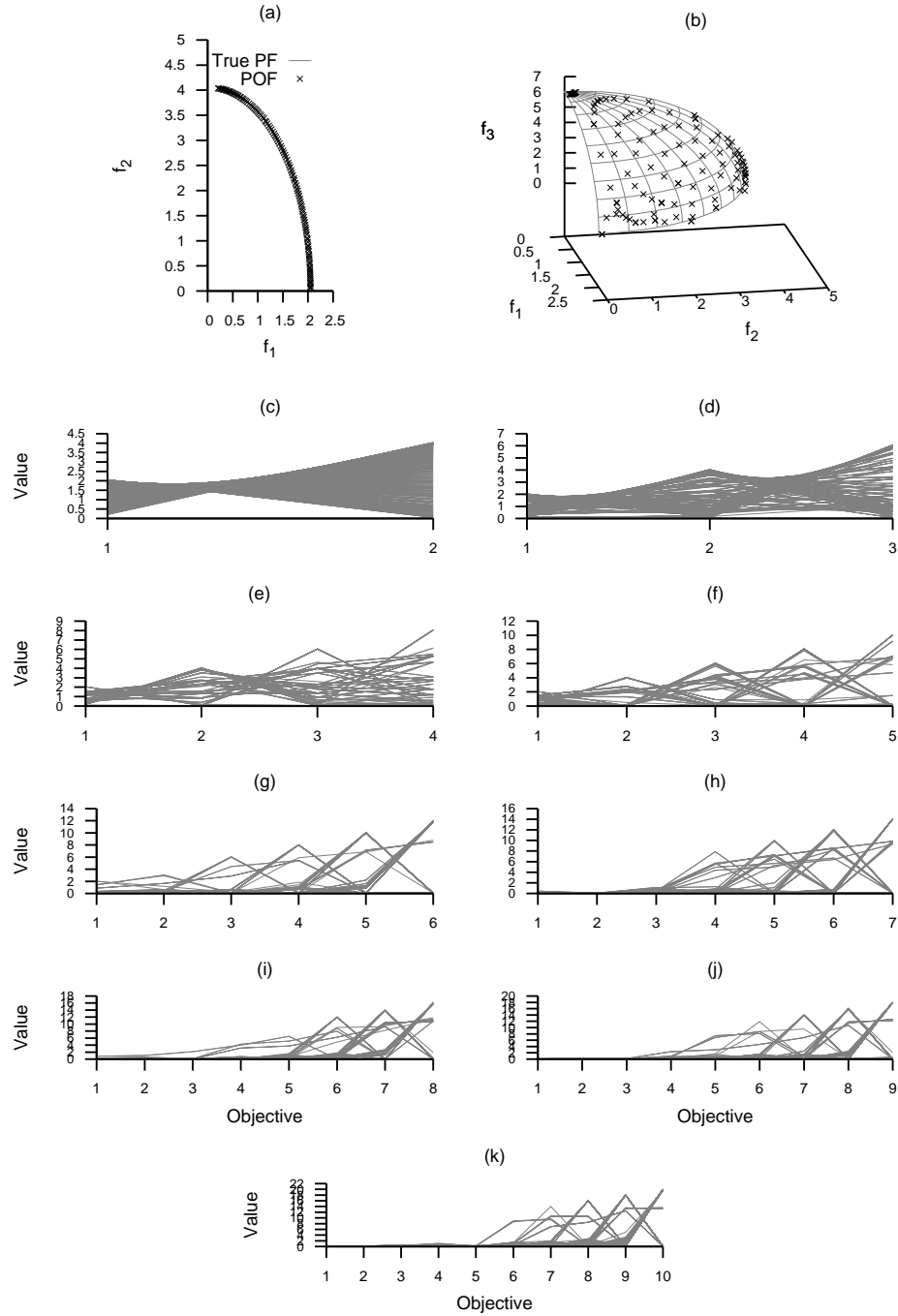


Figure A.191: Plots of the approximations obtained by MOMBI-NTCH from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

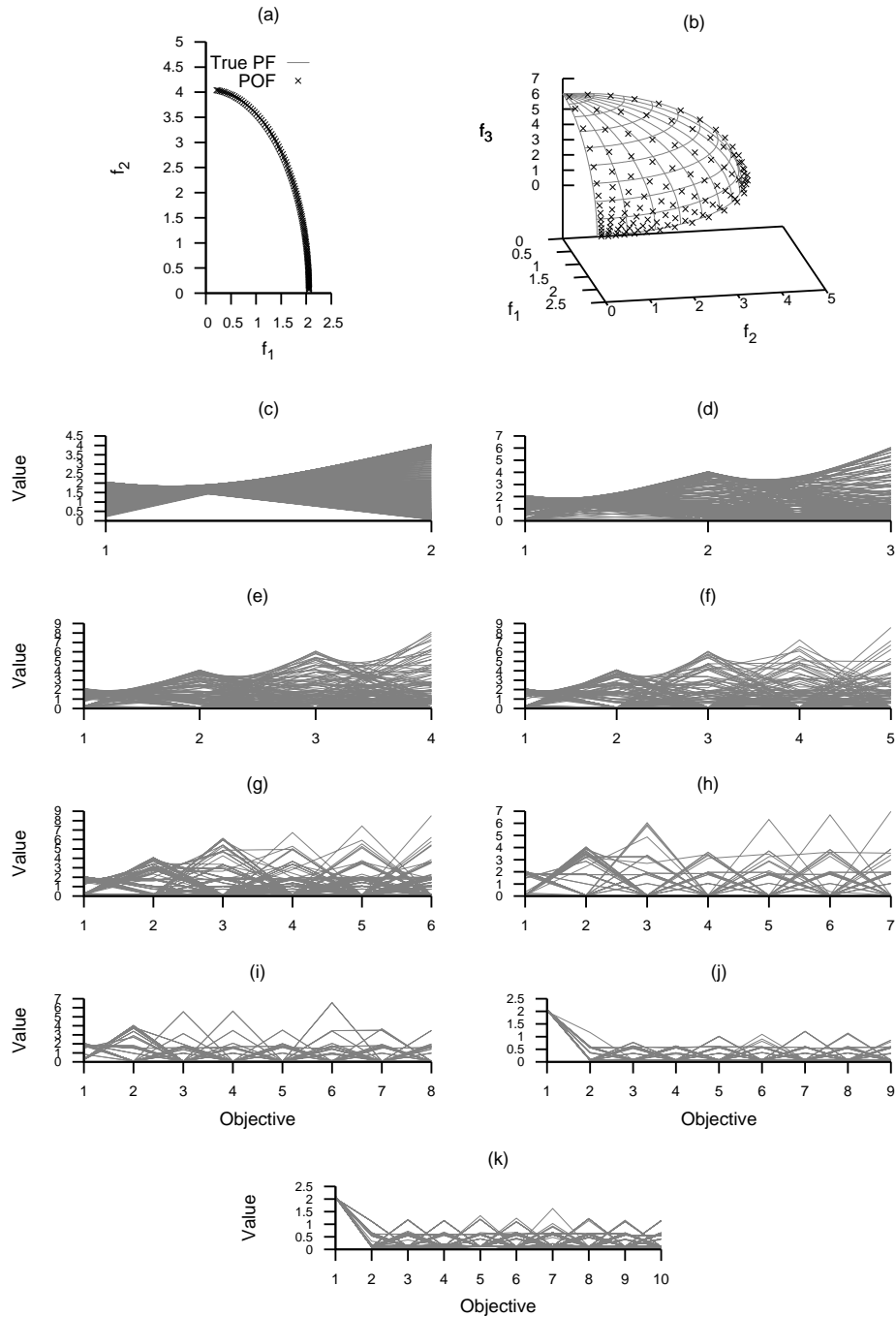


Figure A.192: Plots of the approximations obtained by MOMBI-PBI from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



Table A.74: Comparison of hypervolume indicator values for different optimizers on the WFG6 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_P$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	8.198399e+00	7.955993e+00	8.082958e+00	7.739815e+00	8.290557e+00	8.383870e+00	8.219880e+00	8.252860e+00	8.414499e+00	8.296825e+00	8.311365e+00	8.311444e+00	8.205897e+00
	avg.	8.197206e+00	7.971961e+00	8.047853e+00	7.747859e+00	8.267252e+00	8.375367e+00	8.213968e+00	8.241236e+00	8.411896e+00	8.291430e+00	8.306684e+00	8.313134e+00	8.204390e+00
	min.	7.922492e+00	7.344567e+00	7.419382e+00	7.214627e+00	7.829334e+00	8.131968e+00	7.848123e+00	7.855328e+00	8.249492e+00	7.945132e+00	8.026150e+00	8.104609e+00	7.934502e+00
	max.	8.365323e+00	8.406431e+00	8.418566e+00	8.234679e+00	8.416587e+00	8.556755e+00	8.368359e+00	8.398362e+00	8.501731e+00	8.460240e+00	8.483398e+00	8.452581e+00	8.355683e+00
	std.	6.707658e-02	2.132452e-01	2.065814e-01	2.097158e-01	9.423930e-02	7.334322e-02	7.654552e-02	9.204397e-02	5.186673e-02	6.856778e-02	7.285841e-02	6.007773e-02	7.214439e-02
3D	med.	6.533248e+01	7.075609e+01	6.178565e+01	7.037546e+01	7.438432e+01	6.820303e+01	7.061023e+01	7.082778e+01	7.027634e+01	7.344659e+01	7.055486e+01	7.177054e+01	7.064981e+01
	avg.	6.532154e+01	7.069896e+01	5.983697e+01	7.029907e+01	7.437958e+01	6.816260e+01	7.056013e+01	7.084200e+01	7.011261e+01	7.346859e+01	7.057881e+01	7.177643e+01	7.072292e+01
	min.	6.315394e+01	6.830847e+01	4.485882e+01	6.328253e+01	7.319357e+01	6.438016e+01	6.838058e+01	6.915319e+01	6.704986e+01	7.228541e+01	6.938603e+01	7.068194e+01	6.945001e+01
	max.	6.701344e+01	7.238491e+01	7.057151e+01	7.328161e+01	7.523925e+01	7.054648e+01	7.216472e+01	7.271915e+01	7.231757e+01	7.453073e+01	7.178119e+01	7.323433e+01	7.237079e+01
	std.	8.952243e-01	8.222284e-01	6.402817e+00	1.307385e+00	3.838560e-01	1.505403e+00	7.477918e-01	6.898669e-01	1.137004e+00	3.755876e-01	5.212065e-01	5.334335e-01	6.127926e-01
4D	med.	5.310829e+02	6.301578e+02	4.033159e+02	6.916158e+02	3.860228e+02	5.442857e+02	6.896138e+02	7.005123e+02	6.802800e+02	7.287785e+02	6.416354e+02	6.631739e+02	6.995114e+02
	avg.	5.306902e+02	6.321811e+02	4.309106e+02	6.756297e+02	3.928837e+02	5.562915e+02	6.898694e+02	7.006026e+02	6.621593e+02	7.284762e+02	6.409725e+02	6.636700e+02	6.999433e+02
	min.	4.983728e+02	6.084806e+02	2.907193e+02	4.779654e+02	3.751016e+02	4.265367e+02	6.678222e+02	6.834021e+02	5.415568e+02	7.154691e+02	6.130462e+02	6.406045e+02	6.841927e+02
	max.	5.607935e+02	6.569206e+02	6.188954e+02	7.212529e+02	5.170160e+02	6.617834e+02	7.090052e+02	7.131034e+02	6.996298e+02	7.381682e+02	6.686926e+02	6.688423e+02	7.159170e+02
	std.	1.468210e+01	9.593374e+00	8.287990e+01	4.327641e+01	2.356036e+01	7.421731e+01	9.370016e+01	6.364672e+01	3.597864e+01	3.721526e+01	1.144702e+01	1.092326e+01	6.205130e+00
5D	med.	4.629934e+03	6.947170e+03	3.444497e+03	6.083947e+03	8.463090e+03	5.036667e+03	7.873426e+03	8.108202e+03	7.776712e+03	8.314185e+03	7.512366e+03	8.113862e+03	7.742084e+03
	avg.	4.635416e+03	6.950653e+03	3.886520e+03	6.190272e+03	7.760370e+03	5.069824e+03	7.876071e+03	8.107614e+03	7.705731e+03	8.315038e+03	7.490810e+03	8.093530e+03	7.739955e+03
	min.	4.120170e+03	6.391129e+03	3.181937e+03	4.703992e+03	4.164542e+03	4.593307e+03	7.602497e+03	7.912308e+03	5.961302e+03	8.136684e+03	7.137780e+03	7.658902e+03	7.506620e+03
	max.	5.100349e+03	7.564287e+03	5.793984e+03	7.701776e+03	8.637264e+03	5.650397e+03	8.114454e+03	8.314738e+03	8.038419e+03	8.459291e+03	7.672111e+03	8.378182e+03	7.959509e+03
	std.	2.019826e+02	2.513995e+02	6.449523e+02	8.157317e+02	1.598301e+03	2.050387e+02	1.018171e+02	7.909874e+01	3.446156e+02	5.874370e+01	1.133328e+02	1.642679e+02	9.287244e+01
6D	med.	4.761073e+04	8.409319e+04	3.847487e+04	6.359488e+04	5.556630e+04	4.597019e+04	1.042823e+05	1.085614e+05	1.002484e+05	1.081247e+05	9.387435e+04	9.616293e+04	9.634051e+04
	avg.	4.750881e+04	8.403717e+04	4.118283e+04	6.770052e+04	6.781128e+04	4.591750e+04	1.042277e+05	1.084473e+05	1.004079e+05	1.081302e+05	9.476508e+04	9.638075e+04	9.561168e+04
	min.	4.158537e+04	7.283864e+04	3.316718e+04	5.723719e+04	5.366528e+04	3.948072e+04	9.975875e+04	1.052566e+05	9.559366e+04	1.059814e+05	8.743242e+04	8.632237e+04	8.639624e+04
	max.	5.316428e+04	9.211034e+04	5.885318e+04	8.692755e+04	1.135627e+05	5.205203e+04	1.078238e+05	1.109185e+05	1.056982e+05	1.100730e+05	1.046794e+05	1.072251e+05	1.018491e+05
	std.	2.218968e+03	3.702314e+03	5.465761e+03	9.363197e+03	2.279236e+04	2.744808e+03	1.397019e+03	1.063683e+03	2.109452e+03	8.361316e+02	4.452830e+03	4.040541e+03	2.861954e+03
7D	med.	5.758959e+05	9.257741e+05	5.157778e+05	8.174321e+05	7.651943e+05	5.631466e+05	1.460292e+06	1.618442e+06	1.515949e+06	1.636875e+06	1.142412e+06	1.096667e+06	1.273273e+06
	avg.	5.781532e+05	9.320311e+05	5.275256e+05	8.233028e+05	7.861047e+05	5.643614e+05	1.459999e+06	1.618315e+06	1.512933e+06	1.637251e+06	1.138200e+06	1.099457e+06	1.215035e+06
	min.	4.749569e+05	7.301262e+05	3.410576e+05	7.805534e+05	6.813088e+05	4.917092e+05	1.367939e+06	1.575810e+06	1.139877e+06	1.591222e+06	9.356240e+05	1.014342e+06	6.496085e+05
	max.	6.848574e+05	1.133770e+06	6.874671e+05	1.148869e+06	1.691054e+06	6.604783e+05	1.523406e+06	1.671660e+06	1.598693e+06	1.676982e+06	1.467280e+06	1.289128e+06	1.481108e+06
	std.	3.556166e+04	9.074537e+04	5.168059e+04	4.450323e+04	1.289444e+05	3.557264e+04	3.117700e+04	2.096478e+04	5.016319e+04	1.635700e+04	1.197600e+05	3.915788e+04	2.341787e+05
8D	med.	9.296541e+06	1.337458e+07	8.169218e+06	1.390621e+07	1.384185e+07	9.512056e+06	2.655225e+07	2.827464e+07	2.403116e+07	2.815551e+07	1.918951e+07	1.752966e+07	2.007328e+07
	avg.	9.324613e+06	1.337349e+07	8.341145e+06	1.397680e+07	1.464813e+07	9.449651e+06	2.649272e+07	2.825463e+07	2.409941e+07	2.813432e+07	1.885493e+07	1.761584e+07	1.837050e+07
	min.	7.916147e+06	1.068787e+07	5.966225e+06	1.308321e+07	1.342127e+07	7.587994e+06	2.539712e+07	2.738672e+07	2.229417e+07	2.677226e+07	1.485032e+07	1.579089e+07	1.104695e+07
	max.	1.078135e+07	1.777074e+07	1.258902e+07	1.938983e+07	2.811992e+07	1.085002e+07	2.732889e+07	2.882212e+07	2.603136e+07	2.891979e+07	2.338487e+07	2.180354e+07	2.373431e+07
	std.	4.865645e+05	1.532617e+06	1.046179e+06	7.087609e+05	3.038526e+06	6.836613e+05	4.688279e+05	2.634557e+05	7.489747e+05	3.208167e+05	1.932117e+06	8.711573e+05	4.701845e+06
9D	med.	1.797243e+08	2.277564e+08	1.522483e+08	2.616315e+08	3.200054e+08	1.823990e+08	5.245650e+08	5.395201e+08	3.944129e+08	5.127369e+08	3.579743e+08	2.992875e+08	3.463672e+08
	avg.	1.794065e+08	2.274561e+08	1.506790e+08	2.607860e+08	3.2517368e+08	1.794530e+08	5.249169e+08	5.398921e+08	3.934499e+08	5.131126e+08	3.552834e+08	2.983975e+08	3.188133e+08
	min.	1.620398e+08	1.813144e+08	8.576120e+07	2.430622e+08	2.995552e+08	1.459607e+08	5.076285e+08	5.191176e+08	3.553208e+08	4.943923e+08	2.850316e+08	2.126582e+08	2.079309e+08
	max.	1.935352e+08	3.090712e+08	2.441470e+08	3.574412e+08	5.203575e+08	2.080311e+08	5.416467e+08	5.535201e+08	4.328732e+08	5.329118e+08	4.375812e+08	3.782036e+08	4.174641e+08
	std.	6.719862e+06	2.398846e+07	2.378363e+07	1.362119e+07	6.786068e+07	1.300977e+07	6.466826e+07	5.857796e+06	1.377885e+07	7.422836e+06	3.585370e+06	2.691810e+07	7.398673e+07
10D	med.	3.837978e+09	4.246700e+09	3.076697e+09	5.310315e+09	8.845401e+09	3.662930e+09	1.113458e+10	1.119161e+10	7.087531e+09	1.001971e+10	6.819535e+09	4.795547e+09	6.700709e+09
	avg.	3.832608e+09	4.364400e+09	3.053759e+09	5.320243e+09	8.888280e+09	3.630691e+09	1.113564e+10	1.118317e+10	7.093735e+09	1.001362e+10	6.766917e+09	4.875523e+09	6.397196e+09
	min.	3.528675e+09	3.571576e+09	1.329755e+09	4.987042e+09	8.010218e+09	2.861973e+09	1.077319e+09	1.056327e+10	6.586990e+09	9.576448e+09	4.923735e+09	4.177946e+09	4.297880e+09
	max.	4.129361e+09	5.663319e+09	4.248431e+09	6.444359e+09	1.001556e+10	4.201883e+09	1.148347e+10	1.147126e+10	7.537655e+09	1.045908e+10	8.176068e+09	6.441128e+09	8.108881e+09
	std.	1.304981e+08	5.610065e+08	5.054195e+08	2.094273e+08	4.208213e+08	3.024254e+08	1.317243e+08	1.608895e+08	2.160166e+08	1.681741e+08	6.598270e+08	7.228997e+08	1.200734e+09

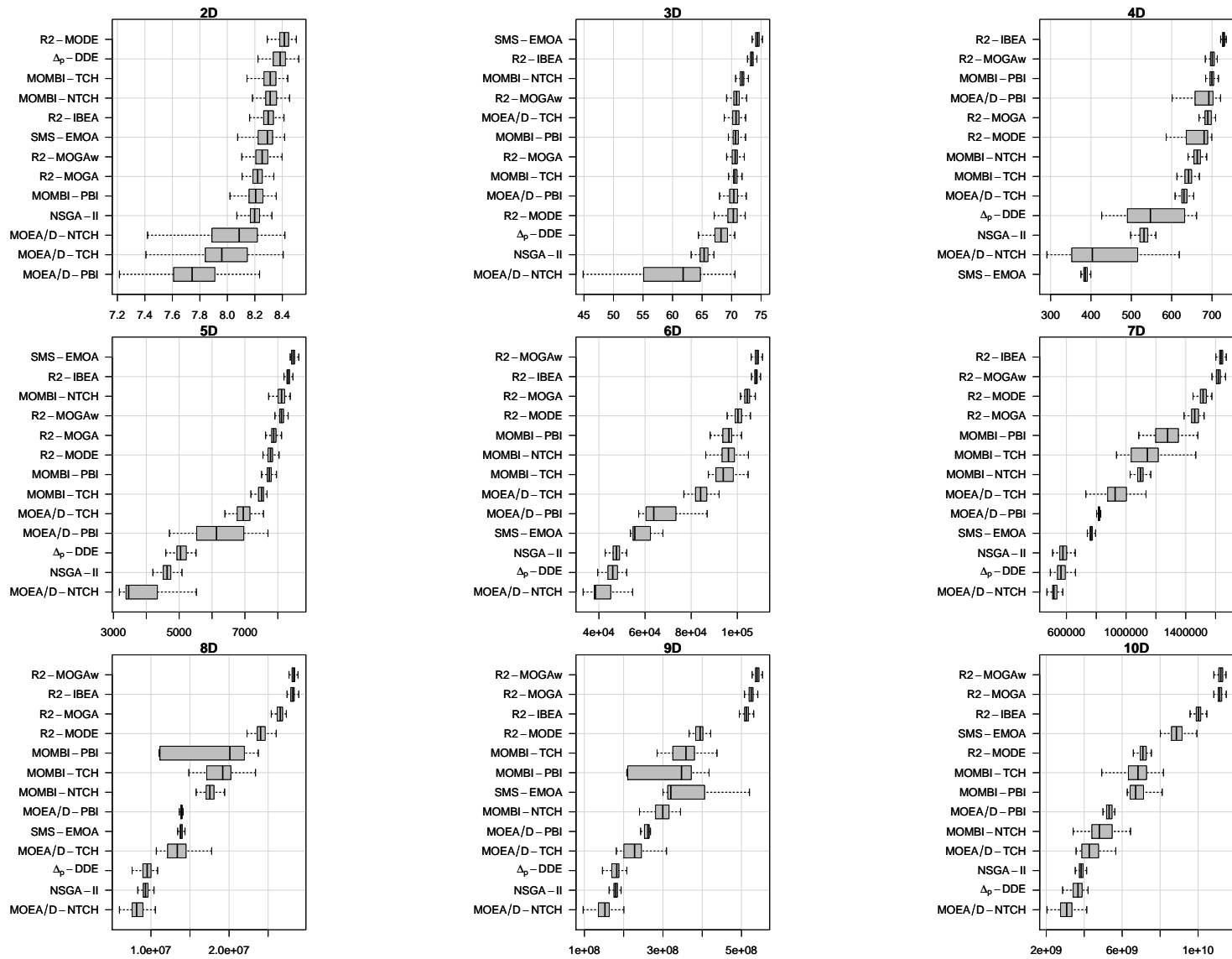


Figure A.193: Box-plot of hypervolume indicator values for different optimizers on the WFG6 test problem.

Table A.75: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG6 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D													
NSGA-II	—	1.86e-15	2.97e-08	6.23e-33	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	> 0.05	—	> 0.05	2.15e-11	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	5.21e-03	—	2.68e-17	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	2.64e-12	2.34e-22	2.11e-17	3.37e-33	—	> 0.05	3.14e-09	1.45e-03	> 0.05	> 0.05	> 0.05	> 0.05	1.17e-10
$\Delta_p$ -DDE	5.38e-29	3.86e-31	8.96e-30	1.53e-34	1.13e-18	—	5.25e-28	2.81e-23	> 0.05	4.58e-16	9.68e-12	2.70e-11	7.67e-29
R2-MOGA	3.58e-03	4.27e-17	1.42e-10	1.02e-32	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	8.58e-08	8.62e-20	6.57e-14	8.11e-33	> 0.05	> 0.05	4.06e-04	—	> 0.05	> 0.05	> 0.05	> 0.05	2.10e-05
R2-MODE	9.46e-34	2.51e-33	1.12e-32	1.28e-34	3.60e-28	2.87e-05	1.52e-33	8.77e-31	—	1.20e-26	5.90e-22	1.52e-22	1.13e-33
R2-IBEA	2.85e-19	1.86e-25	2.30e-21	3.65e-34	> 0.05	> 0.05	4.23e-16	5.22e-07	> 0.05	—	> 0.05	> 0.05	3.94e-17
MOMBI-TCH	9.84e-21	1.11e-26	5.28e-23	2.48e-34	1.56e-03	> 0.05	9.74e-18	2.45e-09	> 0.05	> 0.05	—	> 0.05	4.73e-19
MOMBI-NTCH	9.55e-24	1.57e-27	3.64e-24	1.84e-34	2.26e-04	> 0.05	1.35e-20	5.18e-11	> 0.05	1.09e-02	> 0.05	—	1.25e-21
MOMBI-PBI	> 0.05	3.61e-16	4.71e-09	7.43e-33	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—
3D													
NSGA-II	—	> 0.05	3.27e-12	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	5.54e-34	4.42e-03	> 0.05	1.41e-27	> 0.05	> 0.05	> 0.05	8.95e-05	> 0.05	4.67e-02	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	2.44e-33	> 0.05	7.43e-33	—	> 0.05	1.70e-20	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	1.28e-34	1.28e-34	1.28e-34	1.32e-34	—	1.28e-34	1.28e-34	1.28e-34	1.28e-34	2.40e-28	1.28e-34	1.32e-34	1.28e-34
$\Delta_p$ -DDE	6.23e-27	> 0.05	2.60e-27	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	> 0.05	5.22e-34	> 0.05	> 0.05	9.24e-27	—	> 0.05	4.06e-03	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	1.28e-34	> 0.05	3.99e-34	1.25e-04	> 0.05	3.20e-30	7.55e-03	—	1.52e-06	> 0.05	1.81e-03	> 0.05	> 0.05
R2-MODE	1.28e-34	> 0.05	4.52e-33	> 0.05	> 0.05	3.10e-18	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	1.28e-34	1.36e-34	1.28e-34	3.44e-34	> 0.05	1.28e-34	1.28e-34	1.44e-34	1.32e-34	—	1.28e-34	9.75e-34	1.32e-34
MOMBI-TCH	1.28e-34	> 0.05	6.06e-34	3.31e-02	> 0.05	3.87e-29	> 0.05	> 0.05	2.58e-03	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.28e-34	1.05e-19	1.28e-34	1.26e-20	> 0.05	1.28e-34	1.81e-24	3.18e-19	2.31e-26	> 0.05	2.47e-28	—	1.26e-22
MOMBI-PBI	1.28e-34	> 0.05	4.78e-34	2.64e-03	> 0.05	1.29e-29	> 0.05	> 0.05	8.52e-05	> 0.05	> 0.05	> 0.05	—
4D													
NSGA-II	—	> 0.05	7.14e-17	> 0.05	3.05e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	1.49e-34	> 0.05	1.28e-34	1.25e-10	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	3.08e-33	2.92e-19	6.63e-34	—	1.36e-34	5.86e-26	> 0.05	> 0.05	3.04e-06	> 0.05	1.71e-16	7.56e-09	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	> 0.05	> 0.05	8.76e-17	> 0.05	3.57e-33	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.28e-34	—	> 0.05	1.27e-10	> 0.05	1.32e-34	5.28e-30	> 0.05
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.20e-06	1.28e-34	1.28e-34	8.21e-15	—	1.74e-28	> 0.05	1.28e-34	1.49e-34	> 0.05
R2-MODE	8.16e-34	6.61e-13	3.55e-34	> 0.05	1.28e-34	1.06e-22	> 0.05	> 0.05	—	> 0.05	2.38e-08	1.11e-02	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.40e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.32e-34
MOMBI-TCH	1.28e-34	3.54e-08	1.36e-34	> 0.05	1.28e-34	7.38e-16	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.28e-34	1.37e-32	1.28e-34	> 0.05	1.28e-34	3.02e-30	> 0.05	> 0.05	> 0.05	> 0.05	2.91e-26	—	> 0.05
MOMBI-PBI	1.28e-34	1.28e-34	1.28e-34	1.02e-05	1.28e-34	1.28e-34	1.58e-13	> 0.05	3.15e-28	> 0.05	1.28e-34	1.40e-34	—

Table A.76: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG6 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
5D													
NSGA-II	–	> 0.05	1.47e – 19	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e – 34	–	1.28e – 34	2.20e – 10	> 0.05	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.50e – 32	> 0.05	1.89e – 32	–	> 0.05	1.98e – 22	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	7.44e – 17	3.75e – 16	6.68e – 28	3.75e – 16	–	3.75e – 16	3.75e – 16	3.75e – 16	3.75e – 16	2.35e – 15	3.75e – 16	3.83e – 16	3.75e – 16
$\Delta_p$ -DDE	8.10e – 27	> 0.05	7.99e – 26	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e – 34	1.28e – 34	1.28e – 34	1.49e – 34	> 0.05	1.28e – 34	–	> 0.05	9.59e – 10	> 0.05	3.15e – 34	> 0.05	5.58e – 17
R2-MOGAw	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	1.28e – 34	2.94e – 30	–	2.91e – 33	> 0.05	1.28e – 34	> 0.05	1.49e – 34
R2-MODE	1.28e – 34	2.90e – 28	1.28e – 34	3.86e – 31	> 0.05	1.28e – 34	> 0.05	> 0.05	–	> 0.05	1.34e – 24	> 0.05	3.94e – 02
R2-IBEA	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	> 0.05	1.28e – 34	1.28e – 34	1.19e – 32	1.28e – 34	–	1.28e – 34	6.60e – 24	1.28e – 34
MOMBI-TCH	1.28e – 34	1.04e – 31	1.28e – 34	5.51e – 32	> 0.05	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NTCH	1.28e – 34	1.28e – 34	1.28e – 34	1.32e – 34	> 0.05	1.28e – 34	9.22e – 20	> 0.05	6.01e – 26	> 0.05	1.49e – 34	–	2.16e – 28
MOMBI-PBI	1.28e – 34	1.32e – 34	1.28e – 34	3.65e – 34	> 0.05	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	2.63e – 30	> 0.05	–
6D													
NSGA-II	–	> 0.05	6.18e – 16	> 0.05	> 0.05	2.59e – 05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e – 34	–	1.28e – 34	3.19e – 22	3.95e – 12	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e – 34	> 0.05	1.36e – 34	–	1.56e – 10	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	1.28e – 34	> 0.05	2.23e – 33	> 0.05	–	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	> 0.05	> 0.05	2.06e – 12	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	3.63e – 12	1.28e – 34	–	> 0.05	3.58e – 26	> 0.05	5.51e – 32	5.00e – 30	1.63e – 34
R2-MOGAw	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	6.79e – 15	1.28e – 34	1.13e – 33	–	1.36e – 34	6.37e – 03	1.28e – 34	1.89e – 34	1.28e – 34
R2-MODE	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	3.95e – 12	1.28e – 34	> 0.05	> 0.05	–	> 0.05	2.77e – 17	1.79e – 14	7.99e – 26
R2-IBEA	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	2.00e – 14	1.28e – 34	7.69e – 34	> 0.05	1.28e – 34	–	1.28e – 34	1.84e – 34	1.28e – 34
MOMBI-TCH	1.28e – 34	3.89e – 32	1.28e – 34	1.28e – 34	3.95e – 12	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NTCH	1.28e – 34	1.52e – 33	1.28e – 34	1.36e – 34	3.82e – 12	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	1.98e – 03	–	> 0.05
MOMBI-PBI	1.28e – 34	1.71e – 33	1.28e – 34	1.44e – 34	3.95e – 12	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	1.53e – 02	> 0.05	–
7D													
NSGA-II	–	> 0.05	1.86e – 16	> 0.05	> 0.05	5.47e – 03	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e – 34	–	1.28e – 34	3.11e – 22	2.75e – 27	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e – 34	> 0.05	1.28e – 34	–	4.08e – 27	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	1.32e – 34	> 0.05	1.32e – 34	> 0.05	–	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	> 0.05	> 0.05	8.94e – 11	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	4.63e – 32	1.28e – 34	–	> 0.05	> 0.05	> 0.05	2.44e – 33	1.28e – 34	6.98e – 30
R2-MOGAw	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	2.92e – 32	1.28e – 34	1.28e – 34	–	2.33e – 34	> 0.05	1.28e – 34	1.28e – 34	1.28e – 34
R2-MODE	1.28e – 34	1.28e – 34	1.28e – 34	1.32e – 34	4.63e – 32	1.28e – 34	4.77e – 22	> 0.05	–	> 0.05	8.40e – 34	1.78e – 34	5.23e – 33
R2-IBEA	1.28e – 34	1.28e – 34	1.28e – 34	1.28e – 34	1.22e – 32	1.28e – 34	1.28e – 34	4.37e – 11	1.32e – 34	–	1.28e – 34	1.28e – 34	1.28e – 34
MOMBI-TCH	1.28e – 34	8.75e – 25	1.28e – 34	2.17e – 33	4.63e – 32	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	–	8.46e – 03	> 0.05
MOMBI-NTCH	1.28e – 34	1.64e – 28	1.28e – 34	1.29e – 32	4.63e – 32	1.28e – 34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	3.65e – 34	9.64e – 20	8.92e – 34	8.62e – 20	3.52e – 18	1.89e – 34	> 0.05	> 0.05	> 0.05	> 0.05	1.00e – 09	4.36e – 18	–

Table A.77: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG6 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
8D													
NSGA-II	—	> 0.05	$5.71e-16$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$1.40e-34$	—	$4.92e-34$	> 0.05	> 0.05	$1.40e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$1.28e-34$	$2.64e-03$	$1.28e-34$	—	> 0.05	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	$1.28e-34$	$3.65e-04$	$1.28e-34$	> 0.05	—	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	$4.94e-02$	> 0.05	$3.83e-16$	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.59e-31$	$1.28e-34$	—	> 0.05	$4.78e-34$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MOGAw	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$3.24e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.50e-03$	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MODE	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.60e-27$	$1.28e-34$	> 0.05	> 0.05	—	> 0.05	$2.26e-34$	$1.28e-34$	$2.83e-32$
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$5.54e-34$	$1.28e-34$	$3.34e-34$	> 0.05	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$4.26e-33$	$1.28e-34$	$1.87e-33$	$3.13e-27$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	—	$4.23e-07$	> 0.05
MOMBI-NTCH	$1.28e-34$	$2.66e-33$	$1.28e-34$	$6.23e-33$	$2.97e-27$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	$1.28e-34$	$4.14e-09$	$6.83e-34$	$4.34e-08$	$7.85e-06$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	$2.12e-02$	$1.67e-07$	—
9D													
NSGA-II	—	> 0.05	$7.05e-23$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$2.10e-33$	—	$5.08e-33$	> 0.05	> 0.05	$1.88e-30$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$1.28e-34$	$1.42e-17$	$1.36e-34$	—	> 0.05	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	$1.28e-34$	$1.78e-34$	$1.28e-34$	$1.66e-33$	—	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.50e-14$	> 0.05
$\Delta_p$ -DDE	> 0.05	> 0.05	$3.21e-20$	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.96e-34$	$1.28e-34$	—	> 0.05	$1.28e-34$	$8.97e-21$	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MOGAw	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.32e-34$	$1.28e-34$	$4.56e-29$	—	$1.28e-34$	$4.50e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MODE	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.32e-34$	$5.15e-10$	$1.28e-34$	> 0.05	> 0.05	—	> 0.05	$3.78e-17$	$2.07e-34$	$1.80e-19$
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.74e-33$	$1.28e-34$	> 0.05	> 0.05	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$3.24e-34$	$1.28e-34$	$9.19e-34$	$3.56e-03$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	—	$8.21e-22$	$5.47e-03$
MOMBI-NTCH	$1.28e-34$	$7.88e-29$	$1.40e-34$	$1.01e-22$	> 0.05	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	$1.28e-34$	$6.82e-14$	$3.05e-34$	$2.33e-07$	> 0.05	$1.32e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$3.82e-06$	—
10D													
NSGA-II	—	> 0.05	$5.15e-25$	> 0.05	> 0.05	$8.33e-07$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$2.04e-13$	—	$1.88e-30$	> 0.05	> 0.05	$2.23e-20$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$1.28e-34$	$1.45e-24$	$1.28e-34$	—	> 0.05	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.38e-06$	> 0.05
SMS-EMOA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	> 0.05	> 0.05	$1.28e-34$	> 0.05	$1.49e-34$	$1.28e-34$	$1.40e-34$
$\Delta_p$ -DDE	> 0.05	> 0.05	$7.29e-17$	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MOGAw	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.89e-03$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MODE	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05	> 0.05	—	> 0.05	$9.96e-05$	$1.28e-34$	$1.61e-07$
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.30e-33$	$1.28e-34$	> 0.05	> 0.05	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$3.65e-34$	$1.28e-34$	$1.55e-31$	> 0.05	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	—	$4.57e-31$	> 0.05
MOMBI-NTCH	$9.32e-24$	$1.10e-07$	$1.54e-32$	> 0.05	> 0.05	$2.47e-27$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	$1.28e-34$	$2.18e-22$	$1.28e-34$	$2.18e-11$	> 0.05	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.65e-15$	—

Table A.78: Comparison of R2 indicator values for different optimizers on the WFG6 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	6.062184e-01	6.216965e-01	6.108048e-01	6.372144e-01	6.024985e-01	5.988759e-01	6.058917e-01	6.045430e-01	5.985769e-01	6.026027e-01	6.019824e-01	6.025599e-01	6.065004e-01
	avg.	6.065248e-01	6.237660e-01	6.170947e-01	6.402068e-01	6.037343e-01	5.995661e-01	6.062890e-01	6.052932e-01	5.987867e-01	6.028407e-01	6.023452e-01	6.024500e-01	6.067942e-01
	min.	5.988898e-01	5.974269e-01	5.970769e-01	6.055323e-01	5.975398e-01	5.927496e-01	5.993385e-01	5.989279e-01	5.950880e-01	5.961169e-01	5.950415e-01	5.965523e-01	6.001705e-01
	max.	6.217767e-01	6.956465e-01	6.828979e-01	7.086830e-01	6.307495e-01	6.099887e-01	6.255286e-01	6.292535e-01	6.051197e-01	6.236637e-01	6.150225e-01	6.117761e-01	6.215495e-01
	std.	3.267805e-03	1.846061e-02	1.576594e-02	2.170480e-02	4.763326e-03	3.052958e-03	3.956395e-03	4.588026e-03	2.083945e-03	3.149571e-03	3.059542e-03	2.554547e-03	3.221174e-03
3D	med.	3.922180e-01	3.237909e-01	4.135694e-01	3.321472e-01	3.188129e-01	3.474404e-01	3.261551e-01	3.247710e-01	3.283607e-01	3.227037e-01	3.239715e-01	3.239969e-01	3.293845e-01
	avg.	3.917716e-01	3.239476e-01	4.295851e-01	3.338134e-01	3.188548e-01	3.485030e-01	3.266168e-01	3.247640e-01	3.288012e-01	3.226823e-01	3.241230e-01	3.240376e-01	3.294457e-01
	min.	3.600506e-01	3.194804e-01	3.314364e-01	3.231264e-01	3.166958e-01	3.332667e-01	3.230210e-01	3.203568e-01	3.227428e-01	3.193935e-01	3.208658e-01	3.200938e-01	3.239392e-01
	max.	4.381538e-01	3.308422e-01	7.168234e-01	3.805302e-01	3.226383e-01	3.721566e-01	3.307581e-01	3.288760e-01	3.392639e-01	3.258488e-01	3.274737e-01	3.272653e-01	3.341481e-01
	std.	1.702936e-02	2.250142e-03	9.779642e-02	7.589051e-03	1.110941e-03	8.348595e-03	1.825397e-03	1.804619e-03	3.194579e-03	1.198958e-03	1.406202e-03	1.518758e-03	2.065983e-03
4D	med.	3.809937e-01	2.281994e-01	6.041542e-01	2.317523e-01	4.339845e-01	2.900438e-01	2.176628e-01	2.176038e-01	2.207749e-01	2.191134e-01	2.276226e-01	2.267836e-01	2.215449e-01
	avg.	3.821512e-01	2.281901e-01	5.701786e-01	2.415784e-01	4.295963e-01	2.934882e-01	2.178442e-01	2.177256e-01	2.264736e-01	2.190945e-01	2.277049e-01	2.266114e-01	2.216679e-01
	min.	3.358453e-01	2.256378e-01	2.402110e-01	2.160507e-01	3.150261e-01	2.319602e-01	2.144155e-01	2.138111e-01	2.176939e-01	2.163826e-01	2.254358e-01	2.219006e-01	2.173570e-01
	max.	4.277813e-01	2.319143e-01	8.043549e-01	3.644087e-01	4.541226e-01	3.737299e-01	2.212308e-01	2.239590e-01	2.668918e-01	2.236065e-01	2.306526e-01	2.308126e-01	2.262437e-01
	std.	1.996858e-02	1.139482e-03	1.946439e-01	2.373151e-02	2.212833e-02	4.485156e-02	1.512951e-03	2.017098e-03	1.059797e-02	1.223493e-03	1.049402e-03	2.149069e-03	1.696958e-03
5D	med.	4.226521e-01	1.806087e-01	7.904191e-01	2.334239e-01	1.686053e-01	3.126552e-01	1.669826e-01	1.667437e-01	1.686211e-01	1.725915e-01	1.760699e-01	1.699234e-01	1.850849e-01
	avg.	4.207410e-01	1.834703e-01	6.569407e-01	2.427331e-01	1.985086e-01	3.114674e-01	1.672700e-01	1.674317e-01	1.702773e-01	1.728373e-01	1.762421e-01	1.700671e-01	1.851476e-01
	min.	3.640553e-01	1.760171e-01	2.484277e-01	1.876513e-01	1.374988e-01	2.796590e-01	1.646320e-01	1.639039e-01	1.649487e-01	1.687777e-01	1.741009e-01	1.666131e-01	1.796659e-01
	max.	4.700529e-01	2.147907e-01	8.075484e-01	3.253830e-01	3.557783e-01	3.401597e-01	1.720939e-01	1.758647e-01	2.171991e-01	1.776817e-01	1.795445e-01	1.748822e-01	1.895051e-01
	std.	2.224653e-02	7.938291e-03	1.685675e-01	3.513649e-02	6.766091e-02	1.255247e-02	1.468906e-03	2.266366e-03	8.127402e-03	1.820096e-03	1.254712e-03	1.731555e-03	1.890629e-03
6D	med.	4.478990e-01	1.927762e-01	8.169766e-01	2.520005e-01	2.863581e-01	3.420897e-01	1.247725e-01	1.264278e-01	1.282819e-01	1.384340e-01	1.655373e-01	1.807980e-01	1.487673e-01
	avg.	4.469288e-01	1.968067e-01	7.293677e-01	2.417495e-01	2.523663e-01	3.416636e-01	1.251924e-01	1.269463e-01	1.283947e-01	1.387513e-01	1.695000e-01	1.875890e-01	1.539756e-01
	min.	3.974776e-01	1.490540e-01	3.558727e-01	1.851132e-01	1.268516e-01	3.185822e-01	1.225387e-01	1.231055e-01	1.241446e-01	1.334722e-01	1.297457e-01	1.309264e-01	1.422452e-01
	max.	5.035632e-01	2.376126e-01	8.397794e-01	2.853393e-01	3.008727e-01	3.652974e-01	1.337395e-01	1.343335e-01	1.357386e-01	1.451315e-01	2.284631e-01	2.269945e-01	1.820538e-01
	std.	2.004078e-02	2.323646e-02	1.361631e-01	3.331741e-02	6.482255e-02	9.890282e-03	1.964607e-03	2.398512e-03	2.009857e-03	2.554336e-03	2.551174e-02	8.731300e-03	8.731300e-03
7D	med.	4.750445e-01	3.237540e-01	8.430194e-01	2.512203e-01	2.739655e-01	3.578494e-01	1.195931e-01	1.110871e-01	1.151421e-01	1.192495e-01	2.737100e-01	3.452310e-01	1.490360e-01
	avg.	4.771432e-01	3.175739e-01	7.742881e-01	2.498710e-01	2.699826e-01	3.580977e-01	1.200465e-01	1.115009e-01	1.148371e-01	1.200333e-01	2.744810e-01	3.374866e-01	1.693837e-01
	min.	4.117515e-01	1.953284e-01	3.720497e-01	1.698386e-01	1.119381e-01	3.364708e-01	1.126826e-01	1.046777e-01	1.093310e-01	1.140358e-01	1.557628e-01	1.979462e-01	1.272354e-01
	max.	5.317581e-01	4.860984e-01	9.014235e-01	2.611485e-01	2.931531e-01	3.769220e-01	1.295870e-01	1.235005e-01	1.439215e-01	1.413180e-01	3.802692e-01	3.773917e-01	2.891316e-01
	std.	2.274746e-02	5.118345e-02	1.271465e-01	1.143324e-02	2.385093e-02	9.840525e-03	3.737951e-03	3.955471e-03	4.692870e-03	3.876500e-03	5.184548e-02	3.240466e-02	4.831181e-02
8D	med.	4.449091e-01	3.835761e-01	8.566213e-01	2.196643e-01	2.181823e-01	3.217311e-01	9.312884e-02	9.214389e-02	1.000434e-01	1.061944e-01	2.857534e-01	3.651011e-01	1.474663e-01
	avg.	4.471704e-01	3.753922e-01	7.366249e-01	2.186917e-01	2.118586e-01	3.220656e-01	9.389536e-02	9.242904e-02	9.977216e-02	1.068509e-01	2.888547e-01	3.603583e-01	1.682188e-01
	min.	3.986336e-01	2.410337e-01	3.906455e-01	1.480092e-01	1.005846e-01	3.010042e-01	8.922476e-02	8.844789e-02	9.252088e-02	1.010230e-01	1.644931e-01	2.366933e-01	1.143654e-01
	max.	4.892466e-01	4.385279e-01	9.012469e-01	2.308493e-01	2.28058e-01	3.427994e-01	1.024224e-01	9.716536e-02	1.073126e-01	1.261068e-01	3.814242e-01	3.972334e-01	2.528528e-01
	std.	1.685694e-02	3.574146e-02	1.578663e-01	9.907290e-03	2.710351e-02	8.236168e-03	3.004526e-03	2.050910e-03	3.077189e-03	3.597288e-03	4.669435e-02	2.742605e-02	5.383574e-02
9D	med.	4.117352e-01	4.089710e-01	6.117648e-01	1.965836e-01	1.601749e-01	2.952659e-01	7.637991e-02	7.870373e-02	1.082747e-01	1.083392e-01	2.991961e-01	3.838746e-01	1.372870e-01
	avg.	4.121916e-01	4.092249e-01	6.802822e-01	1.972763e-01	1.464433e-01	2.947535e-01	7.680272e-02	7.879539e-02	1.088463e-01	1.083208e-01	3.006155e-01	3.761670e-01	1.581220e-01
	min.	3.629754e-01	3.230525e-01	4.112920e-01	1.348806e-01	9.278793e-02	2.802414e-01	7.407747e-02	7.599281e-02	9.831802e-02	9.506907e-02	1.769106e-01	2.245557e-01	1.099941e-01
	max.	4.468800e-01	5.399526e-01	9.167643e-01	2.082344e-01	1.734837e-01	3.067693e-01	8.141290e-02	8.322336e-02	1.216581e-01	1.173850e-01	4.169249e-01	4.189033e-01	2.252796e-01
	std.	1.441315e-02	3.149910e-02	1.440014e-01	8.936074e-03	2.742681e-02	5.548244e-03	1.566999e-03	1.509272e-03	4.831853e-03	4.269283e-03	5.245776e-02	3.449174e-02	4.382146e-02
10D	med.	3.848576e-01	4.330214e-01	5.944731e-01	1.816210e-01	1.069138e-01	2.716625e-01	6.688605e-02	7.118173e-02	1.262206e-01	1.193967e-01	3.077512e-01	3.883424e-01	1.290840e-01
	avg.	3.852852e-01	4.331952e-01	6.462034e-01	1.814206e-01	1.083943e-01	2.721279e-01	6.715032e-02	7.134066e-02	1.265972e-01	1.201852e-01	3.651942e-01	4.421231e-01	1.421231e-01
	min.	3.578743e-01	3.231600e-01	4.217907e-01	1.461359e-01	9.106430e-02	2.609995e-01	6.570169e-02	6.862308e-02	1.145962e-01	1.070553e-01	2.125865e-01	2.641389e-01	1.014482e-01
	max.	4.248171e-01	5.064916e-01	9.431864e-01	1.899377e-01	1.267769e-01	2.919519e-01	7.147659e-02	7.662786e-02	1.426304e-01	1.374264e-01	4.158188e-01	4.289426e-01	2.035997e-01
	std.	1.269199e-02	2.647878e-02	1.329147e-01	5.922809e-03	8.644188e-03	5.679097e-03	1.102176e-03	1.485585e-03	5.302720e-03	5.671050e-03	4.329998e-02	4.620799e-02	3.429514e-02

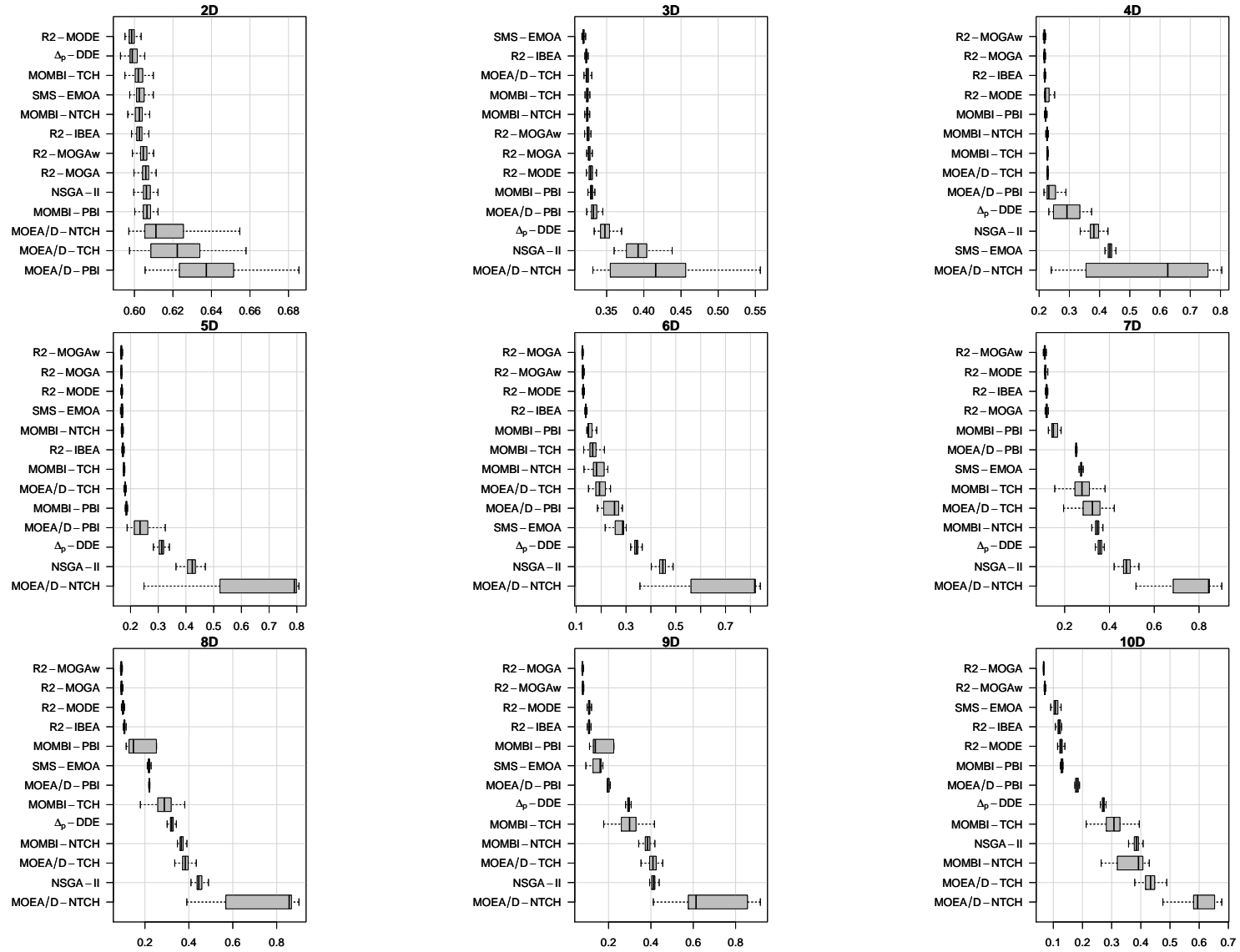


Figure A.194: Box-plot of R2 indicator values for different optimizers on the WFG6 test problem.

Table A.79: Comparison of runtime (in milliseconds) for different optimizers on the WFG6 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_\mu$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	4.260210e + 02	8.953010e + 02	1.411445e + 03	1.334311e + 03	3.358123e + 04	1.647387e + 03	2.131921e + 03	1.674300e + 03	2.105040e + 03	4.592690e + 05	2.168191e + 03	2.022995e + 03	2.973893e + 03
	avg.	4.395059e + 02	8.986296e + 02	1.410403e + 03	1.324508e + 03	3.243114e + 04	1.645415e + 03	2.087149e + 03	1.680750e + 03	2.080520e + 03	4.803539e + 05	2.143049e + 03	2.060602e + 03	2.981165e + 03
	min.	3.692130e + 02	6.858850e + 02	1.099960e + 03	9.433090e + 02	2.137271e + 04	1.294461e + 03	1.488596e + 03	1.265013e + 03	1.461461e + 03	4.578870e + 05	1.652061e + 03	1.770317e + 03	2.449307e + 03
	max.	5.915300e + 02	1.215739e + 03	1.917658e + 03	1.993190e + 03	3.602126e + 04	2.177872e + 03	2.690555e + 03	2.061605e + 03	2.585888e + 03	5.677980e + 05	2.495849e + 03	2.424911e + 03	3.333691e + 03
	std.	5.157592e + 01	9.536621e + 01	1.377336e + 02	1.250858e + 02	2.980795e + 03	1.525732e + 02	2.819774e + 02	1.694544e + 02	2.748557e + 02	3.892710e + 04	2.024720e + 02	1.533840e + 02	1.695442e + 02
3D	med.	5.976090e + 02	9.302490e + 02	1.744093e + 03	1.401218e + 03	8.503595e + 05	1.938522e + 03	2.341584e + 03	2.039762e + 03	2.611465e + 03	5.658520e + 05	2.246939e + 03	2.076991e + 03	3.184828e + 03
	avg.	5.963015e + 02	9.272620e + 02	1.719928e + 03	1.400295e + 03	8.397337e + 05	1.923307e + 03	2.351791e + 03	2.017189e + 03	2.553396e + 03	5.640358e + 05	2.244412e + 03	2.098161e + 03	3.183973e + 03
	min.	4.981980e + 02	7.581100e + 02	1.218626e + 03	1.163890e + 03	6.428398e + 05	1.494794e + 03	1.785486e + 03	1.475134e + 03	1.721310e + 03	5.156750e + 05	1.887595e + 03	1.835357e + 03	2.696688e + 03
	max.	7.486940e + 02	1.293383e + 03	2.218250e + 03	1.710235e + 03	9.448498e + 05	2.344571e + 03	2.811905e + 03	2.356470e + 03	3.076853e + 03	5.868700e + 05	2.745007e + 03	3.138499e + 03	3.595442e + 03
	std.	5.904755e + 01	9.391088e + 01	1.497021e + 02	1.011912e + 02	6.173272e + 04	1.884142e + 02	2.351782e + 02	1.789250e + 02	2.919061e + 02	1.522140e + 04	1.625057e + 02	1.723493e + 02	1.687575e + 02
4D	med.	9.393500e + 02	1.235276e + 03	2.049834e + 03	1.806282e + 03	2.786360e + 06	2.705464e + 03	2.851021e + 03	2.597451e + 03	3.691463e + 03	5.936210e + 05	2.392028e + 03	2.743419e + 03	3.725613e + 03
	avg.	9.394324e + 02	1.226350e + 03	2.047916e + 03	1.763726e + 03	2.699613e + 06	2.680981e + 03	2.877255e + 03	2.647222e + 03	3.605888e + 03	5.934876e + 05	2.457979e + 03	2.713367e + 03	3.744635e + 03
	min.	7.703250e + 02	9.913190e + 02	1.397658e + 03	1.443461e + 03	1.827589e + 06	1.716216e + 03	2.099525e + 03	2.002281e + 03	2.504759e + 03	5.634800e + 05	1.963055e + 03	2.183819e + 03	3.315291e + 03
	max.	1.141140e + 03	1.482276e + 03	2.681254e + 03	1.978000e + 03	3.022353e + 06	3.178143e + 03	3.751413e + 03	3.237325e + 03	4.611322e + 03	6.022980e + 05	3.662411e + 03	3.162653e + 03	4.199709e + 03
	std.	8.690843e + 01	1.008560e + 02	1.524609e + 02	1.199897e + 02	2.628631e + 05	2.708009e + 02	3.811060e + 02	2.627174e + 02	4.624658e + 02	4.303513e + 03	3.175108e + 02	1.706526e + 02	1.894709e + 02
5D	med.	1.302472e + 03	1.569027e + 03	2.722364e + 03	1.855752e + 03	2.338290e + 06	6.096329e + 03	3.847701e + 03	3.501393e + 03	5.064981e + 03	6.739010e + 05	2.888132e + 03	3.169412e + 03	4.414423e + 03
	avg.	1.303612e + 03	1.558608e + 03	2.693867e + 03	1.860244e + 03	2.530093e + 06	5.967462e + 03	3.877351e + 03	3.532018e + 03	5.028640e + 03	6.736726e + 05	2.956192e + 03	3.190493e + 03	4.390977e + 03
	min.	1.079534e + 03	1.270397e + 03	1.966429e + 03	1.517385e + 03	1.698708e + 06	3.765863e + 03	3.808465e + 03	2.763761e + 03	3.478967e + 03	6.584900e + 05	2.477082e + 03	2.702743e + 03	3.450797e + 03
	max.	1.493815e + 03	2.070098e + 03	3.470897e + 03	2.328708e + 03	4.295610e + 06	7.471471e + 03	4.572344e + 03	4.222398e + 03	5.946901e + 03	6.818370e + 05	4.347144e + 03	3.750948e + 03	4.895048e + 03
	std.	1.017293e + 02	1.233542e + 02	2.179997e + 02	1.831364e + 02	7.141047e + 05	8.289009e + 02	3.661746e + 02	3.199221e + 02	4.825007e + 02	3.335950e + 03	3.144224e + 02	2.106607e + 02	2.110195e + 02
6D	med.	1.709506e + 03	1.901288e + 03	3.041464e + 03	2.344995e + 03	3.973586e + 06	6.563831e + 03	4.804252e + 03	4.480372e + 03	6.423705e + 03	6.536820e + 05	3.065600e + 03	3.643302e + 03	4.902583e + 03
	avg.	1.707543e + 03	1.900680e + 03	3.018665e + 03	2.293787e + 03	3.672377e + 06	6.510618e + 03	4.820797e + 03	4.451784e + 03	6.366915e + 03	6.816006e + 05	3.152202e + 03	3.632604e + 03	4.901172e + 03
	min.	1.480025e + 03	1.612557e + 03	2.613442e + 03	1.792484e + 03	1.979064e + 06	3.826261e + 03	3.418958e + 03	3.497627e + 03	4.832643e + 03	5.962450e + 05	2.603195e + 03	3.062624e + 03	4.123556e + 03
	max.	1.978183e + 03	2.537881e + 03	3.711564e + 03	2.752182e + 03	4.384563e + 06	8.996266e + 03	5.706501e + 03	5.137111e + 03	7.751965e + 03	7.283290e + 05	4.828924e + 03	4.245069e + 03	5.665665e + 03
	std.	1.178596e + 02	1.482716e + 02	2.061637e + 02	2.242537e + 02	7.350421e + 05	8.838121e + 02	4.605219e + 02	3.466159e + 02	6.660609e + 02	3.672778e + 04	3.582302e + 02	2.291333e + 02	2.884617e + 02
7D	med.	1.944261e + 03	2.237958e + 03	3.498721e + 03	2.484007e + 03	2.871914e + 06	1.681875e + 04	4.999370e + 03	4.446596e + 03	5.523112e + 03	2.968900e + 05	2.827936e + 03	3.241196e + 03	4.264598e + 03
	avg.	1.953004e + 03	2.233800e + 03	3.464543e + 03	2.476216e + 03	2.764092e + 06	1.689467e + 04	4.909898e + 03	4.435397e + 03	5.585036e + 03	3.005482e + 05	2.808006e + 03	3.274218e + 03	4.249224e + 03
	min.	1.612576e + 03	1.795756e + 03	2.466139e + 03	1.972361e + 03	1.811271e + 06	1.026722e + 04	3.488386e + 03	3.529460e + 03	4.147927e + 03	2.727160e + 05	2.236740e + 03	2.733108e + 03	3.387033e + 03
	max.	2.225433e + 03	2.677328e + 03	4.081261e + 03	2.964444e + 03	3.061467e + 06	2.569360e + 04	5.564748e + 03	5.201310e + 03	6.617901e + 03	3.656010e + 05	3.196141e + 03	3.802907e + 03	4.875555e + 03
	std.	1.372500e + 02	1.898438e + 02	2.077027e + 02	2.516520e + 02	2.769146e + 05	3.101953e + 04	4.775460e + 02	3.480308e + 02	5.069704e + 02	3.041829e + 04	1.964091e + 02	2.142634e + 02	2.870708e + 02
8D	med.	2.618596e + 03	2.705788e + 03	4.068432e + 03	2.914851e + 03	4.575500e + 06	3.743680e + 04	6.888323e + 03	6.396943e + 03	9.104181e + 03	1.275520e + 06	3.914613e + 03	4.734092e + 03	6.201692e + 03
	avg.	2.641206e + 03	2.705572e + 03	4.006208e + 03	2.893294e + 03	4.415153e + 06	3.773804e + 04	6.864282e + 03	6.367607e + 03	8.994260e + 03	1.245010e + 06	3.952180e + 03	4.758552e + 03	6.219122e + 03
	min.	2.107183e + 03	2.290890e + 03	2.971693e + 03	2.268134e + 03	2.582362e + 06	1.958333e + 04	4.906912e + 03	4.709733e + 03	6.658829e + 03	1.171073e + 06	3.129274e + 03	4.016259e + 03	4.965723e + 03
	max.	3.002776e + 03	3.212372e + 03	4.474423e + 03	3.535347e + 03	5.090239e + 06	5.795859e + 04	7.981276e + 03	7.418536e + 03	1.029459e + 04	1.326490e + 06	4.499148e + 03	5.296891e + 03	7.124517e + 03
	std.	1.766329e + 02	1.878453e + 02	2.742932e + 02	2.970782e + 02	5.248488e + 05	7.677428e + 03	7.453802e + 02	5.859986e + 02	8.736229e + 02	5.161654e + 04	2.837466e + 02	2.968299e + 02	3.819344e + 02
9D	med.	3.481459e + 03	3.278051e + 03	5.036689e + 03	3.526710e + 03	6.304984e + 06	6.876309e + 04	8.645990e + 03	8.924431e + 03	1.181394e + 04	3.225605e + 06	5.447668e + 03	6.784035e + 03	8.582903e + 03
	avg.	3.491934e + 03	3.278521e + 03	4.908700e + 03	3.453421e + 03	6.062075e + 06	6.994497e + 04	8.519990e + 03	8.872024e + 03	1.176783e + 04	3.010553e + 06	5.420757e + 03	6.629453e + 03	8.614265e + 03
	min.	2.942473e + 03	2.587419e + 03	3.978305e + 03	2.651514e + 03	3.079247e + 06	4.089600e + 04	6.323871e + 03	6.649150e + 03	8.168168e + 03	2.262864e + 06	4.521135e + 03	5.283312e + 03	7.067459e + 03
	max.	3.945298e + 03	3.822850e + 03	5.292356e + 03	3.884668e + 03	8.352401e + 06	1.199108e + 05	1.098646e + 04	1.023089e + 04	1.380935e + 04	3.294167e + 06	5.949655e + 03	7.310248e + 03	1.049518e + 04
	std.	2.185095e + 02	2.362789e + 02	3.114853e + 02	3.251412e + 02	1.444130e + 06	1.123997e + 04	1.390041e + 03	7.454686e + 02	1.282667e + 03	3.464957e + 05	3.408980e + 02	4.235726e + 02	5.202592e + 02
10D	med.	4.021007e + 03	3.236706e + 03	5.645391e + 03	3.978054e + 03	7.214859e + 06	1.404293e + 05	8.991639e + 03	1.186563e + 04	1.065650e + 04	4.954626e + 06	5.574573e + 03	6.960842e + 03	1.123257e + 04
	avg.	4.103516e + 03	3.302589e + 03	5.501508e + 03	4.014456e + 03	7.174186e + 06	1.426605e + 05	1.001328e + 04	1.155340e + 04	1.310501e + 04	4.795340e + 06	5.838143e + 03	7.391842e + 03	1.120606e + 04
	min.	3.558855e + 03	2.810536e + 03	3.934810e + 03	3.174255e + 03	5.171000e + 06	9.541271e + 04	8.882904e + 03	8.975969e + 03	9.549429e + 03	4.263310e + 06	5.281597e + 03	6.540305e + 03	9.277416e + 03
	max.	4.998232e + 03	4.327607e + 03	6.250688e + 03	4.618862e + 03	9.021323e + 06	2.014981e + 05	1.492098e + 04	1.367848e + 04	2.015281e + 04	5.355692e + 06	7.357782e + 03	9.919206e + 03	1.193664e + 04
	std.	5.152638e + 02	3.407750e + 02	4.448886e + 02	3.882887e + 02	8.678081e + 05	2.090722e + 04	1.939401e + 03	1.244643e + 03	4.041456e + 03	2.991257e + 05	5.576122e + 02	9.874067e + 02	4.739053e + 02



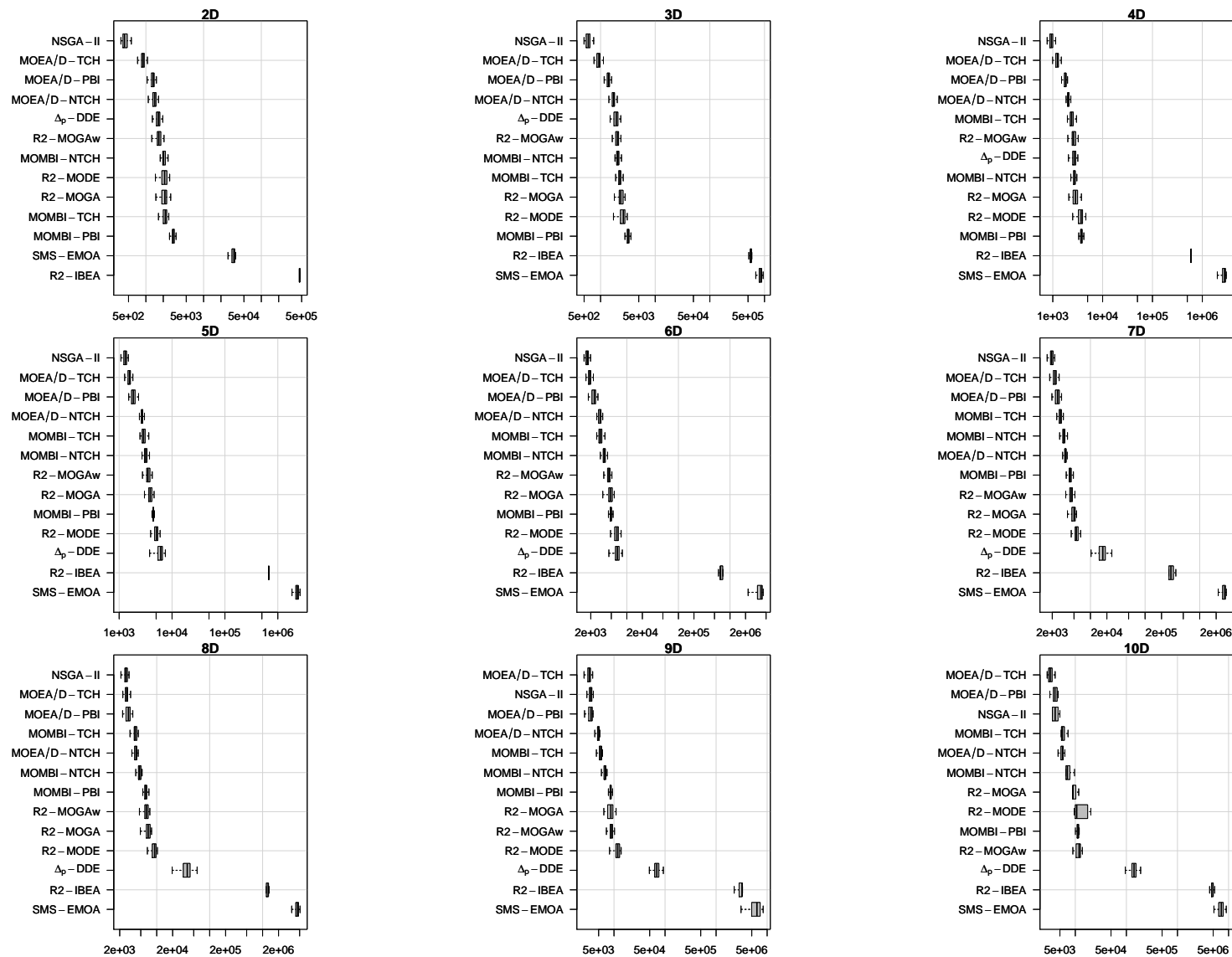


Figure A.195: Box-plot of runtime (in logarithmic scale) for different optimizers on the WFG6 test problem.

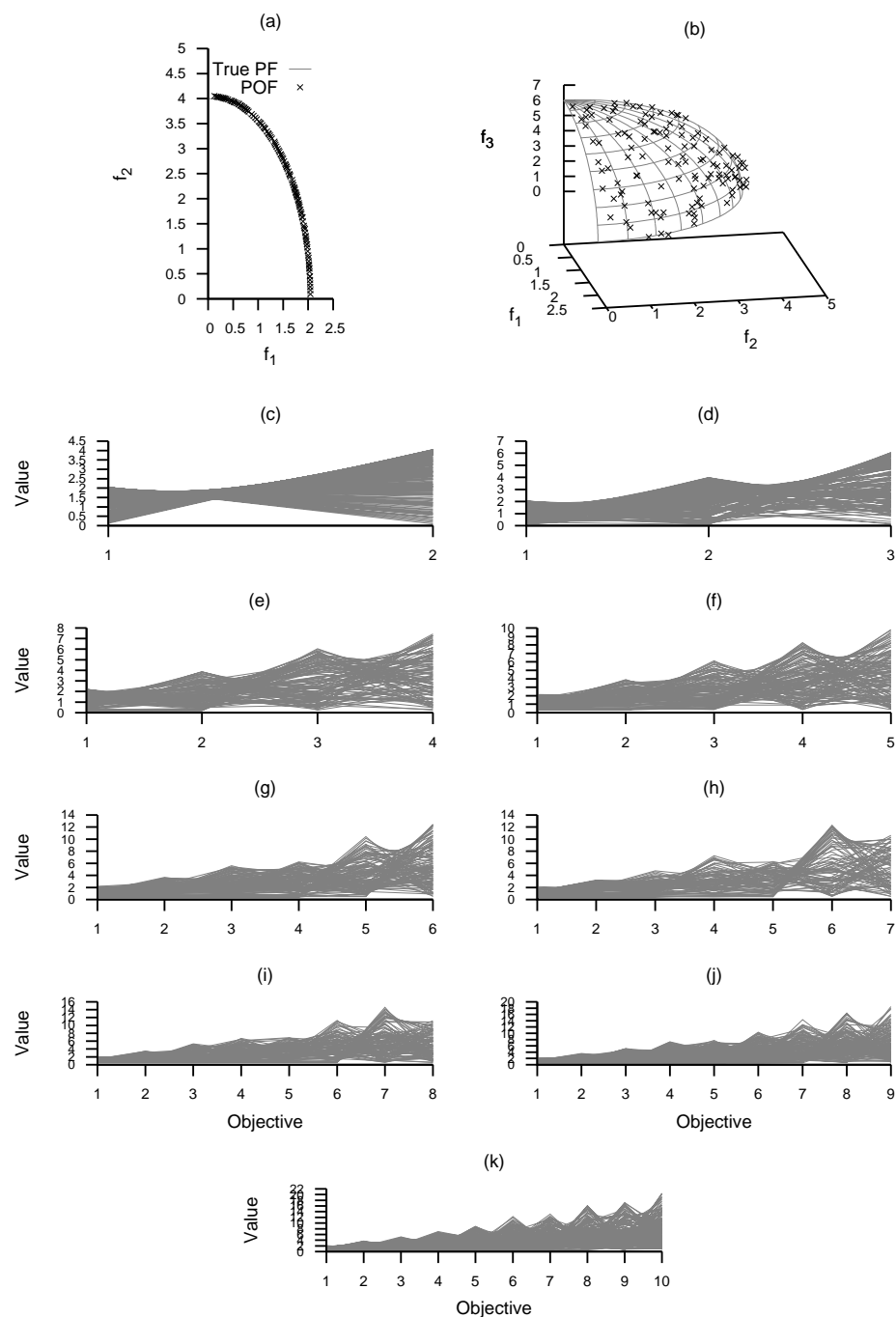


Figure A.196: Plots of the approximations obtained by NSGA-II from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

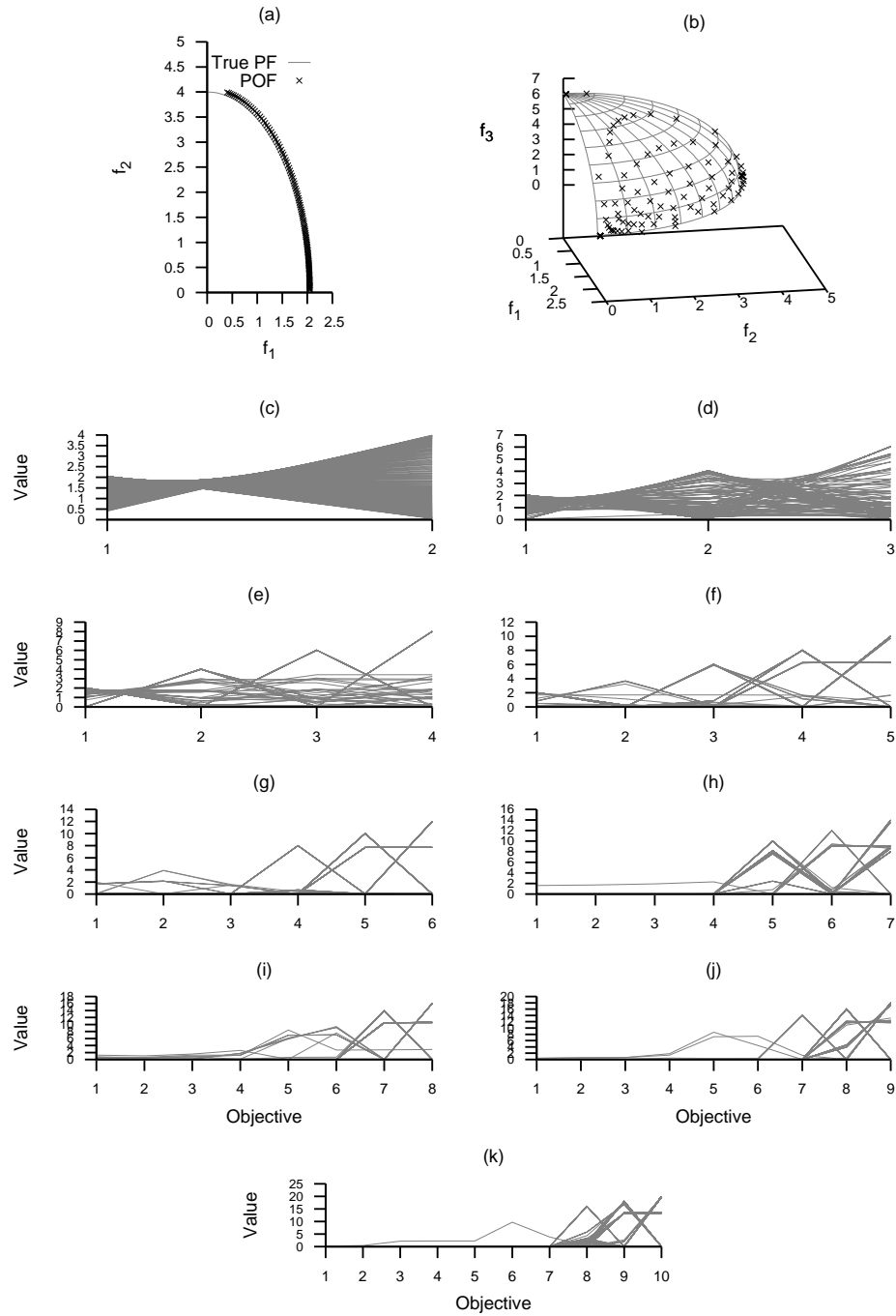


Figure A.197: Plots of the approximations obtained by MOEA/D-TCH from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

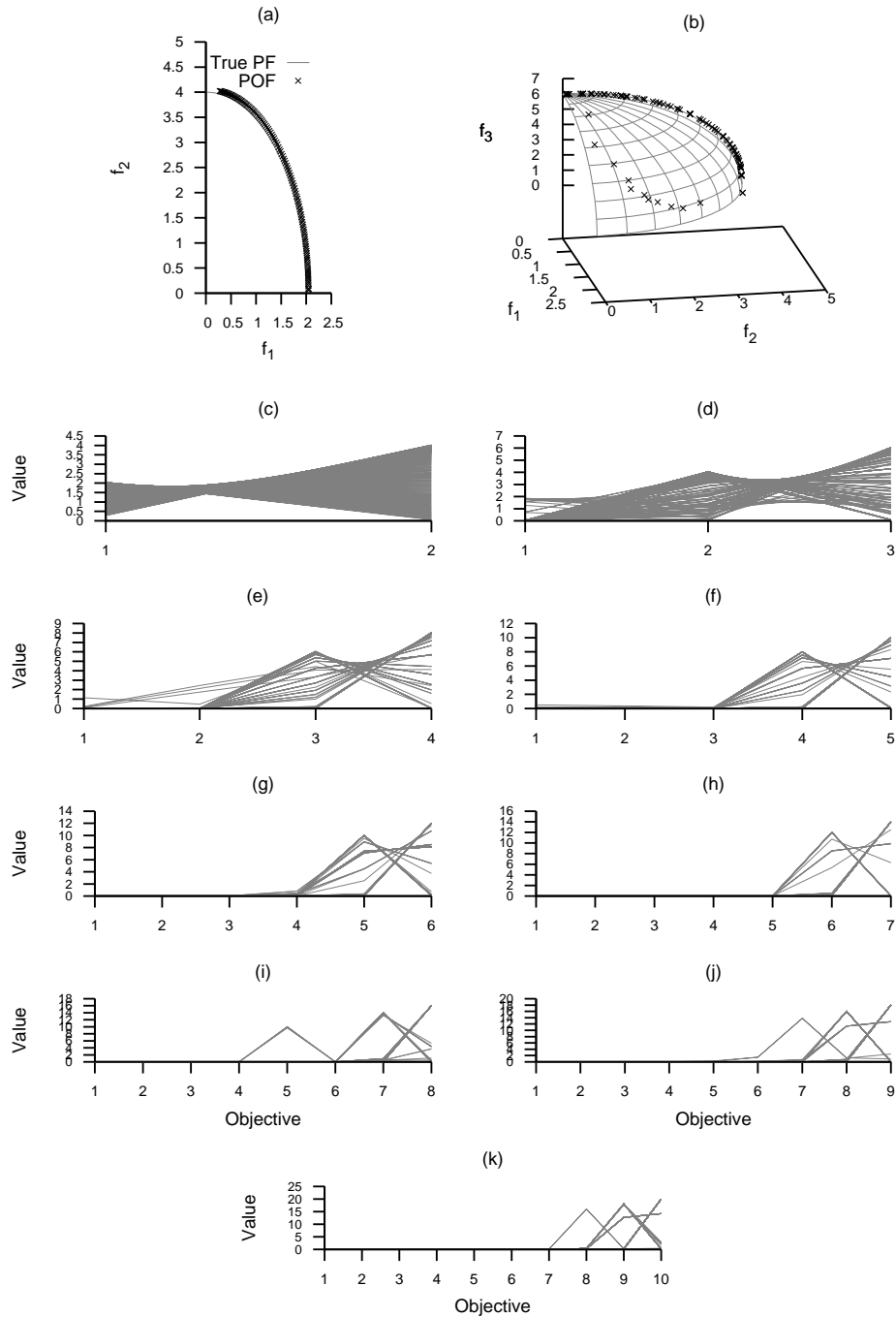


Figure A.198: Plots of the approximations obtained by MOEA/D-NTCH from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

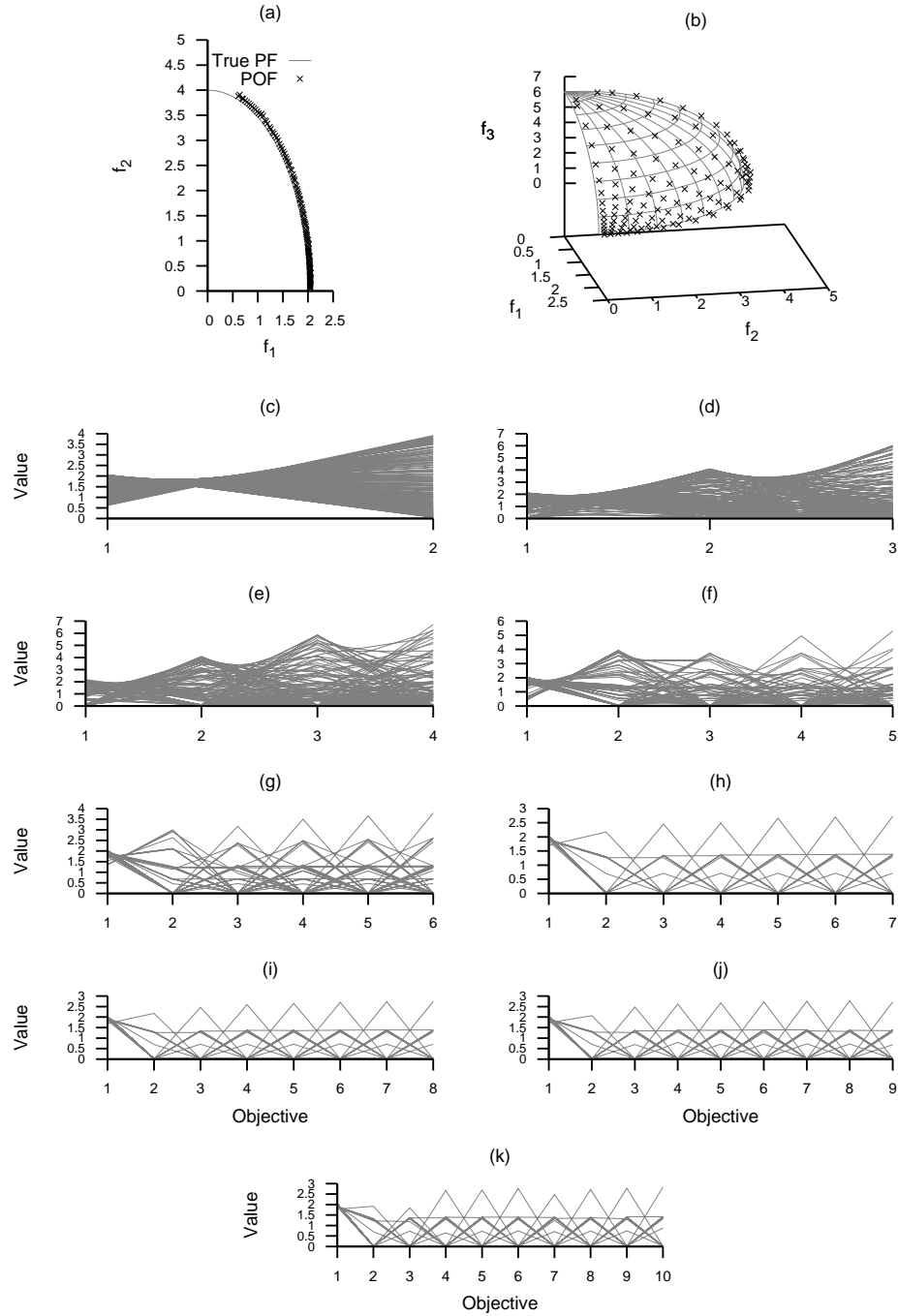


Figure A.199: Plots of the approximations obtained by MOEA/D-PBI from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

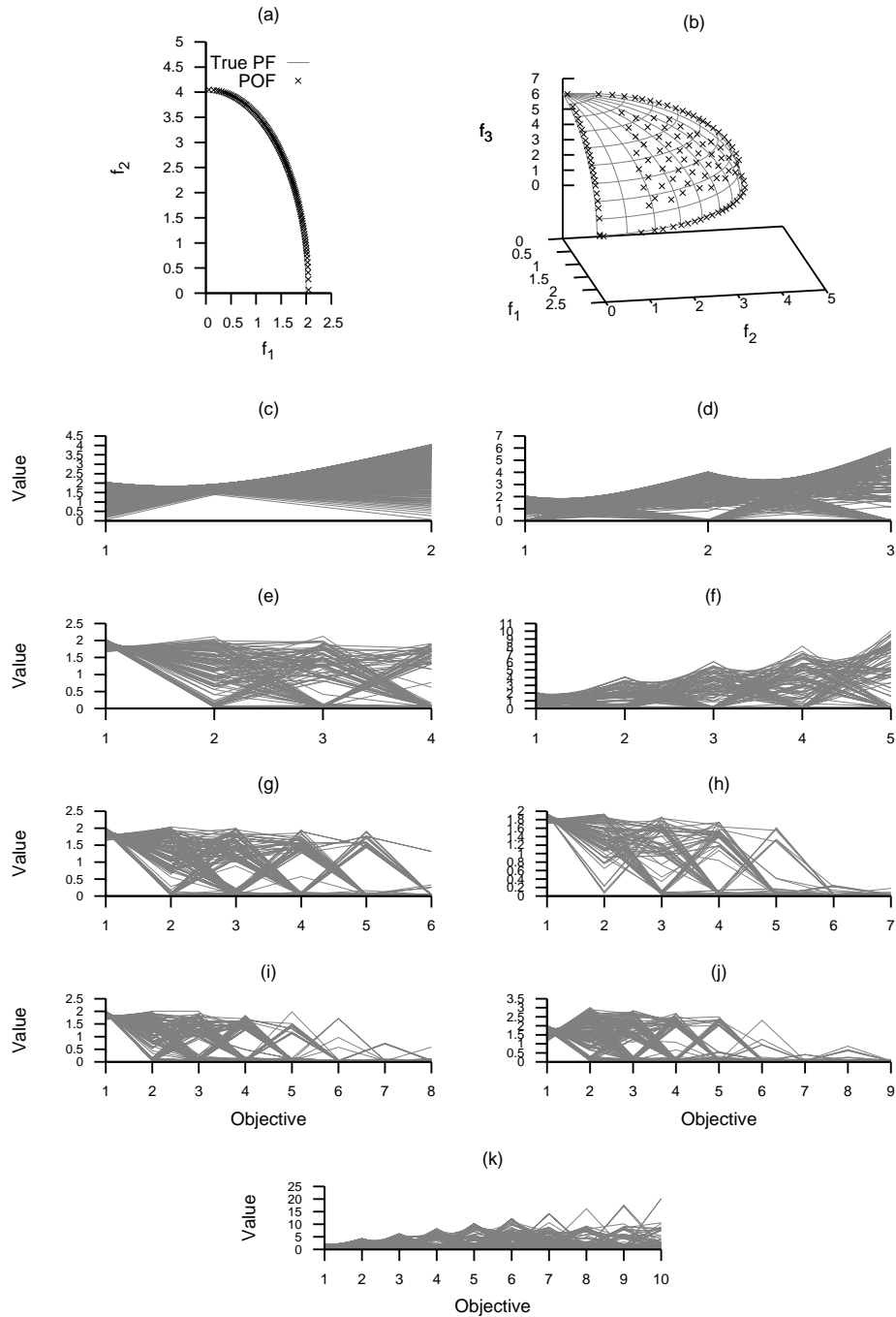


Figure A.200: Plots of the approximations obtained by SMS-EMOA from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

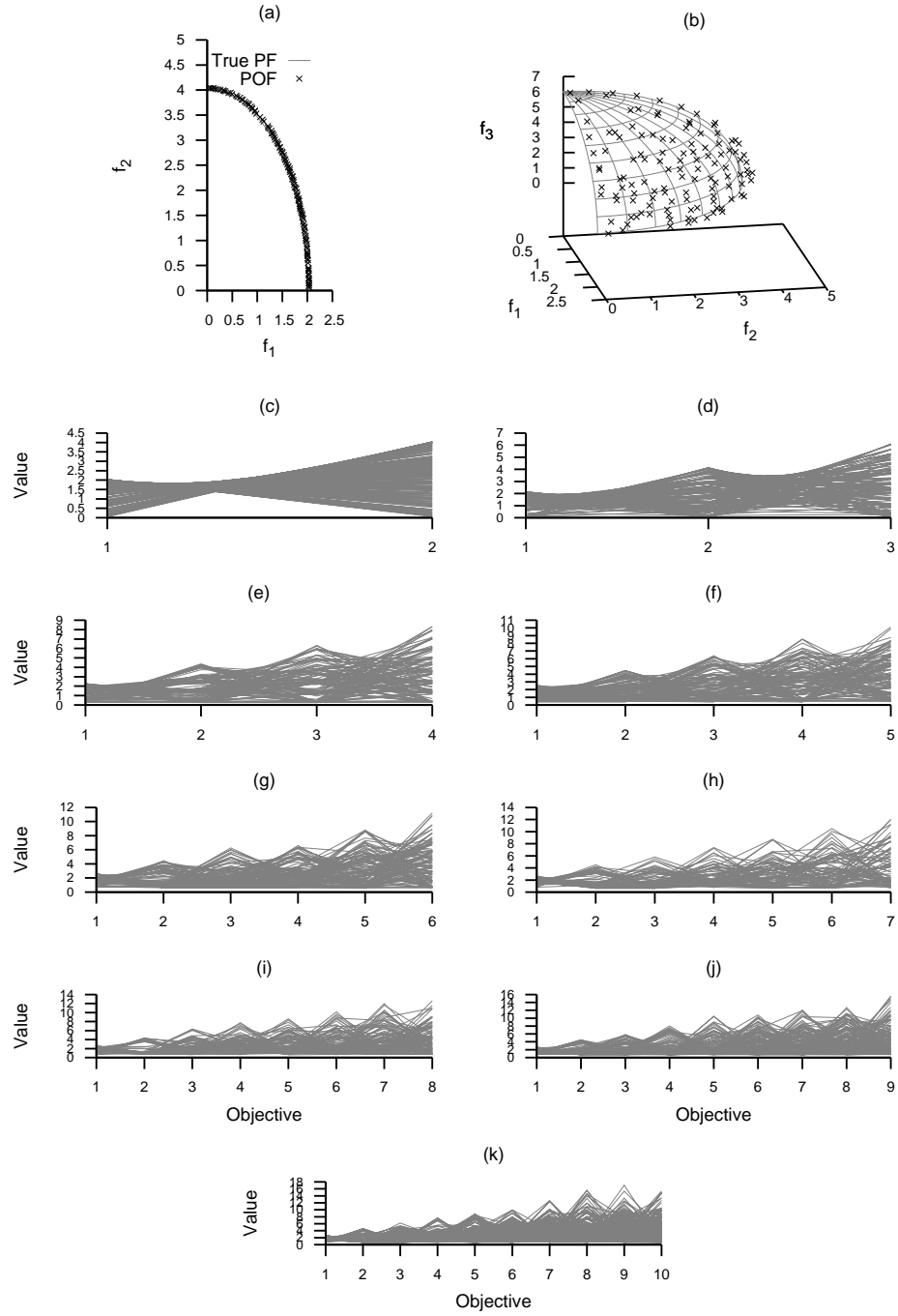


Figure A.201: Plots of the approximations obtained by  $\Delta_p$ -DDE from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

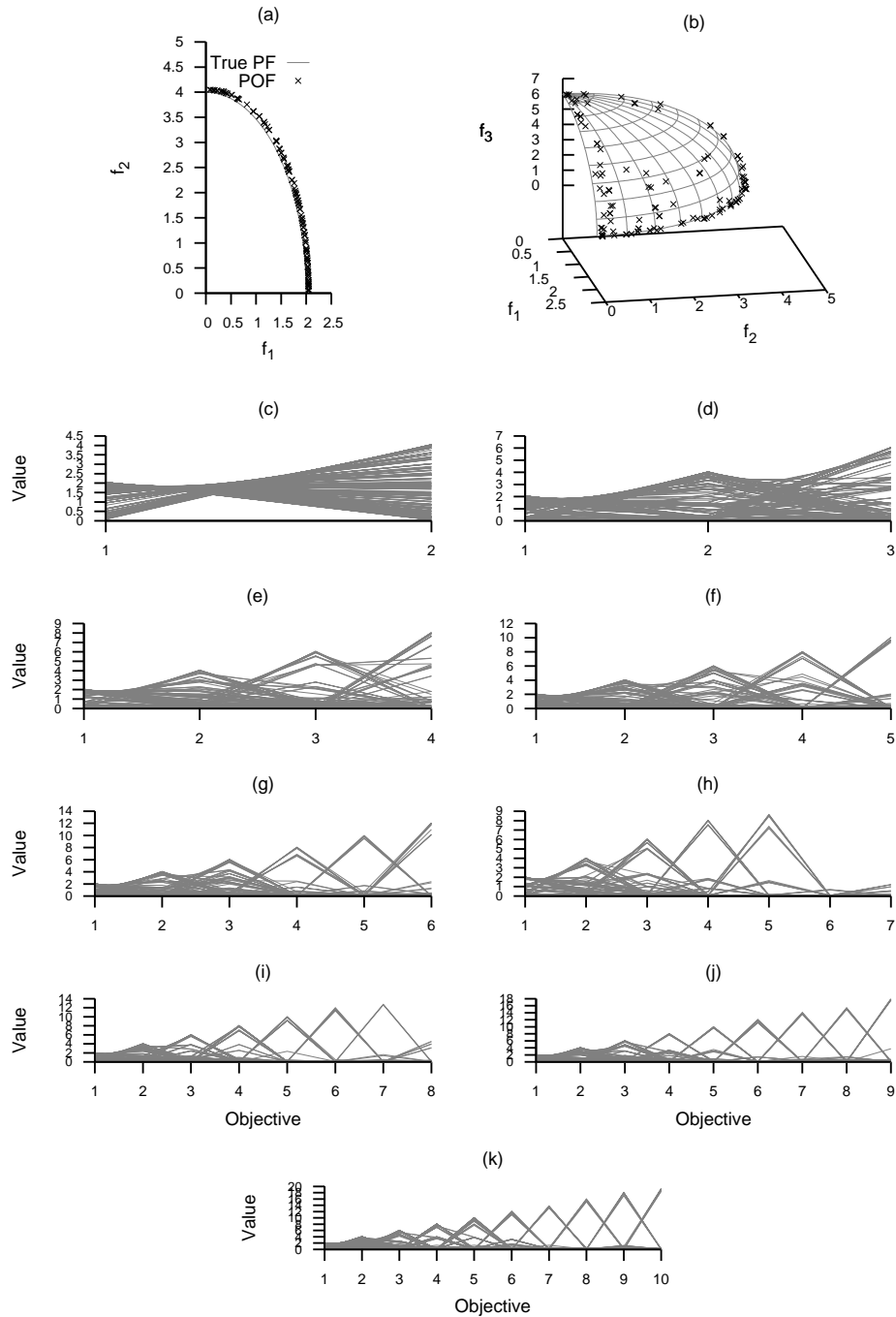


Figure A.202: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



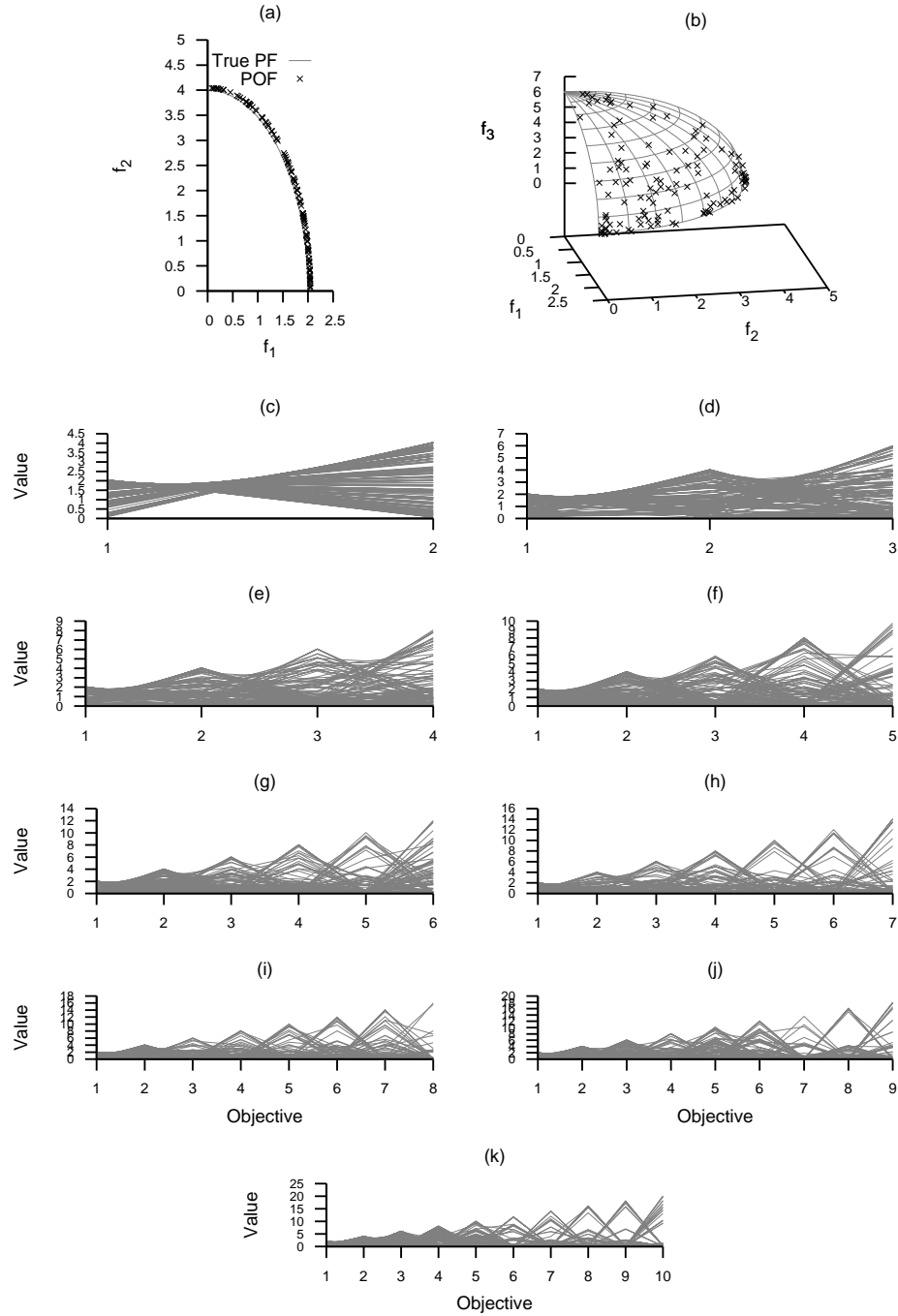


Figure A.203: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

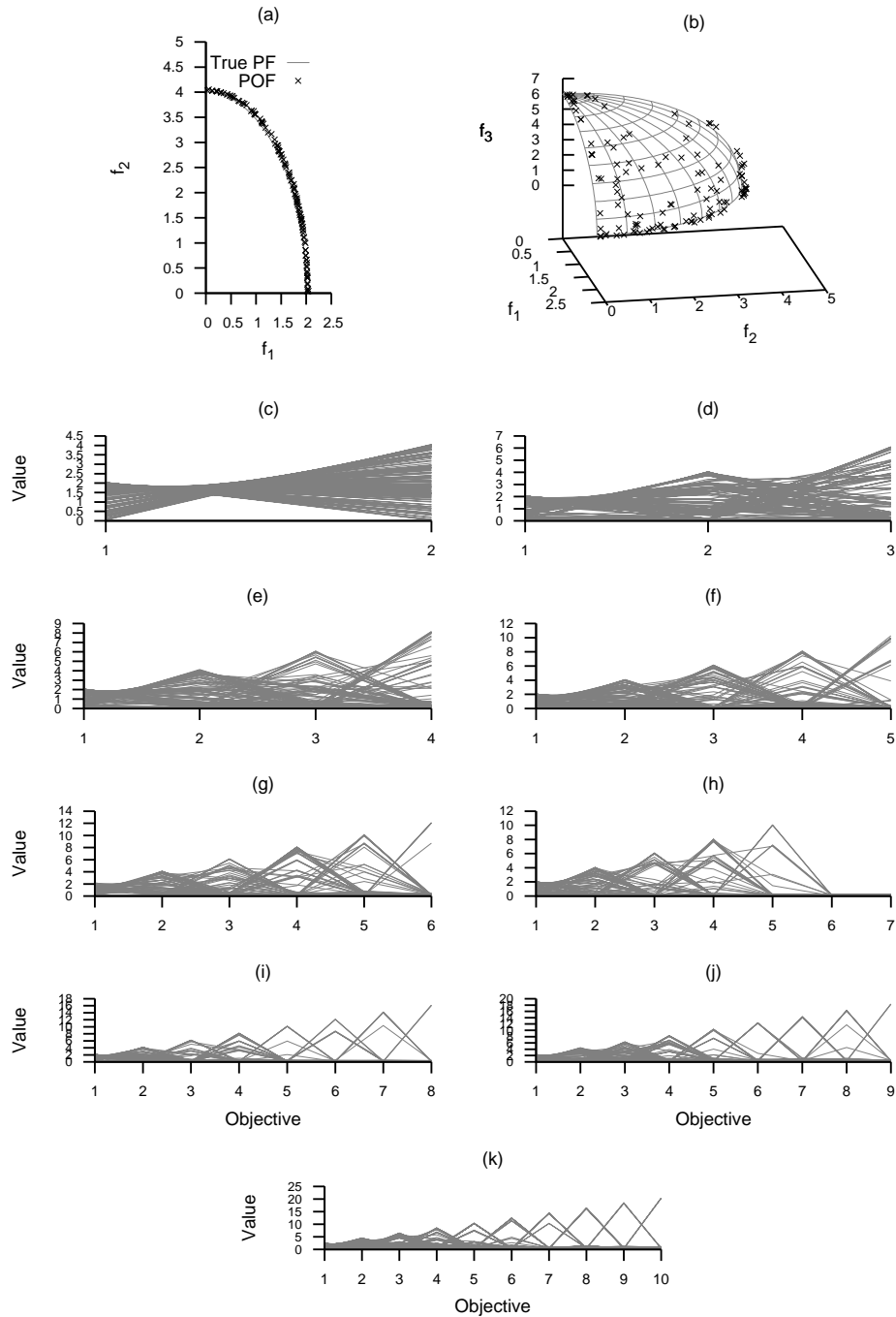


Figure A.204: Plots of the approximations obtained by *R2-MODE* from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

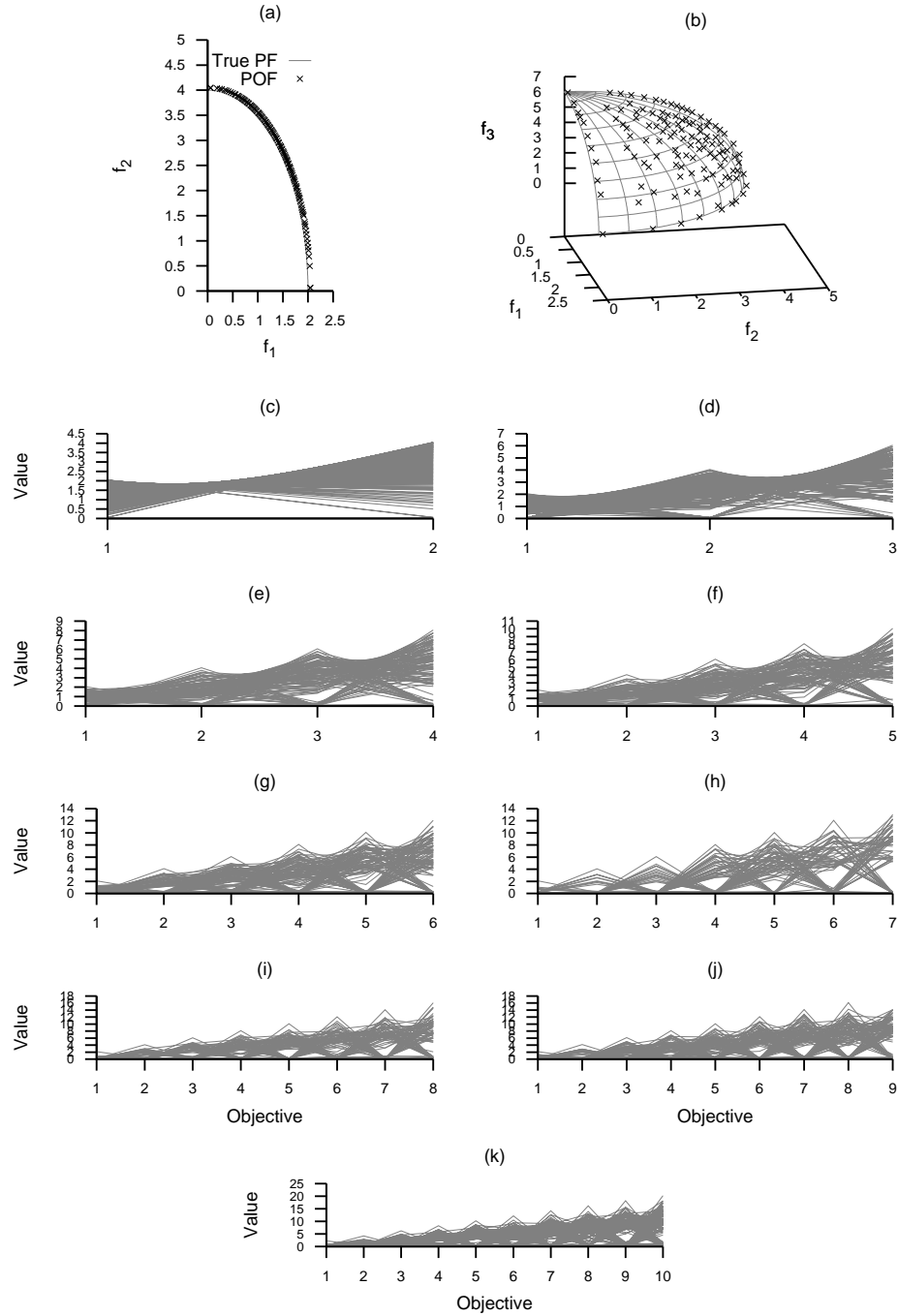


Figure A.205: Plots of the approximations obtained by *R2*-IBEA from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

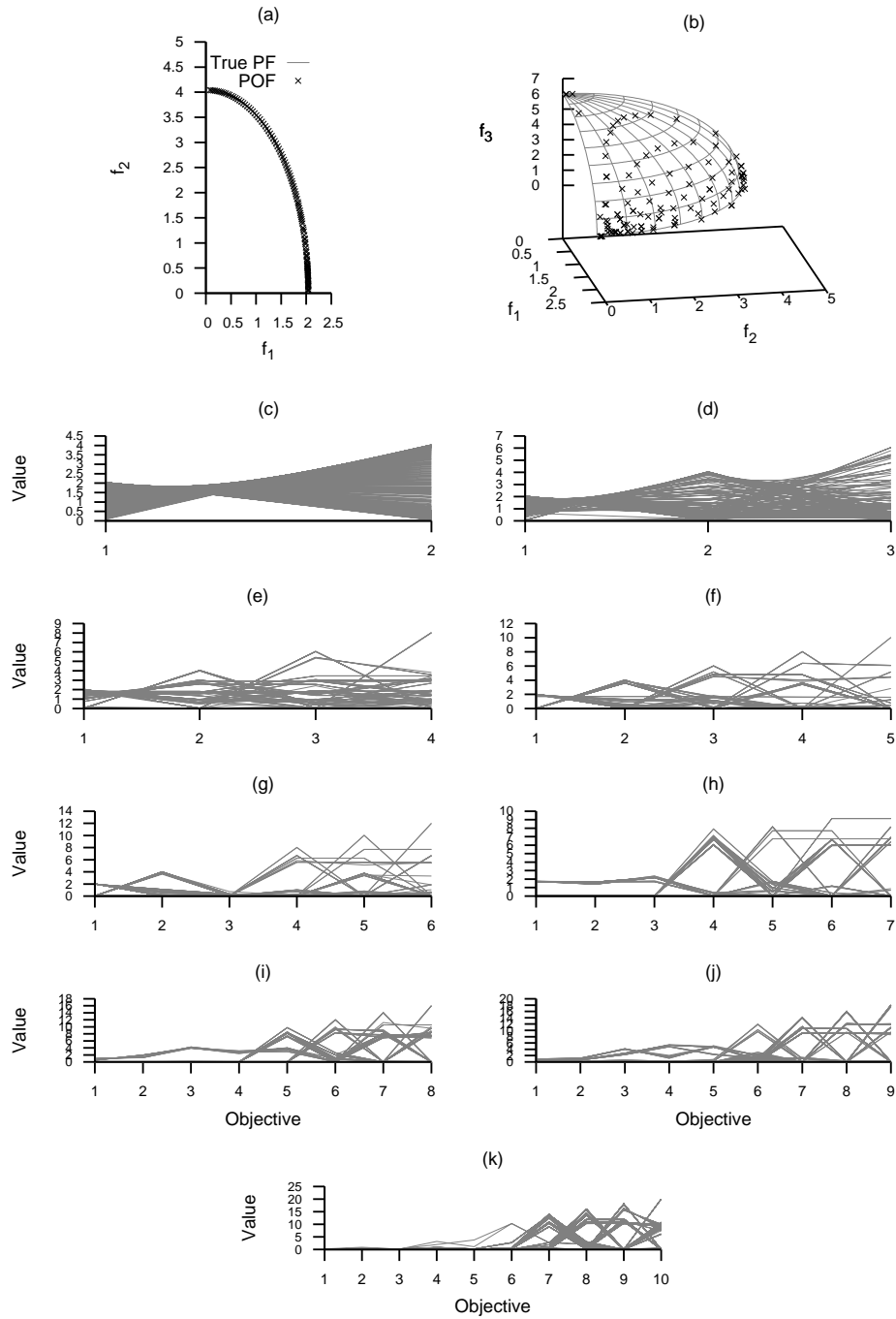


Figure A.206: Plots of the approximations obtained by MOMBI-TCH from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

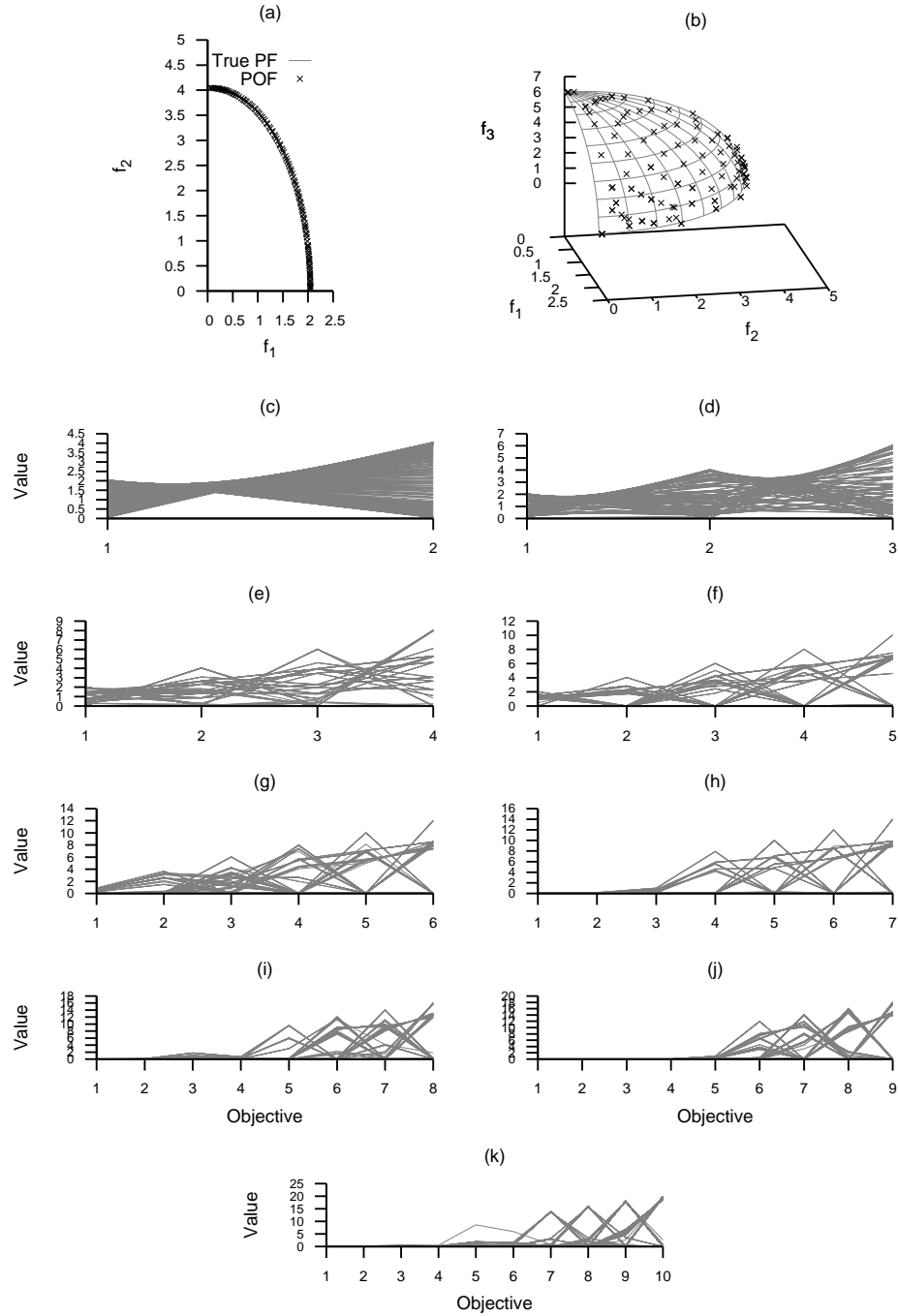


Figure A.207: Plots of the approximations obtained by MOMBI-NTCH from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

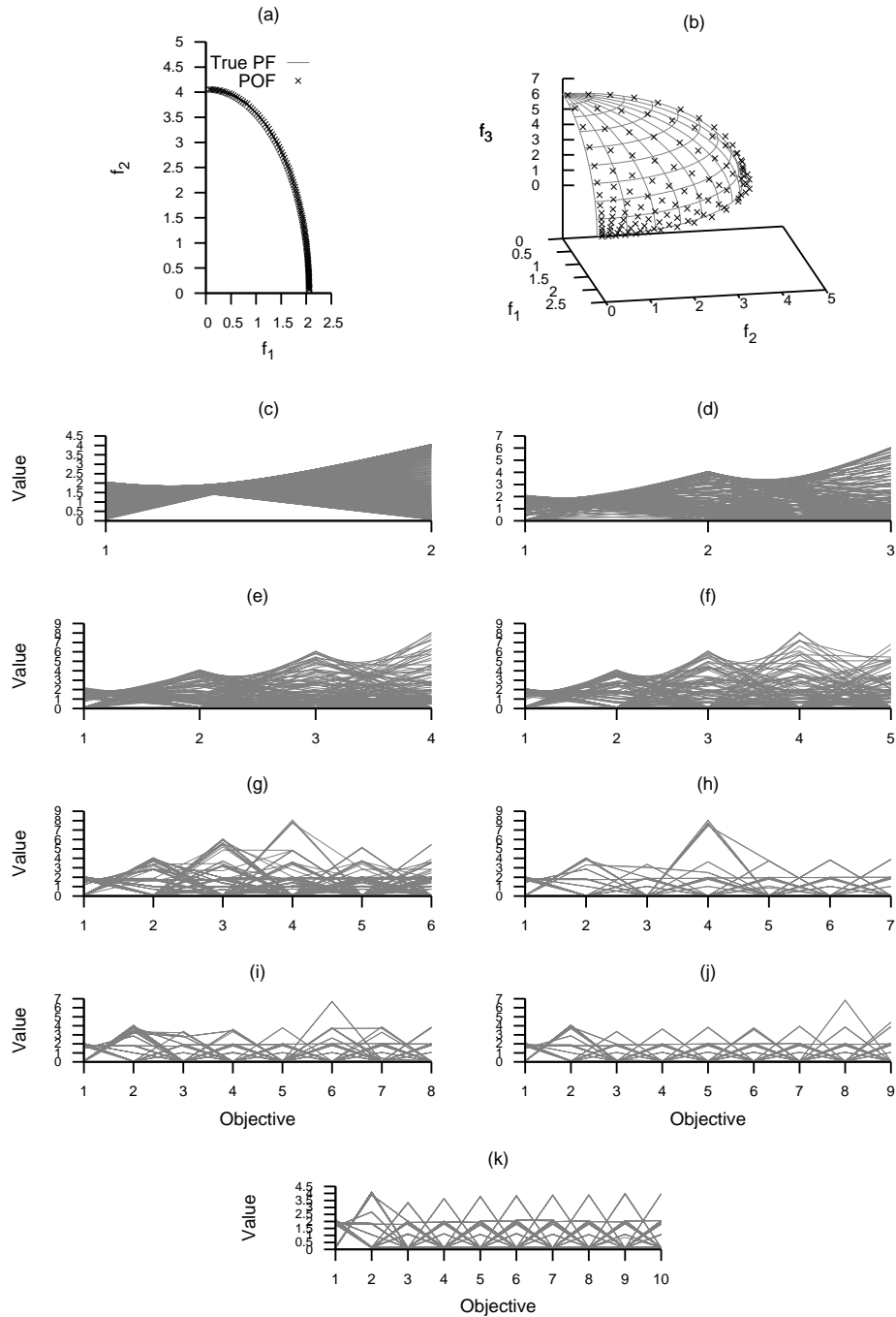


Figure A.208: Plots of the approximations obtained by MOMBI-PBI from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.80: Comparison of hypervolume indicator values for different optimizers on the WFG7 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	8.504703e+00	8.626862e+00	8.640761e+00	8.501081e+00	8.648595e+00	8.645529e+00	8.510212e+00	8.594195e+00	8.578840e+00	8.664916e+00	8.640381e+00	8.647797e+00	8.575772e+00
	avg.	8.496168e+00	8.607483e+00	8.635188e+00	8.479757e+00	8.647091e+00	8.636725e+00	8.510422e+00	8.563717e+00	8.571050e+00	8.662777e+00	8.636950e+00	8.647571e+00	8.574442e+00
	min.	8.360640e+00	8.311574e+00	8.447057e+00	8.223255e+00	8.605502e+00	8.566737e+00	8.418814e+00	8.095398e+00	8.446053e+00	8.636817e+00	8.537964e+00	8.627091e+00	8.504507e+00
	max.	8.571561e+00	8.655344e+00	8.664380e+00	8.564254e+00	8.674638e+00	8.663867e+00	8.591234e+00	8.642467e+00	8.595462e+00	8.676610e+00	8.664411e+00	8.664633e+00	8.611422e+00
	std.	4.441915e-02	6.058798e-02	2.784170e-02	7.629052e-02	1.648421e-02	2.322885e-02	3.310268e-02	9.550412e-02	2.695979e-02	8.636309e-03	1.829080e-02	8.378594e-03	1.806206e-02
3D	med.	6.848436e+01	7.377252e+01	5.806926e+01	7.158144e+01	7.758681e+01	7.433119e+01	7.290086e+01	7.345501e+01	7.290508e+01	7.666449e+01	7.377041e+01	7.494624e+01	7.404968e+01
	avg.	6.847171e+01	7.385688e+01	6.071312e+01	7.084538e+01	7.758593e+01	7.430486e+01	7.292731e+01	7.345815e+01	7.281803e+01	7.666302e+01	7.383202e+01	7.496039e+01	7.397895e+01
	min.	6.648365e+01	7.349459e+01	4.782078e+01	6.508053e+01	7.752450e+01	7.310984e+01	7.115957e+01	7.187388e+01	7.073237e+01	7.640184e+01	7.338281e+01	7.455641e+01	7.251642e+01
	max.	7.019949e+01	7.470177e+01	7.516605e+01	7.418575e+01	7.762396e+01	7.524816e+01	7.392816e+01	7.466967e+01	7.375654e+01	7.685646e+01	7.460577e+01	7.539181e+01	7.465891e+01
	std.	8.051748e-01	2.554404e-01	5.597093e+00	2.260749e+00	1.971970e-02	4.330730e-01	5.444382e-01	5.523644e-01	5.556086e-01	8.888271e-02	2.842080e-01	1.980329e-01	4.207457e-01
4D	med.	4.996780e+02	6.815890e+02	4.926428e+02	7.109104e+02	3.934979e+02	6.729799e+02	7.088413e+02	7.201050e+02	6.915485e+02	7.567138e+02	6.780766e+02	7.036777e+02	7.278043e+02
	avg.	4.950940e+02	6.806759e+02	4.793916e+02	7.079086e+02	3.933686e+02	6.695096e+02	7.093104e+02	7.194620e+02	6.915284e+02	7.566763e+02	6.797429e+02	7.041609e+02	7.266777e+02
	min.	4.585869e+02	6.454274e+02	3.529497e+02	6.168809e+02	3.845495e+02	6.211562e+02	6.915378e+02	6.961591e+02	6.748466e+02	7.491660e+02	6.534668e+02	6.845851e+02	7.111254e+02
	max.	5.295568e+02	7.097558e+02	6.445533e+02	7.364975e+02	4.140168e+02	6.947232e+02	7.275123e+02	7.336746e+02	7.138966e+02	7.616801e+02	7.154148e+02	7.285928e+02	7.376116e+02
	std.	1.661458e+01	1.242416e+01	8.774536e+01	1.979033e+01	4.102186e+00	1.548993e+01	7.188398e+00	7.290635e+00	7.570164e+00	2.170323e+00	1.221340e+01	8.774531e+00	5.766343e+00
5D	med.	4.581494e+03	7.613603e+03	3.518326e+03	6.824832e+03	4.389039e+03	5.261429e+03	8.147017e+03	8.280371e+03	7.709541e+03	8.619556e+03	7.692143e+03	8.299983e+03	7.779964e+03
	avg.	4.581974e+03	7.573569e+03	3.987697e+03	6.692119e+03	6.324375e+03	5.266323e+03	8.141599e+03	8.271861e+03	7.711053e+03	8.623928e+03	7.669825e+03	8.227213e+03	7.797698e+03
	min.	4.055973e+03	6.646088e+03	3.312491e+03	5.429259e+03	4.230460e+03	4.976617e+03	7.841723e+03	7.972437e+03	7.505675e+03	8.528252e+03	7.147984e+03	7.572698e+03	7.423948e+03
	max.	5.063850e+03	7.957100e+03	6.271653e+03	7.647823e+03	8.845006e+03	5.559716e+03	8.410013e+03	8.403445e+03	7.962784e+03	8.720661e+03	8.062887e+03	8.659450e+03	8.197369e+03
	std.	1.860412e+02	2.447673e+02	8.130933e+02	5.364329e+02	2.214465e+03	1.226506e+02	1.061709e+02	7.116472e+01	9.641079e+01	3.843964e+01	1.912596e+02	2.267532e+02	1.666955e+02
6D	med.	5.268889e+04	8.801274e+04	3.975091e+04	7.031332e+04	5.595178e+04	5.218829e+04	1.080754e+05	1.104212e+05	9.974094e+04	1.115394e+05	9.629348e+04	1.000582e+05	9.370454e+04
	avg.	5.273139e+04	8.779621e+04	4.176570e+04	7.248123e+04	5.603654e+04	5.246955e+04	1.081103e+05	1.103203e+05	9.968901e+04	1.114732e+05	9.645915e+04	9.997789e+04	9.001702e+04
	min.	4.768233e+04	6.975562e+04	3.481339e+04	6.354243e+04	5.497228e+04	4.477344e+04	1.053113e+05	1.081891e+05	9.667935e+04	1.098409e+05	8.434412e+04	8.891664e+04	4.498396e+04
	max.	5.675878e+04	9.880760e+04	6.705377e+04	8.938255e+04	5.890542e+04	6.042130e+04	1.115843e+05	1.121306e+05	1.031626e+05	1.132433e+05	1.054070e+05	1.078565e+05	1.015285e+05
	std.	1.829317e+03	5.267269e+03	6.412307e+03	6.570827e+03	6.164381e+02	3.070249e+03	1.410753e+03	8.600368e+02	1.491739e+03	7.191713e+02	4.542967e+03	3.251826e+03	1.230366e+04
7D	med.	6.588675e+05	9.412831e+05	5.275699e+05	8.689784e+05	7.718860e+05	6.954845e+05	1.497727e+06	1.628666e+06	1.415037e+06	1.632747e+06	1.108584e+06	1.154726e+06	6.749686e+05
	avg.	6.601036e+05	9.588548e+05	5.365159e+05	8.817116e+05	7.637689e+05	6.951539e+05	1.494538e+06	1.625337e+06	1.414173e+06	1.623872e+06	1.127245e+06	1.171606e+06	9.855901e+05
	min.	5.694477e+05	7.165991e+05	4.643722e+05	8.330884e+05	7.002583e+05	5.796203e+05	1.409066e+06	1.580052e+06	1.313644e+06	1.536628e+06	9.296996e+05	1.032891e+06	6.743840e+05
	max.	7.374033e+05	1.251383e+06	7.309912e+05	1.173028e+06	8.530466e+05	7.908863e+05	1.573716e+06	1.691777e+06	1.486610e+06	1.682867e+06	1.440584e+06	1.348358e+06	1.528107e+06
	std.	3.809771e+04	1.114411e+05	4.236611e+04	5.382654e+04	2.607919e+04	4.521402e+04	3.436046e+04	2.201532e+04	3.476070e+04	4.084226e+04	1.066234e+05	6.155011e+04	3.363489e+05
8D	med.	1.128588e+07	1.371264e+07	8.119357e+06	1.474089e+07	1.410866e+07	1.155439e+07	2.711783e+07	2.837560e+07	2.415682e+07	2.836729e+07	1.849510e+07	1.831931e+07	1.145757e+07
	avg.	1.126592e+07	1.366528e+07	8.472570e+06	1.479252e+07	1.412459e+07	1.149401e+07	2.707942e+07	2.835807e+07	2.413456e+07	2.828529e+07	1.891702e+07	1.832524e+07	1.435635e+07
	min.	9.501038e+06	9.492094e+06	7.028833e+06	1.376811e+07	1.377528e+07	9.527608e+06	2.576277e+07	2.757953e+07	2.297867e+07	2.718126e+07	1.281851e+07	1.459351e+07	1.144279e+07
	max.	1.259708e+07	1.845966e+07	1.192321e+07	2.065369e+07	1.497527e+07	1.307830e+07	2.799873e+07	2.929733e+07	2.556623e+07	2.912573e+07	2.356691e+07	2.225555e+07	2.496964e+07
	std.	5.358108e+05	1.524736e+06	1.101527e+06	8.215860e+05	1.433778e+05	7.254081e+05	4.732436e+05	2.898557e+05	4.850471e+05	4.975036e+05	2.087757e+06	1.101044e+06	4.468953e+06
9D	med.	2.192067e+08	2.372689e+08	1.444611e+08	2.753702e+08	3.127783e+08	2.103898e+08	5.389564e+08	5.444820e+08	4.386912e+08	5.325717e+08	3.438641e+08	3.242924e+08	2.172197e+08
	avg.	2.192494e+08	2.400425e+08	1.502446e+08	2.721544e+08	3.139217e+08	2.091964e+08	5.391399e+08	5.444462e+08	4.382711e+08	5.309585e+08	3.460286e+08	3.247308e+08	2.551916e+08
	min.	2.001196e+08	1.771660e+08	1.236465e+08	2.518592e+08	2.930513e+08	1.678184e+08	5.193023e+08	5.279954e+08	4.220771e+08	5.104017e+08	2.180577e+08	2.677608e+08	2.168127e+08
	max.	2.348297e+08	3.100721e+08	2.191585e+08	2.989193e+08	3.521590e+08	2.390311e+08	5.549639e+08	5.572454e+08	4.527455e+08	5.482985e+08	4.438543e+08	3.896908e+08	4.378971e+08
	std.	6.942984e+06	3.098686e+07	2.000005e+07	9.767556e+06	9.031780e+06	1.667939e+07	6.992072e+06	5.623291e+06	6.556562e+06	9.048002e+06	3.622318e+07	2.313491e+07	6.996237e+07
10D	med.	4.751688e+09	4.239049e+09	2.820428e+09	5.555775e+09	8.587573e+09	4.043786e+09	1.149293e+10	1.128382e+10	8.557179e+09	1.079123e+10	6.746903e+09	6.546829e+09	4.537529e+09
	avg.	4.744373e+09	4.416705e+09	2.990753e+09	5.591367e+09	9.384584e+09	4.065421e+09	1.147875e+10	1.126482e+10	8.568650e+09	1.079130e+10	6.778277e+09	6.592880e+09	4.956644e+09
	min.	4.400981e+09	3.370583e+09	2.441213e+09	5.150938e+09	7.680944e+09	3.453819e+09	1.061731e+10	1.077925e+10	8.227219e+09	1.051288e+10	5.475389e+09	5.683925e+09	4.514491e+09
	max.	5.076689e+09	5.984284e+09	4.523598e+09	7.524643e+09	1.102677e+10	4.832496e+09	1.176877e+10	1.153472e+10	9.014280e+09	1.116177e+10	8.176208e+09	7.680751e+09	8.289543e+09
	std.	1.297099e+08	5.895958e+08	4.621721e+08	3.194888e+08	1.191104e+09	1.255603e+08	1.375012e+08	1.379507e+08	1.384150e+08	1.457900e+08	4.789153e+08	4.010393e+08	9.882030e+08

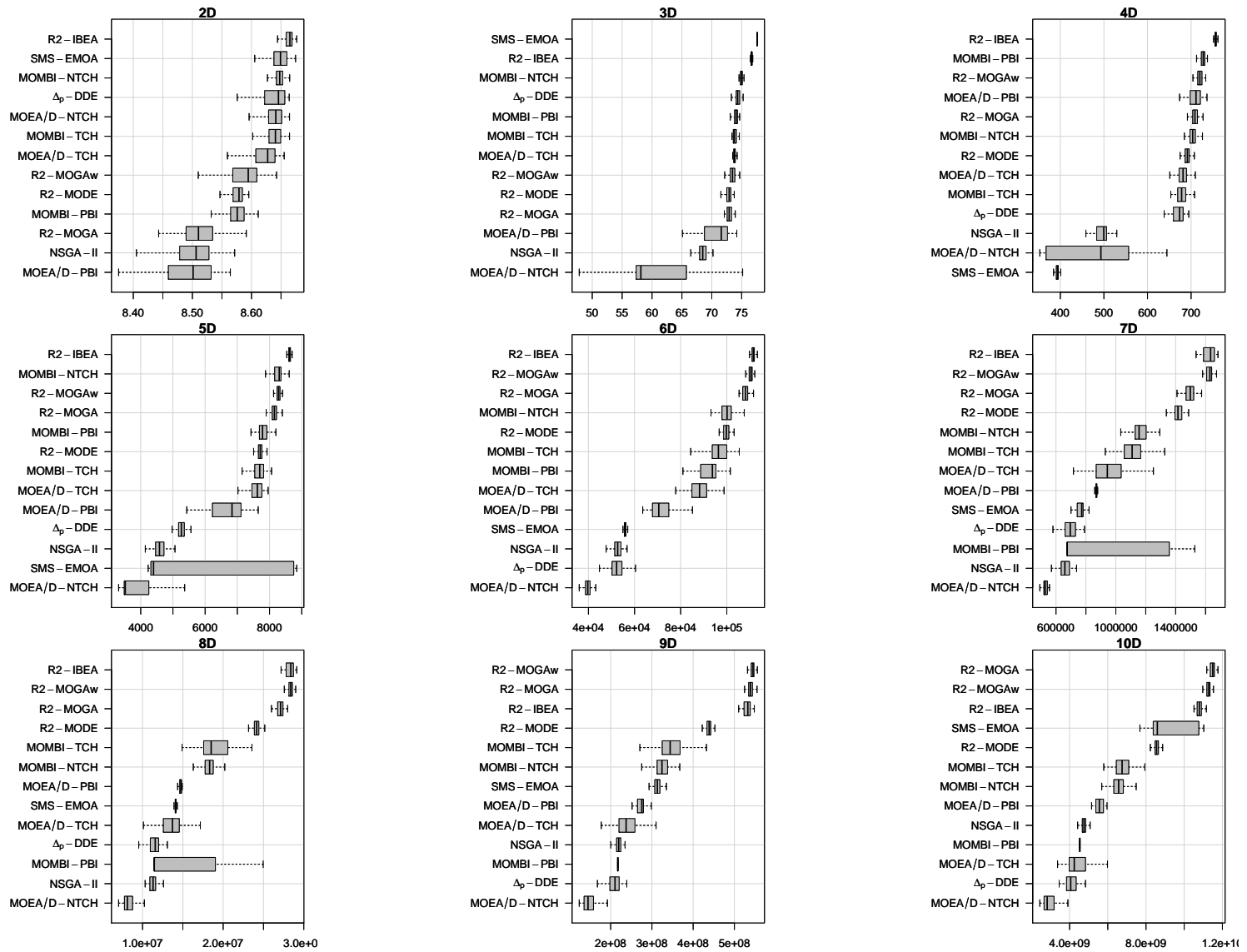


Figure A.209: Box-plot of hypervolume indicator values for different optimizers on the WFG7 test problem.



Table A.81: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG7 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	4.66e-27	—	> 0.05	2.16e-27	> 0.05	> 0.05	3.78e-26	2.67e-12	7.31e-19	> 0.05	> 0.05	> 0.05	3.17e-18
MOEA/D-NTCH	2.04e-33	2.52e-08	—	2.30e-33	> 0.05	> 0.05	4.39e-33	8.41e-26	3.89e-30	> 0.05	> 0.05	> 0.05	7.38e-30
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	1.28e-34	3.91e-16	7.43e-05	1.28e-34	—	1.66e-03	1.28e-34	1.95e-31	1.28e-34	> 0.05	1.24e-05	> 0.05	1.40e-34
$\Delta_p$ -DDE	1.32e-34	8.64e-08	> 0.05	1.28e-34	> 0.05	—	1.63e-34	7.28e-24	2.52e-31	> 0.05	> 0.05	> 0.05	6.25e-31
R2-MOGA	4.46e-02	> 0.05	> 0.05	1.69e-02	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	4.77e-22	> 0.05	> 0.05	7.83e-22	> 0.05	> 0.05	8.97e-21	—	2.48e-05	> 0.05	> 0.05	> 0.05	9.17e-05
R2-MODE	8.10e-29	> 0.05	> 0.05	1.19e-28	> 0.05	> 0.05	2.37e-26	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	1.28e-34	1.50e-32	4.41e-26	1.28e-34	3.64e-14	2.18e-22	1.28e-34	1.40e-34	1.28e-34	—	1.66e-27	3.76e-22	1.28e-34
MOMBI-TCH	1.89e-34	8.60e-09	> 0.05	2.26e-34	> 0.05	> 0.05	2.63e-34	8.97e-28	1.12e-32	> 0.05	—	> 0.05	2.38e-32
MOMBI-NTCH	1.28e-34	7.83e-22	1.38e-05	1.28e-34	> 0.05	2.58e-02	1.28e-34	6.06e-34	1.28e-34	> 0.05	6.28e-07	—	1.28e-34
MOMBI-PBI	9.24e-32	> 0.05	> 0.05	5.42e-31	> 0.05	> 0.05	2.86e-29	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—
3D													
NSGA-II	—	> 0.05	4.60e-20	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	2.51e-33	1.32e-28	> 0.05	> 0.05	9.00e-27	7.42e-08	5.23e-33	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.06e-13	> 0.05	2.34e-27	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34
$\Delta_p$ -DDE	1.28e-34	1.63e-14	2.44e-33	1.83e-32	> 0.05	—	1.07e-31	1.55e-21	3.90e-33	> 0.05	5.19e-15	> 0.05	1.59e-07
R2-MOGA	1.28e-34	> 0.05	2.82e-33	5.88e-14	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	1.28e-34	> 0.05	2.58e-33	4.55e-22	> 0.05	> 0.05	1.37e-10	—	1.83e-13	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	1.28e-34	> 0.05	2.99e-33	1.76e-12	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	1.28e-34	> 0.05	2.51e-33	3.06e-28	> 0.05	> 0.05	9.81e-26	1.90e-07	9.02e-31	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.28e-34	2.40e-34	1.66e-33	1.28e-34	> 0.05	4.77e-26	1.28e-34	1.49e-34	1.28e-34	> 0.05	1.36e-34	—	1.58e-34
MOMBI-PBI	1.28e-34	1.54e-05	2.51e-33	5.00e-30	> 0.05	> 0.05	6.56e-27	1.62e-12	4.09e-29	> 0.05	2.05e-05	> 0.05	—
4D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	1.28e-34	> 0.05	1.28e-34	2.19e-07	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	4.38e-06	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	1.25e-23	1.44e-34	—	1.28e-34	4.23e-28	> 0.05	> 0.05	1.21e-16	> 0.05	2.95e-23	3.88e-04	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e-34	> 0.05	1.53e-34	> 0.05	1.28e-34	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.20e-31	1.28e-34	> 0.05	1.28e-34	1.63e-34	—	> 0.05	3.32e-28	> 0.05	3.86e-31	7.51e-06	> 0.05
R2-MOGAw	1.28e-34	2.96e-34	1.28e-34	1.65e-07	1.28e-34	1.28e-34	3.47e-16	—	8.92e-34	> 0.05	5.71e-34	4.35e-23	> 0.05
R2-MODE	1.28e-34	3.01e-12	1.28e-34	> 0.05	1.28e-34	4.64e-26	> 0.05	> 0.05	—	> 0.05	1.10e-13	> 0.05	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	1.28e-34	> 0.05	1.28e-34	> 0.05	1.28e-34	2.43e-05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.28e-34	1.99e-28	1.28e-34	> 0.05	1.28e-34	4.00e-34	> 0.05	> 0.05	1.26e-19	> 0.05	1.03e-27	—	> 0.05
MOMBI-PBI	1.28e-34	1.28e-34	1.28e-34	2.97e-18	1.28e-34	1.28e-34	1.25e-29	2.28e-12	1.49e-34	> 0.05	1.49e-34	1.13e-31	—

Table A.82: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG7 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
5D													
NSGA-II	—	> 0.05	$6.92e-12$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$1.28e-34$	—	$1.28e-34$	$4.12e-28$	> 0.05	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$1.28e-34$	> 0.05	$1.71e-33$	—	> 0.05	$1.63e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	$1.01e-19$	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	$1.44e-34$	> 0.05	$2.19e-19$	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$1.28e-34$	$3.15e-34$	$1.28e-34$	$1.28e-34$	> 0.05	$1.28e-34$	—	> 0.05	$2.40e-34$	> 0.05	$1.61e-33$	> 0.05	$1.56e-29$
R2-MOGAw	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	$1.28e-34$	$4.14e-19$	—	$1.28e-34$	> 0.05	$1.40e-34$	> 0.05	$3.34e-34$
R2-MODE	$1.28e-34$	$2.48e-05$	$1.28e-34$	$9.75e-34$	> 0.05	$1.28e-34$	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.09e-32$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$4.45e-03$	$1.28e-34$	$1.38e-31$	> 0.05	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	$1.28e-34$	$6.60e-30$	$1.28e-34$	$1.36e-34$	> 0.05	$1.28e-34$	$1.00e-08$	> 0.05	$8.56e-29$	> 0.05	$5.54e-28$	—	$3.14e-24$
MOMBI-PBI	$1.28e-34$	$3.57e-11$	$1.28e-34$	$9.46e-34$	> 0.05	$1.28e-34$	> 0.05	> 0.05	$1.90e-05$	> 0.05	$7.02e-06$	> 0.05	—
6D													
NSGA-II	—	> 0.05	$9.42e-23$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$1.28e-34$	—	$1.28e-34$	$5.11e-28$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$1.28e-34$	> 0.05	$2.33e-34$	—	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	$3.02e-30$	> 0.05	$2.05e-27$	> 0.05	—	$5.05e-29$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	> 0.05	> 0.05	$1.29e-22$	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	> 0.05	$1.28e-34$	> 0.05	$1.36e-34$	$3.08e-33$	$1.28e-34$
R2-MOGAw	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$4.67e-23$	—	$1.28e-34$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MODE	$1.28e-34$	$1.61e-33$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	—	> 0.05	$7.78e-09$	> 0.05	$6.56e-27$
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.33e-32$	$1.70e-17$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$4.35e-23$	$1.28e-34$	$2.55e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	$3.14e-08$
MOMBI-NTCH	$1.28e-34$	$3.37e-32$	$1.28e-34$	$1.32e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	$1.08e-08$	—	$1.34e-24$
MOMBI-PBI	$2.97e-27$	$5.29e-09$	$2.66e-33$	$9.32e-26$	$2.97e-27$	$2.54e-27$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—
7D													
NSGA-II	—	> 0.05	$6.24e-30$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$1.49e-34$	—	$1.32e-34$	$4.84e-08$	$3.75e-31$	$7.03e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$1.28e-34$	> 0.05	$1.28e-34$	—	$1.44e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	$1.31e-33$	> 0.05	$1.84e-34$	> 0.05	—	$1.49e-23$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	$5.04e-08$	> 0.05	$4.37e-32$	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	> 0.05	$1.09e-28$	> 0.05	$1.73e-34$	$1.28e-34$	$1.44e-29$
R2-MOGAw	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MODE	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	—	> 0.05	$1.38e-31$	$1.89e-34$	$4.10e-21$
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$9.75e-34$	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$7.98e-19$	$1.28e-34$	$1.23e-31$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	$3.33e-02$
MOMBI-NTCH	$1.28e-34$	$4.71e-28$	$1.28e-34$	$1.68e-32$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	$1.26e-07$	—	$7.76e-03$
MOMBI-PBI	$2.85e-14$	> 0.05	$6.06e-34$	> 0.05	> 0.05	$6.41e-05$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—

Table A.83: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG7 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
8D													
NSGA-II	—	> 0.05	6.00e-29	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.47e-28	—	5.22e-34	> 0.05	> 0.05	4.10e-25	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	7.29e-05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	1.76e-12	1.28e-34	—	1.09e-25	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.23e-06
SMS-EMOA	1.28e-34	8.35e-04	1.28e-34	> 0.05	—	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.73e-06
$\Delta_p$ -DDE	1.73e-03	> 0.05	4.00e-30	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	> 0.05	1.28e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	3.15e-34	—	1.28e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-MODE	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	—	> 0.05	1.95e-34	1.28e-34	1.94e-32
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	8.02e-30	> 0.05	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	1.28e-34	9.24e-32	1.28e-34	2.06e-31	2.58e-33	1.40e-34	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	5.62e-11
MOMBI-NTCH	1.28e-34	5.71e-33	1.28e-34	2.59e-31	1.32e-34	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	8.93e-08
MOMBI-PBI	2.65e-10	> 0.05	4.08e-31	> 0.05	> 0.05	4.75e-03	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—
9D													
NSGA-II	—	> 0.05	6.43e-34	> 0.05	> 0.05	1.37e-06	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.62e-08	—	1.61e-33	> 0.05	> 0.05	6.34e-14	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	2.91e-02
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	5.94e-16	1.28e-34	—	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.07e-10
SMS-EMOA	1.28e-34	4.26e-33	1.28e-34	1.44e-34	—	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	9.23e-11
$\Delta_p$ -DDE	> 0.05	> 0.05	3.86e-31	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	> 0.05	1.28e-34	4.15e-10	1.28e-34	1.28e-34	1.28e-34
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	3.79e-09	—	1.28e-34	3.85e-23	1.28e-34	1.28e-34	1.28e-34
R2-MODE	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	—	> 0.05	8.60e-33	1.28e-34	5.71e-34
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	6.43e-34	3.00e-32	1.32e-34	3.64e-31	2.23e-16	3.55e-34	> 0.05	> 0.05	> 0.05	> 0.05	—	1.74e-07	5.17e-16
MOMBI-NTCH	1.28e-34	9.95e-33	1.28e-34	2.82e-31	7.11e-07	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	5.46e-12
MOMBI-PBI	> 0.05	> 0.05	1.24e-33	> 0.05	> 0.05	7.46e-09	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—
10D													
NSGA-II	—	5.39e-08	1.53e-34	> 0.05	> 0.05	1.37e-32	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	6.97e-13
MOEA/D-TCH	> 0.05	—	1.07e-30	> 0.05	> 0.05	9.78e-06	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.28e-34	8.50e-28	1.28e-34	—	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	3.40e-17
SMS-EMOA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	1.68e-34	1.28e-34	2.40e-34
$\Delta_p$ -DDE	> 0.05	> 0.05	3.80e-28	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.23e-19	1.28e-34	1.32e-34	1.28e-34	1.28e-34	1.28e-34
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.28e-34	4.64e-34	1.28e-34	> 0.05	—	1.28e-34	1.33e-32	1.28e-34	1.28e-34	1.28e-34
R2-MODE	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	—	> 0.05	1.28e-34	1.28e-34	1.32e-34
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	3.77e-14	1.28e-34	> 0.05	> 0.05	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	1.28e-34	1.84e-34	1.28e-34	1.84e-31	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	—	1.50e-03	1.78e-21
MOMBI-NTCH	1.28e-34	1.89e-34	1.28e-34	3.64e-31	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	2.98e-19
MOMBI-PBI	> 0.05	8.93e-08	1.53e-34	> 0.05	> 0.05	1.51e-28	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—

Table A.84: Comparison of R2 indicator values for different optimizers on the WFG7 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	5.913470e-01	5.887410e-01	5.886869e-01	5.935770e-01	5.878833e-01	5.891144e-01	5.917615e-01	5.902394e-01	5.923199e-01	5.881130e-01	5.884961e-01	5.886310e-01	5.909428e-01
	avg.	5.919664e-01	5.893687e-01	5.888785e-01	5.943522e-01	5.879265e-01	5.891479e-01	5.918456e-01	5.911569e-01	5.924375e-01	5.881439e-01	5.885521e-01	5.886506e-01	5.909834e-01
	min.	5.896397e-01	5.884765e-01	5.883965e-01	5.917615e-01	5.876572e-01	5.886575e-01	5.900113e-01	5.890200e-01	5.915369e-01	5.879257e-01	5.882211e-01	5.883553e-01	5.902338e-01
	max.	6.014459e-01	6.016659e-01	5.994507e-01	6.061164e-01	5.885804e-01	5.901024e-01	5.943678e-01	6.067948e-01	5.950359e-01	5.885203e-01	5.904693e-01	5.891076e-01	5.922331e-01
	std.	1.959254e-03	2.078797e-03	1.143166e-03	2.830431e-03	1.886597e-04	2.561203e-04	8.635871e-04	3.270873e-03	6.441269e-04	1.181056e-04	2.663489e-04	1.672607e-04	3.973192e-04
3D	med.	3.765193e-01	3.160344e-01	4.440265e-01	3.273441e-01	3.109093e-01	3.282133e-01	3.185091e-01	3.167011e-01	3.207964e-01	3.144188e-01	3.159766e-01	3.158733e-01	3.195516e-01
	avg.	3.783644e-01	3.160023e-01	4.352386e-01	3.289156e-01	3.109174e-01	3.298507e-01	3.186521e-01	3.170267e-01	3.209041e-01	3.144426e-01	3.159975e-01	3.158723e-01	3.199177e-01
	min.	3.440841e-01	3.156076e-01	3.176853e-01	3.194655e-01	3.103871e-01	3.177134e-01	3.158786e-01	3.148203e-01	3.188909e-01	3.134009e-01	3.153982e-01	3.150038e-01	3.182202e-01
	max.	4.230576e-01	3.163363e-01	7.010860e-01	3.468202e-01	3.114274e-01	3.543444e-01	3.249276e-01	3.235787e-01	3.259372e-01	3.160294e-01	3.164273e-01	3.166624e-01	3.252778e-01
	std.	1.715388e-02	1.577798e-04	7.878153e-02	6.120790e-03	2.206867e-04	7.331587e-03	1.244793e-03	1.464610e-03	1.084285e-03	5.032063e-04	2.373294e-04	3.411559e-04	1.507591e-03
4D	med.	3.987798e-01	2.234079e-01	4.395404e-01	2.247641e-01	4.447128e-01	2.386002e-01	2.139401e-01	2.130412e-01	2.186797e-01	2.133166e-01	2.232283e-01	2.206099e-01	2.151985e-01
	avg.	4.016218e-01	2.231903e-01	5.061774e-01	2.260539e-01	4.440817e-01	2.447058e-01	2.142544e-01	2.133799e-01	2.185818e-01	2.133595e-01	2.231142e-01	2.204110e-01	2.156459e-01
	min.	3.545981e-01	2.190505e-01	2.468044e-01	2.120103e-01	4.228730e-01	2.247230e-01	2.112009e-01	2.102217e-01	2.161744e-01	2.115678e-01	2.180296e-01	2.167617e-01	2.125975e-01
	max.	4.553587e-01	2.258430e-01	7.636856e-01	2.618330e-01	5.265151e-01	2.830416e-01	2.189883e-01	2.188604e-01	2.233102e-01	2.160760e-01	2.257252e-01	2.234012e-01	2.258274e-01
	std.	1.995997e-02	1.582766e-03	1.893729e-01	8.579405e-03	6.110985e-03	1.626829e-02	1.507204e-03	1.861165e-03	1.148488e-03	8.495695e-04	1.709845e-03	1.319418e-03	2.506090e-03
5D	med.	4.030198e-01	1.741944e-01	7.935666e-01	2.163474e-01	3.439643e-01	3.016014e-01	1.636052e-01	1.639820e-01	1.702640e-01	1.677064e-01	1.750772e-01	1.677181e-01	1.845662e-01
	avg.	4.034878e-01	1.749984e-01	6.755695e-01	2.221027e-01	2.685713e-01	3.011049e-01	1.639157e-01	1.643416e-01	1.702925e-01	1.680068e-01	1.751906e-01	1.683782e-01	1.843213e-01
	min.	3.640515e-01	1.711587e-01	2.223236e-01	1.871536e-01	1.604439e-01	2.761896e-01	1.602987e-01	1.611396e-01	1.676661e-01	1.653836e-01	1.721657e-01	1.645581e-01	1.755288e-01
	max.	4.521835e-01	1.858973e-01	8.060478e-01	2.838182e-01	3.633016e-01	3.227283e-01	1.708831e-01	1.701322e-01	1.727049e-01	1.744656e-01	1.787356e-01	1.849538e-01	1.899726e-01
	std.	1.796325e-02	2.676141e-03	1.885651e-01	1.960991e-02	9.514580e-02	9.006353e-03	1.805451e-03	1.949800e-03	1.006111e-03	1.596743e-03	1.550930e-03	2.432127e-03	2.965618e-03
6D	med.	4.057817e-01	1.919913e-01	8.144276e-01	2.284507e-01	2.966605e-01	3.094862e-01	1.243465e-01	1.246464e-01	1.312530e-01	1.365476e-01	1.645906e-01	1.665953e-01	1.613215e-01
	avg.	4.056370e-01	1.975959e-01	7.587133e-01	2.247194e-01	2.961918e-01	3.079906e-01	1.250966e-01	1.249439e-01	1.318042e-01	1.366718e-01	1.705798e-01	1.785110e-01	1.749535e-01
	min.	3.735650e-01	1.488463e-01	2.401319e-01	1.802096e-01	2.808715e-01	2.719160e-01	1.214201e-01	1.210227e-01	1.286900e-01	1.311231e-01	1.487007e-01	1.323880e-01	1.418472e-01
	max.	4.443634e-01	3.772378e-01	8.393102e-01	2.619483e-01	3.030455e-01	3.363956e-01	1.322076e-01	1.369715e-01	1.411387e-01	1.432550e-01	2.153361e-01	2.236578e-01	3.334280e-01
	std.	1.413018e-02	3.397331e-02	1.271877e-01	2.248444e-02	4.442200e-03	1.241110e-02	2.536143e-03	2.636641e-03	2.449717e-03	2.503209e-03	1.623275e-02	2.389270e-02	4.199832e-02
7D	med.	4.343148e-01	3.440468e-01	8.429179e-01	2.419546e-01	2.774333e-01	3.187206e-01	1.232536e-01	1.120113e-01	1.223056e-01	1.246457e-01	2.574549e-01	3.229825e-01	2.857606e-01
	avg.	4.359014e-01	3.353358e-01	8.105448e-01	2.386901e-01	2.771794e-01	3.198555e-01	1.231663e-01	1.124541e-01	1.236163e-01	1.308452e-01	2.677419e-01	3.027522e-01	2.199816e-01
	min.	3.777385e-01	2.000330e-01	5.294144e-01	1.684323e-01	2.531187e-01	2.842846e-01	1.153656e-01	1.054097e-01	1.134670e-01	1.141125e-01	1.724947e-01	1.853558e-01	1.252304e-01
	max.	4.937049e-01	5.073692e-01	8.640804e-01	2.519046e-01	2.909911e-01	3.434849e-01	1.322372e-01	1.252486e-01	1.345976e-01	1.633117e-01	3.822358e-01	3.697643e-01	2.858367e-01
	std.	2.388592e-02	6.034955e-02	8.412839e-02	1.356145e-02	5.140655e-03	1.190305e-02	3.996287e-03	3.990852e-03	4.912387e-03	1.403245e-02	4.854880e-02	5.154710e-02	6.970364e-02
8D	med.	3.958258e-01	3.910733e-01	8.615399e-01	2.117140e-01	2.237142e-01	2.903940e-01	9.513872e-02	9.237544e-02	1.033036e-01	1.088902e-01	2.709355e-01	3.581000e-01	2.501423e-01
	avg.	3.970969e-01	3.863597e-01	8.060663e-01	2.111544e-01	2.233931e-01	2.916418e-01	9.599811e-02	9.247797e-02	1.036557e-01	1.130569e-01	2.809094e-01	3.370049e-01	2.155685e-01
	min.	3.626414e-01	2.295467e-01	5.154715e-01	1.450662e-01	2.076465e-01	2.722252e-01	8.962729e-02	8.807676e-02	9.624155e-02	1.023517e-01	1.579253e-01	1.902961e-01	1.117124e-01
	max.	4.414216e-01	6.363271e-01	8.856623e-01	2.252227e-01	2.287352e-01	3.145797e-01	1.038440e-01	9.906414e-02	1.118305e-01	1.331580e-01	4.110527e-01	3.873650e-01	2.502372e-01
	std.	1.715577e-02	7.112413e-02	1.169835e-01	1.018793e-02	3.278038e-03	8.507205e-03	3.229683e-03	2.232276e-03	3.046774e-03	9.235392e-03	6.430625e-02	4.211061e-02	5.229950e-02
9D	med.	3.640612e-01	4.166012e-01	8.595639e-01	1.897329e-01	1.680306e-01	2.677992e-01	7.790277e-02	7.799115e-02	9.700807e-02	1.071347e-01	3.342514e-01	3.762671e-01	2.224687e-01
	avg.	3.653368e-01	4.194343e-01	7.992658e-01	1.925958e-01	1.677123e-01	2.687101e-01	7.812664e-02	7.807035e-02	9.708638e-02	1.095208e-01	3.164166e-01	3.680929e-01	1.997043e-01
	min.	3.340274e-01	2.870587e-01	5.176894e-01	1.755643e-01	1.472567e-01	2.438267e-01	7.350934e-02	7.489295e-02	9.321636e-02	9.616931e-02	1.570549e-01	2.303026e-01	1.102203e-01
	max.	4.095773e-01	5.617522e-01	8.814387e-01	2.045575e-01	1.792331e-01	2.877699e-01	8.421477e-02	8.324902e-02	1.010541e-01	1.315250e-01	4.367891e-01	4.189919e-01	2.225863e-01
	std.	1.300808e-02	6.252047e-02	1.161636e-01	6.173502e-03	4.996007e-03	7.665020e-03	1.845232e-03	1.636700e-03	1.774969e-03	8.176933e-03	6.676923e-02	3.553534e-02	4.108912e-02
10D	med.	3.355777e-01	4.680029e-01	8.772046e-01	1.775378e-01	1.171438e-01	2.528987e-01	6.661583e-02	7.026106e-02	9.928312e-02	1.130913e-01	3.689437e-01	3.852454e-01	2.004894e-01
	avg.	3.355397e-01	4.641696e-01	8.119527e-01	1.759446e-01	1.071718e-01	2.523461e-01	6.676906e-02	7.044428e-02	9.923467e-02	1.143710e-01	3.552823e-01	3.792880e-01	1.881742e-01
	min.	3.091716e-01	1.843003e-01	5.611475e-01	1.221256e-01	8.679442e-02	2.369723e-01	6.496670e-02	6.795303e-02	9.525059e-02	1.013938e-01	2.286995e-01	2.542836e-01	1.049623e-01
	max.	3.601877e-01	6.108167e-01	8.950633e-01	1.871958e-01	1.277782e-01	2.693399e-01	9.665291e-02	7.438315e-02	1.032157e-01	1.462061e-01	4.297338e-01	4.246900e-01	2.007823e-01
	std.	1.048693e-02	7.198241e-02	1.159063e-01	9.156060e-03	1.536245e-02	7.270639e-03	1.139569e-03	1.300813e-03	1.502417e-03	7.598627e-03	4.200752e-02	2.641330e-02	2.857735e-02

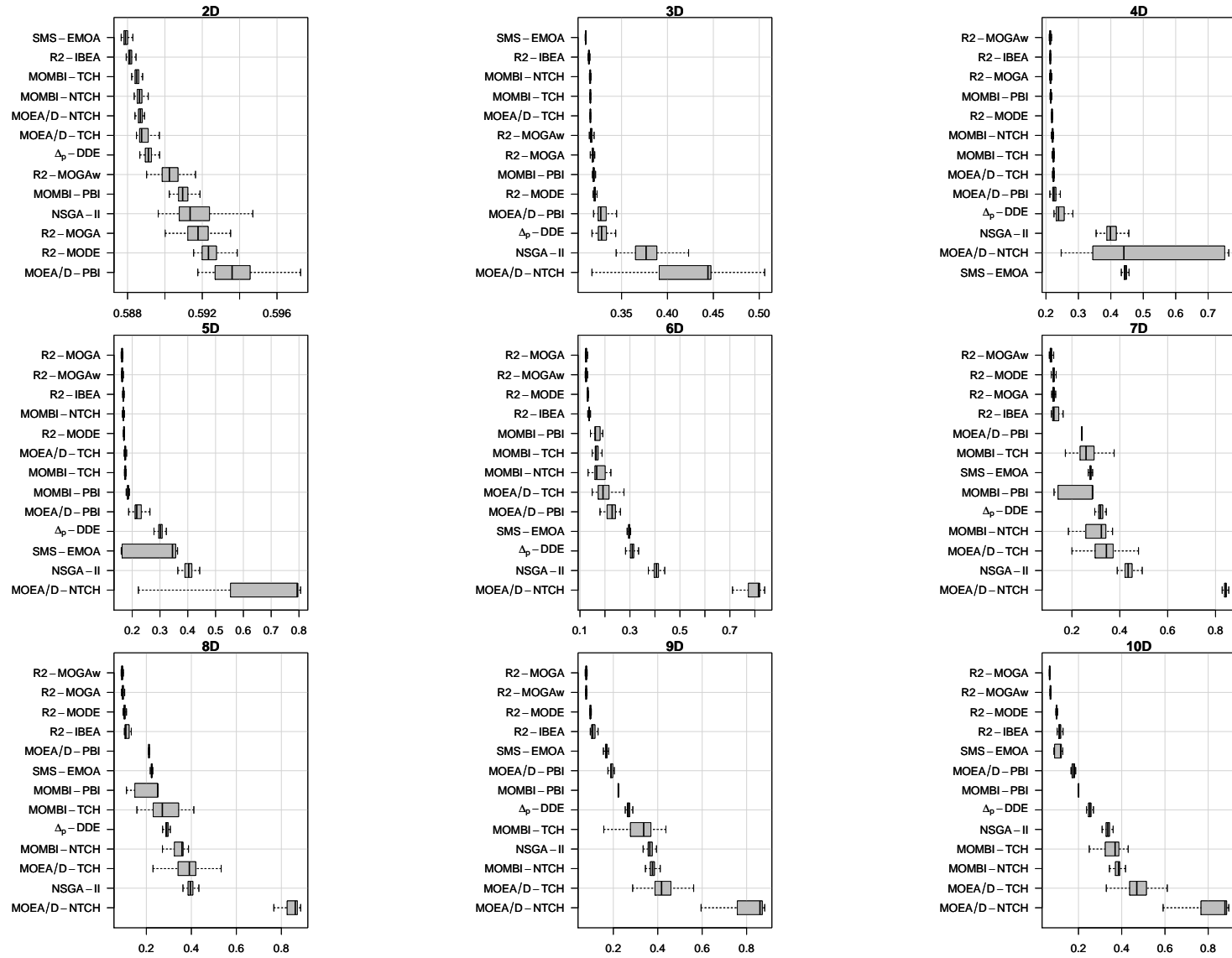


Figure A.210: Box-plot of R2 indicator values for different optimizers on the WFG7 test problem.

Table A.85: Comparison of runtime (in milliseconds) for different optimizers on the WFG7 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_\mu$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	4.196190e+02	8.560720e+02	1.411405e+03	1.174941e+03	3.901854e+04	1.605808e+03	1.904364e+03	1.561175e+03	1.819359e+03	5.369370e+05	2.144533e+03	1.965213e+03	3.074559e+03
	avg.	4.275783e+02	8.724441e+02	1.408891e+03	1.182793e+03	3.917198e+04	1.612666e+03	1.890371e+03	1.558726e+03	1.786995e+03	5.349004e+05	2.134526e+03	1.998731e+03	3.055208e+03
	min.	3.539550e+02	6.538890e+02	1.089324e+03	9.117270e+02	2.727001e+04	1.156802e+03	1.393753e+03	1.197661e+03	1.197570e+03	4.898440e+05	1.670276e+03	1.729639e+03	2.647373e+03
	max.	5.829840e+02	1.184948e+03	1.840500e+03	1.641108e+03	4.377536e+04	2.220698e+03	2.316836e+03	1.959046e+03	2.280689e+03	6.044740e+05	2.608513e+03	2.565226e+03	3.200270e+03
	std.	5.140437e+01	1.051328e+02	1.146811e+02	1.148231e+02	3.845551e+03	1.720147e+02	2.164173e+02	1.731543e+02	2.281623e+02	4.012817e+04	1.782650e+02	1.490547e+02	8.976354e+01
3D	med.	5.764570e+02	8.793030e+02	1.706126e+03	1.339226e+03	1.123041e+06	1.905519e+03	2.207061e+03	1.915398e+03	2.264394e+03	5.669360e+05	2.128146e+03	2.095647e+03	3.134486e+03
	avg.	5.789482e+02	8.795873e+02	1.671606e+03	1.310192e+03	1.084926e+06	1.906811e+03	2.129941e+03	1.892704e+03	2.240158e+03	5.642883e+05	2.132862e+03	2.118058e+03	3.104906e+03
	min.	4.866490e+02	6.616090e+02	1.227247e+03	9.916570e+02	8.218466e+05	1.420358e+03	1.529007e+03	1.488132e+03	1.697294e+03	5.138950e+05	1.663932e+03	1.814906e+03	2.504612e+03
	max.	7.805790e+02	1.217928e+03	1.968213e+03	1.584773e+03	1.198246e+06	2.543617e+03	2.669108e+03	2.221430e+03	2.602306e+03	5.872020e+05	2.678505e+03	2.602839e+03	3.421362e+03
	std.	5.755353e+01	1.047895e+02	1.355518e+02	1.136545e+02	1.037255e+05	2.025527e+02	2.905260e+02	1.734503e+02	2.039501e+02	1.555138e+04	1.705703e+02	1.536880e+02	1.980046e+02
4D	med.	8.448630e+02	1.072154e+03	1.918952e+03	1.664171e+03	3.418901e+06	2.485935e+03	2.851205e+03	2.485231e+03	3.257268e+03	5.936130e+05	2.241755e+03	2.473392e+03	4.199952e+03
	avg.	8.460850e+02	1.092929e+03	1.908499e+03	1.611032e+03	3.328435e+06	2.447409e+03	2.809616e+03	2.481392e+03	3.147053e+03	5.932559e+05	2.288689e+03	2.483828e+03	4.152941e+03
	min.	6.911140e+02	8.480360e+02	1.498856e+03	1.183146e+03	2.092638e+06	1.641551e+03	2.118171e+03	2.001335e+03	2.177919e+03	5.637610e+05	1.809941e+03	2.033319e+03	3.267360e+03
	max.	1.037493e+03	1.496274e+03	2.209084e+03	1.787182e+03	3.845740e+06	3.253455e+03	3.329219e+03	2.809361e+03	3.665122e+03	6.027470e+05	3.013751e+03	3.232456e+03	4.653926e+03
	std.	7.287625e+01	1.140819e+02	1.376167e+02	1.413228e+02	3.796100e+05	3.173200e+02	2.512882e+02	2.060501e+02	3.615139e+02	4.755770e+03	2.764971e+02	1.803171e+02	2.450320e+02
5D	med.	1.092017e+03	1.257298e+03	2.480448e+03	1.626499e+03	3.150177e+06	3.140958e+03	3.632980e+03	3.809380e+03	4.389820e+03	6.738920e+05	2.648345e+03	2.901141e+03	4.097075e+03
	avg.	1.096349e+03	1.258052e+03	2.435510e+03	1.671211e+03	3.141266e+06	3.131676e+03	3.536400e+03	3.137287e+03	3.808151e+03	6.737915e+05	2.755632e+03	2.887232e+03	4.103678e+03
	min.	9.242310e+02	1.065151e+03	1.869205e+03	1.322982e+03	1.648472e+06	3.174709e+03	2.441797e+03	2.326109e+03	2.764367e+03	6.626910e+05	2.169886e+03	2.444149e+03	3.338965e+03
	max.	1.300366e+03	1.573689e+03	2.803125e+03	2.083241e+03	4.983846e+06	6.712812e+03	4.238852e+03	3.654095e+03	4.643956e+03	6.814850e+05	4.290475e+03	3.377740e+03	4.626189e+03
	std.	8.493077e+01	1.110858e+02	2.050503e+02	1.894970e+02	1.049017e+06	7.592904e+02	4.401752e+02	3.191248e+02	3.910724e+02	3.406326e+03	4.029659e+02	1.837761e+02	2.348741e+02
6D	med.	1.379964e+03	1.476412e+03	2.662966e+03	1.774492e+03	4.460867e+06	4.940896e+03	4.318079e+03	3.856636e+03	4.900903e+03	6.549220e+05	2.715364e+03	2.898030e+03	4.539484e+03
	avg.	1.361738e+03	1.468838e+03	2.623555e+03	1.811011e+03	4.337211e+06	4.930791e+03	4.246829e+03	3.811504e+03	4.886154e+03	6.826825e+05	2.748137e+03	2.929086e+03	4.470928e+03
	min.	1.104018e+03	1.163732e+03	2.020743e+03	1.386129e+03	3.088337e+06	3.206628e+03	3.004849e+03	2.975471e+03	3.749083e+03	5.950900e+05	2.235659e+03	2.618699e+03	3.718346e+03
	max.	1.610363e+03	1.768565e+03	3.012266e+03	2.188117e+03	4.882698e+06	7.073290e+03	4.850679e+03	4.468532e+03	5.726891e+03	7.290370e+05	3.701797e+03	3.636542e+03	5.187597e+03
	std.	1.228485e+02	1.131443e+02	2.037989e+02	1.960646e+02	4.141508e+05	7.511577e+02	3.964604e+02	2.824493e+02	4.773792e+02	3.700979e+04	2.332395e+02	1.934196e+02	2.438895e+02
7D	med.	1.395117e+03	1.637240e+03	2.903134e+03	2.045212e+03	3.045751e+06	1.412447e+04	4.147097e+03	3.437683e+03	3.967740e+03	2.979820e+05	2.342967e+03	2.568520e+03	3.620227e+03
	avg.	1.391876e+03	1.634474e+03	2.855843e+03	1.957202e+03	2.995075e+06	1.425404e+04	4.030772e+03	3.459558e+03	3.967008e+03	3.001329e+05	2.314847e+03	2.597584e+03	3.608035e+03
	min.	1.092533e+03	1.287274e+03	2.051035e+03	1.425780e+03	1.967529e+06	8.016062e+03	3.011764e+03	2.777392e+03	2.790561e+03	3.001329e+05	2.724250e+05	1.903819e+03	2.113543e+03
	max.	1.632982e+03	2.213443e+03	3.353794e+03	2.343190e+03	3.394925e+06	1.944305e+04	4.473610e+03	4.016691e+03	4.696789e+03	3.636990e+05	2.847287e+03	2.981454e+03	4.098664e+03
	std.	1.122167e+02	1.621870e+02	2.466482e+02	2.249165e+02	4.024291e+05	2.158087e+03	3.421579e+02	2.861122e+02	3.382885e+02	3.031309e+04	1.822675e+02	1.687611e+02	2.130984e+02
8D	med.	1.860728e+03	1.865504e+03	3.127776e+03	2.056791e+03	4.816439e+06	2.689471e+04	5.441089e+03	5.028285e+03	6.351655e+03	1.273819e+06	2.810299e+03	3.393452e+03	4.855133e+03
	avg.	1.860209e+03	1.867232e+03	3.118606e+03	2.078361e+03	4.723245e+06	2.827821e+04	5.246156e+03	4.985590e+03	6.394676e+03	1.243456e+06	2.869547e+03	3.443737e+03	4.840882e+03
	min.	1.542930e+03	1.392775e+03	2.308499e+03	1.603661e+03	3.113904e+06	1.630774e+04	3.937270e+03	3.746657e+03	4.472583e+03	1.166352e+06	2.391686e+03	3.011804e+03	4.292748e+03
	max.	2.183030e+03	2.393078e+03	3.534761e+03	2.550713e+03	5.268591e+06	4.546571e+04	6.361786e+03	5.919859e+03	7.409557e+03	1.321963e+06	3.707721e+03	4.046044e+03	5.077839e+03
	std.	1.343934e+02	1.588953e+02	2.523943e+02	2.116160e+02	4.374900e+05	5.791552e+03	7.050577e+02	4.719912e+02	6.987443e+02	5.189461e+04	2.587420e+02	2.739787e+02	1.598273e+02
9D	med.	2.467048e+03	2.071171e+03	3.950292e+03	2.407709e+03	7.539931e+06	5.716589e+04	7.398815e+03	7.155952e+03	9.900079e+03	3.217838e+06	4.386492e+03	4.735996e+03	7.282815e+03
	avg.	2.439548e+03	2.069439e+03	3.892693e+03	2.382007e+03	7.483691e+06	5.839434e+04	7.262798e+03	7.011171e+03	9.912999e+03	3.002700e+06	4.314845e+03	4.793886e+03	7.266560e+03
	min.	1.910328e+03	1.628231e+03	2.909681e+03	1.854579e+03	5.366484e+06	3.626576e+04	5.154619e+03	5.204656e+03	7.140644e+03	2.248155e+06	3.203753e+03	4.047249e+03	5.900808e+03
	max.	2.790609e+03	2.531719e+03	4.306382e+03	2.789711e+03	8.342975e+06	8.402154e+04	8.583747e+03	8.101746e+03	1.108908e+04	3.278176e+06	4.673395e+03	5.537160e+03	8.478091e+03
	std.	1.820328e+02	1.773199e+02	2.362372e+02	2.494243e+02	5.559975e+05	9.645586e+03	8.456498e+02	6.826149e+02	7.927764e+02	3.509391e+05	2.980244e+02	3.916168e+02	4.565737e+02
10D	med.	2.790622e+03	2.333907e+03	4.160734e+03	2.671917e+03	1.023130e+07	1.326591e+05	9.636087e+03	9.641789e+03	8.693820e+03	4.451226e+06	4.577642e+03	5.712821e+03	9.133507e+03
	avg.	2.840326e+03	2.297912e+03	4.060263e+03	2.620443e+03	1.032812e+07	1.359612e+05	9.857575e+03	9.652135e+03	1.025196e+04	4.651597e+06	4.879419e+03	5.995448e+03	9.286586e+03
	min.	2.426110e+03	1.833195e+03	2.980395e+03	1.936636e+03	7.740979e+06	8.413646e+04	7.411481e+03	7.629354e+03	7.585892e+03	4.277189e+06	4.066117e+03	5.326972e+03	8.062776e+03
	max.	3.502970e+03	2.936089e+03	4.574087e+03	3.173834e+03	1.256273e+07	2.426625e+05	1.229011e+04	1.143858e+04	1.530217e+04	5.427535e+06	6.298523e+03	7.812036e+03	1.188365e+04
	std.	3.709630e+02	2.443443e+02	3.676978e+02	3.528066e+02	1.110991e+06	2.659797e+04	1.765736e+03	9.515835e+02	2.733552e+03	4.064420e+05	7.640950e+02	7.179128e+02	8.200739e+02

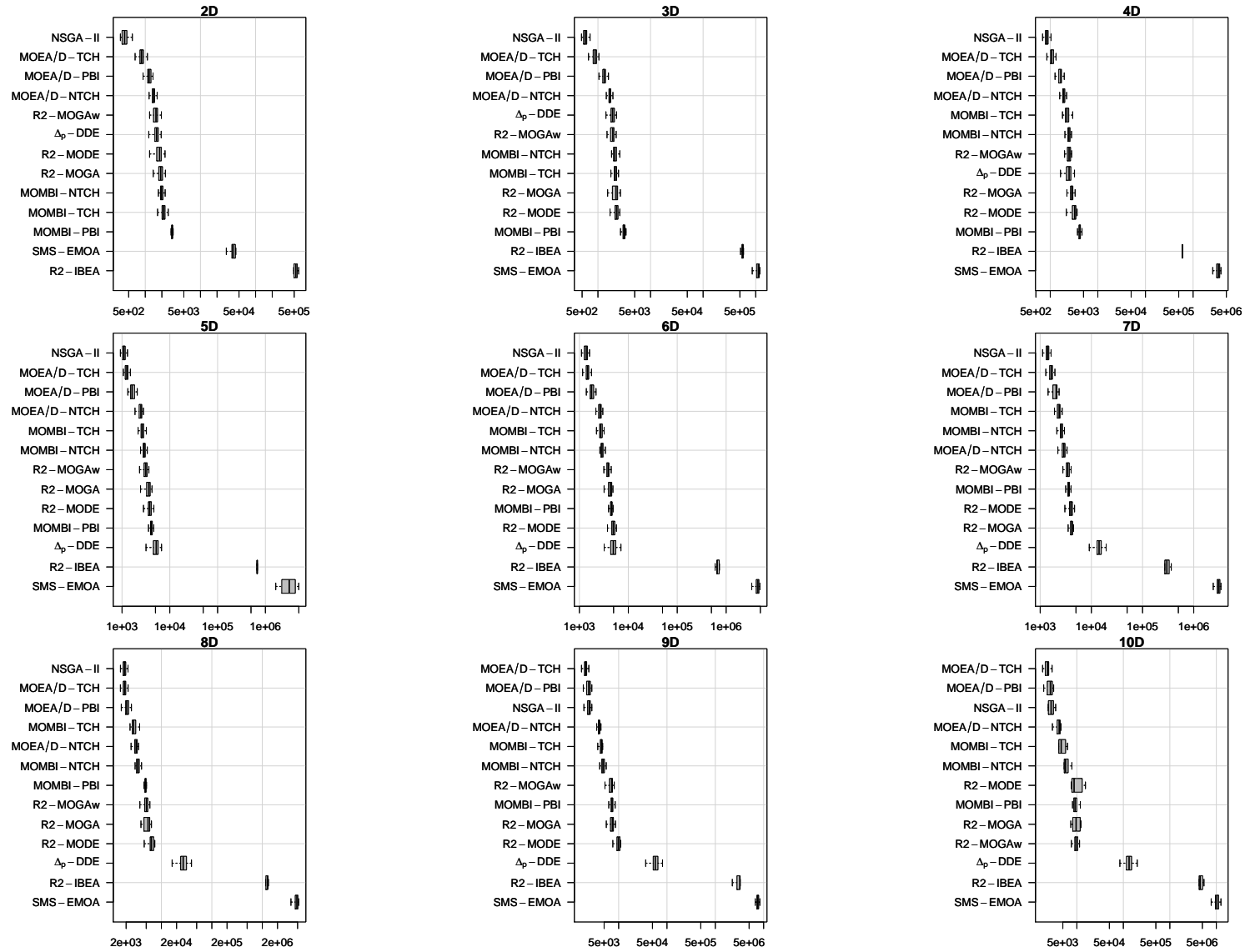


Figure A.211: Box-plot of runtime (in logarithmic scale) for different optimizers on the WFG7 test problem.

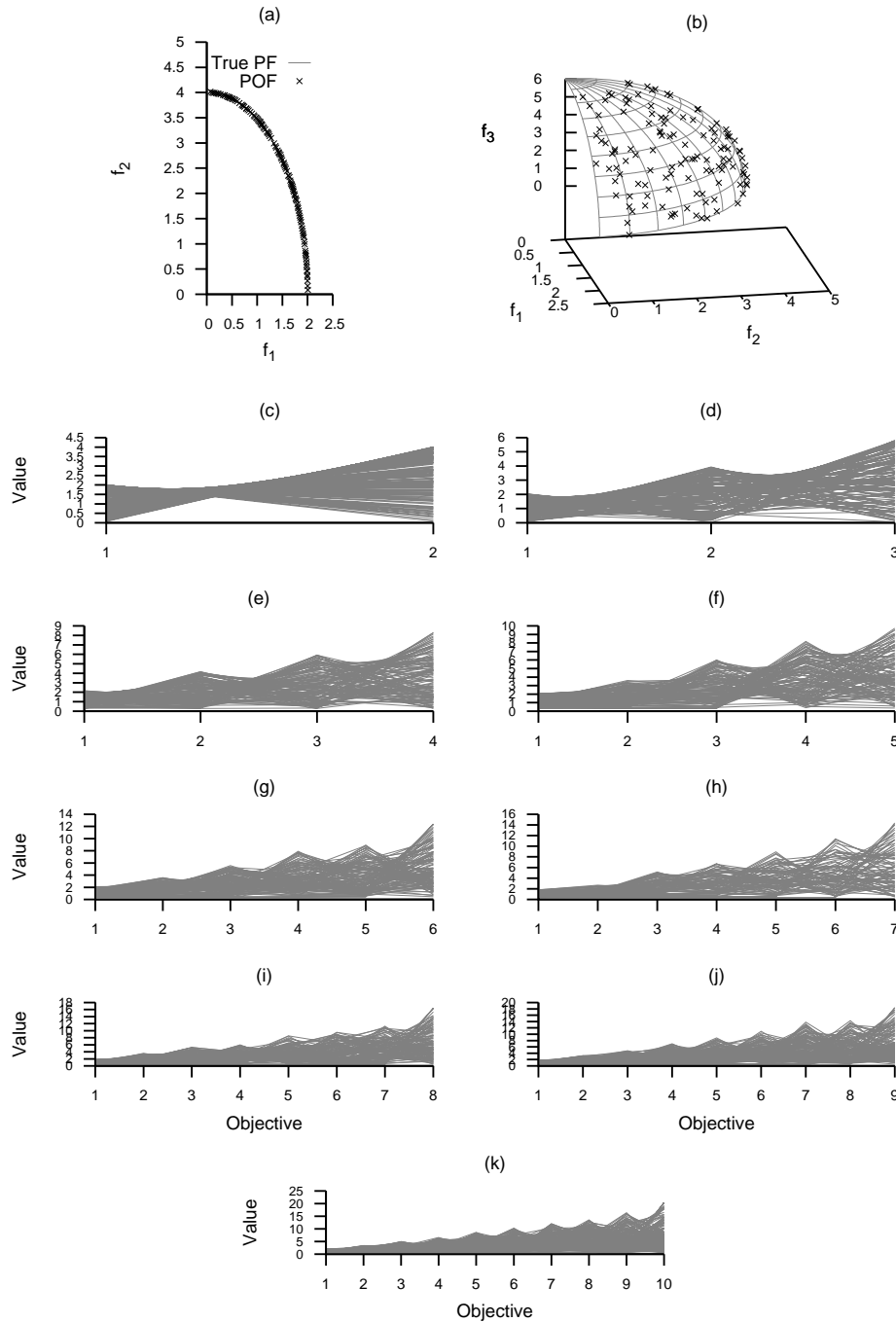


Figure A.212: Plots of the approximations obtained by NSGA-II from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



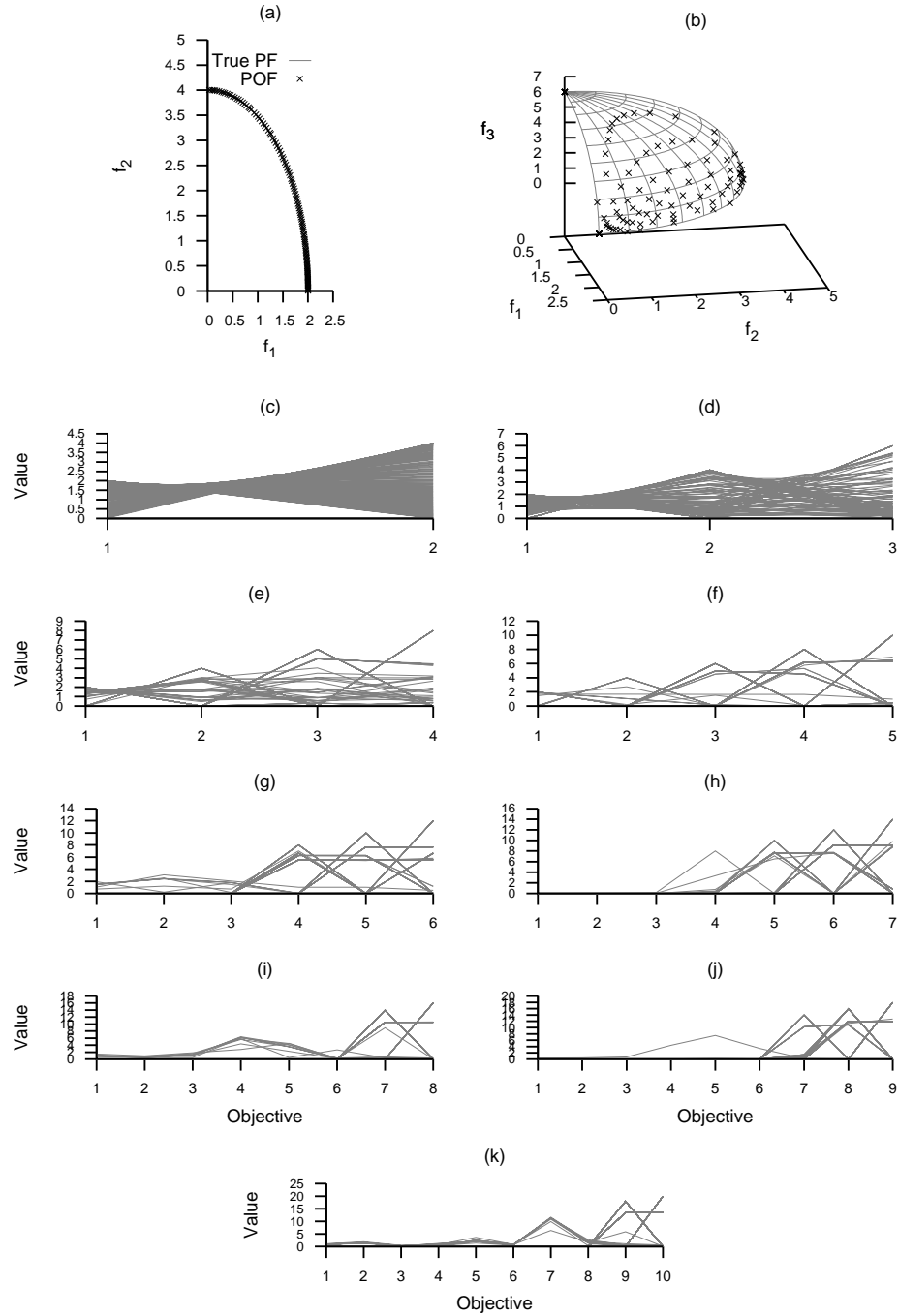


Figure A.213: Plots of the approximations obtained by MOEA/D-TCH from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

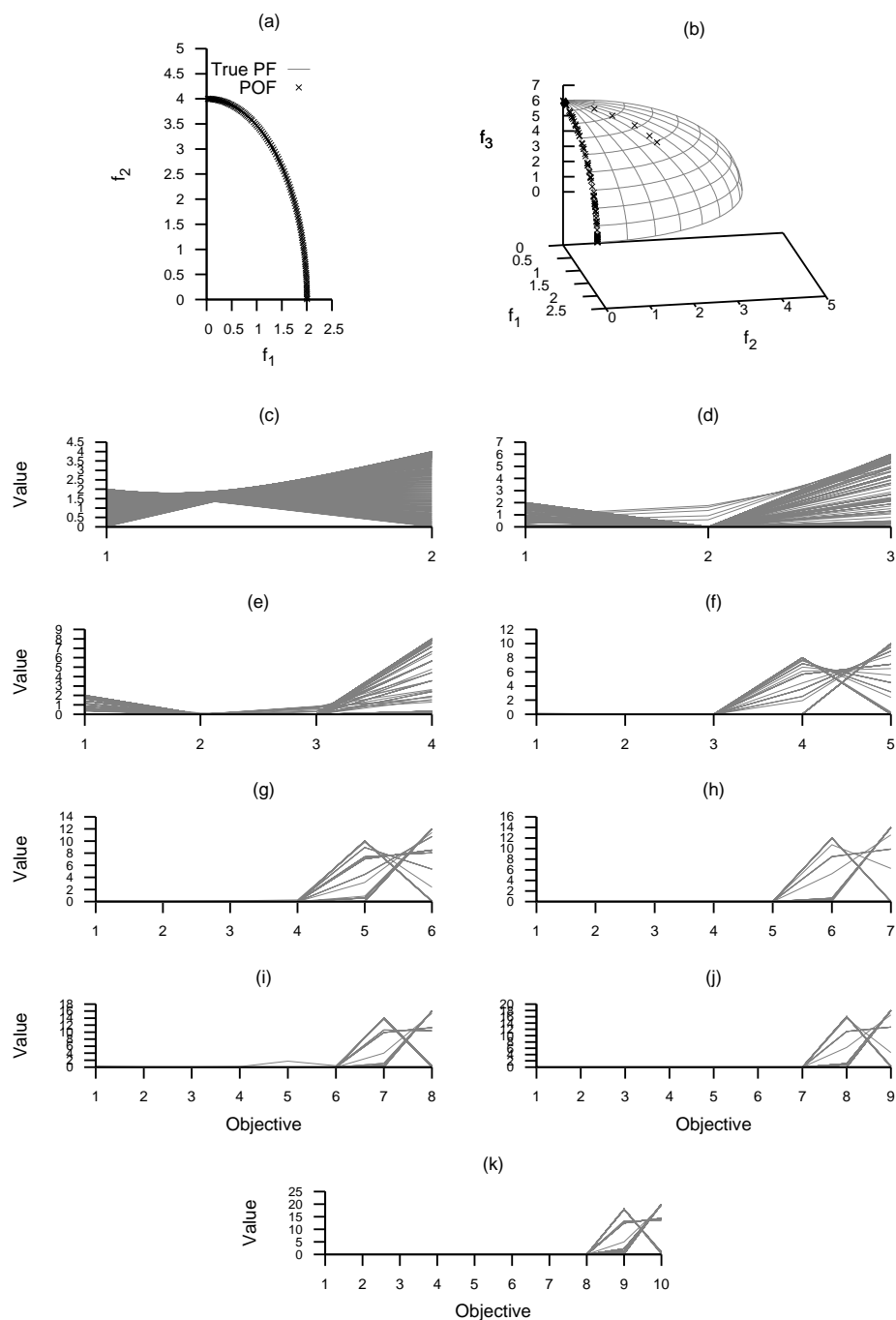


Figure A.214: Plots of the approximations obtained by MOEA/D-NTCH from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

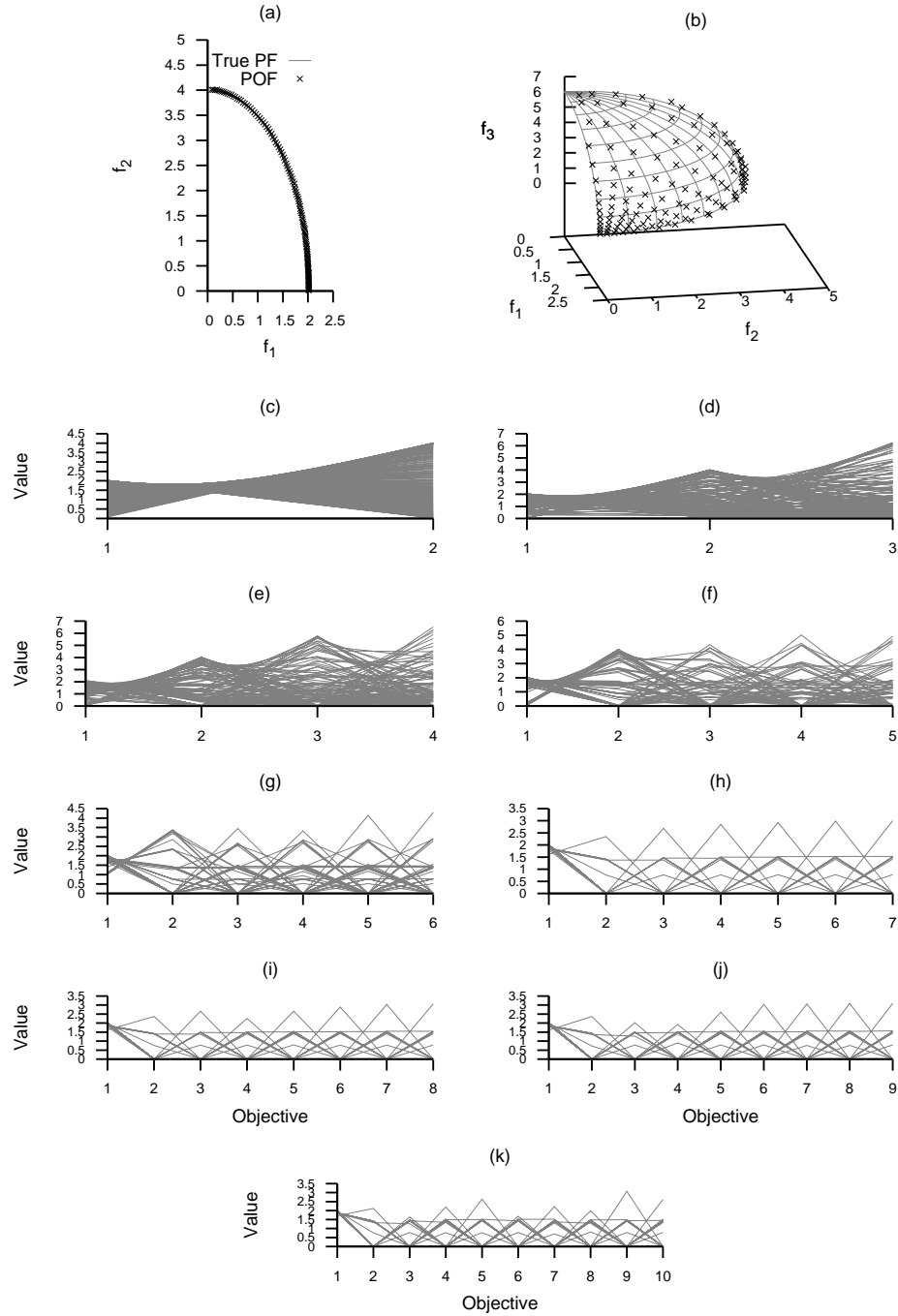


Figure A.215: Plots of the approximations obtained by MOEA/D-PBI from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

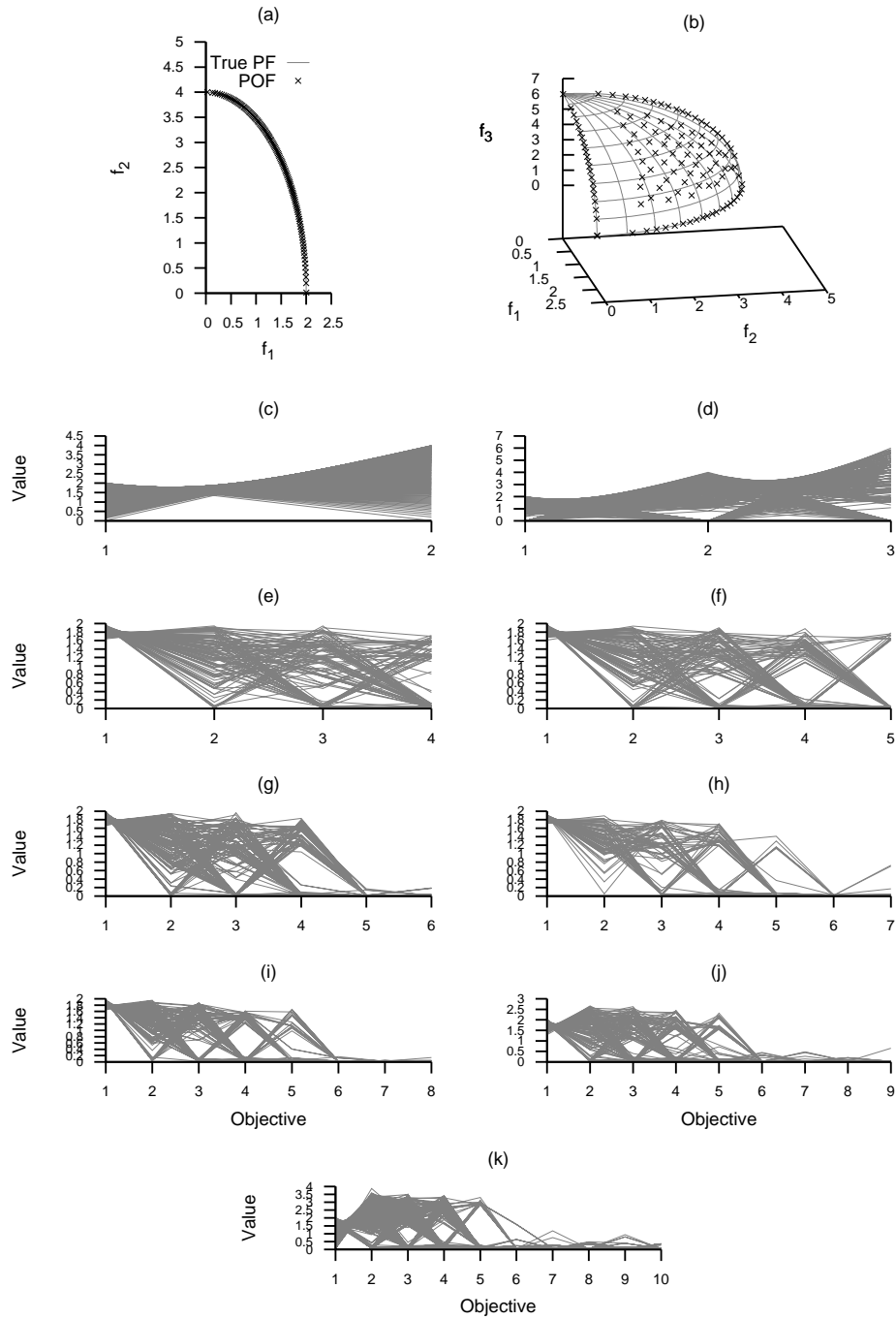


Figure A.216: Plots of the approximations obtained by SMS-EMOA from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

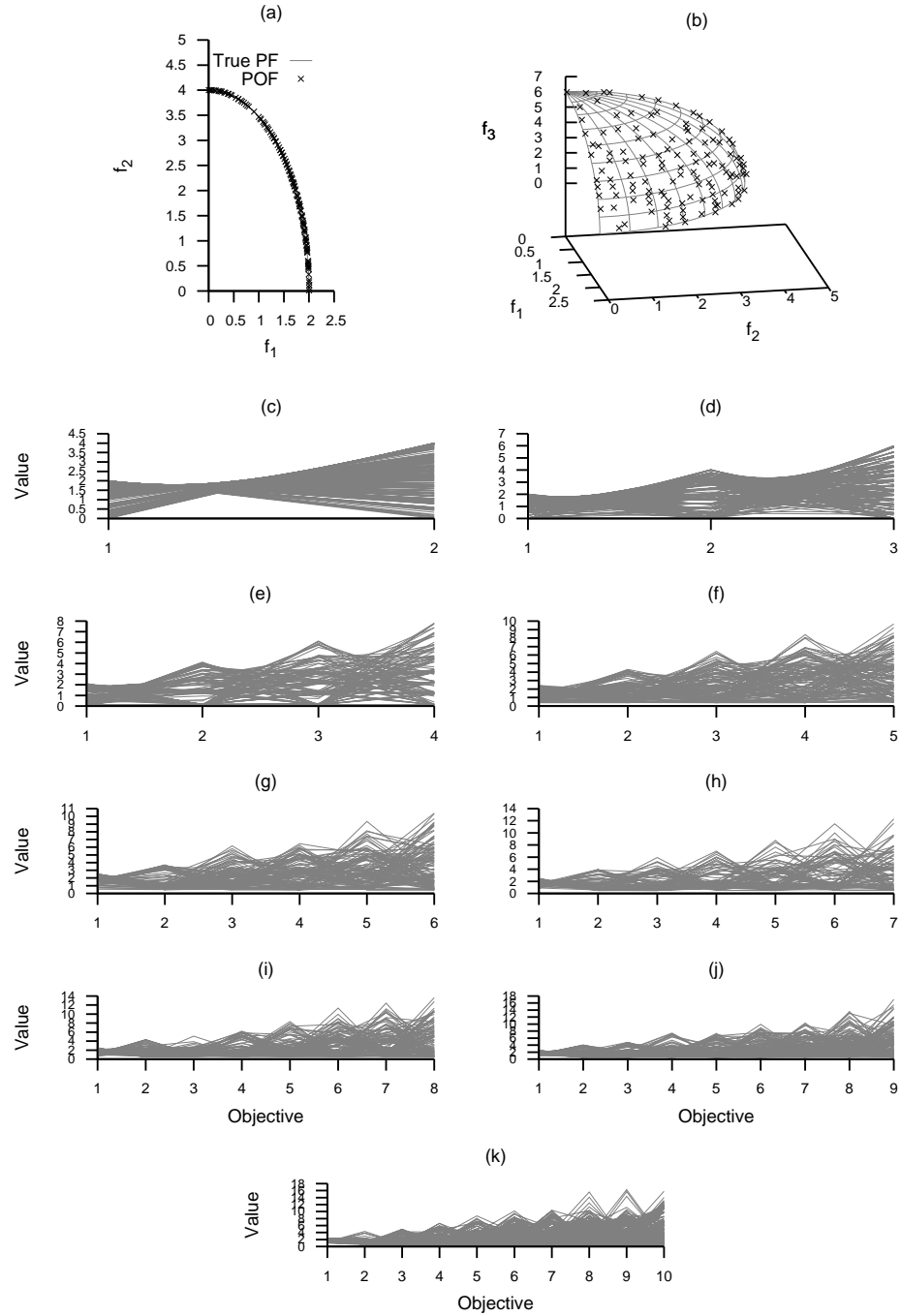


Figure A.217: Plots of the approximations obtained by  $\Delta_p$ -DDE from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

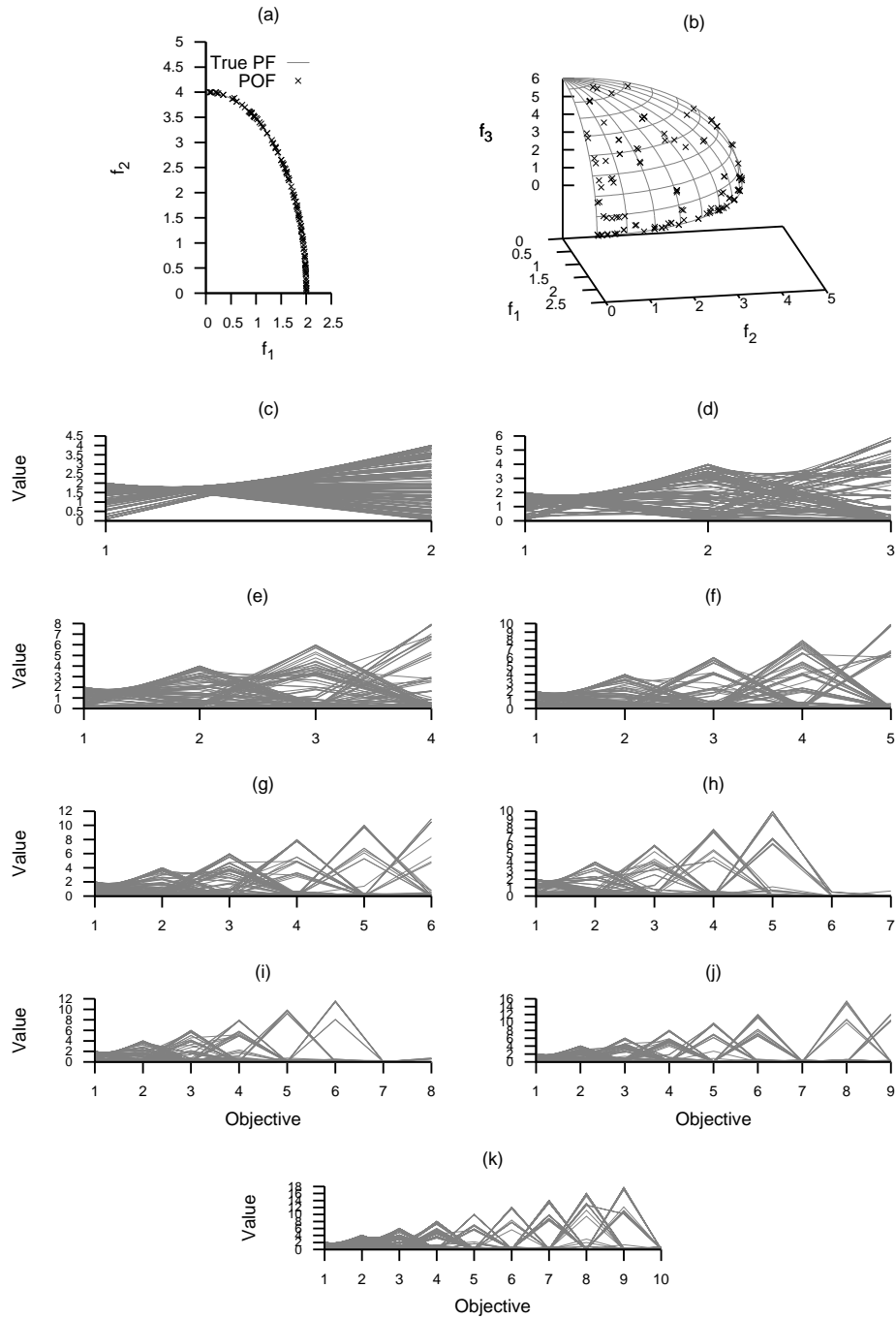


Figure A.218: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

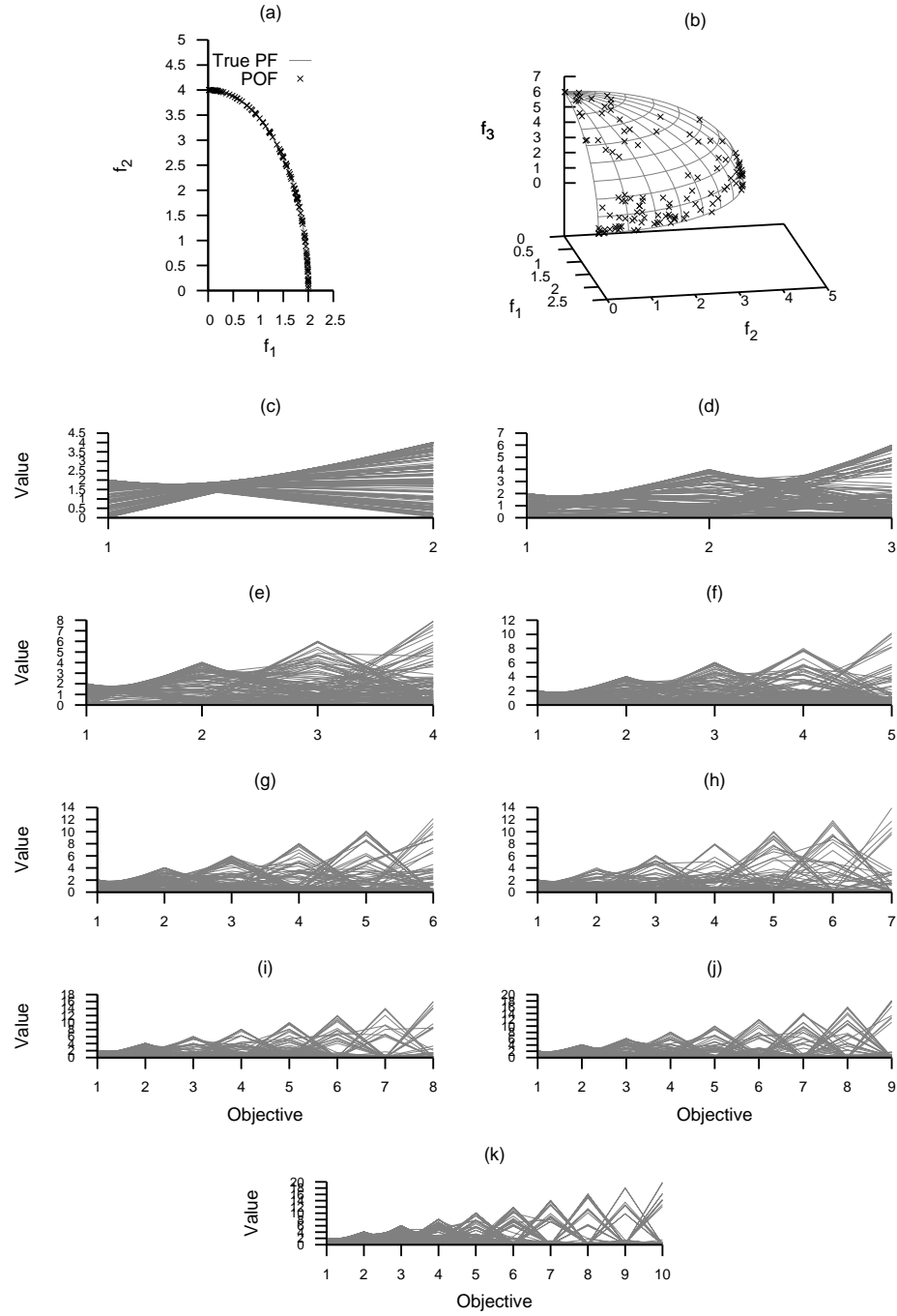


Figure A.219: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

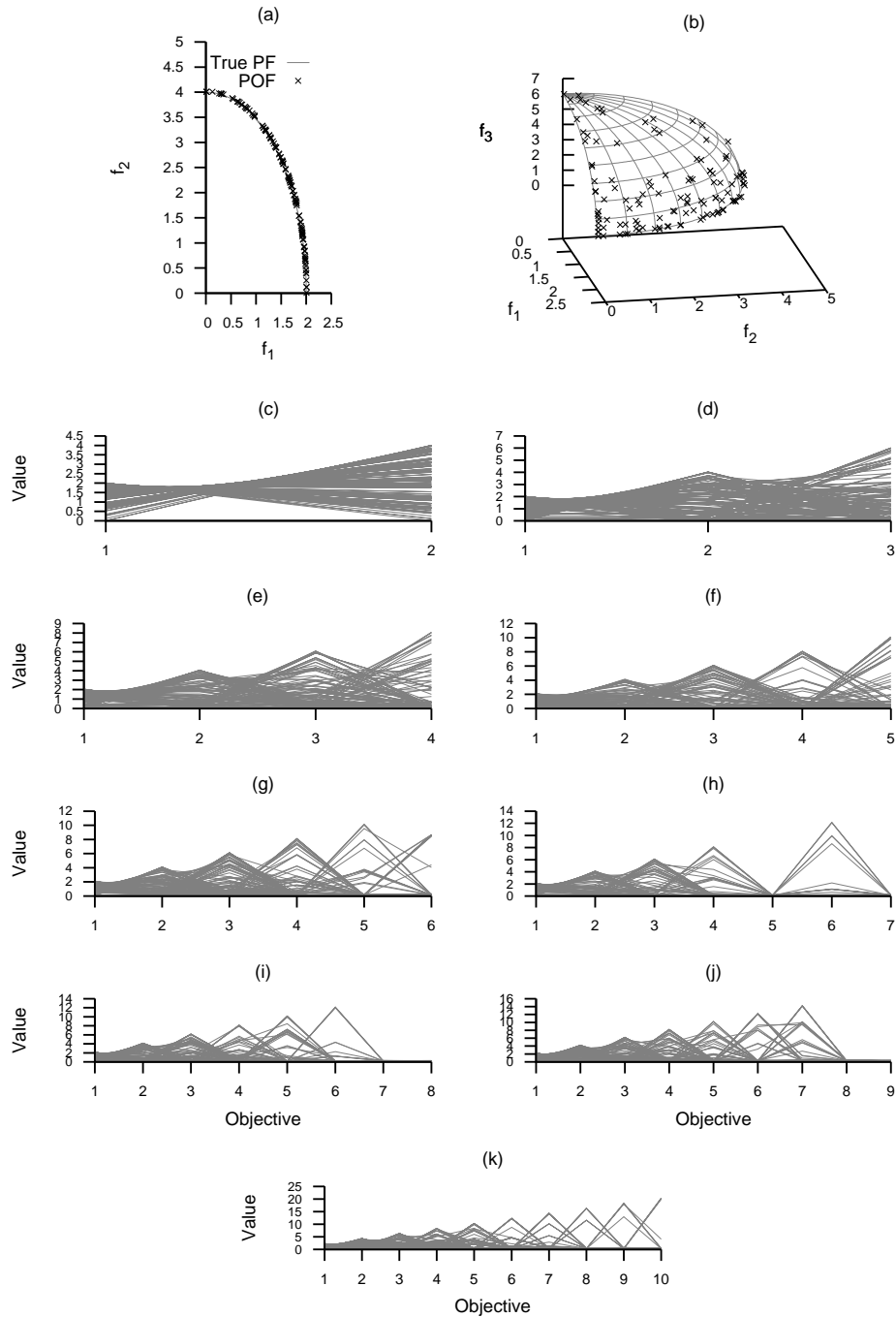


Figure A.220: Plots of the approximations obtained by *R2-MODE* from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



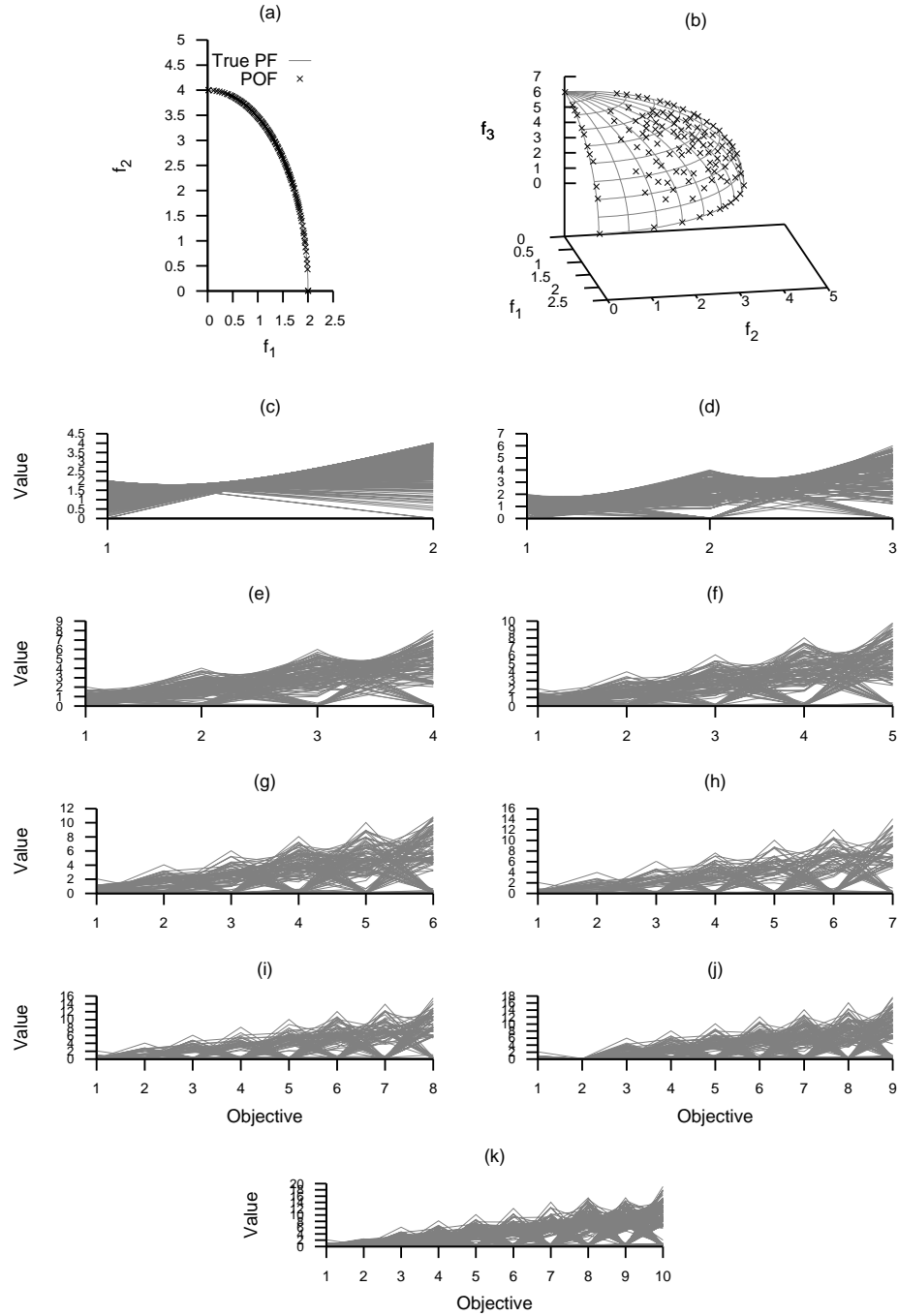


Figure A.221: Plots of the approximations obtained by *R2-IBEA* from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

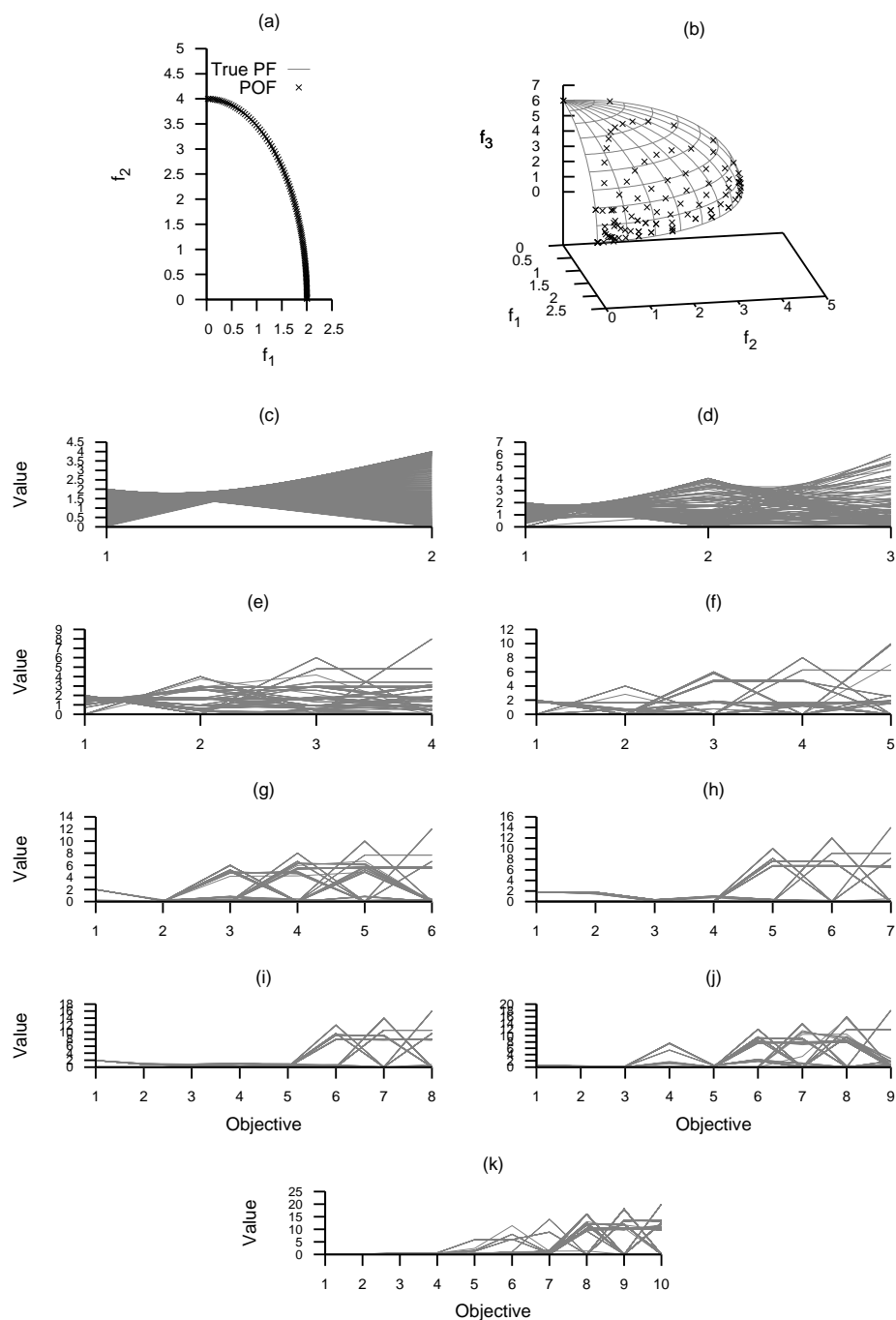


Figure A.222: Plots of the approximations obtained by MOMBI-TCH from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

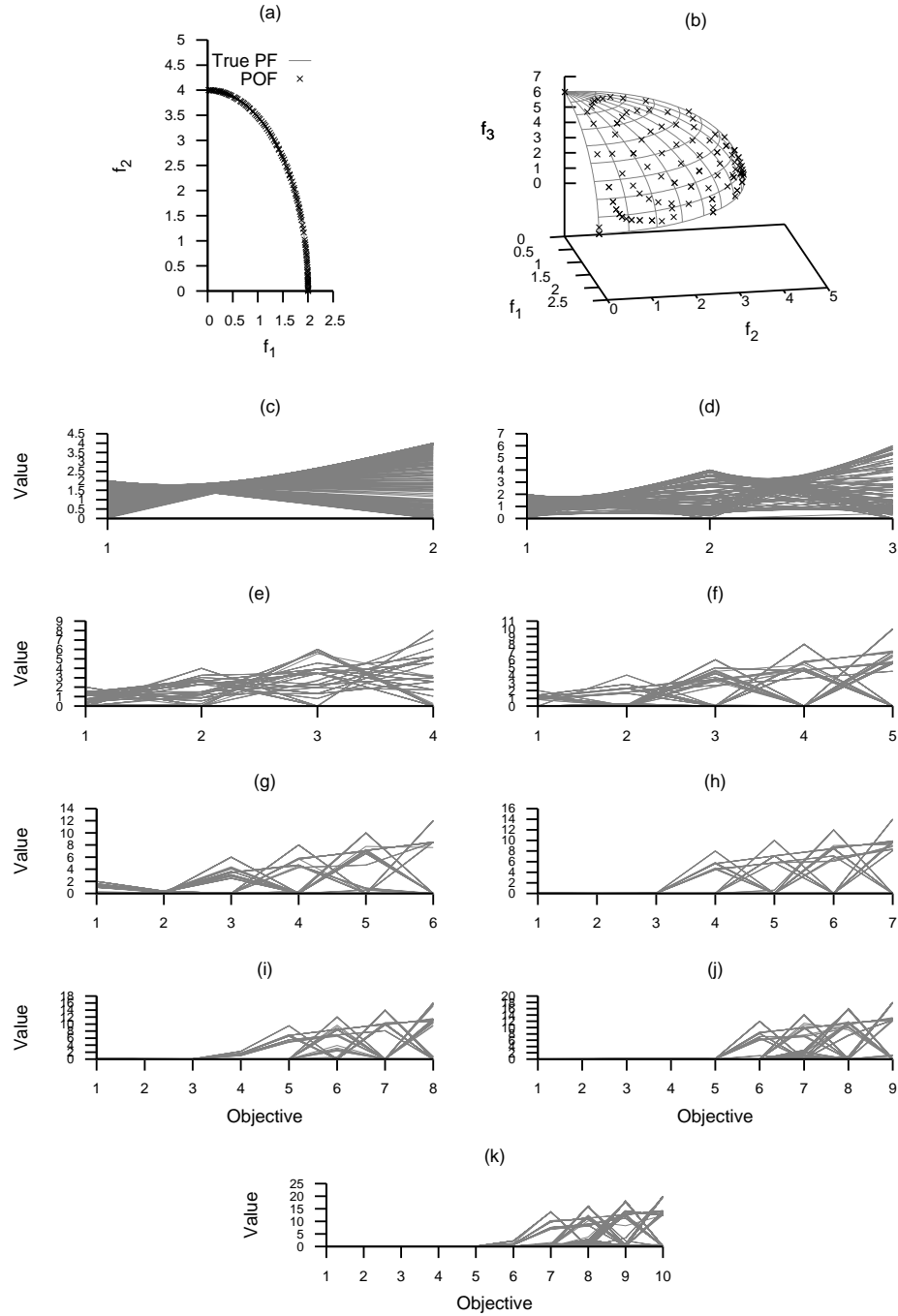


Figure A.223: Plots of the approximations obtained by MOMBI-NTCH from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

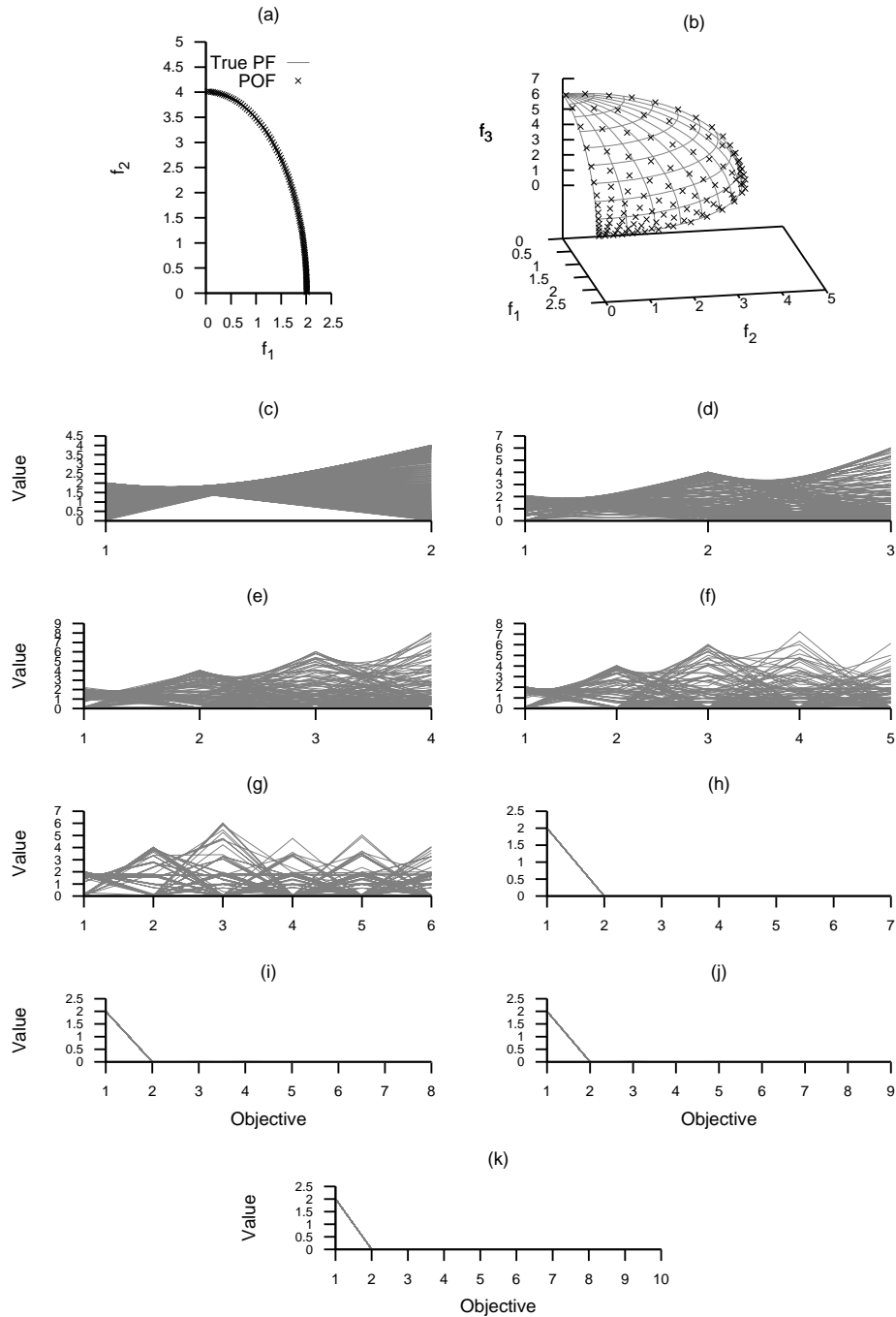


Figure A.224: Plots of the approximations obtained by MOMBI-PBI from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.86: Comparison of hypervolume indicator values for different optimizers on the WFG8 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	7.414018e+00	7.656952e+00	7.690919e+00	7.471072e+00	7.589346e+00	7.497658e+00	7.474729e+00	7.576929e+00	7.378723e+00	7.616577e+00	7.606740e+00	7.630038e+00	7.423013e+00
	avg.	7.410571e+00	7.639845e+00	7.694180e+00	7.452049e+00	7.582036e+00	7.498123e+00	7.476148e+00	7.573019e+00	7.379619e+00	7.613844e+00	7.604978e+00	7.626727e+00	7.413928e+00
	min.	7.249999e+00	7.423127e+00	7.509650e+00	7.156570e+00	7.403324e+00	7.440070e+00	7.406427e+00	7.497625e+00	7.329458e+00	7.496403e+00	7.533713e+00	7.567827e+00	7.311988e+00
	max.	7.559110e+00	7.758234e+00	7.785776e+00	7.584452e+00	7.655364e+00	7.555113e+00	7.538277e+00	7.648406e+00	7.432824e+00	7.694835e+00	7.662401e+00	7.674512e+00	7.498454e+00
	std.	5.872359e-02	6.728818e-02	4.521205e-02	9.989329e-02	4.636608e-02	2.254993e-02	2.800474e-02	3.551417e-02	2.282147e-02	3.352068e-02	2.634150e-02	2.348470e-02	4.488246e-02
3D	med.	6.136014e+01	6.820326e+01	5.646411e+01	6.708345e+01	7.097425e+01	6.397082e+01	6.531562e+01	6.742250e+01	6.411767e+01	6.975463e+01	6.728580e+01	6.812604e+01	6.730882e+01
	avg.	6.139574e+01	6.817618e+01	5.612454e+01	6.703574e+01	7.094368e+01	6.396532e+01	6.528575e+01	6.740995e+01	6.407731e+01	6.973631e+01	6.729550e+01	6.813741e+01	6.728380e+01
	min.	6.001858e+01	6.701647e+01	3.336945e+01	5.917705e+01	7.016162e+01	6.218915e+01	6.370779e+01	6.576586e+01	6.252444e+01	6.920995e+01	6.680781e+01	6.756006e+01	6.638516e+01
	max.	6.326637e+01	6.866650e+01	6.044412e+01	6.870213e+01	7.125347e+01	6.517403e+01	6.647188e+01	6.865250e+01	6.548347e+01	7.009516e+01	6.789230e+01	6.856292e+01	6.809308e+01
	std.	6.615193e-01	3.154008e-01	3.422397e+00	1.036261e+00	1.748198e-01	5.482895e-01	5.620513e-01	5.419310e-01	5.768989e-01	1.883877e-01	2.100657e-01	2.123212e-01	3.372698e-01
4D	med.	5.178129e+02	5.779936e+02	4.038885e+02	6.398205e+02	3.275459e+02	5.861289e+02	6.276188e+02	6.497390e+02	5.938900e+02	6.892077e+02	5.726299e+02	6.012084e+02	6.600620e+02
	avg.	5.171332e+02	5.777390e+02	4.004869e+02	6.340418e+02	3.279101e+02	5.862035e+02	6.256156e+02	6.505546e+02	5.927055e+02	6.891638e+02	5.777320e+02	6.028895e+02	6.598377e+02
	min.	4.820578e+02	5.625277e+02	3.085031e+02	4.223093e+02	3.157218e+02	5.605313e+02	5.920013e+02	6.256638e+02	5.639649e+02	6.800161e+02	5.567845e+02	5.844026e+02	6.458711e+02
	max.	5.377159e+02	5.912511e+02	5.519041e+02	6.738891e+02	3.480212e+02	6.073269e+02	6.446698e+02	6.652269e+02	6.138718e+02	6.962646e+02	6.072617e+02	6.335064e+02	6.782308e+02
	std.	1.139422e+01	6.103535e+00	4.017219e+01	3.226526e+01	6.447711e+00	9.694184e+00	9.742541e+00	7.022852e+00	9.604922e+00	2.966325e+00	1.362981e+01	1.035031e+01	5.742691e+00
5D	med.	4.701915e+03	5.570846e+03	3.778863e+03	5.442972e+03	3.509819e+03	4.897645e+03	7.083948e+03	7.352185e+03	6.497748e+03	7.782207e+03	5.774852e+03	5.863087e+03	7.288308e+03
	avg.	4.689386e+03	5.562041e+03	3.735984e+03	5.314447e+03	3.517666e+03	4.885363e+03	7.060591e+03	7.348311e+03	6.509441e+03	7.781706e+03	5.805813e+03	5.900438e+03	7.288489e+03
	min.	4.171368e+03	4.937473e+03	2.014974e+03	3.515850e+03	3.389561e+03	4.528120e+03	6.731320e+03	7.031697e+03	6.273247e+03	7.662703e+03	5.606126e+03	5.684910e+03	7.057470e+03
	max.	5.177757e+03	6.076709e+03	4.723501e+03	6.592384e+03	3.756003e+03	5.240091e+03	7.295738e+03	7.631910e+03	6.874916e+03	7.908423e+03	6.138275e+03	6.304071e+03	7.494259e+03
	std.	1.809007e+02	1.395862e+02	4.097390e+02	7.984237e+02	6.992227e+01	1.260683e+02	1.230738e+02	9.449962e+01	1.142037e+02	5.245949e+01	1.263648e+02	1.457249e+02	9.843935e+01
6D	med.	5.088712e+04	5.223261e+04	4.072945e+04	4.878435e+04	4.586166e+04	4.691687e+04	9.172090e+04	9.604253e+04	8.272414e+04	1.004391e+05	5.860406e+04	6.274318e+04	8.662297e+04
	avg.	5.105257e+04	5.308794e+04	4.056820e+04	4.829028e+04	4.586564e+04	4.715374e+04	9.174727e+04	9.595053e+04	8.277720e+04	1.005372e+05	6.013998e+04	6.253890e+04	8.592590e+04
	min.	4.621694e+04	4.731843e+04	2.964448e+04	2.854790e+04	4.374862e+04	4.083625e+04	8.719301e+04	9.265241e+04	7.785012e+04	9.840714e+04	5.630949e+04	5.076812e+04	6.125103e+04
	max.	5.696151e+04	6.545067e+04	5.419895e+04	6.934252e+04	4.891951e+04	5.534253e+04	9.745408e+04	9.936997e+04	8.580723e+04	1.035575e+05	6.848363e+04	7.391515e+04	9.065210e+04
	std.	2.081902e+03	4.135705e+03	4.568342e+03	1.143986e+04	8.763283e+02	3.497003e+03	1.881598e+03	1.474205e+03	1.581094e+03	9.739120e+03	3.038988e+03	4.384717e+03	3.543793e+03
7D	med.	6.295993e+05	5.429104e+05	5.197118e+05	5.179028e+05	5.959733e+05	5.992080e+05	1.231875e+06	1.390706e+06	1.175122e+06	1.492982e+06	6.888843e+05	7.093740e+05	1.181845e+06
	avg.	6.330144e+05	5.629083e+05	5.066791e+05	5.242716e+05	5.945667e+05	6.017049e+05	1.234433e+06	1.388300e+06	1.173963e+06	1.492344e+06	6.915676e+05	7.117015e+05	1.159868e+06
	min.	5.556352e+05	4.822746e+05	3.075501e+05	3.031279e+05	5.568856e+05	5.010612e+05	1.156930e+06	1.287122e+06	1.089088e+06	1.455914e+06	5.543968e+05	5.629852e+05	8.118912e+05
	max.	7.129799e+05	7.299513e+05	7.060748e+05	1.114366e+06	6.188820e+05	7.037062e+05	1.330749e+06	1.455884e+06	1.251766e+06	1.543029e+06	9.120445e+05	9.213501e+05	1.313261e+06
	std.	3.606480e+04	6.030944e+04	6.912706e+04	1.569529e+05	1.142818e+04	4.682738e+04	3.862599e+04	3.150896e+04	3.450404e+04	1.867656e+04	7.870499e+04	7.824725e+04	9.786009e+04
8D	med.	1.076408e+07	8.008727e+06	8.256383e+06	8.771063e+06	1.158736e+07	1.002094e+07	2.226608e+07	2.378661e+07	1.952435e+07	2.614665e+07	1.086252e+07	1.058468e+07	1.850981e+07
	avg.	1.067027e+07	8.322891e+06	8.109114e+06	9.097558e+06	1.158884e+07	1.000957e+07	2.228269e+07	2.375972e+07	1.953351e+07	2.612709e+07	1.108777e+07	1.056747e+07	1.822260e+07
	min.	9.635240e+06	6.839033e+06	4.298967e+06	4.360080e+06	1.093574e+07	8.223773e+06	2.045365e+07	2.248448e+07	1.838993e+07	2.535326e+07	8.404531e+06	8.213546e+06	6.880557e+06
	max.	1.148794e+07	1.150289e+07	1.233601e+07	1.999896e+07	1.214117e+07	1.170195e+07	2.345003e+07	2.507242e+07	2.086694e+07	2.691552e+07	1.470861e+07	1.571690e+07	2.099143e+07
	std.	4.409612e+05	1.034676e+06	1.260024e+06	3.002025e+06	2.616136e+05	8.119872e+05	5.680952e+05	4.634828e+05	5.105384e+05	3.253082e+05	1.433053e+06	1.636957e+06	1.764341e+06
9D	med.	2.038997e+08	1.367203e+08	1.495139e+08	1.512546e+08	2.768925e+08	1.805773e+08	4.359670e+08	4.481271e+08	3.549264e+08	4.894094e+08	1.986093e+08	1.848086e+08	3.205919e+08
	avg.	2.042755e+08	1.416472e+08	1.477315e+08	1.498197e+08	2.770711e+08	1.812214e+08	4.351253e+08	4.485837e+08	3.551497e+08	4.887379e+08	2.012299e+08	1.856252e+08	3.106811e+08
	min.	1.864115e+08	1.194040e+08	7.581260e+07	7.768961e+07	2.548909e+08	1.434406e+08	3.999600e+08	4.156698e+08	3.310152e+08	4.687732e+08	1.443829e+08	1.355544e+08	1.335061e+08
	max.	2.220711e+08	2.112974e+08	2.061202e+08	2.876264e+08	3.033815e+08	2.089691e+08	4.546783e+08	4.686136e+08	3.788361e+08	5.067261e+08	2.800170e+08	2.657768e+08	3.574663e+08
	std.	7.372729e+06	1.839811e+07	2.202065e+07	4.564036e+07	9.827313e+06	1.490355e+07	1.073683e+07	6.686111e+06	8.669565e+06	6.072263e+06	3.204641e+07	3.128124e+07	4.224189e+07
10D	med.	4.312222e+09	2.561497e+09	3.043866e+09	3.176005e+09	7.925282e+09	3.545467e+09	9.232464e+09	9.150706e+09	7.087128e+09	9.785364e+09	4.255596e+09	3.530811e+09	6.172762e+09
	avg.	4.321592e+09	2.707882e+09	3.026470e+09	3.208768e+09	7.765067e+09	3.514929e+09	9.191803e+09	9.169301e+09	7.079780e+09	9.787128e+09	4.169900e+09	3.456948e+09	5.905880e+09
	min.	3.967435e+09	2.098695e+09	1.951931e+09	1.472281e+09	6.692926e+09	2.756863e+09	8.686943e+09	8.840177e+09	6.701380e+09	9.511568e+09	2.544255e+09	2.583693e+09	1.701902e+09
	max.	4.709810e+09	3.898766e+09	4.279190e+09	7.665369e+09	8.802851e+09	4.231838e+09	9.636592e+09	9.549076e+09	7.366548e+09	1.005151e+10	5.770651e+09	5.503842e+09	6.987059e+09
	std.	1.550633e+08	4.121497e+08	3.710229e+08	1.166053e+09	6.802840e+08	3.094936e+08	2.138387e+08	1.522943e+08	1.522943e+08	1.179608e+08	7.443742e+08	6.545885e+08	9.424136e+08

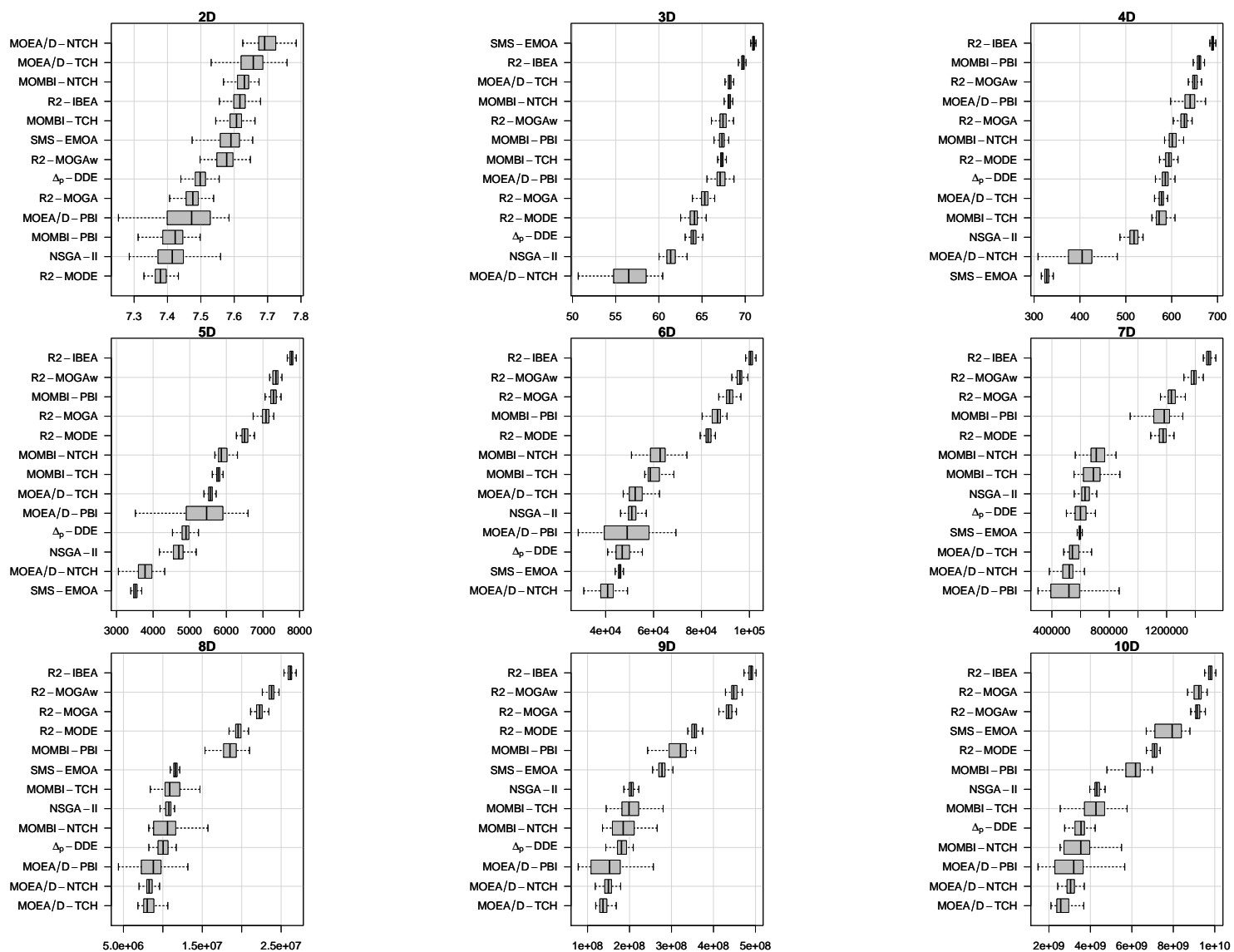


Table A.87: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG8 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.06e-07	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	9.39e-33	—	> 0.05	1.43e-28	1.66e-14	9.22e-28	2.42e-29	9.70e-17	1.44e-34	1.73e-08	3.69e-11	5.62e-06	5.88e-33
MOEA/D-NTCH	1.40e-34	4.00e-11	—	5.38e-34	1.59e-30	3.55e-34	1.89e-34	2.59e-31	1.28e-34	2.34e-27	2.06e-29	4.41e-26	1.28e-34
MOEA/D-PBI	2.02e-06	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	1.12e-12	> 0.05	> 0.05	> 0.05	7.74e-07
SMS-EMOA	9.95e-33	> 0.05	> 0.05	4.10e-25	—	2.08e-26	7.80e-30	1.69e-02	1.95e-34	> 0.05	> 0.05	> 0.05	1.27e-33
$\Delta_p$ -DDE	1.17e-25	> 0.05	> 0.05	1.18e-02	> 0.05	—	1.49e-08	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	3.75e-31
R2-MOGA	6.12e-18	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	3.05e-34	> 0.05	> 0.05	> 0.05	9.91e-22
R2-MOGAw	7.25e-34	> 0.05	> 0.05	1.46e-24	> 0.05	3.56e-29	2.38e-32	—	1.28e-34	> 0.05	> 0.05	> 0.05	1.32e-34
R2-MODE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	2.33e-34	> 0.05	> 0.05	2.52e-32	5.46e-08	7.43e-33	6.63e-34	4.20e-15	1.28e-34	—	4.27e-03	> 0.05	1.32e-34
MOMBI-TCH	1.53e-34	> 0.05	> 0.05	5.71e-33	1.59e-04	1.78e-34	1.36e-34	5.53e-11	1.28e-34	> 0.05	—	> 0.05	1.28e-34
MOMBI-NTCH	1.28e-34	> 0.05	> 0.05	2.07e-34	3.34e-14	1.28e-34	1.28e-34	8.76e-23	1.28e-34	1.32e-03	1.24e-08	—	1.28e-34
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	2.45e-10	> 0.05	> 0.05	> 0.05	—
3D													
NSGA-II	—	> 0.05	1.68e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	1.28e-34	1.44e-26	> 0.05	1.28e-34	1.28e-34	8.45e-24	1.28e-34	> 0.05	2.04e-30	1.90e-02	6.07e-30
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	2.51e-33	> 0.05	1.78e-34	—	> 0.05	2.66e-33	5.90e-31	> 0.05	2.82e-33	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34
$\Delta_p$ -DDE	2.88e-34	> 0.05	1.28e-34	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	1.29e-28	—	> 0.05	3.98e-26	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	1.28e-34	> 0.05	1.28e-34	2.14e-04	> 0.05	1.28e-34	3.65e-34	—	1.28e-34	> 0.05	3.48e-03	> 0.05	5.28e-03
R2-MODE	2.26e-34	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	1.28e-34	> 0.05	1.28e-34	6.87e-03	> 0.05	1.28e-34	1.28e-34	> 0.05	1.28e-34	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.28e-34	> 0.05	1.28e-34	4.08e-27	> 0.05	1.28e-34	1.28e-34	1.68e-24	1.28e-34	> 0.05	3.24e-34	—	2.82e-33
MOMBI-PBI	1.28e-34	> 0.05	1.28e-34	2.04e-02	> 0.05	1.28e-34	1.40e-34	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	—
4D													
NSGA-II	—	> 0.05	2.51e-33	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.28e-34	—	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	1.53e-27	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	2.92e-32	4.44e-29	3.24e-34	—	1.28e-34	2.34e-27	6.81e-11	> 0.05	2.99e-26	> 0.05	7.88e-29	1.25e-23	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e-34	9.21e-12	1.28e-34	> 0.05	1.28e-34	—	> 0.05	> 0.05	> 0.05	> 0.05	8.85e-07	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	5.38e-34	—	> 0.05	7.01e-33	> 0.05	3.76e-34	5.29e-26	> 0.05
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.96e-09	1.28e-34	1.28e-34	1.92e-33	—	1.28e-34	> 0.05	1.28e-34	1.36e-34	> 0.05
R2-MODE	1.28e-34	1.80e-22	1.28e-34	> 0.05	1.28e-34	1.45e-06	> 0.05	> 0.05	—	> 0.05	2.46e-14	> 0.05	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	1.28e-34	> 0.05	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	1.28e-34	5.22e-34	1.28e-34	> 0.05	1.28e-34	1.25e-21	> 0.05	> 0.05	6.09e-10	> 0.05	1.15e-24	—	> 0.05
MOMBI-PBI	1.28e-34	1.28e-34	1.28e-34	6.40e-23	1.28e-34	1.28e-34	1.28e-34	1.33e-17	1.28e-34	> 0.05	1.28e-34	1.28e-34	—

Table A.88: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG8 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
5D													
NSGA-II	—	> 0.05	$6.81e-33$	> 0.05	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$1.73e-34$	—	$1.28e-34$	> 0.05	$1.28e-34$	$5.88e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	$1.42e-13$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	$1.36e-12$	> 0.05	$2.02e-26$	—	$1.52e-33$	$4.62e-10$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	$7.47e-15$	> 0.05	$2.13e-34$	> 0.05	$1.28e-34$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	> 0.05	$1.89e-34$	> 0.05	$1.28e-34$	$1.28e-34$	> 0.05
R2-MOGAw	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$4.50e-32$	—	$1.28e-34$	> 0.05	$1.28e-34$	$1.28e-34$	$2.64e-05$
R2-MODE	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.82e-31$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	—	> 0.05	$1.28e-34$	$1.44e-34$	> 0.05
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$4.09e-29$	$1.28e-34$	$1.14e-06$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.59e-31$	$1.28e-34$	$2.38e-09$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	$2.90e-07$	—	> 0.05
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.63e-26$	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	$1.28e-34$	—
6D													
NSGA-II	—	> 0.05	$7.14e-32$	> 0.05	$8.92e-34$	$3.60e-15$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	$8.56e-04$	—	$1.73e-32$	$2.22e-03$	$2.79e-34$	$2.45e-19$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	> 0.05	$1.54e-06$	—	$4.42e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	$4.04e-23$	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	> 0.05	> 0.05	$2.97e-21$	> 0.05	$9.34e-03$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	$1.28e-34$	$2.12e-31$
R2-MOGAw	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.36e-29$	—	$1.28e-34$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$
R2-MODE	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	—	> 0.05	$1.28e-34$	$1.28e-34$	> 0.05
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.48e-34$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$1.68e-34$	$1.06e-22$	$1.28e-34$	$1.69e-14$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	$4.14e-33$	$1.97e-26$	$1.44e-34$	$2.61e-19$	$1.28e-34$	$3.15e-34$	> 0.05	> 0.05	> 0.05	> 0.05	$9.89e-06$	—	> 0.05
MOMBI-PBI	$1.28e-34$	$1.58e-34$	$1.28e-34$	$1.89e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$1.83e-17$	> 0.05	$4.24e-34$	$1.10e-33$	—
7D													
NSGA-II	—	$2.32e-16$	$1.77e-30$	$1.33e-17$	$6.45e-17$	$7.28e-07$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	> 0.05	—	$2.97e-07$	$4.55e-04$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	$1.00e-10$	$6.50e-26$	$2.45e-10$	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	> 0.05	$1.49e-08$	$4.04e-22$	$5.62e-11$	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	> 0.05	$1.95e-20$	> 0.05	$1.28e-34$	$1.28e-34$	$1.19e-10$
R2-MOGAw	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.01e-34$	—	$1.28e-34$	> 0.05	$1.28e-34$	$1.28e-34$	$1.53e-34$
R2-MODE	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.40e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	—	> 0.05	$1.28e-34$	$1.28e-34$	> 0.05
R2-IBEA	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	—	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$3.00e-09$	$1.13e-23$	$7.34e-32$	$2.18e-20$	$5.67e-21$	$1.07e-16$	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	$1.13e-14$	$2.63e-26$	$3.09e-32$	$3.65e-21$	$3.84e-20$	$1.59e-20$	> 0.05	> 0.05	> 0.05	> 0.05	$1.81e-02$	—	> 0.05
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$6.83e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	$1.89e-34$	$1.73e-34$	—



Table A.89: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG8 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
8D													
NSGA-II	—	2.90e-28	1.04e-31	3.40e-15	> 0.05	3.10e-10	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	1.36e-02	3.12e-03	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	4.63e-32	6.83e-34	2.51e-33	1.26e-18	—	2.59e-31	> 0.05	> 0.05	> 0.05	> 0.05	1.99e-05	4.84e-10	> 0.05
$\Delta_p$ -DDE	> 0.05	6.33e-22	3.23e-26	1.33e-08	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	> 0.05	1.40e-34	> 0.05	1.28e-34	1.28e-34	1.32e-34
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.20e-31	—	1.28e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-MODE	1.28e-34	1.28e-34	1.28e-34	1.61e-33	1.28e-34	1.28e-34	> 0.05	> 0.05	—	> 0.05	1.28e-34	1.28e-34	2.85e-14
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	2.15e-02	8.32e-27	3.19e-29	2.39e-13	> 0.05	2.67e-09	> 0.05	> 0.05	> 0.05	> 0.05	—	3.00e-03	> 0.05
MOMBI-NTCH	> 0.05	1.25e-21	1.28e-23	1.31e-08	> 0.05	1.24e-02	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	2.51e-33	2.44e-33	1.81e-33	1.84e-31	2.51e-33	2.51e-33	> 0.05	> 0.05	> 0.05	> 0.05	3.37e-33	3.47e-33	—
9D													
NSGA-II	—	1.29e-32	5.39e-33	4.71e-21	> 0.05	1.01e-25	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	1.24e-07	> 0.05
MOEA/D-TCH	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	9.26e-06	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	1.28e-34	1.28e-34	1.28e-34	1.92e-33	—	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	2.51e-33	2.26e-34	> 0.05
$\Delta_p$ -DDE	> 0.05	1.05e-26	3.14e-24	1.00e-10	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	> 0.05	1.28e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	4.27e-17	—	1.28e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-MODE	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	> 0.05	—	> 0.05	1.28e-34	1.28e-34	1.14e-27
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	> 0.05	8.79e-29	1.48e-24	2.40e-15	> 0.05	3.46e-07	> 0.05	> 0.05	> 0.05	> 0.05	—	1.42e-04	> 0.05
MOMBI-NTCH	> 0.05	5.74e-23	6.17e-15	6.00e-10	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	8.06e-31	1.54e-32	2.52e-31	6.74e-32	1.46e-18	8.06e-31	> 0.05	> 0.05	> 0.05	> 0.05	6.60e-30	1.54e-30	—
10D													
NSGA-II	—	1.28e-34	3.65e-34	1.39e-17	> 0.05	7.25e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	5.03e-24	> 0.05
MOEA/D-TCH	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	6.28e-09	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	4.17e-04	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	1.28e-34	1.28e-34	1.28e-34	5.06e-34	—	1.28e-34	> 0.05	> 0.05	1.17e-10	> 0.05	1.28e-34	1.28e-34	1.71e-33
$\Delta_p$ -DDE	> 0.05	2.28e-25	1.37e-18	8.28e-05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	2.19e-34	1.28e-34	—	> 0.05	1.28e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	—	1.28e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-MODE	1.28e-34	1.28e-34	1.28e-34	2.51e-33	> 0.05	1.28e-34	> 0.05	> 0.05	—	> 0.05	1.28e-34	1.28e-34	1.92e-33
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	2.48e-34	1.40e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	> 0.05	1.15e-28	8.76e-23	3.39e-12	> 0.05	5.84e-13	> 0.05	> 0.05	> 0.05	> 0.05	—	1.73e-11	> 0.05
MOMBI-NTCH	> 0.05	2.34e-17	9.06e-06	4.06e-03	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	1.22e-28	3.16e-31	5.28e-30	1.27e-27	> 0.05	1.25e-29	> 0.05	> 0.05	> 0.05	> 0.05	2.08e-26	7.38e-30	—

Table A.90: Comparison of R2 indicator values for different optimizers on the WFG8 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	6.641160e-01	6.366718e-01	6.364565e-01	6.470789e-01	6.464178e-01	6.462751e-01	6.446753e-01	6.411892e-01	6.506232e-01	6.430751e-01	6.399495e-01	6.400146e-01	6.490217e-01
	avg.	6.654519e-01	6.378997e-01	6.373616e-01	6.499995e-01	6.476742e-01	6.463679e-01	6.448314e-01	6.416302e-01	6.506399e-01	6.439286e-01	6.401307e-01	6.406152e-01	6.496341e-01
	min.	6.470628e-01	6.320527e-01	6.316537e-01	6.406955e-01	6.384644e-01	6.427142e-01	6.418965e-01	6.378456e-01	6.476618e-01	6.379157e-01	6.359056e-01	6.370029e-01	6.449793e-01
	max.	6.900674e-01	6.541211e-01	6.618029e-01	6.811928e-01	6.803194e-01	6.519751e-01	6.493231e-01	6.497886e-01	6.528222e-01	6.652628e-01	6.484678e-01	6.513542e-01	6.582463e-01
	std.	9.095514e-03	4.291531e-03	4.117389e-03	9.077938e-03	6.618074e-03	1.744722e-03	1.477848e-03	2.215190e-03	1.157507e-03	4.286528e-03	1.986016e-03	2.414530e-03	2.810682e-03
3D	med.	4.183488e-01	3.434536e-01	3.872177e-01	3.504220e-01	3.403462e-01	3.665793e-01	3.508970e-01	3.451697e-01	3.570212e-01	3.506177e-01	3.445136e-01	3.459429e-01	3.491413e-01
	avg.	4.180984e-01	3.440064e-01	4.049988e-01	3.512216e-01	3.405237e-01	3.674138e-01	3.509547e-01	3.455031e-01	3.570170e-01	3.506717e-01	3.445554e-01	3.459401e-01	3.493598e-01
	min.	3.897262e-01	3.423355e-01	3.754574e-01	3.445852e-01	3.388890e-01	3.598320e-01	3.481146e-01	3.433688e-01	3.536569e-01	3.462197e-01	3.432537e-01	3.438946e-01	3.473461e-01
	max.	4.453826e-01	3.514065e-01	7.947888e-01	3.953817e-01	3.450415e-01	3.776129e-01	3.542922e-01	3.536623e-01	3.600491e-01	3.556701e-01	3.458497e-01	3.487436e-01	3.540320e-01
	std.	1.173685e-02	1.873010e-03	6.478060e-02	5.777749e-03	9.948010e-04	3.725698e-03	1.142160e-03	1.370492e-03	1.421924e-03	1.921228e-03	5.319063e-04	8.205285e-04	1.226743e-03
4D	med.	4.004808e-01	2.493800e-01	4.671667e-01	2.510267e-01	4.740304e-01	2.814719e-01	2.407511e-01	2.389478e-01	2.531369e-01	2.492529e-01	2.481506e-01	2.499598e-01	2.420717e-01
	avg.	4.005993e-01	2.498480e-01	5.016000e-01	2.553271e-01	4.751906e-01	2.821083e-01	2.409231e-01	2.390849e-01	2.529302e-01	2.486702e-01	2.484181e-01	2.506637e-01	2.421560e-01
	min.	3.743815e-01	2.464607e-01	3.279691e-01	2.379215e-01	4.514836e-01	2.644359e-01	2.380183e-01	2.354353e-01	2.474652e-01	2.434029e-01	2.457445e-01	2.470497e-01	2.370629e-01
	max.	4.442850e-01	2.560838e-01	7.916881e-01	3.584085e-01	4.963844e-01	3.279172e-01	2.453343e-01	2.461132e-01	2.568304e-01	2.529794e-01	2.530989e-01	2.564817e-01	2.471030e-01
	std.	1.430875e-02	2.206162e-03	9.537618e-02	1.664007e-02	9.713372e-03	9.226527e-03	1.386877e-03	2.040643e-03	1.942015e-03	2.414092e-03	1.482602e-03	2.029756e-03	1.867703e-03
5D	med.	4.222028e-01	2.125974e-01	5.243703e-01	2.586742e-01	3.808077e-01	3.242866e-01	1.884953e-01	1.918732e-01	2.043135e-01	2.054029e-01	2.027619e-01	2.035163e-01	1.973540e-01
	avg.	4.238333e-01	2.159059e-01	5.449998e-01	2.684363e-01	3.805136e-01	3.237639e-01	1.885564e-01	1.919957e-01	2.045358e-01	2.053569e-01	2.030439e-01	2.036191e-01	1.996499e-01
	min.	3.895631e-01	2.055437e-01	4.101504e-01	2.170551e-01	3.619763e-01	3.047426e-01	1.868835e-01	2.007503e-01	2.007503e-01	1.998899e-01	1.989806e-01	2.002250e-01	1.926697e-01
	max.	4.676755e-01	3.120964e-01	8.843636e-01	3.589779e-01	3.913690e-01	3.493868e-01	1.924065e-01	2.040977e-01	2.083972e-01	2.105524e-01	2.089656e-01	2.081597e-01	2.121023e-01
	std.	1.551368e-02	1.548830e-02	8.503812e-02	3.555466e-02	5.440121e-03	7.731663e-03	1.617318e-03	2.984957e-03	1.695829e-03	2.216154e-03	1.946793e-03	1.761478e-03	5.571552e-03
6D	med.	4.457544e-01	2.927324e-01	5.625063e-01	2.793203e-01	3.140278e-01	3.428287e-01	1.472283e-01	1.527915e-01	1.663545e-01	1.739181e-01	2.424996e-01	2.414498e-01	1.739975e-01
	avg.	4.455712e-01	3.008519e-01	5.980336e-01	2.877331e-01	3.140783e-01	3.418499e-01	1.476234e-01	1.529730e-01	1.665391e-01	1.740044e-01	2.425930e-01	2.436513e-01	1.745957e-01
	min.	4.020425e-01	2.506339e-01	4.374455e-01	2.072636e-01	2.966915e-01	3.062918e-01	1.433564e-01	1.480734e-01	1.624472e-01	1.660406e-01	2.382464e-01	2.338621e-01	1.645099e-01
	max.	5.051233e-01	4.608435e-01	8.508436e-01	3.921370e-01	3.244332e-01	3.667596e-01	1.559707e-01	1.588648e-01	1.739601e-01	1.806298e-01	2.505914e-01	3.476735e-01	2.431860e-01
	std.	1.846130e-02	3.786188e-02	1.066209e-01	4.956964e-02	4.529951e-03	1.217300e-02	2.311305e-03	2.320833e-03	2.033856e-03	2.775142e-03	1.911985e-03	1.438678e-02	9.129233e-03
7D	med.	4.806985e-01	4.093405e-01	6.652396e-01	3.049925e-01	2.954465e-01	3.460223e-01	1.441697e-01	1.415685e-01	1.474571e-01	1.560876e-01	3.708747e-01	3.668095e-01	1.579927e-01
	avg.	4.811314e-01	4.304280e-01	7.005614e-01	3.051980e-01	2.952076e-01	3.463017e-01	1.436675e-01	1.457951e-01	1.478105e-01	1.567003e-01	3.720270e-01	3.683204e-01	1.619510e-01
	min.	4.389215e-01	3.373268e-01	4.594106e-01	1.657655e-01	2.855406e-01	3.213822e-01	1.347179e-01	1.312850e-01	1.391268e-01	1.490900e-01	3.557907e-01	3.557174e-01	1.435975e-01
	max.	5.306384e-01	5.481261e-01	9.122558e-01	3.858777e-01	3.056215e-01	3.724718e-01	1.545220e-01	1.920934e-01	1.582601e-01	1.868334e-01	3.978873e-01	4.018617e-01	2.105738e-01
	std.	1.895217e-02	5.065513e-02	1.214109e-01	4.824431e-02	3.736711e-03	1.110693e-02	4.516128e-03	1.195867e-02	4.660650e-03	4.664802e-03	9.294447e-03	8.423090e-03	1.399978e-02
8D	med.	4.520759e-01	4.439141e-01	6.850377e-01	2.667959e-01	2.320468e-01	3.131856e-01	1.179929e-01	1.247627e-01	1.358328e-01	1.396305e-01	3.936658e-01	3.928490e-01	1.481485e-01
	avg.	4.524455e-01	4.641378e-01	7.045519e-01	2.639680e-01	2.320285e-01	3.140735e-01	1.234664e-01	1.296399e-01	1.356397e-01	1.415895e-01	3.955755e-01	3.931928e-01	1.522456e-01
	min.	4.086446e-01	3.972378e-01	5.225393e-01	1.432211e-01	2.233888e-01	2.939214e-01	1.105165e-01	1.155506e-01	1.287093e-01	1.292175e-01	3.733890e-01	3.704902e-01	1.361550e-01
	max.	5.080265e-01	5.570774e-01	9.340297e-01	3.586437e-01	2.411712e-01	3.403658e-01	1.777920e-01	1.598100e-01	1.402250e-01	1.637558e-01	4.274806e-01	4.278956e-01	2.973151e-01
	std.	1.583222e-02	4.877763e-02	1.168529e-01	4.775812e-02	4.039771e-03	8.340624e-03	1.242928e-02	1.172054e-02	2.137714e-03	6.861884e-03	1.236583e-02	1.136527e-02	1.745016e-02
9D	med.	4.247240e-01	4.692579e-01	6.475339e-01	2.452548e-01	1.721986e-01	2.917809e-01	1.126860e-01	1.189277e-01	1.329512e-01	1.357755e-01	4.108646e-01	4.060809e-01	1.424370e-01
	avg.	4.248380e-01	4.882938e-01	6.785154e-01	2.526096e-01	1.723200e-01	2.916960e-01	1.150754e-01	1.231471e-01	1.331195e-01	1.369934e-01	4.128213e-01	4.090922e-01	1.492614e-01
	min.	3.901991e-01	4.126897e-01	5.534805e-01	1.596038e-01	1.582290e-01	2.751839e-01	9.827360e-02	1.060209e-01	1.280398e-01	1.275382e-01	3.772730e-01	3.741066e-01	1.335904e-01
	max.	4.607499e-01	5.972685e-01	9.253149e-01	3.237971e-01	1.834129e-01	3.086212e-01	1.566185e-01	1.582441e-01	1.386368e-01	1.495981e-01	4.528954e-01	4.548837e-01	2.621511e-01
	std.	1.351548e-02	5.056692e-02	7.794850e-02	4.029282e-02	4.426912e-03	6.645386e-03	1.261100e-02	1.305304e-02	2.186271e-03	4.742409e-03	1.497260e-02	1.398837e-02	2.197657e-02
10D	med.	3.967162e-01	5.216695e-01	6.679904e-01	2.193887e-01	1.269159e-01	2.718896e-01	1.151989e-01	1.229718e-01	1.361750e-01	1.406833e-01	4.135900e-01	4.147587e-01	1.408334e-01
	avg.	3.972634e-01	5.174239e-01	6.908741e-01	2.273903e-01	1.273903e-01	2.723617e-01	1.161451e-01	1.248375e-01	1.361846e-01	1.414492e-01	4.181846e-01	4.185105e-01	1.467802e-01
	min.	3.699385e-01	4.026366e-01	5.056285e-01	1.189657e-01	1.167858e-01	2.582043e-01	9.330656e-02	1.055673e-01	1.317728e-01	1.326043e-01	3.897938e-01	3.948661e-01	1.297690e-01
	max.	4.231404e-01	6.190336e-01	8.885114e-01	2.988218e-01	1.409787e-01	2.901025e-01	1.411849e-01	1.467435e-01	1.396662e-01	1.506482e-01	4.641633e-01	4.585843e-01	2.763801e-01
	std.	1.287214e-02	5.251936e-02	8.077108e-02	4.296244e-02	7.072456e-03	5.898946e-03	1.174130e-02	1.028397e-02	1.675913e-03	4.048310e-03	1.596031e-02	1.231327e-02	2.372494e-02

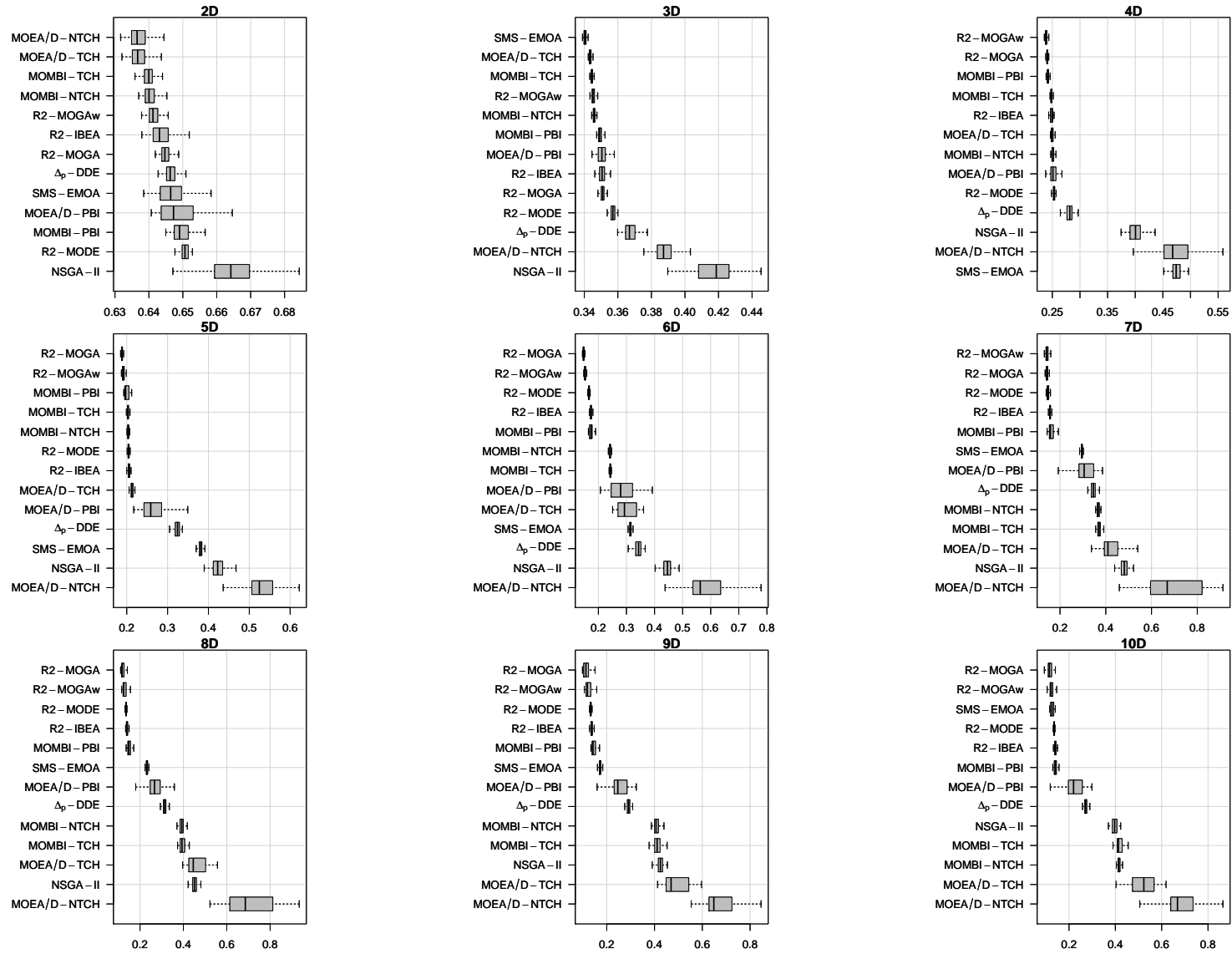


Figure A.226: Box-plot of R2 indicator values for different optimizers on the WFG8 test problem.

Table A.91: Comparison of runtime (in milliseconds) for different optimizers on the WFG8 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	5.037510e + 02	9.460480e + 02	1.578427e + 03	1.444021e + 03	2.066206e + 04	1.703897e + 03	2.545825e + 03	1.934263e + 03	2.723980e + 03	5.372240e + 05	2.326549e + 03	2.143058e + 03	3.233019e + 03
	avg.	5.053726e + 02	9.649560e + 02	1.574743e + 03	1.434388e + 03	2.022226e + 04	1.708429e + 03	2.532578e + 03	1.967369e + 03	2.696144e + 03	5.374012e + 05	2.310251e + 03	2.177644e + 03	3.213924e + 03
	min.	4.301490e + 02	7.957340e + 02	1.121834e + 03	1.197986e + 03	1.447304e + 04	1.380965e + 03	1.828031e + 03	1.628285e + 03	1.974525e + 03	4.891600e + 05	1.868724e + 03	1.865153e + 03	2.819712e + 03
	max.	6.303060e + 02	1.485392e + 03	1.883924e + 03	1.849078e + 03	2.231220e + 04	2.322790e + 03	3.019090e + 03	2.404315e + 03	3.151580e + 03	6.155880e + 05	2.922306e + 03	3.312517e + 03	3.497653e + 03
	std.	4.913809e + 01	1.055118e + 02	1.189903e + 02	1.029593e + 02	1.449883e + 03	1.677840e + 02	2.564919e + 02	1.771684e + 02	2.517861e + 02	4.161179e + 04	1.740414e + 02	1.934003e + 02	1.170809e + 02
3D	med.	6.782540e + 02	9.970800e + 02	1.741720e + 03	1.525824e + 03	6.157839e + 05	1.941043e + 03	2.970724e + 03	2.308791e + 03	2.721850e + 03	6.013510e + 05	2.279790e + 03	2.226047e + 03	3.401070e + 03
	avg.	6.797128e + 02	1.028519e + 03	1.734576e + 03	1.487618e + 03	5.979291e + 05	1.952001e + 03	2.929042e + 03	2.289240e + 03	2.723283e + 03	5.987608e + 05	2.285269e + 03	2.251029e + 03	3.352117e + 03
	min.	5.728750e + 02	8.496880e + 02	1.391377e + 03	1.166048e + 03	4.114333e + 05	1.407915e + 03	2.060964e + 03	1.771832e + 03	2.191340e + 03	5.542750e + 05	1.908216e + 03	1.938348e + 03	2.676055e + 03
	max.	8.458640e + 02	1.357384e + 03	2.221262e + 03	2.008969e + 03	6.530069e + 05	2.316526e + 03	3.458891e + 03	2.725502e + 03	3.317403e + 03	6.175080e + 05	3.103178e + 03	2.673544e + 03	3.756343e + 03
	std.	5.469765e + 01	1.071505e + 02	1.503701e + 02	1.160391e + 02	4.913409e + 04	1.913234e + 02	2.881218e + 02	1.968023e + 02	2.328913e + 02	1.399463e + 04	2.116209e + 02	1.530268e + 02	1.901873e + 02
4D	med.	1.010498e + 03	1.308149e + 03	2.203132e + 03	1.960022e + 03	2.952582e + 06	2.709706e + 03	4.025680e + 03	3.077145e + 03	4.232739e + 03	5.932520e + 05	2.485426e + 03	2.790838e + 03	4.408459e + 03
	avg.	1.020942e + 03	1.320603e + 03	2.181264e + 03	1.879200e + 03	2.886648e + 06	2.649860e + 03	3.966448e + 03	3.040003e + 03	4.100168e + 03	5.931601e + 05	2.538575e + 03	2.784784e + 03	4.395782e + 03
	min.	8.660930e + 02	1.095747e + 03	1.568092e + 03	1.328271e + 03	1.838753e + 06	1.918171e + 03	2.965394e + 03	2.325761e + 03	2.803192e + 03	5.695710e + 05	2.074246e + 03	2.267698e + 03	3.702366e + 03
	max.	1.272976e + 03	1.625421e + 03	2.925823e + 03	2.626619e + 03	3.338370e + 06	3.210396e + 03	5.62317e + 03	3.505290e + 03	4.551226e + 03	6.007720e + 05	3.472628e + 03	3.117005e + 03	4.942445e + 03
	std.	8.075516e + 01	1.158787e + 02	1.825997e + 02	2.040883e + 02	7.727014e + 05	2.903692e + 02	3.372411e + 02	2.779977e + 02	3.770573e + 02	4.067473e + 03	2.534574e + 02	1.658204e + 02	2.507418e + 02
5D	med.	1.403479e + 03	1.613720e + 03	2.219629e + 03	2.218872e + 03	3.924938e + 06	5.337681e + 03	5.199100e + 03	4.300547e + 03	5.300837e + 03	6.753880e + 05	2.937444e + 03	3.218775e + 03	4.260508e + 03
	avg.	1.409870e + 03	1.612764e + 03	2.854936e + 03	2.172588e + 03	3.865040e + 06	5.317007e + 03	5.075419e + 03	4.276441e + 03	5.346023e + 03	6.750196e + 05	3.001099e + 03	3.204554e + 03	4.231908e + 03
	min.	1.128524e + 03	1.338172e + 03	2.057356e + 03	1.724403e + 03	2.750571e + 06	3.392800e + 03	3.541225e + 03	3.524746e + 03	3.956452e + 03	6.479550e + 05	2.448996e + 03	2.675395e + 03	3.685873e + 03
	max.	1.653094e + 03	1.965198e + 03	3.492751e + 03	2.599937e + 03	4.525765e + 06	6.986741e + 03	5.830717e + 03	4.868229e + 03	5.999684e + 03	6.811660e + 05	4.567337e + 03	3.594204e + 03	4.440743e + 03
	std.	1.215828e + 02	1.312209e + 02	2.656440e + 02	2.315402e + 02	3.346736e + 05	6.962296e + 02	5.060545e + 02	3.052298e + 02	4.235958e + 02	4.151224e + 03	3.360820e + 02	1.958375e + 02	1.560614e + 02
6D	med.	1.734041e + 03	1.851054e + 03	3.202681e + 03	2.467012e + 03	5.383751e + 06	6.306213e + 03	6.509632e + 03	5.081145e + 03	6.613711e + 03	6.577040e + 05	2.810592e + 03	3.581925e + 03	5.166769e + 03
	avg.	1.733914e + 03	1.877768e + 03	3.151826e + 03	2.402768e + 03	5.282588e + 06	6.271775e + 03	6.334605e + 03	5.078814e + 03	6.589672e + 03	6.829179e + 05	2.857722e + 03	3.603648e + 03	5.095743e + 03
	min.	1.437701e + 03	1.505289e + 03	2.515875e + 03	1.768937e + 03	3.997042e + 06	3.700903e + 03	4.762530e + 03	4.179333e + 03	5.385318e + 03	6.011760e + 05	2.491810e + 03	3.030784e + 03	4.042132e + 03
	max.	2.063443e + 03	2.335397e + 03	3.713190e + 03	3.046130e + 03	5.890629e + 06	8.712269e + 03	7.625169e + 03	6.002894e + 03	7.528338e + 03	7.305740e + 05	3.654712e + 03	4.123617e + 03	5.623665e + 03
	std.	1.253944e + 02	1.580046e + 02	2.580762e + 02	3.187059e + 02	4.085469e + 05	9.315664e + 02	6.063542e + 02	3.622885e + 02	5.561514e + 02	3.544012e + 04	2.239527e + 02	2.409646e + 02	2.873842e + 02
7D	med.	1.920106e + 03	2.136071e + 03	3.644718e + 03	2.639691e + 03	3.036663e + 06	1.635161e + 04	6.097362e + 03	5.259522e + 03	6.389332e + 03	2.733640e + 05	2.461605e + 03	3.202908e + 03	4.393095e + 03
	avg.	1.941121e + 03	2.149384e + 03	3.600359e + 03	2.655868e + 03	2.957015e + 06	1.656549e + 04	5.928741e + 03	5.243923e + 03	6.460935e + 03	2.888052e + 05	2.518653e + 03	3.185103e + 03	4.292486e + 03
	min.	1.551867e + 03	1.707744e + 03	2.566178e + 03	2.032077e + 03	2.028185e + 06	8.801357e + 03	4.305106e + 03	4.310218e + 03	4.835609e + 03	2.729560e + 05	2.158239e + 03	2.611371e + 03	3.367444e + 03
	max.	2.340969e + 03	2.760688e + 03	4.643847e + 03	3.196470e + 03	3.366777e + 06	2.587236e + 04	6.997483e + 03	6.147251e + 03	7.471558e + 03	3.733870e + 05	3.171682e + 03	3.939761e + 03	5.021992e + 03
	std.	1.789269e + 02	1.989040e + 02	3.178630e + 02	2.975270e + 02	2.761277e + 05	2.906432e + 03	6.493791e + 02	4.031274e + 02	5.619667e + 02	3.089984e + 04	2.110483e + 02	2.091857e + 02	2.919960e + 02
8D	med.	2.594460e + 03	2.572124e + 03	4.285537e + 03	2.813278e + 03	4.703319e + 06	3.321484e + 04	8.207277e + 03	7.389436e + 03	9.042178e + 03	1.280929e + 06	3.386013e + 03	4.323352e + 03	6.195594e + 03
	avg.	2.603571e + 03	2.562253e + 03	4.246124e + 03	2.887660e + 03	4.890345e + 06	3.473961e + 04	8.167846e + 03	7.371326e + 03	9.043706e + 03	1.281142e + 06	3.415704e + 03	4.366887e + 03	6.104764e + 03
	min.	2.152451e + 03	2.004633e + 03	3.123711e + 03	2.280062e + 03	3.966824e + 06	1.967887e + 04	5.839930e + 03	6.016670e + 03	6.431669e + 03	1.255931e + 06	2.998043e + 03	3.676024e + 03	4.799518e + 03
	max.	3.041427e + 03	3.005142e + 03	4.788911e + 03	3.921507e + 03	5.939921e + 06	5.508741e + 04	9.486885e + 03	8.459949e + 03	1.032874e + 04	1.291843e + 06	4.113233e + 03	4.934711e + 03	6.698881e + 03
	std.	1.954864e + 02	2.138435e + 02	2.550209e + 02	3.461175e + 02	5.637391e + 05	7.879509e + 03	1.039311e + 03	5.490002e + 02	8.122554e + 02	5.197719e + 03	2.547811e + 02	2.827714e + 02	3.353808e + 02
9D	med.	3.454482e + 03	2.953608e + 03	5.147734e + 03	3.630872e + 03	9.228558e + 06	6.947527e + 04	7.863898e + 03	1.009232e + 04	1.367005e + 04	2.772608e + 06	4.486149e + 03	6.313074e + 03	9.553782e + 03
	avg.	3.446597e + 03	2.927583e + 03	5.009152e + 03	3.519200e + 03	8.964626e + 06	7.232134e + 04	7.876105e + 03	1.013203e + 04	1.306086e + 04	2.674995e + 06	4.563223e + 03	6.214038e + 03	9.561574e + 03
	min.	2.904428e + 03	2.287745e + 03	3.404578e + 03	2.675612e + 03	6.896269e + 06	4.922501e + 04	7.599080e + 03	8.914806e + 03	9.407986e + 03	2.254171e + 06	3.862184e + 03	5.002606e + 03	7.919916e + 03
	max.	3.874899e + 03	3.502598e + 03	5.619426e + 03	4.093929e + 03	1.013206e + 07	1.042677e + 05	8.831208e + 03	1.163387e + 04	1.420540e + 04	2.840976e + 06	5.547698e + 03	6.823889e + 03	1.084433e + 04
	std.	2.256839e + 02	2.583899e + 02	3.924205e + 02	3.622914e + 02	8.787044e + 05	1.279611e + 04	1.763299e + 02	6.431613e + 02	1.137440e + 03	1.831456e + 05	4.430360e + 02	3.731977e + 02	5.566844e + 02
10D	med.	3.207352e + 03	2.607754e + 03	5.058648e + 03	3.879013e + 03	1.335742e + 07	1.877010e + 05	1.194515e + 04	1.248216e + 04	1.231136e + 04	4.459717e + 06	4.880560e + 03	6.144766e + 03	9.645921e + 03
	avg.	3.396503e + 03	2.840132e + 03	4.946111e + 03	3.852278e + 03	1.311129e + 07	1.910159e + 05	1.267196e + 04	1.233444e + 04	1.437270e + 04	4.694997e + 06	5.147151e + 03	6.656141e + 03	9.878966e + 03
	min.	3.169777e + 03	2.382613e + 03	3.596965e + 03	2.805139e + 03	1.040417e + 07	1.285748e + 05	1.053617e + 04	9.953033e + 03	1.177866e + 04	4.262162e + 06	4.678702e + 03	5.890544e + 03	8.614301e + 03
	max.	4.638466e + 03	4.230844e + 03	7.561201e + 03	4.631031e + 03	1.554263e + 07	2.831686e + 05	1.554691e + 04	1.546917e + 04	1.936604e + 04	5.288015e + 06	7.185119e + 03	8.851703e + 03	1.301860e + 04
	std.	3.774626e + 02	4.883514e + 02	8.212009e + 02	5.098913e + 02	1.166488e + 06	3.349660e + 04	2.012056e + 03	1.617752e + 03	2.677788e + 03	4.476540e + 05	5.307513e + 02	7.982950e + 02	1.122111e + 03

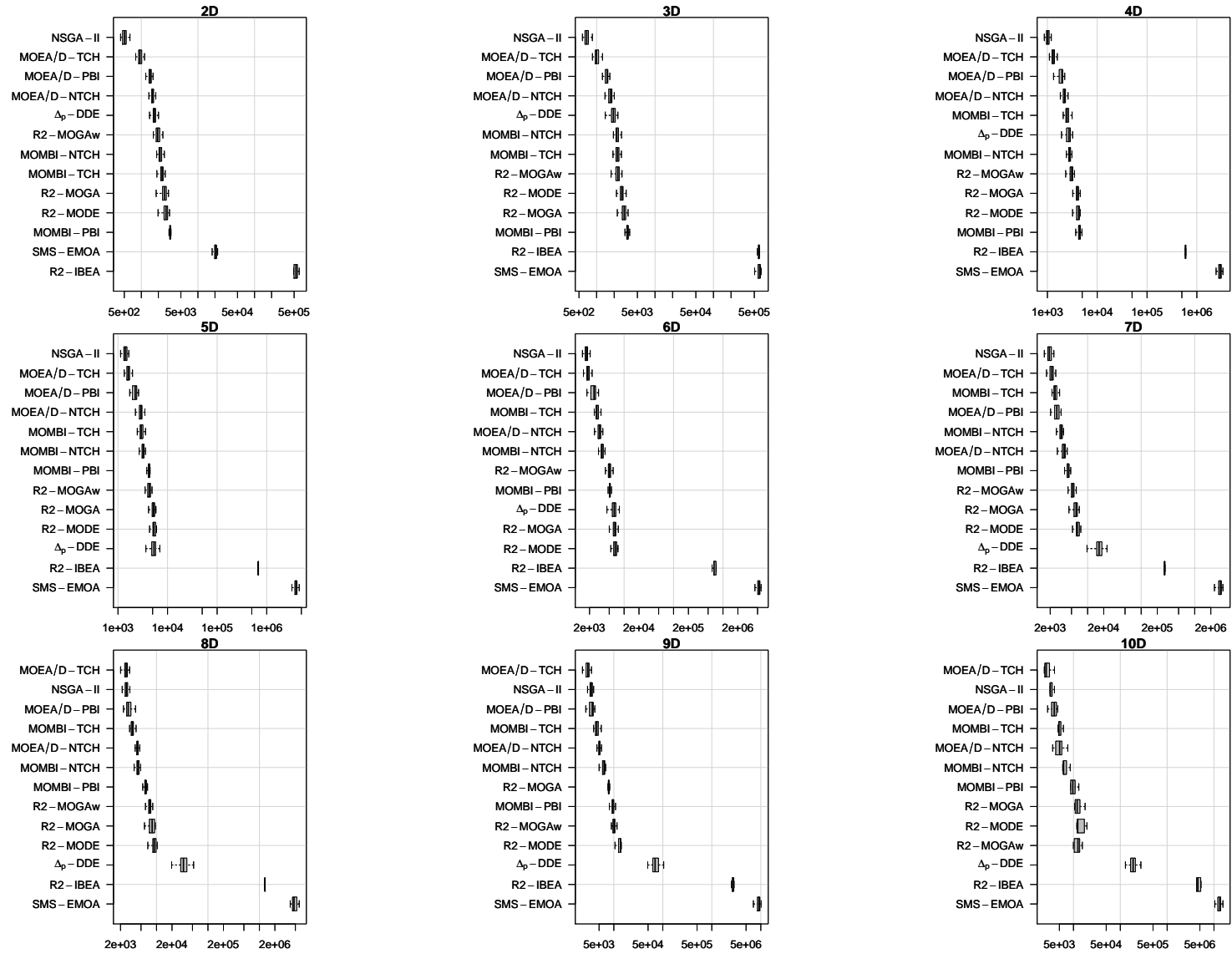


Figure A.227: Box-plot of runtime (in logarithmic scale) for different optimizers on the WFG8 test problem.

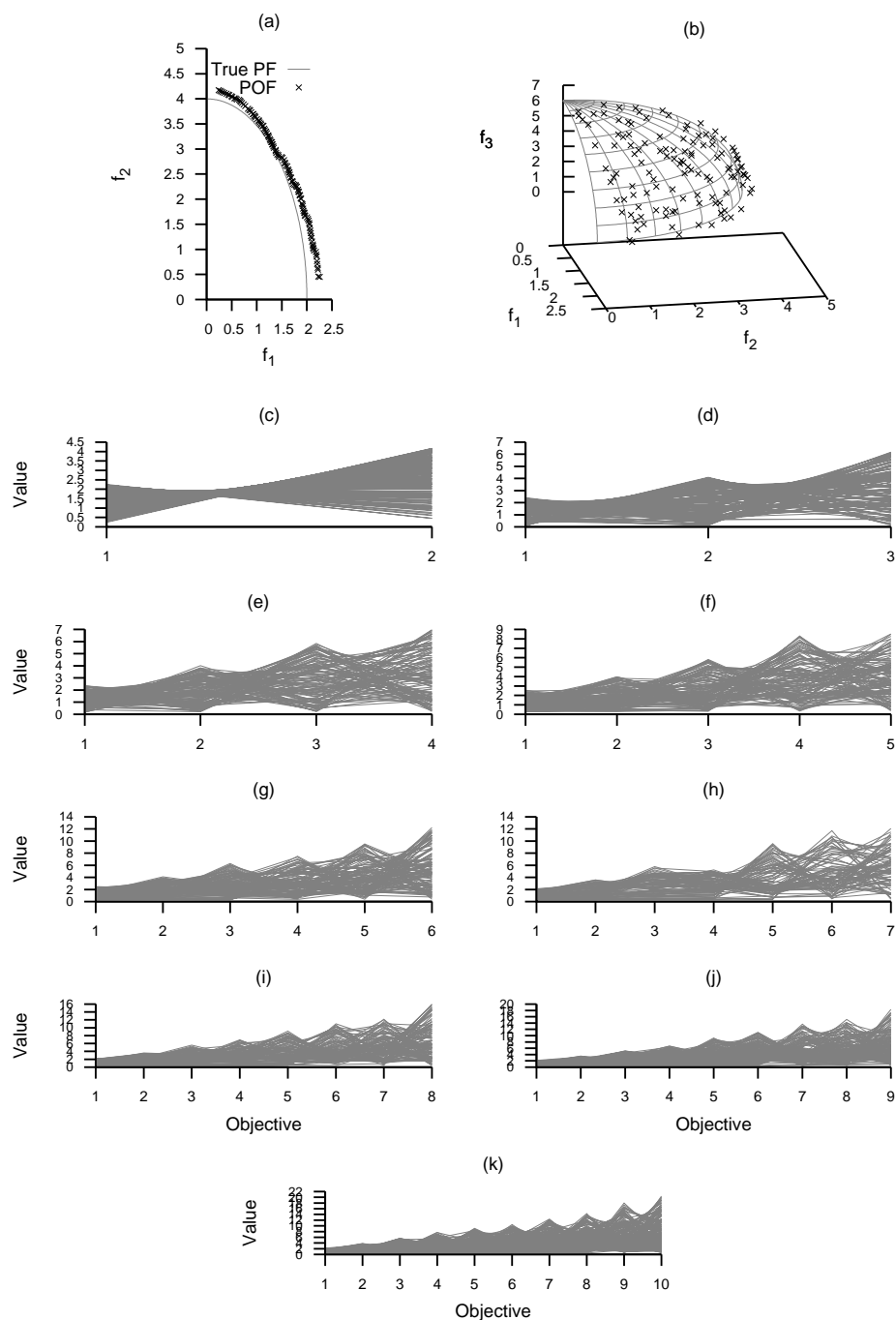


Figure A.228: Plots of the approximations obtained by NSGA-II from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

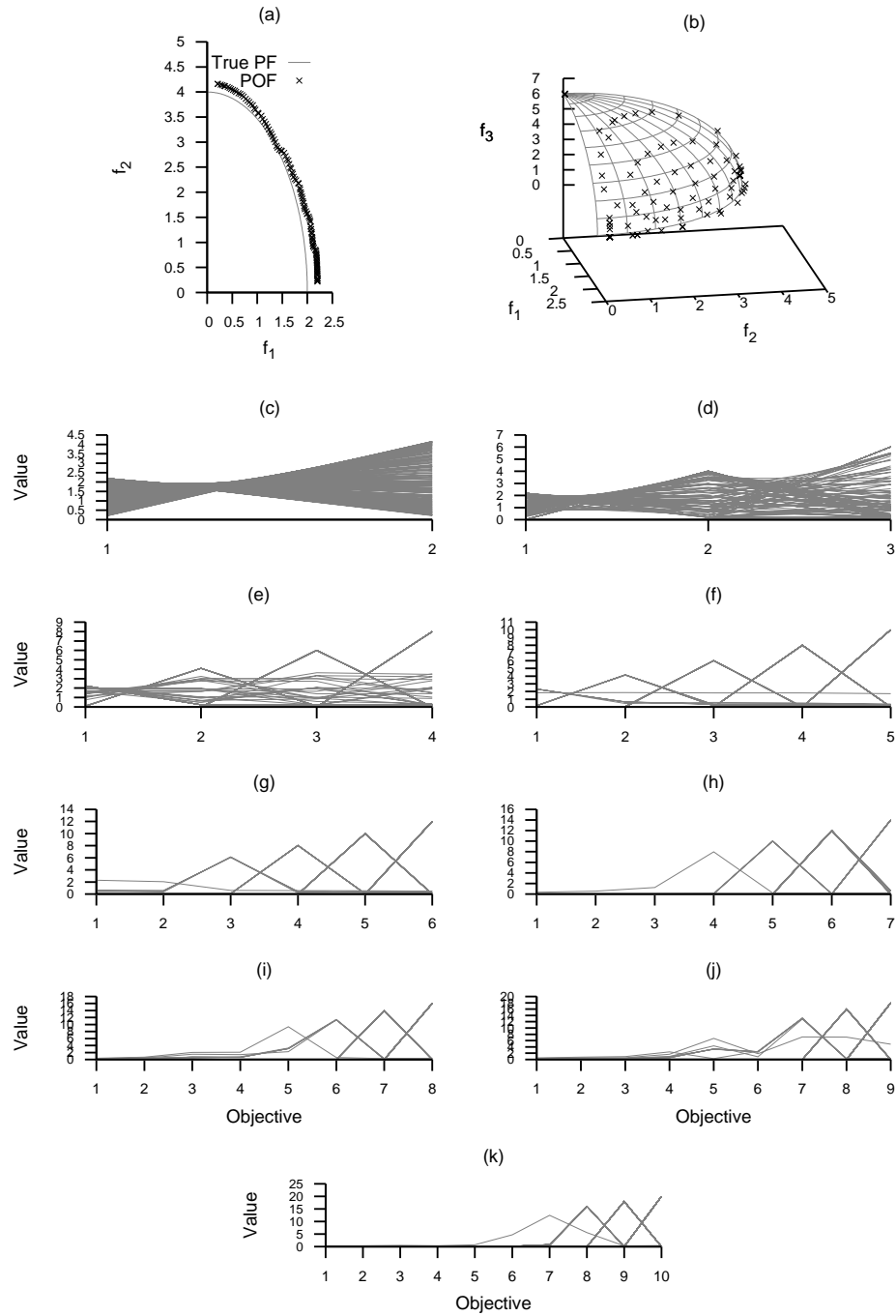


Figure A.229: Plots of the approximations obtained by MOEA/D-TCH from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

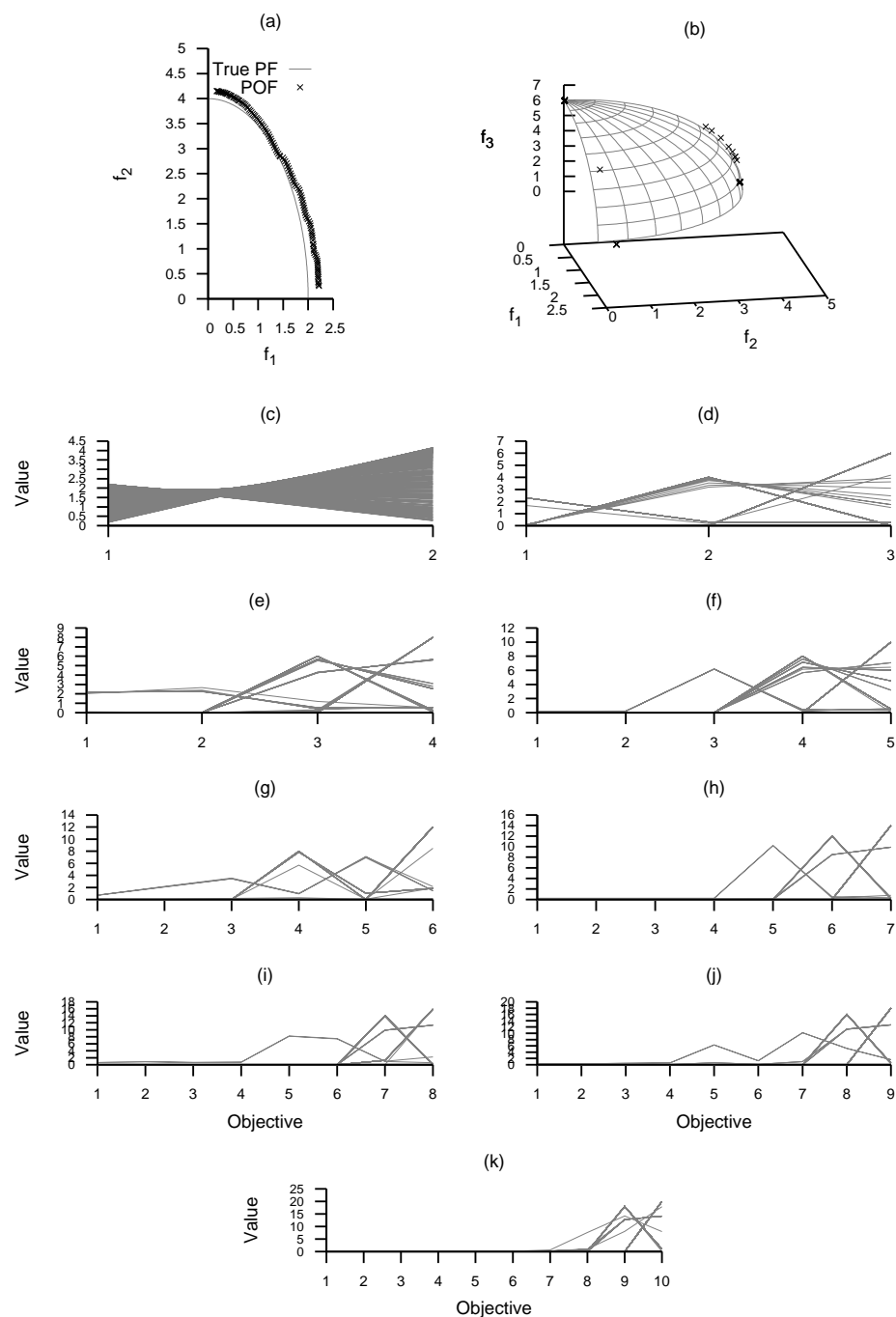


Figure A.230: Plots of the approximations obtained by MOEA/D-NTCH from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



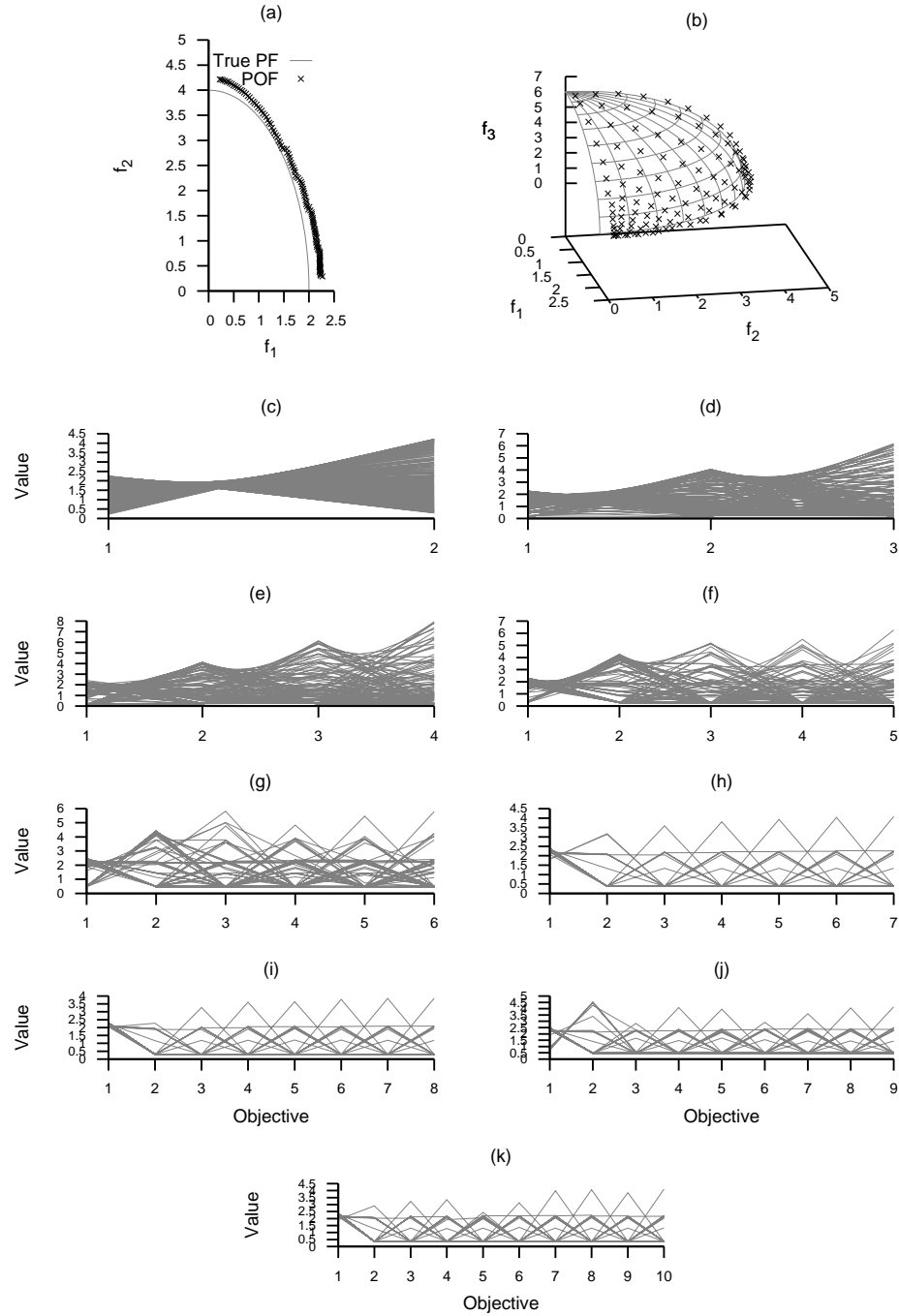


Figure A.231: Plots of the approximations obtained by MOEA/D-PBI from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

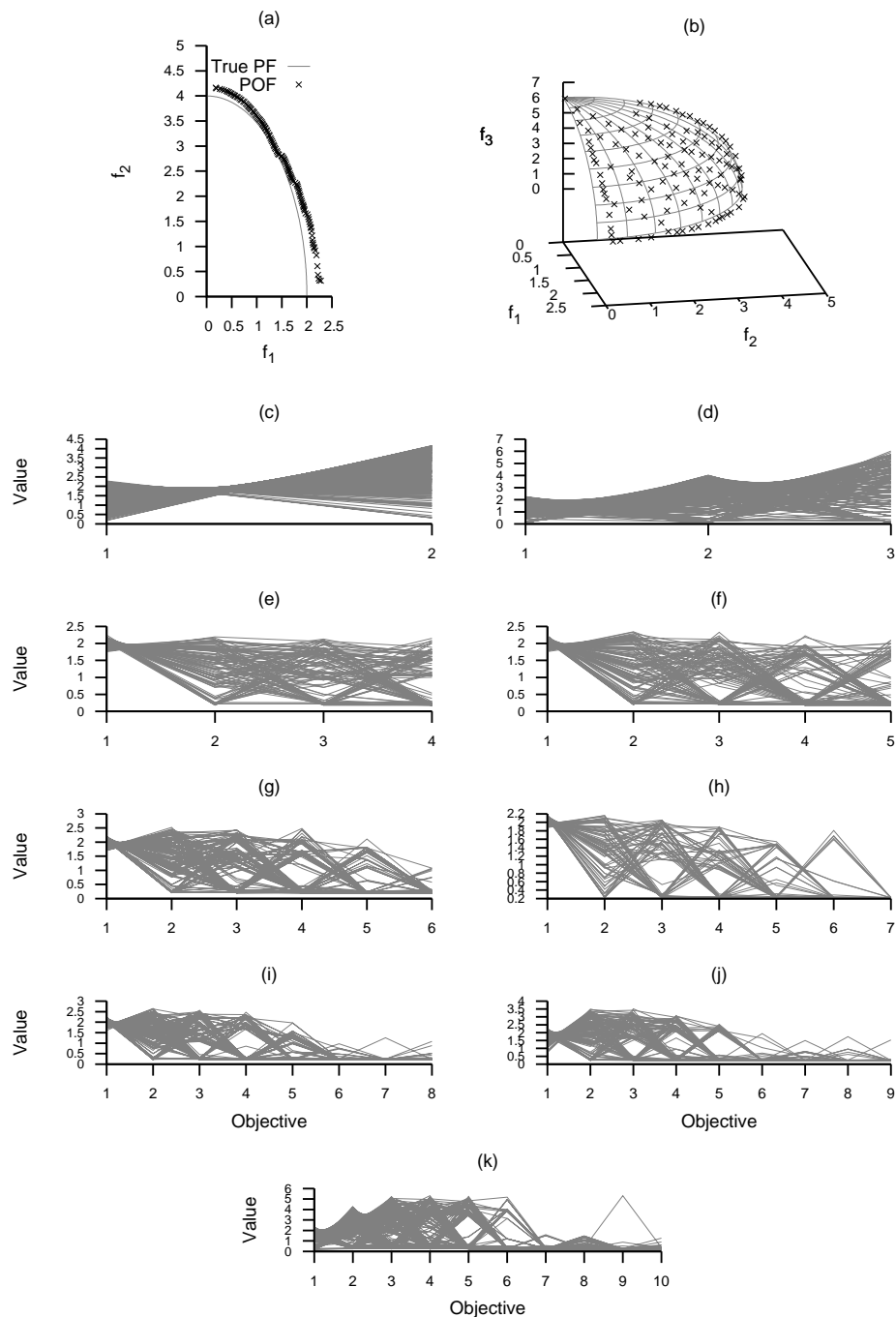


Figure A.232: Plots of the approximations obtained by SMS-EMOA from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

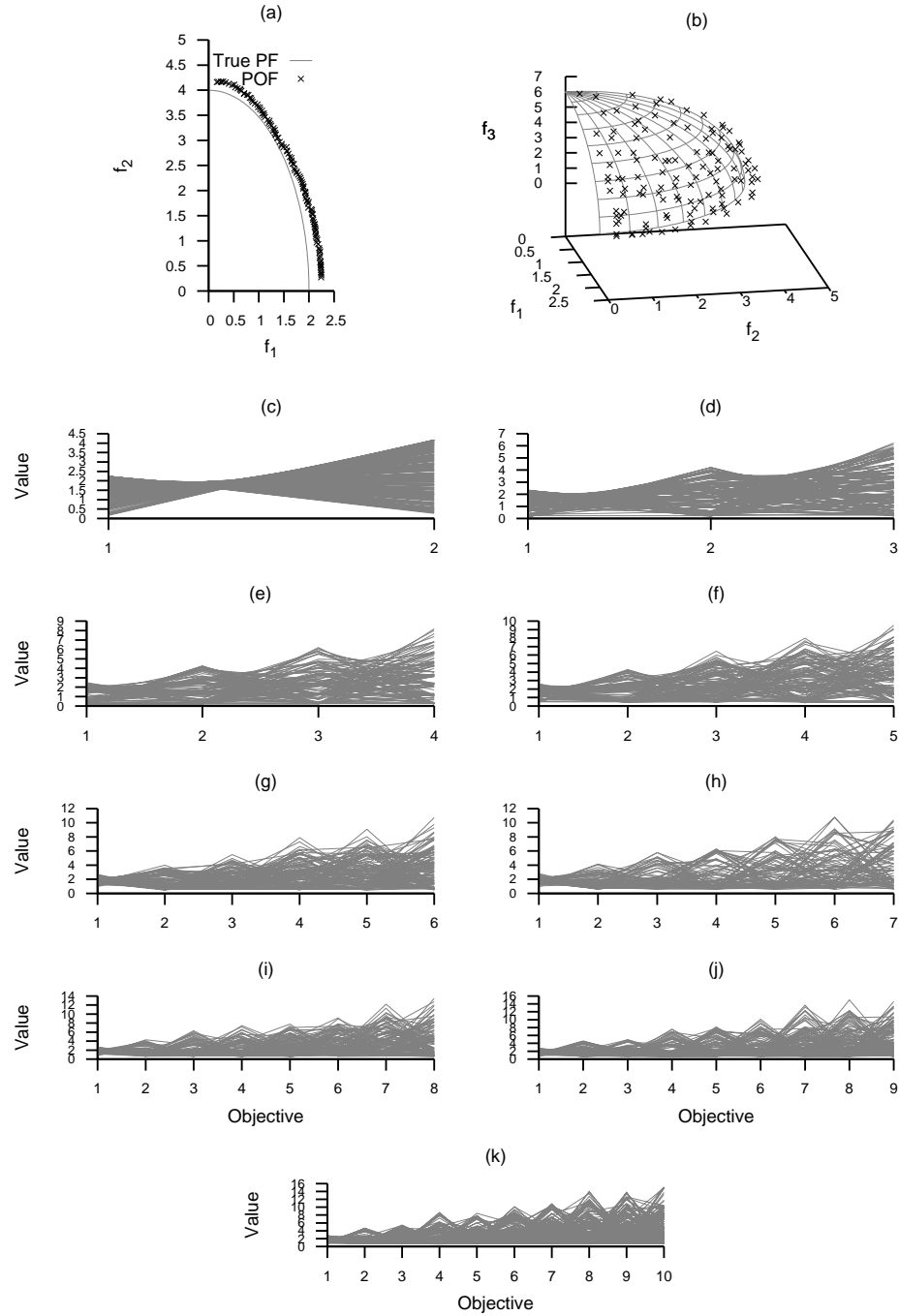


Figure A.233: Plots of the approximations obtained by  $\Delta_p$ -DDE from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

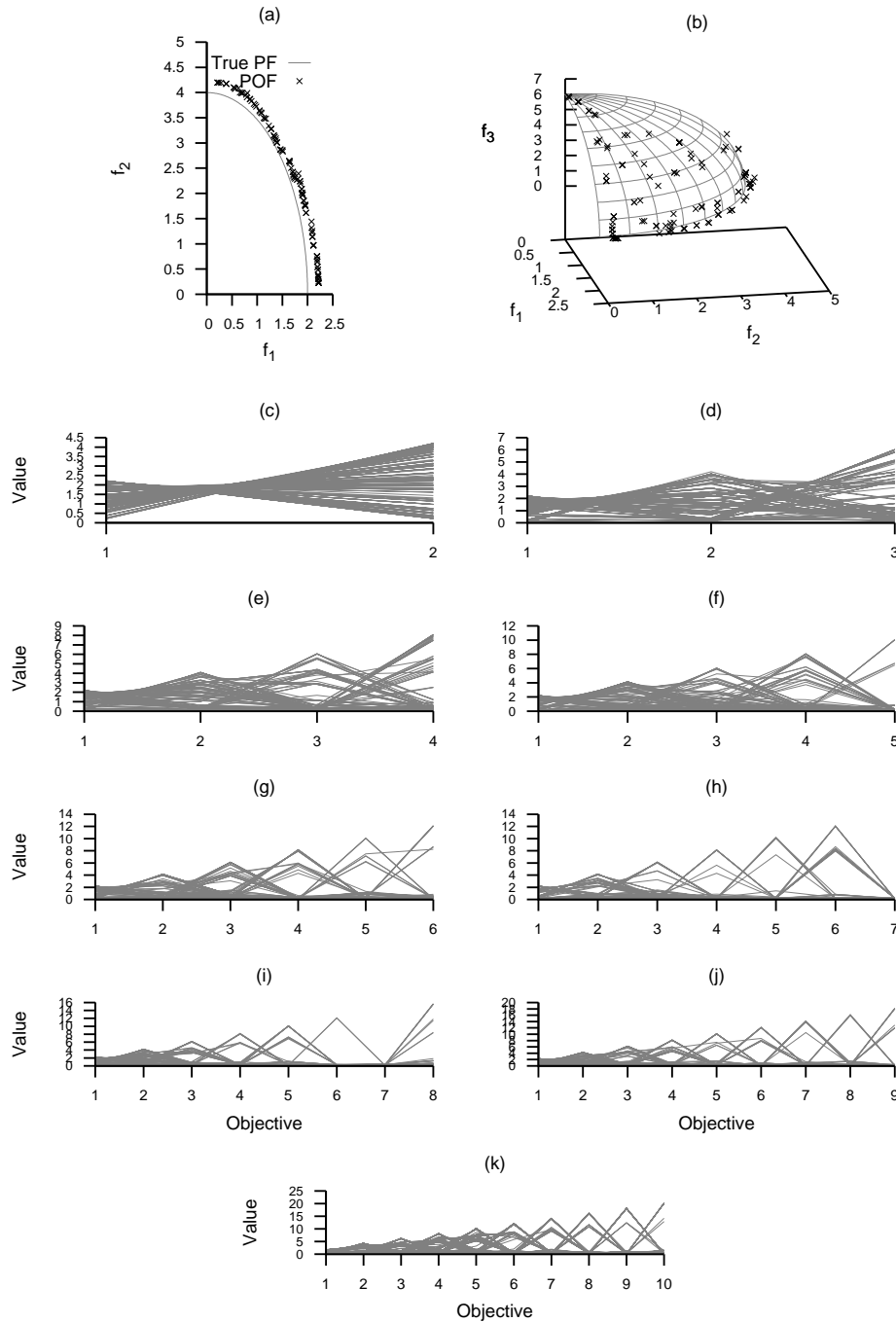


Figure A.234: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

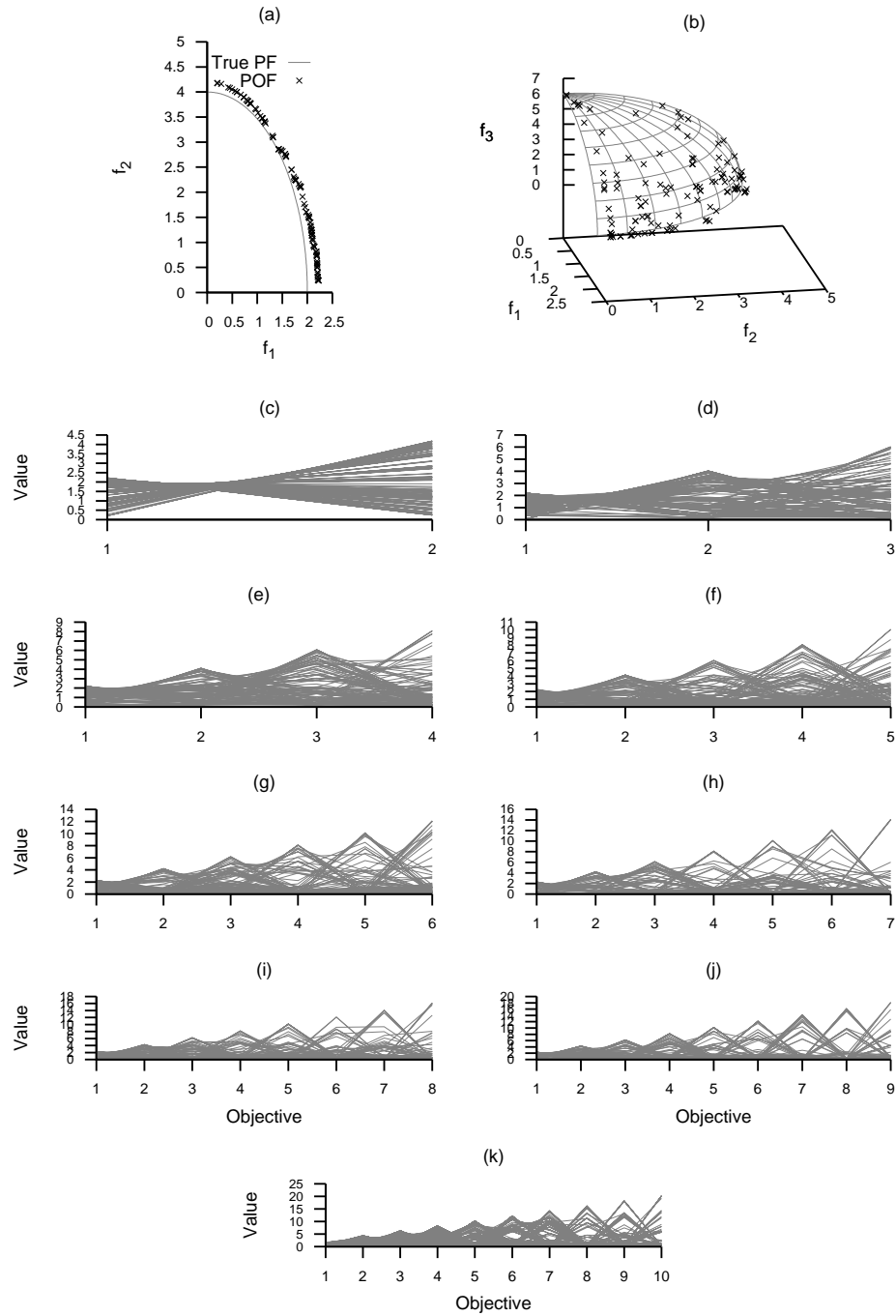


Figure A.235: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

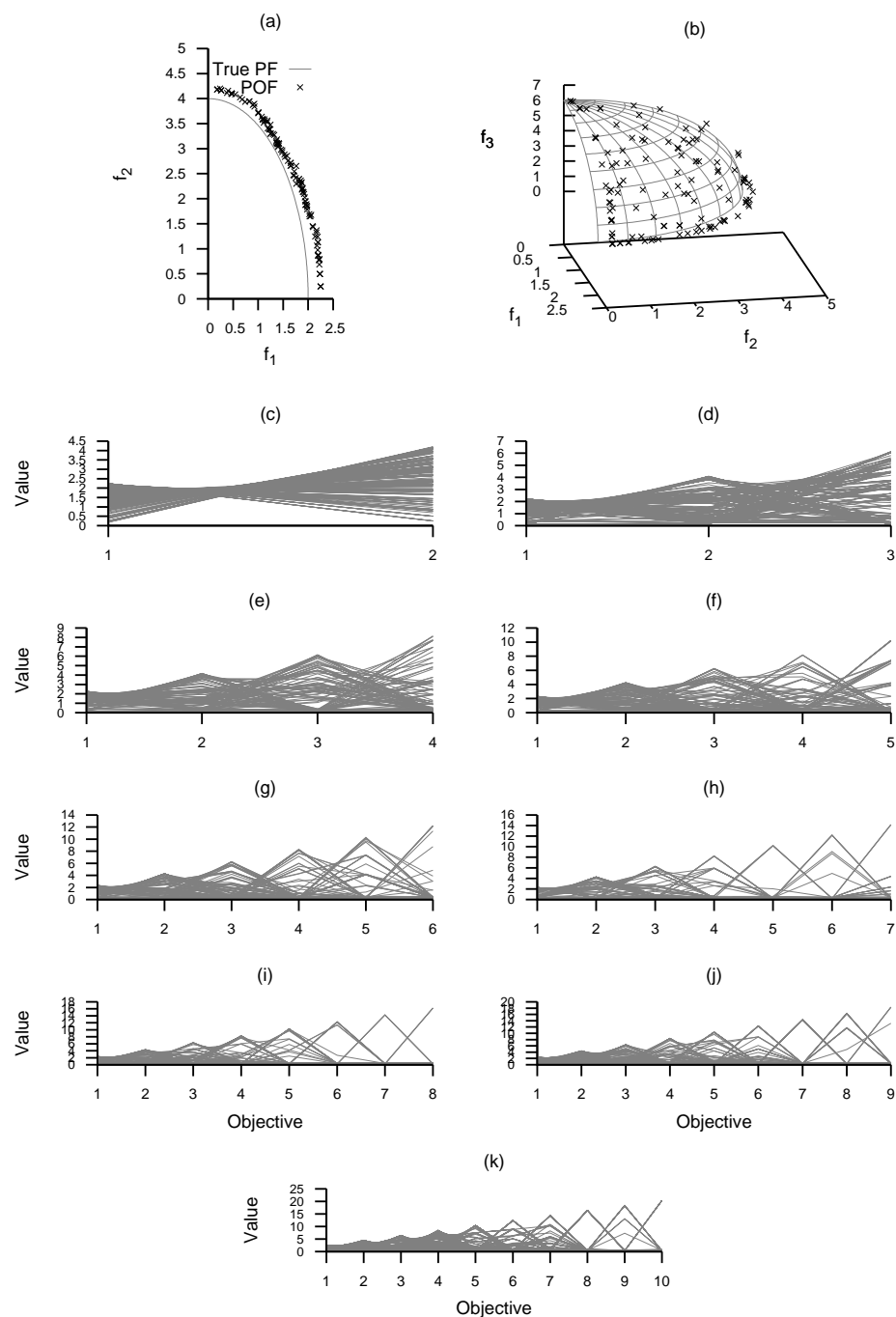


Figure A.236: Plots of the approximations obtained by *R2-MODE* from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

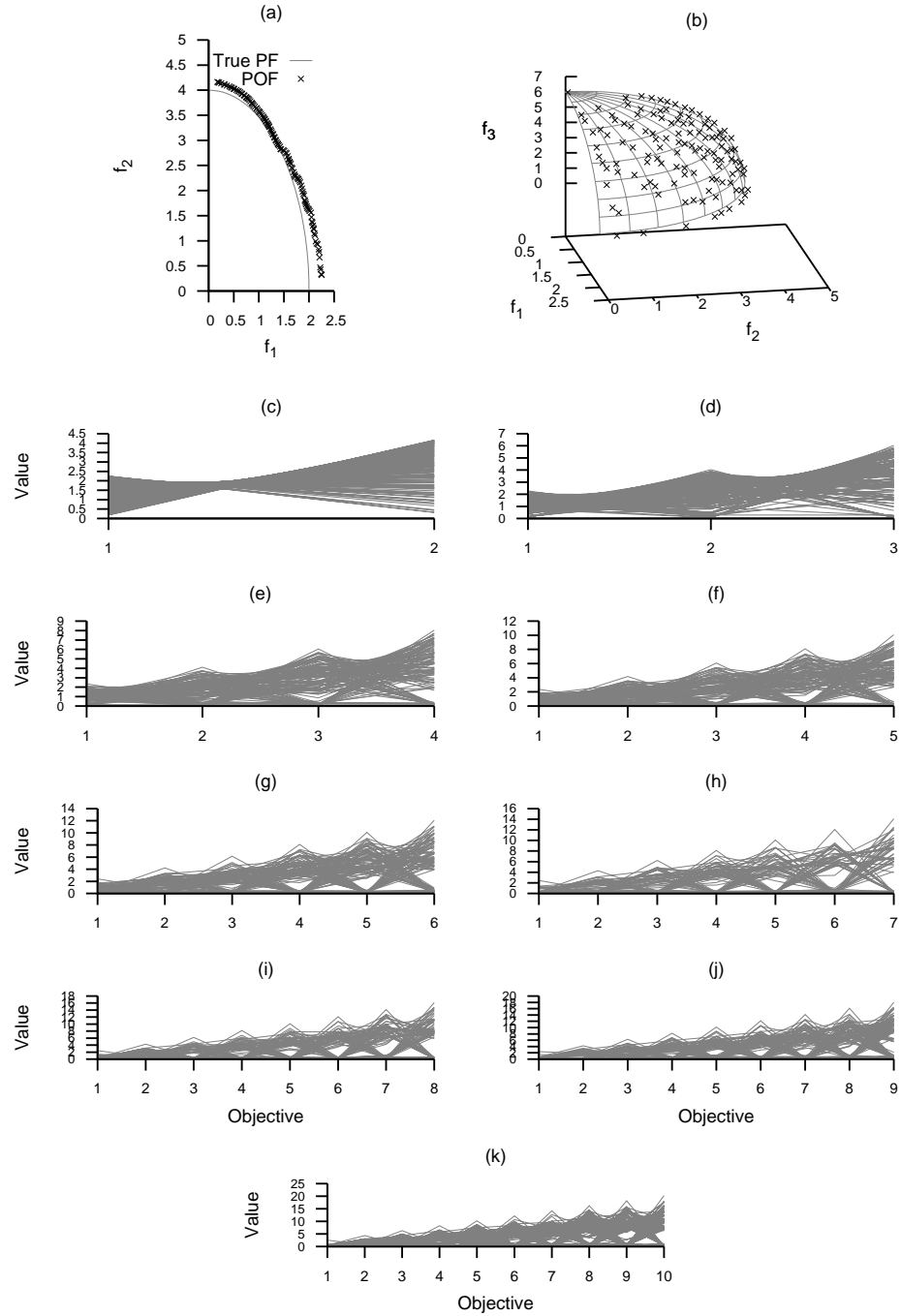


Figure A.237: Plots of the approximations obtained by *R2-IBEA* from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

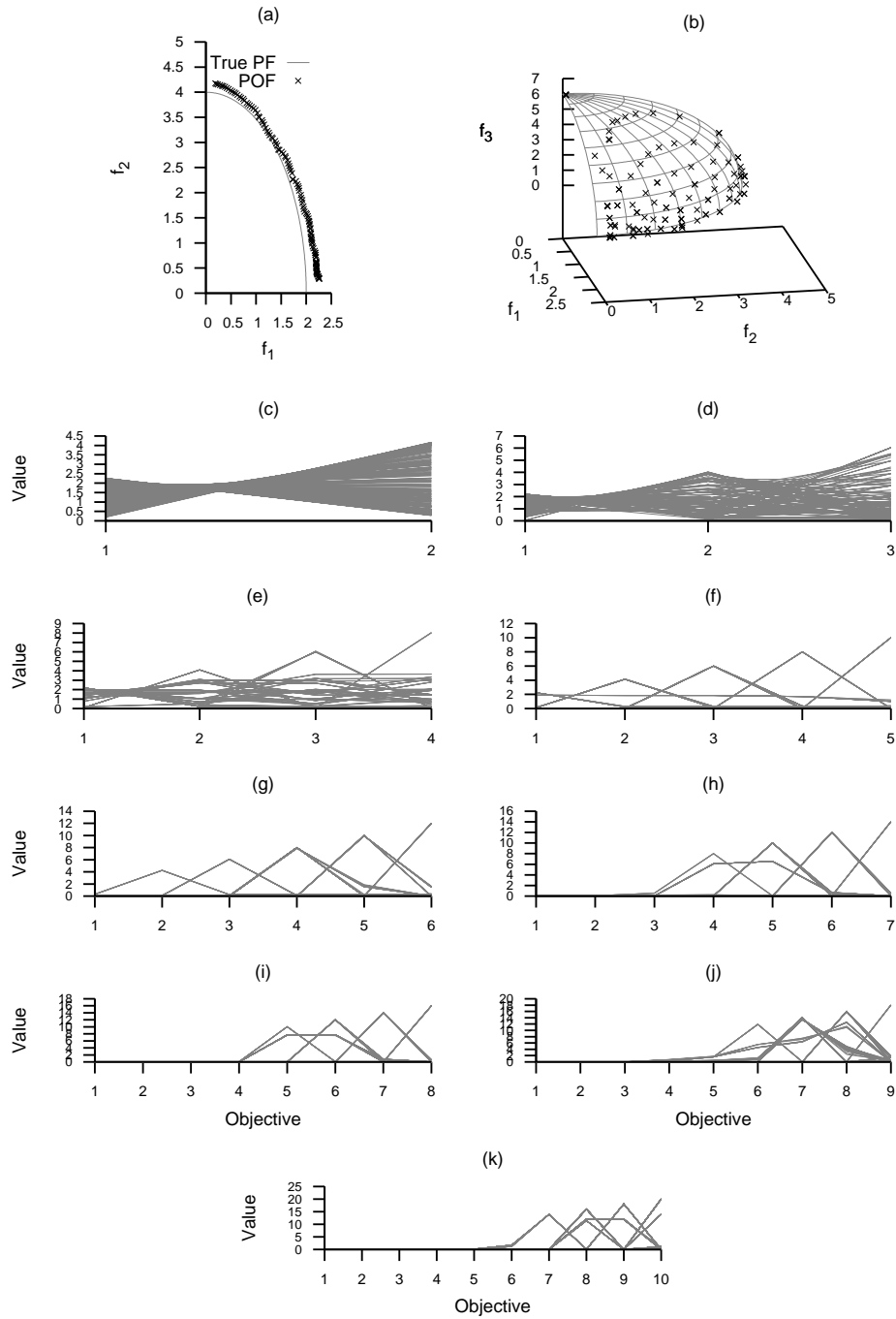


Figure A.238: Plots of the approximations obtained by MOMBI-TCH from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



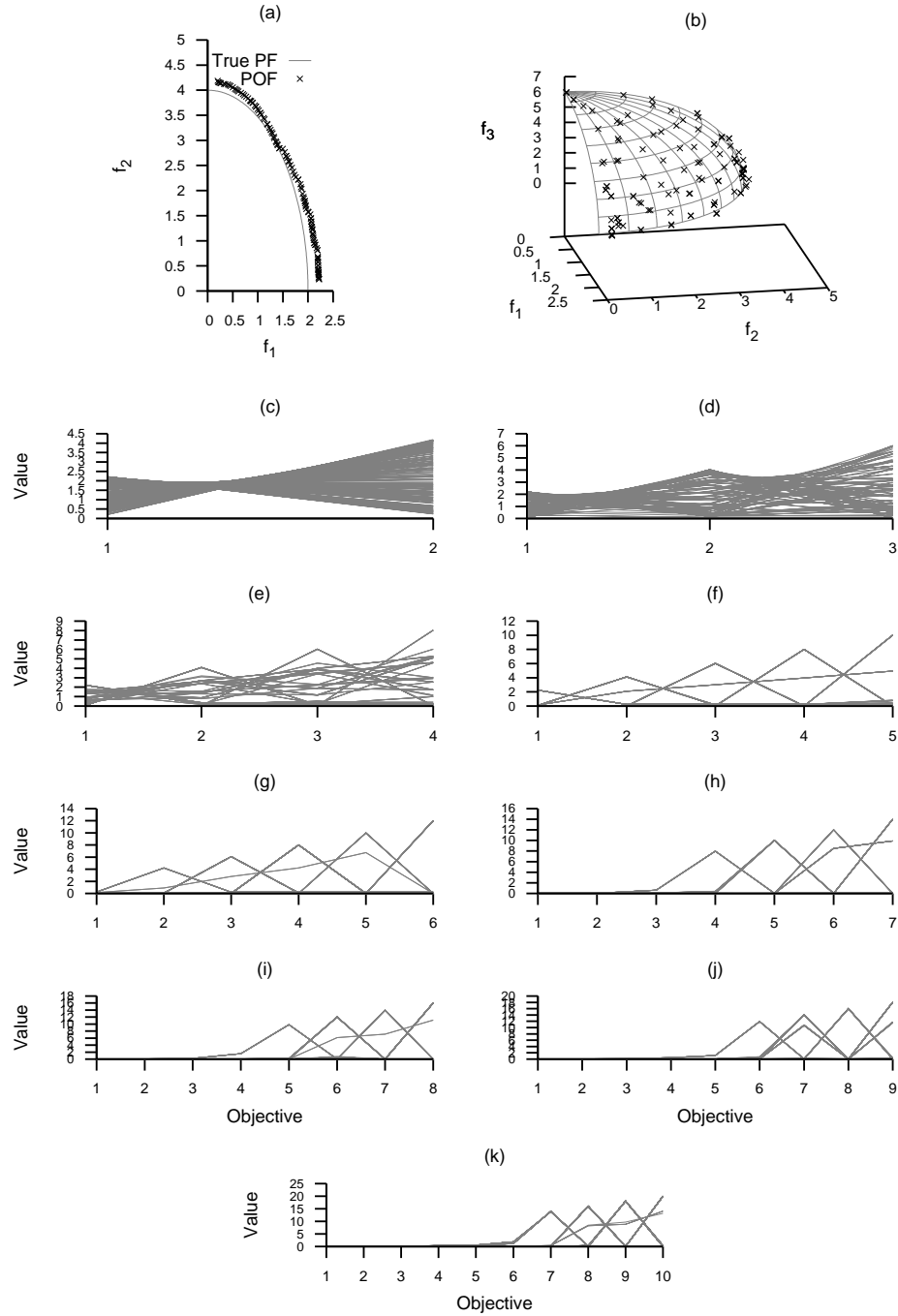


Figure A.239: Plots of the approximations obtained by MOMBI-NTCH from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

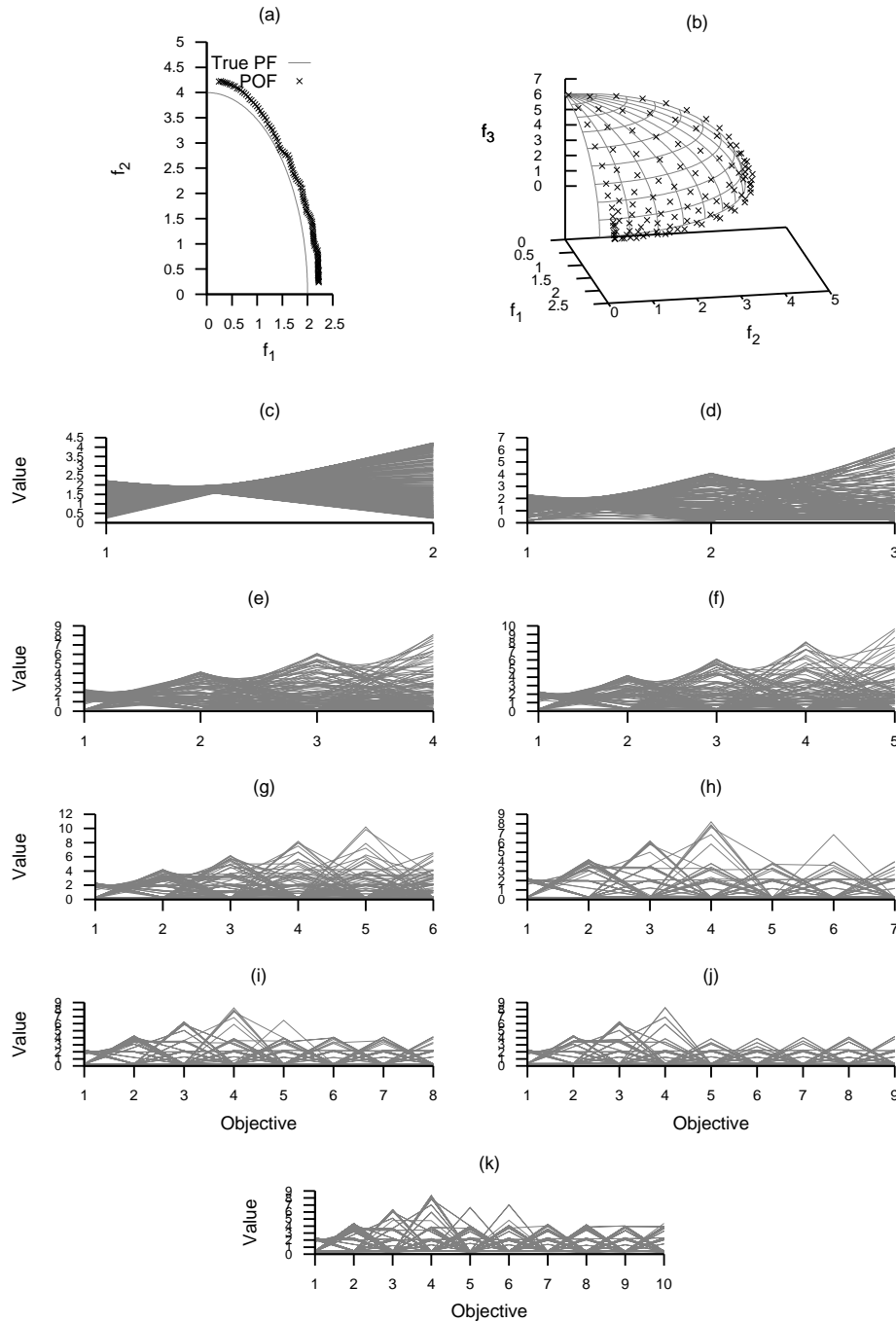


Figure A.240: Plots of the approximations obtained by MOMBI-PBI from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.92: Comparison of hypervolume indicator values for different optimizers on the WFG9 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	7.691902e+00	8.046242e+00	7.879194e+00	7.865473e+00	8.123898e+00	7.646212e+00	7.672756e+00	7.671188e+00	7.641120e+00	7.703374e+00	7.699809e+00	7.700877e+00	7.695820e+00
	avg.	7.875740e+00	7.973569e+00	7.891093e+00	7.838712e+00	8.011115e+00	7.639040e+00	7.870638e+00	7.879265e+00	7.641998e+00	7.950802e+00	7.893297e+00	7.919926e+00	7.944253e+00
	min.	7.668239e+00	7.571492e+00	7.022037e+00	7.219605e+00	7.684438e+00	7.519595e+00	7.620548e+00	7.638597e+00	7.606183e+00	7.677094e+00	7.680532e+00	7.690265e+00	7.678573e+00
	max.	8.287337e+00	8.280538e+00	8.327027e+00	8.162905e+00	8.385896e+00	7.695877e+00	8.338148e+00	8.347446e+00	7.674065e+00	8.383730e+00	8.439334e+00	8.369776e+00	8.286741e+00
	std.	2.602582e-01	2.076009e-01	2.346923e-01	2.130769e-01	3.070772e-01	3.338346e-02	2.910519e-01	1.454194e-01	3.060959e-02	2.938971e-01	3.035863e-01	2.761519e-01	
3D	med.	6.527962e+01	6.534327e+01	5.836650e+01	6.516279e+01	7.369133e+01	6.668867e+01	6.605526e+01	6.602463e+01	6.566029e+01	7.254621e+01	6.527684e+01	6.653916e+01	6.696926e+01
	avg.	6.514404e+01	6.681222e+01	5.870186e+01	6.509685e+01	7.243494e+01	6.660841e+01	6.691457e+01	6.717851e+01	6.565937e+01	7.106331e+01	6.722452e+01	6.793165e+01	6.751350e+01
	min.	6.128491e+01	6.456790e+01	4.721122e+01	5.899720e+01	6.827014e+01	6.561104e+01	6.406720e+01	6.468427e+01	6.415069e+01	6.747216e+01	6.494306e+01	6.610249e+01	6.518482e+01
	max.	6.780847e+01	7.047450e+01	6.731869e+01	6.949716e+01	7.43484e+01	6.708276e+01	7.168178e+01	7.085422e+01	6.679318e+01	7.344112e+01	7.078491e+01	7.202515e+01	7.064832e+01
	std.	1.300269e+00	2.045506e+00	3.217780e+00	1.983030e+00	2.376786e+00	3.038930e-01	2.084607e+00	2.112440e+00	4.291575e-01	2.381627e+00	2.428549e+00	2.315506e+00	1.230659e+00
4D	med.	5.300733e+02	5.714567e+02	5.832796e+02	5.886619e+02	3.748838e+02	6.647851e+02	6.523326e+02	6.533806e+02	6.403066e+02	7.012552e+02	5.761338e+02	6.100624e+02	6.485794e+02
	avg.	5.413780e+02	5.765696e+02	5.338624e+02	5.739912e+02	3.788375e+02	6.646283e+02	6.556279e+02	6.551010e+02	6.396910e+02	6.961588e+02	5.820332e+02	6.111755e+02	6.460903e+02
	min.	4.582109e+02	5.471758e+02	3.303547e+02	3.908799e+02	3.417029e+02	6.492662e+02	6.241218e+02	6.358438e+02	6.193526e+02	6.791572e+02	5.673522e+02	6.009936e+02	5.991620e+02
	max.	6.128625e+02	6.203928e+02	6.380364e+02	6.536377e+02	4.608396e+02	6.753173e+02	6.961417e+02	6.818845e+02	6.555038e+02	7.100820e+02	6.280856e+02	6.460136e+02	6.665745e+02
	std.	4.120578e+01	1.351225e+01	9.340550e+01	5.748357e+01	2.273445e+01	4.549004e+00	5.116911e+01	9.342123e+00	7.609203e+00	1.015836e+01	1.412893e+01	7.492596e+00	1.486217e+01
5D	med.	5.764846e+03	5.865680e+03	4.786984e+03	6.458545e+03	4.140161e+03	6.913458e+03	7.512507e+03	7.465690e+03	7.204375e+03	7.879252e+03	6.453111e+03	6.164764e+03	6.697752e+03
	avg.	5.727189e+03	5.797702e+03	4.803194e+03	5.944315e+03	4.939073e+03	6.908978e+03	7.540150e+03	7.445514e+03	7.194991e+03	7.884235e+03	6.455636e+03	6.169278e+03	6.712606e+03
	min.	4.280141e+03	4.236082e+03	3.187078e+03	2.913353e+03	3.851469e+03	6.676350e+03	7.216084e+03	7.129465e+03	6.835440e+03	7.726729e+03	5.861417e+03	5.815316e+03	6.204190e+03
	max.	6.262670e+03	6.320721e+03	6.300747e+03	7.221621e+03	8.199542e+03	7.133778e+03	7.880198e+03	7.624332e+03	7.437330e+03	8.046273e+03	6.773108e+03	6.524031e+03	7.435088e+03
	std.	2.449390e+02	3.727852e+02	9.231974e+02	1.176378e+03	1.562664e+03	9.322844e+01	1.755420e+02	1.092773e+02	1.183286e+02	6.125363e+01	1.511765e+02	1.740056e+02	2.639866e+02
6D	med.	5.858313e+04	6.039915e+04	5.454309e+04	7.531459e+04	5.643296e+04	8.312766e+04	9.948890e+04	9.717952e+04	9.246754e+04	1.010312e+05	7.520660e+04	6.849397e+04	7.792511e+04
	avg.	5.817116e+04	6.166974e+04	5.180986e+04	6.812625e+04	6.254205e+04	8.292610e+04	9.931126e+04	9.656728e+04	9.227328e+04	1.010852e+05	7.592627e+04	6.996235e+04	7.728142e+04
	min.	4.867960e+04	4.250618e+04	3.429049e+04	2.752715e+04	5.058179e+04	5.126471e+04	9.512043e+04	9.034999e+04	8.521438e+04	9.732801e+04	5.612416e+04	5.640955e+04	6.897675e+04
	max.	6.716692e+04	7.688261e+04	6.910443e+04	9.004344e+04	1.060216e+05	8.970307e+04	1.031064e+05	1.013236e+05	9.557828e+04	1.042365e+05	8.989481e+04	8.492864e+04	8.486564e+04
	std.	3.928474e+03	8.064318e+03	1.003475e+04	1.794691e+04	1.582459e+04	4.780684e+03	1.649863e+03	2.487816e+03	2.074095e+03	1.479435e+03	6.866258e+03	6.460904e+03	3.846485e+03
7D	med.	6.323621e+05	7.053351e+05	6.762812e+05	7.587291e+05	8.051304e+05	1.088514e+06	1.420512e+06	1.341180e+06	1.310690e+06	1.456702e+06	8.101270e+05	6.741161e+05	9.671365e+05
	avg.	6.384818e+05	6.864183e+05	6.530240e+05	7.513025e+05	8.581351e+05	1.042663e+06	1.422545e+06	1.343000e+06	1.307359e+06	1.459950e+06	8.230703e+05	7.194633e+05	9.689602e+05
	min.	5.036688e+05	4.004194e+05	4.890448e+05	1.407016e+05	6.589928e+05	5.394781e+05	1.325909e+06	1.221795e+06	1.163620e+06	1.400465e+06	6.317589e+05	5.261187e+05	6.581308e+05
	max.	7.738779e+05	9.150218e+05	8.400844e+05	1.168134e+06	1.401082e+06	1.188216e+06	1.485754e+06	1.457133e+06	1.379770e+06	1.536552e+06	1.028823e+06	9.061386e+05	1.127031e+06
	std.	5.519328e+04	9.500074e+04	9.549866e+04	2.979684e+05	1.606389e+05	1.477581e+05	3.013138e+04	5.260217e+04	4.275769e+04	3.284949e+04	6.389270e+04	7.610115e+04	7.280027e+04
8D	med.	1.013170e+07	1.135647e+07	1.057083e+07	1.242834e+07	1.542715e+07	1.785157e+07	2.436908e+07	2.265258e+07	2.169063e+07	2.502594e+07	1.351279e+07	1.093869e+07	1.426530e+07
	avg.	1.026419e+07	1.088985e+07	1.026451e+07	1.101311e+07	1.671322e+07	1.747521e+07	2.428314e+07	2.282905e+07	2.145821e+07	2.506680e+07	1.357910e+07	1.096904e+07	1.418359e+07
	min.	8.361535e+06	5.089832e+06	6.169576e+06	1.840888e+06	1.291891e+07	8.584098e+06	2.094311e+07	2.068587e+07	1.844275e+07	2.389601e+07	7.661959e+06	8.517427e+06	8.519897e+06
	max.	1.211684e+07	1.455851e+07	1.374816e+07	1.875135e+07	2.668800e+07	1.941610e+07	2.542654e+07	2.517900e+07	2.287393e+07	2.615148e+07	1.779995e+07	1.391191e+07	1.623293e+07
	std.	7.222541e+05	1.947084e+06	1.490817e+06	4.752779e+06	3.590589e+06	1.785026e+06	6.283132e+05	1.052510e+06	8.783492e+05	4.563667e+05	1.574957e+06	1.263360e+06	1.192018e+06
9D	med.	1.916078e+08	2.128270e+08	1.862439e+08	2.042259e+08	3.385112e+08	2.713920e+08	4.569334e+08	4.209202e+08	3.850236e+08	4.601000e+08	2.468089e+08	2.041964e+08	2.415614e+08
	avg.	1.925167e+08	2.054503e+08	1.858569e+08	1.853978e+08	3.478025e+08	2.614818e+08	4.530753e+08	4.222975e+08	3.825263e+08	4.599595e+08	2.459670e+08	2.048265e+08	2.432177e+08
	min.	1.649565e+08	1.123482e+08	1.418733e+08	2.883366e+07	2.974242e+08	1.530858e+08	3.647748e+08	3.271847e+08	3.271847e+08	4.353291e+08	1.150329e+08	1.427793e+08	1.897724e+08
	max.	2.204062e+08	2.643534e+08	2.361205e+08	3.188717e+08	4.694565e+08	3.218445e+08	4.852099e+08	4.863847e+08	4.142138e+08	4.820497e+08	2.987134e+08	2.879048e+08	2.932679e+08
	std.	1.179596e+07	2.300492e+07	8.127499e+07	3.859251e+07	4.158010e+07	1.458010e+07	1.698714e+07	1.262651e+07	1.879681e+07	9.849063e+06	2.730898e+06	3.179065e+07	2.302510e+07
10D	med.	3.977346e+09	4.046560e+09	3.858338e+09	2.880690e+09	8.336035e+09	4.033182e+09	9.161036e+09	8.284437e+09	7.403559e+09	9.200227e+09	5.274879e+09	4.305756e+09	4.079922e+09
	avg.	3.975575e+09	3.856809e+09	3.786417e+09	2.920718e+09	8.250245e+09	4.016097e+09	9.173132e+09	8.392398e+09	7.423016e+09	9.184543e+09	5.290645e+09	4.182478e+09	4.101724e+09
	min.	3.256858e+09	1.908503e+09	2.798560e+09	8.316655e+08	7.338042e+09	8.093801e+09	8.590508e+09	6.573627e+09	5.750508e+09	6.573627e+09	2.061043e+09	2.755909e+09	1.803577e+09
	max.	4.674208e+09	5.271410e+09	4.510414e+09	6.008565e+09	9.096199e+09	5.96258e+09	1.000591e+10	9.524578e+09	8.173462e+09	9.693040e+09	6.922321e+09	6.210660e+09	5.261407e+09
	std.	2.687454e+08	8.365429e+08	4.180281e+08	1.394625e+09	4.985583e+08	3.322876e+08	5.050724e+08	3.812067e+08	3.812067e+08	2.052846e+08	6.758185e+08	7.653200e+08	4.588281e+08

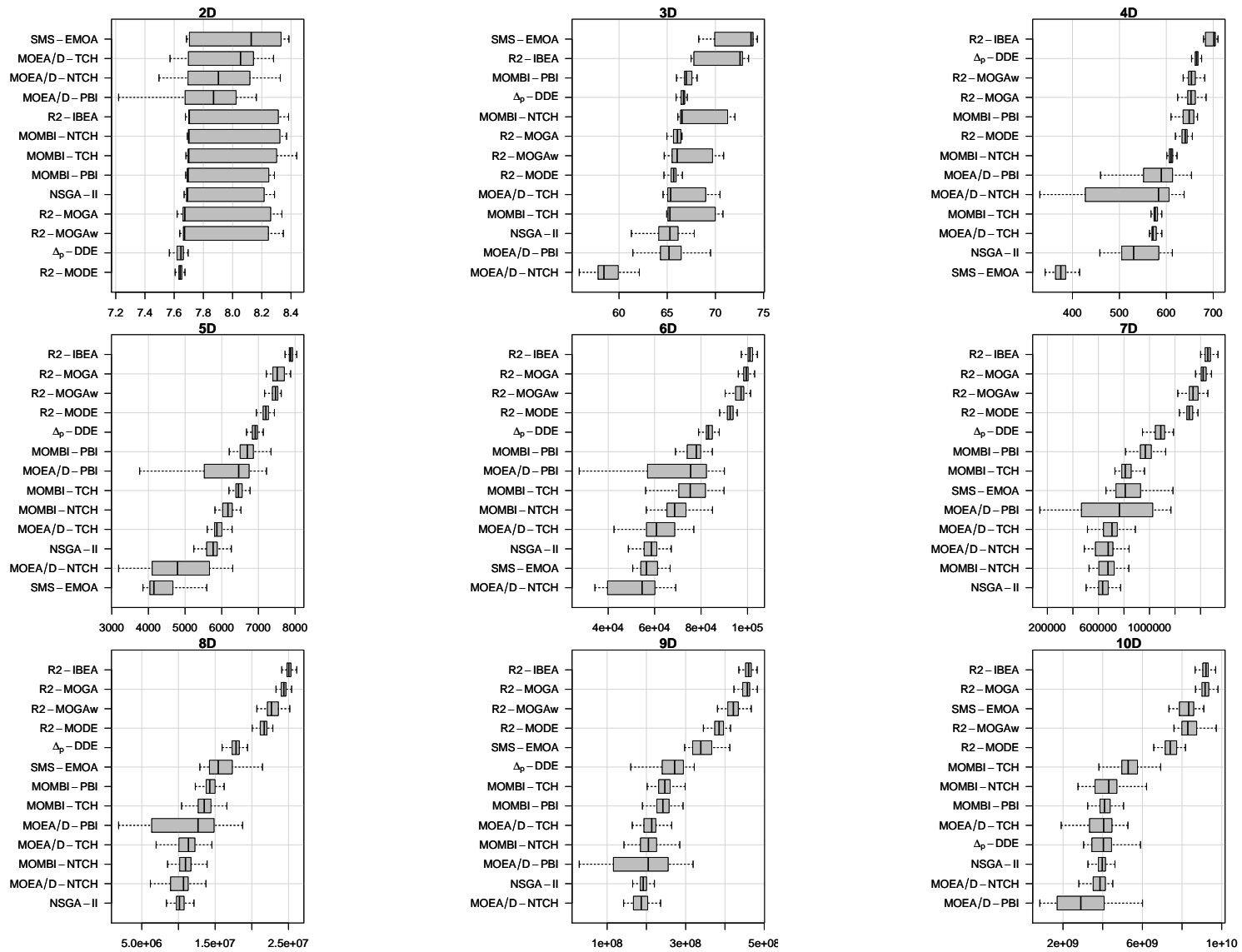


Figure A.241: Box-plot of hypervolume indicator values for different optimizers on the WFG9 test problem.

Table A.93: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG9 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	4.60e-30	3.79e-05	3.79e-05	1.63e-34	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.12e-03	—	9.10e-03	1.02e-07	> 0.05	1.59e-32	4.20e-05	1.99e-04	2.51e-33	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	9.72e-03	> 0.05	2.97e-27	1.62e-03	4.75e-03	1.29e-28	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	> 0.05	5.51e-18	> 0.05	> 0.05	3.59e-20	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	5.55e-12	9.77e-04	9.40e-07	3.29e-06	—	4.78e-34	1.18e-12	4.61e-12	1.28e-34	2.41e-03	1.71e-06	8.44e-05	7.46e-07
$\Delta_p$ -DDE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	2.66e-17	—	> 0.05	1.40e-25	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGAw	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	2.91e-18	> 0.05	—	1.52e-26	> 0.05	> 0.05	> 0.05	> 0.05
R2-MODE	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	9.73e-10	> 0.05	4.03e-03	1.72e-03	> 0.05	4.64e-34	6.81e-11	2.69e-10	1.28e-34	—	1.24e-02	> 0.05	4.60e-05
MOMBI-TCH	9.42e-08	> 0.05	> 0.05	> 0.05	> 0.05	7.03e-34	5.55e-10	3.91e-09	1.28e-34	> 0.05	—	> 0.05	1.32e-03
MOMBI-NTCH	4.99e-10	> 0.05	4.93e-02	2.13e-02	> 0.05	1.95e-34	3.45e-11	2.34e-10	1.28e-34	> 0.05	3.89e-02	—	2.14e-05
MOMBI-PBI	1.14e-03	> 0.05	> 0.05	1.81e-04	> 0.05	7.78e-32	1.38e-06	1.52e-06	1.28e-34	> 0.05	> 0.05	> 0.05	—
3D													
NSGA-II	—	> 0.05	1.60e-28	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.69e-06	—	1.13e-30	9.40e-07	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	> 0.05	1.05e-27	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	1.28e-34	5.70e-25	1.28e-34	7.92e-34	—	1.28e-34	8.28e-28	6.44e-24	1.28e-34	8.82e-16	4.44e-22	1.27e-25	4.01e-28
$\Delta_p$ -DDE	2.10e-19	> 0.05	1.12e-32	7.04e-11	> 0.05	—	5.37e-09	2.09e-03	1.60e-29	> 0.05	4.39e-02	> 0.05	> 0.05
R2-MOGA	7.04e-11	8.70e-04	4.44e-31	2.11e-09	> 0.05	> 0.05	—	> 0.05	9.79e-08	> 0.05	6.27e-04	> 0.05	> 0.05
R2-MOGAw	6.29e-11	8.35e-04	2.66e-31	1.77e-10	> 0.05	> 0.05	> 0.05	—	8.76e-06	> 0.05	1.63e-02	> 0.05	> 0.05
R2-MODE	3.34e-04	> 0.05	1.74e-29	8.21e-04	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05
R2-IBEA	8.92e-34	2.39e-20	1.28e-34	6.93e-32	> 0.05	1.28e-34	7.99e-26	6.21e-21	1.28e-34	—	1.08e-18	2.61e-23	2.11e-25
MOMBI-TCH	1.47e-06	2.28e-02	1.10e-30	1.61e-07	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	6.72e-23	2.46e-06	4.39e-33	1.36e-15	> 0.05	> 0.05	3.44e-14	4.58e-09	2.31e-31	> 0.05	9.87e-07	—	> 0.05
MOMBI-PBI	6.14e-25	3.43e-04	1.43e-33	6.58e-17	> 0.05	2.89e-16	1.49e-11	3.50e-05	9.95e-33	> 0.05	6.70e-03	1.61e-05	—
4D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	1.32e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.42e-06	—	> 0.05	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	4.02e-02	> 0.05	—	> 0.05	2.57e-24	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.91e-08	1.16e-02	1.09e-02	—	8.92e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e-34	1.28e-34	1.28e-34	1.32e-34	1.28e-34	—	5.10e-12	6.57e-14	2.26e-34	> 0.05	1.28e-34	1.28e-34	3.26e-25
R2-MOGA	1.28e-34	1.28e-34	3.05e-34	1.78e-32	1.28e-34	> 0.05	—	> 0.05	4.74e-17	> 0.05	1.36e-34	1.20e-33	1.31e-03
R2-MOGAw	1.28e-34	1.28e-34	1.44e-34	4.14e-33	1.28e-34	> 0.05	> 0.05	—	1.03e-23	> 0.05	1.28e-34	2.88e-34	8.95e-05
R2-MODE	1.28e-34	1.32e-34	7.88e-33	4.01e-28	1.28e-34	> 0.05	> 0.05	> 0.05	—	> 0.05	1.89e-34	3.07e-31	> 0.05
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.28e-34	4.32e-31	1.63e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	5.76e-08	1.41e-08	> 0.05	> 0.05	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05
MOMBI-NTCH	6.61e-33	8.48e-30	4.02e-13	1.69e-08	1.28e-34	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	3.43e-25	—	> 0.05
MOMBI-PBI	1.58e-34	2.33e-34	1.34e-31	5.53e-29	1.28e-34	> 0.05	> 0.05	> 0.05	2.90e-06	> 0.05	7.46e-34	1.68e-30	—

Table A.94: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG9 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
5D													
NSGA-II	—	> 0.05	4.02e-12	> 0.05	6.82e-14	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	3.15e-05	—	9.21e-15	> 0.05	4.98e-14	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	2.94e-02	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	4.58e-08	8.85e-07	5.88e-14	—	2.27e-07	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	7.76e-03	> 0.05
SMS-EMOA	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	1.28e-34	1.28e-34	1.28e-34	1.53e-18	1.82e-14	—	> 0.05	> 0.05	> 0.05	> 0.05	1.68e-34	1.28e-34	1.18e-11
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	1.32e-34	1.82e-14	1.28e-34	—	5.01e-04	6.24e-30	> 0.05	1.28e-34	1.28e-34	6.83e-34
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.73e-34	1.82e-14	1.36e-34	> 0.05	—	8.76e-27	> 0.05	1.28e-34	1.28e-34	2.04e-33
R2-MODE	1.28e-34	1.28e-34	1.28e-34	2.24e-31	1.82e-14	8.96e-30	> 0.05	> 0.05	—	> 0.05	1.28e-34	1.28e-34	6.07e-27
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.63e-14	1.28e-34	5.20e-32	1.28e-34	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	7.69e-34	9.39e-33	7.03e-34	> 0.05	1.82e-14	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	4.44e-22	> 0.05
MOMBI-NTCH	4.95e-29	1.70e-20	4.71e-28	> 0.05	1.82e-14	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	1.36e-34	1.84e-34	1.53e-34	4.12e-07	1.82e-14	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	5.35e-13	3.11e-30	—
6D													
NSGA-II	—	> 0.05	8.57e-06	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	3.85e-04	—	8.39e-11	> 0.05	1.20e-03	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	5.68e-09	1.09e-06	2.45e-12	—	6.94e-05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	2.87e-02	> 0.05
SMS-EMOA	> 0.05	> 0.05	4.16e-05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
$\Delta_p$ -DDE	2.75e-32	2.31e-32	3.08e-33	8.98e-14	1.59e-18	—	> 0.05	> 0.05	> 0.05	> 0.05	4.22e-14	3.13e-27	2.10e-24
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	4.44e-22	1.28e-34	—	7.38e-16	1.53e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.31e-21	1.28e-34	> 0.05	—	2.23e-22	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-MODE	1.28e-34	1.28e-34	1.28e-34	2.88e-34	6.66e-21	4.24e-34	> 0.05	> 0.05	—	> 0.05	3.15e-34	1.28e-34	1.28e-34
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	4.91e-23	1.28e-34	7.22e-13	1.00e-29	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	2.04e-33	1.35e-23	4.14e-33	4.46e-02	4.94e-19	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	9.88e-10	> 0.05
MOMBI-NTCH	4.20e-29	1.02e-11	2.53e-28	> 0.05	4.74e-17	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	1.28e-34	2.70e-30	1.32e-34	2.06e-02	1.15e-19	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	4.42e-02	5.48e-16	—
7D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.32e-07	—	3.00e-03	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	2.24e-02	> 0.05
MOEA/D-NTCH	1.83e-02	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	3.22e-04	3.00e-02	3.64e-03	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	7.50e-03	> 0.05
SMS-EMOA	1.52e-29	1.97e-16	1.17e-21	1.71e-02	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	3.41e-21	> 0.05
$\Delta_p$ -DDE	1.57e-27	3.02e-25	1.78e-26	6.34e-14	4.63e-14	—	> 0.05	> 0.05	> 0.05	> 0.05	9.42e-23	5.57e-26	1.68e-16
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	2.26e-34	1.28e-34	—	1.84e-22	8.66e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.28e-34	9.39e-33	1.28e-34	> 0.05	—	3.14e-06	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-MODE	1.28e-34	1.28e-34	1.28e-34	1.36e-34	5.35e-32	1.40e-34	> 0.05	> 0.05	—	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.32e-34	1.28e-34	2.76e-13	5.58e-31	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	3.18e-33	1.28e-24	5.25e-28	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	1.14e-27	> 0.05
MOMBI-NTCH	7.87e-04	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	3.65e-34	2.74e-33	8.40e-34	7.34e-06	1.65e-11	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	2.91e-26	1.48e-33	—

Table A.95: Wilcoxon rank sum test applied to hypervolume indicator values on the WFG9 test problem (continuation).

Optimizer	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
8D													
NSGA-II	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	3.59e-07	—	3.65e-04	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	1.40e-02	4.28e-02	1.18e-02	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	1.28e-34	1.81e-33	1.95e-34	6.57e-14	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	3.83e-16	4.64e-34	1.72e-09
$\Delta_p$ -DDE	1.59e-31	5.90e-31	1.30e-31	8.06e-28	1.05e-08	—	> 0.05	> 0.05	> 0.05	> 0.05	1.74e-28	7.40e-31	3.37e-29
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	2.01e-25	1.28e-34	—	2.09e-20	3.08e-33	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.28e-34	6.33e-22	1.28e-34	> 0.05	—	1.23e-17	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-MODE	1.28e-34	1.28e-34	1.28e-34	1.36e-34	8.15e-19	3.44e-34	> 0.05	> 0.05	—	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	2.16e-27	1.28e-34	7.54e-20	4.32e-31	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	2.42e-30	4.44e-22	2.34e-28	5.94e-03	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	6.13e-24	> 0.05
MOMBI-NTCH	4.53e-06	> 0.05	1.26e-03	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	3.47e-32	2.34e-28	1.01e-31	2.51e-05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	3.64e-05	8.02e-30	—
9D													
NSGA-II	—	> 0.05	1.90e-02	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	1.64e-09	—	2.79e-09	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	1.28e-34	1.28e-34	1.28e-34	1.20e-33	—	5.50e-32	> 0.05	> 0.05	> 0.05	> 0.05	1.36e-34	1.28e-34	1.28e-34
$\Delta_p$ -DDE	9.20e-25	4.33e-19	2.46e-25	6.97e-13	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	2.96e-05	1.98e-18	9.75e-07
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	4.44e-31	1.28e-34	—	1.59e-18	4.24e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.28e-34	2.43e-26	1.28e-34	> 0.05	—	5.57e-26	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-MODE	1.28e-34	1.28e-34	1.28e-34	1.28e-34	3.88e-15	1.28e-34	> 0.05	> 0.05	—	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	2.18e-32	1.28e-34	2.32e-03	2.28e-25	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	2.06e-31	3.59e-22	4.73e-30	7.32e-08	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	1.89e-18	> 0.05
MOMBI-NTCH	3.75e-04	> 0.05	7.10e-06	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	1.26e-32	2.87e-20	9.28e-31	5.55e-07	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	2.55e-17	—
10D													
NSGA-II	—	> 0.05	2.29e-03	2.05e-08	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-TCH	> 0.05	—	5.94e-03	4.02e-07	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-NTCH	> 0.05	> 0.05	—	3.61e-06	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOEA/D-PBI	> 0.05	> 0.05	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
SMS-EMOA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	—	1.28e-34	> 0.05	> 0.05	2.51e-22	> 0.05	1.28e-34	1.28e-34	1.28e-34
$\Delta_p$ -DDE	> 0.05	> 0.05	2.62e-03	1.55e-08	> 0.05	—	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
R2-MOGA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	5.99e-28	1.28e-34	—	4.82e-21	1.53e-34	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-MOGAw	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	> 0.05	—	1.09e-28	> 0.05	1.28e-34	1.28e-34	1.28e-34
R2-MODE	1.28e-34	1.28e-34	1.28e-34	1.28e-34	> 0.05	1.28e-34	> 0.05	> 0.05	—	> 0.05	1.73e-34	1.28e-34	1.28e-34
R2-IBEA	1.28e-34	1.28e-34	1.28e-34	1.28e-34	1.23e-30	1.28e-34	> 0.05	1.20e-22	1.28e-34	—	1.28e-34	1.28e-34	1.28e-34
MOMBI-TCH	5.00e-30	1.11e-26	6.61e-31	3.87e-27	> 0.05	2.02e-26	> 0.05	> 0.05	> 0.05	> 0.05	—	4.94e-21	3.39e-27
MOMBI-NTCH	5.51e-04	4.71e-03	2.83e-06	1.03e-10	> 0.05	2.60e-02	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—	> 0.05
MOMBI-PBI	3.85e-03	> 0.05	6.68e-07	6.67e-10	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	—

Table A.96: Comparison of R2 indicator values for different optimizers on the WFG9 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	6.374737e-01	6.141825e-01	6.240781e-01	6.264305e-01	6.118747e-01	6.429325e-01	6.381006e-01	6.381383e-01	6.413719e-01	6.368733e-01	6.368422e-01	6.368137e-01	6.370227e-01
	avg.	6.280430e-01	6.201722e-01	6.266428e-01	6.312073e-01	6.197323e-01	6.433286e-01	6.268301e-01	6.264398e-01	6.413234e-01	6.230863e-01	6.259769e-01	6.247761e-01	6.231082e-01
	min.	6.046449e-01	6.033492e-01	6.026045e-01	6.090651e-01	5.991636e-01	6.373306e-01	6.000670e-01	6.012128e-01	6.380039e-01	5.992450e-01	5.940072e-01	5.998328e-01	6.019202e-01
	max.	6.405920e-01	6.556853e-01	6.835943e-01	6.903178e-01	6.397224e-01	6.549550e-01	6.401242e-01	6.409063e-01	6.454122e-01	6.403187e-01	6.391735e-01	6.378948e-01	6.392631e-01
	std.	1.414766e-02	1.322962e-02	1.527188e-02	1.766426e-02	1.740742e-02	3.302205e-03	1.649069e-02	1.644013e-02	1.550670e-03	1.747772e-02	1.669699e-02	1.689185e-02	1.558241e-02
3D	med.	4.003432e-01	3.512588e-01	3.820262e-01	3.604678e-01	3.235054e-01	3.587748e-01	3.506886e-01	3.499651e-01	3.526949e-01	3.290028e-01	3.506422e-01	3.502332e-01	3.502730e-01
	avg.	4.004718e-01	3.440713e-01	3.849863e-01	3.612567e-01	3.290203e-01	3.598278e-01	3.459650e-01	3.434246e-01	3.526223e-01	3.354779e-01	3.417765e-01	3.442458e-01	3.477874e-01
	min.	3.524828e-01	3.288790e-01	3.468199e-01	3.400375e-01	3.197542e-01	3.535132e-01	3.245698e-01	3.269116e-01	3.499639e-01	3.249412e-01	3.243362e-01	3.265962e-01	3.343349e-01
	max.	4.394737e-01	3.540128e-01	4.827479e-01	3.976134e-01	3.487248e-01	3.730331e-01	3.536423e-01	3.523760e-01	3.545167e-01	3.540583e-01	3.515955e-01	3.515195e-01	3.578989e-01
	std.	1.642581e-02	9.294537e-03	2.107819e-02	9.798760e-03	9.973942e-03	4.189443e-03	9.731293e-03	9.892632e-03	9.956899e-04	1.048829e-02	1.071851e-02	9.752584e-03	5.683055e-03
4D	med.	3.790563e-01	2.532590e-01	3.070367e-01	2.786638e-01	4.341579e-01	2.581518e-01	2.391983e-01	2.404687e-01	2.454407e-01	2.383626e-01	2.516576e-01	2.522740e-01	2.497414e-01
	avg.	3.816079e-01	2.523398e-01	3.635767e-01	2.902078e-01	4.287801e-01	2.577345e-01	2.373910e-01	2.399188e-01	2.463940e-01	2.396257e-01	2.502520e-01	2.518783e-01	2.517441e-01
	min.	3.227030e-01	2.397717e-01	2.464914e-01	2.440027e-01	3.514701e-01	2.441719e-01	2.247648e-01	2.295866e-01	2.410252e-01	2.321709e-01	2.385685e-01	2.388213e-01	2.422842e-01
	max.	4.661925e-01	2.971162e-01	7.705314e-01	4.20574e-01	4.657006e-01	2.679582e-01	2.448469e-01	2.512845e-01	2.564323e-01	2.518356e-01	2.551774e-01	2.551930e-01	2.726223e-01
	std.	3.107193e-02	4.577984e-03	1.050953e-01	3.638257e-02	2.332720e-02	4.829890e-03	5.171303e-03	3.845328e-03	3.589527e-03	4.880153e-03	3.734567e-03	2.687279e-03	7.062089e-03
5D	med.	3.734816e-01	2.116464e-01	4.882519e-01	2.342705e-01	3.440259e-01	2.628732e-01	1.900229e-01	1.956817e-01	2.008520e-01	2.034399e-01	2.078007e-01	2.098962e-01	2.308127e-01
	avg.	3.730449e-01	2.188856e-01	4.819874e-01	2.600905e-01	3.122738e-01	2.628899e-01	1.880968e-01	1.963817e-01	2.030077e-01	2.035072e-01	2.084951e-01	2.100380e-01	2.315306e-01
	min.	3.119251e-01	2.022393e-01	2.824535e-01	2.052404e-01	1.870647e-01	2.467463e-01	1.898126e-01	1.898126e-01	1.957271e-01	1.955750e-01	2.037590e-01	2.033086e-01	2.019446e-01
	max.	4.363222e-01	3.270606e-01	8.028374e-01	4.423013e-01	3.632231e-01	2.803048e-01	1.959311e-01	2.103603e-01	2.242563e-01	2.157680e-01	2.172513e-01	2.230716e-01	2.519438e-01
	std.	2.185414e-02	2.121697e-02	1.288608e-01	6.128956e-02	6.145287e-02	6.627812e-03	3.947119e-03	3.132023e-03	6.509934e-03	3.309511e-03	2.747320e-03	3.453494e-03	9.633806e-03
6D	med.	4.259070e-01	2.557619e-01	5.021388e-01	2.043017e-01	2.758521e-01	2.450401e-01	1.534205e-01	1.651279e-01	1.708628e-01	1.813804e-01	2.312219e-01	2.522152e-01	2.082659e-01
	avg.	4.274117e-01	2.555926e-01	5.421843e-01	2.350653e-01	2.605326e-01	2.471286e-01	1.537774e-01	1.665387e-01	1.734993e-01	1.822332e-01	2.237643e-01	2.415545e-01	2.088866e-01
	min.	3.667797e-01	1.783273e-01	2.958914e-01	1.729502e-01	1.583561e-01	2.264244e-01	1.486148e-01	1.578459e-01	1.642792e-01	1.693129e-01	1.671882e-01	1.688180e-01	1.878672e-01
	max.	4.968168e-01	3.878446e-01	8.209674e-01	4.234532e-01	2.989433e-01	3.618535e-01	1.673124e-01	1.822304e-01	2.016635e-01	2.083603e-01	3.015940e-01	3.399488e-01	2.303942e-01
	std.	2.650694e-02	5.111104e-02	1.626368e-01	6.739625e-02	3.921429e-02	1.750561e-02	2.676265e-03	5.092697e-03	8.303624e-03	5.781209e-03	3.283568e-02	3.102292e-02	9.741353e-03
7D	med.	4.895159e-01	3.824001e-01	5.367198e-01	2.425664e-01	2.414761e-01	2.791945e-01	1.428781e-01	1.630427e-01	1.590064e-01	1.736585e-01	3.710862e-01	3.810805e-01	2.109034e-01
	avg.	4.920525e-01	3.871230e-01	5.775091e-01	2.617111e-01	2.388728e-01	2.889232e-01	1.433164e-01	1.656957e-01	1.616387e-01	1.740593e-01	3.590209e-01	3.750106e-01	2.116305e-01
	min.	4.278245e-01	2.843763e-01	2.856546e-01	1.666064e-01	1.639971e-01	2.537182e-01	1.380978e-01	1.475615e-01	1.495376e-01	1.629562e-01	2.290386e-01	2.707370e-01	1.855073e-01
	max.	5.595732e-01	5.316876e-01	8.525053e-01	5.521646e-01	2.836435e-01	4.028439e-01	1.514234e-01	2.139010e-01	1.998672e-01	1.877218e-01	4.052509e-01	4.074818e-01	2.697209e-01
	std.	2.721197e-02	4.166528e-02	1.509273e-01	8.553538e-02	2.912009e-02	3.282748e-02	2.752582e-03	1.134325e-02	9.458890e-03	5.236277e-03	3.248438e-02	1.920693e-02	1.346590e-02
8D	med.	4.727067e-01	4.014261e-01	5.466208e-01	2.178756e-01	2.062664e-01	2.730290e-01	1.351996e-01	1.554726e-01	1.716756e-01	1.673530e-01	3.819850e-01	3.967130e-01	2.112251e-01
	avg.	4.734402e-01	4.115522e-01	5.829778e-01	2.584216e-01	1.990148e-01	2.793883e-01	1.370990e-01	1.584381e-01	1.774000e-01	1.677934e-01	3.655130e-01	3.910892e-01	2.123148e-01
	min.	4.260233e-01	2.654214e-01	3.625807e-01	1.615960e-01	1.407221e-01	2.524606e-01	1.306132e-01	1.363020e-01	1.551275e-01	1.566085e-01	2.441317e-01	2.815605e-01	1.921586e-01
	max.	5.241628e-01	5.482886e-01	8.691738e-01	4.826262e-01	2.279857e-01	4.023260e-01	2.112521e-01	2.021054e-01	1.875266e-01	1.875266e-01	4.232503e-01	4.272062e-01	2.876566e-01
	std.	2.159949e-02	4.423018e-02	1.323850e-01	8.409084e-02	2.196932e-02	2.594070e-02	9.142413e-03	1.268867e-02	1.795544e-02	5.938464e-03	3.919736e-02	2.235743e-02	1.429348e-02
9D	med.	4.469952e-01	4.152061e-01	5.615548e-01	2.182564e-01	1.830486e-01	3.153237e-01	1.349760e-01	1.563224e-01	2.042996e-01	1.757429e-01	4.025969e-01	4.108421e-01	2.054320e-01
	avg.	4.469467e-01	4.256691e-01	5.779252e-01	2.528688e-01	1.825434e-01	3.189942e-01	1.362468e-01	1.574324e-01	2.101192e-01	1.762068e-01	3.920095e-01	4.023417e-01	2.055935e-01
	min.	4.063688e-01	3.218087e-01	4.179049e-01	1.600867e-01	1.546356e-01	2.640531e-01	1.261923e-01	1.319333e-01	1.772872e-01	1.620166e-01	2.950859e-01	3.252500e-01	1.728362e-01
	max.	4.910980e-01	5.619244e-01	8.571031e-01	5.439618e-01	2.004796e-01	3.745534e-01	1.863067e-01	1.852886e-01	2.745587e-01	1.943619e-01	4.598710e-01	4.455896e-01	2.411606e-01
	std.	1.479351e-02	3.855074e-02	1.121638e-01	8.457385e-02	9.300792e-03	2.684693e-02	7.763022e-03	1.083619e-02	2.146304e-02	5.907802e-03	3.197682e-02	2.571197e-02	1.190278e-02
10D	med.	4.219886e-01	4.362721e-01	5.684489e-01	2.552287e-01	1.797671e-01	3.449674e-01	1.391726e-01	1.654302e-01	2.366405e-01	1.905899e-01	4.012377e-01	4.128850e-01	2.151577e-01
	avg.	4.226874e-01	4.489533e-01	5.597542e-01	2.712065e-01	1.799813e-01	3.499357e-01	1.399814e-01	1.653945e-01	2.387209e-01	1.912743e-01	3.870916e-01	4.128776e-01	2.148776e-01
	min.	3.865636e-01	3.525529e-01	4.180108e-01	1.603418e-01	1.589261e-01	2.980909e-01	1.275629e-01	1.387302e-01	2.007910e-01	1.784184e-01	2.960074e-01	3.149260e-01	1.852206e-01
	max.	4.549693e-01	6.39675e-01	8.571038e-01	4.669688e-01	1.979171e-01	3.756298e-01	1.777364e-01	1.942655e-01	2.832127e-01	2.083465e-01	4.511844e-01	4.605085e-01	3.193956e-01
	std.	1.189746e-02	4.398402e-02	8.439147e-02	7.635390e-02	8.250617e-03	1.418844e-02	8.441883e-03	1.191077e-02	1.911682e-02	6.689295e-03	3.514536e-02	2.916996e-02	1.529778e-02



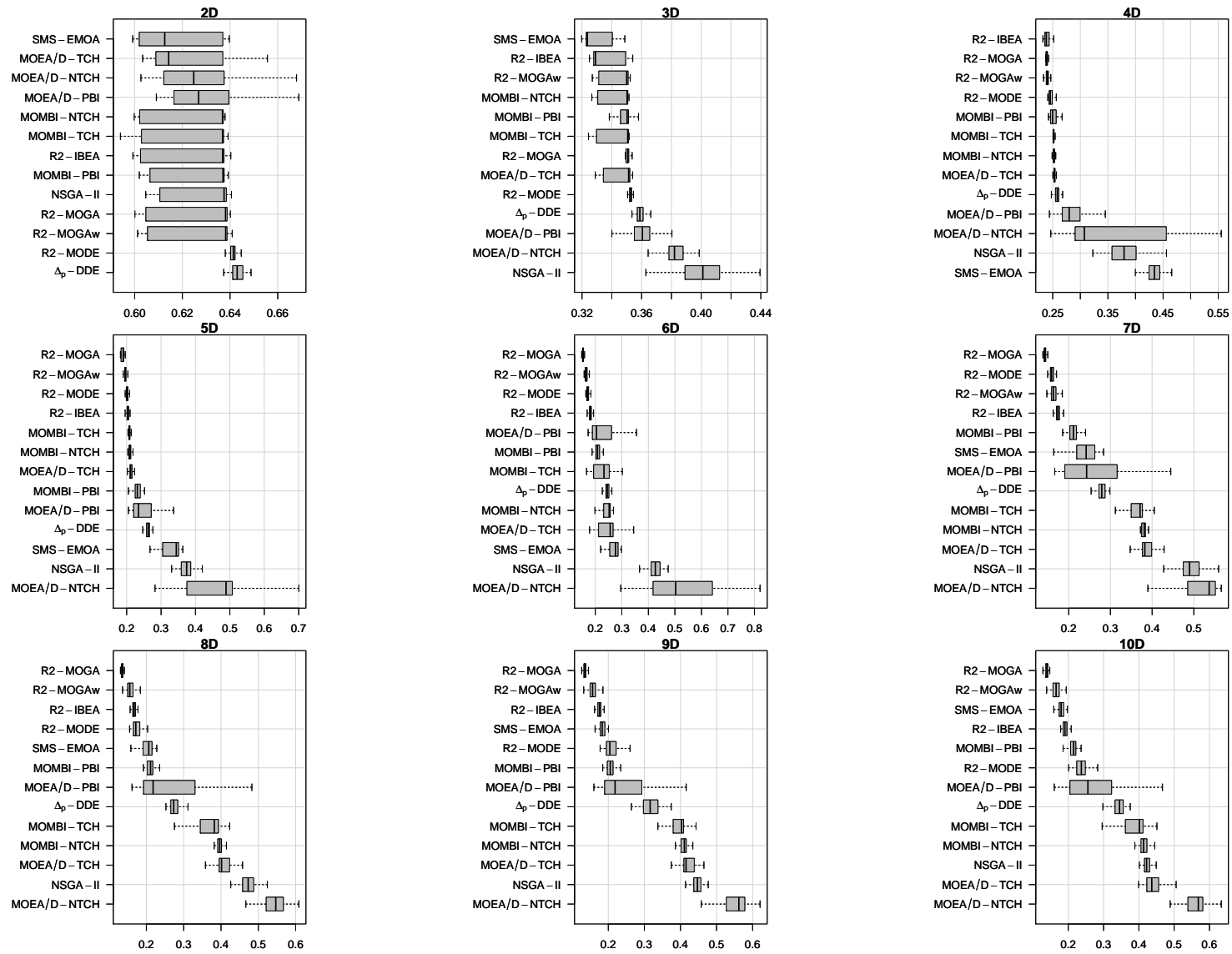


Figure A.242: Box-plot of R2 indicator values for different optimizers on the WFG9 test problem.

Table A.97: Comparison of runtime (in milliseconds) for different optimizers on the WFG9 test problem.

Dim.	Stat.	NSGA-II	MOEA/D TCH	MOEA/D NTCH	MOEA/D PBI	SMS-EMOA	$\Delta_p$ -DDE	R2-MOGA	R2-MOGAw	R2-MODE	R2-IBEA	MOMBI TCH	MOMBI NTCH	MOMBI PBI
2D	med.	7.192840e + 02	1.130402e + 03	1.764393e + 03	1.600620e + 03	3.925776e + 04	1.904283e + 03	2.474890e + 03	2.198544e + 03	2.592772e + 03	5.399460e + 05	2.369813e + 03	2.291440e + 03	3.347902e + 03
	avg.	7.214116e + 02	1.157607e + 03	1.760396e + 03	1.588276e + 03	4.795838e + 04	1.906657e + 03	2.470216e + 03	2.219562e + 03	2.596391e + 03	5.373937e + 05	2.371150e + 03	2.310871e + 03	3.319258e + 03
	min.	6.278880e + 02	9.493610e + 02	1.406653e + 03	1.151858e + 03	2.922216e + 04	1.649812e + 03	1.818981e + 03	1.850676e + 03	1.880385e + 03	4.888070e + 05	1.993054e + 03	1.967070e + 03	2.897781e + 03
	max.	8.969350e + 02	1.601898e + 03	2.511381e + 03	2.327696e + 03	6.240651e + 04	2.314574e + 03	3.143434e + 03	2.672411e + 03	3.022182e + 03	6.178340e + 05	2.841859e + 03	2.715129e + 03	3.861625e + 03
	std.	5.188032e + 01	1.252588e + 02	1.467916e + 02	1.945073e + 02	1.219291e + 04	1.298081e + 02	2.677614e + 02	1.773750e + 02	2.654672e + 02	4.132423e + 04	1.918736e + 02	1.527153e + 02	1.674054e + 02
3D	med.	8.774470e + 02	1.193648e + 03	2.039704e + 03	1.736035e + 03	1.117016e + 06	2.072782e + 03	2.852315e + 03	2.557808e + 03	2.980237e + 03	5.820110e + 05	2.446716e + 03	2.536833e + 03	3.630984e + 03
	avg.	8.887733e + 02	1.198713e + 03	2.020585e + 03	1.709085e + 03	1.096010e + 06	2.065788e + 03	2.838989e + 03	2.557755e + 03	2.998436e + 03	5.819289e + 05	2.454751e + 03	2.547638e + 03	3.611426e + 03
	min.	7.691360e + 02	9.847760e + 02	1.514931e + 03	1.319411e + 03	7.676752e + 05	1.518003e + 03	2.066717e + 03	1.998539e + 03	2.260535e + 03	5.683680e + 05	2.076116e + 03	2.186154e + 03	2.759733e + 03
	max.	1.042736e + 03	1.525176e + 03	2.387325e + 03	2.090751e + 03	1.311382e + 06	2.477946e + 03	3.241309e + 03	3.115404e + 03	3.486518e + 03	5.947040e + 05	3.257350e + 03	3.450557e + 03	4.006917e + 03
	std.	6.384703e + 01	1.101050e + 02	1.485269e + 02	1.175534e + 02	1.194502e + 05	2.023121e + 02	2.514821e + 02	2.307326e + 02	2.762007e + 02	4.226292e + 03	1.974394e + 02	1.986334e + 02	1.838090e + 02
4D	med.	1.365699e + 03	1.713082e + 03	2.694023e + 03	2.204725e + 03	3.785914e + 06	2.787588e + 03	3.940359e + 03	3.677244e + 03	4.320690e + 03	5.939470e + 05	2.942485e + 03	3.056217e + 03	4.193481e + 03
	avg.	1.383305e + 03	1.746758e + 03	2.653738e + 03	2.183388e + 03	3.784411e + 06	2.736548e + 03	3.968582e + 03	3.679295e + 03	4.348462e + 03	5.936051e + 05	2.959054e + 03	3.072286e + 03	4.163198e + 03
	min.	1.180328e + 03	1.467095e + 03	2.071932e + 03	1.661204e + 03	2.835462e + 06	1.855707e + 03	3.005023e + 03	2.908263e + 03	3.304308e + 03	5.773870e + 05	2.456110e + 03	2.625444e + 03	3.395185e + 03
	max.	1.643490e + 03	2.132277e + 03	3.560926e + 03	2.682901e + 03	4.499426e + 06	3.274070e + 03	4.636689e + 03	4.327432e + 03	5.122252e + 03	6.012740e + 05	3.923666e + 03	3.511107e + 03	4.619231e + 03
	std.	9.698830e + 01	1.421389e + 02	2.048855e + 02	1.989543e + 02	3.608629e + 05	2.485403e + 02	3.581732e + 02	2.839773e + 02	3.892389e + 02	3.831109e + 03	2.989573e + 02	1.876520e + 02	2.522115e + 02
5D	med.	1.889681e + 03	2.227688e + 03	3.489571e + 03	2.550505e + 03	4.681305e + 06	5.760096e + 03	5.518078e + 03	5.174560e + 03	5.977288e + 03	6.754550e + 05	3.436670e + 03	3.448432e + 03	5.126275e + 03
	avg.	1.897484e + 03	2.271914e + 03	3.492821e + 03	2.605415e + 03	4.645937e + 06	5.651620e + 03	5.447080e + 03	5.158201e + 03	5.975200e + 03	6.753681e + 05	3.462781e + 03	3.490345e + 03	5.143773e + 03
	min.	1.645536e + 03	1.913147e + 03	2.911188e + 03	2.091222e + 03	2.694878e + 06	3.985465e + 03	4.026752e + 03	4.539501e + 03	5.439501e + 03	6.296752e + 05	3.070114e + 03	3.471414e + 03	4.514142e + 03
	max.	2.298359e + 03	2.732816e + 03	4.424126e + 03	3.214659e + 03	5.531271e + 06	7.212566e + 03	6.346004e + 03	6.040483e + 03	6.904888e + 03	6.826490e + 05	5.559857e + 03	4.200042e + 03	6.082301e + 03
	std.	1.329118e + 02	1.878477e + 02	2.638158e + 02	2.405385e + 02	7.526093e + 05	6.484323e + 02	5.514919e + 02	4.074979e + 02	5.297221e + 02	4.933571e + 03	3.643096e + 02	2.473066e + 02	2.676484e + 02
6D	med.	2.533786e + 03	2.904138e + 03	4.325300e + 03	3.316259e + 03	5.507849e + 06	7.408391e + 03	7.165551e + 03	6.715202e + 03	7.618406e + 03	6.572410e + 05	3.579250e + 03	4.425430e + 03	5.938982e + 03
	avg.	2.554816e + 03	2.933778e + 03	4.277074e + 03	3.262703e + 03	5.382523e + 06	7.285439e + 03	7.282330e + 03	6.729820e + 03	7.699552e + 03	6.834738e + 05	3.646867e + 03	4.458278e + 03	5.930740e + 03
	min.	2.173682e + 03	2.513934e + 03	3.287095e + 03	2.523083e + 03	3.466406e + 06	5.373017e + 03	5.619156e + 03	5.650167e + 03	6.132209e + 03	5.953150e + 05	3.229801e + 03	3.883393e + 03	5.024529e + 03
	max.	2.995271e + 03	3.391721e + 03	4.923873e + 03	3.836908e + 03	6.441502e + 06	9.233846e + 03	8.399468e + 03	7.860767e + 03	9.013502e + 03	7.295100e + 05	4.332554e + 03	5.068267e + 03	6.741998e + 03
	std.	1.947456e + 02	1.979391e + 02	3.032149e + 02	3.189174e + 02	5.587366e + 05	6.805960e + 02	6.488364e + 02	6.137516e + 02	6.969519e + 02	3.618031e + 04	2.592418e + 02	2.667618e + 02	3.681548e + 02
7D	med.	3.004238e + 03	3.570111e + 03	4.967457e + 03	3.876554e + 03	4.022010e + 06	2.087519e + 04	7.980553e + 03	7.335407e + 03	8.950352e + 03	3.312830e + 05	4.064730e + 03	3.948905e + 03	5.658185e + 03
	avg.	3.012625e + 03	3.619978e + 03	4.934538e + 03	3.780127e + 03	3.973815e + 06	2.099483e + 04	7.770934e + 03	7.358554e + 03	8.951391e + 03	3.312636e + 05	4.018588e + 03	4.031591e + 03	5.682689e + 03
	min.	2.505831e + 03	2.980423e + 03	3.813804e + 03	2.885106e + 03	2.998222e + 06	1.302417e + 04	5.573955e + 03	5.519842e + 03	6.979076e + 03	3.207090e + 05	3.328393e + 03	3.493842e + 03	4.806977e + 03
	max.	3.489163e + 03	4.243729e + 03	5.534479e + 03	4.574495e + 03	4.688841e + 06	3.179705e + 04	8.981936e + 03	8.631752e + 03	9.967942e + 03	3.380520e + 05	4.600801e + 03	4.725166e + 03	6.623583e + 03
	std.	1.994171e + 02	2.633453e + 02	3.424317e + 02	3.673238e + 02	2.878678e + 05	3.287762e + 03	8.590249e + 02	6.137516e + 02	6.969519e + 02	3.186006e + 03	2.759589e + 02	3.195335e + 02	4.056784e + 02
8D	med.	3.993369e + 03	4.349874e + 03	6.046946e + 03	4.474010e + 03	5.916715e + 06	5.842457e + 04	1.105263e + 04	1.062235e + 04	1.293526e + 04	1.285496e + 06	4.768991e + 03	6.306760e + 03	9.077633e + 03
	avg.	4.041834e + 03	4.369598e + 03	5.950163e + 03	4.454157e + 03	5.783686e + 06	6.145178e + 04	1.075726e + 04	1.056563e + 04	1.288598e + 04	1.285563e + 06	4.888574e + 03	6.238861e + 03	9.004002e + 03
	min.	3.343315e + 03	3.675382e + 03	4.829459e + 03	3.405061e + 03	3.442897e + 06	3.403055e + 04	7.895213e + 03	9.284393e + 03	9.417447e + 03	1.255533e + 06	4.355209e + 03	4.995345e + 03	7.162046e + 03
	max.	4.716239e + 03	5.064191e + 03	6.680847e + 03	5.224358e + 03	7.220612e + 06	1.134710e + 05	1.310672e + 04	1.203258e + 04	1.435294e + 04	1.294386e + 06	5.631740e + 03	6.946921e + 03	1.060814e + 04
	std.	2.889103e + 02	2.626317e + 02	3.673021e + 02	4.580820e + 02	9.063075e + 05	1.459154e + 04	1.395752e + 03	5.978679e + 02	1.082482e + 03	5.444053e + 03	3.561547e + 02	3.850393e + 02	4.656284e + 02
9D	med.	5.134081e + 03	5.290670e + 03	7.089575e + 03	4.799098e + 03	1.093376e + 07	1.568057e + 05	1.040179e + 04	1.411817e + 04	1.715072e + 04	2.776831e + 06	7.108653e + 03	7.336645e + 03	9.700902e + 03
	avg.	5.189303e + 03	5.370589e + 03	6.991086e + 03	4.918607e + 03	1.090544e + 07	1.628049e + 05	1.047516e + 04	1.417089e + 04	1.691554e + 04	2.680729e + 06	7.126569e + 03	7.540661e + 03	9.680108e + 03
	min.	4.462718e + 03	4.690250e + 03	5.230945e + 03	3.871651e + 03	7.531984e + 06	1.032202e + 05	9.851604e + 03	1.160222e + 04	1.276560e + 04	2.259893e + 06	5.412874e + 03	6.459254e + 03	8.527388e + 03
	max.	5.923836e + 03	6.219566e + 03	7.960573e + 03	6.195811e + 03	1.327175e + 07	2.333416e + 05	1.130856e + 04	1.631984e + 04	1.944322e + 04	2.852953e + 06	7.905838e + 03	9.492292e + 03	1.003308e + 04
	std.	3.383692e + 02	3.874915e + 02	5.169930e + 02	6.250932e + 02	1.468840e + 06	3.026116e + 04	3.583671e + 02	9.565439e + 02	1.648246e + 03	1.839235e + 06	4.870952e + 02	7.031027e + 02	3.203743e + 02
10D	med.	5.078370e + 03	4.844155e + 03	5.754995e + 03	4.618519e + 03	1.321701e + 07	4.822912e + 05	1.391672e + 04	1.367200e + 04	1.584915e + 04	4.489565e + 06	6.758430e + 03	7.934840e + 03	1.387976e + 04
	avg.	5.285561e + 03	5.046520e + 03	6.494865e + 03	4.915030e + 03	1.312186e + 07	4.969255e + 05	1.476386e + 04	1.476648e + 04	1.722070e + 04	4.689465e + 06	7.117316e + 03	8.004669e + 03	1.340089e + 04
	min.	5.043765e + 03	4.571916e + 03	5.490435e + 03	4.366647e + 03	9.450986e + 06	2.976068e + 05	1.332024e + 04	1.326350e + 04	1.508449e + 04	4.259661e + 06	6.685372e + 03	7.733450e + 03	1.097450e + 04
	max.	7.271970e + 03	6.877007e + 03	8.775643e + 03	7.053356e + 03	8.068743e + 07	8.068743e + 05	2.184924e + 04	2.126640e + 04	2.494461e + 04	5.290493e + 06	9.923358e + 03	1.018370e + 04	1.501850e + 04
	std.	5.259886e + 02	4.422963e + 02	1.192188e + 03	6.225032e + 02	1.302945e + 06	1.098780e + 05	1.947518e + 03	1.962458e + 03	2.925531e + 03	4.458576e + 05	7.484181e + 02	3.844350e + 02	9.856861e + 02

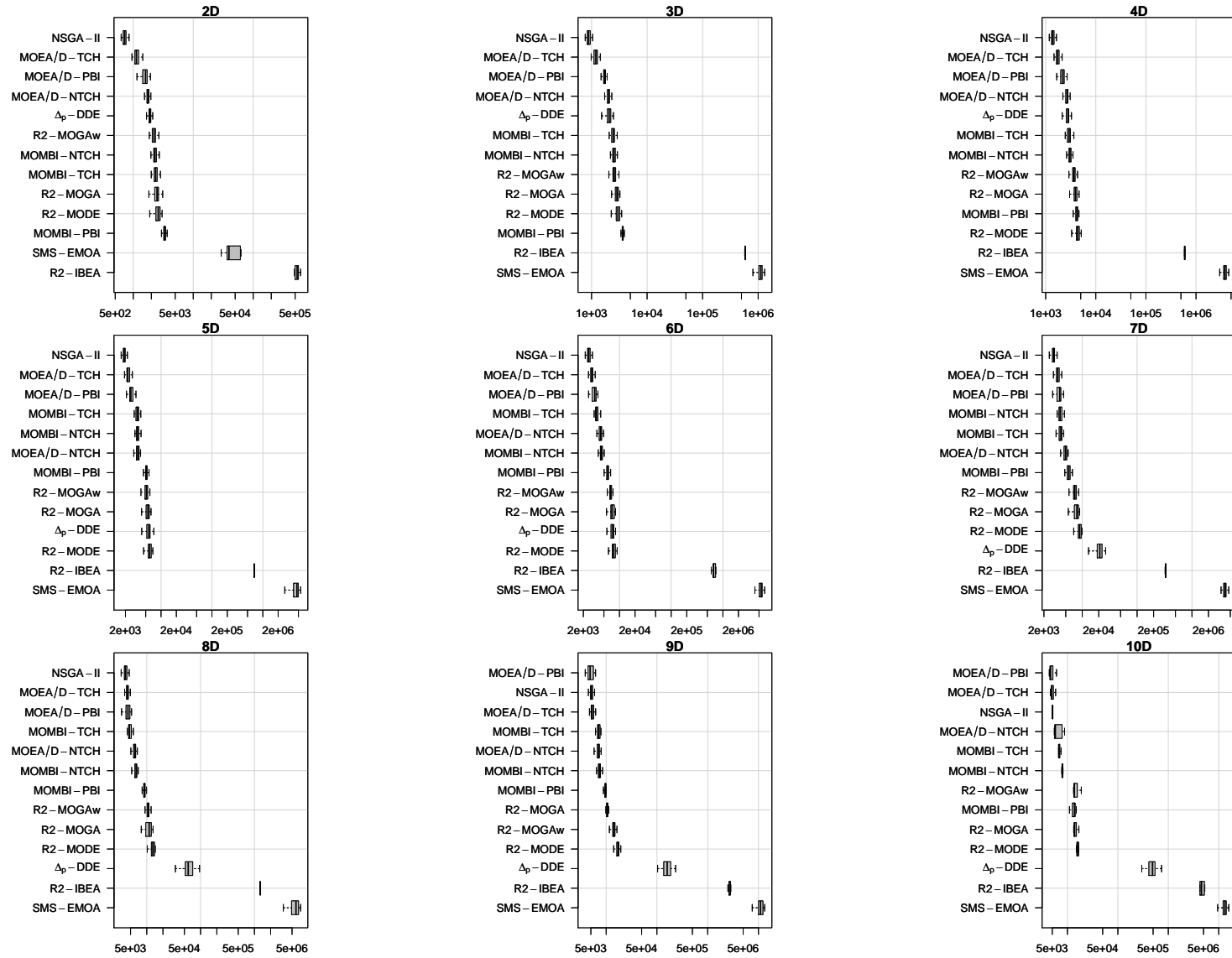


Figure A.243: Box-plot of runtime (in logarithmic scale) for different optimizers on the WFG9 test problem.

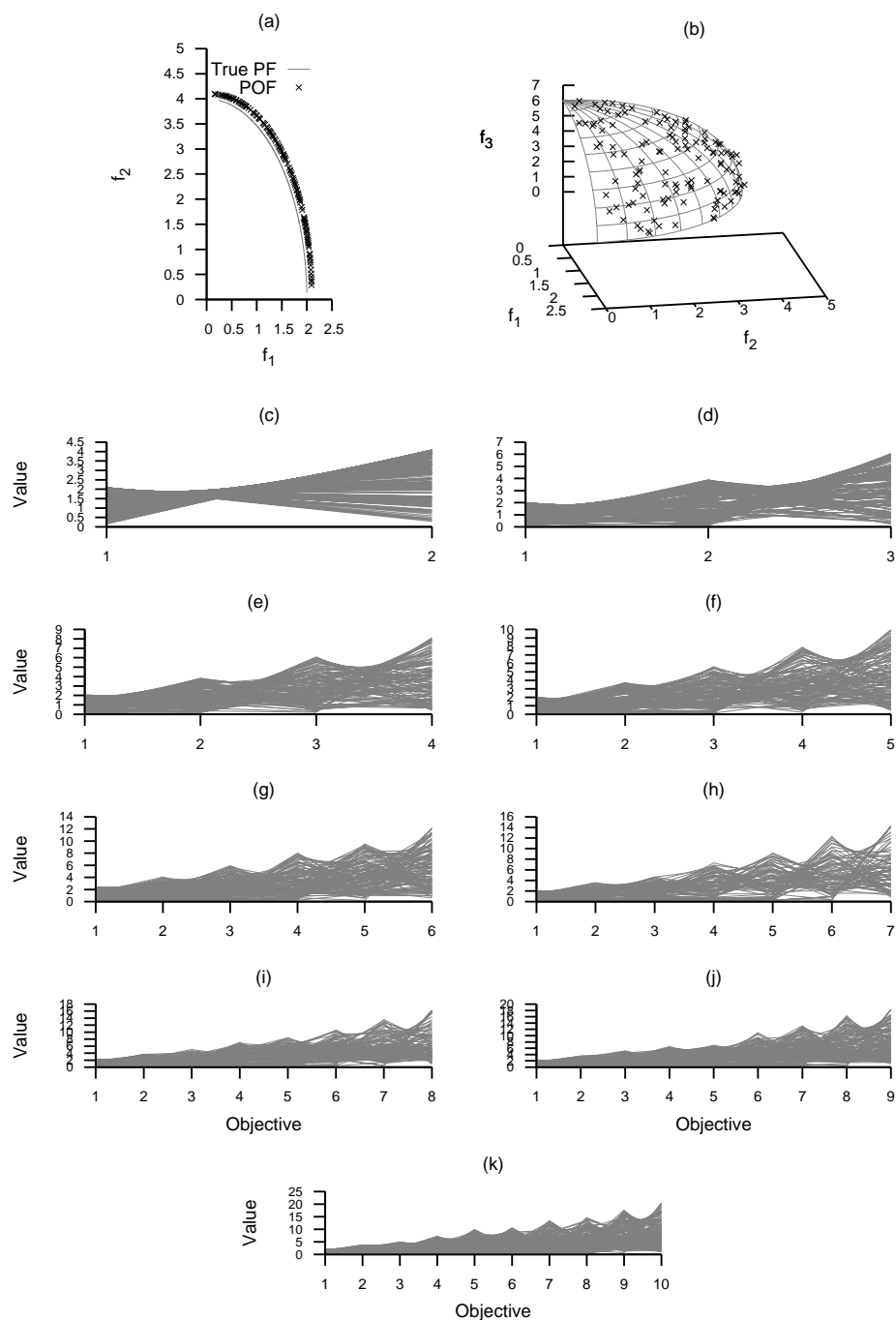


Figure A.244: Plots of the approximations obtained by NSGA-II from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

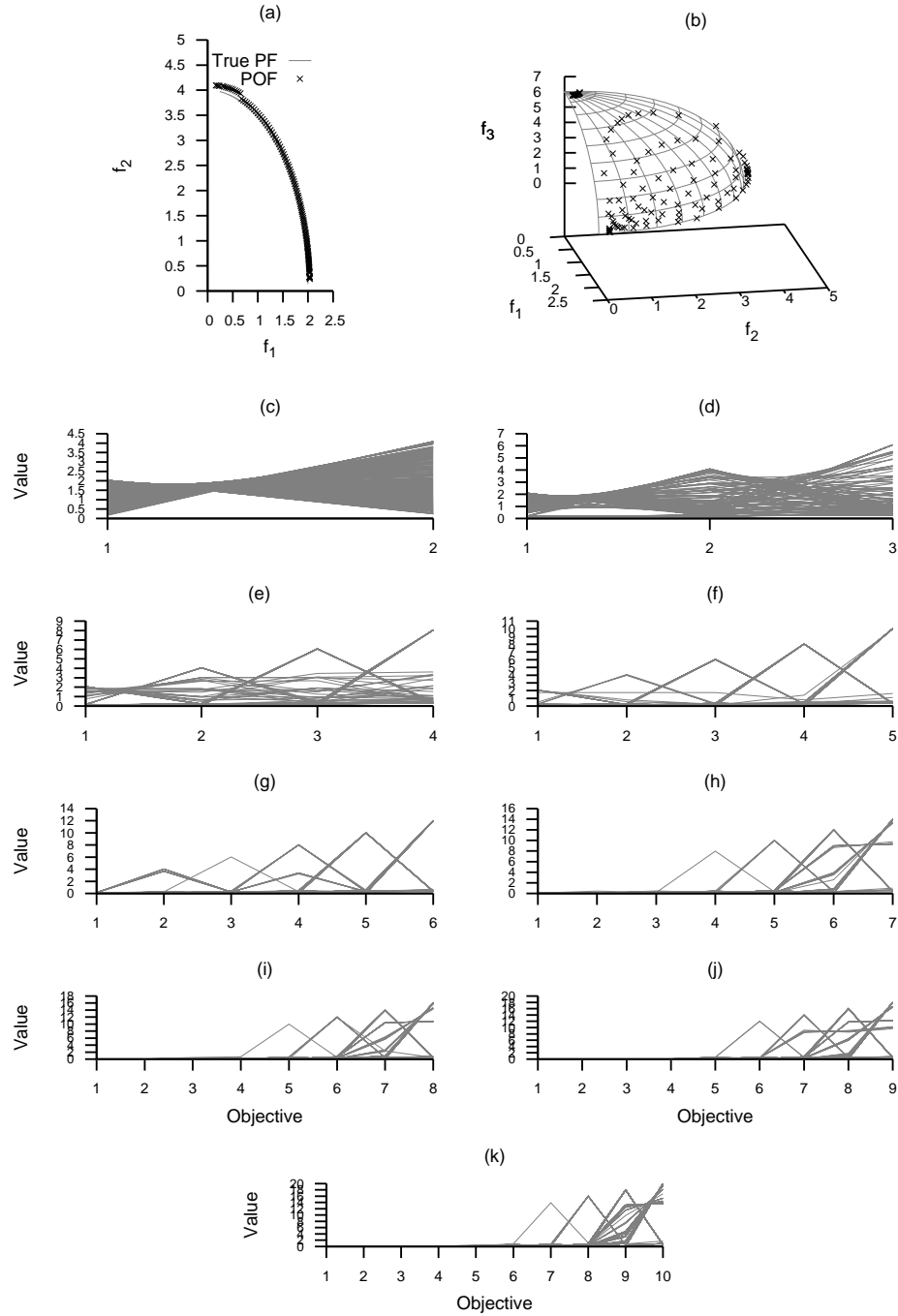


Figure A.245: Plots of the approximations obtained by MOEA/D-TCH from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

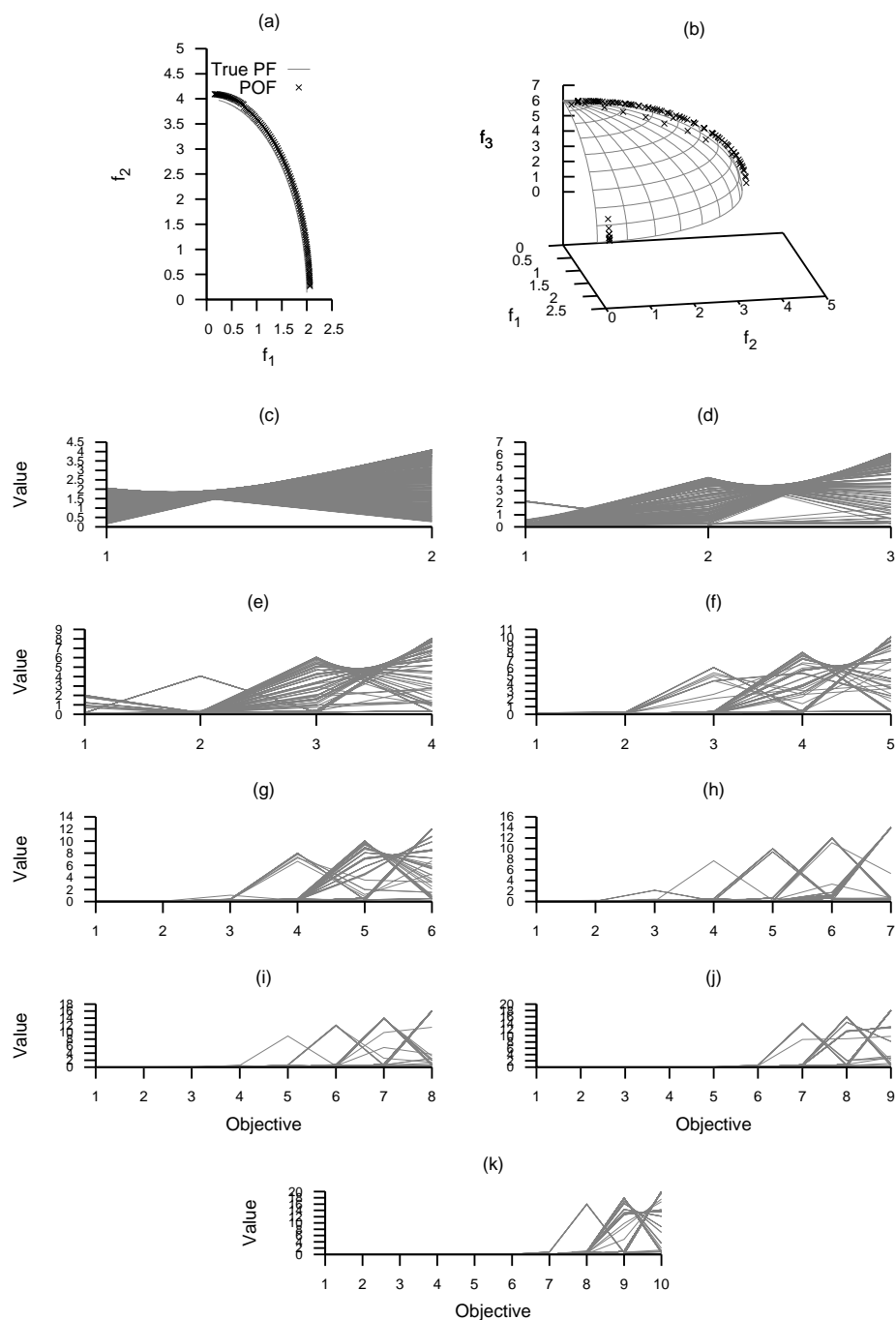


Figure A.246: Plots of the approximations obtained by MOEA/D-NTCH from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

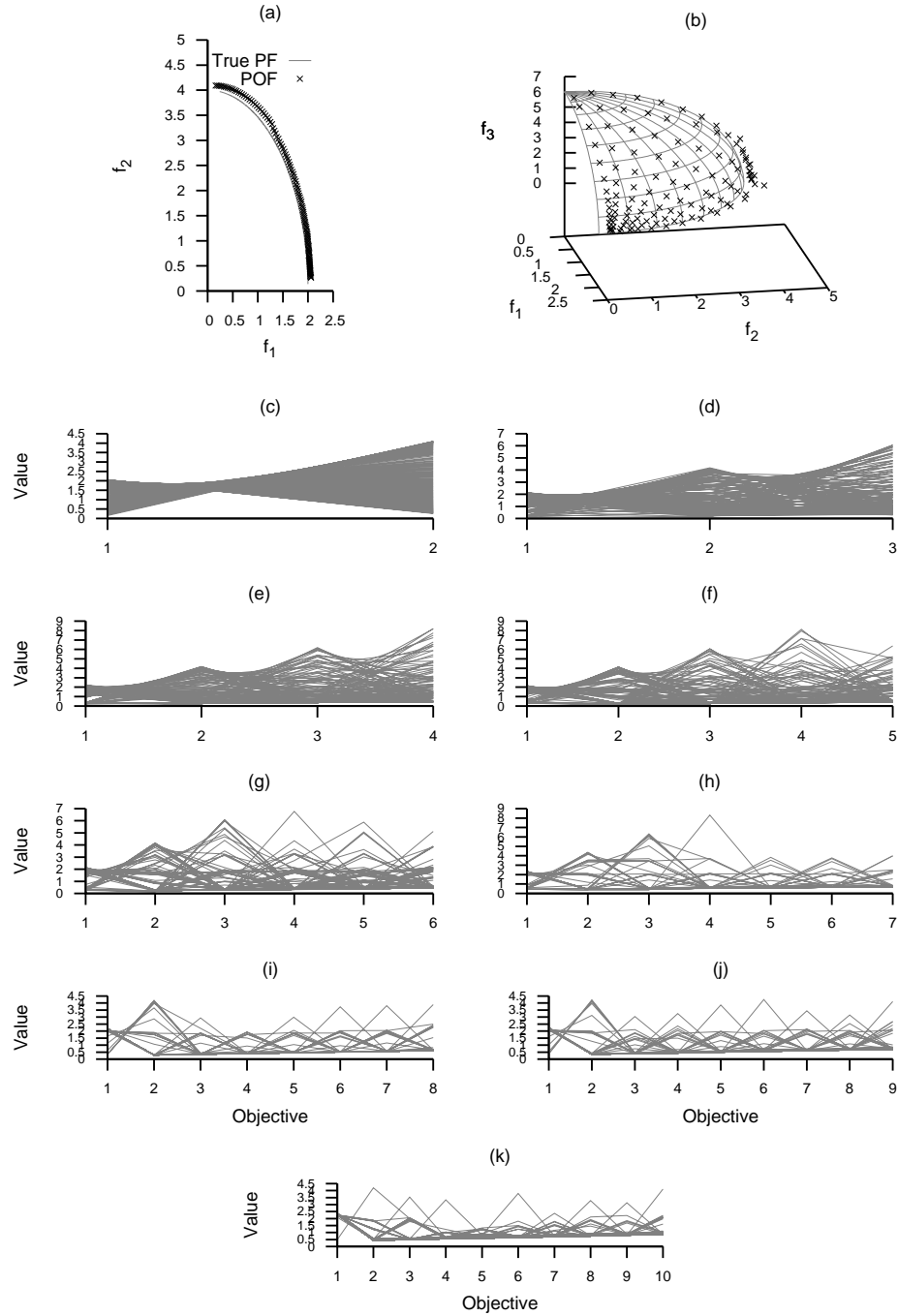


Figure A.247: Plots of the approximations obtained by MOEA/D-PBI from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

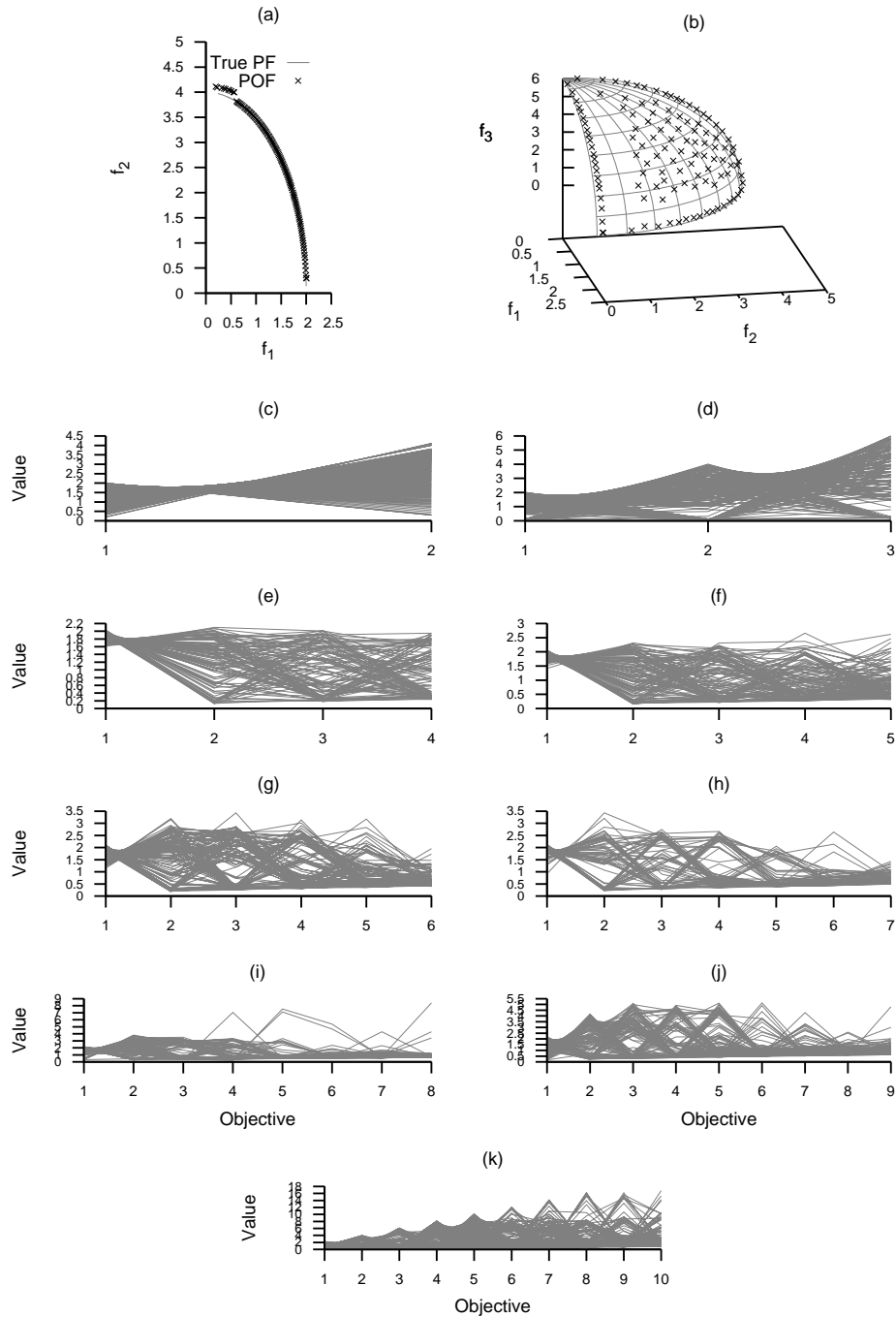


Figure A.248: Plots of the approximations obtained by SMS-EMOA from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



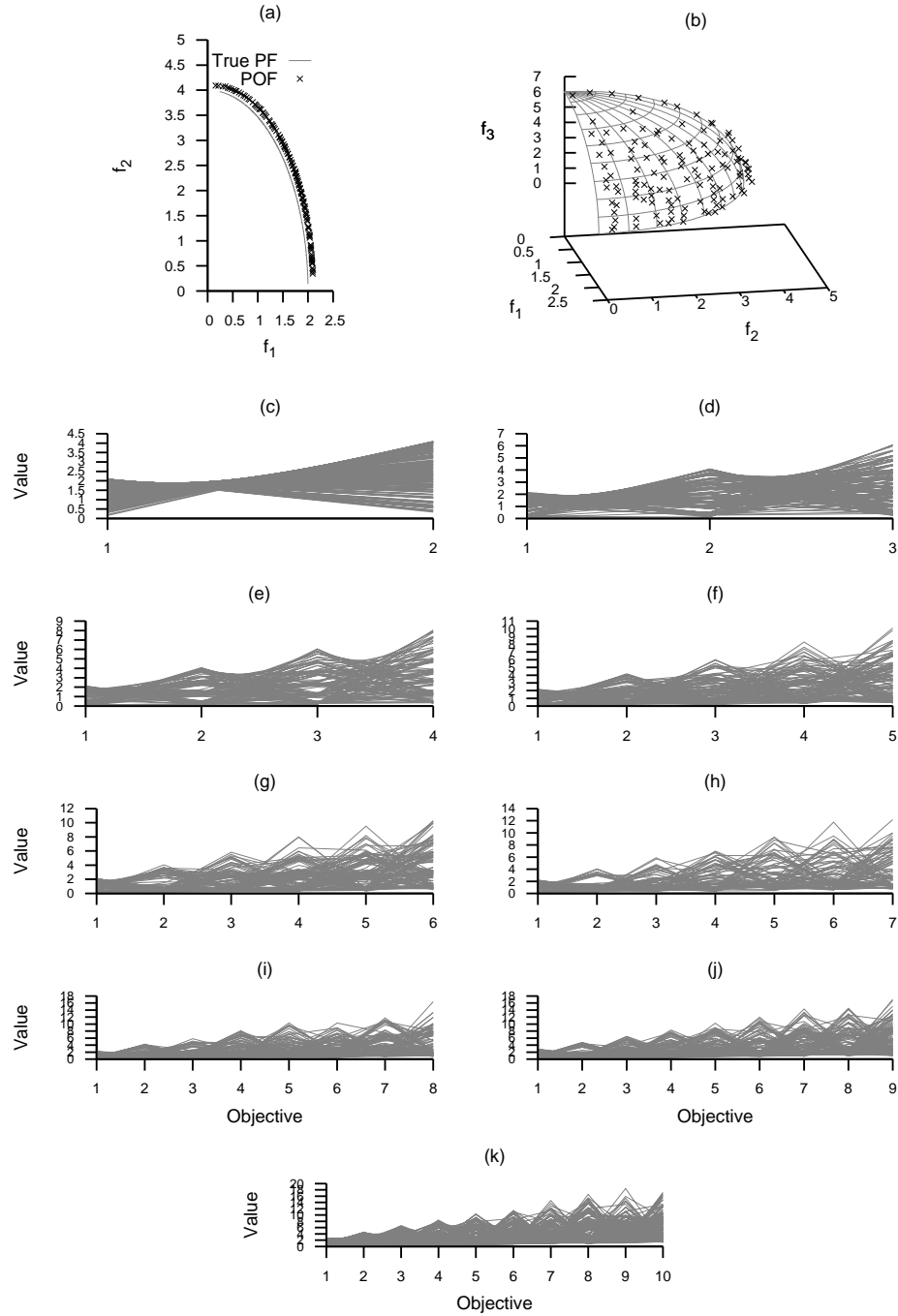


Figure A.249: Plots of the approximations obtained by  $\Delta_p$ -DDE from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

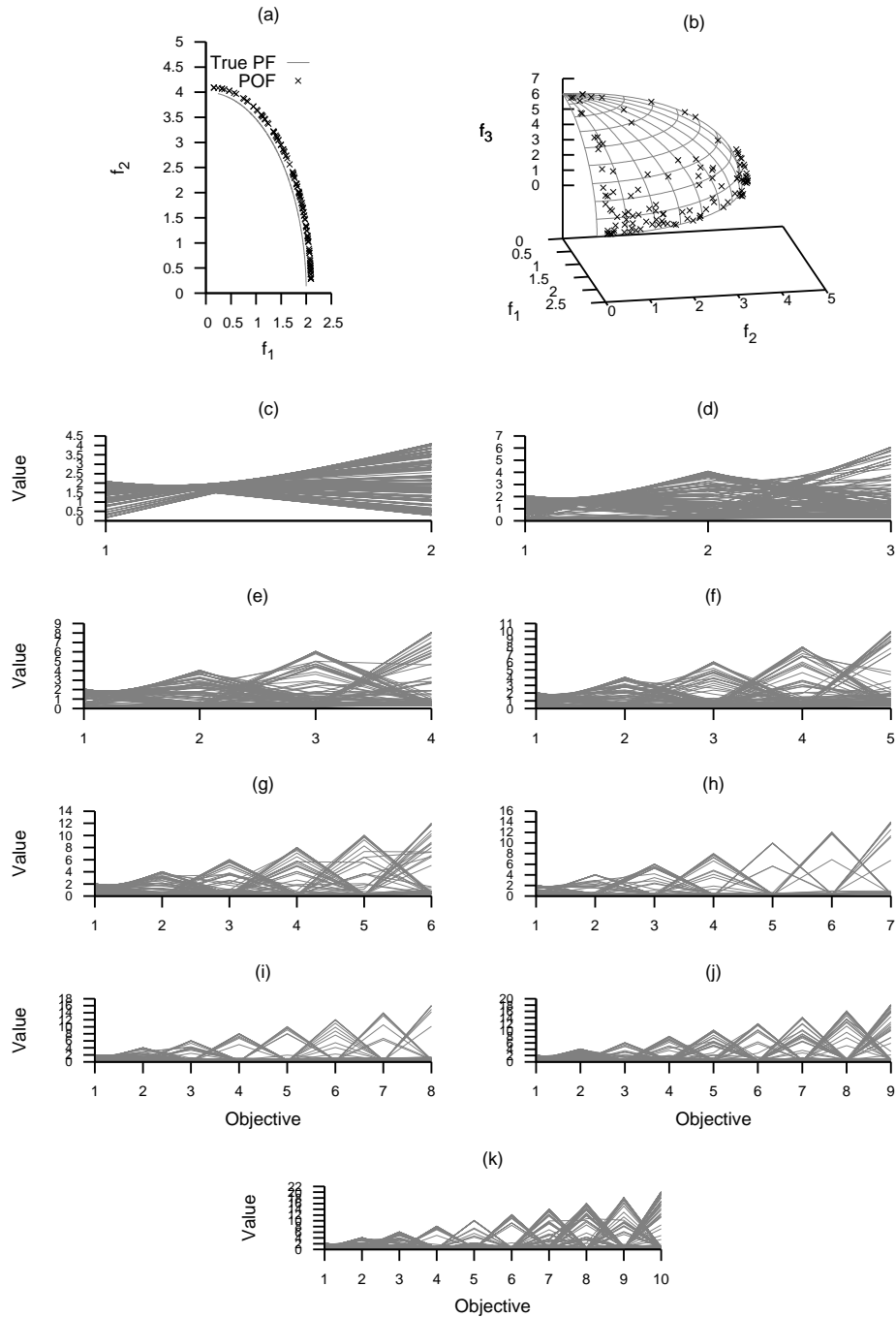


Figure A.250: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

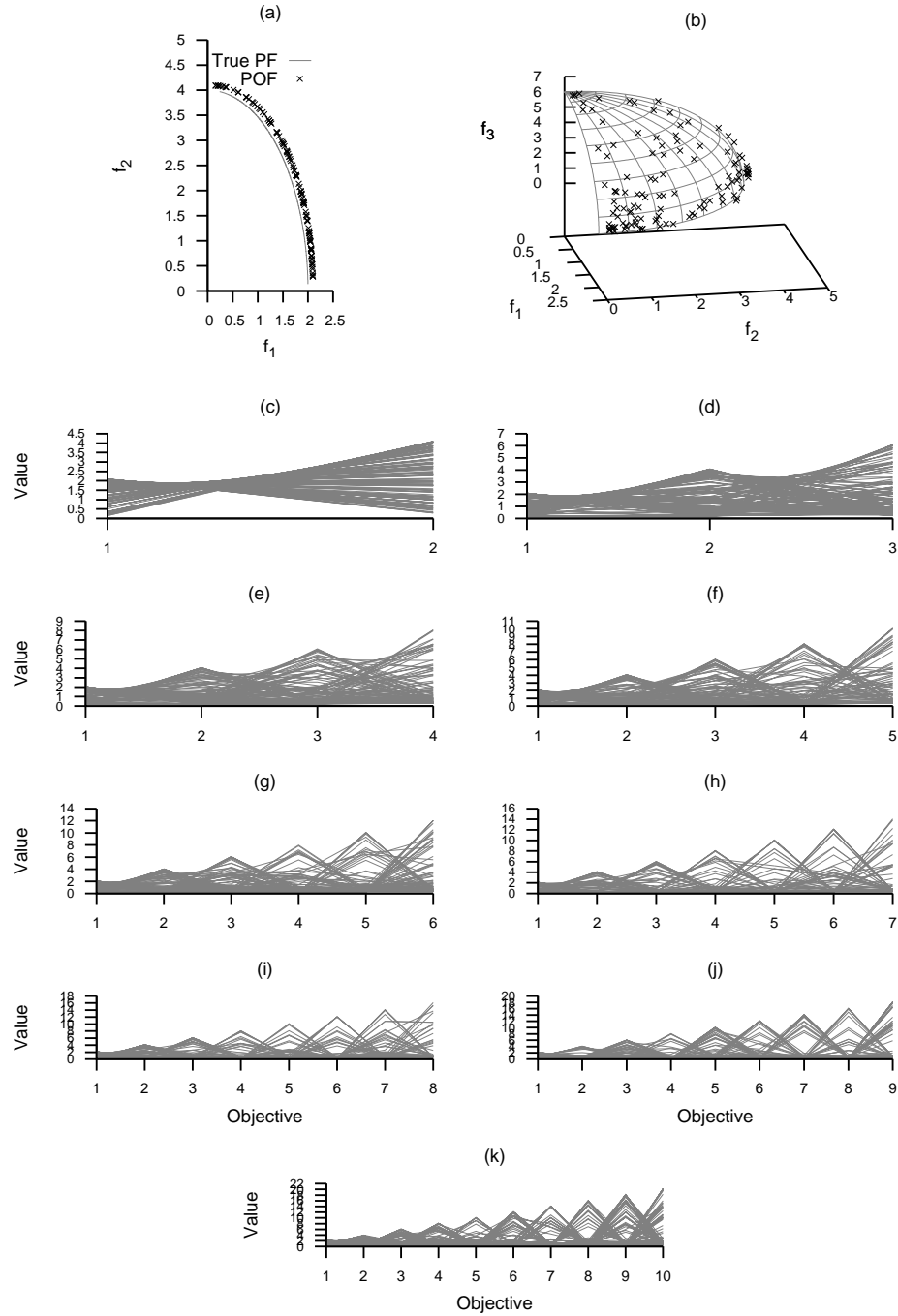


Figure A.251: Plots of the approximations obtained by *R2-MOGA* from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

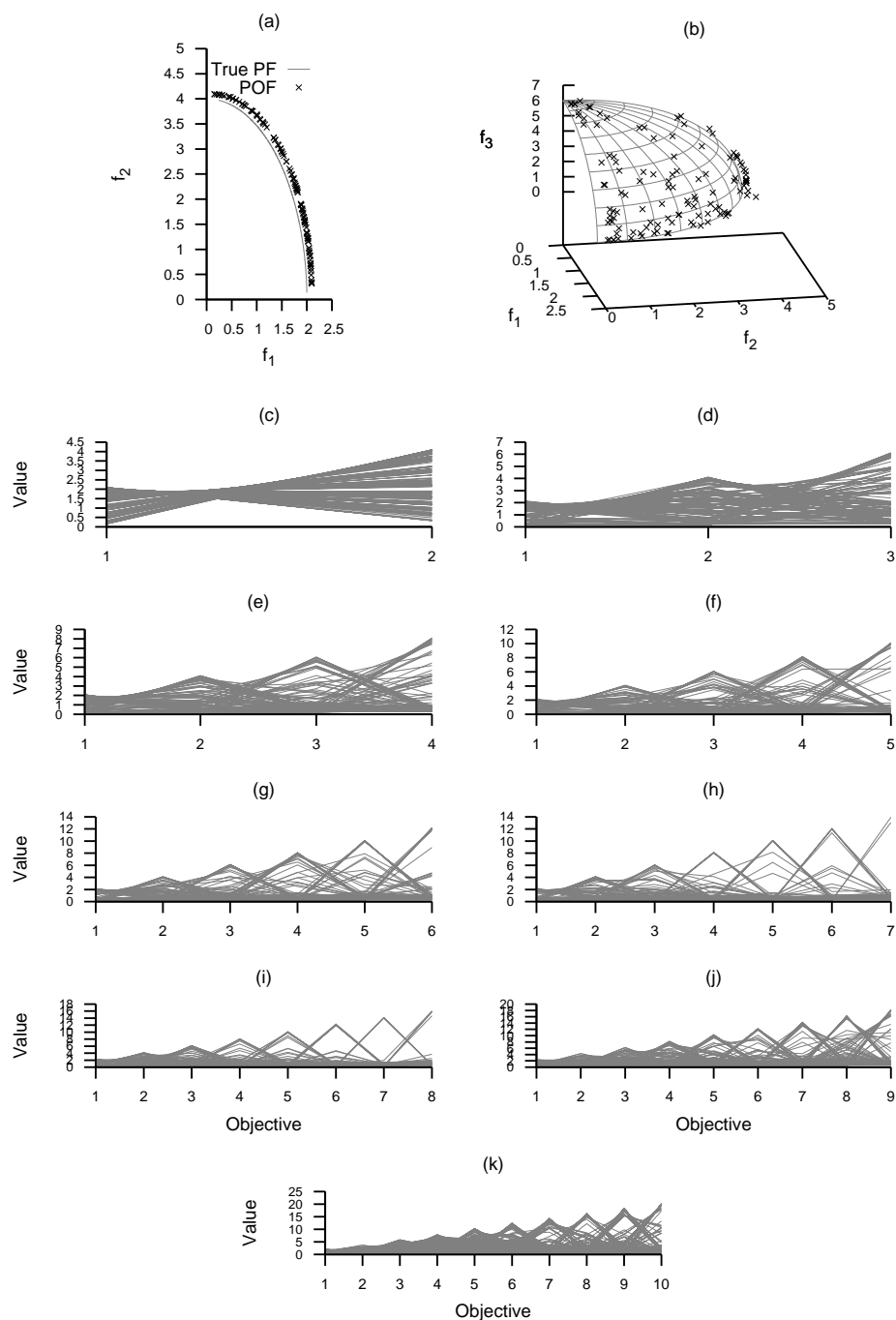


Figure A.252: Plots of the approximations obtained by *R2-MODE* from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

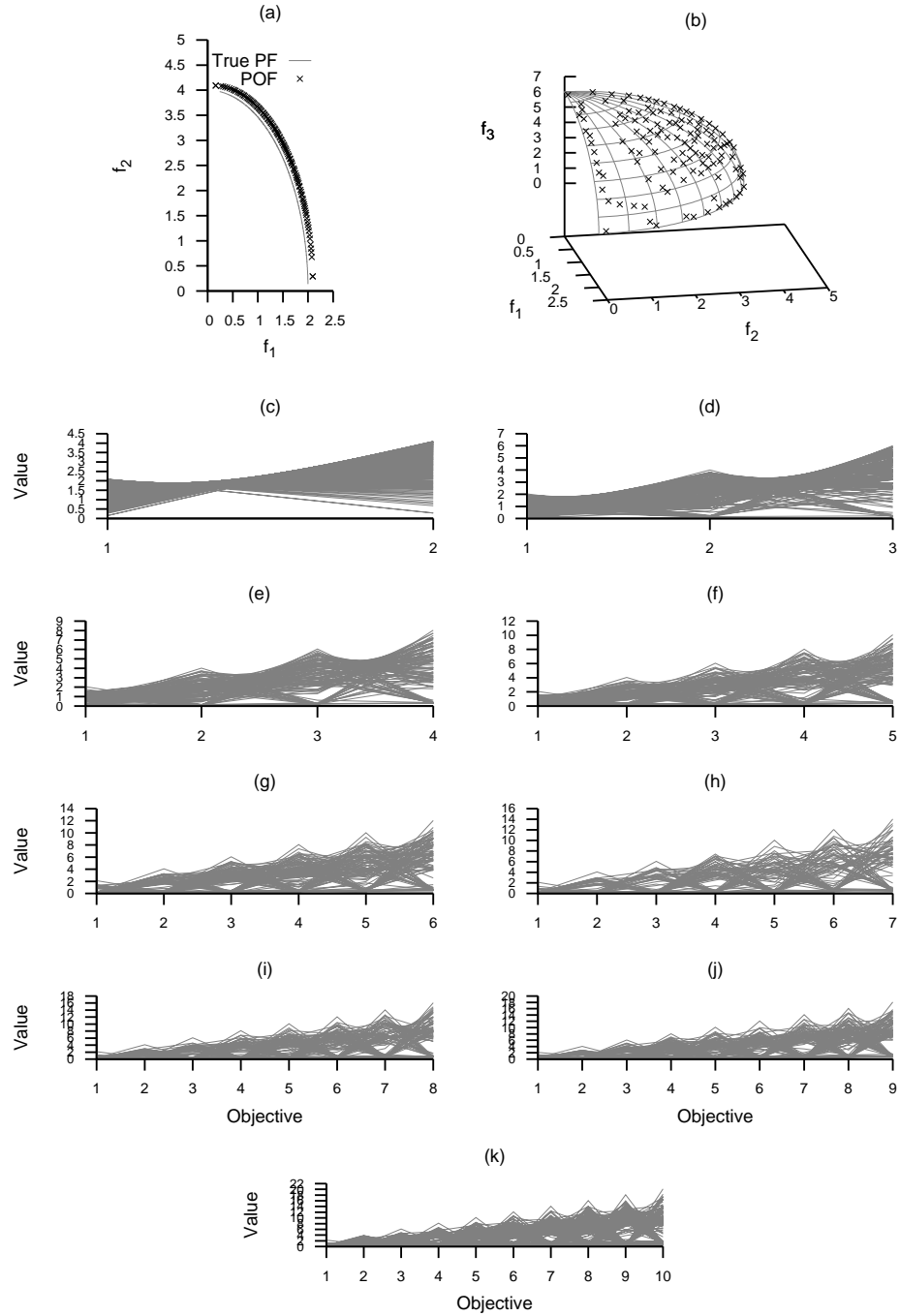


Figure A.253: Plots of the approximations obtained by *R2-IBEA* from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

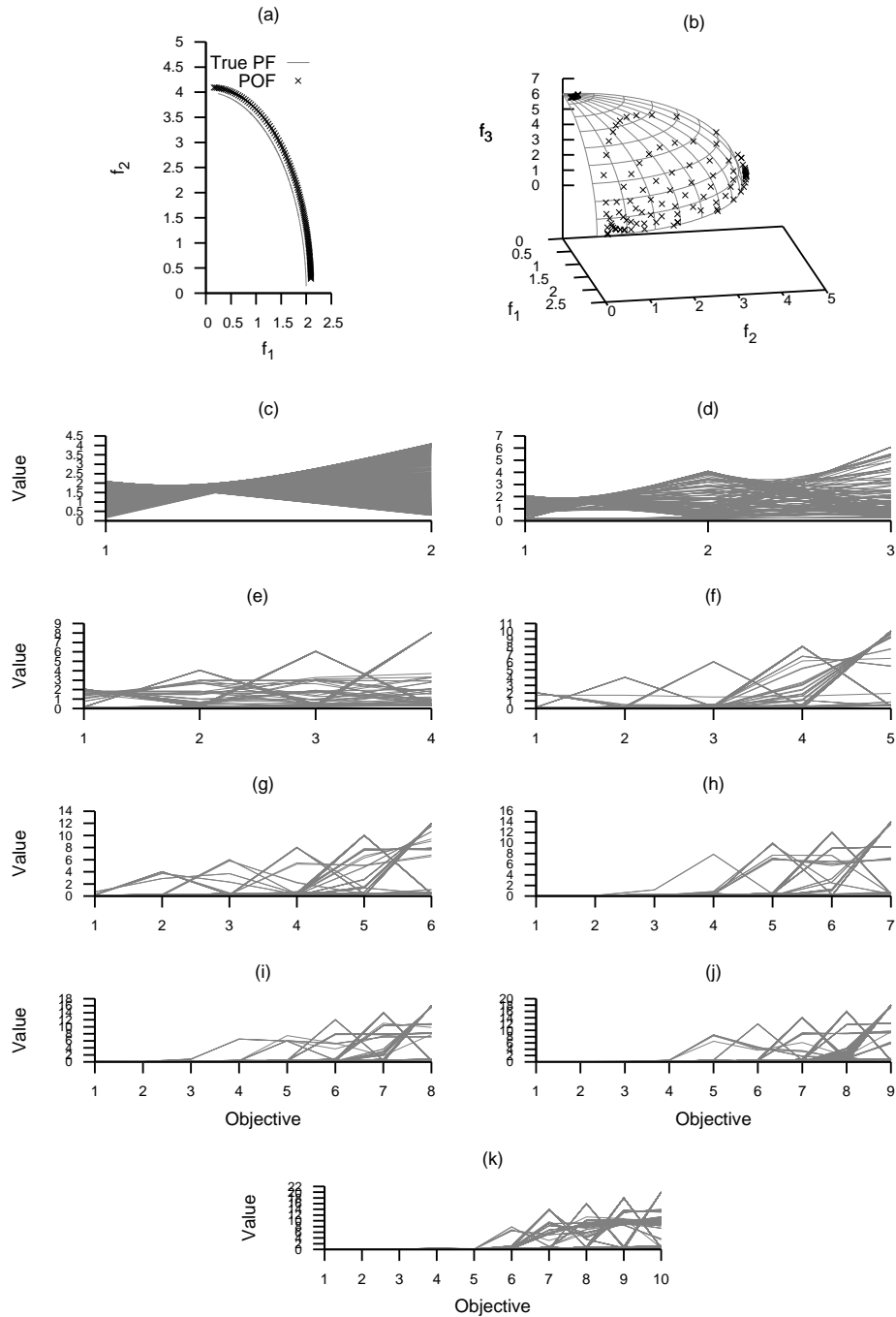


Figure A.254: Plots of the approximations obtained by MOMBI-TCH from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

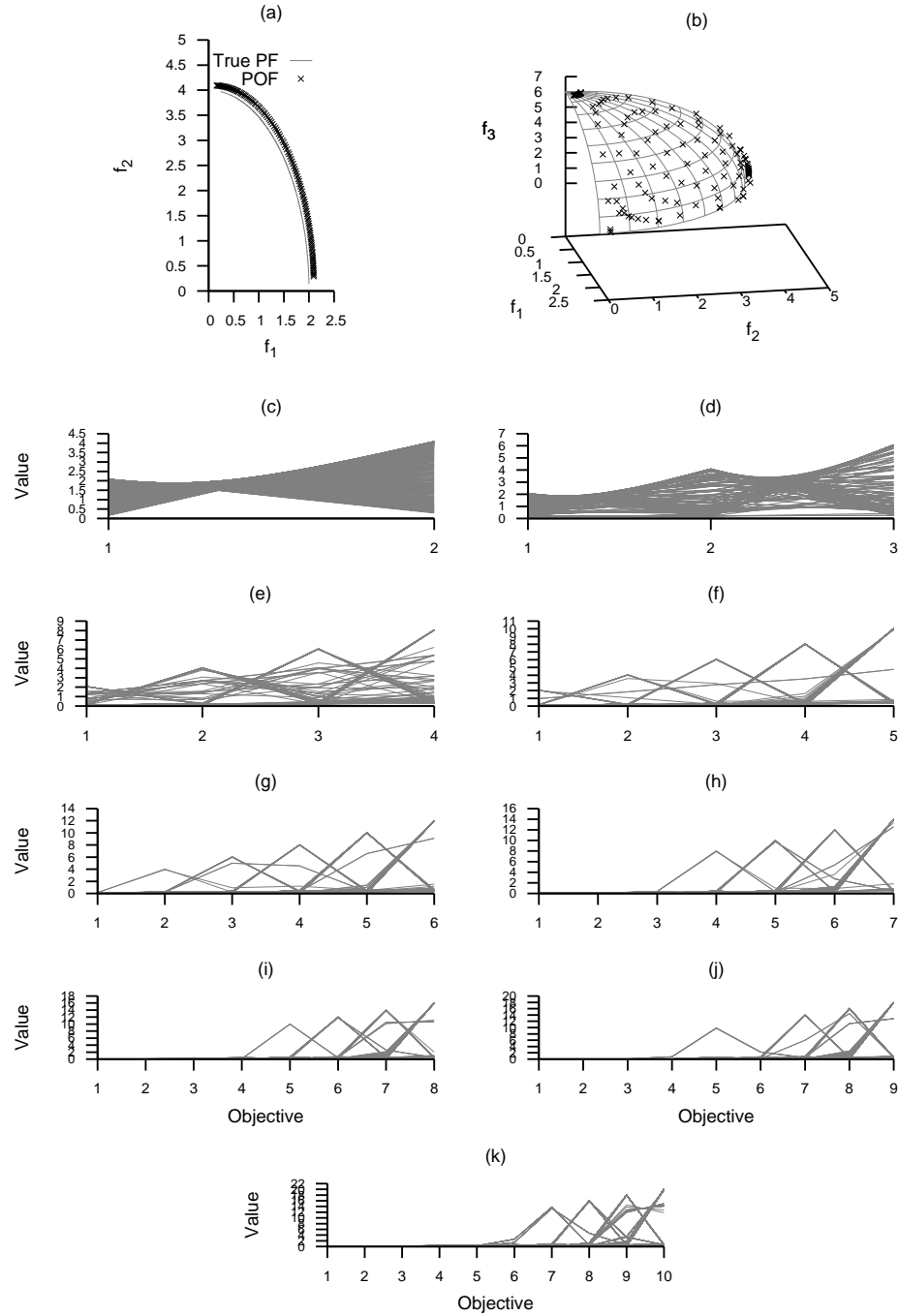


Figure A.255: Plots of the approximations obtained by MOMBI-NTCH from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

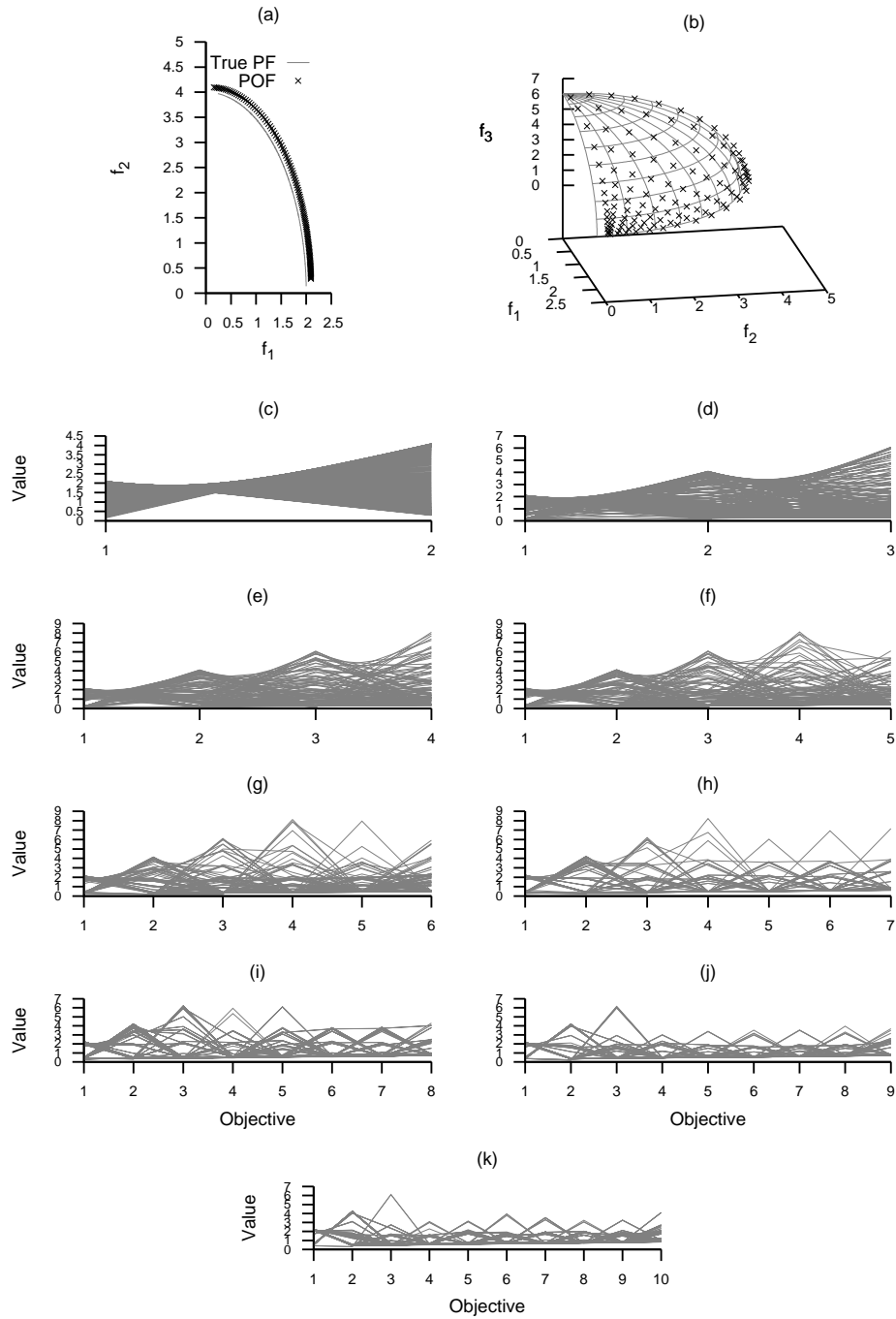


Figure A.256: Plots of the approximations obtained by MOMBI-PBI from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



Table A.98: Comparison of hypervolume indicator values for different optimizers on the DTLZ1 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	7.499102e-01	7.499329e-01	8.738113e-01	8.732149e-01	8.738090e-01	8.733514e-01	8.738256e-01	8.733775e-01	8.738114e-01	8.733622e-01	8.736974e-01
	avg.	7.500480e-01	7.505643e-01	8.737806e-01	8.731702e-01	8.737701e-01	8.732996e-01	8.737867e-01	8.733532e-01	8.737658e-01	8.733316e-01	8.736630e-01
	min.	7.492005e-01	7.494780e-01	8.733358e-01	8.717594e-01	8.733204e-01	8.724558e-01	8.731281e-01	8.726061e-01	8.730440e-01	8.725153e-01	8.730834e-01
	max.	7.617345e-01	7.616159e-01	8.739211e-01	8.735808e-01	8.739289e-01	8.735642e-01	8.739315e-01	8.736383e-01	8.739267e-01	8.736360e-01	8.739260e-01
	std.	1.268357e-03	2.233793e-03	1.055189e-04	2.935284e-04	1.302104e-04	1.985998e-04	1.349311e-04	1.641038e-04	1.684355e-04	1.975251e-04	1.833416e-04
3D	med.	9.028278e-01	8.748754e-01	9.688241e-01	9.620131e-01	9.689163e-01	9.692995e-01	9.689483e-01	9.677696e-01	9.680661e-01	9.689783e-01	9.742867e-01
	avg.	9.047166e-01	8.749721e-01	9.688408e-01	9.605063e-01	9.689146e-01	9.690667e-01	9.689541e-01	9.675823e-01	9.681591e-01	9.689054e-01	9.742420e-01
	min.	8.841992e-01	8.740353e-01	9.685928e-01	9.455386e-01	9.682246e-01	9.656587e-01	9.671168e-01	9.641093e-01	9.668524e-01	9.655217e-01	9.736435e-01
	max.	9.386655e-01	8.839229e-01	9.695763e-01	9.678835e-01	9.697169e-01	9.711462e-01	9.699389e-01	9.698959e-01	9.704580e-01	9.706321e-01	9.744374e-01
	std.	1.132507e-02	9.928248e-04	1.373682e-04	5.096420e-03	2.608073e-04	1.144123e-03	2.999153e-04	1.337222e-03	7.534431e-04	1.064124e-03	1.617626e-04
4D	med.	9.561706e-01	9.373980e-01	9.884162e-01	9.829275e-01	9.884644e-01	9.863286e-01	9.884437e-01	9.826095e-01	9.827528e-01	9.840921e-01	9.943865e-01
	avg.	9.565988e-01	9.373796e-01	9.884218e-01	9.826314e-01	9.884805e-01	9.860388e-01	9.884559e-01	9.823616e-01	9.829046e-01	9.837076e-01	9.943673e-01
	min.	9.450139e-01	9.365343e-01	9.881301e-01	9.724379e-01	9.880942e-01	9.740749e-01	9.852238e-01	9.608561e-01	9.795038e-01	9.771062e-01	9.939344e-01
	max.	9.717251e-01	9.431202e-01	9.886941e-01	9.880841e-01	9.889809e-01	9.897144e-01	9.886979e-01	9.877788e-01	9.886568e-01	9.879071e-01	9.944752e-01
	std.	5.348394e-03	6.169256e-04	7.832945e-05	3.261257e-03	1.515741e-04	2.489194e-03	9.149275e-05	3.162739e-03	1.673782e-03	2.002676e-03	8.047841e-05
5D	med.	9.816650e-01	9.686670e-01	9.967628e-01	9.904465e-01	9.968031e-01	9.930505e-01	9.968296e-01	9.863240e-01	9.863240e-01	9.772221e-01	9.986269e-01
	avg.	9.819103e-01	9.688154e-01	9.967669e-01	9.902278e-01	9.967962e-01	9.934813e-01	9.968356e-01	9.770910e-01	9.865058e-01	9.776074e-01	9.986192e-01
	min.	9.759746e-01	9.677887e-01	9.966805e-01	9.799350e-01	9.963873e-01	9.858207e-01	9.967222e-01	9.665860e-01	9.829394e-01	9.727499e-01	9.984602e-01
	max.	9.880377e-01	9.729738e-01	9.969191e-01	9.955372e-01	9.969618e-01	9.961868e-01	9.969470e-01	9.865807e-01	9.911020e-01	9.845705e-01	9.986858e-01
	std.	2.661765e-03	7.135776e-04	3.885717e-05	2.745202e-03	5.892449e-05	2.111478e-03	3.404310e-05	4.475760e-03	1.584174e-03	2.872541e-03	3.995588e-05
6D	med.	9.863341e-01	9.686705e-01	9.983603e-01	9.866687e-01	9.978304e-01	9.950702e-01	9.978869e-01	9.590170e-01	9.862735e-01	9.605208e-01	9.995873e-01
	avg.	9.864854e-01	9.692401e-01	9.983451e-01	9.850570e-01	9.977573e-01	9.948682e-01	9.977098e-01	9.572817e-01	9.869139e-01	9.604550e-01	9.995815e-01
	min.	9.755554e-01	9.678756e-01	9.977862e-01	9.677502e-01	9.963648e-01	9.852808e-01	9.966096e-01	9.212678e-01	9.754454e-01	9.419970e-01	9.994148e-01
	max.	9.939501e-01	9.850834e-01	9.986887e-01	9.944929e-01	9.986282e-01	9.982505e-01	9.986560e-01	9.743908e-01	9.959841e-01	9.781317e-01	9.996393e-01
	std.	4.448077e-03	2.166773e-03	1.746397e-04	6.725272e-03	6.144780e-04	2.125100e-03	6.827810e-04	1.114390e-02	4.158628e-03	9.511016e-03	3.434115e-05
7D	med.	9.630093e-01	9.373482e-01	9.950244e-01	9.303641e-01	9.928364e-01	9.802395e-01	9.922070e-01	8.924983e-01	9.675817e-01	8.999725e-01	9.997984e-01
	avg.	9.632877e-01	9.294829e-01	9.949552e-01	9.364212e-01	9.925053e-01	9.792139e-01	9.916484e-01	8.996716e-01	9.677218e-01	9.027918e-01	9.997937e-01
	min.	8.724864e-01	6.859978e-01	9.936439e-01	8.900783e-01	9.857861e-01	9.604555e-01	9.863141e-01	8.238279e-01	9.460125e-01	6.886191e-01	9.997099e-01
	max.	9.902840e-01	9.523363e-01	9.956582e-01	9.786272e-01	9.950179e-01	9.912602e-01	9.955921e-01	9.440373e-01	9.904946e-01	9.412304e-01	9.998324e-01
	std.	1.285935e-02	3.875479e-02	3.951056e-04	2.439733e-02	1.957281e-03	5.024918e-03	2.386654e-03	2.160518e-02	1.002039e-02	2.861393e-02	2.505791e-05
8D	med.	9.804928e-01	9.373745e-01	9.962777e-01	9.803071e-01	9.947075e-01	9.861348e-01	9.942457e-01	9.373716e-01	9.799733e-01	9.381065e-01	9.996902e-01
	avg.	9.766892e-01	9.377754e-01	9.963444e-01	9.797272e-01	9.942505e-01	9.867069e-01	9.938460e-01	9.336992e-01	9.758086e-01	9.368723e-01	9.996684e-01
	min.	8.132439e-01	8.746592e-01	9.946063e-01	8.981213e-01	9.310579e-01	9.749248e-01	9.828052e-01	8.778707e-01	9.517698e-01	8.759496e-01	9.996512e-01
	max.	9.891169e-01	9.495426e-01	9.965625e-01	9.873620e-01	9.969101e-01	9.937108e-01	9.963476e-01	9.443792e-01	9.921650e-01	9.529496e-01	9.999205e-01
	std.	2.194805e-02	1.245412e-02	3.550606e-04	1.624988e-02	6.526375e-03	3.126610e-03	2.325452e-03	1.872307e-02	1.064428e-02	1.951132e-02	5.313092e-05
9D	med.	9.804928e-01	9.373745e-01	9.962777e-01	9.803071e-01	9.947075e-01	9.861348e-01	9.942457e-01	9.373716e-01	9.799733e-01	9.381065e-01	9.996902e-01
	avg.	9.766892e-01	9.377754e-01	9.963444e-01	9.797272e-01	9.942505e-01	9.867069e-01	9.938460e-01	9.336992e-01	9.758086e-01	9.368723e-01	9.996684e-01
	min.	9.479008e-01	9.350071e-01	9.946398e-01	9.360912e-01	9.865709e-01	9.805126e-01	9.865951e-01	8.818792e-01	9.499091e-01	8.886394e-01	9.992891e-01
	max.	9.935354e-01	9.487383e-01	9.974578e-01	9.875555e-01	9.970298e-01	9.940561e-01	9.970739e-01	9.472665e-01	9.918328e-01	9.490044e-01	9.999052e-01
	std.	1.075523e-02	1.947436e-03	4.142554e-04	6.098559e-03	1.822598e-03	2.572836e-03	2.116299e-03	1.064636e-02	1.154900e-02	9.577722e-03	1.343164e-04
10D	med.	9.821717e-01	9.373755e-01	9.966769e-01	9.841339e-01	9.954456e-01	9.868389e-01	9.946785e-01	9.374550e-01	9.821383e-01	9.383349e-01	9.993754e-01
	avg.	9.793596e-01	9.388469e-01	9.967345e-01	9.831440e-01	9.949729e-01	9.871164e-01	9.944919e-01	9.369573e-01	9.792170e-01	9.382383e-01	9.992360e-01
	min.	9.488493e-01	9.355892e-01	9.951120e-01	9.647156e-01	9.884206e-01	9.815663e-01	9.871841e-01	9.141754e-01	9.509724e-01	8.916564e-01	9.972886e-01
	max.	9.948870e-01	9.732685e-01	9.976026e-01	9.888149e-01	9.979086e-01	9.948524e-01	9.975966e-01	9.463765e-01	9.958183e-01	9.515828e-01	9.997364e-01
	std.	1.187574e-02	5.107644e-03	3.843876e-04	4.277574e-03	1.732963e-03	2.146802e-03	1.954508e-03	6.284422e-03	1.150213e-02	5.662998e-03	4.413970e-04

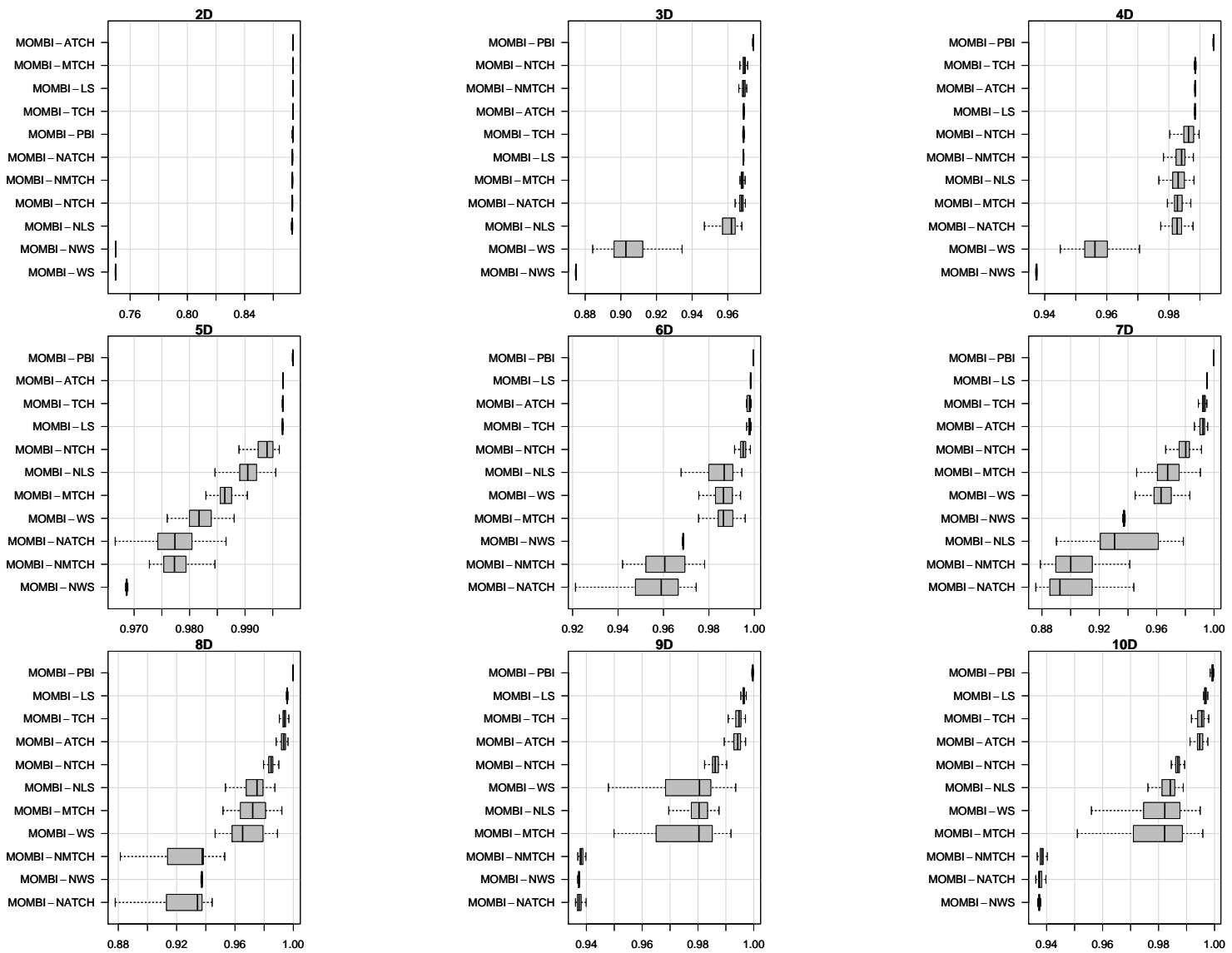


Figure A.257: Box-plot of the hypervolume indicator values for different optimizers on the DTLZ1 test problem.

Table A.99: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ1 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
2D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	$2.29e-02$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$6.63e-34$	> 0.05	$1.61e-33$	> 0.05	$3.57e-33$	> 0.05	$5.39e-33$	$1.32e-07$
MOMBI-NLS	$1.28e-34$	$1.28e-34$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	> 0.05	$3.96e-33$	–	$3.68e-32$	> 0.05	$1.64e-31$	> 0.05	$1.74e-31$	$1.36e-06$
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	> 0.05	$3.12e-04$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	> 0.05	$5.08e-33$	> 0.05	$2.45e-32$	–	$1.07e-31$	> 0.05	$1.11e-31$	$5.60e-09$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	> 0.05	$1.06e-07$	> 0.05	$3.98e-02$	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	> 0.05	$9.28e-31$	> 0.05	$3.87e-29$	> 0.05	$3.10e-28$	–	$1.51e-28$	$3.03e-07$
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	> 0.05	$3.25e-06$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	$1.28e-34$	$1.28e-34$	> 0.05	$5.61e-28$	> 0.05	$1.89e-25$	> 0.05	$1.24e-23$	> 0.05	$2.28e-23$	–
3D											
MOMBI-WS	–	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	> 0.05	> 0.05	> 0.05	$3.79e-16$	$8.21e-15$	> 0.05	> 0.05
MOMBI-NLS	$1.28e-34$	$1.28e-34$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$6.80e-05$	$1.28e-34$	–	> 0.05	> 0.05	$4.55e-17$	$8.48e-16$	> 0.05	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$9.06e-06$	$7.69e-34$	$1.68e-04$	–	$5.56e-04$	$8.21e-15$	$1.26e-11$	> 0.05	> 0.05
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$3.52e-08$	$1.40e-34$	> 0.05	> 0.05	–	$4.31e-18$	$7.44e-17$	> 0.05	> 0.05
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	> 0.05	$1.68e-30$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	> 0.05	$1.39e-33$	> 0.05	> 0.05	> 0.05	$2.70e-03$	–	> 0.05	> 0.05
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$3.64e-02$	$5.71e-34$	> 0.05	> 0.05	> 0.05	$3.16e-12$	$1.08e-08$	–	> 0.05
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–
4D											
MOMBI-WS	–	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	> 0.05	$8.57e-21$	> 0.05	$1.28e-34$	$2.44e-33$	$1.28e-34$	> 0.05
MOMBI-NLS	$1.28e-34$	$1.28e-34$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$3.40e-04$	$1.28e-34$	–	$2.24e-21$	> 0.05	$1.28e-34$	$1.92e-33$	$1.28e-34$	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	> 0.05	$6.69e-16$	> 0.05	–	> 0.05	$1.38e-19$	$1.99e-20$	$1.58e-13$	> 0.05
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$2.05e-03$	$1.28e-34$	> 0.05	$2.41e-21$	–	$1.28e-34$	$2.17e-33$	$1.28e-34$	> 0.05
MOMBI-NATCH	$2.33e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	> 0.05	$2.44e-02$	> 0.05	> 0.05	> 0.05	$9.58e-05$	$2.89e-04$	–	> 0.05
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–

Table A.100: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ1 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
5D											
MOMBI-WS	–	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$4.28e-15$	$> 0.05$	$5.51e-19$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$
MOMBI-NLS	$1.04e-31$	$1.28e-34$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$7.25e-34$	$8.57e-21$	$3.05e-34$	$> 0.05$
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$6.93e-10$	$1.28e-34$	–	$1.28e-34$	$> 0.05$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$
MOMBI-NTCH	$1.78e-34$	$1.28e-34$	$> 0.05$	$3.62e-17$	$> 0.05$	–	$> 0.05$	$1.32e-34$	$7.43e-33$	$1.28e-34$	$> 0.05$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$6.07e-25$	$1.28e-34$	$6.25e-12$	$1.28e-34$	–	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$
MOMBI-NATCH	$> 0.05$	$6.16e-29$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$2.17e-25$	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$5.31e-33$	–	$3.55e-34$	$> 0.05$
MOMBI-NMTCH	$> 0.05$	$1.32e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–
6D											
MOMBI-WS	–	$4.50e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.63e-34$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.61e-16$	$> 0.05$	$1.30e-05$	$> 0.05$
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$1.02e-14$	$5.07e-34$	$4.41e-15$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$
MOMBI-NLS	$> 0.05$	$3.09e-32$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$5.07e-34$	$> 0.05$	$3.79e-33$	$> 0.05$
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	–	$1.64e-26$	$> 0.05$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$
MOMBI-NTCH	$2.74e-31$	$1.28e-34$	$> 0.05$	$2.28e-30$	$> 0.05$	–	$> 0.05$	$1.28e-34$	$2.94e-30$	$1.28e-34$	$> 0.05$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.77e-25$	–	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$
MOMBI-NATCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$> 0.05$	$4.12e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$	–	$1.58e-34$	$> 0.05$
MOMBI-NMTCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.75e-02$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–
7D											
MOMBI-WS	–	$3.47e-33$	$> 0.05$	$5.74e-13$	$> 0.05$	$> 0.05$	$> 0.05$	$2.37e-33$	$> 0.05$	$2.44e-33$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$8.29e-03$	$> 0.05$	$> 0.05$	$> 0.05$	$2.06e-18$	$> 0.05$	$2.15e-14$	$> 0.05$
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$7.83e-31$	$1.28e-34$	$2.82e-27$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$
MOMBI-NLS	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$1.84e-19$	$> 0.05$	$1.46e-16$	$> 0.05$
MOMBI-TCH	$1.78e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	–	$4.78e-34$	$4.99e-03$	$1.28e-34$	$2.07e-34$	$1.28e-34$	$> 0.05$
MOMBI-NTCH	$9.43e-25$	$1.28e-34$	$> 0.05$	$2.18e-31$	$> 0.05$	–	$> 0.05$	$1.28e-34$	$1.03e-16$	$1.28e-34$	$> 0.05$
MOMBI-ATCH	$2.79e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$	$2.30e-33$	–	$1.28e-34$	$3.55e-34$	$1.28e-34$	$> 0.05$
MOMBI-NATCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$3.83e-03$	$1.73e-34$	$> 0.05$	$3.27e-17$	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$	–	$1.28e-34$	$> 0.05$
MOMBI-NMTCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$2.69e-02$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–

Table A.101: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ1 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
8D											
MOMBI-WS	–	$4.91e-32$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$4.63e-32$	$> 0.05$	$8.98e-32$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$4.69e-06$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$1.13e-20$	$1.28e-34$	$8.50e-28$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$
MOMBI-NLS	$2.47e-03$	$2.84e-26$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$4.69e-29$	$> 0.05$	$8.28e-28$	$> 0.05$
MOMBI-TCH	$2.74e-33$	$2.23e-33$	$> 0.05$	$2.23e-33$	–	$5.20e-32$	$9.84e-03$	$5.88e-34$	$6.23e-33$	$9.75e-34$	$> 0.05$
MOMBI-NTCH	$3.43e-25$	$1.28e-34$	$> 0.05$	$2.56e-26$	$> 0.05$	–	$> 0.05$	$1.28e-34$	$1.55e-21$	$1.28e-34$	$> 0.05$
MOMBI-ATCH	$2.40e-34$	$1.28e-34$	$> 0.05$	$1.84e-34$	$> 0.05$	$7.20e-31$	–	$1.28e-34$	$1.76e-33$	$1.28e-34$	$> 0.05$
MOMBI-NATCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$6.68e-03$	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$	–	$1.53e-34$	$> 0.05$
MOMBI-NMTCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.59e-03$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–
9D											
MOMBI-WS	–	$1.32e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.40e-34$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$4.77e-25$	$1.28e-34$	$5.70e-25$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$
MOMBI-NLS	$> 0.05$	$2.44e-33$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$1.39e-33$	$> 0.05$	$2.04e-33$	$> 0.05$
MOMBI-TCH	$8.92e-34$	$1.28e-34$	$> 0.05$	$1.49e-34$	–	$7.34e-32$	$> 0.05$	$1.28e-34$	$4.78e-34$	$1.28e-34$	$> 0.05$
MOMBI-NTCH	$1.06e-18$	$1.28e-34$	$> 0.05$	$9.43e-25$	$> 0.05$	–	$> 0.05$	$1.28e-34$	$1.28e-15$	$1.28e-34$	$> 0.05$
MOMBI-ATCH	$2.91e-33$	$1.28e-34$	$> 0.05$	$1.49e-34$	$> 0.05$	$1.01e-30$	–	$1.28e-34$	$1.61e-33$	$1.28e-34$	$> 0.05$
MOMBI-NATCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$> 0.05$	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$	–	$1.28e-34$	$> 0.05$
MOMBI-NMTCH	$> 0.05$	$6.22e-14$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$8.81e-08$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–
10D											
MOMBI-WS	–	$4.92e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.44e-34$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$7.64e-21$	$1.28e-34$	$5.81e-23$	$1.28e-34$	$1.32e-34$	$1.28e-34$	$> 0.05$
MOMBI-NLS	$> 0.05$	$1.49e-34$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$
MOMBI-TCH	$7.34e-32$	$1.28e-34$	$> 0.05$	$1.49e-34$	–	$5.08e-33$	$2.75e-02$	$1.28e-34$	$1.89e-32$	$1.28e-34$	$> 0.05$
MOMBI-NTCH	$1.13e-07$	$1.28e-34$	$> 0.05$	$1.40e-14$	$> 0.05$	–	$> 0.05$	$1.28e-34$	$6.33e-08$	$1.28e-34$	$> 0.05$
MOMBI-ATCH	$6.43e-31$	$1.28e-34$	$> 0.05$	$2.26e-34$	$> 0.05$	$5.20e-32$	–	$1.28e-34$	$1.89e-31$	$1.28e-34$	$> 0.05$
MOMBI-NATCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$> 0.05$	$4.12e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$	–	$1.32e-34$	$> 0.05$
MOMBI-NMTCH	$> 0.05$	$9.21e-12$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$6.77e-07$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.68e-34$	$1.28e-34$	$1.32e-34$	$1.28e-34$	$1.40e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–

Table A.102: Comparison of R2 indicator values for different optimizers on the DTLZ1 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	1.250221e-01	1.250163e-01	8.369820e-02	8.386538e-02	8.371409e-02	8.385249e-02	8.371175e-02	8.384388e-02	8.373182e-02	8.385941e-02	8.371635e-02
	avg.	1.249867e-01	1.248529e-01	8.371131e-02	8.388333e-02	8.372884e-02	8.386765e-02	8.372551e-02	8.385349e-02	8.374716e-02	8.387195e-02	8.373929e-02
	min.	1.219549e-01	1.218983e-01	8.367856e-02	8.377495e-02	8.368068e-02	8.376172e-02	8.367723e-02	8.375428e-02	8.369493e-02	8.377493e-02	8.367817e-02
	max.	1.251997e-01	1.251304e-01	8.391189e-02	8.428382e-02	8.387587e-02	8.414359e-02	8.394290e-02	8.409915e-02	8.398594e-02	8.411696e-02	8.392338e-02
	std.	3.277107e-04	5.827286e-04	3.736521e-05	8.696792e-05	4.282189e-05	6.513487e-05	4.521515e-05	5.336213e-05	5.551000e-05	6.268689e-05	5.624534e-05
3D	med.	5.082485e-02	5.555261e-02	3.299942e-02	3.540260e-02	3.302183e-02	3.341733e-02	3.300273e-02	3.324191e-02	3.290946e-02	3.318821e-02	3.227950e-02
	avg.	5.048222e-02	5.553815e-02	3.300123e-02	3.576947e-02	3.303757e-02	3.341071e-02	3.301590e-02	3.330056e-02	3.291350e-02	3.322690e-02	3.230350e-02
	min.	4.369927e-02	5.416726e-02	3.293280e-02	3.384948e-02	3.293377e-02	3.293918e-02	3.291751e-02	3.289149e-02	3.280452e-02	3.292649e-02	3.222502e-02
	max.	5.410865e-02	5.567709e-02	3.308017e-02	4.024770e-02	3.330247e-02	3.402440e-02	3.320161e-02	3.391853e-02	3.308863e-02	3.379888e-02	3.256077e-02
	std.	2.158061e-03	1.514249e-04	2.638213e-05	1.442053e-03	6.383624e-05	2.138387e-04	5.694183e-05	2.139802e-04	5.654246e-05	2.022500e-04	6.758045e-05
4D	med.	2.852337e-02	3.120427e-02	2.067122e-02	2.207390e-02	2.066227e-02	2.126167e-02	2.068065e-02	2.120022e-02	2.087161e-02	2.106089e-02	1.812413e-02
	avg.	2.841021e-02	3.120663e-02	2.066933e-02	2.226426e-02	2.066300e-02	2.129038e-02	2.067520e-02	2.130614e-02	2.086195e-02	2.108857e-02	1.814022e-02
	min.	2.564135e-02	3.046037e-02	2.055480e-02	2.095669e-02	2.045131e-02	2.001447e-02	2.049293e-02	2.035999e-02	1.992222e-02	2.031951e-02	1.806785e-02
	max.	3.004109e-02	3.131175e-02	2.076938e-02	2.467113e-02	2.083683e-02	2.484103e-02	2.077825e-02	2.689293e-02	2.130416e-02	2.227604e-02	1.850672e-02
	std.	8.768144e-04	7.979630e-05	3.456031e-05	8.147470e-04	6.643922e-05	6.305181e-04	4.756459e-05	6.826678e-04	2.333287e-04	3.646866e-04	6.557684e-05
5D	med.	1.833907e-02	2.035124e-02	1.430786e-02	1.624479e-02	1.433614e-02	1.554745e-02	1.430869e-02	1.898600e-02	1.695748e-02	1.907802e-02	1.220423e-02
	avg.	1.825653e-02	2.033216e-02	1.430007e-02	1.636810e-02	1.434300e-02	1.562031e-02	1.430497e-02	1.886676e-02	1.694517e-02	1.899977e-02	1.221775e-02
	min.	1.682529e-02	1.976477e-02	1.416102e-02	1.466556e-02	1.423039e-02	1.420109e-02	1.419250e-02	1.687659e-02	1.584148e-02	1.721067e-02	1.215597e-02
	max.	1.933811e-02	2.046444e-02	1.439533e-02	1.902732e-02	1.469482e-02	1.817083e-02	1.447436e-02	2.114919e-02	1.766810e-02	1.974120e-02	1.242517e-02
	std.	5.146750e-04	9.458097e-05	3.806663e-05	7.987582e-04	5.064963e-05	6.728384e-04	4.133582e-05	8.705071e-04	3.349990e-04	5.146585e-04	5.116746e-05
6D	med.	1.443351e-02	1.678270e-02	1.247372e-02	1.406544e-02	1.150296e-02	1.254882e-02	1.163648e-02	1.823229e-02	1.434794e-02	1.832000e-02	8.632511e-03
	avg.	1.402087e-02	1.673756e-02	1.220877e-02	1.446829e-02	1.160249e-02	1.266418e-02	1.167386e-02	1.848578e-02	1.402633e-02	1.818651e-02	8.644993e-03
	min.	1.188570e-02	1.372270e-02	1.088757e-02	1.205165e-02	1.087346e-02	1.157861e-02	1.071464e-02	1.615388e-02	1.125889e-02	1.564722e-02	8.580063e-03
	max.	1.635841e-02	1.693068e-02	1.263643e-02	1.756071e-02	1.255867e-02	1.482012e-02	1.279933e-02	2.221540e-02	1.616536e-02	2.057186e-02	8.893578e-03
	std.	1.191943e-03	3.545128e-04	4.757693e-04	1.446066e-03	3.699470e-04	5.926070e-04	4.526589e-04	1.498232e-03	1.122830e-03	1.413972e-03	4.879750e-05
7D	med.	1.548116e-02	1.765394e-02	1.248233e-02	1.914325e-02	1.224978e-02	1.367108e-02	1.203762e-02	2.250559e-02	1.503058e-02	2.190542e-02	6.841354e-03
	avg.	1.520486e-02	1.760213e-02	1.247470e-02	1.831676e-02	1.216415e-02	1.379823e-02	1.216076e-02	2.175110e-02	1.483966e-02	2.123805e-02	6.849752e-03
	min.	1.124695e-02	1.645853e-02	1.186803e-02	1.334244e-02	1.086849e-02	1.290458e-02	1.065480e-02	1.713418e-02	1.147181e-02	1.738785e-02	6.810513e-03
	max.	1.702178e-02	1.789205e-02	1.305695e-02	2.281144e-02	1.371423e-02	1.552155e-02	1.365901e-02	2.853267e-02	1.713786e-02	2.348764e-02	6.949707e-03
	std.	1.068590e-03	2.201881e-04	2.069580e-04	2.635105e-03	7.589905e-04	5.791267e-04	7.398971e-04	2.132940e-03	1.271067e-03	1.970351e-03	2.677237e-05
8D	med.	1.324676e-02	1.554876e-02	1.036881e-02	1.224220e-02	1.019968e-02	1.140010e-02	1.045715e-02	1.595129e-02	1.247068e-02	1.553427e-02	5.226346e-03
	avg.	1.286818e-02	1.553167e-02	1.036887e-02	1.277110e-02	1.033023e-02	1.141933e-02	1.040267e-02	1.684867e-02	1.242876e-02	1.649465e-02	5.231511e-03
	min.	9.904890e-03	1.473203e-02	9.541820e-03	1.075784e-02	9.122911e-03	9.767834e-03	9.092824e-03	1.507225e-02	9.216409e-03	1.428744e-02	5.166102e-03
	max.	1.515979e-02	1.563526e-02	1.090829e-02	1.927909e-02	2.038663e-02	1.249347e-02	1.170612e-02	2.057618e-02	1.456981e-02	2.037553e-02	5.391744e-03
	std.	1.323130e-03	1.028801e-04	2.134539e-04	1.621969e-03	1.203351e-03	3.851181e-04	6.433107e-04	1.664534e-03	1.329171e-03	1.731001e-03	3.380665e-05
9D	med.	1.006132e-02	1.384116e-02	8.700264e-03	1.048738e-02	9.072458e-03	9.961506e-03	8.942962e-03	1.384027e-02	1.020200e-02	1.380122e-02	4.036310e-03
	avg.	1.048893e-02	1.381935e-02	8.697359e-03	1.048189e-02	9.060204e-03	9.986422e-03	8.921439e-03	1.413679e-02	1.059982e-02	1.393866e-02	4.048937e-03
	min.	8.019706e-03	1.315375e-02	8.156883e-03	9.566892e-03	7.719002e-03	9.136399e-03	7.643449e-03	1.325586e-02	8.427585e-03	1.312988e-02	3.979084e-03
	max.	1.318731e-02	1.397014e-02	9.456080e-03	1.405531e-02	1.051368e-02	1.084005e-02	1.033110e-02	1.809686e-02	1.307334e-02	1.772265e-02	4.192183e-03
	std.	1.245488e-03	1.154010e-04	1.845507e-04	5.998334e-04	5.731677e-04	2.773646e-04	5.853407e-04	8.203342e-04	1.343947e-03	7.969170e-04	4.925594e-05
10D	med.	8.861302e-03	1.246619e-02	7.408450e-03	9.006248e-03	7.808807e-03	8.832738e-03	7.717094e-03	1.246153e-02	8.898656e-03	1.241806e-02	3.291724e-03
	avg.	9.084486e-03	1.237656e-02	7.406562e-03	9.090917e-03	7.857563e-03	8.830119e-03	7.766006e-03	1.250523e-02	9.109441e-03	1.243514e-02	3.292841e-03
	min.	6.548312e-03	9.637418e-03	6.533864e-03	8.435308e-03	6.759960e-03	7.684716e-03	6.653663e-03	1.198505e-02	6.758134e-03	1.167359e-02	3.197968e-03
	max.	1.184285e-02	1.255416e-02	8.057331e-03	1.078617e-02	9.009505e-03	9.643741e-03	9.288301e-03	1.388609e-02	1.170764e-02	1.592553e-02	3.456552e-03
	std.	1.330531e-03	3.485326e-04	1.753365e-04	4.283549e-04	5.105638e-04	2.288299e-04	5.309948e-04	3.664999e-04	1.298019e-03	3.974172e-04	5.414324e-05

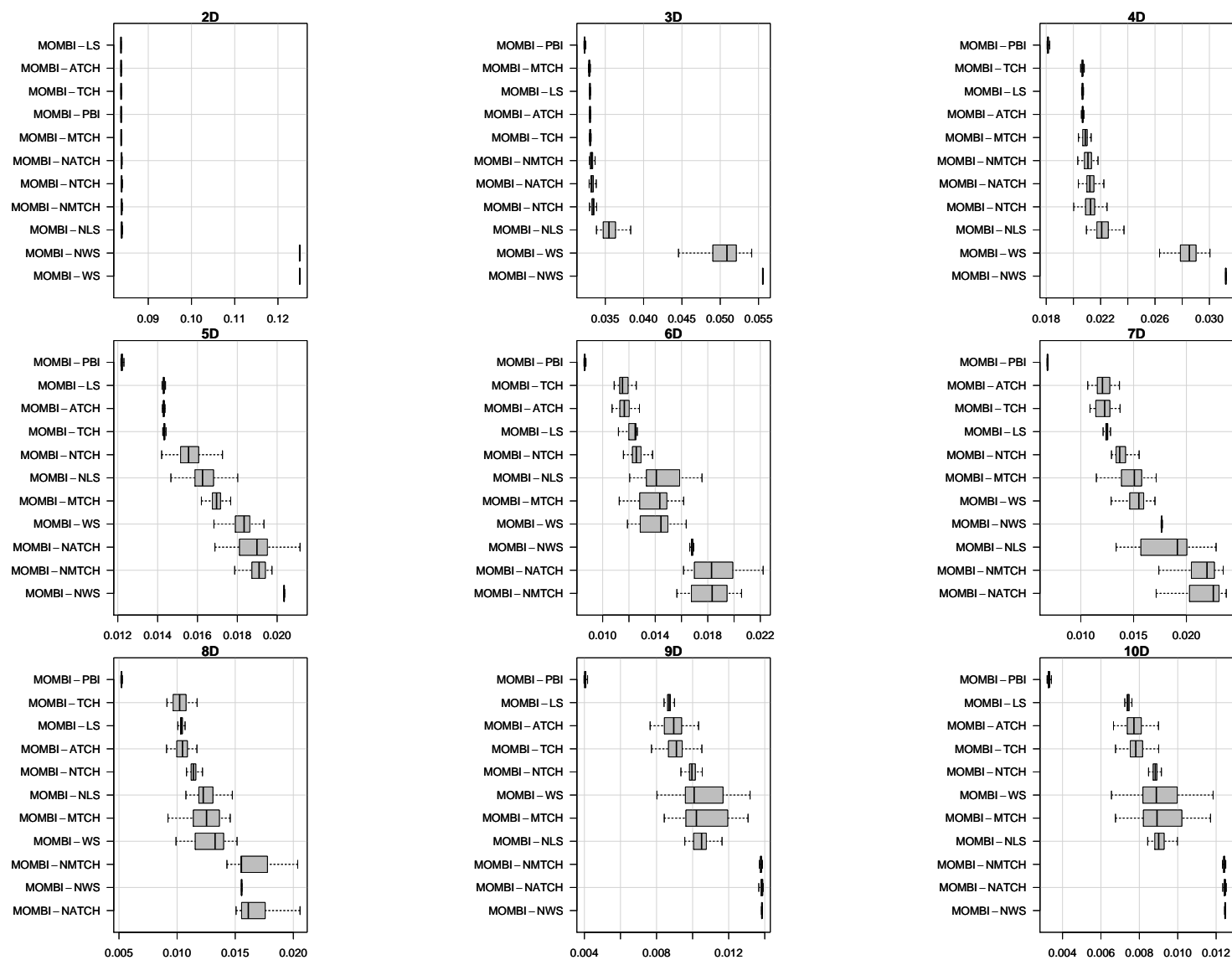


Figure A.258: Box-plot of the R2 indicator values for different optimizers on the DTLZ1 test problem.

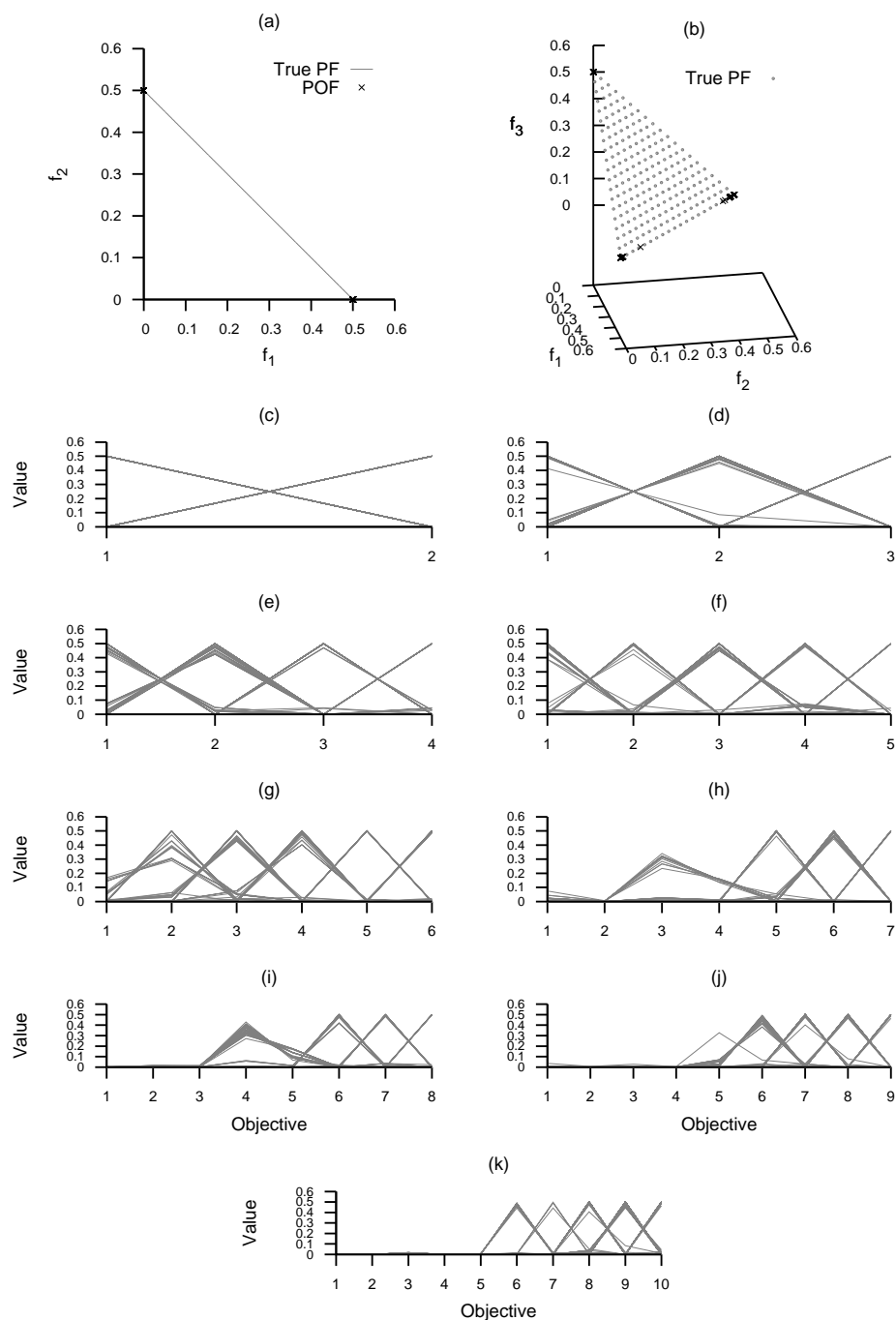


Figure A.259: Plots of the approximations obtained by MOMBI-WS from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



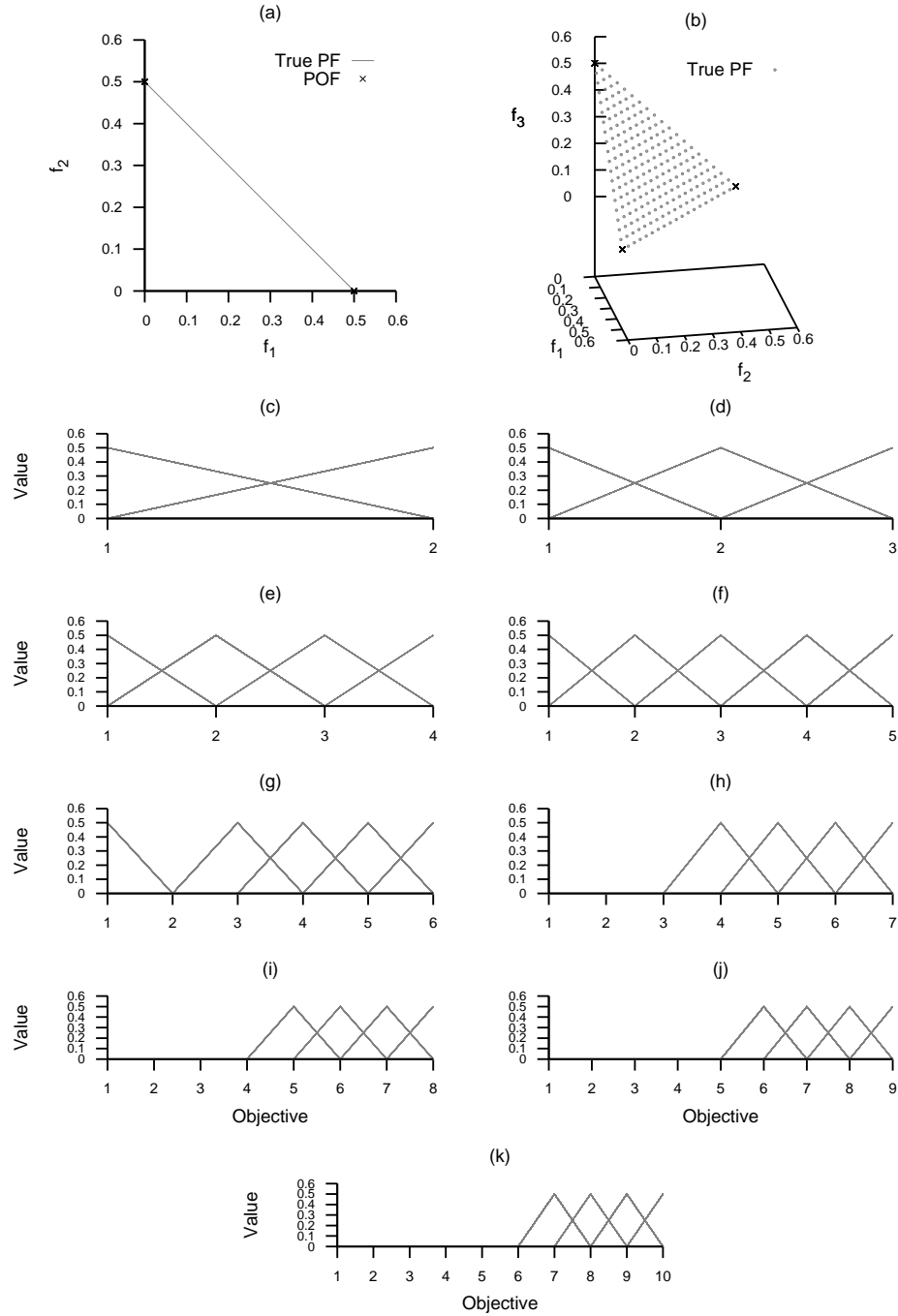


Figure A.260: Plots of the approximations obtained by MOMBI-NWS from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

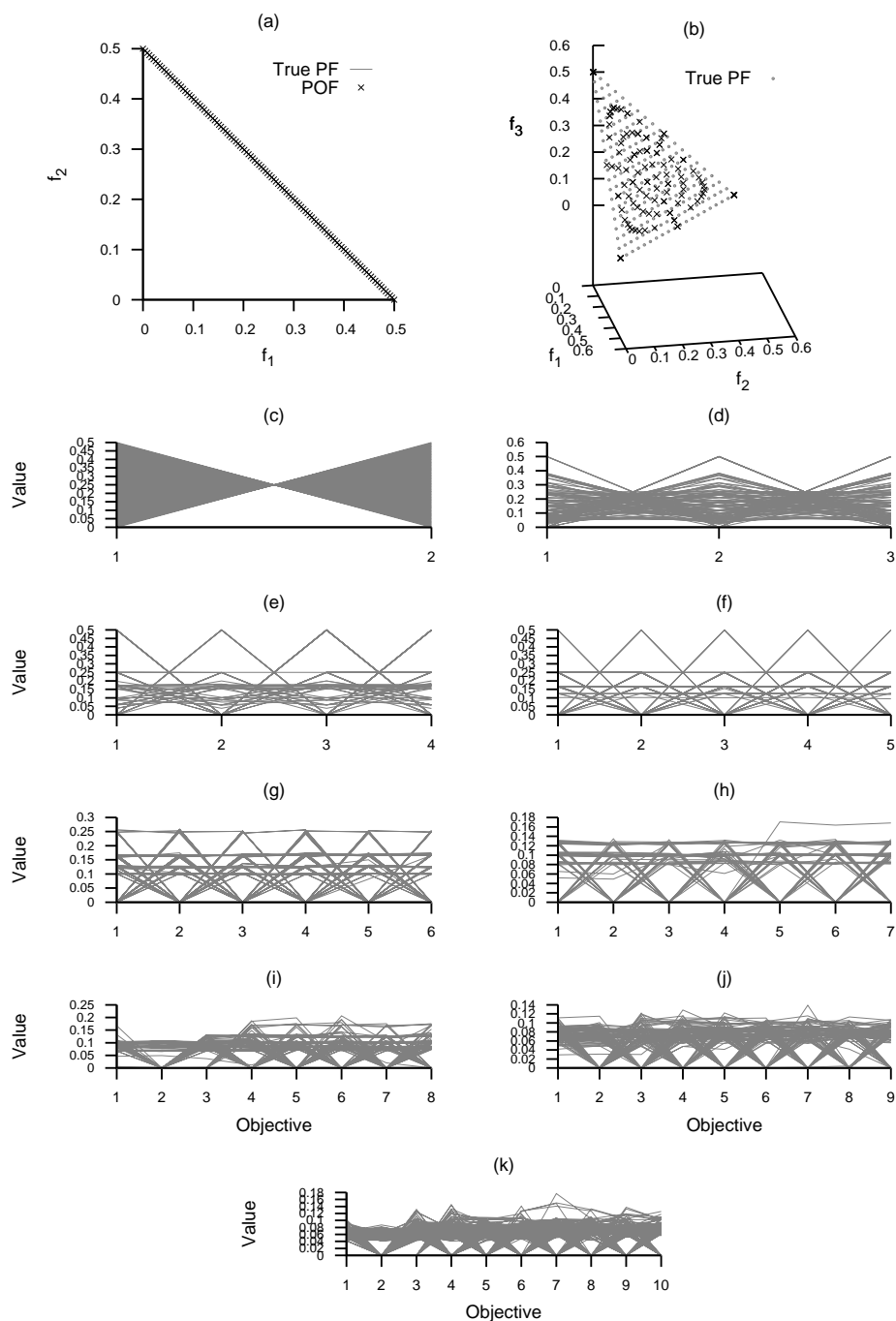


Figure A.261: Plots of the approximations obtained by MOMBI-LS from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

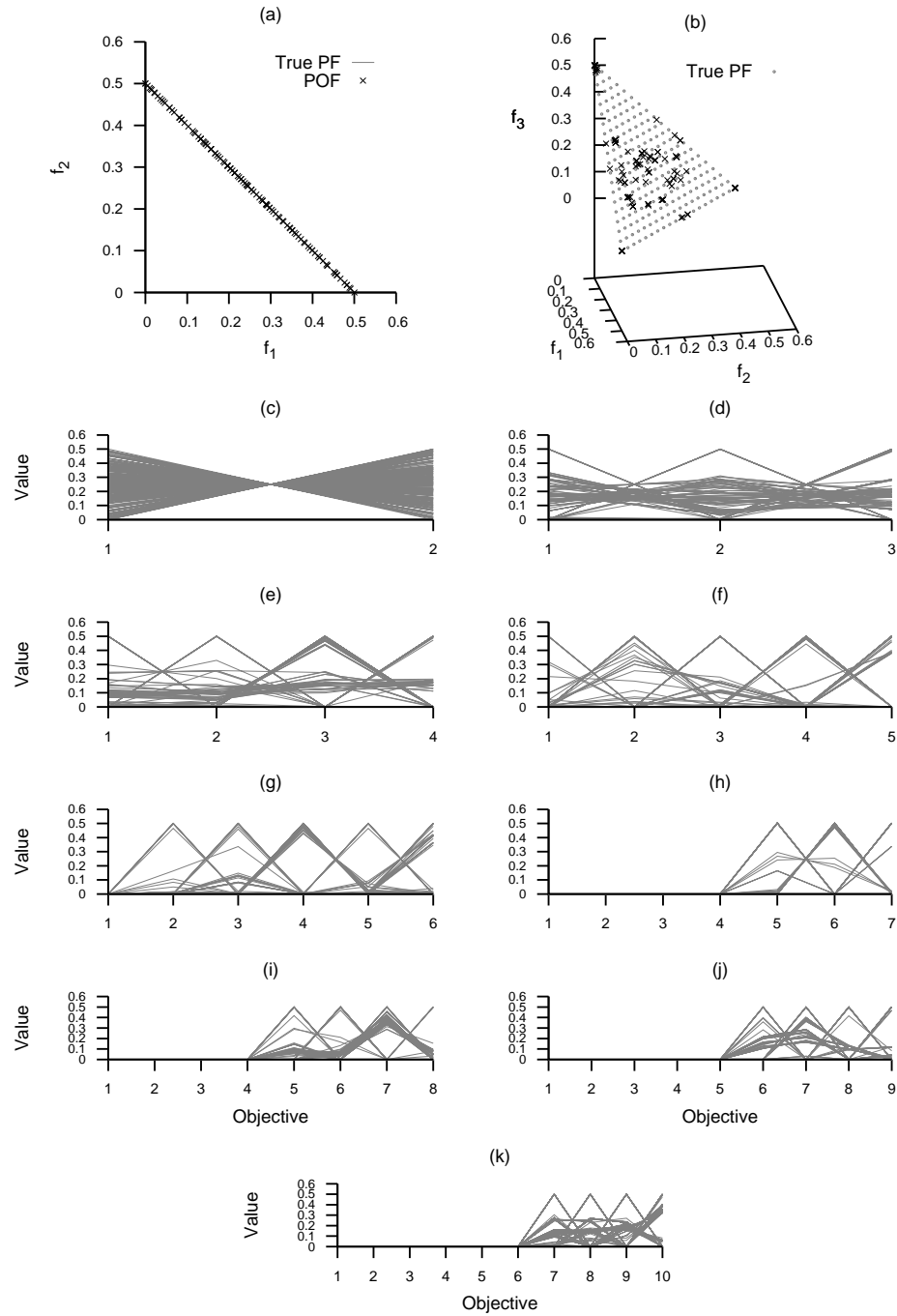


Figure A.262: Plots of the approximations obtained by MOMBI-NLS from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

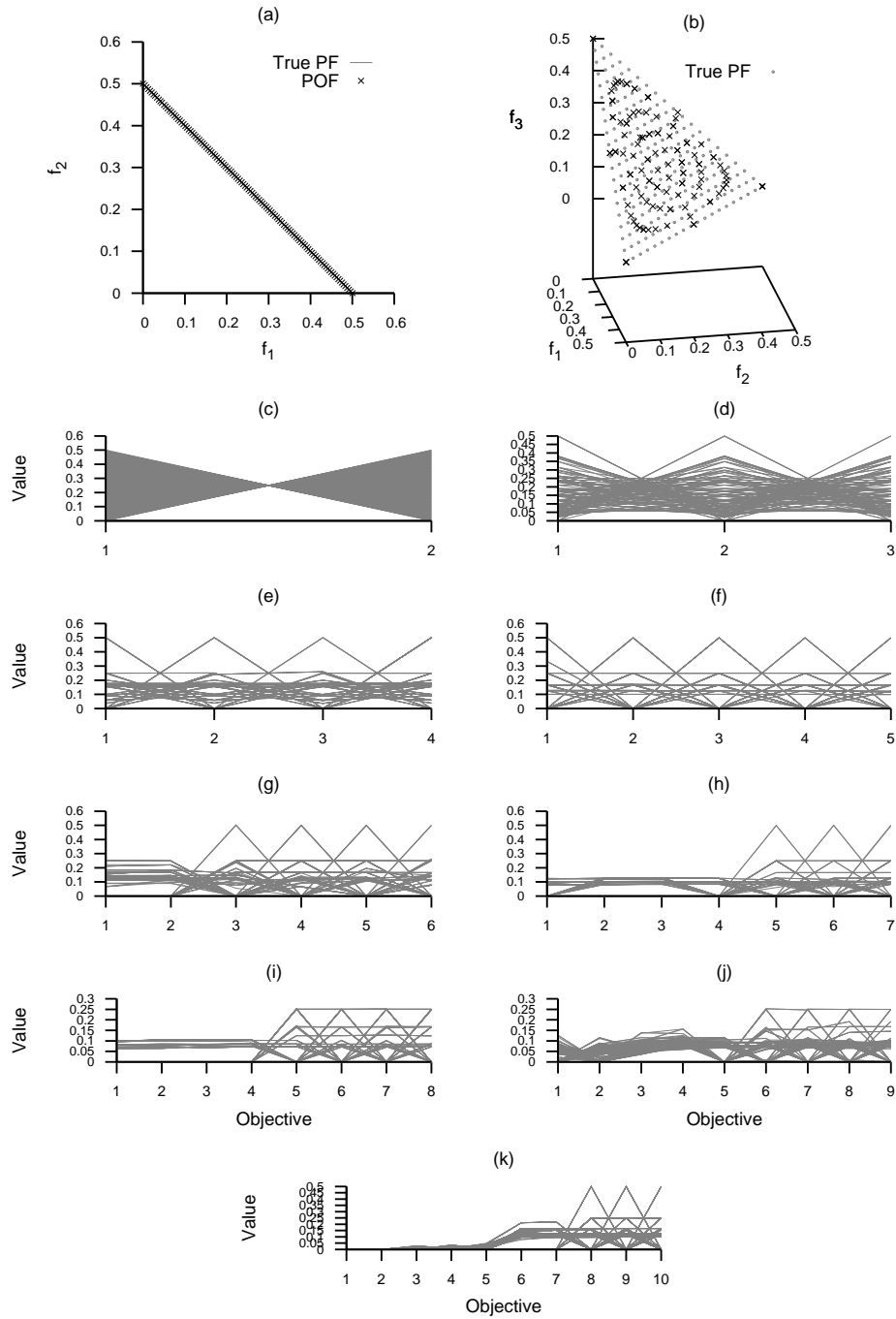


Figure A.263: Plots of the approximations obtained by MOMBI-ATCH from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

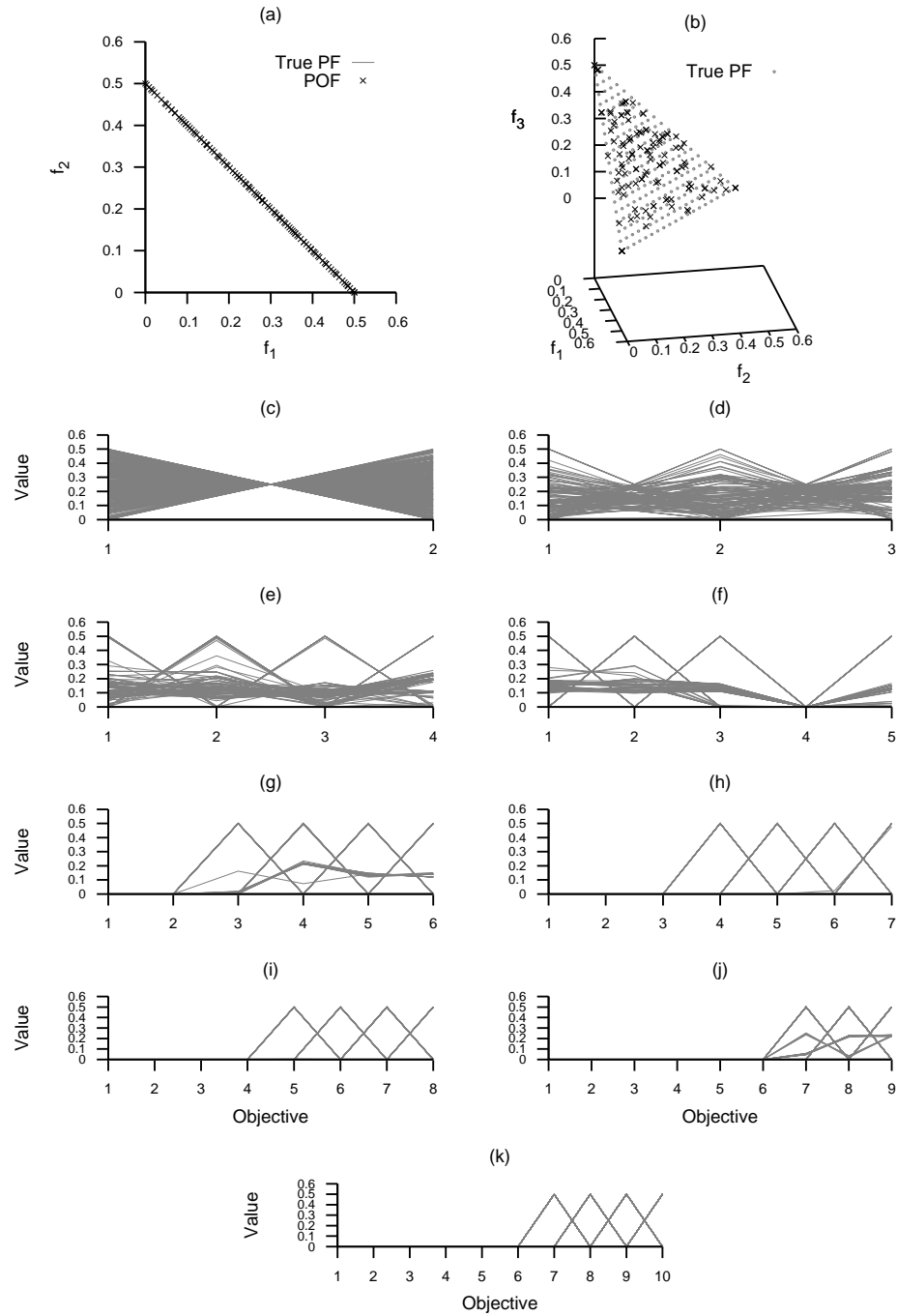


Figure A.264: Plots of the approximations obtained by MOMBI-NATCH from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

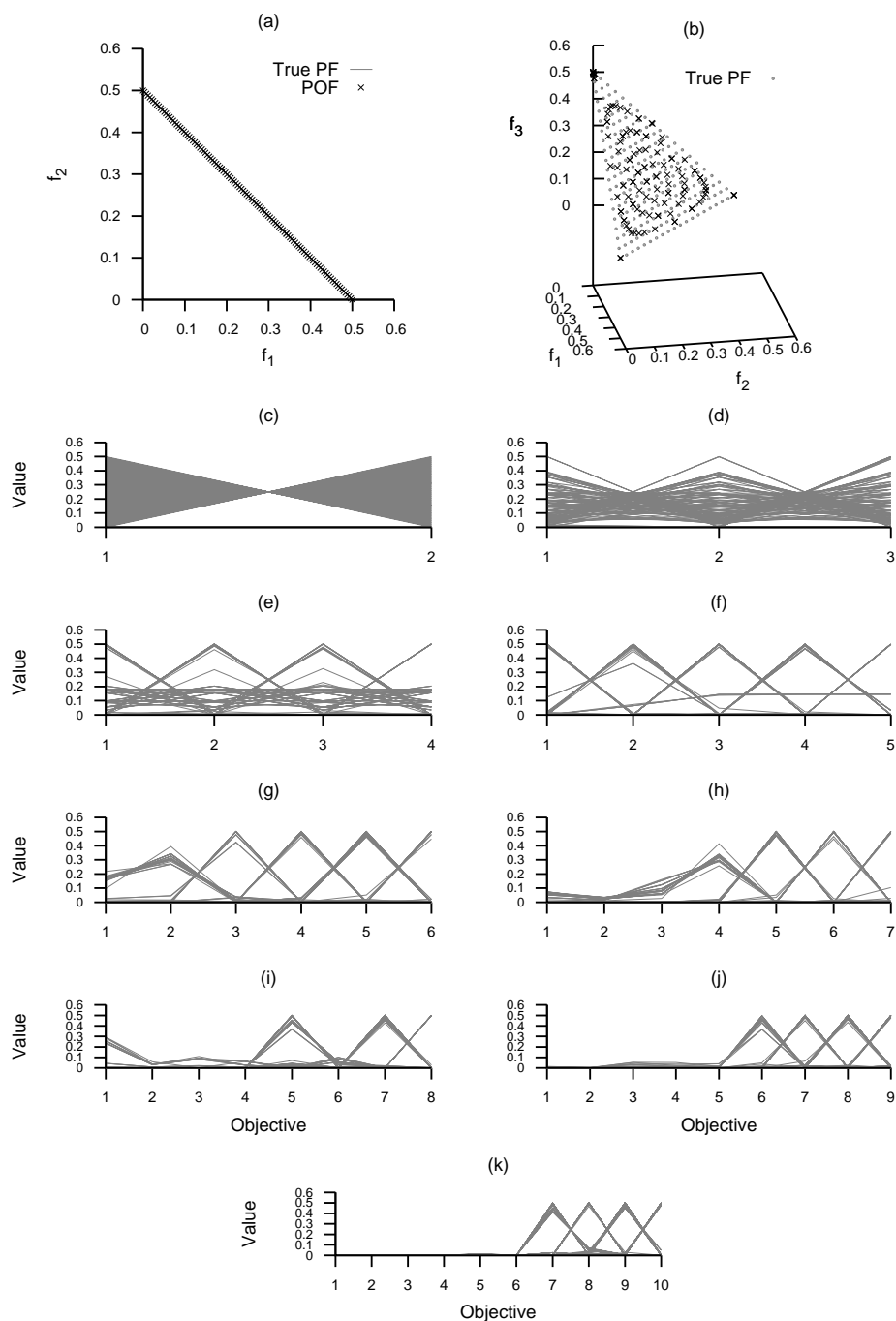


Figure A.265: Plots of the approximations obtained by MOMBI-MTCH from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

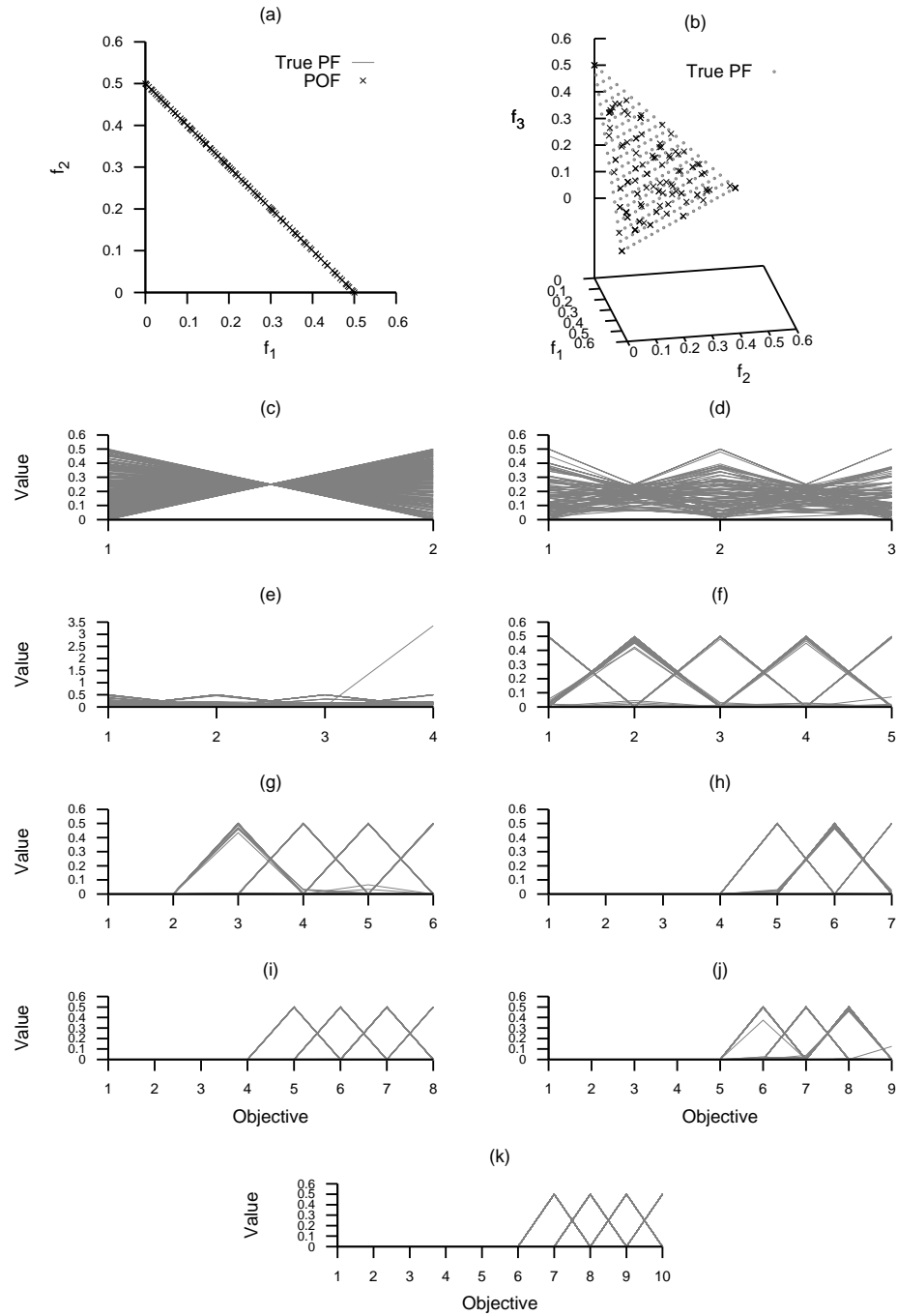


Figure A.266: Plots of the approximations obtained by MOMBI-NMTCH from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.103: Comparison of hypervolume indicator values for different optimizers on the DTLZ2 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	3.000000e+00	3.000000e+00	3.000000e+00	3.000000e+00	3.210817e+00	3.209389e+00	3.209806e+00	3.209055e+00	3.210798e+00	3.209436e+00	3.210539e+00
	avg.	3.000000e+00	3.000000e+00	3.000000e+00	3.000000e+00	3.210809e+00	3.209364e+00	3.209828e+00	3.209061e+00	3.210797e+00	3.209411e+00	3.210525e+00
	min.	3.000000e+00	3.000000e+00	3.000000e+00	3.000000e+00	3.210634e+00	3.208928e+00	3.209711e+00	3.208548e+00	3.210761e+00	3.208725e+00	3.210045e+00
	max.	3.000000e+00	3.000000e+00	3.000000e+00	3.000002e+00	3.210875e+00	3.209828e+00	3.210140e+00	3.209633e+00	3.210825e+00	3.209959e+00	3.210819e+00
	std.	0.000000e+00	0.000000e+00	0.000000e+00	2.241437e-07	5.344664e-05	2.030409e-04	7.762029e-05	1.948324e-04	5.703565e-06	2.456584e-04	1.685171e-04
3D	med.	7.000000e+00	7.000000e+00	7.009515e+00	7.000000e+00	7.383660e+00	7.388885e+00	7.397252e+00	7.395917e+00	7.378203e+00	7.387712e+00	7.421727e+00
	avg.	7.000000e+00	7.000000e+00	7.014729e+00	7.000000e+00	7.384076e+00	7.389081e+00	7.397292e+00	7.395898e+00	7.378906e+00	7.387756e+00	7.421722e+00
	min.	7.000000e+00	7.000000e+00	7.000240e+00	7.000000e+00	7.382117e+00	7.383709e+00	7.393735e+00	7.388852e+00	7.373839e+00	7.380159e+00	7.421178e+00
	max.	7.000000e+00	7.000000e+00	7.071601e+00	7.000000e+00	7.388551e+00	7.394650e+00	7.401605e+00	7.400149e+00	7.385040e+00	7.395354e+00	7.422009e+00
	std.	0.000000e+00	0.000000e+00	1.464602e-02	0.000000e+00	1.394472e-03	2.095594e-03	1.855040e-03	2.355233e-03	2.864178e-03	3.309552e-03	1.338885e-04
4D	med.	1.500000e+01	1.500000e+01	1.502344e+01	1.500000e+01	1.542125e+01	1.542188e+01	1.535442e+01	1.535322e+01	1.528510e+01	1.528549e+01	1.556707e+01
	avg.	1.500000e+01	1.500000e+01	1.502849e+01	1.500002e+01	1.542192e+01	1.542250e+01	1.535100e+01	1.535190e+01	1.528211e+01	1.527877e+01	1.556704e+01
	min.	1.500000e+01	1.500000e+01	1.500270e+01	1.500000e+01	1.542061e+01	1.542070e+01	1.496012e+01	1.531949e+01	1.478023e+01	1.478262e+01	1.556629e+01
	max.	1.500000e+01	1.500000e+01	1.510385e+01	1.500237e+01	1.542960e+01	1.543046e+01	1.538263e+01	1.537609e+01	1.530766e+01	1.531943e+01	1.556754e+01
	std.	0.000000e+00	0.000000e+00	1.876452e-02	2.370000e-04	1.747971e-03	1.926998e-03	4.084009e-02	1.053333e-02	5.115773e-02	6.835398e-02	2.312808e-02
5D	med.	3.100000e+01	3.100000e+01	3.104614e+01	3.100000e+01	3.153407e+01	3.153462e+01	3.106250e+01	3.106250e+01	3.105267e+01	3.105321e+01	3.166721e+01
	avg.	3.100000e+01	3.099000e+01	3.105023e+01	3.100000e+01	3.153416e+01	3.153484e+01	3.105817e+01	3.106026e+01	3.105709e+01	3.105748e+01	3.166721e+01
	min.	3.100000e+01	3.000000e+01	3.097757e+01	3.100000e+01	3.153365e+01	3.153397e+01	3.093775e+01	3.105161e+01	3.105160e+01	3.105148e+01	3.166628e+01
	max.	3.100000e+01	3.100000e+01	3.116503e+01	3.100001e+01	3.154020e+01	3.153846e+01	3.107550e+01	3.111815e+01	3.109523e+01	3.109403e+01	3.166791e+01
	std.	0.000000e+00	1.000000e-01	3.182874e-02	1.694367e-06	7.362071e-04	7.164234e-04	1.435429e-02	9.921606e-03	7.872253e-03	8.488562e-03	3.210210e-04
6D	med.	6.200000e+01	6.200000e+01	6.210013e+01	6.200000e+01	6.304662e+01	6.299141e+01	6.200000e+01	6.200000e+01	6.200000e+01	6.200000e+01	6.373820e+01
	avg.	6.198000e+01	6.201991e+01	6.226777e+01	6.200002e+01	6.301022e+01	6.297849e+01	6.200000e+01	6.200000e+01	6.200062e+01	6.200992e+01	6.373816e+01
	min.	6.000000e+01	6.199999e+01	6.179042e+01	6.199999e+01	6.283157e+01	6.277087e+01	6.200000e+01	6.199999e+01	6.199999e+01	6.199999e+01	6.373638e+01
	max.	6.200000e+01	6.299720e+01	6.306979e+01	6.200138e+01	6.324268e+01	6.316268e+01	6.200000e+01	6.200000e+01	6.206245e+01	6.297996e+01	6.373926e+01
	std.	2.000000e-01	1.400792e-01	3.585205e-01	1.392040e-04	1.161170e-01	7.419412e-02	0.000000e+00	1.355494e-06	6.245010e-03	9.799014e-02	5.630192e-04
7D	med.	1.200000e+02	1.200000e+02	1.200218e+02	1.200000e+02	1.222130e+02	1.228876e+02	1.200000e+02	1.200000e+02	1.200000e+02	1.200000e+02	1.277497e+02
	avg.	1.195200e+02	1.193606e+02	1.198781e+02	1.200002e+02	1.220508e+02	1.225810e+02	1.200000e+02	1.200000e+02	1.196800e+02	1.196000e+02	1.277494e+02
	min.	8.800000e+01	8.800000e+01	1.182428e+02	1.200000e+02	1.196176e+02	1.208709e+02	1.200000e+02	1.200000e+02	8.800000e+01	1.040000e+02	1.277458e+02
	max.	1.200001e+02	1.200294e+02	1.232907e+02	1.200234e+02	1.237597e+02	1.231339e+02	1.200000e+02	1.200000e+02	1.200000e+02	1.200005e+02	1.277521e+02
	std.	3.376927e+00	3.537847e+00	7.285207e-01	2.340029e-03	1.343584e+00	5.622711e-01	0.000000e+00	0.000000e+00	3.200000e+00	2.088933e+00	1.361677e-03
8D	med.	2.400000e+02	2.400000e+02	2.404689e+02	2.400000e+02	2.438127e+02	2.457684e+02	2.400000e+02	2.400000e+02	2.400000e+02	2.400000e+02	2.558165e+02
	avg.	2.393600e+02	2.391907e+02	2.417453e+02	2.400002e+02	2.440659e+02	2.452670e+02	2.400000e+02	2.400000e+02	2.400000e+02	2.400005e+02	2.558164e+02
	min.	2.240000e+02	1.760000e+02	2.359809e+02	2.399999e+02	2.376711e+02	2.420399e+02	2.400000e+02	2.400000e+02	2.400000e+02	2.399999e+02	2.558080e+02
	max.	2.400000e+02	2.475473e+02	2.501938e+02	2.400067e+02	2.478016e+02	2.462900e+02	2.400000e+02	2.400000e+02	2.400000e+02	2.400478e+02	2.558203e+02
	std.	3.151142e+00	6.857751e+00	3.271583e+00	1.037997e-03	2.562898e+00	9.513752e-01	0.000000e+00	0.000000e+00	0.000000e+00	4.780110e-03	2.316092e-03
9D	med.	4.800000e+02	4.800000e+02	4.822105e+02	4.800000e+02	4.909975e+02	4.912586e+02	4.800000e+02	4.800000e+02	4.800000e+02	4.800000e+02	5.118535e+02
	avg.	4.800000e+02	4.803135e+02	4.864112e+02	4.801641e+02	4.894250e+02	4.903181e+02	4.800000e+02	4.800000e+02	4.800000e+02	4.800001e+02	5.118523e+02
	min.	4.799999e+02	4.799999e+02	4.788534e+02	4.799999e+02	4.731516e+02	4.834090e+02	4.800000e+02	4.799999e+02	4.799999e+02	4.799999e+02	5.118240e+02
	max.	4.800000e+02	4.957759e+02	5.060964e+02	4.964118e+02	4.954755e+02	4.939305e+02	4.800000e+02	4.800000e+02	4.800000e+02	4.800073e+02	5.118632e+02
	std.	1.084395e-05	2.205226e+00	7.005656e+00	1.641178e+00	5.155163e+00	2.170805e+00	0.000000e+00	1.084395e-05	1.533566e-05	7.793322e-04	5.978400e-03
10D	med.	9.600000e+02	9.600000e+02	9.859972e+02	9.600000e+02	9.786679e+02	9.832145e+02	9.600000e+02	9.600000e+02	9.600000e+02	9.600000e+02	1.023846e+03
	avg.	9.600000e+02	9.608974e+02	9.799869e+02	9.606109e+02	9.782661e+02	9.825038e+02	9.600000e+02	9.600000e+02	9.600000e+02	9.603147e+02	1.023841e+03
	min.	9.599991e+02	9.599997e+02	9.547715e+02	9.599997e+02	9.445224e+02	9.679557e+02	9.599997e+02	9.599997e+02	9.599988e+02	9.599997e+02	1.023755e+03
	max.	9.600000e+02	9.912954e+02	9.924265e+02	9.922212e+02	9.922212e+02	9.961464e+02	9.600000e+02	9.600000e+02	9.600000e+02	9.914229e+02	1.023870e+03
	std.	1.172952e-04	5.130950e+00	1.400526e+01	4.211491e+00	1.051349e+01	4.121702e+00	9.577117e-05	8.538534e-05	1.323673e-04	3.142246e+00	1.953483e-02



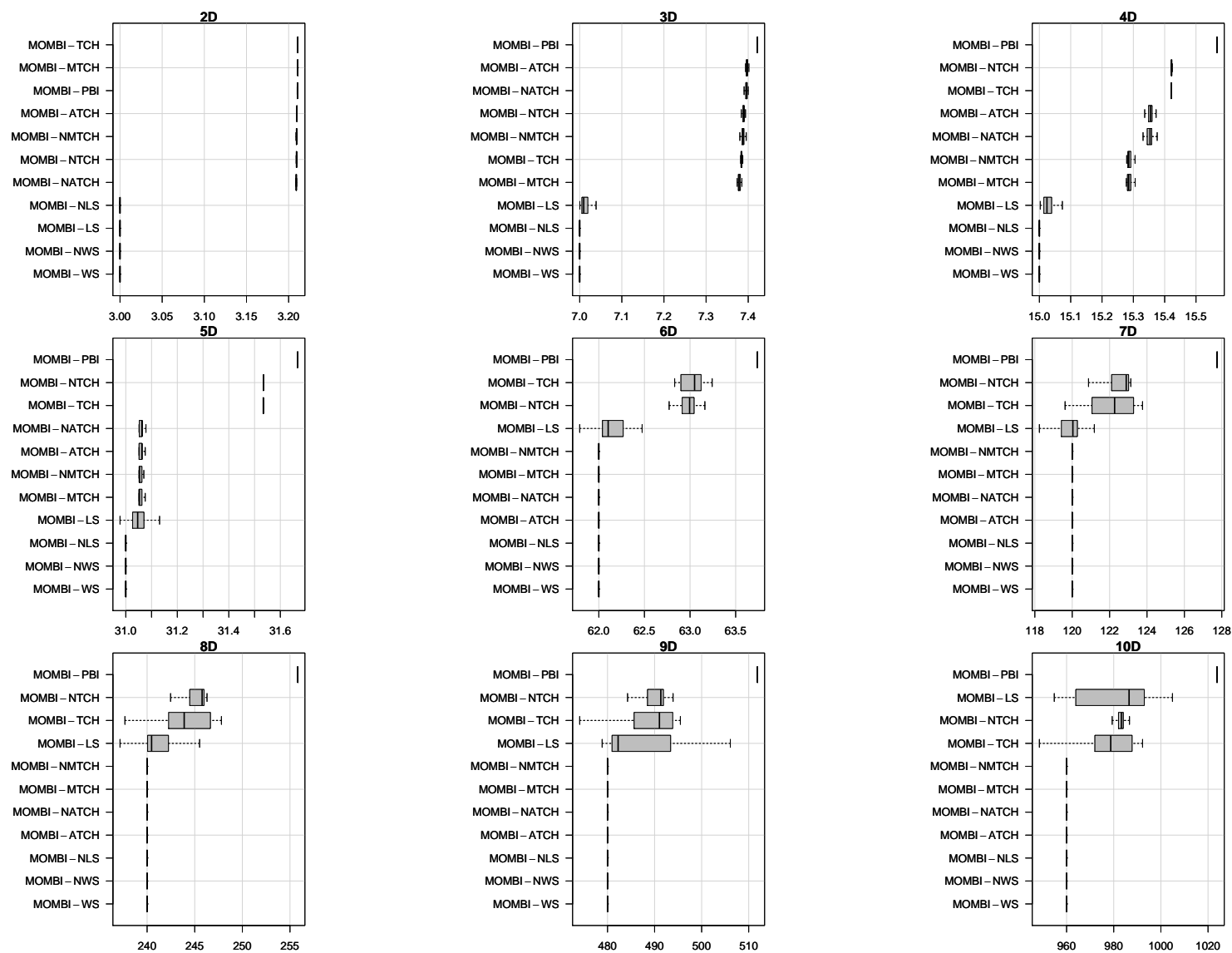


Figure A.267: Box-plot of the hypervolume indicator values for different optimizers on the DTLZ2 test problem.

Table A.104: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ2 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
2D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$2.81e-39$	$2.81e-39$	$2.81e-39$	$5.75e-39$	–	$1.28e-34$	$1.27e-34$	$1.28e-34$	$7.71e-04$	$1.28e-34$	$1.13e-30$
MOMBI-NTCH	$2.82e-39$	$2.82e-39$	$2.82e-39$	$5.76e-39$	> 0.05	–	> 0.05	$1.38e-18$	> 0.05	> 0.05	> 0.05
MOMBI-ATCH	$2.80e-39$	$2.80e-39$	$2.80e-39$	$5.73e-39$	> 0.05	$1.81e-33$	–	$1.28e-34$	> 0.05	$3.85e-29$	> 0.05
MOMBI-NATCH	$2.82e-39$	$2.82e-39$	$2.82e-39$	$5.76e-39$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$2.06e-39$	$2.06e-39$	$2.06e-39$	$4.24e-39$	> 0.05	$1.01e-34$	$1.00e-34$	$1.01e-34$	–	$1.01e-34$	$4.17e-32$
MOMBI-NMTCH	$2.82e-39$	$2.82e-39$	$2.82e-39$	$5.76e-39$	> 0.05	> 0.05	> 0.05	$2.64e-19$	> 0.05	–	> 0.05
MOMBI-PBI	$2.82e-39$	$2.82e-39$	$2.82e-39$	$5.76e-39$	> 0.05	$1.28e-34$	$1.53e-34$	$1.28e-34$	> 0.05	$1.28e-34$	–
3D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$2.82e-39$	$2.82e-39$	–	$2.82e-39$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$2.82e-39$	$2.82e-39$	$1.28e-34$	$2.82e-39$	–	> 0.05	> 0.05	> 0.05	$1.14e-26$	> 0.05	> 0.05
MOMBI-NTCH	$2.82e-39$	$2.82e-39$	$1.28e-34$	$2.82e-39$	$1.64e-31$	–	> 0.05	> 0.05	$1.53e-34$	$2.42e-04$	> 0.05
MOMBI-ATCH	$2.82e-39$	$2.82e-39$	$1.28e-34$	$2.82e-39$	$1.28e-34$	$1.58e-34$	–	$3.30e-05$	$1.28e-34$	$6.24e-34$	> 0.05
MOMBI-NATCH	$2.82e-39$	$2.82e-39$	$1.28e-34$	$2.82e-39$	$1.28e-34$	$5.43e-32$	> 0.05	–	$1.28e-34$	$3.97e-31$	> 0.05
MOMBI-MTCH	$2.82e-39$	$2.82e-39$	$1.28e-34$	$2.82e-39$	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$2.82e-39$	$2.82e-39$	$1.28e-34$	$2.82e-39$	$1.31e-18$	> 0.05	> 0.05	> 0.05	$4.01e-32$	–	> 0.05
MOMBI-PBI	$2.82e-39$	$2.82e-39$	$1.28e-34$	$2.82e-39$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–
4D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$2.82e-39$	$2.82e-39$	–	$4.05e-39$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$2.81e-39$	$2.81e-39$	$1.28e-34$	$4.04e-39$	–	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05
MOMBI-NTCH	$2.81e-39$	$2.81e-39$	$1.28e-34$	$4.04e-39$	$3.47e-09$	–	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05
MOMBI-ATCH	$8.42e-38$	$8.42e-38$	$2.51e-33$	$1.19e-37$	> 0.05	> 0.05	–	> 0.05	$2.44e-33$	$2.36e-33$	> 0.05
MOMBI-NATCH	$2.82e-39$	$2.82e-39$	$1.28e-34$	$4.05e-39$	> 0.05	> 0.05	> 0.05	–	$1.28e-34$	$1.28e-34$	> 0.05
MOMBI-MTCH	$8.42e-38$	$8.42e-38$	$2.51e-33$	$1.19e-37$	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$2.35e-36$	$2.35e-36$	$4.63e-32$	$3.28e-36$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	$2.80e-39$	$2.80e-39$	$1.27e-34$	$4.02e-39$	$1.27e-34$	$1.27e-34$	$1.27e-34$	$1.27e-34$	$1.27e-34$	$1.27e-34$	–

Table A.105: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ2 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
5D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$8.42e-38$	$1.15e-37$	–	$2.32e-37$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$4.14e-02$	$2.30e-02$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$2.80e-39$	$4.02e-39$	$1.27e-34$	$8.04e-39$	–	> 0.05	$9.13e-35$	$1.25e-34$	$1.14e-34$	$1.27e-34$	> 0.05
MOMBI-NTCH	$2.81e-39$	$4.04e-39$	$1.28e-34$	$8.08e-39$	$8.66e-26$	–	$9.17e-35$	$1.26e-34$	$1.15e-34$	$1.27e-34$	> 0.05
MOMBI-ATCH	$5.56e-38$	$7.64e-38$	$1.87e-05$	$1.55e-37$	> 0.05	> 0.05	–	> 0.05	$1.62e-02$	> 0.05	> 0.05
MOMBI-NATCH	$2.77e-39$	$3.98e-39$	$7.83e-06$	$7.97e-39$	> 0.05	> 0.05	$6.78e-03$	–	$3.53e-05$	$6.94e-03$	> 0.05
MOMBI-MTCH	$2.45e-39$	$3.52e-39$	$1.63e-04$	$7.07e-39$	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$2.80e-39$	$4.02e-39$	$1.78e-04$	$8.06e-39$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	$2.81e-39$	$4.04e-39$	$1.28e-34$	$8.08e-39$	$1.27e-34$	$1.27e-34$	$9.17e-35$	$1.26e-34$	$1.15e-34$	$1.27e-34$	–
6D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	$2.98e-02$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$3.00e-02$	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.63e-24$	$2.07e-22$	–	$3.26e-24$	> 0.05	> 0.05	$1.15e-24$	$2.19e-24$	$3.18e-24$	$3.59e-23$	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$8.11e-39$	$1.70e-37$	$3.05e-27$	$1.56e-38$	–	$1.99e-03$	$2.82e-39$	$8.11e-39$	$5.76e-39$	$6.88e-38$	> 0.05
MOMBI-NTCH	$8.11e-39$	$3.20e-37$	$6.67e-26$	$1.56e-38$	> 0.05	–	$2.82e-39$	$8.11e-39$	$5.76e-39$	$1.03e-37$	> 0.05
MOMBI-ATCH	$4.14e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	$4.14e-02$	> 0.05	> 0.05	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$4.81e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$4.88e-02$	> 0.05	–	> 0.05
MOMBI-PBI	$8.09e-39$	$1.13e-38$	$1.28e-34$	$1.56e-38$	$1.28e-34$	$1.28e-34$	$2.81e-39$	$8.09e-39$	$5.75e-39$	$2.14e-38$	–
7D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$8.25e-03$	$5.44e-03$	–	$1.98e-02$	> 0.05	> 0.05	$1.84e-02$	$1.84e-02$	$1.41e-02$	$6.18e-03$	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	$4.24e-02$	$2.95e-02$	> 0.05
MOMBI-TCH	$1.96e-21$	$2.61e-21$	$1.91e-21$	$3.88e-21$	–	> 0.05	$2.67e-21$	$2.67e-21$	$2.28e-21$	$1.65e-21$	> 0.05
MOMBI-NTCH	$1.13e-38$	$6.89e-38$	$3.37e-33$	$5.76e-39$	> 0.05	–	$2.82e-39$	$2.82e-39$	$4.05e-39$	$1.56e-38$	> 0.05
MOMBI-ATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	$1.11e-38$	$6.76e-38$	$1.26e-34$	$5.64e-39$	$1.26e-34$	$1.26e-34$	$2.76e-39$	$2.76e-39$	$3.97e-39$	$1.53e-38$	–

Table A.106: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ2 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
8D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$2.37e-13$	$5.34e-12$	–	$1.34e-12$	> 0.05	> 0.05	$8.86e-13$	$8.86e-13$	$8.86e-13$	$1.14e-12$	> 0.05
MOMBI-NLS	$1.67e-02$	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.78e-29$	$6.21e-27$	$1.37e-08$	$4.04e-29$	–	> 0.05	$1.44e-29$	$1.44e-29$	$1.44e-29$	$2.45e-29$	> 0.05
MOMBI-NTCH	$1.13e-38$	$1.52e-35$	$1.29e-12$	$1.13e-38$	$7.81e-03$	–	$2.82e-39$	$2.82e-39$	$2.82e-39$	$5.76e-39$	> 0.05
MOMBI-ATCH	$2.22e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NATCH	$2.22e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$2.22e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$4.95e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	$1.12e-38$	$2.13e-38$	$1.27e-34$	$1.12e-38$	$1.27e-34$	$1.27e-34$	$2.80e-39$	$2.80e-39$	$2.80e-39$	$5.73e-39$	–
9D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$2.03e-33$	$1.40e-30$	–	$7.07e-32$	> 0.05	> 0.05	$1.50e-33$	$2.74e-33$	$3.66e-33$	$4.85e-33$	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$4.57e-32$	$6.08e-29$	$1.52e-04$	$1.84e-30$	–	> 0.05	$3.41e-32$	$6.08e-32$	$8.02e-32$	$1.05e-31$	> 0.05
MOMBI-NTCH	$4.05e-39$	$2.02e-35$	$5.68e-06$	$3.20e-37$	> 0.05	–	$2.82e-39$	$5.76e-39$	$8.11e-39$	$1.13e-38$	> 0.05
MOMBI-ATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	$4.14e-02$	> 0.05	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$3.00e-02$	–	> 0.05
MOMBI-PBI	$4.04e-39$	$2.90e-38$	$1.28e-34$	$1.13e-38$	$1.28e-34$	$1.28e-34$	$2.81e-39$	$5.75e-39$	$8.09e-39$	$1.13e-38$	–
10D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.01e-32$	$1.66e-29$	–	$5.65e-30$	$4.53e-02$	> 0.05	$4.81e-33$	$4.82e-33$	$8.08e-34$	$1.07e-31$	> 0.05
MOMBI-NLS	> 0.05	$4.65e-02$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$3.68e-30$	$2.34e-26$	> 0.05	$3.89e-27$	–	> 0.05	$1.86e-30$	$1.87e-30$	$3.62e-31$	$7.05e-29$	> 0.05
MOMBI-NTCH	$9.07e-37$	$6.54e-32$	> 0.05	$7.85e-33$	$3.17e-02$	–	$3.91e-37$	$3.92e-37$	$5.20e-38$	$3.02e-35$	> 0.05
MOMBI-ATCH	> 0.05	$4.39e-02$	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NATCH	> 0.05	$3.32e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$9.99e-03$	$8.36e-04$	> 0.05	> 0.05	> 0.05	> 0.05	$4.15e-02$	$4.57e-02$	–	> 0.05	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	$8.99e-37$	$1.03e-35$	$1.27e-34$	$2.32e-35$	$1.27e-34$	$1.27e-34$	$3.88e-37$	$3.89e-37$	$5.16e-38$	$1.32e-36$	–

Table A.107: Comparison of R2 indicator values for different optimizers on the DTLZ2 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	2.500000e-01	2.500000e-01	2.500000e-01	2.500000e-01	2.180308e-01	2.182540e-01	2.181216e-01	2.182367e-01	2.180398e-01	2.182550e-01	2.180291e-01
	avg.	2.500000e-01	2.500000e-01	2.500000e-01	2.500000e-01	2.180308e-01	2.182590e-01	2.181189e-01	2.182360e-01	2.180397e-01	2.182592e-01	2.180293e-01
	min.	2.500000e-01	2.500000e-01	2.499999e-01	2.499998e-01	2.180275e-01	2.181768e-01	2.180863e-01	2.181429e-01	2.180361e-01	2.181700e-01	2.180259e-01
	max.	2.500000e-01	2.500000e-01	2.500000e-01	2.500000e-01	2.180331e-01	2.183614e-01	2.181264e-01	2.183130e-01	2.180429e-01	2.183853e-01	2.180342e-01
	std.	0.000000e+00	0.000000e+00	9.905845e-09	2.230780e-08	1.231029e-06	3.766276e-05	7.908700e-06	3.480767e-05	1.317523e-06	4.473304e-05	1.858377e-06
3D	med.	1.110692e-01	1.110692e-01	1.106991e-01	1.110692e-01	9.479799e-02	9.484754e-02	9.499186e-02	9.513172e-02	9.486747e-02	9.488492e-02	9.375475e-02
	avg.	1.110692e-01	1.110692e-01	1.105158e-01	1.110692e-01	9.479141e-02	9.483848e-02	9.499251e-02	9.515625e-02	9.486189e-02	9.488573e-02	9.375487e-02
	min.	1.110692e-01	1.110692e-01	1.084303e-01	1.110692e-01	9.470296e-02	9.462702e-02	9.485395e-02	9.495681e-02	9.475525e-02	9.466674e-02	9.374729e-02
	max.	1.110692e-01	1.110692e-01	1.110602e-01	1.110692e-01	9.483710e-02	9.498280e-02	9.511901e-02	9.550618e-02	9.490746e-02	9.511215e-02	9.376177e-02
	std.	3.242449e-09	3.242449e-09	5.363816e-04	3.242449e-09	2.989691e-05	6.990199e-05	5.008267e-05	1.023010e-04	3.074648e-05	7.968112e-05	3.219655e-06
4D	med.	6.238390e-02	6.238390e-02	6.199880e-02	6.238390e-02	5.605324e-02	5.602445e-02	5.709061e-02	5.708327e-02	5.725677e-02	5.718694e-02	5.308346e-02
	avg.	6.238390e-02	6.238390e-02	6.195126e-02	6.238350e-02	5.604590e-02	5.601650e-02	5.708553e-02	5.708293e-02	5.725005e-02	5.717479e-02	5.308349e-02
	min.	6.238390e-02	6.238390e-02	6.100319e-02	6.234430e-02	5.585720e-02	5.583301e-02	5.664286e-02	5.661493e-02	5.704553e-02	5.682400e-02	5.307339e-02
	max.	6.238390e-02	6.238390e-02	6.233889e-02	6.238390e-02	5.608181e-02	5.607247e-02	5.733467e-02	5.731110e-02	5.730948e-02	5.725758e-02	5.309646e-02
	std.	0.000000e+00	0.000000e+00	2.711172e-04	3.959969e-06	3.851688e-05	4.330903e-05	1.347790e-04	1.171178e-04	4.171458e-05	6.784018e-05	4.565198e-06
5D	med.	4.068163e-02	4.068163e-02	4.033882e-02	4.068163e-02	3.729399e-02	3.729399e-02	4.025005e-02	4.024632e-02	4.024904e-02	4.024808e-02	3.525871e-02
	avg.	4.068163e-02	4.068163e-02	4.031404e-02	4.068163e-02	3.729930e-02	3.729171e-02	4.022401e-02	4.021545e-02	4.022688e-02	4.020666e-02	3.525849e-02
	min.	4.068163e-02	4.068163e-02	3.961440e-02	4.068157e-02	3.723644e-02	3.723558e-02	3.998636e-02	3.980273e-02	3.987898e-02	3.989176e-02	3.524231e-02
	max.	4.068163e-02	4.068165e-02	4.063473e-02	4.068163e-02	3.730778e-02	3.730368e-02	4.038540e-02	4.038264e-02	4.040906e-02	4.031885e-02	3.527074e-02
	std.	2.340036e-09	3.104409e-09	2.018705e-04	1.014615e-08	7.628398e-06	9.783172e-06	9.487780e-05	1.014769e-04	8.045130e-05	8.523471e-05	5.221627e-06
6D	med.	3.352445e-02	3.372373e-02	3.333117e-02	3.352445e-02	3.411872e-02	3.311628e-02	3.356277e-02	3.352445e-02	3.372373e-02	3.352445e-02	2.432634e-02
	avg.	3.361982e-02	3.351160e-02	3.274296e-02	3.361298e-02	3.475861e-02	3.351589e-02	3.361764e-02	3.360246e-02	3.362378e-02	3.354372e-02	2.432674e-02
	min.	3.329674e-02	2.781405e-02	2.760717e-02	3.287427e-02	2.974131e-02	3.040000e-02	3.329674e-02	3.329674e-02	3.329674e-02	2.783160e-02	2.431928e-02
	max.	3.372373e-02	3.372377e-02	3.742333e-02	3.372373e-02	4.122630e-02	4.159664e-02	3.372373e-02	3.372377e-02	3.372377e-02	3.372377e-02	2.433920e-02
	std.	1.048430e-04	8.245394e-04	2.299253e-03	1.320437e-04	2.868855e-03	2.190757e-03	1.134481e-04	1.242915e-04	1.160987e-04	5.883852e-04	4.217322e-06
7D	med.	3.528776e-02	3.528776e-02	3.535933e-02	3.528776e-02	3.768943e-02	3.436736e-02	3.528776e-02	3.528776e-02	3.528776e-02	3.528776e-02	1.840721e-02
	avg.	3.534022e-02	3.530755e-02	3.617359e-02	3.533822e-02	4.114697e-02	3.565327e-02	3.535099e-02	3.531667e-02	3.534387e-02	3.532752e-02	1.840739e-02
	min.	3.528764e-02	3.525458e-02	3.024216e-02	3.528722e-02	3.430803e-02	3.230975e-02	3.528776e-02	3.528776e-02	3.528776e-02	3.528658e-02	1.840483e-02
	max.	3.570223e-02	3.570223e-02	4.030859e-02	3.579891e-02	5.195072e-02	4.560467e-02	3.570223e-02	3.571675e-02	3.570223e-02	3.570223e-02	1.841123e-02
	std.	1.345992e-04	8.231506e-05	1.739811e-03	1.299498e-04	6.192330e-03	3.663617e-03	1.447316e-04	9.951597e-05	1.359159e-04	1.209872e-04	1.178195e-06
8D	med.	3.108295e-02	3.108295e-02	3.084236e-02	3.108295e-02	3.935602e-02	3.027331e-02	3.108295e-02	3.108295e-02	3.108295e-02	3.108295e-02	1.401876e-02
	avg.	3.106297e-02	3.096384e-02	3.022854e-02	3.106654e-02	3.666891e-02	3.164337e-02	3.107034e-02	3.107574e-02	3.106497e-02	3.106074e-02	1.401879e-02
	min.	3.088313e-02	2.513597e-02	2.351880e-02	3.088313e-02	2.974694e-02	2.839520e-02	3.074329e-02	3.088313e-02	3.088313e-02	3.088313e-02	1.401249e-02
	max.	3.108295e-02	3.136058e-02	3.470606e-02	3.136057e-02	4.728679e-02	4.030218e-02	3.136058e-02	3.136058e-02	3.108295e-02	3.108298e-02	1.402618e-02
	std.	6.024800e-05	8.157750e-04	2.619905e-03	6.528687e-05	5.393034e-03	3.282372e-03	6.483200e-05	5.236945e-05	5.747293e-05	6.279756e-05	2.637193e-06
9D	med.	2.766842e-02	2.766842e-02	2.728554e-02	2.766842e-02	2.894477e-02	2.826892e-02	2.766842e-02	2.766842e-02	2.766842e-02	2.766842e-02	1.103101e-02
	avg.	2.767586e-02	2.756479e-02	2.591054e-02	2.762287e-02	3.175329e-02	2.890363e-02	2.768193e-02	2.766935e-02	2.767777e-02	2.766932e-02	1.103151e-02
	min.	2.766842e-02	2.225750e-02	1.727147e-02	2.217458e-02	2.687202e-02	2.539048e-02	2.766842e-02	2.766842e-02	2.766842e-02	2.766672e-02	1.101944e-02
	max.	2.776139e-02	2.783983e-02	2.909330e-02	2.776633e-02	4.134631e-02	3.463259e-02	2.783983e-02	2.776139e-02	2.776634e-02	2.776139e-02	1.104661e-02
	std.	2.534919e-05	7.616205e-04	2.429686e-03	5.510608e-03	4.627953e-03	2.892479e-03	3.862328e-05	9.296960e-06	2.818461e-05	9.301351e-06	5.468121e-06
10D	med.	2.491999e-02	2.491999e-02	2.170473e-02	2.491999e-02	2.954966e-02	2.450914e-02	2.491999e-02	2.491999e-02	2.491999e-02	2.491999e-02	8.878397e-03
	avg.	2.492868e-02	2.478166e-02	2.227665e-02	2.484747e-02	2.905554e-02	2.564697e-02	2.493465e-02	2.492418e-02	2.493496e-02	2.487509e-02	8.879348e-03
	min.	2.491999e-02	2.006167e-02	1.708038e-02	1.999737e-02	2.349235e-02	2.213010e-02	2.491999e-02	2.491999e-02	2.491999e-02	2.001535e-02	8.864845e-03
	max.	2.515995e-02	2.512946e-02	2.727548e-02	2.512945e-02	3.768290e-02	3.229353e-02	2.512945e-02	2.512945e-02	2.515995e-02	2.512945e-02	8.897091e-03
	std.	4.283659e-05	8.278492e-04	2.239516e-03	6.555059e-04	4.167112e-03	2.492782e-03	5.371159e-05	2.947180e-05	5.490442e-05	4.917668e-04	7.986777e-06

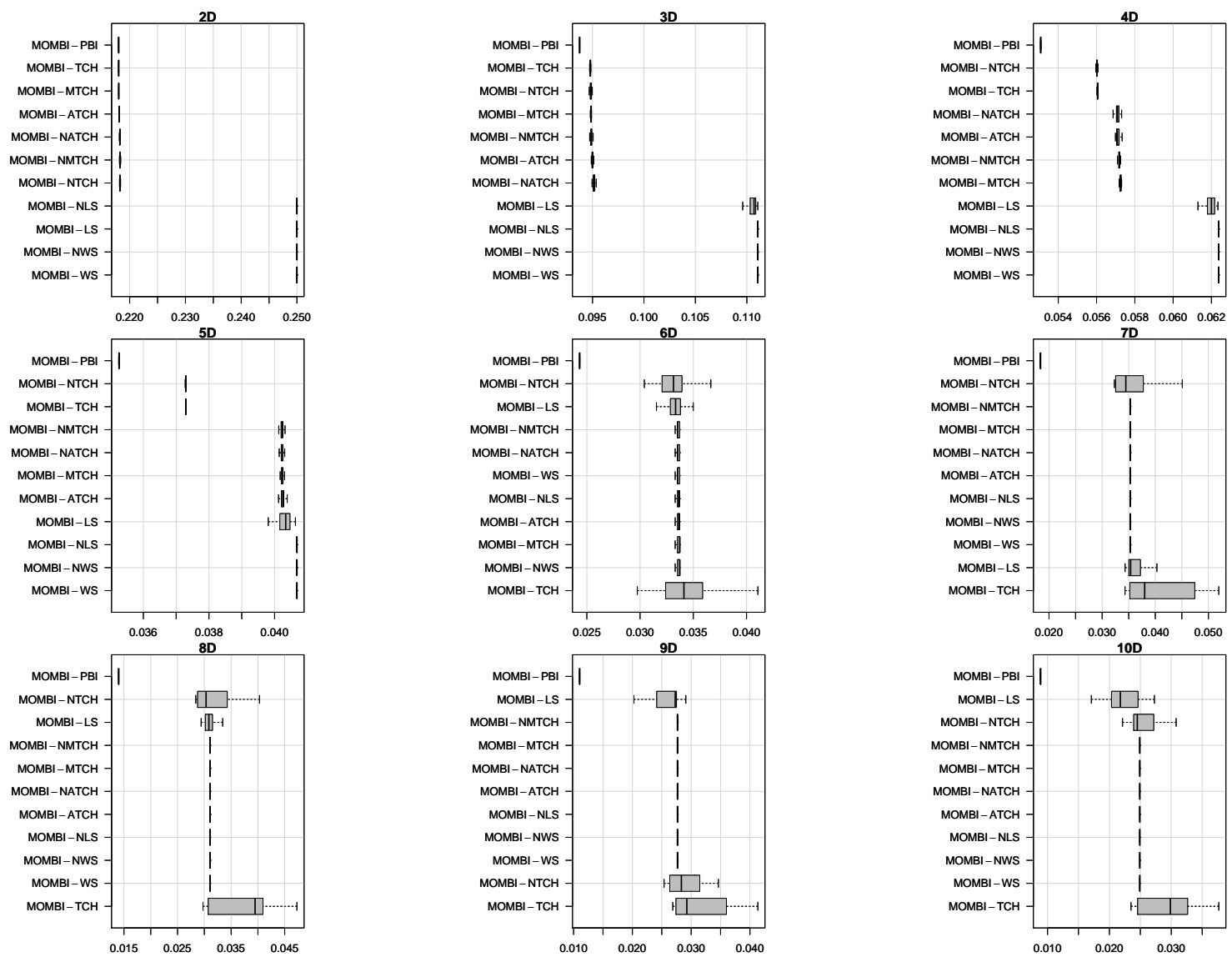


Figure A.268: Box-plot of the R2 indicator values for different optimizers on the DTLZ2 test problem.

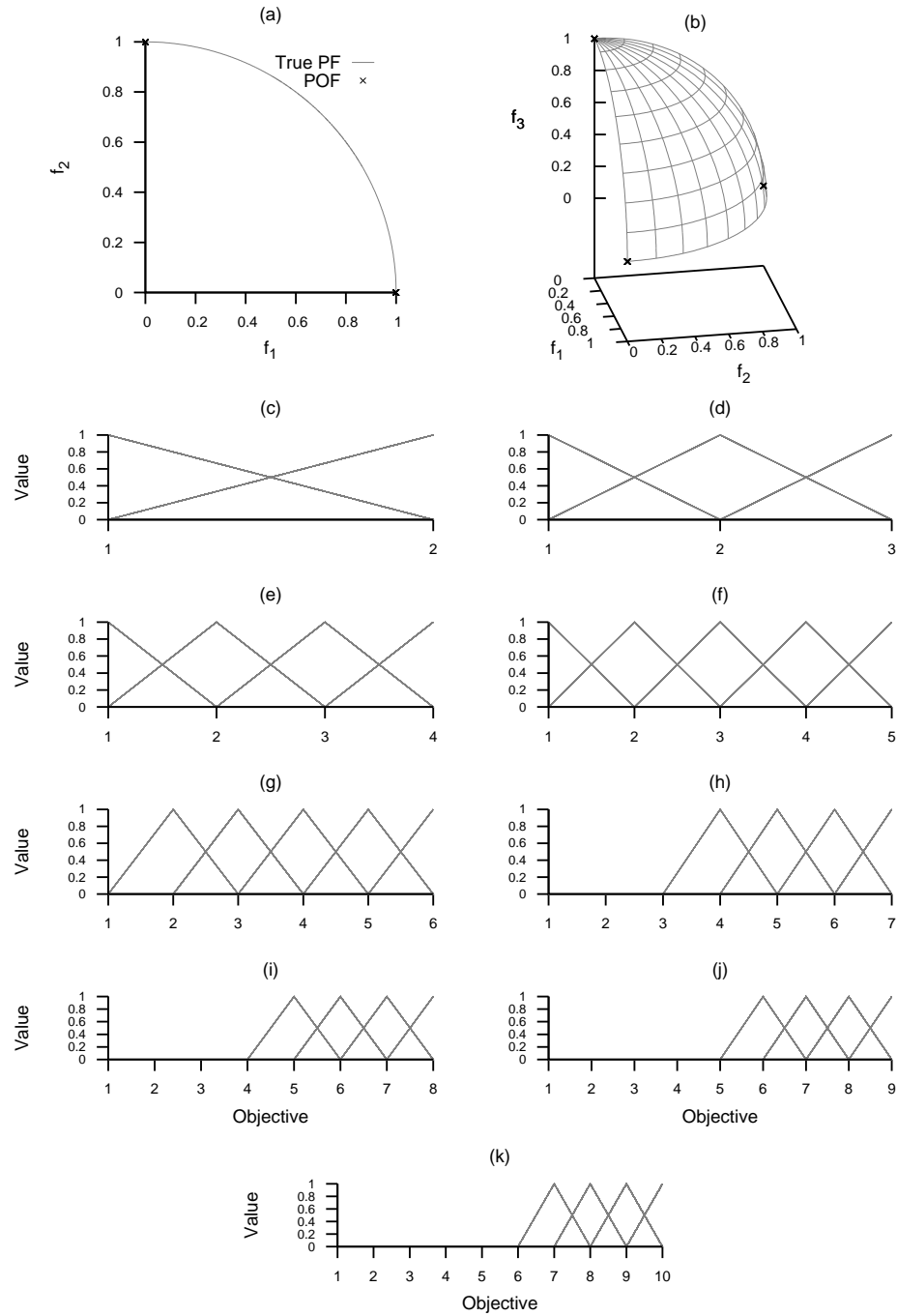


Figure A.269: Plots of the approximations obtained by MOMBI-WS from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

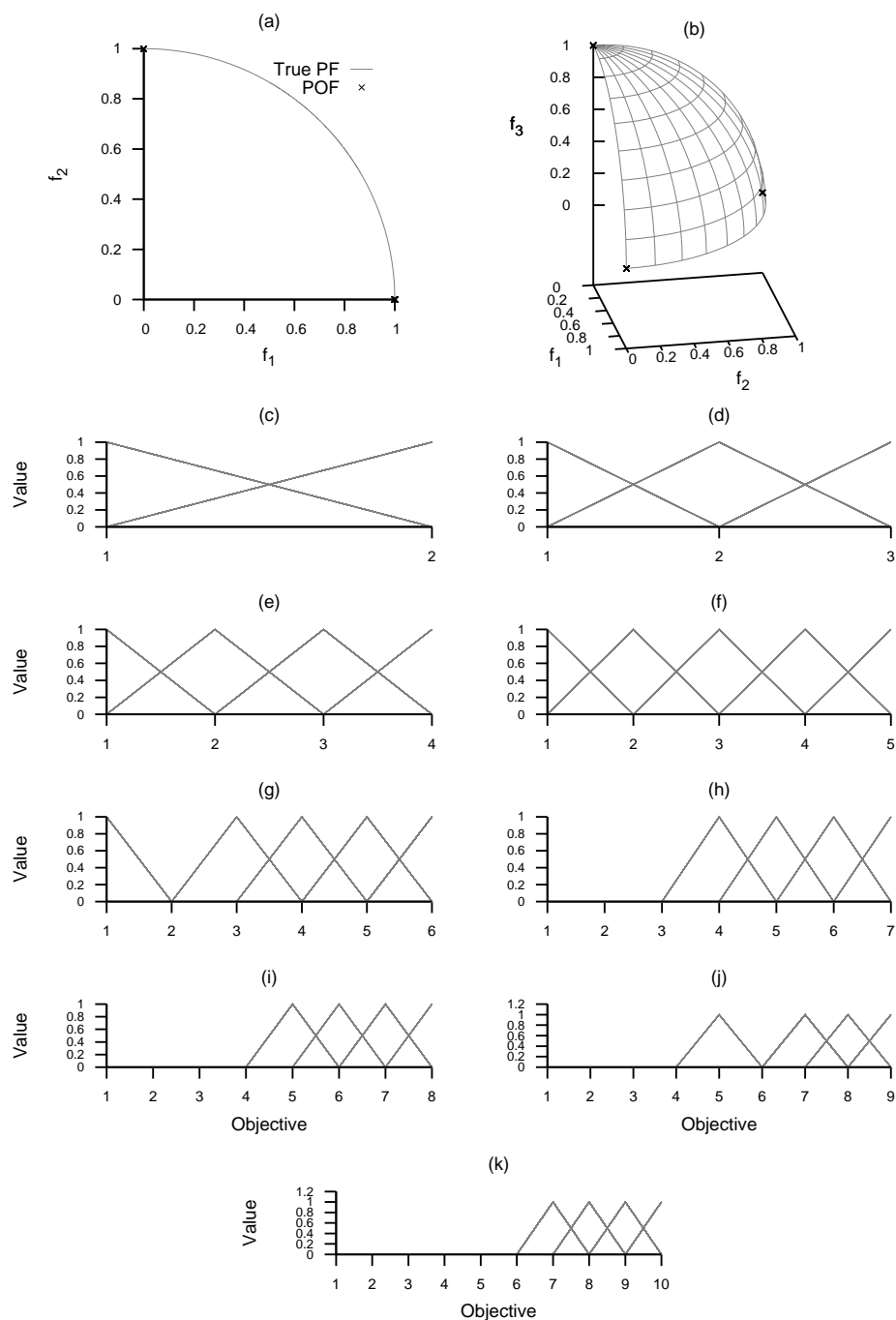


Figure A.270: Plots of the approximations obtained by MOMBI-NWS from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



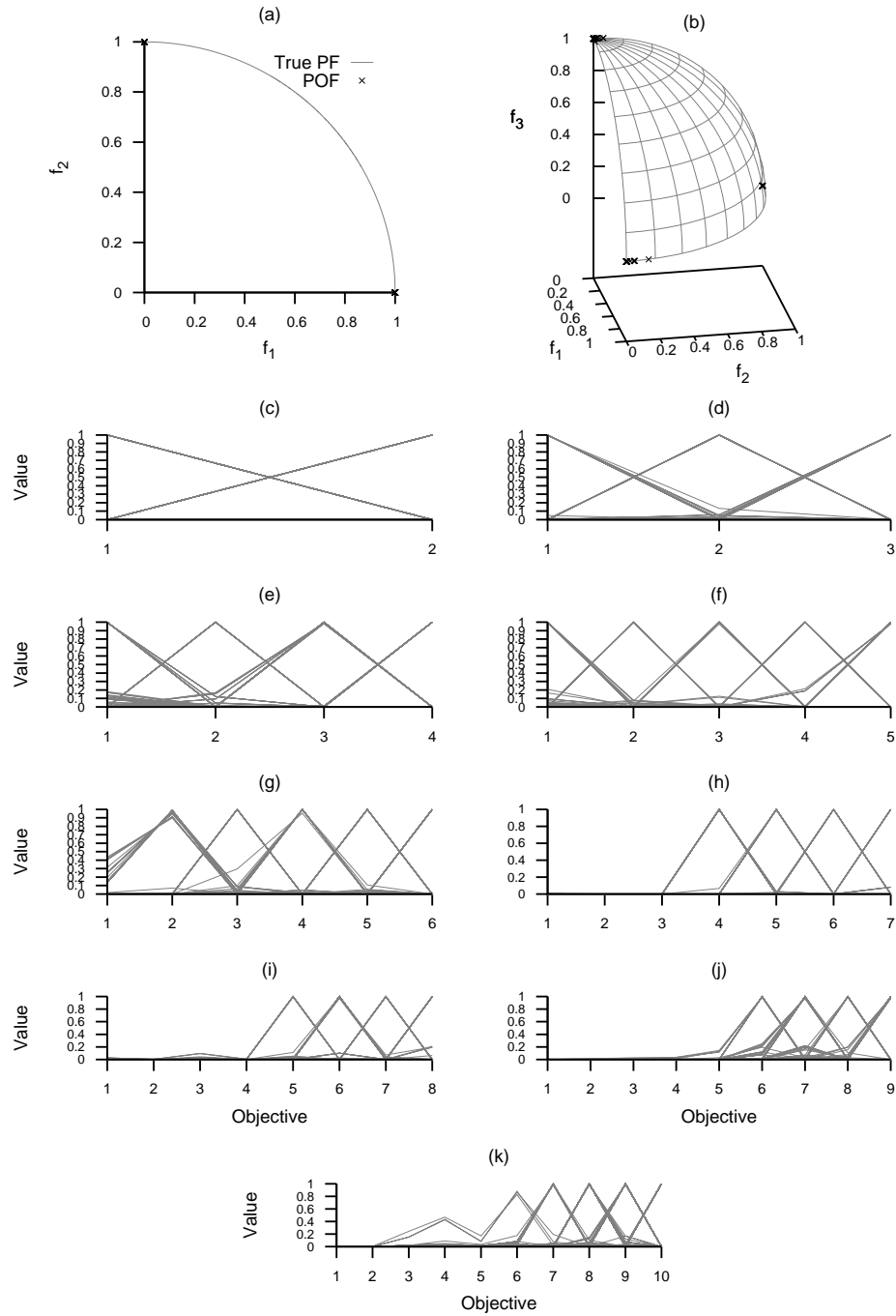


Figure A.271: Plots of the approximations obtained by MOMBI-LS from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

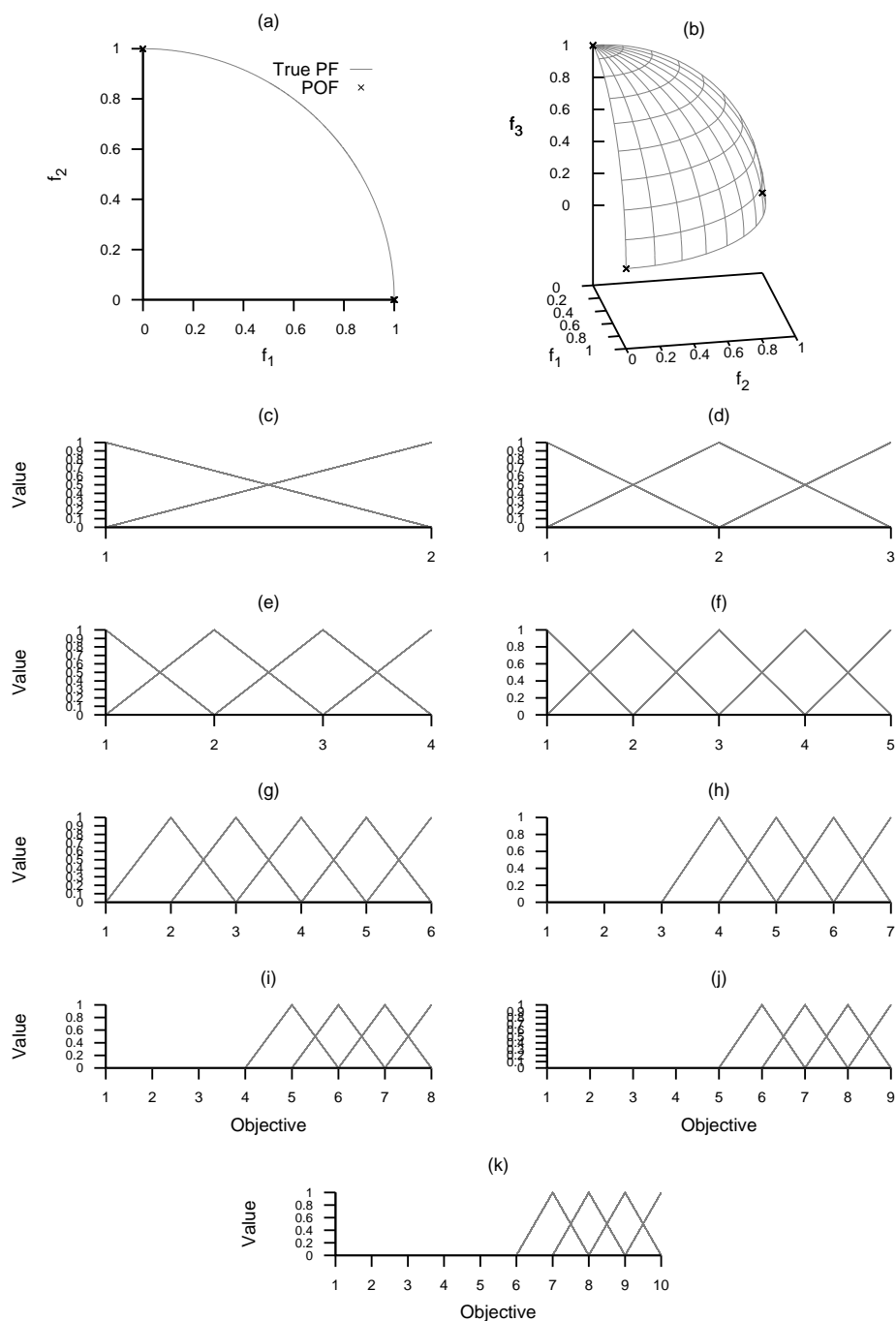


Figure A.272: Plots of the approximations obtained by MOMBI-NLS from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

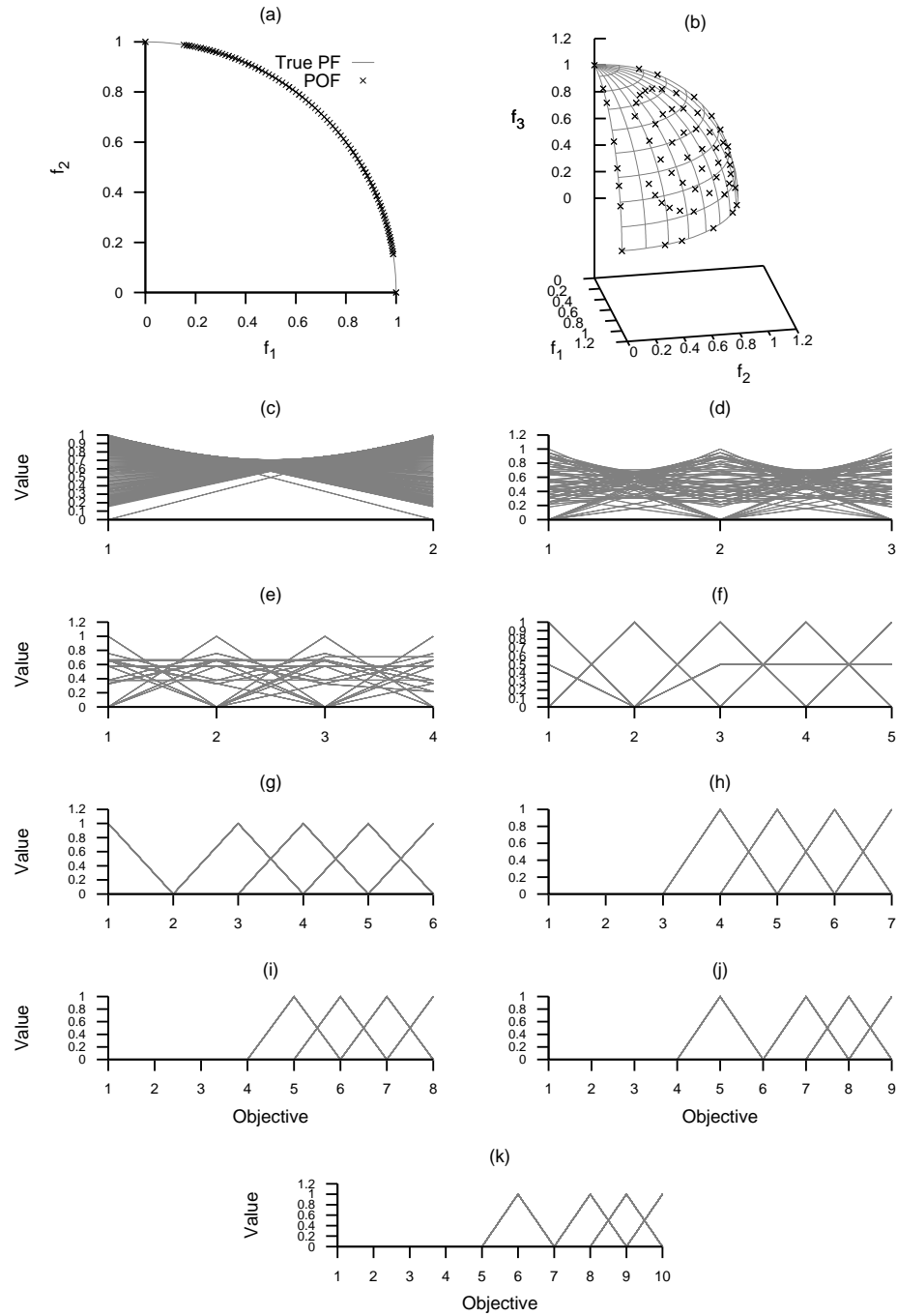


Figure A.273: Plots of the approximations obtained by MOMBI-ATCH from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

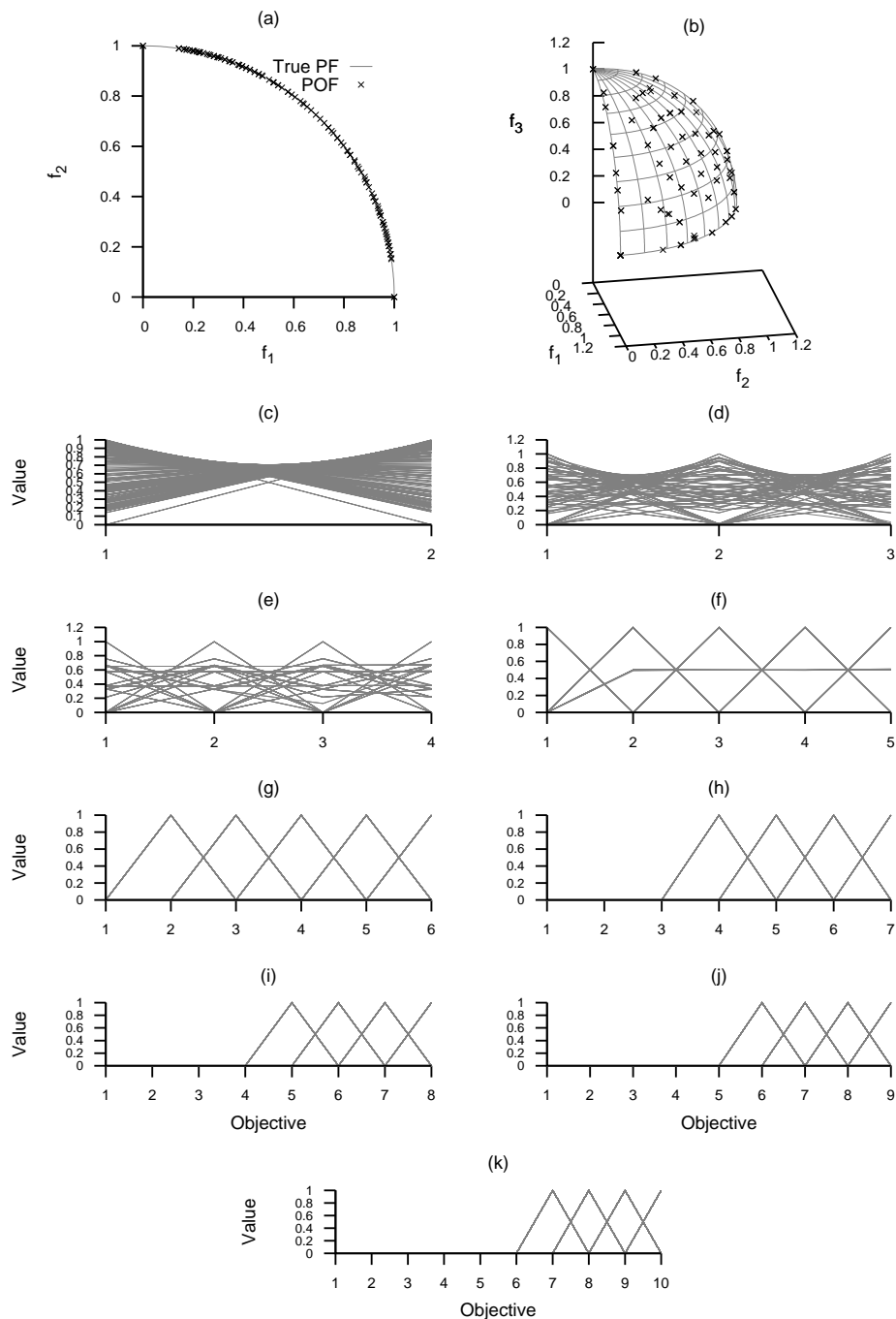


Figure A.274: Plots of the approximations obtained by MOMBI-NATCH from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

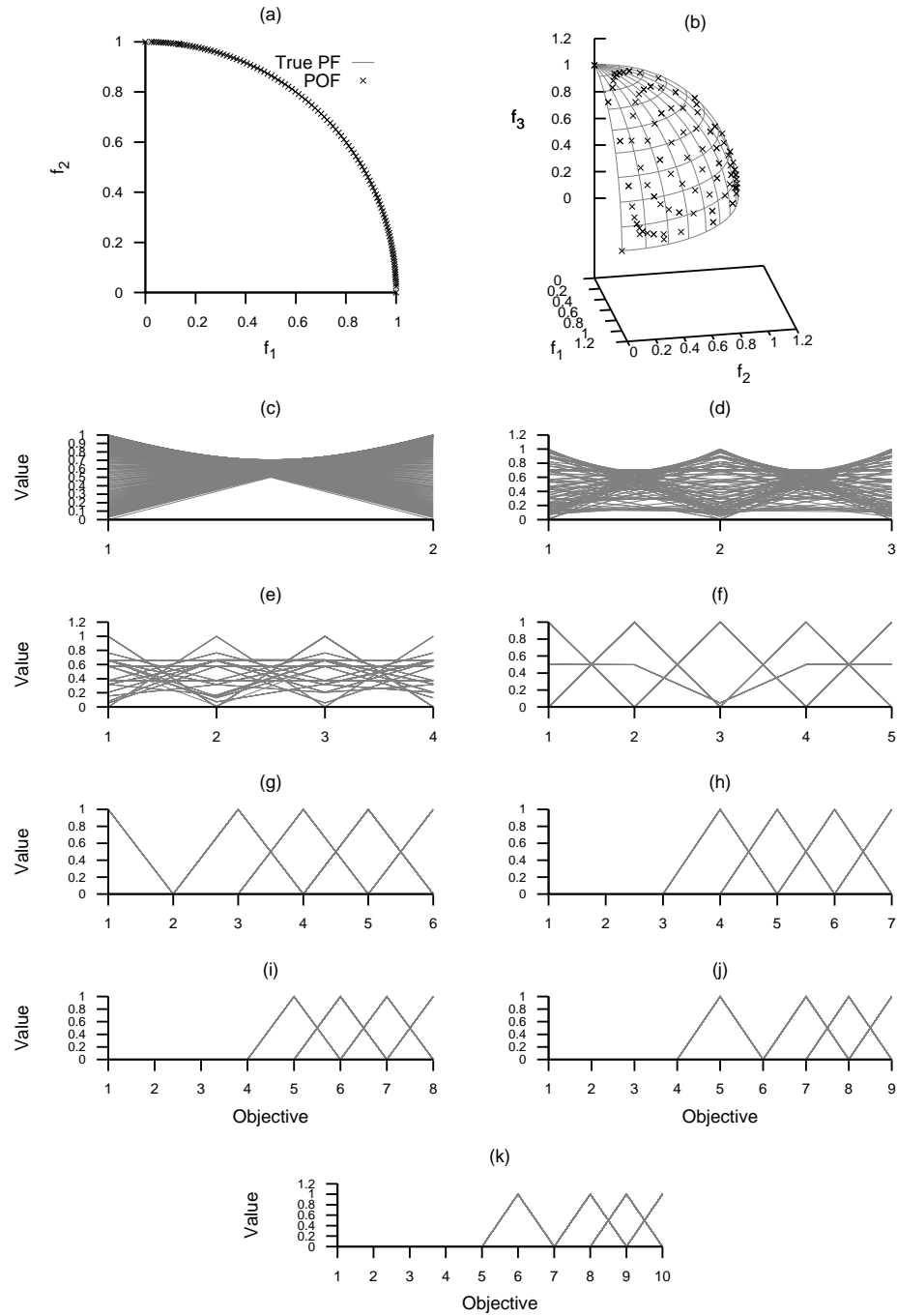


Figure A.275: Plots of the approximations obtained by MOMBI-MTCH from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

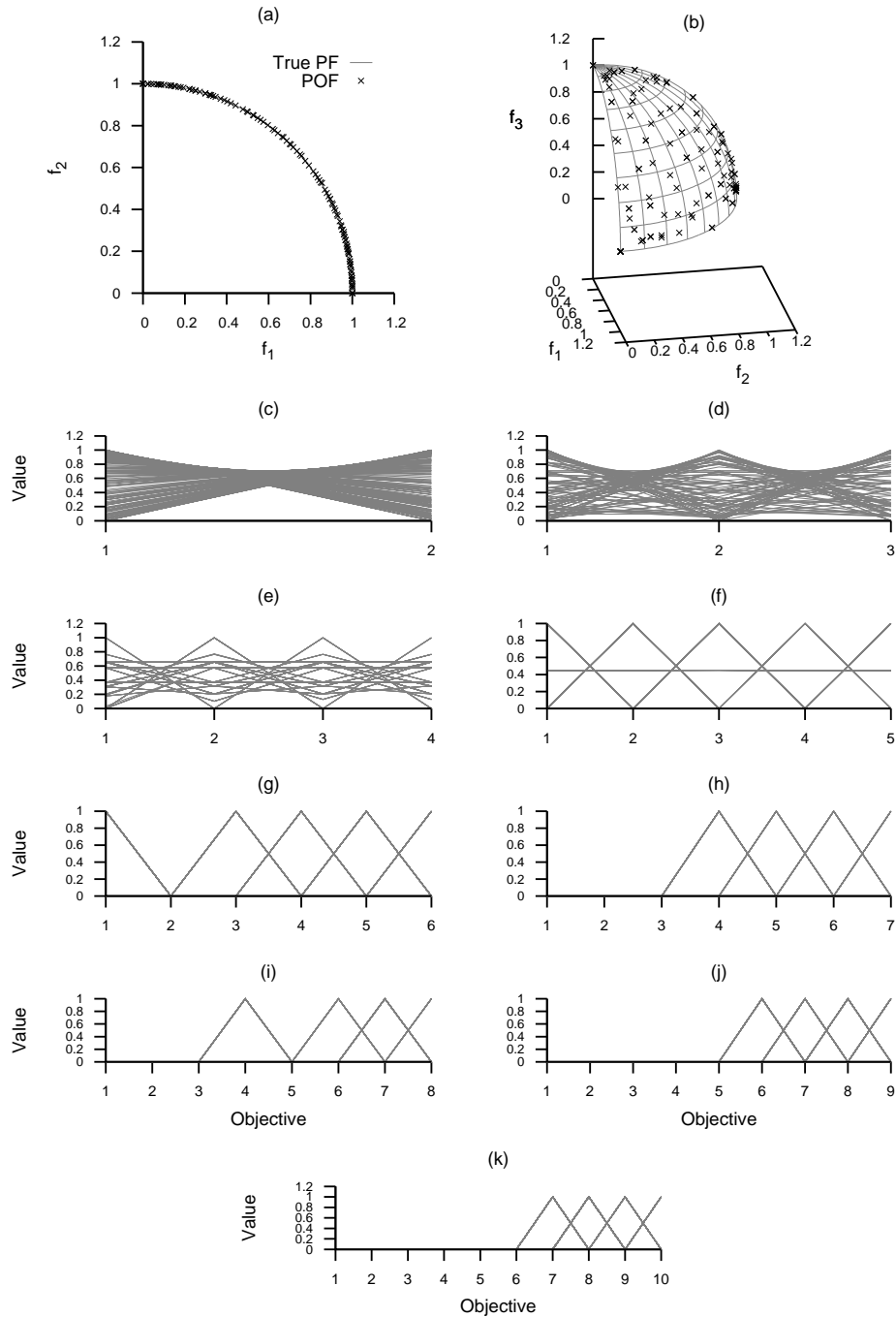


Figure A.276: Plots of the approximations obtained by MOMBI-NMTCH from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.108: Comparison of hypervolume indicator values for different optimizers on the DTLZ3 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	4.799082e + 01	4.799241e + 01	4.799482e + 01	4.799519e + 01	4.820384e + 01	4.820344e + 01	4.820322e + 01	4.820232e + 01	4.820368e + 01	4.820323e + 01	4.819496e + 01
	avg.	4.795878e + 01	4.799004e + 01	4.799427e + 01	4.799475e + 01	4.820280e + 01	4.820249e + 01	4.820103e + 01	4.819875e + 01	4.817871e + 01	4.817841e + 01	4.817025e + 01
	min.	4.495421e + 01	4.795768e + 01	4.797243e + 01	4.796563e + 01	4.819307e + 01	4.818562e + 01	4.815828e + 01	4.798495e + 01	4.582866e + 01	4.582997e + 01	4.583700e + 01
	max.	4.799873e + 01	4.799924e + 01	4.801256e + 01	4.800605e + 01	4.820880e + 01	4.820806e + 01	4.820863e + 01	4.820867e + 01	4.820967e + 01	4.820813e + 01	4.820817e + 01
	std.	3.035776e - 01	7.305578e - 03	5.691895e - 03	5.351674e - 03	4.143223e - 03	3.427684e - 03	6.839577e - 03	2.233186e - 02	2.374497e - 01	2.372643e - 01	2.358605e - 01
3D	med.	3.419811e + 02	3.419819e + 02	3.420086e + 02	3.419929e + 02	3.423685e + 02	3.423669e + 02	3.423816e + 02	3.423772e + 02	3.423684e + 02	3.423696e + 02	3.423670e + 02
	avg.	3.419057e + 02	3.419270e + 02	3.420122e + 02	3.419912e + 02	3.423648e + 02	3.423215e + 02	3.423801e + 02	3.423761e + 02	3.423675e + 02	3.423262e + 02	3.423366e + 02
	min.	3.348838e + 02	3.369313e + 02	3.419637e + 02	3.419702e + 02	3.422668e + 02	3.378986e + 02	3.423403e + 02	3.423481e + 02	3.423371e + 02	3.380332e + 02	3.398713e + 02
	max.	3.419977e + 02	3.419982e + 02	3.420823e + 02	3.420027e + 02	3.423856e + 02	3.423903e + 02	3.423981e + 02	3.423946e + 02	3.423854e + 02	3.423913e + 02	3.424061e + 02
	std.	7.095474e - 01	5.049417e - 01	1.665570e - 02	6.989885e - 03	1.794653e - 02	4.469076e - 01	9.516855e - 03	8.819773e - 03	1.056119e - 02	4.337496e - 01	2.513773e - 01
4D	med.	2.399967e + 03	2.399962e + 03	2.400022e + 03	2.399980e + 03	2.400388e + 03	2.400288e + 03	2.400339e + 03	2.400327e + 03	2.400290e + 03	2.400322e + 03	2.400411e + 03
	avg.	2.399805e + 03	2.399650e + 03	2.400027e + 03	2.399977e + 03	2.400294e + 03	2.400101e + 03	2.400335e + 03	2.400158e + 03	2.400260e + 03	2.400313e + 03	2.399870e + 03
	min.	2.384453e + 03	2.384707e + 03	2.399953e + 03	2.399918e + 03	2.391935e + 03	2.389409e + 03	2.400272e + 03	2.395999e + 03	2.397597e + 03	2.400190e + 03	2.371101e + 03
	max.	2.399995e + 03	2.399997e + 03	2.400115e + 03	2.400001e + 03	2.400423e + 03	2.400414e + 03	2.400380e + 03	2.400428e + 03	2.400346e + 03	2.400411e + 03	2.400530e + 03
	std.	1.550968e + 00	2.139998e + 00	2.642184e - 02	1.569584e - 02	8.456463e - 01	1.400188e + 00	2.449368e - 02	7.987952e - 01	2.705843e - 01	5.080251e - 02	3.087098e + 00
5D	med.	1.680596e + 04	1.680589e + 04	1.680603e + 04	1.680594e + 04	1.680644e + 04	1.680598e + 04	1.680604e + 04	1.680595e + 04	1.680598e + 04	1.680597e + 04	1.680570e + 04
	avg.	1.680559e + 04	1.678687e + 04	1.680603e + 04	1.680594e + 04	1.680477e + 04	1.680442e + 04	1.680602e + 04	1.680470e + 04	1.680602e + 04	1.680455e + 04	1.680108e + 04
	min.	1.677342e + 04	1.576592e + 04	1.680596e + 04	1.680578e + 04	1.676760e + 04	1.677485e + 04	1.680581e + 04	1.677279e + 04	1.680589e + 04	1.677232e + 04	1.659717e + 04
	max.	1.680599e + 04	1.680599e + 04	1.680612e + 04	1.680599e + 04	1.680652e + 04	1.680611e + 04	1.680613e + 04	1.680605e + 04	1.680613e + 04	1.680603e + 04	1.680651e + 04
	std.	3.252246e + 00	1.092679e + 02	2.839840e - 02	3.997056e - 02	6.237501e + 00	6.455193e + 00	4.811829e - 02	4.834826e + 00	4.101287e - 02	6.146566e + 00	2.185032e + 01
6D	med.	1.176416e + 05	1.176412e + 05	1.176423e + 05	1.176416e + 05	1.176435e + 05	1.176418e + 05	1.176417e + 05	1.176415e + 05	1.176417e + 05	1.176416e + 05	1.176441e + 05
	avg.	1.176417e + 05	1.172826e + 05	1.176420e + 05	1.176394e + 05	1.176004e + 05	1.175772e + 05	1.176379e + 05	1.176257e + 05	1.176394e + 05	1.176338e + 05	1.176270e + 05
	min.	1.176393e + 05	9.082952e + 04	1.175463e + 05	1.175952e + 05	1.144103e + 05	1.160869e + 05	1.174200e + 05	1.174063e + 05	1.173946e + 05	1.173946e + 05	1.171821e + 05
	max.	1.176479e + 05	1.176418e + 05	1.176480e + 05	1.176420e + 05	1.176456e + 05	1.176432e + 05	1.176420e + 05	1.176420e + 05	1.176420e + 05	1.176419e + 05	1.176480e + 05
	std.	1.146474e + 00	2.700591e + 03	9.887412e + 00	9.661623e + 00	3.240174e + 02	2.146278e + 02	2.660562e + 01	4.695818e + 01	2.210700e + 01	3.447875e + 01	6.585741e + 01
7D	med.	8.231899e + 05	8.231848e + 05	8.231905e + 05	8.231886e + 05	8.231932e + 05	8.231950e + 05	8.231895e + 05	8.231840e + 05	8.231900e + 05	8.231870e + 05	8.235313e + 05
	avg.	8.230719e + 05	8.229728e + 05	8.231311e + 05	8.228939e + 05	8.227246e + 05	8.229331e + 05	8.231891e + 05	8.227822e + 05	8.231465e + 05	8.228046e + 05	8.231568e + 05
	min.	8.179256e + 05	8.178407e + 05	8.179447e + 05	8.210018e + 05	8.150565e + 05	8.168645e + 05	8.231685e + 05	8.179583e + 05	8.211159e + 05	8.178068e + 05	8.028747e + 05
	max.	8.231989e + 05	8.231984e + 05	8.234970e + 05	8.231990e + 05	8.232328e + 05	8.232536e + 05	8.231984e + 05	8.231980e + 05	8.232026e + 05	8.231981e + 05	8.235414e + 05
	std.	6.330562e + 02	1.031842e + 03	5.288416e + 02	7.305593e + 02	1.505674e + 03	1.152114e + 03	6.280731e + 00	1.009494e + 03	2.910740e + 02	1.005506e + 03	2.215570e + 03
8D	med.	5.762311e + 06	5.762236e + 06	5.762468e + 06	5.762323e + 06	5.762343e + 06	5.762394e + 06	5.762337e + 06	5.762292e + 06	5.762329e + 06	5.762294e + 06	5.764650e + 06
	avg.	5.761936e + 06	5.753002e + 06	5.762671e + 06	5.761575e + 06	5.759918e + 06	5.757651e + 06	5.762313e + 06	5.760418e + 06	5.762312e + 06	5.760070e + 06	5.763522e + 06
	min.	5.726319e + 06	5.566642e + 06	5.760872e + 06	5.725452e + 06	5.711206e + 06	5.600515e + 06	5.761899e + 06	5.724946e + 06	5.762034e + 06	5.725257e + 06	5.720126e + 06
	max.	5.762390e + 06	5.762380e + 06	5.764477e + 06	5.762393e + 06	5.763272e + 06	5.762784e + 06	5.762393e + 06	5.762445e + 06	5.762389e + 06	5.762388e + 06	5.764781e + 06
	std.	3.598531e + 03	2.907924e + 04	7.586427e + 02	5.169814e + 03	9.037175e + 03	1.852697e + 04	7.857090e + 01	8.083216e + 03	6.654576e + 01	8.717984e + 03	5.313367e + 03
9D	med.	4.033610e + 07	4.033574e + 07	4.033848e + 07	4.033615e + 07	4.033671e + 07	4.033679e + 07	4.033625e + 07	4.033600e + 07	4.033632e + 07	4.033588e + 07	4.035225e + 07
	avg.	4.032246e + 07	4.029450e + 07	4.034219e + 07	4.032826e + 07	4.019237e + 07	4.025289e + 07	4.033367e + 07	4.032287e + 07	4.033119e + 07	4.032803e + 07	4.026886e + 07
	min.	4.006345e + 07	4.002807e + 07	4.033069e + 07	4.006722e + 07	3.105052e + 07	3.628453e + 07	4.008269e + 07	4.007352e + 07	4.008104e + 07	4.007417e + 07	3.238091e + 07
	max.	4.035093e + 07	4.033668e + 07	4.035260e + 07	4.033687e + 07	4.034588e + 07	4.034589e + 07	4.033674e + 07	4.033664e + 07	4.035112e + 07	4.033675e + 07	4.035346e + 07
	std.	5.697450e + 04	9.447178e + 04	6.378796e + 03	4.470951e + 04	9.452982e + 05	4.263875e + 05	2.535516e + 04	5.638471e + 04	3.589405e + 04	4.408532e + 04	7.968960e + 05
10D	med.	2.823503e + 08	2.823394e + 08	2.824414e + 08	2.823528e + 08	2.823763e + 08	2.823590e + 08	2.823531e + 08	2.823507e + 08	2.823531e + 08	2.823489e + 08	2.824631e + 08
	avg.	2.820403e + 08	2.811227e + 08	2.824035e + 08	2.822915e + 08	2.817766e + 08	2.816348e + 08	2.823342e + 08	2.819401e + 08	2.822979e + 08	2.820811e + 08	2.824412e + 08
	min.	2.728391e + 08	2.515892e + 08	2.816672e + 08	2.805069e + 08	2.440098e + 08	2.577412e + 08	2.805851e + 08	2.717070e + 08	2.805851e + 08	2.728538e + 08	2.815578e + 08
	max.	2.824572e + 08	2.823556e + 08	2.824702e + 08	2.823619e + 08	2.824434e + 08	2.823865e + 08	2.823571e + 08	2.823568e + 08	2.823570e + 08	2.823574e + 08	2.824725e + 08
	std.	1.256733e + 06	3.667446e + 06	1.072426e + 05	3.140725e + 05	3.851130e + 06	2.842990e + 06	1.767389e + 05	1.868344e + 06	3.075061e + 05	1.064537e + 06	1.007577e + 05

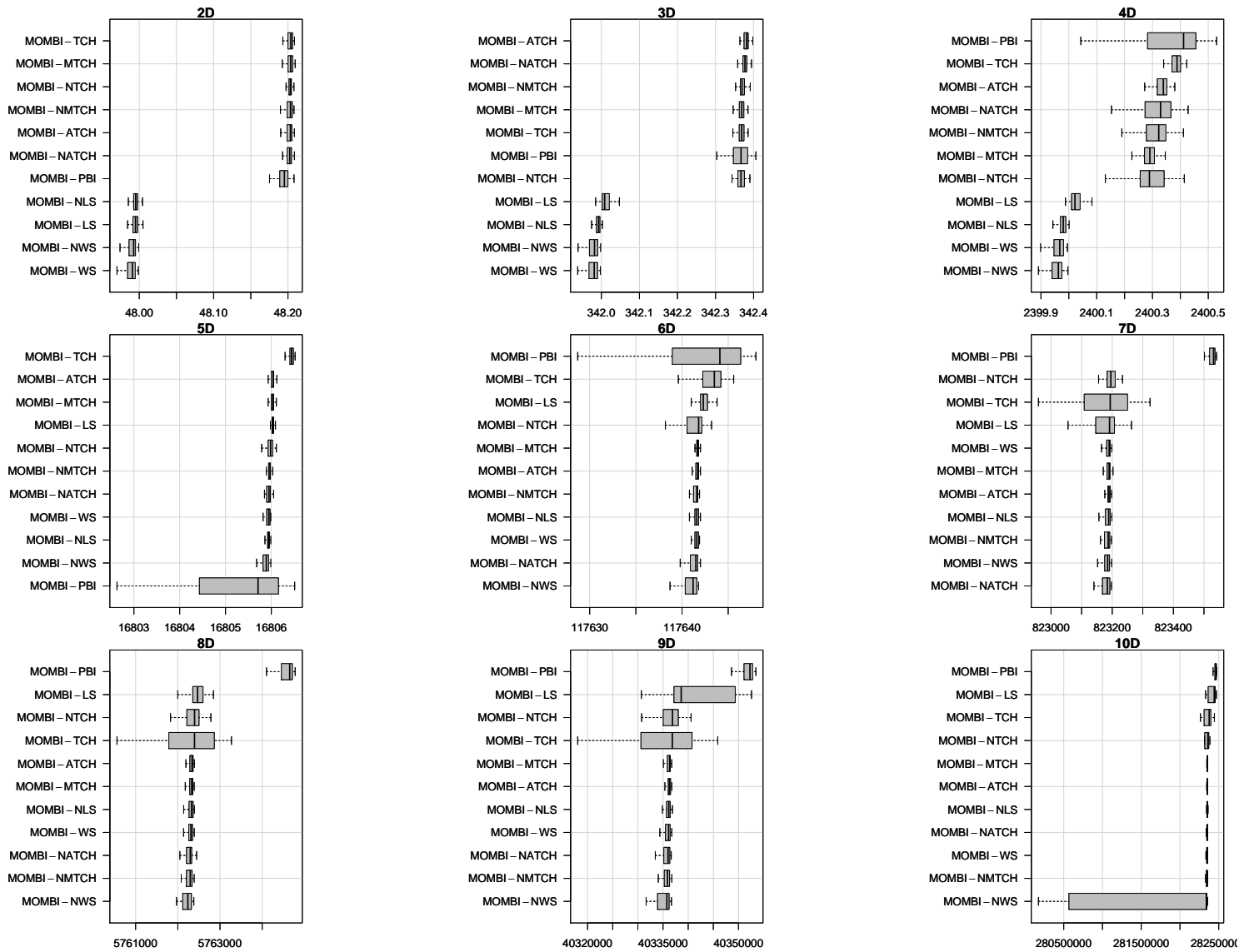


Figure A.277: Box-plot of the hypervolume indicator values for different optimizers on the DTLZ3 test problem.



Table A.109: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ3 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
2D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$8.36e-08$	$6.18e-06$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$1.08e-09$	$1.34e-07$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	> 0.05	$4.23e-02$	$5.82e-03$	> 0.05	> 0.05	$3.91e-16$
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	–	> 0.05	$4.66e-02$	> 0.05	> 0.05	$6.05e-16$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$1.52e-11$
MOMBI-NATCH	$1.16e-33$	$1.43e-33$	$2.16e-33$	$2.30e-33$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	$1.47e-10$
MOMBI-MTCH	$2.44e-33$	$2.51e-33$	$2.51e-33$	$2.51e-33$	> 0.05	> 0.05	> 0.05	$6.94e-03$	–	> 0.05	$3.28e-14$
MOMBI-NMTCH	$2.44e-33$	$2.51e-33$	$2.51e-33$	$2.51e-33$	> 0.05	> 0.05	> 0.05	$2.85e-02$	> 0.05	–	$2.46e-13$
MOMBI-PBI	$2.44e-33$	$2.51e-33$	$2.51e-33$	$2.51e-33$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
3D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.54e-30$	$2.41e-30$	–	$5.76e-25$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$4.17e-14$	$3.24e-13$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	$2.43e-33$	$2.43e-33$	$2.51e-33$	$2.50e-33$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$5.05e-18$	$3.51e-17$	–	$1.63e-04$	$3.10e-17$	$1.44e-14$	$4.38e-06$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$3.18e-11$	$1.67e-10$	> 0.05	–	$4.62e-10$	$1.23e-07$	$1.28e-03$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$2.43e-33$	$2.43e-33$	$2.50e-33$	$2.50e-33$	> 0.05	$1.44e-02$	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	$2.43e-33$	$2.43e-33$	$2.51e-33$	$2.50e-33$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
4D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$2.11e-33$	$1.52e-33$	–	$6.08e-32$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$3.78e-07$	$1.24e-10$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$2.41e-33$	$2.34e-33$	$2.13e-32$	$2.47e-33$	–	$1.96e-21$	$4.07e-21$	$9.08e-15$	$9.86e-29$	$2.17e-20$	> 0.05
MOMBI-NTCH	$4.35e-32$	$4.10e-32$	$4.61e-32$	$4.58e-32$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-ATCH	$1.27e-34$	$1.27e-34$	$1.27e-34$	$1.26e-34$	> 0.05	$1.70e-07$	–	$4.01e-02$	$6.74e-23$	$1.10e-03$	> 0.05
MOMBI-NATCH	$1.18e-29$	$1.05e-29$	$1.31e-29$	$1.31e-29$	> 0.05	$1.54e-02$	> 0.05	–	$1.48e-05$	> 0.05	> 0.05
MOMBI-MTCH	$2.42e-33$	$2.34e-33$	$2.49e-33$	$2.48e-33$	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$1.27e-34$	$1.27e-34$	$1.27e-34$	$1.27e-34$	> 0.05	$2.58e-02$	> 0.05	> 0.05	$7.79e-06$	–	> 0.05
MOMBI-PBI	$4.19e-20$	$3.09e-20$	$2.09e-19$	$7.66e-20$	$1.79e-02$	$4.52e-08$	$3.46e-06$	$8.63e-07$	$1.64e-09$	$3.94e-07$	–

Table A.110: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ3 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
5D											
MOMBI-WS	–	$7.74e-07$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.44e-03$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$3.06e-33$	$3.66e-34$	–	$1.10e-33$	$> 0.05$	$1.77e-10$	$> 0.05$	$2.18e-29$	$> 0.05$	$3.90e-30$	$2.44e-06$
MOMBI-NLS	$> 0.05$	$2.97e-08$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$4.85e-04$
MOMBI-TCH	$4.03e-23$	$1.64e-24$	$4.58e-22$	$5.93e-23$	–	$4.27e-23$	$3.21e-22$	$2.38e-23$	$3.47e-22$	$2.44e-23$	$3.56e-21$
MOMBI-NTCH	$1.77e-07$	$5.27e-14$	$> 0.05$	$2.29e-07$	$> 0.05$	–	$> 0.05$	$3.80e-06$	$> 0.05$	$5.32e-04$	$1.91e-03$
MOMBI-ATCH	$3.41e-26$	$5.54e-30$	$> 0.05$	$7.77e-27$	$> 0.05$	$3.27e-07$	–	$4.10e-23$	$> 0.05$	$9.93e-22$	$4.33e-06$
MOMBI-NATCH	$> 0.05$	$1.37e-06$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$2.61e-02$
MOMBI-MTCH	$3.78e-27$	$1.05e-30$	$> 0.05$	$2.19e-27$	$> 0.05$	$3.80e-07$	$> 0.05$	$1.45e-23$	–	$1.85e-22$	$4.59e-06$
MOMBI-NMTCH	$1.65e-04$	$1.10e-13$	$> 0.05$	$1.99e-05$	$> 0.05$	$> 0.05$	$> 0.05$	$6.68e-03$	$> 0.05$	–	$3.84e-03$
MOMBI-PBI	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–
6D											
MOMBI-WS	–	$3.86e-10$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$9.44e-04$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$8.55e-18$	$1.50e-24$	–	$6.21e-20$	$> 0.05$	$2.41e-10$	$3.62e-19$	$1.17e-22$	$8.87e-19$	$3.98e-21$	$> 0.05$
MOMBI-NLS	$> 0.05$	$3.76e-10$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$4.83e-04$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-TCH	$1.72e-10$	$2.00e-15$	$2.99e-03$	$1.75e-12$	–	$1.33e-13$	$3.38e-12$	$2.43e-14$	$5.36e-12$	$4.02e-13$	$> 0.05$
MOMBI-NTCH	$> 0.05$	$8.53e-07$	$> 0.05$	$3.77e-02$	$> 0.05$	–	$> 0.05$	$3.46e-04$	$> 0.05$	$5.74e-03$	$> 0.05$
MOMBI-ATCH	$1.14e-02$	$7.09e-14$	$> 0.05$	$4.81e-02$	$> 0.05$	$> 0.05$	–	$1.71e-06$	$> 0.05$	$8.22e-04$	$> 0.05$
MOMBI-NATCH	$> 0.05$	$4.97e-03$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$1.18e-03$	$6.41e-16$	$> 0.05$	$8.13e-03$	$> 0.05$	$> 0.05$	$> 0.05$	$6.75e-08$	–	$7.89e-05$	$> 0.05$
MOMBI-NMTCH	$> 0.05$	$1.55e-06$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$2.82e-02$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$1.36e-03$	$1.29e-07$	$> 0.05$	$9.65e-05$	$7.27e-03$	$7.61e-07$	$2.61e-04$	$6.85e-07$	$4.27e-04$	$1.91e-05$	–
7D											
MOMBI-WS	–	$9.57e-04$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$2.43e-04$	$> 0.05$	$2.95e-03$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$> 0.05$	$> 0.05$	–	$2.13e-02$	$> 0.05$	$> 0.05$	$> 0.05$	$2.50e-03$	$> 0.05$	$7.81e-03$	$> 0.05$
MOMBI-NLS	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$1.19e-02$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-TCH	$> 0.05$	$> 0.05$	$> 0.05$	$4.51e-02$	–	$> 0.05$	$> 0.05$	$1.54e-02$	$> 0.05$	$2.91e-02$	$> 0.05$
MOMBI-NTCH	$2.60e-05$	$4.34e-08$	$2.20e-02$	$1.42e-06$	$> 0.05$	–	$1.96e-04$	$3.40e-09$	$5.35e-05$	$3.52e-08$	$> 0.05$
MOMBI-ATCH	$> 0.05$	$9.41e-06$	$> 0.05$	$4.23e-02$	$> 0.05$	$> 0.05$	–	$4.18e-06$	$> 0.05$	$1.96e-04$	$> 0.05$
MOMBI-NATCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$> 0.05$	$2.56e-04$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$6.00e-05$	–	$1.14e-03$	$> 0.05$
MOMBI-NMTCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$2.55e-26$	$2.18e-26$	$2.99e-26$	$8.98e-27$	$5.60e-27$	$1.87e-26$	$4.07e-26$	$6.55e-27$	$3.22e-26$	$7.67e-27$	–

Table A.111: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ3 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
8D											
MOMBI-WS	–	$8.84e-07$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$8.51e-03$	$> 0.05$	$1.35e-02$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$2.28e-12$	$5.36e-17$	–	$8.67e-12$	$3.24e-02$	$6.79e-04$	$2.35e-11$	$2.77e-14$	$1.93e-11$	$2.93e-14$	$> 0.05$
MOMBI-NLS	$> 0.05$	$1.39e-08$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$2.40e-04$	$> 0.05$	$5.27e-04$	$> 0.05$
MOMBI-TCH	$> 0.05$	$1.98e-03$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NTCH	$5.19e-05$	$1.47e-09$	$> 0.05$	$1.56e-04$	$> 0.05$	–	$4.96e-04$	$1.53e-06$	$3.73e-04$	$8.48e-07$	$> 0.05$
MOMBI-ATCH	$9.68e-03$	$1.12e-10$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$3.75e-06$	$> 0.05$	$1.13e-05$	$> 0.05$
MOMBI-NATCH	$> 0.05$	$5.52e-03$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$3.10e-02$	$6.46e-10$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.93e-05$	–	$6.34e-05$	$> 0.05$
MOMBI-NMTCH	$> 0.05$	$4.16e-03$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$1.78e-28$	$3.37e-29$	$6.92e-26$	$1.64e-28$	$2.21e-28$	$5.83e-29$	$1.93e-28$	$1.06e-28$	$2.04e-28$	$1.00e-28$	–
9D											
MOMBI-WS	–	$2.83e-04$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$8.27e-21$	$2.21e-24$	–	$6.88e-21$	$3.22e-08$	$5.30e-10$	$7.03e-20$	$1.45e-22$	$1.20e-19$	$2.18e-22$	$> 0.05$
MOMBI-NLS	$> 0.05$	$4.40e-06$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$7.57e-03$	$> 0.05$	$4.45e-03$	$> 0.05$
MOMBI-TCH	$> 0.05$	$6.48e-04$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$2.90e-02$	$> 0.05$	$4.42e-02$	$> 0.05$
MOMBI-NTCH	$2.33e-03$	$2.14e-06$	$> 0.05$	$7.20e-03$	$> 0.05$	–	$3.31e-02$	$2.64e-04$	$2.28e-02$	$3.52e-04$	$> 0.05$
MOMBI-ATCH	$2.70e-03$	$4.02e-10$	$> 0.05$	$2.10e-02$	$> 0.05$	$> 0.05$	–	$9.61e-06$	$> 0.05$	$1.99e-06$	$> 0.05$
MOMBI-NATCH	$> 0.05$	$1.16e-02$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$3.39e-02$	$9.65e-08$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$2.83e-04$	–	$1.09e-04$	$> 0.05$
MOMBI-NMTCH	$> 0.05$	$1.18e-02$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$2.42e-26$	$2.60e-27$	$4.93e-21$	$2.55e-26$	$4.30e-27$	$3.30e-27$	$3.48e-26$	$1.87e-26$	$4.82e-26$	$2.55e-26$	–
10D											
MOMBI-WS	–	$6.70e-11$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$3.63e-02$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$9.90e-24$	$2.16e-30$	–	$3.21e-25$	$4.55e-11$	$1.07e-16$	$6.05e-25$	$1.60e-26$	$4.35e-25$	$1.32e-27$	$> 0.05$
MOMBI-NLS	$1.28e-02$	$3.26e-18$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$3.77e-03$	$> 0.05$	$1.36e-05$	$> 0.05$
MOMBI-TCH	$4.26e-05$	$5.89e-11$	$> 0.05$	$2.26e-05$	–	$2.33e-05$	$6.83e-05$	$6.60e-06$	$3.28e-05$	$6.85e-07$	$> 0.05$
MOMBI-NTCH	$9.69e-04$	$3.12e-10$	$> 0.05$	$2.09e-03$	$> 0.05$	–	$5.01e-03$	$1.92e-04$	$2.49e-03$	$1.82e-05$	$> 0.05$
MOMBI-ATCH	$1.55e-03$	$6.56e-21$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$1.75e-04$	$> 0.05$	$2.31e-07$	$> 0.05$
MOMBI-NATCH	$> 0.05$	$1.15e-12$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$2.79e-02$	$> 0.05$
MOMBI-MTCH	$4.45e-03$	$2.60e-19$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$9.41e-04$	–	$2.93e-06$	$> 0.05$
MOMBI-NMTCH	$> 0.05$	$1.12e-07$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$7.66e-29$	$3.64e-31$	$3.84e-14$	$7.99e-30$	$1.15e-28$	$1.98e-30$	$1.16e-29$	$6.58e-30$	$9.06e-30$	$3.19e-30$	–

Table A.112: Comparison of R2 indicator values for different optimizers on the DTLZ3 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	2.511196e-01	2.509359e-01	2.506334e-01	2.505735e-01	2.189177e-01	2.189862e-01	2.189490e-01	2.190524e-01	2.189795e-01	2.190509e-01	2.196327e-01
	avg.	2.538690e-01	2.512398e-01	2.507028e-01	2.506414e-01	2.190609e-01	2.191258e-01	2.192621e-01	2.195128e-01	2.213330e-01	2.213982e-01	2.220063e-01
	min.	2.501584e-01	2.500945e-01	2.481223e-01	2.491384e-01	2.182177e-01	2.184476e-01	2.182667e-01	2.182751e-01	2.181898e-01	2.184125e-01	2.183202e-01
	max.	5.028540e-01	2.552346e-01	2.534176e-01	2.542589e-01	2.204203e-01	2.214743e-01	2.259363e-01	2.439631e-01	4.370944e-01	4.369987e-01	4.362868e-01
	std.	2.516598e-02	9.060835e-04	7.271029e-04	6.788525e-04	5.694114e-04	4.800630e-04	9.890922e-04	2.589182e-03	2.180802e-02	2.178741e-02	2.166905e-02
3D	med.	1.117499e-01	1.117283e-01	1.107260e-01	1.113284e-01	9.570354e-02	9.614886e-02	9.573983e-02	9.602085e-02	9.563074e-02	9.617511e-02	9.567975e-02
	avg.	1.130360e-01	1.127790e-01	1.105994e-01	1.113896e-01	9.583654e-02	9.717005e-02	9.585460e-02	9.611636e-02	9.569680e-02	9.716380e-02	9.641343e-02
	min.	1.111557e-01	1.111363e-01	1.080536e-01	1.109442e-01	9.498448e-02	9.516375e-02	9.505943e-02	9.531592e-02	9.482888e-02	9.541585e-02	9.407236e-02
	max.	2.232086e-01	2.004886e-01	1.123916e-01	1.121593e-01	9.956025e-02	1.930106e-01	9.789305e-02	9.784874e-02	9.714796e-02	1.912670e-01	1.528000e-01
	std.	1.115010e-02	8.883443e-03	6.156191e-04	2.568602e-04	6.993556e-04	9.697785e-03	4.766186e-04	4.653289e-04	5.234220e-04	9.518897e-03	5.802189e-03
4D	med.	6.287900e-02	6.295482e-02	6.204665e-02	6.269002e-02	5.672968e-02	5.866139e-02	5.745962e-02	5.814964e-02	5.753435e-02	5.816715e-02	5.477428e-02
	avg.	6.362430e-02	6.432166e-02	6.198225e-02	6.273383e-02	5.745227e-02	5.966052e-02	5.758148e-02	5.895651e-02	5.789477e-02	5.823138e-02	5.700291e-02
	min.	6.245926e-02	6.243757e-02	6.076130e-02	6.235188e-02	5.601076e-02	5.645836e-02	5.672738e-02	5.621575e-02	5.685756e-02	5.644277e-02	5.357825e-02
	max.	1.258240e-01	1.253348e-01	6.310759e-02	6.362390e-02	1.113452e-01	1.175161e-01	5.888074e-02	8.966366e-02	8.272607e-02	6.025114e-02	1.459362e-01
	std.	6.298250e-03	8.765041e-03	3.748600e-04	2.398969e-04	5.516267e-03	8.141844e-03	4.249370e-04	4.249382e-03	2.550045e-03	8.800714e-04	1.126194e-02
5D	med.	4.111386e-02	4.151827e-02	4.043668e-02	4.110450e-02	3.836833e-02	4.084032e-02	4.053007e-02	4.107154e-02	4.058708e-02	4.095572e-02	3.688675e-02
	avg.	4.170802e-02	4.891676e-02	4.043120e-02	4.115051e-02	4.171020e-02	4.317424e-02	4.062856e-02	4.297554e-02	4.065283e-02	4.295531e-02	4.229359e-02
	min.	4.073580e-02	4.074894e-02	3.990140e-02	4.075014e-02	3.749982e-02	3.980911e-02	3.975393e-02	4.046015e-02	3.972346e-02	4.053805e-02	3.553395e-02
	max.	8.215262e-02	1.632651e-01	4.095456e-02	4.231094e-02	8.681867e-02	8.175051e-02	4.232813e-02	8.245822e-02	4.161253e-02	8.224460e-02	1.217864e-01
	std.	4.145848e-03	2.032470e-02	1.939023e-04	2.978209e-04	1.083386e-02	8.995379e-03	3.774293e-04	6.491982e-03	3.396475e-04	8.155923e-03	1.392804e-02
6D	med.	3.398527e-02	3.439310e-02	3.335076e-02	3.391846e-02	3.531130e-02	3.385205e-02	3.390131e-02	3.410331e-02	3.391831e-02	3.397727e-02	2.542165e-02
	avg.	3.387485e-02	4.149854e-02	3.324349e-02	3.435724e-02	3.915461e-02	4.042907e-02	3.453267e-02	3.674697e-02	3.427655e-02	3.528127e-02	2.921352e-02
	min.	2.798116e-02	3.360174e-02	2.743809e-02	3.343546e-02	3.113753e-02	3.271242e-02	3.342722e-02	3.352073e-02	3.340658e-02	3.335040e-02	2.461868e-02
	max.	3.574492e-02	1.754054e-01	6.025829e-02	4.260736e-02	1.306031e-01	9.917337e-02	6.734509e-02	6.853497e-02	6.770954e-02	6.877819e-02	8.033997e-02
	std.	1.042611e-03	2.050678e-02	3.555501e-03	1.879583e-03	1.276158e-02	1.508735e-02	4.320291e-03	7.376955e-03	3.387167e-03	5.165596e-03	1.064526e-02
7D	med.	3.555451e-02	3.567303e-02	3.551390e-02	3.563829e-02	3.870824e-02	3.543263e-02	3.560222e-02	3.576102e-02	3.555736e-02	3.563263e-02	1.907615e-02
	avg.	3.600057e-02	3.715566e-02	3.643876e-02	3.732402e-02	4.256365e-02	3.732684e-02	3.566033e-02	3.818849e-02	3.566139e-02	3.800025e-02	2.389739e-02
	min.	3.531515e-02	3.532724e-02	2.799959e-02	3.531377e-02	3.472938e-02	3.408839e-02	3.534750e-02	3.533844e-02	3.532908e-02	3.533662e-02	1.848337e-02
	max.	7.098790e-02	7.125553e-02	7.052274e-02	4.828632e-02	8.428961e-02	7.665440e-02	3.649455e-02	7.086644e-02	3.676953e-02	7.136024e-02	9.103367e-02
	std.	3.543630e-03	6.936875e-03	4.009821e-03	4.186798e-03	1.156363e-02	7.923691e-03	2.208980e-04	6.364976e-03	2.924751e-04	6.325248e-03	1.327141e-02
8D	med.	3.132530e-02	3.158622e-02	3.084756e-02	3.128931e-02	3.365271e-02	3.108880e-02	3.126359e-02	3.139062e-02	3.128564e-02	3.138210e-02	1.480337e-02
	avg.	3.168940e-02	3.716717e-02	3.043145e-02	3.198061e-02	3.673349e-02	3.463538e-02	3.133117e-02	3.305093e-02	3.133327e-02	3.333256e-02	1.806544e-02
	min.	3.091053e-02	3.108112e-02	2.463415e-02	3.095181e-02	3.023291e-02	2.994852e-02	3.087324e-02	3.095099e-02	3.088456e-02	3.103778e-02	1.410347e-02
	max.	6.179273e-02	9.368662e-02	3.535671e-02	6.253718e-02	7.758194e-02	8.917630e-02	3.258970e-02	6.274022e-02	3.220625e-02	6.261685e-02	6.333743e-02
	std.	3.050850e-03	1.350824e-02	2.191746e-03	4.385616e-03	9.674480e-03	1.033853e-02	2.633342e-04	6.797505e-03	2.322211e-04	7.381696e-03	9.066083e-03
9D	med.	2.795312e-02	2.812283e-02	2.705158e-02	2.791905e-02	3.044704e-02	2.767610e-02	2.789050e-02	2.798935e-02	2.791356e-02	2.803136e-02	1.195850e-02
	avg.	2.946039e-02	3.259968e-02	2.615711e-02	2.879224e-02	3.663638e-02	3.265519e-02	2.819230e-02	2.946705e-02	2.845312e-02	2.891709e-02	1.502622e-02
	min.	2.245132e-02	2.771843e-02	1.907234e-02	2.763250e-02	2.630044e-02	2.505055e-02	2.769438e-02	2.773411e-02	2.229308e-02	2.769057e-02	1.123617e-02
	max.	5.639814e-02	5.801508e-02	3.048567e-02	5.621468e-02	1.802492e-01	1.093129e-01	5.543943e-02	5.590244e-02	5.552409e-02	5.587067e-02	8.644366e-02
	std.	6.213596e-03	9.976909e-03	2.342573e-03	4.778347e-03	1.950514e-02	1.296318e-02	2.757731e-03	6.047689e-03	3.929036e-03	4.733211e-03	9.890004e-03
10D	med.	2.529563e-02	2.583028e-02	2.197036e-02	2.517608e-02	2.638117e-02	2.494989e-02	2.517511e-02	2.527248e-02	2.516776e-02	2.532866e-02	1.025248e-02
	avg.	2.813749e-02	2.547371e-02	2.272445e-02	2.611312e-02	3.148853e-02	3.092685e-02	2.547764e-02	2.775704e-02	2.599136e-02	2.834814e-02	1.193529e-02
	min.	1.989225e-02	2.502653e-02	1.793583e-02	2.470974e-02	2.191569e-02	2.358280e-02	2.494283e-02	2.496176e-02	2.495381e-02	2.493288e-02	9.334014e-03
	max.	7.496697e-02	1.003074e-01	4.379799e-02	5.039479e-02	1.195715e-01	9.566673e-02	4.988471e-02	7.707541e-02	5.019152e-02	7.493938e-02	4.294708e-02
	std.	9.351934e-03	1.546373e-02	3.742241e-03	4.402574e-03	1.315822e-02	1.296809e-02	2.476542e-03	1.018631e-02	4.262967e-03	8.592820e-03	5.260752e-03

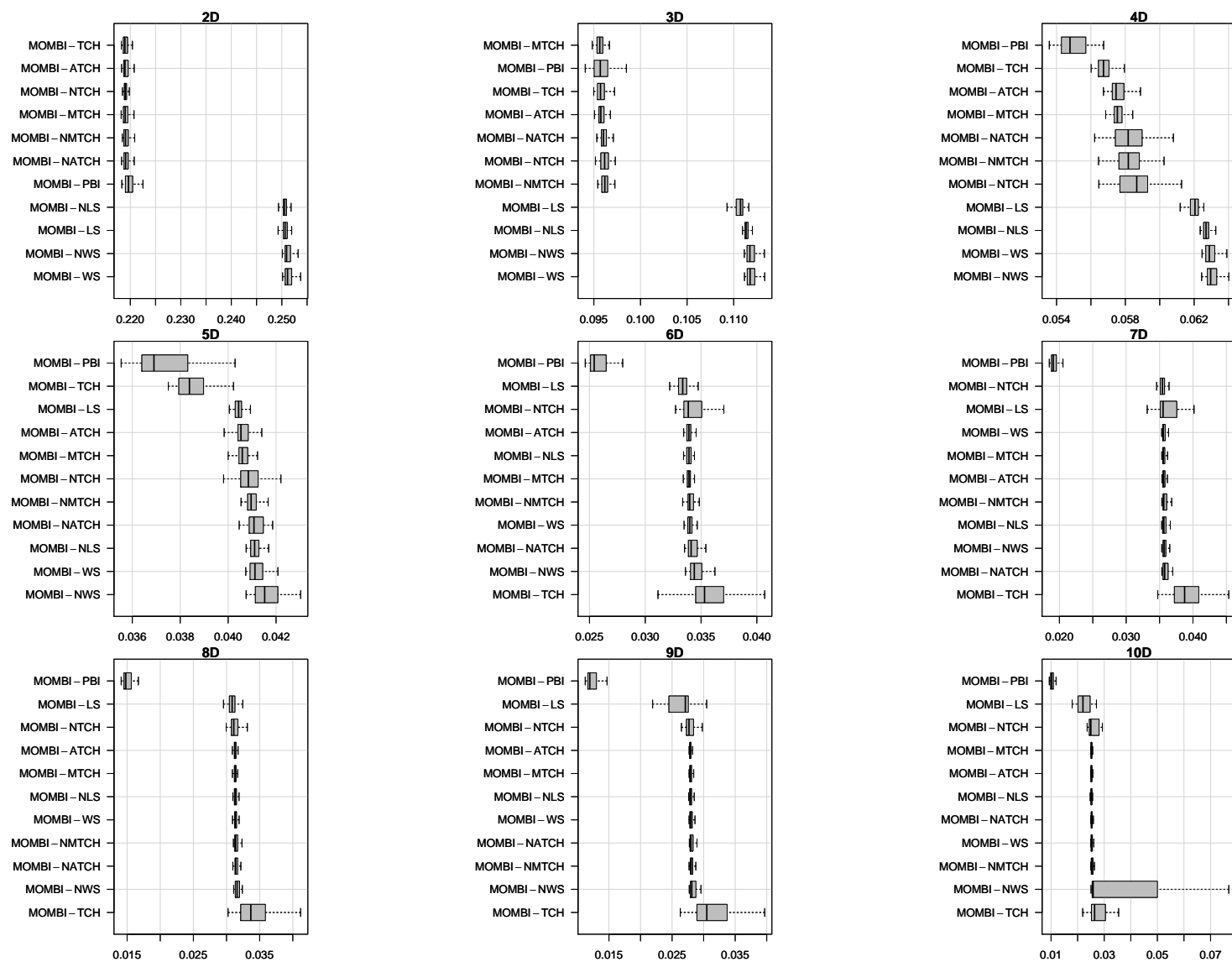


Figure A.278: Box-plot of the R2 indicator values for different optimizers on the DTLZ3 test problem.

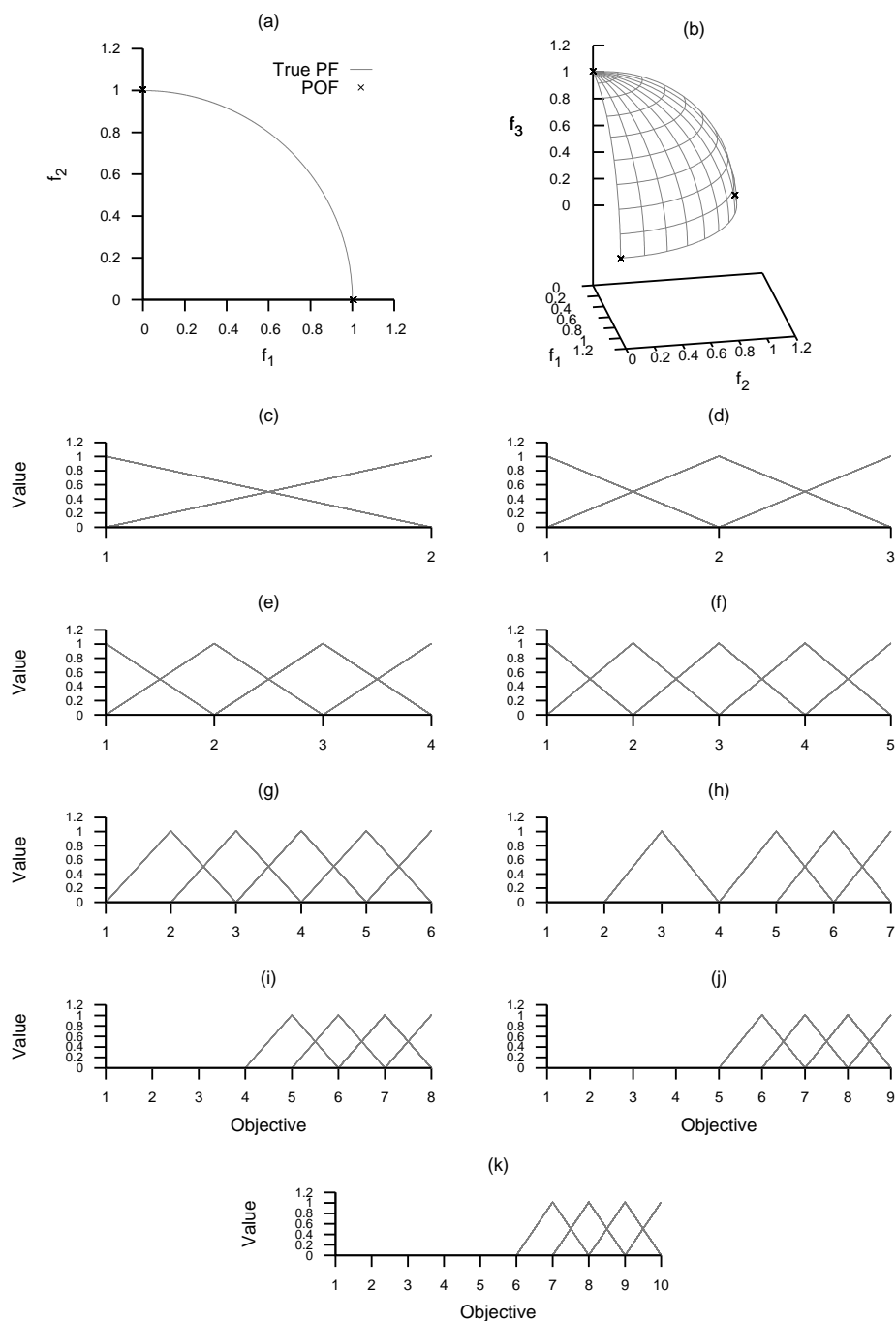


Figure A.279: Plots of the approximations obtained by MOMBI-WS from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

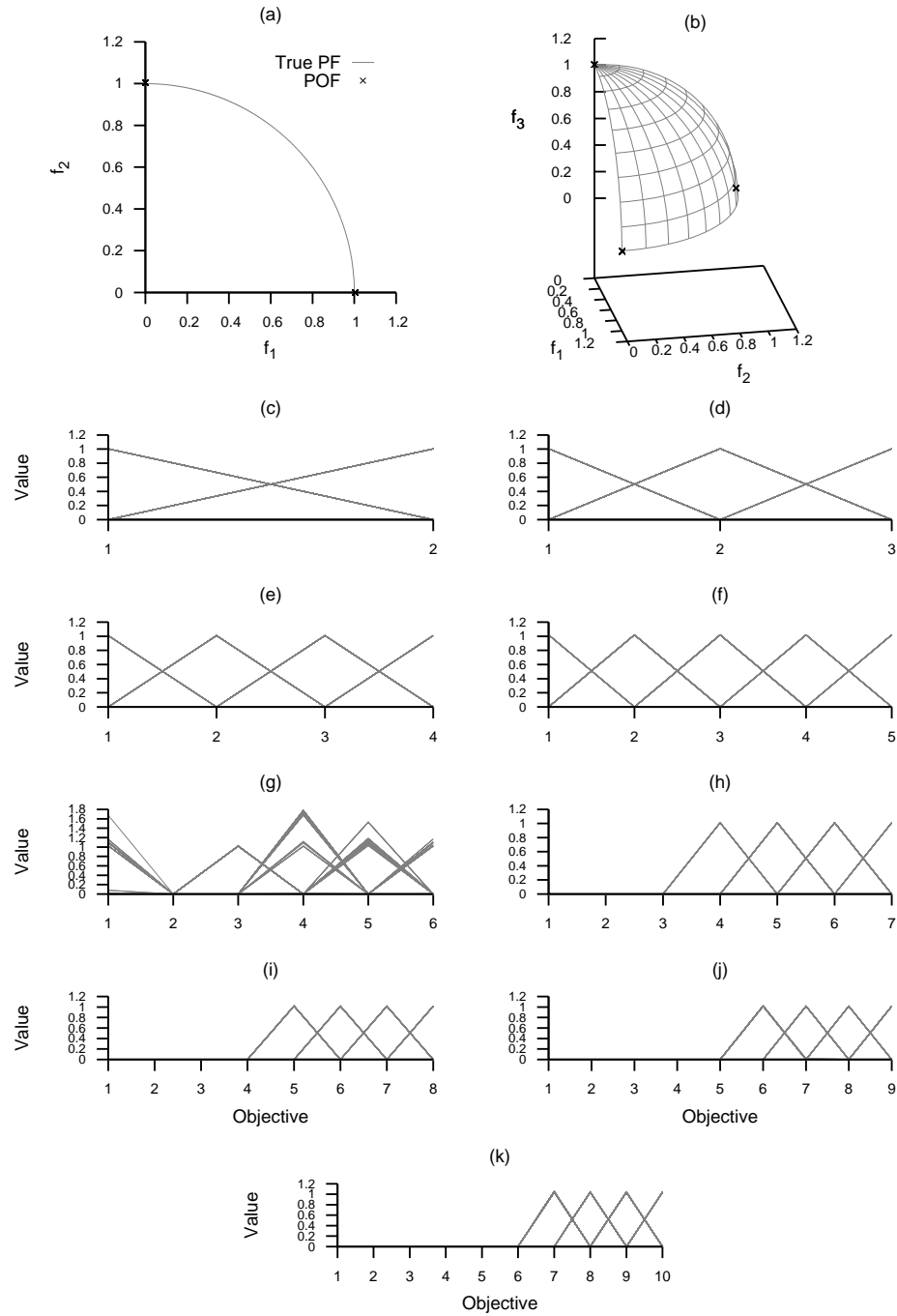


Figure A.280: Plots of the approximations obtained by MOMBI-NWS from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

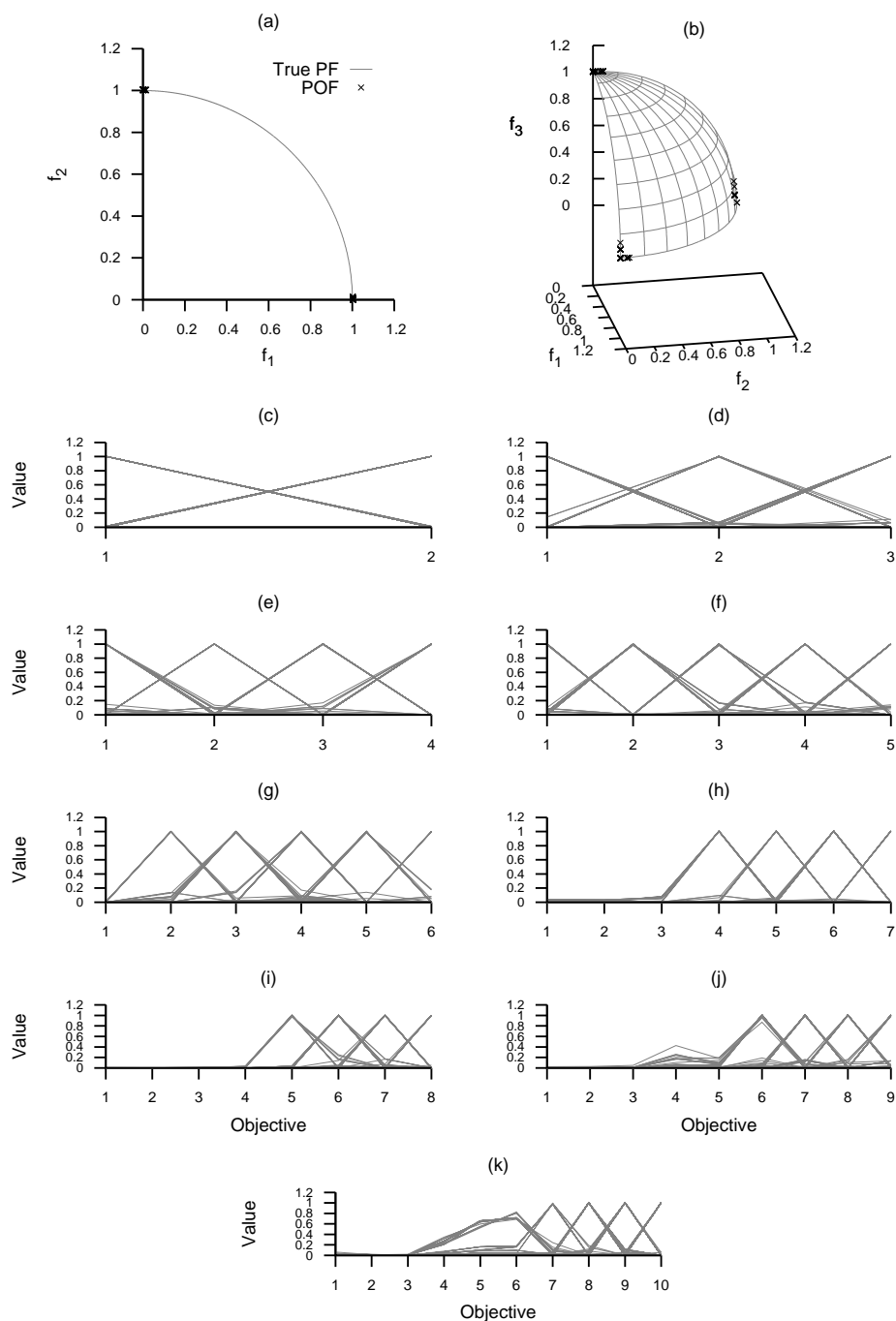


Figure A.281: Plots of the approximations obtained by MOMBI-LS from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



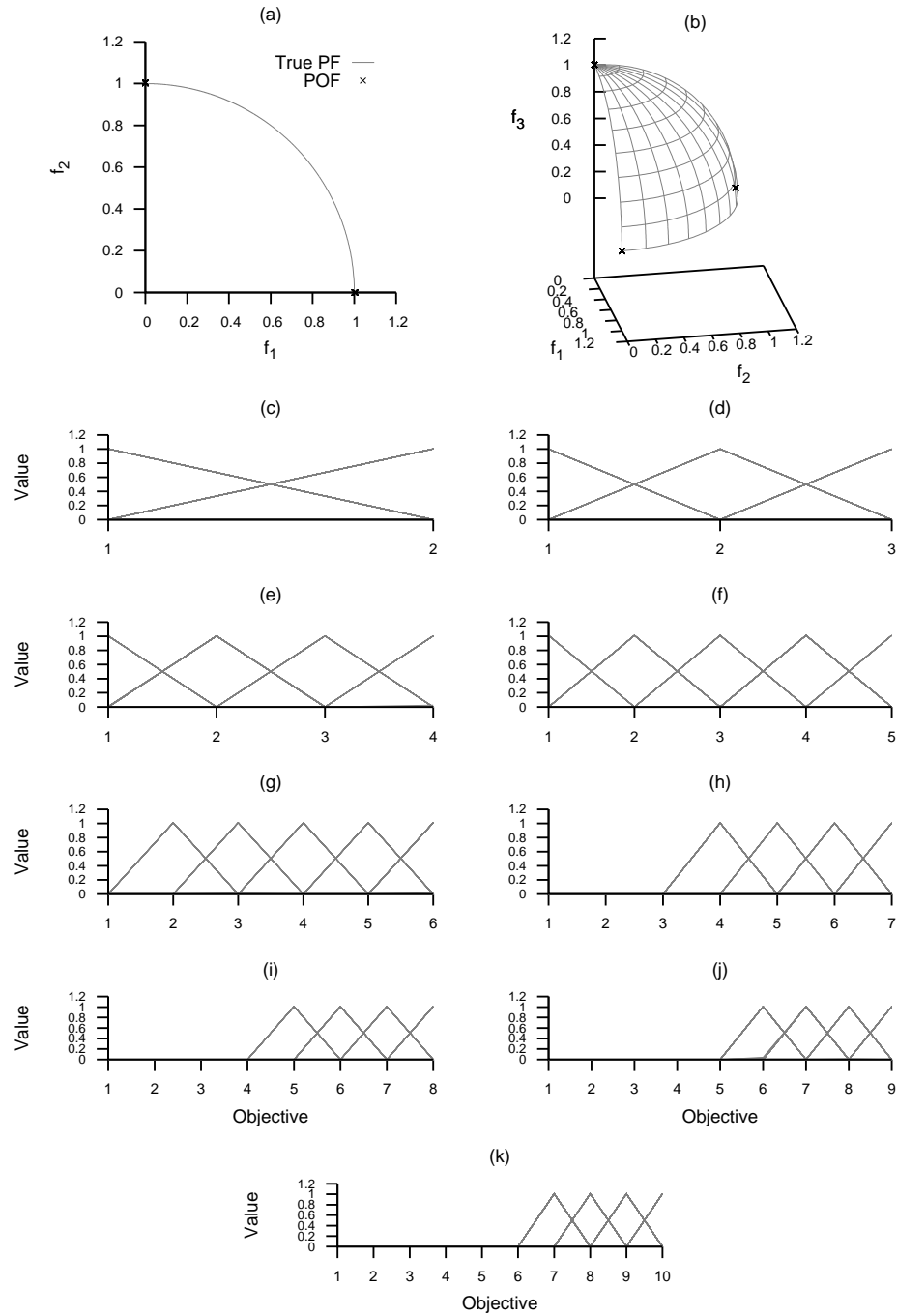


Figure A.282: Plots of the approximations obtained by MOMBI-NLS from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

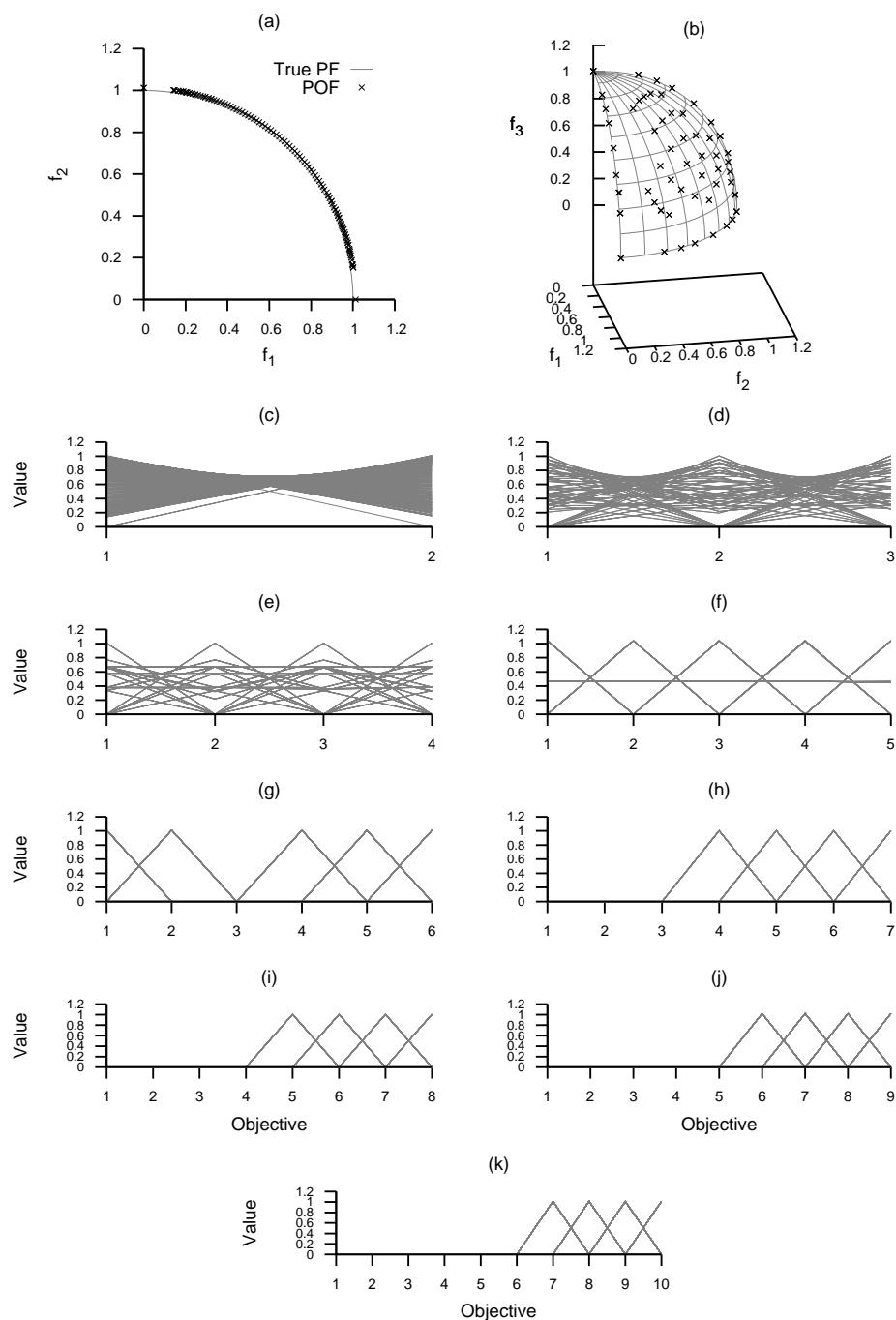


Figure A.283: Plots of the approximations obtained by MOMBI-ATCH from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

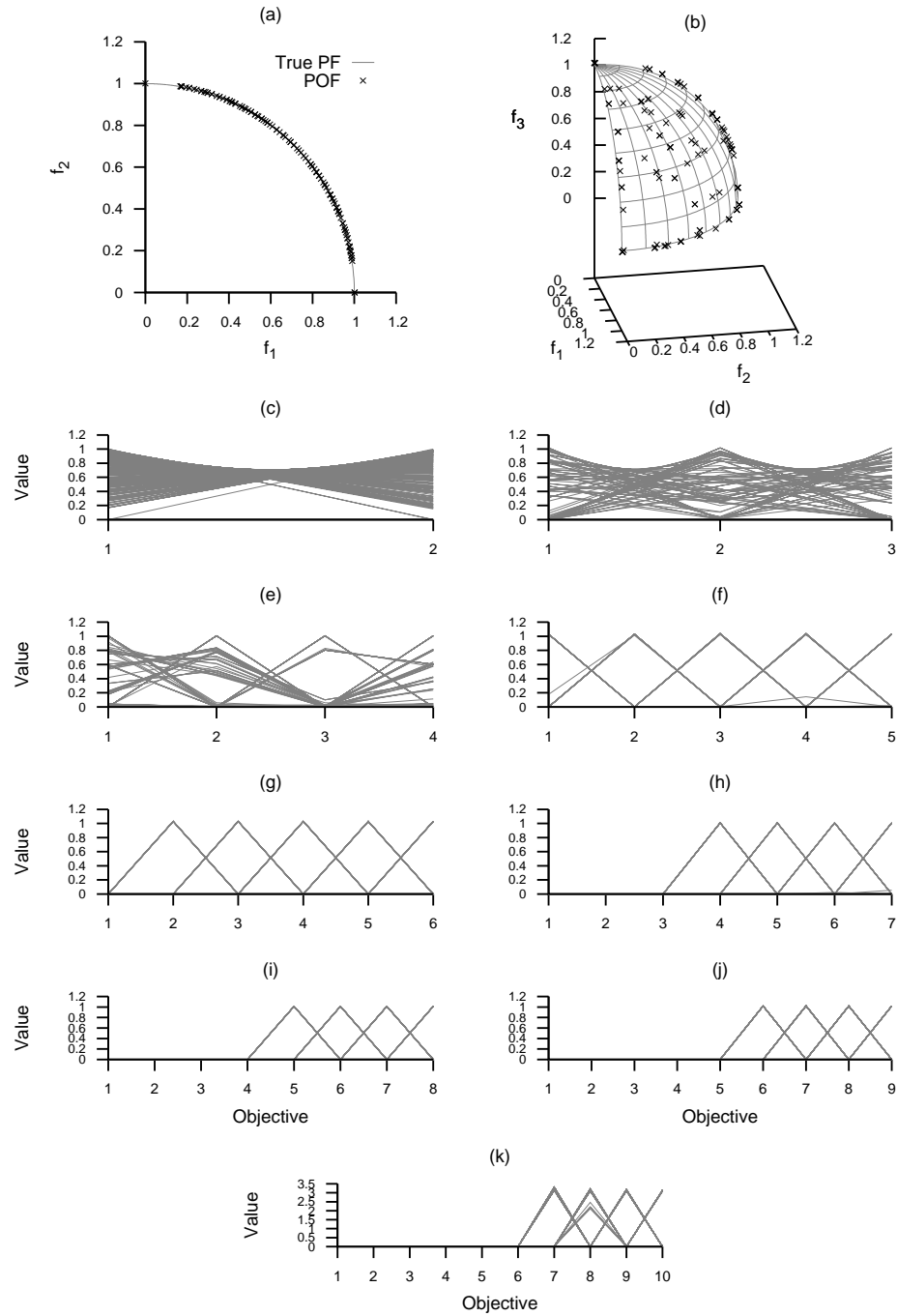


Figure A.284: Plots of the approximations obtained by MOMBI-NATCH from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

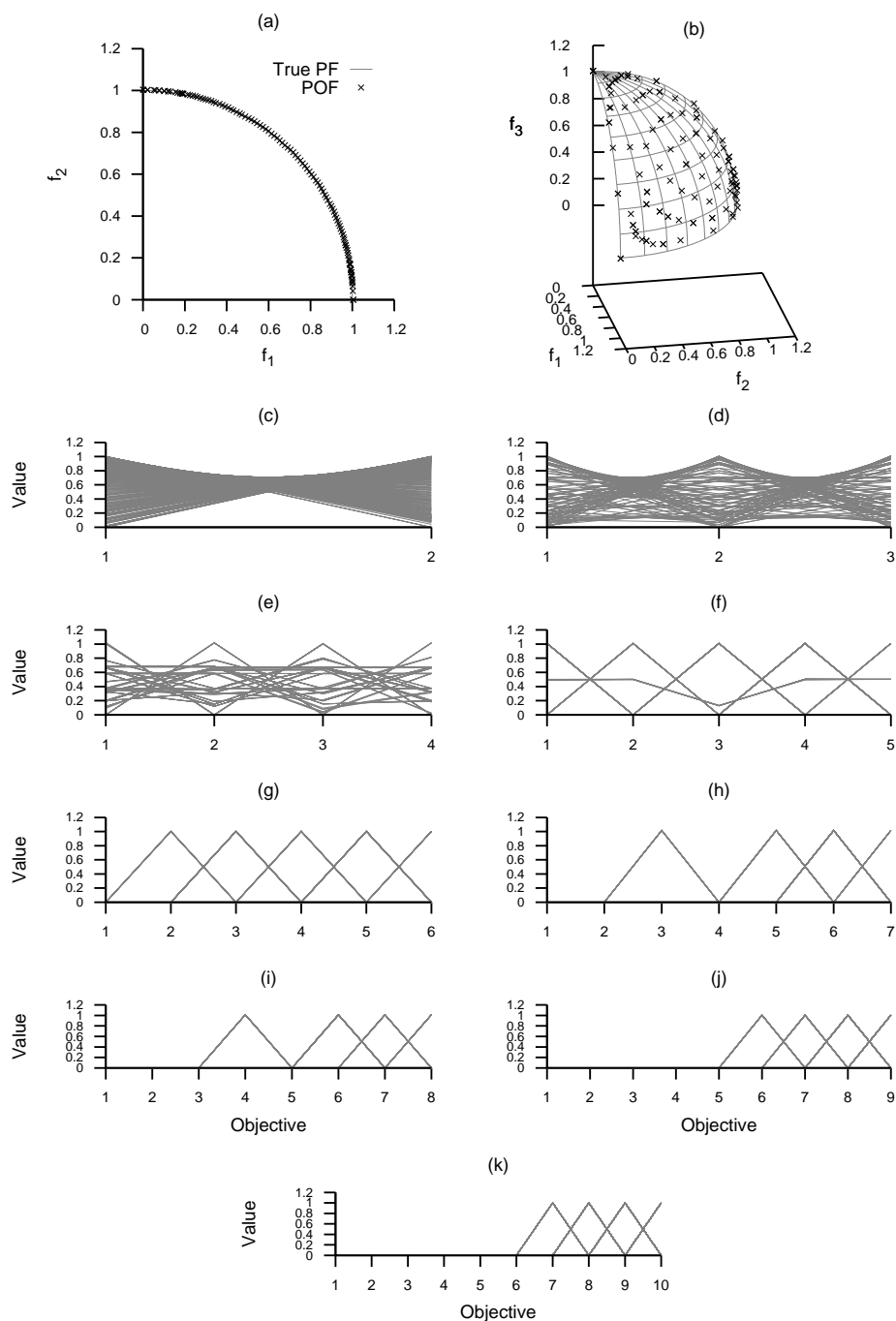


Figure A.285: Plots of the approximations obtained by MOMBI-MTCH from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

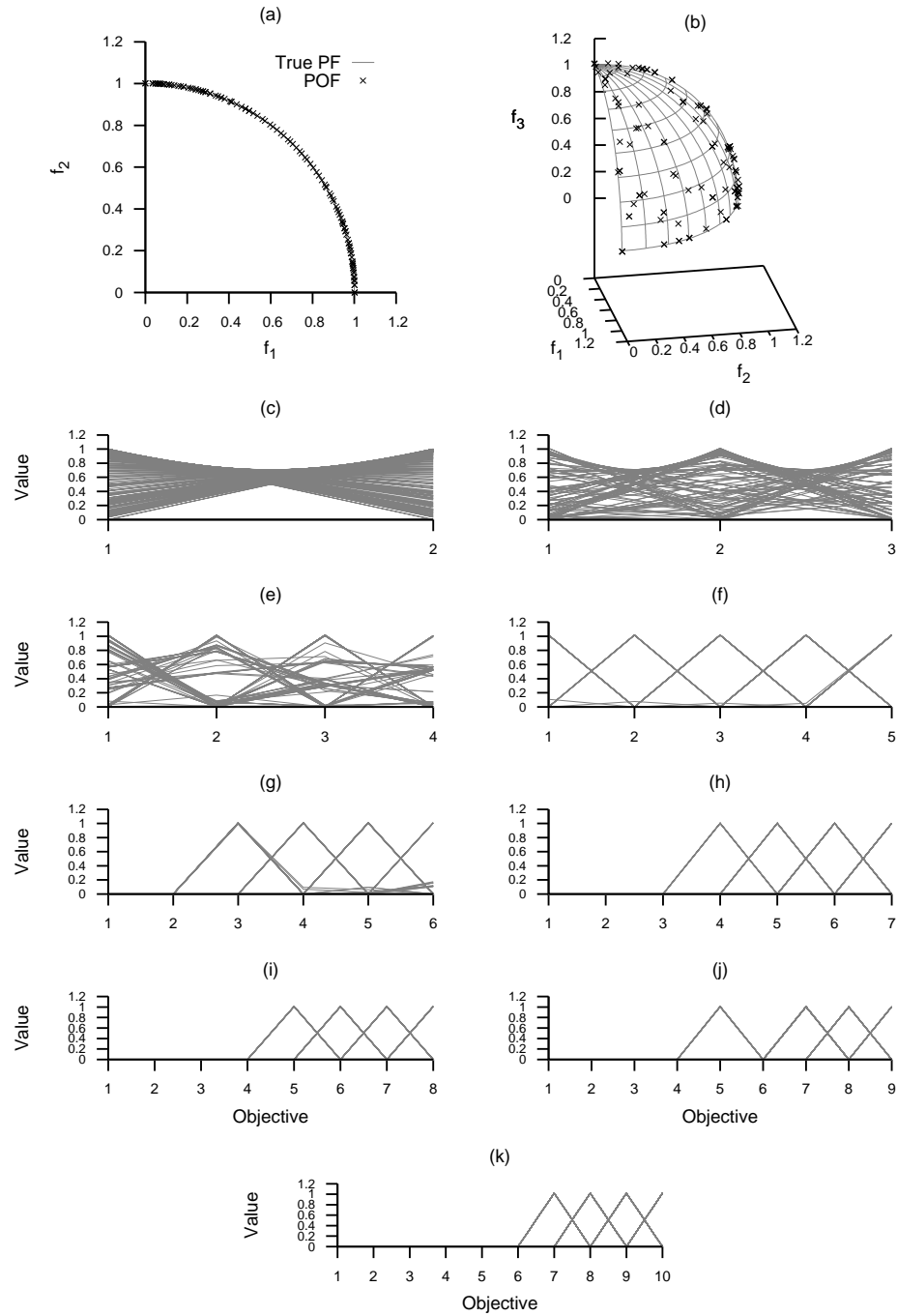


Figure A.286: Plots of the approximations obtained by MOMBI-NMTCH from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.113: Comparison of hypervolume indicator values for different optimizers on the DTLZ4 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	3.000000e+00	3.000000e+00	3.000000e+00	3.000000e+00	3.210825e+00	3.209390e+00	3.209799e+00	3.209024e+00	3.210798e+00	3.209416e+00	3.210565e+00
	avg.	2.980000e+00	2.970000e+00	2.970000e+00	2.950000e+00	3.210821e+00	3.161042e+00	3.185618e+00	3.136472e+00	3.174474e+00	3.197320e+00	3.210552e+00
	min.	2.000000e+00	2.000000e+00	2.000000e+00	2.000000e+00	3.210709e+00	2.000000e+00	2.000000e+00	2.000000e+00	2.000000e+00	2.000000e+00	3.210126e+00
	max.	3.000000e+00	3.000000e+00	3.000000e+00	3.000000e+00	3.210875e+00	3.209960e+00	3.210096e+00	3.209521e+00	3.210812e+00	3.209926e+00	3.210792e+00
	std.	1.407053e-01	1.714466e-01	1.714466e-01	2.190429e-01	4.272330e-05	2.381908e-01	1.702273e-01	2.885711e-01	2.075873e-01	1.209417e-01	1.386925e-04
3D	med.	7.000000e+00	7.000000e+00	7.141577e+00	7.000000e+00	7.384563e+00	7.391205e+00	7.397135e+00	7.396510e+00	7.382070e+00	7.392363e+00	7.421944e+00
	avg.	6.930000e+00	6.890000e+00	7.087171e+00	6.880000e+00	7.365698e+00	7.352006e+00	7.308595e+00	7.116291e+00	7.344235e+00	7.196142e+00	7.421934e+00
	min.	6.000000e+00	6.000000e+00	6.000000e+00	6.000000e+00	6.411863e+00	6.407924e+00	6.409521e+00	4.000000e+00	6.411578e+00	6.403362e+00	7.421680e+00
	max.	7.000000e+00	7.000000e+00	7.300792e+00	7.000000e+00	7.393202e+00	7.398547e+00	7.401589e+00	7.401334e+00	7.393137e+00	7.399770e+00	7.422104e+00
	std.	2.564324e-01	3.144660e-01	2.608921e-01	3.265986e-01	1.369607e-01	1.934810e-01	2.841488e-01	5.326232e-01	1.913776e-01	3.958881e-01	9.531244e-05
4D	med.	1.500000e+01	1.500000e+01	1.518138e+01	1.500000e+01	1.542126e+01	1.542138e+01	1.534202e+01	1.516816e+01	1.528641e+01	1.528454e+01	1.556772e+01
	avg.	1.472000e+01	1.447998e+01	1.511882e+01	1.482000e+01	1.529634e+01	1.523222e+01	1.522909e+01	1.492489e+01	1.519007e+01	1.504057e+01	1.555271e+01
	min.	1.200000e+01	1.200000e+01	1.400000e+01	1.400000e+01	1.457992e+01	1.407898e+01	1.409151e+01	1.275252e+01	1.459273e+01	1.276463e+01	1.481371e+01
	max.	1.500000e+01	1.500000e+01	1.535193e+01	1.500000e+01	1.542777e+01	1.543054e+01	1.538672e+01	1.537178e+01	1.531114e+01	1.532349e+01	1.556793e+01
	std.	5.140449e-01	8.224249e-01	2.660739e-01	3.861229e-01	2.657882e-01	2.327590e-01	2.414354e-01	6.150059e-01	2.166512e-01	4.216130e-01	1.054510e-01
5D	med.	3.000000e+01	3.000000e+01	3.118687e+01	3.100000e+01	3.152316e+01	3.150687e+01	3.105162e+01	3.035705e+01	3.105162e+01	3.105162e+01	3.166884e+01
	avg.	2.986000e+01	2.940000e+01	3.106984e+01	3.055000e+01	3.140671e+01	3.121460e+01	3.068121e+01	3.026364e+01	3.068041e+01	3.059924e+01	3.166883e+01
	min.	2.400000e+01	2.400000e+01	3.005580e+01	2.800000e+01	3.069620e+01	2.912067e+01	2.880009e+01	2.541904e+01	2.874904e+01	2.800972e+01	3.166843e+01
	max.	3.100000e+01	3.100000e+01	3.144381e+01	3.100036e+01	3.153484e+01	3.154114e+01	3.107550e+01	3.106519e+01	3.108751e+01	3.108818e+01	3.166921e+01
	std.	1.287233e+00	1.669694e+00	3.586152e-01	7.436623e-01	2.463208e-01	3.750560e-01	5.324364e-01	1.068374e+00	5.342867e-01	6.335171e-01	1.748137e-04
6D	med.	6.000000e+01	6.000000e+01	6.239829e+01	6.200000e+01	6.290219e+01	6.299952e+01	6.200000e+01	6.200000e+01	6.200000e+01	6.025000e+01	6.374108e+01
	avg.	5.964000e+01	5.930000e+01	6.226583e+01	6.120000e+01	6.290071e+01	6.273470e+01	6.138802e+01	6.061725e+01	6.076250e+01	6.008517e+01	6.373957e+01
	min.	4.800000e+01	4.800000e+01	5.413594e+01	4.600000e+01	6.152763e+01	5.794277e+01	5.600000e+01	5.076265e+01	4.800000e+01	4.800000e+01	6.367677e+01
	max.	6.200000e+01	6.200000e+01	6.353680e+01	6.200000e+01	6.321483e+01	6.309517e+01	6.200000e+01	6.200000e+01	6.200000e+01	6.200000e+01	6.374152e+01
	std.	3.497532e+00	3.307475e+00	9.509072e-01	2.666667e+00	2.204929e-01	6.761371e-01	1.152757e+00	2.162599e+00	2.141278e+00	2.704586e+00	7.654837e-03
7D	med.	1.120000e+02	1.129350e+02	1.205329e+02	1.200000e+02	1.208132e+02	1.220016e+02	1.120000e+02	1.200000e+02	1.120000e+02	1.120000e+02	1.277531e+02
	avg.	1.069600e+02	1.132128e+02	1.194605e+02	1.172069e+02	1.208996e+02	1.212000e+02	1.092000e+02	1.136861e+02	1.074537e+02	1.129042e+02	1.277396e+02
	min.	6.400000e+01	9.600000e+01	9.600000e+01	9.600000e+01	1.167376e+02	1.053045e+02	6.400000e+01	9.599881e+01	6.400000e+01	6.400000e+01	1.274381e+02
	max.	1.200000e+02	1.203801e+02	1.250463e+02	1.207036e+02	1.235095e+02	1.231622e+02	1.200000e+02	1.200000e+02	1.200000e+02	1.200000e+02	1.277543e+02
	std.	1.040000e+01	8.616841e+00	4.773684e+00	5.136845e+00	1.064559e+00	2.743257e+00	1.183728e+01	8.414765e+00	1.133219e+01	9.368532e+00	6.168573e-02
8D	med.	2.240000e+02	2.240000e+02	2.423657e+02	2.400000e+02	2.397117e+02	2.439501e+02	2.400000e+02	2.400000e+02	2.240000e+02	2.240000e+02	2.558159e+02
	avg.	2.148800e+02	2.127957e+02	2.418182e+02	2.365211e+02	2.396473e+02	2.435255e+02	2.281600e+02	2.313584e+02	2.150400e+02	2.240000e+02	2.557871e+02
	min.	1.280000e+02	1.280000e+02	2.240000e+02	1.920000e+02	2.340803e+02	2.320328e+02	1.280000e+02	1.920000e+02	1.280000e+02	1.280000e+02	2.553921e+02
	max.	2.400000e+02	2.400000e+02	2.510924e+02	2.438409e+02	2.466684e+02	2.465206e+02	2.400000e+02	2.400000e+02	2.400000e+02	2.400000e+02	2.558254e+02
	std.	3.001995e+01	2.724551e+01	4.240231e+00	9.557529e+00	2.464897e+00	2.059505e+00	1.897511e+01	1.190720e+01	2.985281e+01	2.145420e+01	6.950513e-02
9D	med.	4.480000e+02	4.480000e+02	4.873448e+02	4.800000e+02	4.738668e+02	4.855129e+02	4.800000e+02	4.800000e+02	4.800000e+02	4.800000e+02	5.117019e+02
	avg.	4.464000e+02	4.471990e+02	4.870110e+02	4.709501e+02	4.739067e+02	4.847846e+02	4.681600e+02	4.662400e+02	4.595200e+02	4.608000e+02	5.116571e+02
	min.	2.560000e+02	2.560000e+02	4.477587e+02	3.520000e+02	4.264901e+02	4.526099e+02	2.560000e+02	3.520000e+02	2.560000e+02	3.840000e+02	5.108706e+02
	max.	4.800000e+02	4.959486e+02	5.036156e+02	4.843332e+02	4.961251e+02	4.950181e+02	4.800000e+02	4.800000e+02	4.800000e+02	4.800000e+02	5.118733e+02
	std.	4.691019e+01	4.220231e+01	9.014491e+00	2.278538e+01	8.999551e+00	6.244581e+00	3.039362e+01	2.622862e+01	4.514499e+01	2.982508e+01	1.840653e-01
10D	med.	9.600000e+02	9.600000e+02	9.854109e+02	9.600000e+02	9.412917e+02	9.700222e+02	9.600000e+02	9.600000e+02	9.600000e+02	9.600000e+02	1.023911e+03
	avg.	9.600000e+02	9.606397e+02	9.880725e+02	9.601192e+02	9.409625e+02	9.703322e+02	9.600000e+02	9.600000e+02	9.600000e+02	9.600000e+02	1.023911e+03
	min.	9.600000e+02	9.599997e+02	9.667015e+02	9.599999e+02	9.251059e+02	9.543056e+02	9.600000e+02	9.599997e+02	9.600000e+02	9.599997e+02	1.023906e+03
	max.	9.600000e+02	9.919993e+02	1.016785e+03	9.651814e+02	9.739483e+02	9.925290e+02	9.600000e+02	9.600000e+02	9.600000e+02	9.600000e+02	1.023914e+03
	std.	0.000000e+00	4.500258e+00	1.220739e+01	6.912591e-01	1.037739e+01	8.594754e+00	0.000000e+00	3.253185e-05	0.000000e+00	4.199843e-05	1.591440e-03

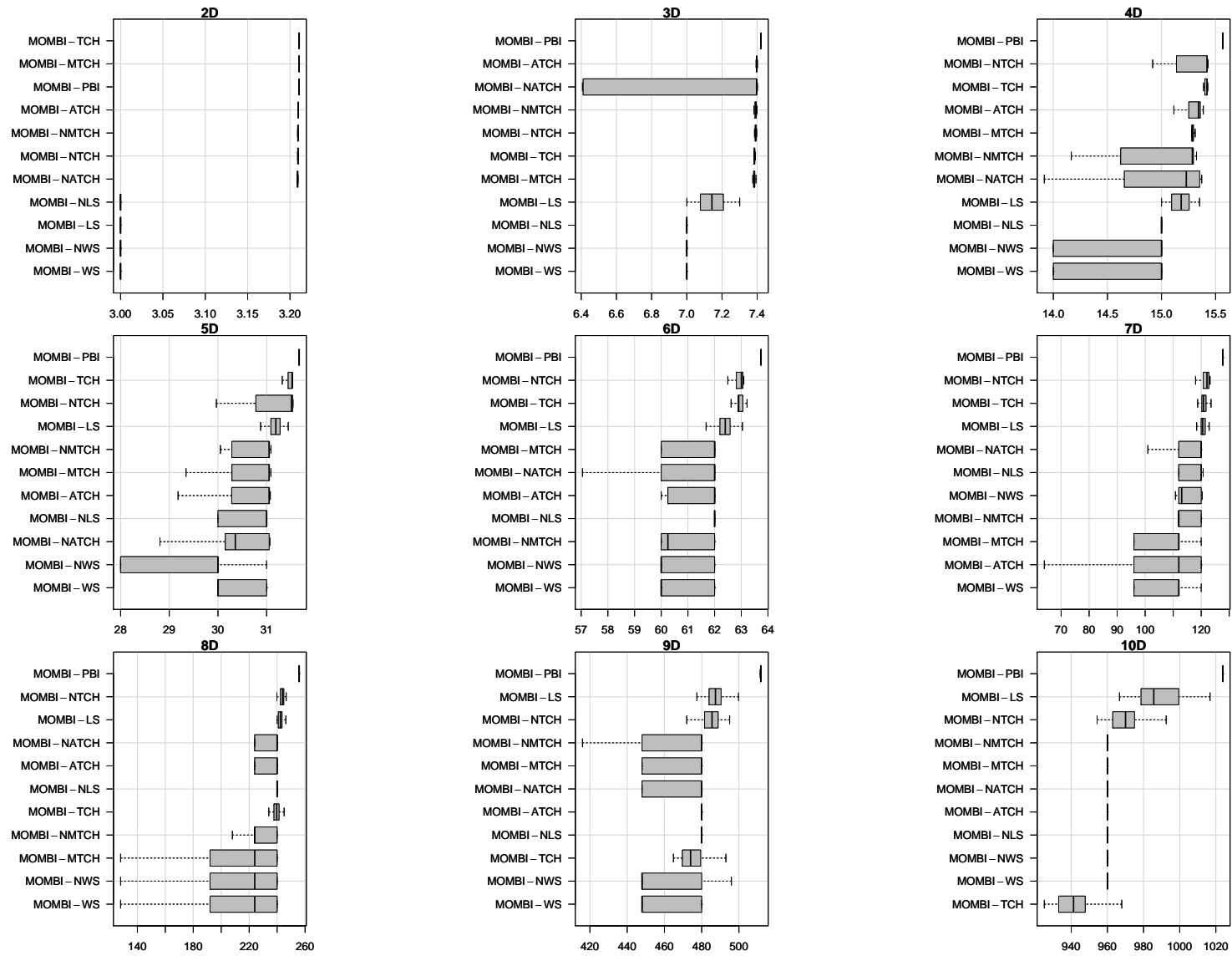


Figure A.287: Box-plot of the hypervolume indicator values for different optimizers on the DTLZ4 test problem.

Table A.114: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ4 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
2D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$5.74e-39$	$8.08e-39$	$8.08e-39$	$1.56e-38$	–	$1.28e-34$	$1.27e-34$	$1.28e-34$	$5.36e-08$	$1.28e-34$	$3.36e-33$
MOMBI-NTCH	$2.41e-33$	$3.02e-33$	$3.02e-33$	$4.64e-33$	> 0.05	–	> 0.05	$9.70e-23$	> 0.05	> 0.05	> 0.05
MOMBI-ATCH	$4.23e-36$	$5.61e-36$	$5.61e-36$	$9.61e-36$	> 0.05	$5.88e-29$	–	$3.86e-32$	> 0.05	$1.96e-27$	> 0.05
MOMBI-NATCH	$1.05e-30$	$1.25e-30$	$1.25e-30$	$1.73e-30$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$6.06e-35$	$7.84e-35$	$7.84e-35$	$1.28e-34$	> 0.05	$4.42e-31$	$4.81e-31$	$4.05e-31$	–	$5.04e-31$	$5.41e-31$
MOMBI-NMTCH	$1.62e-37$	$2.21e-37$	$2.21e-37$	$4.01e-37$	> 0.05	> 0.05	> 0.05	$2.51e-23$	> 0.05	–	> 0.05
MOMBI-PBI	$5.76e-39$	$8.10e-39$	$8.10e-39$	$1.56e-38$	> 0.05	$1.28e-34$	$1.27e-34$	$1.28e-34$	> 0.05	$1.28e-34$	–
3D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$2.51e-29$	$2.92e-29$	–	$2.98e-29$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-35$	$2.83e-35$	$3.47e-32$	$3.38e-35$	–	> 0.05	> 0.05	> 0.05	$1.19e-05$	> 0.05	> 0.05
MOMBI-NTCH	$4.53e-33$	$7.30e-33$	$7.58e-30$	$8.09e-33$	$2.63e-24$	–	> 0.05	> 0.05	$7.11e-24$	> 0.05	> 0.05
MOMBI-ATCH	$3.92e-27$	$3.17e-27$	$2.10e-24$	$2.97e-27$	$6.44e-24$	$2.61e-21$	–	$1.84e-02$	$6.44e-24$	$8.76e-18$	> 0.05
MOMBI-NATCH	$1.65e-11$	$3.85e-12$	$3.45e-10$	$2.66e-12$	$2.34e-09$	$6.94e-08$	> 0.05	–	$2.38e-09$	$8.23e-07$	> 0.05
MOMBI-MTCH	$4.53e-33$	$7.30e-33$	$7.58e-30$	$8.09e-33$	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$3.02e-16$	$8.72e-17$	$1.82e-14$	$6.34e-17$	$1.75e-11$	> 0.05	> 0.05	> 0.05	$1.05e-11$	–	> 0.05
MOMBI-PBI	$2.89e-38$	$8.91e-38$	$1.28e-34$	$1.16e-37$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–
4D											
MOMBI-WS	–	$2.58e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.49e-28$	$3.35e-29$	–	$2.36e-28$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	$4.13e-04$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.41e-23$	$4.97e-25$	$1.71e-16$	$8.64e-23$	–	> 0.05	$8.36e-15$	$4.64e-18$	$5.70e-16$	$3.88e-19$	> 0.05
MOMBI-NTCH	$5.45e-19$	$4.71e-21$	$2.27e-10$	$8.68e-18$	> 0.05	–	$1.37e-08$	$1.02e-12$	$2.71e-09$	$6.97e-13$	> 0.05
MOMBI-ATCH	$3.23e-21$	$5.38e-23$	$4.38e-12$	$3.32e-20$	> 0.05	> 0.05	–	$2.74e-03$	$2.81e-12$	$1.39e-15$	> 0.05
MOMBI-NATCH	$1.45e-04$	$5.46e-07$	> 0.05	$2.28e-03$	> 0.05	> 0.05	> 0.05	–	> 0.05	$2.30e-03$	> 0.05
MOMBI-MTCH	$1.02e-19$	$1.09e-21$	$4.55e-07$	$1.42e-18$	> 0.05	> 0.05	> 0.05	> 0.05	–	$4.72e-02$	> 0.05
MOMBI-NMTCH	$5.23e-10$	$1.02e-12$	$2.24e-02$	$1.78e-08$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	$2.33e-34$	$4.30e-34$	$2.87e-32$	$8.34e-35$	$2.15e-32$	$1.52e-32$	$2.71e-32$	$5.00e-33$	$2.28e-32$	$8.23e-33$	–



Table A.115: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ4 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
5D											
MOMBI-WS	—	$2.43e-02$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NWS	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$3.67e-28$	$2.42e-29$	—	$3.66e-21$	$> 0.05$	$> 0.05$	$1.19e-17$	$7.12e-21$	$1.50e-17$	$2.15e-18$	$> 0.05$
MOMBI-NLS	$1.42e-07$	$6.01e-11$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-TCH	$4.45e-31$	$1.05e-31$	$5.73e-21$	$7.56e-27$	—	$> 0.05$	$3.00e-26$	$3.85e-29$	$2.51e-26$	$7.88e-27$	$> 0.05$
MOMBI-NTCH	$5.92e-23$	$5.38e-25$	$7.93e-08$	$1.69e-13$	$> 0.05$	—	$1.04e-12$	$7.71e-18$	$7.29e-13$	$1.02e-13$	$> 0.05$
MOMBI-ATCH	$1.50e-16$	$1.89e-19$	$> 0.05$	$2.62e-06$	$> 0.05$	$> 0.05$	—	$1.80e-02$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NATCH	$6.46e-08$	$7.67e-11$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$1.26e-17$	$2.21e-20$	$> 0.05$	$9.90e-08$	$> 0.05$	$> 0.05$	$> 0.05$	$2.20e-02$	—	$> 0.05$	$> 0.05$
MOMBI-NMTCH	$9.53e-16$	$9.54e-19$	$> 0.05$	$2.37e-05$	$> 0.05$	$> 0.05$	$> 0.05$	$3.48e-02$	$> 0.05$	—	$> 0.05$
MOMBI-PBI	$2.46e-35$	$4.11e-35$	$1.28e-34$	$7.63e-36$	$1.27e-34$	$1.27e-34$	$1.20e-34$	$1.27e-34$	$1.25e-34$	$1.27e-34$	—
6D											
MOMBI-WS	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NWS	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$9.22e-31$	$1.52e-31$	—	$2.87e-29$	$> 0.05$	$> 0.05$	$2.66e-29$	$9.22e-30$	$9.83e-30$	$3.14e-30$	$> 0.05$
MOMBI-NLS	$3.13e-09$	$4.51e-13$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$6.28e-05$	$3.07e-04$	$2.47e-07$	$> 0.05$
MOMBI-TCH	$1.21e-34$	$9.25e-35$	$1.56e-25$	$5.57e-36$	—	$> 0.05$	$2.44e-35$	$1.15e-34$	$7.70e-35$	$1.46e-34$	$> 0.05$
MOMBI-NTCH	$7.29e-29$	$3.12e-30$	$2.13e-15$	$2.50e-25$	$> 0.05$	—	$6.80e-26$	$2.71e-27$	$5.59e-27$	$3.68e-28$	$> 0.05$
MOMBI-ATCH	$3.15e-07$	$3.52e-11$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$4.14e-03$	$1.19e-02$	$2.00e-05$	$> 0.05$
MOMBI-NATCH	$5.64e-03$	$1.94e-05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$2.63e-03$	$7.42e-06$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$2.99e-02$	$> 0.05$
MOMBI-NMTCH	$> 0.05$	$7.69e-03$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$
MOMBI-PBI	$3.35e-35$	$3.58e-35$	$1.28e-34$	$3.87e-37$	$1.28e-34$	$1.28e-34$	$2.46e-36$	$2.06e-35$	$1.21e-35$	$3.37e-35$	—
7D											
MOMBI-WS	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NWS	$7.24e-07$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.71e-05$	$> 0.05$	$> 0.05$
MOMBI-LS	$1.51e-26$	$2.90e-19$	—	$1.41e-15$	$> 0.05$	$> 0.05$	$1.46e-03$	$6.48e-19$	$2.19e-25$	$3.97e-20$	$> 0.05$
MOMBI-NLS	$6.23e-16$	$4.61e-04$	$> 0.05$	—	$> 0.05$	$> 0.05$	$1.09e-10$	$1.53e-03$	$9.12e-14$	$5.40e-05$	$> 0.05$
MOMBI-TCH	$2.51e-30$	$2.47e-23$	$> 0.05$	$3.13e-19$	—	$> 0.05$	$6.74e-28$	$7.03e-23$	$2.85e-29$	$2.47e-24$	$> 0.05$
MOMBI-NTCH	$1.07e-30$	$6.97e-26$	$2.78e-08$	$3.76e-23$	$1.06e-06$	—	$6.81e-29$	$1.38e-25$	$5.99e-30$	$1.85e-26$	$> 0.05$
MOMBI-ATCH	$2.01e-02$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NATCH	$1.03e-07$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$3.56e-04$	—	$2.92e-06$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	—	$> 0.05$	$> 0.05$
MOMBI-NMTCH	$1.59e-06$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$3.34e-03$	$> 0.05$	$3.92e-05$	—	$> 0.05$
MOMBI-PBI	$3.43e-35$	$3.22e-35$	$1.20e-34$	$4.95e-36$	$1.25e-34$	$1.26e-34$	$3.71e-35$	$2.51e-35$	$4.32e-35$	$3.03e-35$	—

Table A.116: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ4 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
8D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	2.02e – 31	6.80e – 32	–	1.39e – 27	2.76e – 12	> 0.05	1.15e – 29	9.65e – 30	6.15e – 31	3.22e – 30	> 0.05
MOMBI-NLS	1.27e – 14	1.64e – 17	> 0.05	–	> 0.05	> 0.05	5.48e – 06	1.76e – 06	1.83e – 12	9.19e – 09	> 0.05
MOMBI-TCH	3.10e – 14	1.00e – 16	> 0.05	> 0.05	–	> 0.05	4.65e – 05	1.29e – 05	4.23e – 11	9.94e – 08	> 0.05
MOMBI-NTCH	7.12e – 33	3.88e – 33	5.76e – 07	7.99e – 31	3.26e – 22	–	9.28e – 32	6.68e – 32	1.84e – 32	4.65e – 32	> 0.05
MOMBI-ATCH	6.03e – 05	4.11e – 07	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	4.37e – 04	> 0.05	> 0.05
MOMBI-NATCH	7.17e – 06	1.47e – 08	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	9.46e – 05	3.78e – 02	> 0.05
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	8.83e – 03	2.34e – 04	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	2.45e – 02	–	> 0.05
MOMBI-PBI	5.09e – 35	5.84e – 35	1.27e – 34	5.98e – 37	1.27e – 34	1.27e – 34	1.54e – 35	1.57e – 35	4.70e – 35	3.07e – 35	–
9D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	3.90e – 31	3.69e – 30	–	4.38e – 30	5.03e – 24	3.58e – 03	1.99e – 30	2.06e – 30	1.70e – 30	1.18e – 30	> 0.05
MOMBI-NLS	5.20e – 07	5.58e – 07	> 0.05	–	1.03e – 05	> 0.05	> 0.05	3.11e – 02	2.81e – 02	9.81e – 04	> 0.05
MOMBI-TCH	1.55e – 02	5.89e – 03	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	1.57e – 24	5.72e – 24	> 0.05	2.45e – 19	1.26e – 20	–	4.69e – 20	4.86e – 21	4.64e – 21	1.74e – 22	> 0.05
MOMBI-ATCH	7.34e – 06	6.96e – 06	> 0.05	> 0.05	1.65e – 04	> 0.05	–	> 0.05	> 0.05	6.73e – 03	> 0.05
MOMBI-NATCH	4.25e – 04	3.42e – 04	> 0.05	> 0.05	1.61e – 02	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	1.33e – 03	1.18e – 03	> 0.05	> 0.05	1.71e – 02	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	1.77e – 02	1.39e – 02	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	3.09e – 35	3.80e – 35	1.28e – 34	8.92e – 37	1.28e – 34	1.28e – 34	1.28e – 36	4.46e – 36	4.11e – 36	1.14e – 35	–
10D											
MOMBI-WS	–	2.28e – 02	> 0.05	> 0.05	3.41e – 32	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	6.44e – 31	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	2.82e – 39	1.15e – 35	–	1.52e – 37	1.84e – 34	1.66e – 21	2.82e – 39	5.76e – 39	2.82e – 39	5.76e – 39	> 0.05
MOMBI-NLS	8.39e – 04	2.35e – 04	> 0.05	–	7.70e – 31	> 0.05	8.39e – 04	1.64e – 03	8.39e – 04	3.17e – 04	> 0.05
MOMBI-TCH	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	1.63e – 23	1.15e – 20	> 0.05	4.67e – 22	5.67e – 32	–	1.63e – 23	2.46e – 23	1.63e – 23	2.46e – 23	> 0.05
MOMBI-ATCH	> 0.05	2.28e – 02	> 0.05	> 0.05	3.41e – 32	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NATCH	> 0.05	3.36e – 02	> 0.05	> 0.05	6.08e – 32	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	> 0.05	2.28e – 02	> 0.05	> 0.05	3.41e – 32	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	6.08e – 32	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	1.44e – 39	8.27e – 38	7.67e – 35	8.28e – 38	7.67e – 35	7.67e – 35	1.44e – 39	2.98e – 39	1.44e – 39	2.98e – 39	–

Table A.117: Comparison of R2 indicator values for different optimizers on the DTLZ4 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	2.500000e-01	2.500000e-01	2.500000e-01	2.500000e-01	2.180309e-01	2.182518e-01	2.181233e-01	2.182440e-01	2.180399e-01	2.182578e-01	2.180300e-01
	avg.	2.550000e-01	2.575000e-01	2.575000e-01	2.625000e-01	2.180310e-01	2.295197e-01	2.237582e-01	2.351500e-01	2.264985e-01	2.210748e-01	2.180298e-01
	min.	2.500000e-01	2.500000e-01	2.500000e-01	2.500000e-01	2.180280e-01	2.181659e-01	2.180913e-01	2.181492e-01	2.180320e-01	2.181531e-01	2.180245e-01
	max.	5.000000e-01	5.000000e-01	5.000000e-01	5.000000e-01	2.180338e-01	5.000000e-01	5.000000e-01	5.000000e-01	5.000000e-01	5.000000e-01	2.180349e-01
	std.	3.517632e-02	4.286165e-02	4.286165e-02	5.476073e-02	1.072315e-06	5.548971e-02	3.966192e-02	6.725028e-02	4.834114e-02	2.817429e-02	1.680112e-06
3D	med.	1.110692e-01	1.110692e-01	1.059536e-01	1.110692e-01	9.478691e-02	9.483128e-02	9.496119e-02	9.511334e-02	9.484772e-02	9.488187e-02	9.375468e-02
	avg.	1.149516e-01	1.171616e-01	1.089019e-01	1.177349e-01	9.578980e-02	9.686440e-02	9.952338e-02	1.101934e-01	9.687176e-02	1.051061e-01	9.375481e-02
	min.	1.110692e-01	1.110692e-01	1.001554e-01	1.110692e-01	9.462488e-02	9.459429e-02	9.482763e-02	9.496316e-02	9.472839e-02	9.464291e-02	9.374634e-02
	max.	1.672105e-01	1.672105e-01	1.671752e-01	1.672105e-01	1.453211e-01	1.465781e-01	1.464239e-01	3.333219e-01	1.464233e-01	1.468506e-01	9.376484e-02
	std.	1.422336e-02	1.741802e-02	1.349074e-02	1.814300e-02	7.111513e-03	1.008088e-02	1.460586e-02	3.157400e-02	1.000512e-02	2.060498e-02	3.811089e-06
4D	med.	6.238390e-02	6.238390e-02	5.989474e-02	6.238390e-02	5.605001e-02	5.602631e-02	5.712015e-02	5.717166e-02	5.724383e-02	5.719850e-02	5.307823e-02
	avg.	6.406389e-02	6.992838e-02	6.108773e-02	6.552715e-02	5.827292e-02	5.925801e-02	5.840868e-02	6.507639e-02	5.819308e-02	6.188367e-02	5.343493e-02
	min.	6.238367e-02	6.238390e-02	5.727860e-02	6.238390e-02	5.588421e-02	5.584383e-02	5.657340e-02	5.685792e-02	5.681552e-02	5.605676e-02	5.305676e-02
	max.	8.349015e-02	1.256637e-01	8.349015e-02	8.349015e-02	7.327319e-02	7.325827e-02	7.331115e-02	1.109276e-01	7.345964e-02	1.108624e-01	7.091970e-02
	std.	5.725910e-03	1.443274e-02	5.246596e-03	7.520247e-03	5.803996e-03	6.755781e-03	4.365839e-03	1.292759e-02	3.858685e-03	8.659617e-03	2.509608e-03
5D	med.	4.068163e-02	4.068163e-02	3.943977e-02	4.068163e-02	3.730086e-02	3.729837e-02	4.025098e-02	4.037086e-02	4.024924e-02	4.025699e-02	3.524734e-02
	avg.	4.245647e-02	4.503683e-02	4.029831e-02	4.304238e-02	3.820068e-02	3.998466e-02	4.130117e-02	4.538796e-02	4.184948e-02	4.269644e-02	3.524759e-02
	min.	4.068163e-02	4.068163e-02	3.748660e-02	4.067917e-02	3.729360e-02	3.722658e-02	4.006501e-02	4.008429e-02	3.989725e-02	3.987674e-02	3.522725e-02
	max.	5.259538e-02	6.678332e-02	5.213988e-02	6.678332e-02	4.836343e-02	6.025993e-02	5.084245e-02	8.867448e-02	5.082129e-02	6.387135e-02	3.527337e-02
	std.	3.873333e-03	6.002346e-03	3.090726e-03	5.482627e-03	2.774234e-03	4.522956e-03	2.812139e-03	7.201238e-03	3.325935e-03	4.896184e-03	9.096936e-06
6D	med.	3.356277e-02	3.356277e-02	3.207350e-02	3.329674e-02	3.876829e-02	3.052267e-02	3.329674e-02	3.356277e-02	3.356277e-02	3.356277e-02	2.430800e-02
	avg.	3.342420e-02	3.453204e-02	3.211924e-02	3.373313e-02	3.755434e-02	3.335181e-02	3.354513e-02	3.511658e-02	3.348812e-02	3.405338e-02	2.430828e-02
	min.	3.287427e-02	3.287427e-02	2.599707e-02	3.287427e-02	3.003495e-02	2.986229e-02	3.287427e-02	3.287427e-02	3.287427e-02	3.287427e-02	2.429085e-02
	max.	3.372373e-02	4.209235e-02	3.994655e-02	5.497776e-02	4.122722e-02	5.096009e-02	4.136239e-02	5.233265e-02	4.119150e-02	4.166045e-02	2.432353e-02
	std.	3.197455e-04	2.892944e-03	1.602240e-03	2.477812e-03	4.192627e-03	4.600377e-03	1.403347e-03	4.101689e-03	8.362912e-04	2.257038e-03	6.371216e-06
7D	med.	3.561079e-02	3.560829e-02	3.454569e-02	3.561079e-02	4.815946e-02	4.008203e-02	3.560829e-02	3.560829e-02	3.561079e-02	3.561079e-02	1.840360e-02
	avg.	3.559080e-02	3.645278e-02	3.424336e-02	3.560463e-02	4.745189e-02	3.850570e-02	3.558887e-02	3.620639e-02	3.581412e-02	3.579016e-02	1.850363e-02
	min.	3.515607e-02	3.515607e-02	2.741864e-02	3.502401e-02	3.755865e-02	3.212523e-02	3.515607e-02	3.515607e-02	3.515607e-02	3.515607e-02	1.840085e-02
	max.	3.608318e-02	7.153649e-02	3.608318e-02	3.608318e-02	5.331162e-02	4.941235e-02	3.608318e-02	6.463228e-02	4.625063e-02	4.636470e-02	2.095778e-02
	std.	2.441223e-04	4.302323e-03	1.748148e-03	2.474047e-04	3.884764e-03	5.094172e-03	2.542943e-04	3.402451e-03	1.507311e-03	1.530560e-03	4.919104e-04
8D	med.	3.126740e-02	3.127948e-02	2.985930e-02	3.124810e-02	4.550909e-02	3.774974e-02	3.126625e-02	3.121280e-02	3.126740e-02	3.126740e-02	1.400548e-02
	avg.	3.127877e-02	3.129779e-02	2.971541e-02	3.122618e-02	4.503406e-02	3.690827e-02	3.126355e-02	3.125491e-02	3.125699e-02	3.136515e-02	1.400532e-02
	min.	3.090852e-02	3.090852e-02	2.363418e-02	2.942122e-02	3.345738e-02	2.843034e-02	3.090852e-02	3.090852e-02	3.090852e-02	3.090852e-02	1.399804e-02
	max.	3.180963e-02	3.180963e-02	3.150389e-02	3.180963e-02	4.896650e-02	4.352262e-02	3.180963e-02	3.180963e-02	3.167959e-02	4.065152e-02	1.401190e-02
	std.	2.162661e-04	2.239836e-04	1.444600e-03	2.798326e-04	2.891159e-03	4.026575e-03	2.191480e-04	2.117802e-04	2.076615e-04	9.615873e-04	2.442933e-06
9D	med.	2.768119e-02	2.773761e-02	2.586657e-02	2.767667e-02	4.209362e-02	3.541774e-02	2.772112e-02	2.773689e-02	2.773689e-02	2.768119e-02	1.100282e-02
	avg.	2.769976e-02	2.764962e-02	2.532657e-02	2.767354e-02	4.142919e-02	3.463256e-02	2.772983e-02	2.773099e-02	2.775005e-02	2.770721e-02	1.101380e-02
	min.	2.666141e-02	2.197259e-02	1.761251e-02	2.633563e-02	2.714807e-02	2.542430e-02	2.666141e-02	2.666141e-02	2.666141e-02	2.666141e-02	1.099106e-02
	max.	2.831601e-02	2.831601e-02	2.757145e-02	2.846501e-02	4.434706e-02	4.073034e-02	2.846501e-02	2.831601e-02	2.846501e-02	2.846501e-02	1.207755e-02
	std.	2.468006e-04	6.404351e-04	1.940926e-03	3.639033e-04	2.829738e-03	3.479869e-03	2.601809e-04	2.170061e-04	2.478404e-04	2.259916e-04	1.075289e-04
10D	med.	2.499901e-02	2.501222e-02	2.265841e-02	2.498156e-02	3.901890e-02	3.388897e-02	2.504395e-02	2.499901e-02	2.501222e-02	2.501222e-02	8.835126e-03
	avg.	2.497502e-02	2.491970e-02	2.175269e-02	2.497299e-02	3.874401e-02	3.336822e-02	2.504073e-02	2.500959e-02	2.500849e-02	2.500131e-02	8.835488e-03
	min.	2.461639e-02	1.982537e-02	1.591072e-02	2.441976e-02	3.332037e-02	2.268224e-02	2.461639e-02	2.470820e-02	2.461639e-02	2.451115e-02	8.823274e-03
	max.	2.535130e-02	2.535130e-02	2.458150e-02	2.535130e-02	4.051128e-02	3.725855e-02	2.535130e-02	2.535130e-02	2.535130e-02	2.535130e-02	8.845786e-03
	std.	1.488031e-04	7.406922e-04	2.099856e-03	1.592983e-04	1.455348e-03	3.008016e-03	1.588889e-04	1.543364e-04	1.623221e-04	1.747061e-04	5.266284e-06

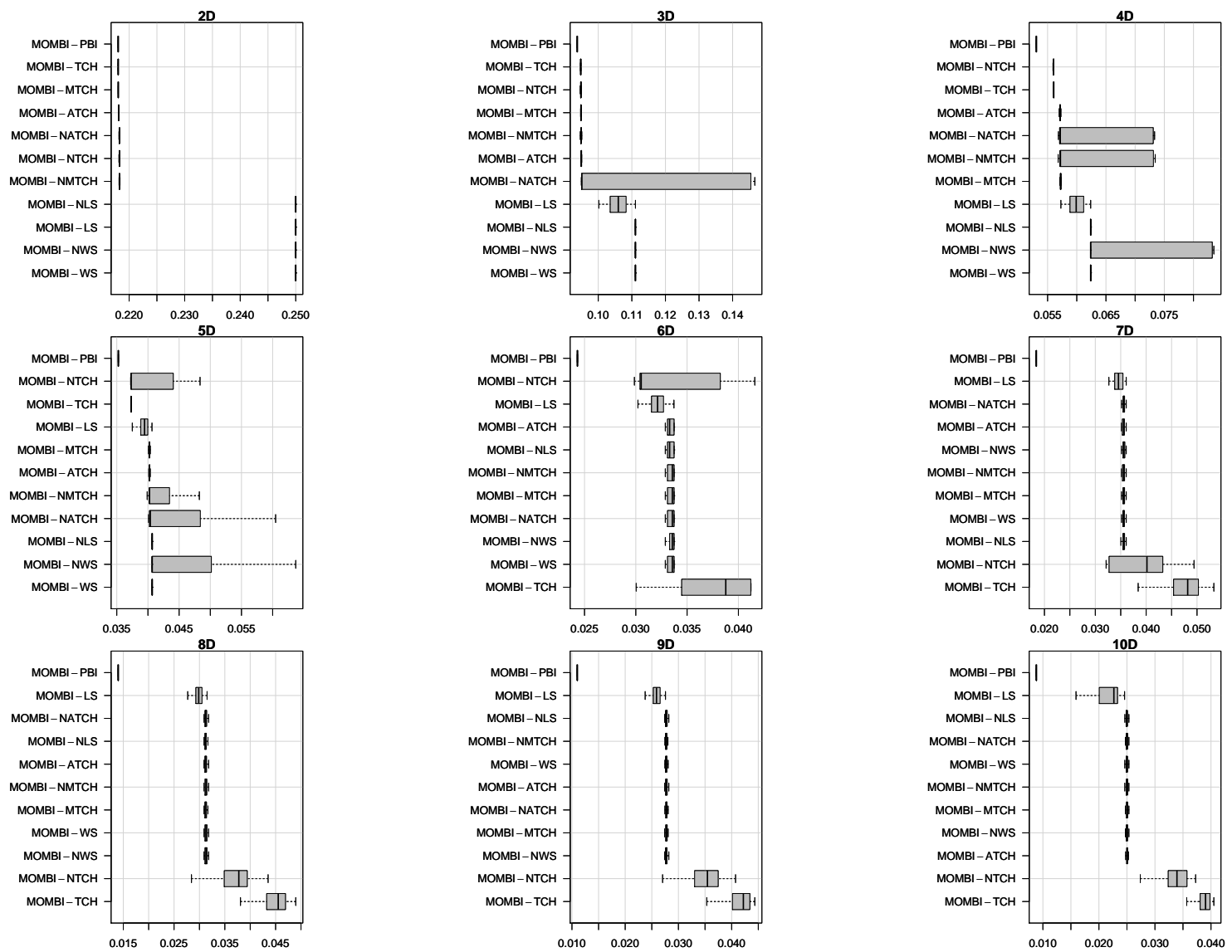


Figure A.288: Box-plot of the R2 indicator values for different optimizers on the DTLZ4 test problem.

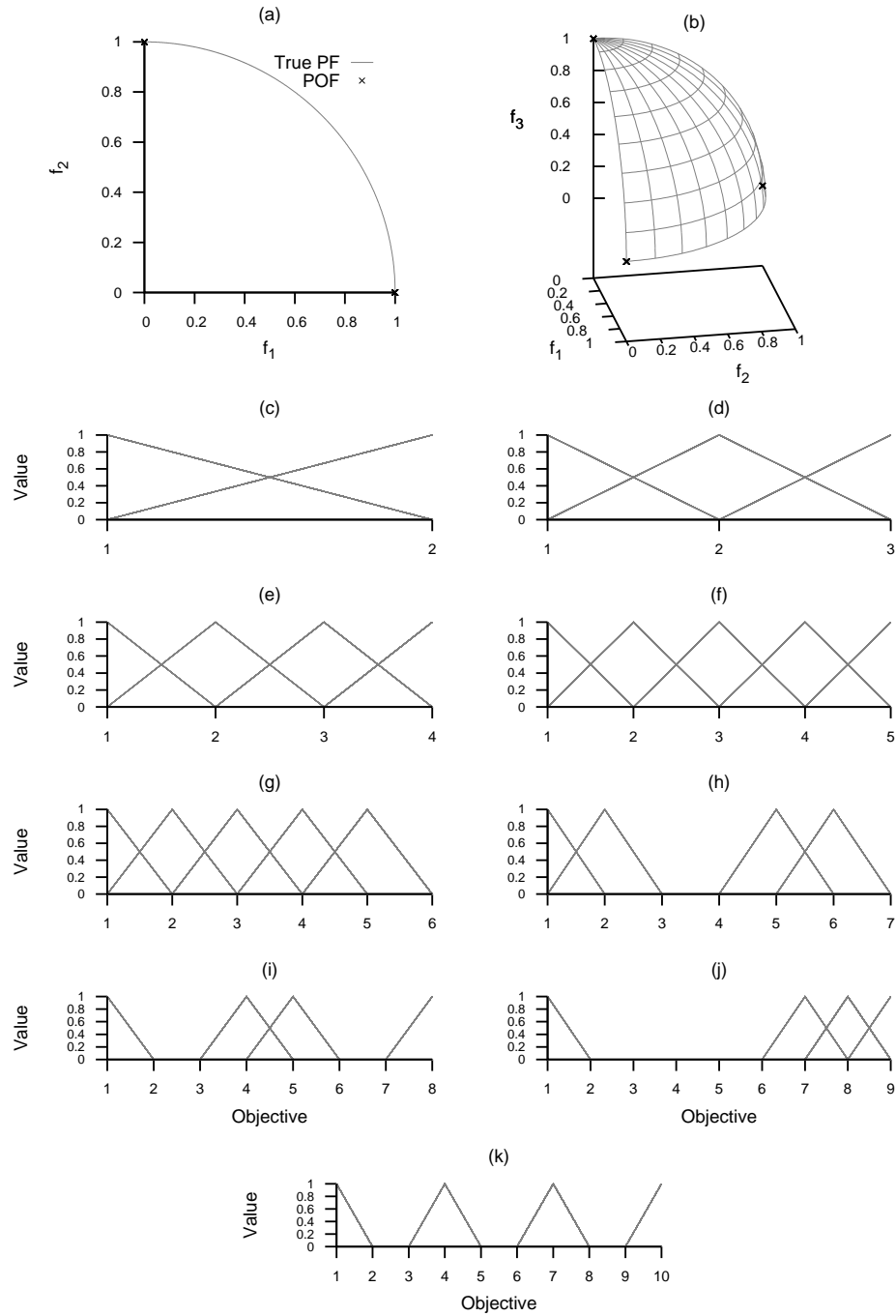


Figure A.289: Plots of the approximations obtained by MOMBI-WS from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

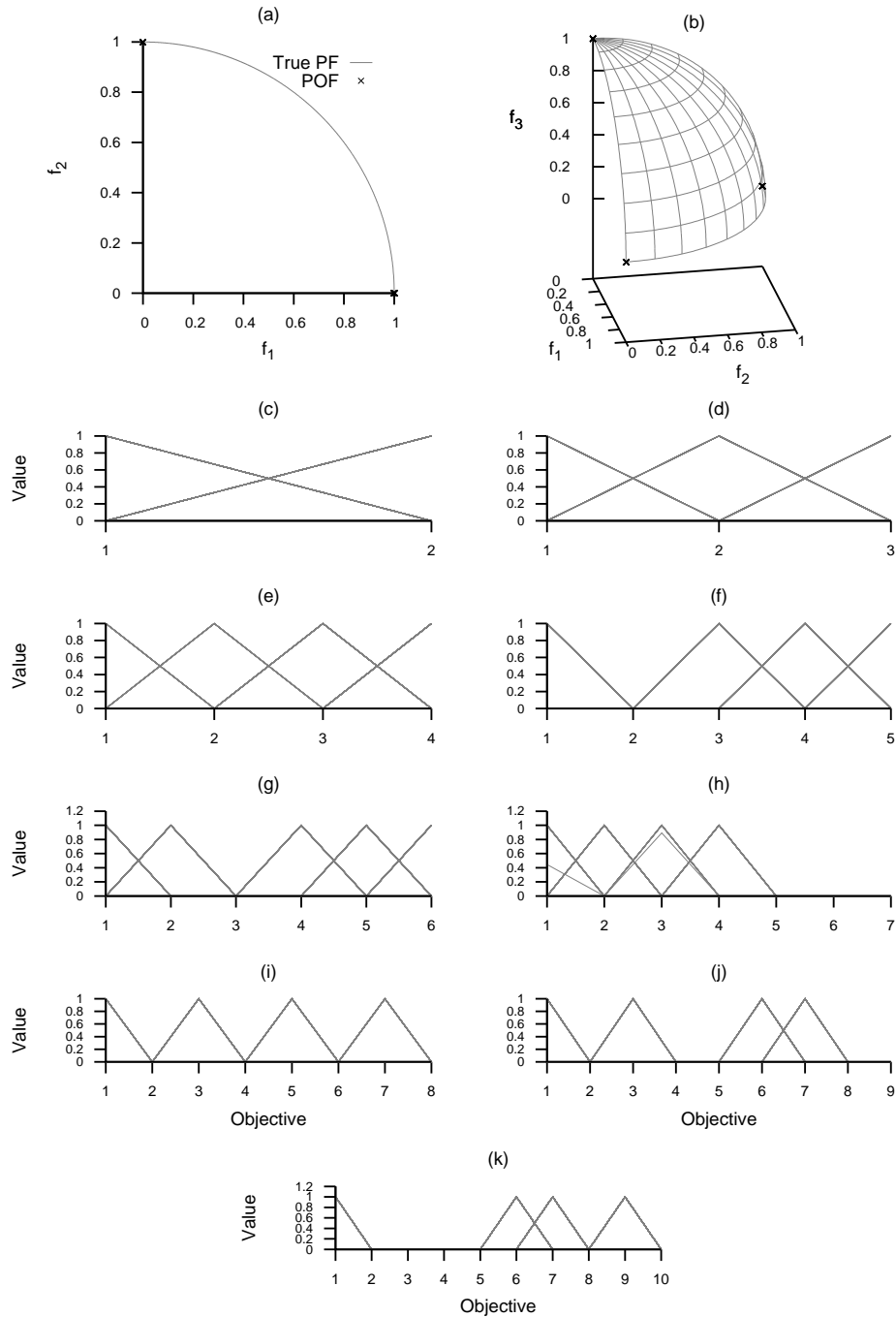


Figure A.290: Plots of the approximations obtained by MOMBI-NWS from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

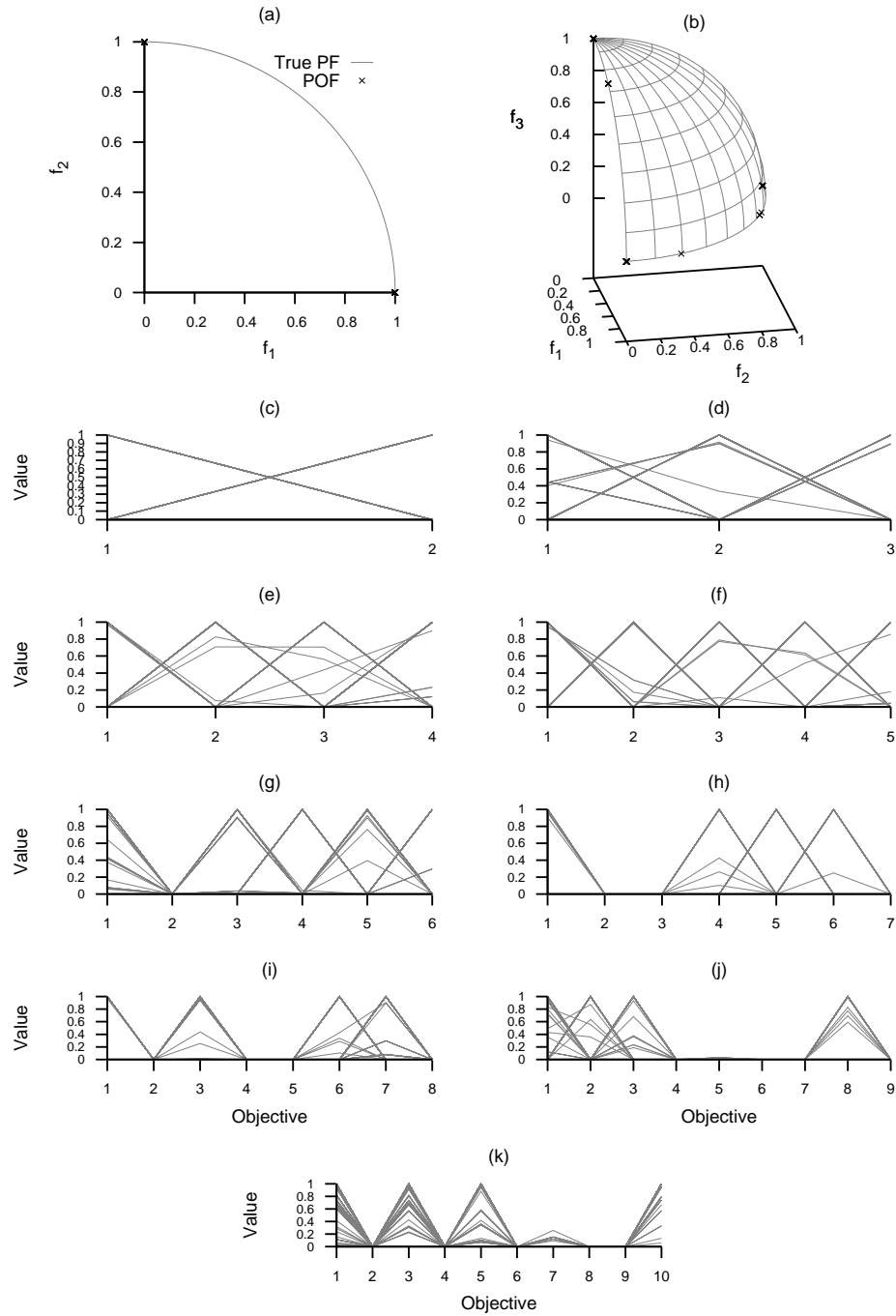


Figure A.291: Plots of the approximations obtained by MOMBI-LS from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

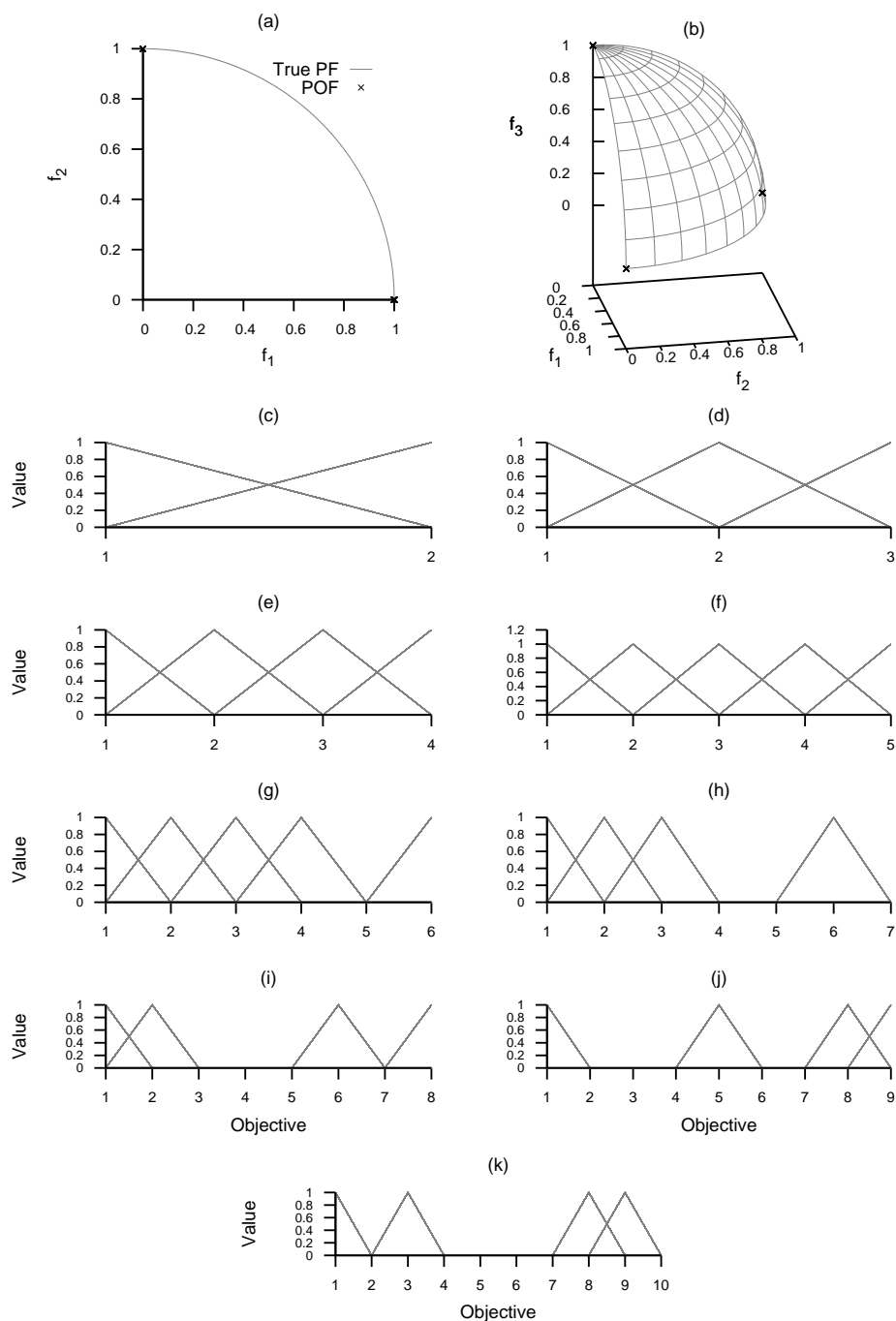


Figure A.292: Plots of the approximations obtained by MOMBI-NLS from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



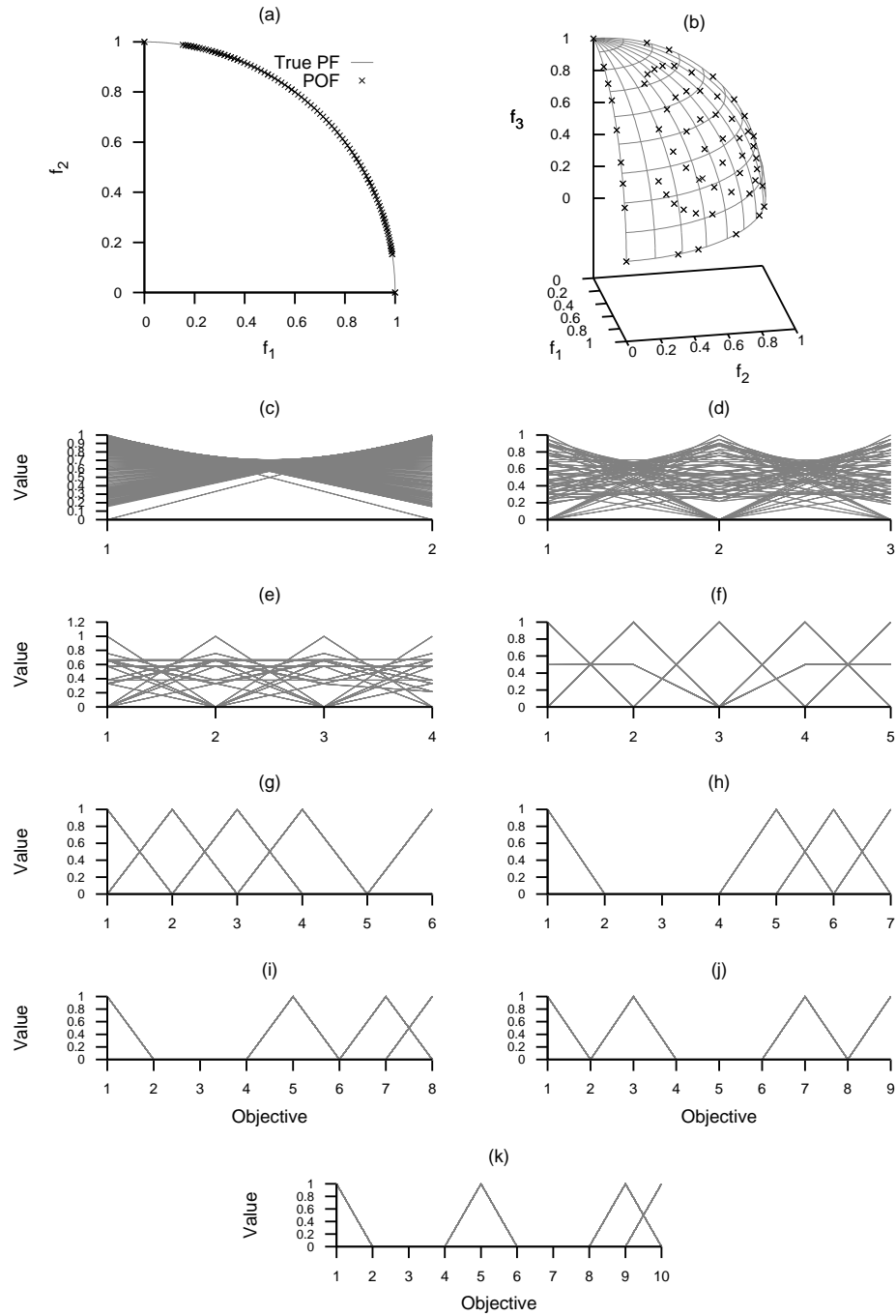


Figure A.293: Plots of the approximations obtained by MOMBI-ATCH from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

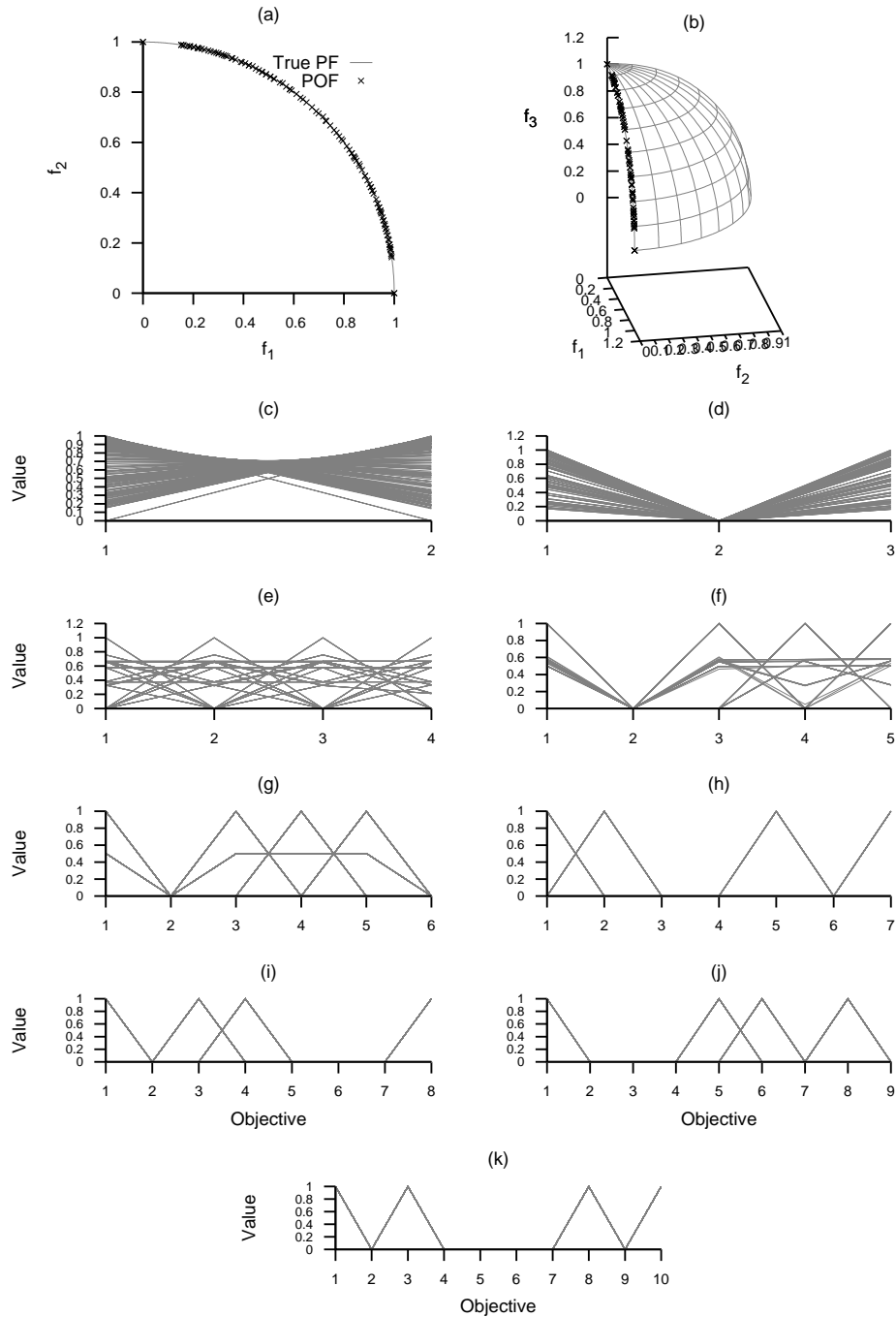


Figure A.294: Plots of the approximations obtained by MOMBI-NATCH from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

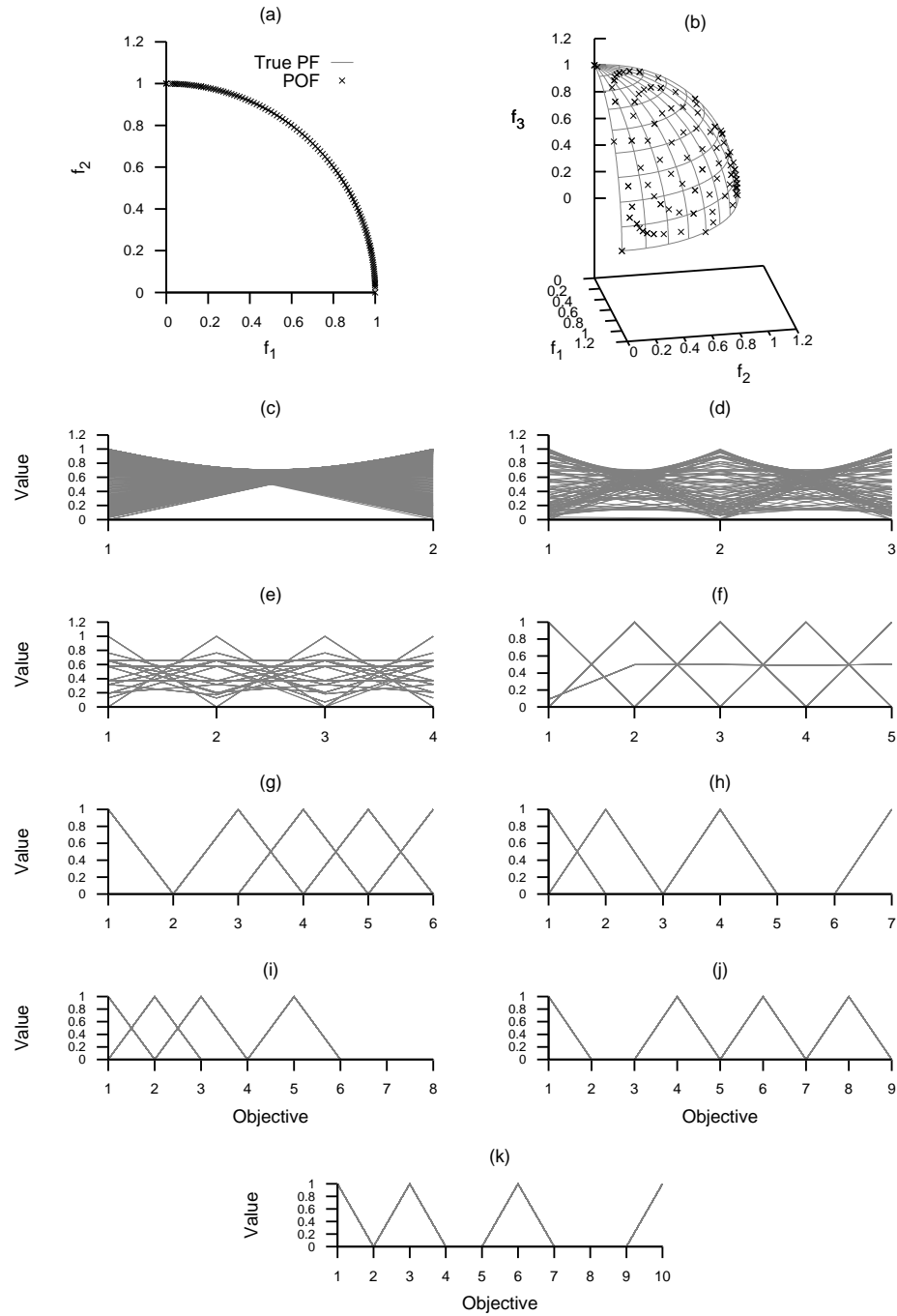


Figure A.295: Plots of the approximations obtained by MOMBI-MTCH from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

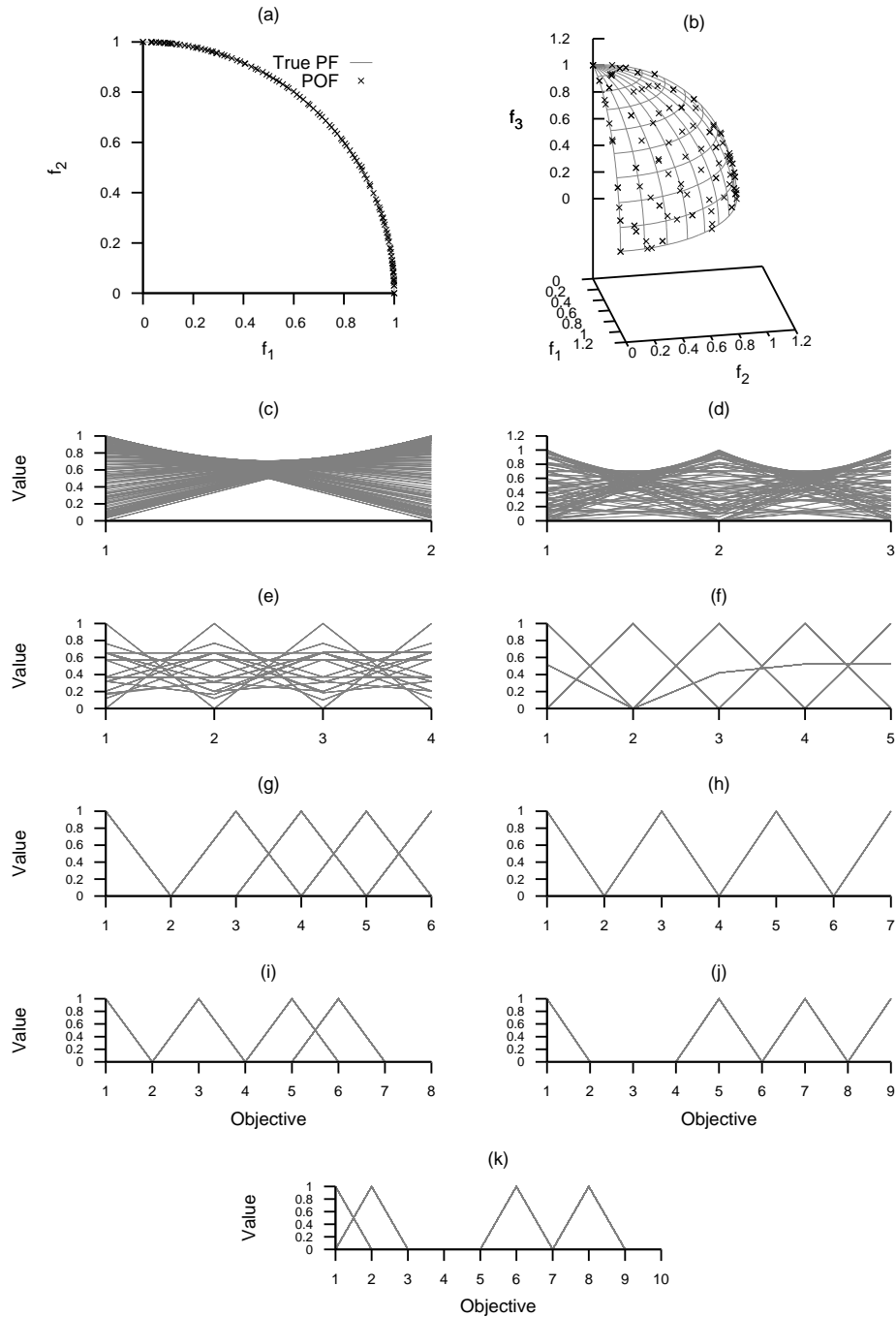


Figure A.296: Plots of the approximations obtained by MOMBI-NMTCH from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.118: Comparison of hypervolume indicator values for different optimizers on the DTLZ5 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	1.500000e+01	1.500000e+01	1.500000e+01	1.500000e+01	1.521071e+01	1.520925e+01	1.520979e+01	1.520908e+01	1.521080e+01	1.520943e+01	1.520987e+01
	avg.	1.500000e+01	1.500000e+01	1.500000e+01	1.500000e+01	1.521069e+01	1.520926e+01	1.520979e+01	1.520908e+01	1.521079e+01	1.520944e+01	1.520979e+01
	min.	1.500000e+01	1.500000e+01	1.500000e+01	1.500000e+01	1.521029e+01	1.520873e+01	1.520949e+01	1.520857e+01	1.521069e+01	1.520876e+01	1.520771e+01
	max.	1.500000e+01	1.500000e+01	1.500000e+01	1.500000e+01	1.521087e+01	1.520985e+01	1.520998e+01	1.520947e+01	1.521082e+01	1.521010e+01	1.521072e+01
	std.	0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00	1.498239e-04	2.412462e-04	5.538455e-05	1.866184e-04	1.624826e-05	2.643041e-04	5.522230e-04
3D	med.	5.884315e+01	5.884315e+01	5.884315e+01	5.884315e+01	5.984253e+01	5.984328e+01	5.983070e+01	5.983054e+01	5.984202e+01	5.984282e+01	5.973309e+01
	avg.	5.884315e+01	5.884315e+01	5.884315e+01	5.884315e+01	5.984209e+01	5.984340e+01	5.983077e+01	5.983081e+01	5.984215e+01	5.984300e+01	5.973300e+01
	min.	5.884315e+01	5.884315e+01	5.884315e+01	5.884315e+01	5.983860e+01	5.983463e+01	5.982986e+01	5.982402e+01	5.984184e+01	5.983466e+01	5.972196e+01
	max.	5.884315e+01	5.884315e+01	5.884315e+01	5.884315e+01	5.984363e+01	5.985322e+01	5.983399e+01	5.984238e+01	5.984323e+01	5.985250e+01	5.973829e+01
	std.	2.443651e-06	2.443651e-06	2.443651e-06	2.443651e-06	1.131695e-03	4.253538e-03	4.521303e-04	3.825647e-03	2.664407e-04	4.246216e-03	3.082657e-03
4D	med.	2.335825e+02	2.351471e+02	2.335830e+02	2.353949e+02	2.388274e+02	2.392515e+02	2.393519e+02	2.393033e+02	2.392167e+02	2.393804e+02	2.381440e+02
	avg.	2.335832e+02	2.350679e+02	2.336115e+02	2.354260e+02	2.388427e+02	2.392180e+02	2.393489e+02	2.392803e+02	2.392088e+02	2.393780e+02	2.381444e+02
	min.	2.335825e+02	2.335825e+02	2.335825e+02	2.335828e+02	2.387520e+02	2.385765e+02	2.392158e+02	2.388813e+02	2.390791e+02	2.391114e+02	2.380934e+02
	max.	2.336498e+02	2.368670e+02	2.340501e+02	2.370241e+02	2.390934e+02	2.394771e+02	2.394508e+02	2.394844e+02	2.393431e+02	2.395439e+02	2.381964e+02
	std.	6.728496e-03	9.634074e-01	7.827934e-02	8.841632e-01	6.242949e-02	1.695007e-01	3.479854e-02	1.269131e-01	6.088671e-02	7.935309e-02	1.727208e-02
5D	med.	9.308495e+02	9.308495e+02	9.399221e+02	9.339654e+02	9.457133e+02	9.497254e+02	9.518612e+02	9.528551e+02	9.521641e+02	9.534972e+02	9.471369e+02
	avg.	9.308664e+02	9.309740e+02	9.401342e+02	9.343904e+02	9.458777e+02	9.495980e+02	9.519152e+02	9.524897e+02	9.524109e+02	9.536680e+02	9.470359e+02
	min.	9.308493e+02	9.308493e+02	9.397786e+02	9.308471e+02	9.450829e+02	9.459158e+02	9.516595e+02	9.475535e+02	9.515607e+02	9.511955e+02	9.460937e+02
	max.	9.324035e+02	9.372719e+02	9.430586e+02	9.414885e+02	9.488152e+02	9.534475e+02	9.530108e+02	9.551880e+02	9.543862e+02	9.565886e+02	9.478844e+02
	std.	1.555086e-01	7.204683e-01	5.684995e-01	2.597547e+00	5.196875e-01	1.588238e+00	2.155428e-01	1.439116e+00	6.127013e-01	1.033731e+00	3.937567e-01
6D	med.	3.711774e+03	3.712957e+03	3.750423e+03	3.744047e+03	3.752972e+03	3.771028e+03	3.785930e+03	3.783677e+03	3.792925e+03	3.790336e+03	3.771074e+03
	avg.	3.711878e+03	3.718371e+03	3.751373e+03	3.743647e+03	3.752949e+03	3.771227e+03	3.786684e+03	3.783219e+03	3.793361e+03	3.790556e+03	3.771084e+03
	min.	3.711746e+03	3.711496e+03	3.749767e+03	3.729283e+03	3.749442e+03	3.740942e+03	3.785768e+03	3.745661e+03	3.790159e+03	3.777270e+03	3.764182e+03
	max.	3.715896e+03	3.749707e+03	3.759700e+03	3.753809e+03	3.766987e+03	3.797604e+03	3.801969e+03	3.806486e+03	3.799284e+03	3.808818e+03	3.784536e+03
	std.	4.840053e-01	1.049225e+01	2.114113e+00	5.506492e+00	1.902042e+00	1.054499e+01	2.358700e+00	9.909918e+00	1.247848e+00	6.540314e+00	3.490121e+00
7D	med.	1.480782e+04	1.493486e+04	1.494238e+04	1.497064e+04	1.492062e+04	1.495067e+04	1.508540e+04	1.492226e+04	1.510313e+04	1.498845e+04	1.493697e+04
	avg.	1.481173e+04	1.486350e+04	1.494422e+04	1.496710e+04	1.492059e+04	1.495211e+04	1.508752e+04	1.487318e+04	1.510427e+04	1.497726e+04	1.493872e+04
	min.	1.480770e+04	8.009838e+03	1.493944e+04	1.486508e+04	1.479388e+04	1.486946e+04	1.508496e+04	1.455069e+04	1.510245e+04	1.465605e+04	1.492733e+04
	max.	1.488144e+04	1.499752e+04	1.497357e+04	1.505459e+04	1.495573e+04	1.506755e+04	1.511415e+04	1.507091e+04	1.511869e+04	1.510223e+04	1.496219e+04
	std.	1.194472e+01	6.929555e+02	5.744193e+00	2.919712e+01	1.376522e+01	4.104625e+01	5.455210e+00	1.438867e+02	2.745033e+00	7.274538e+01	7.426119e+00
8D	med.	5.933896e+04	5.965302e+04	5.982977e+04	5.991644e+04	5.978657e+04	5.980507e+04	6.036441e+04	5.923528e+04	6.042756e+04	5.990610e+04	5.962369e+04
	avg.	5.909717e+04	5.965638e+04	5.985670e+04	5.991874e+04	5.978810e+04	5.980796e+04	6.037093e+04	5.910972e+04	6.042472e+04	5.983433e+04	5.963252e+04
	min.	3.359630e+04	5.953860e+04	5.980335e+04	5.982565e+04	5.977181e+04	5.914743e+04	6.036113e+04	5.666650e+04	5.965449e+04	5.896700e+04	5.957713e+04
	max.	5.957993e+04	5.990215e+04	6.008481e+04	6.003340e+04	5.989905e+04	6.037852e+04	6.052333e+04	6.013095e+04	6.056005e+04	6.027646e+04	5.981212e+04
	std.	2.576136e+03	5.350226e+01	6.082860e+01	4.649591e+01	1.505085e+01	2.085267e+02	2.329061e+01	7.485912e+02	7.943457e+01	3.039689e+02	4.020784e+01
9D	med.	2.380426e+05	2.382460e+05	2.394614e+05	2.397876e+05	2.398795e+05	2.392360e+05	2.415343e+05	2.271201e+05	2.417784e+05	2.382209e+05	2.381501e+05
	avg.	2.380497e+05	2.382400e+05	2.394890e+05	2.398060e+05	2.398821e+05	2.392292e+05	2.415528e+05	2.282360e+05	2.418012e+05	2.383576e+05	2.382023e+05
	min.	2.378449e+05	2.370537e+05	2.393898e+05	2.393963e+05	2.397999e+05	2.370485e+05	2.407041e+05	2.185045e+05	2.417643e+05	2.345108e+05	2.379291e+05
	max.	2.383322e+05	2.390071e+05	2.401474e+05	2.403863e+05	2.401996e+05	2.407366e+05	2.419578e+05	2.397010e+05	2.422650e+05	2.413420e+05	2.388642e+05
	std.	4.882568e+01	2.744860e+02	9.067673e+01	2.059334e+02	4.686006e+01	7.273870e+02	1.174652e+02	5.385842e+03	8.164421e+01	1.616677e+03	1.906735e+02
10D	med.	9.549718e+05	9.510825e+05	9.583739e+05	9.594141e+05	9.613533e+05	9.587430e+05	9.664652e+05	9.031246e+05	9.671629e+05	9.457847e+05	9.516123e+05
	avg.	9.549819e+05	9.508121e+05	9.584477e+05	9.593832e+05	9.611107e+05	9.584653e+05	9.664671e+05	9.045491e+05	9.671737e+05	9.474359e+05	9.519719e+05
	min.	9.539402e+05	9.390160e+05	9.578755e+05	9.578072e+05	9.585495e+05	9.498439e+05	9.663613e+05	8.745780e+05	9.670897e+05	9.302486e+05	9.507467e+05
	max.	9.562041e+05	9.541308e+05	9.602822e+05	9.611066e+05	9.619862e+05	9.634853e+05	9.667138e+05	9.578789e+05	9.675727e+05	9.631783e+05	9.567958e+05
	std.	3.526870e+02	2.054154e+03	3.294785e+02	6.462758e+02	7.941707e+02	3.003023e+03	6.102366e+01	1.658421e+04	6.050973e+01	8.030071e+03	1.114461e+03

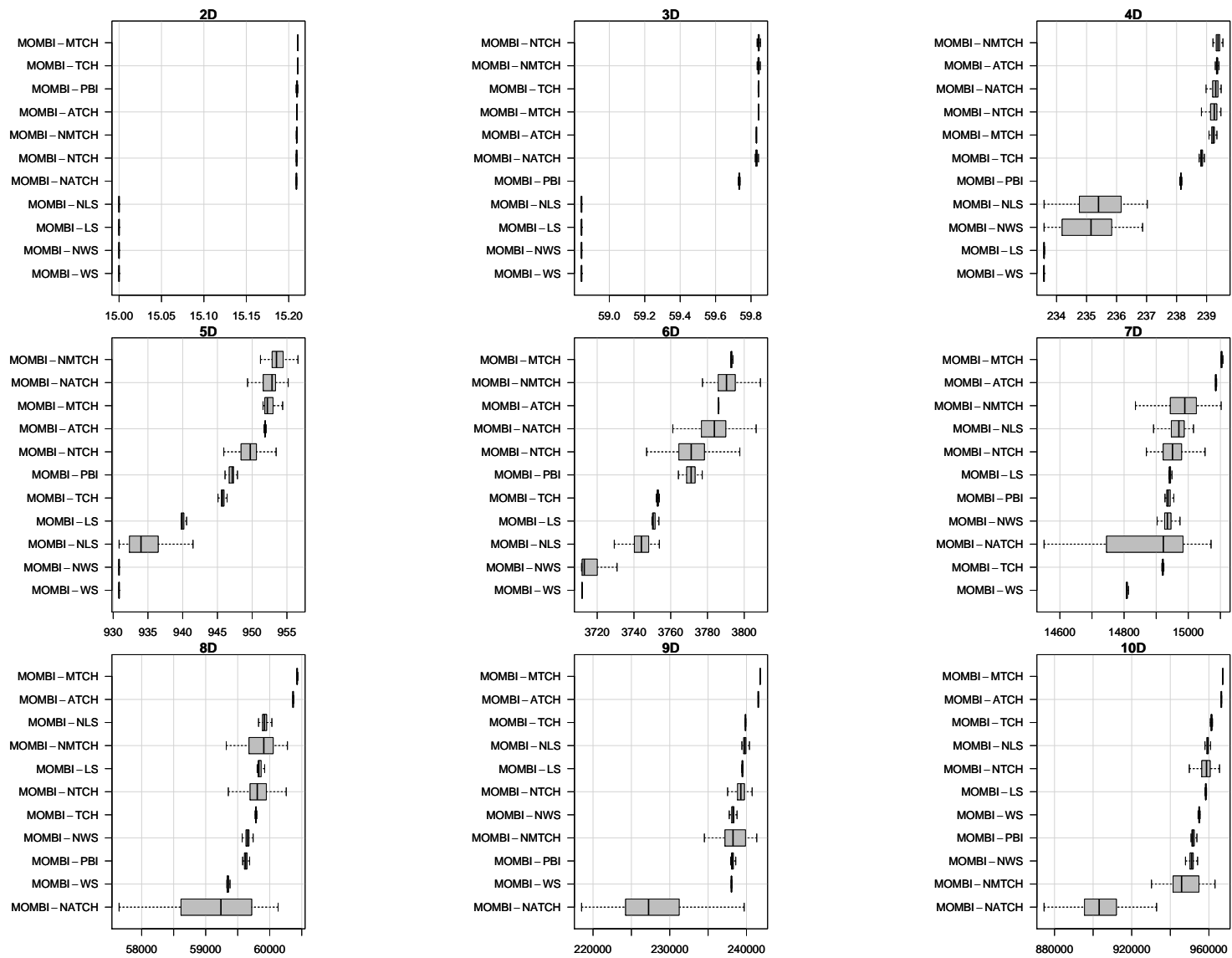


Figure A.297: Box-plot of the hypervolume indicator values for different optimizers on the DTLZ5 test problem.

Table A.119: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ5 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
2D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$2.78e-39$	$2.78e-39$	$2.78e-39$	$2.78e-39$	–	$1.26e-34$	$1.03e-34$	$1.26e-34$	> 0.05	$1.26e-34$	$6.47e-32$
MOMBI-NTCH	$2.80e-39$	$2.80e-39$	$2.80e-39$	$2.80e-39$	> 0.05	–	> 0.05	$7.38e-08$	> 0.05	> 0.05	> 0.05
MOMBI-ATCH	$2.16e-39$	$2.16e-39$	$2.16e-39$	$2.16e-39$	> 0.05	$6.24e-32$	–	$1.04e-34$	> 0.05	$3.80e-23$	> 0.05
MOMBI-NATCH	$2.80e-39$	$2.80e-39$	$2.80e-39$	$2.80e-39$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$1.86e-40$	$1.86e-40$	$1.86e-40$	$1.86e-40$	$1.95e-06$	$1.60e-35$	$1.30e-35$	$1.60e-35$	–	$1.60e-35$	$1.71e-35$
MOMBI-NMTCH	$2.80e-39$	$2.80e-39$	$2.80e-39$	$2.80e-39$	> 0.05	$2.01e-06$	> 0.05	$1.10e-19$	> 0.05	–	> 0.05
MOMBI-PBI	$2.81e-39$	$2.81e-39$	$2.81e-39$	$2.81e-39$	> 0.05	$3.77e-16$	> 0.05	$4.92e-22$	> 0.05	$5.67e-10$	–
3D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$2.81e-39$	$2.81e-39$	$2.81e-39$	$2.81e-39$	–	> 0.05	$1.13e-34$	$5.53e-34$	$2.00e-03$	> 0.05	$1.28e-34$
MOMBI-NTCH	$2.82e-39$	$2.82e-39$	$2.82e-39$	$2.82e-39$	$9.75e-03$	–	$1.13e-34$	$4.13e-33$	$1.39e-02$	> 0.05	$1.28e-34$
MOMBI-ATCH	$2.40e-39$	$2.40e-39$	$2.40e-39$	$2.40e-39$	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$1.13e-34$
MOMBI-NATCH	$2.82e-39$	$2.82e-39$	$2.82e-39$	$2.82e-39$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	$1.28e-34$
MOMBI-MTCH	$2.71e-39$	$2.71e-39$	$2.71e-39$	$2.71e-39$	> 0.05	> 0.05	$1.10e-34$	$1.59e-33$	–	> 0.05	$1.24e-34$
MOMBI-NMTCH	$2.82e-39$	$2.82e-39$	$2.82e-39$	$2.82e-39$	> 0.05	> 0.05	$1.13e-34$	$7.87e-33$	> 0.05	–	$1.28e-34$
MOMBI-PBI	$2.82e-39$	$2.82e-39$	$2.82e-39$	$2.82e-39$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
4D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	$5.64e-36$	–	$1.99e-25$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$6.73e-22$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$2.70e-37$	$5.90e-03$	$6.52e-33$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$2.44e-37$	$1.28e-34$	$1.13e-34$	$1.28e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NTCH	$2.44e-37$	$1.28e-34$	$1.13e-34$	$1.28e-34$	$1.97e-29$	–	> 0.05	> 0.05	$1.17e-02$	> 0.05	$1.28e-34$
MOMBI-ATCH	$2.44e-37$	$1.28e-34$	$1.13e-34$	$1.28e-34$	$1.28e-34$	$3.72e-12$	–	$1.90e-04$	$1.52e-32$	> 0.05	$1.28e-34$
MOMBI-NATCH	$2.44e-37$	$1.28e-34$	$1.13e-34$	$1.28e-34$	$4.00e-34$	$4.30e-03$	> 0.05	–	$6.04e-08$	> 0.05	$1.28e-34$
MOMBI-MTCH	$2.44e-37$	$1.28e-34$	$1.13e-34$	$1.28e-34$	$1.44e-34$	> 0.05	> 0.05	> 0.05	–	> 0.05	$1.28e-34$
MOMBI-NMTCH	$2.44e-37$	$1.28e-34$	$1.13e-34$	$1.28e-34$	$1.28e-34$	$7.83e-15$	$6.32e-04$	$1.05e-08$	$1.65e-29$	–	$1.28e-34$
MOMBI-PBI	$2.44e-37$	$1.28e-34$	$1.13e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–

Table A.120: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ5 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
5D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$2.42e-35$	$1.97e-35$	–	$4.50e-32$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$1.53e-32$	$1.08e-30$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$2.42e-35$	$1.97e-35$	$1.28e-34$	$1.28e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	$2.42e-35$	$1.97e-35$	$1.28e-34$	$1.28e-34$	$3.08e-33$	–	> 0.05	> 0.05	> 0.05	> 0.05	$4.91e-27$
MOMBI-ATCH	$2.41e-35$	$1.97e-35$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$9.81e-26$	–	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NATCH	$2.42e-35$	$1.97e-35$	$1.28e-34$	$1.28e-34$	$1.36e-34$	$7.28e-24$	$1.44e-06$	–	$3.48e-02$	> 0.05	$1.49e-34$
MOMBI-MTCH	$2.42e-35$	$1.97e-35$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.51e-28$	$2.11e-07$	> 0.05	–	> 0.05	$1.28e-34$
MOMBI-NMTCH	$2.42e-35$	$1.97e-35$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.19e-32$	$5.74e-30$	$6.94e-09$	$2.32e-17$	–	$1.28e-34$
MOMBI-PBI	$2.42e-35$	$1.97e-35$	$1.28e-34$	$1.28e-34$	$2.90e-28$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
6D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	$8.53e-18$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.24e-34$	$1.28e-34$	–	$1.54e-24$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$1.24e-34$	$7.44e-28$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.24e-34$	$1.44e-34$	$1.77e-10$	$5.50e-31$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	$1.24e-34$	$1.78e-34$	$2.35e-30$	$3.90e-33$	$2.94e-30$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-ATCH	$1.24e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.11e-26$	–	$1.93e-03$	> 0.05	> 0.05	$1.28e-34$
MOMBI-NATCH	$1.24e-34$	$1.40e-34$	$2.51e-33$	$4.00e-34$	$2.82e-33$	$5.62e-14$	> 0.05	–	> 0.05	> 0.05	$2.03e-22$
MOMBI-MTCH	$1.24e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.31e-31$	$6.24e-30$	$4.04e-15$	–	$2.37e-05$	$1.28e-34$
MOMBI-NMTCH	$1.24e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.61e-27$	$6.76e-07$	$1.14e-08$	> 0.05	–	$3.44e-34$
MOMBI-PBI	$1.24e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.84e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
7D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	$7.84e-33$	–	> 0.05	> 0.05	$3.13e-13$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.24e-34$	$2.18e-05$	–	> 0.05	$1.68e-32$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$5.33e-09$
MOMBI-NLS	$1.31e-34$	$1.28e-15$	$1.14e-12$	–	$1.52e-26$	$3.14e-04$	> 0.05	$2.93e-05$	> 0.05	> 0.05	$6.58e-17$
MOMBI-TCH	$2.42e-33$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	$1.35e-34$	$7.49e-04$	> 0.05	> 0.05	$6.98e-10$	–	> 0.05	$3.01e-03$	> 0.05	> 0.05	$4.23e-03$
MOMBI-ATCH	$1.23e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	> 0.05	$3.46e-32$	$1.28e-34$
MOMBI-NATCH	$8.64e-08$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$1.23e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$8.96e-31$	$1.28e-34$	–	$1.28e-34$	$1.28e-34$
MOMBI-NMTCH	$6.71e-32$	$1.57e-11$	$1.19e-09$	$5.60e-03$	$4.77e-16$	$2.15e-05$	> 0.05	$6.41e-08$	> 0.05	–	$2.03e-11$
MOMBI-PBI	$1.24e-34$	> 0.05	> 0.05	> 0.05	$4.57e-31$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–



Table A.121: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ5 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
8D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	$1.63e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$3.18e-06$	> 0.05	> 0.05	$7.06e-07$
MOMBI-LS	$1.28e-34$	$9.39e-33$	–	> 0.05	$1.54e-32$	$2.79e-03$	> 0.05	$1.66e-21$	> 0.05	> 0.05	$1.36e-34$
MOMBI-NLS	$1.28e-34$	$4.78e-34$	$1.04e-17$	–	$5.54e-34$	$4.55e-08$	> 0.05	$1.48e-25$	> 0.05	> 0.05	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$6.80e-31$	> 0.05	> 0.05	–	> 0.05	> 0.05	$1.63e-14$	> 0.05	> 0.05	$2.37e-33$
MOMBI-NTCH	$2.10e-30$	$2.31e-13$	> 0.05	> 0.05	> 0.05	–	> 0.05	$4.99e-15$	> 0.05	> 0.05	$2.22e-15$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.90e-33$	–	$1.28e-34$	> 0.05	$1.28e-34$	$1.28e-34$
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$1.28e-34$	$5.54e-34$	$2.51e-33$	$2.51e-33$	$2.51e-33$	$1.43e-33$	$6.69e-29$	$3.54e-34$	–	$1.27e-33$	$2.13e-34$
MOMBI-NMTCH	$1.27e-25$	$1.61e-10$	> 0.05	> 0.05	$4.08e-03$	> 0.05	> 0.05	$5.59e-16$	> 0.05	–	$4.77e-12$
MOMBI-PBI	$1.32e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$3.53e-05$	> 0.05	> 0.05	–
9D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$7.22e-23$	> 0.05	> 0.05	> 0.05
MOMBI-NWS	$7.48e-13$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.19e-23$	> 0.05	> 0.05	$3.04e-02$
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	> 0.05	> 0.05	$1.19e-03$	> 0.05	$7.83e-31$	> 0.05	$1.34e-07$	$1.28e-34$
MOMBI-NLS	$1.28e-34$	$1.28e-34$	$1.79e-27$	–	> 0.05	$4.77e-12$	> 0.05	$1.07e-33$	> 0.05	$5.31e-10$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$2.43e-33$	$2.98e-06$	–	$1.41e-15$	> 0.05	$1.28e-34$	> 0.05	$3.50e-10$	$1.28e-34$
MOMBI-NTCH	$3.22e-27$	$1.64e-23$	> 0.05	> 0.05	> 0.05	–	> 0.05	$3.02e-29$	> 0.05	$2.21e-05$	$7.71e-25$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.32e-34$	–	$1.28e-34$	> 0.05	$1.53e-34$	$1.28e-34$
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$5.26e-31$	$1.28e-34$	–	$1.28e-34$	$1.28e-34$
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$4.47e-26$	> 0.05	–	> 0.05
MOMBI-PBI	$6.57e-14$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.37e-23$	> 0.05	> 0.05	–
10D											
MOMBI-WS	–	$1.32e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$4.50e-32$	> 0.05	$4.48e-10$	$4.23e-30$
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$4.98e-31$	> 0.05	$7.94e-06$	> 0.05
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	$1.32e-34$	> 0.05	$1.18e-18$	$1.28e-34$
MOMBI-NLS	$1.28e-34$	$1.28e-34$	$5.54e-23$	–	> 0.05	$1.72e-02$	> 0.05	$1.32e-34$	> 0.05	$3.02e-23$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.87e-33$	$3.70e-25$	–	$2.48e-14$	> 0.05	$1.28e-34$	> 0.05	$1.47e-28$	$1.28e-34$
MOMBI-NTCH	$3.83e-21$	$1.37e-32$	> 0.05	> 0.05	> 0.05	–	> 0.05	$9.75e-34$	> 0.05	$3.48e-19$	$4.57e-31$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	> 0.05	$1.28e-34$	$1.28e-34$
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$1.28e-34$
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.06e-31$	> 0.05	–	> 0.05
MOMBI-PBI	> 0.05	$9.52e-07$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$3.54e-31$	> 0.05	$2.93e-07$	–

Table A.122: Comparison of R2 indicator values for different optimizers on the DTLZ5 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	2.500000e-01	2.500000e-01	2.500000e-01	2.500000e-01	2.180311e-01	2.182541e-01	2.181222e-01	2.182273e-01	2.180398e-01	2.182502e-01	2.180289e-01
	avg.	2.500000e-01	2.500000e-01	2.500000e-01	2.500000e-01	2.180310e-01	2.182502e-01	2.181209e-01	2.182327e-01	2.180397e-01	2.182553e-01	2.180291e-01
	min.	2.500000e-01	2.500000e-01	2.499999e-01	2.499999e-01	2.180279e-01	2.181752e-01	2.181038e-01	2.181699e-01	2.180332e-01	2.181359e-01	2.180252e-01
	max.	2.500000e-01	2.500000e-01	2.500000e-01	2.500000e-01	2.180334e-01	2.183380e-01	2.181262e-01	2.183043e-01	2.180416e-01	2.183783e-01	2.180360e-01
	std.	0.000000e+00	0.000000e+00	1.450067e-08	3.988783e-08	1.105217e-06	3.479241e-05	4.816717e-06	3.078384e-05	1.259025e-06	4.778714e-05	1.860610e-06
3D	med.	1.880164e-01	1.880164e-01	1.880164e-01	1.880164e-01	1.637226e-01	1.637580e-01	1.638746e-01	1.638659e-01	1.638029e-01	1.637952e-01	1.665121e-01
	avg.	1.880164e-01	1.880164e-01	1.880164e-01	1.880164e-01	1.637202e-01	1.637315e-01	1.638727e-01	1.638654e-01	1.637995e-01	1.637837e-01	1.665093e-01
	min.	1.880164e-01	1.880164e-01	1.880164e-01	1.880164e-01	1.636903e-01	1.634209e-01	1.638216e-01	1.635667e-01	1.637715e-01	1.635371e-01	1.664635e-01
	max.	1.880164e-01	1.880164e-01	1.880164e-01	1.880164e-01	1.637248e-01	1.640067e-01	1.638760e-01	1.640873e-01	1.638053e-01	1.640492e-01	1.665322e-01
	std.	6.484898e-09	6.484898e-09	6.484898e-09	6.484898e-09	5.600293e-06	1.231127e-04	7.645950e-06	1.063561e-04	7.142383e-06	1.188863e-04	1.401397e-05
4D	med.	1.386170e-01	1.354712e-01	1.386161e-01	1.344937e-01	1.198290e-01	1.197673e-01	1.195106e-01	1.198644e-01	1.198459e-01	1.197292e-01	1.228701e-01
	avg.	1.386165e-01	1.355421e-01	1.385492e-01	1.345994e-01	1.198228e-01	1.198086e-01	1.195211e-01	1.199469e-01	1.198514e-01	1.197822e-01	1.228663e-01
	min.	1.385637e-01	1.322167e-01	1.372649e-01	1.314747e-01	1.196240e-01	1.191798e-01	1.193099e-01	1.192425e-01	1.196068e-01	1.191843e-01	1.227303e-01
	max.	1.386171e-01	1.386170e-01	1.386170e-01	1.386169e-01	1.200416e-01	1.212535e-01	1.198823e-01	1.212815e-01	1.201464e-01	1.208812e-01	1.229493e-01
	std.	5.332227e-06	1.979533e-03	1.960983e-04	1.819521e-03	7.255273e-05	4.051266e-04	9.970994e-05	3.885915e-04	9.164538e-05	2.847023e-04	4.424071e-01
5D	med.	1.081114e-01	1.081114e-01	1.045146e-01	1.071255e-01	9.717113e-02	9.638179e-02	9.569487e-02	9.503530e-02	9.474863e-02	9.469242e-02	9.648472e-02
	avg.	1.081097e-01	1.080590e-01	1.043725e-01	1.068706e-01	9.710735e-02	9.654845e-02	9.567207e-02	9.521007e-02	9.471743e-02	9.473618e-02	9.643889e-02
	min.	1.079540e-01	1.053389e-01	1.022939e-01	1.042239e-01	9.641774e-02	9.437504e-02	9.521016e-02	9.346246e-02	9.430517e-02	9.323444e-02	9.566003e-02
	max.	1.081115e-01	1.081115e-01	1.046010e-01	1.081114e-01	9.731652e-02	9.926435e-02	9.576505e-02	9.862689e-02	9.487588e-02	9.659209e-02	9.684747e-02
	std.	1.575739e-05	3.158483e-04	3.978996e-04	1.001676e-03	1.708799e-04	1.034965e-03	9.305391e-05	9.204814e-04	9.683645e-05	6.504821e-04	1.339293e-04
6D	med.	8.986999e-02	8.959316e-02	8.871576e-02	8.893057e-02	8.208323e-02	8.214459e-02	8.153869e-02	8.218364e-02	8.109298e-02	8.165993e-02	8.178372e-02
	avg.	8.979306e-02	8.952363e-02	8.860297e-02	8.886482e-02	8.217974e-02	8.217475e-02	8.150460e-02	8.226754e-02	8.108736e-02	8.183690e-02	8.182401e-02
	min.	8.947836e-02	8.816118e-02	8.725888e-02	8.796211e-02	8.139284e-02	8.028016e-02	8.051568e-02	8.021534e-02	8.080900e-02	8.006849e-02	8.122834e-02
	max.	8.987325e-02	9.032048e-02	8.877478e-02	8.939414e-02	8.279955e-02	8.523894e-02	8.158901e-02	8.560571e-02	8.176372e-02	8.434748e-02	8.227848e-02
	std.	8.972345e-05	4.098352e-04	2.903385e-04	3.069657e-04	2.465981e-04	8.541199e-04	1.461873e-04	1.001707e-03	9.398669e-05	8.692438e-04	2.412691e-04
7D	med.	7.713072e-02	7.667093e-02	7.685701e-02	7.652110e-02	7.160730e-02	7.414735e-02	7.114719e-02	7.593687e-02	7.104606e-02	7.422552e-02	7.204697e-02
	avg.	7.712929e-02	7.669936e-02	7.679045e-02	7.652792e-02	7.164742e-02	7.420332e-02	7.113482e-02	7.590015e-02	7.103434e-02	7.436640e-02	7.205479e-02
	min.	7.693129e-02	7.604467e-02	7.571830e-02	7.553926e-02	7.141258e-02	7.082721e-02	7.082142e-02	7.262716e-02	7.076855e-02	7.190460e-02	7.174668e-02
	max.	7.717096e-02	7.716466e-02	7.705477e-02	7.720385e-02	7.360004e-02	7.636608e-02	7.116020e-02	7.928067e-02	7.105996e-02	7.763383e-02	7.237161e-02
	std.	3.862218e-05	2.440119e-04	2.072026e-04	3.060279e-04	2.292866e-04	1.144064e-03	4.954993e-05	1.586593e-03	4.203447e-05	1.346700e-03	1.467678e-04
8D	med.	6.796318e-02	6.740386e-02	6.730070e-02	6.702882e-02	6.284659e-02	6.468170e-02	6.229120e-02	6.842257e-02	6.214552e-02	6.527652e-02	6.374408e-02
	avg.	6.794591e-02	6.743745e-02	6.717202e-02	6.706791e-02	6.284039e-02	6.475608e-02	6.227739e-02	6.917604e-02	6.213867e-02	6.551289e-02	6.372147e-02
	min.	6.753755e-02	6.679478e-02	6.624049e-02	6.678474e-02	6.257827e-02	6.236865e-02	6.168840e-02	6.521958e-02	6.188319e-02	6.272572e-02	6.333756e-02
	max.	6.798731e-02	6.785609e-02	6.764558e-02	6.761608e-02	6.304420e-02	6.710389e-02	6.230021e-02	6.825760e-02	6.216106e-02	6.949823e-02	6.392130e-02
	std.	5.719633e-05	1.658404e-04	4.038459e-04	1.729904e-04	1.008356e-04	1.005822e-03	7.307582e-05	3.167391e-03	3.176815e-05	1.092653e-03	1.258735e-04
9D	med.	5.941419e-02	5.932517e-02	5.890673e-02	5.893475e-02	5.486768e-02	5.660937e-02	5.471997e-02	7.180810e-02	5.455187e-02	5.840848e-02	5.543405e-02
	avg.	5.942319e-02	5.937721e-02	5.886268e-02	5.893840e-02	5.487287e-02	5.669683e-02	5.472454e-02	7.068733e-02	5.454636e-02	5.846626e-02	5.543185e-02
	min.	5.933897e-02	5.890242e-02	5.842225e-02	5.855314e-02	5.442670e-02	5.478925e-02	5.444669e-02	5.798077e-02	5.440226e-02	5.544619e-02	5.517432e-02
	max.	5.965803e-02	6.019658e-02	5.913802e-02	5.951098e-02	5.510892e-02	5.856217e-02	5.618055e-02	7.689549e-02	5.457219e-02	6.174053e-02	5.562490e-02
	std.	3.537105e-05	2.206035e-04	1.606194e-04	1.276758e-04	1.088692e-04	8.468638e-04	1.520065e-04	4.839476e-03	2.544932e-05	1.432356e-03	8.828767e-05
10D	med.	5.369608e-02	5.406314e-02	5.334243e-02	5.329614e-02	4.986227e-02	5.113131e-02	4.962266e-02	6.619530e-02	4.953800e-02	5.507327e-02	5.022183e-02
	avg.	5.370405e-02	5.419308e-02	5.330214e-02	5.331669e-02	4.989926e-02	5.119879e-02	4.962139e-02	6.578846e-02	4.953765e-02	5.464894e-02	5.021471e-02
	min.	5.351984e-02	5.364368e-02	5.242061e-02	5.306575e-02	4.974060e-02	4.956609e-02	4.950811e-02	5.369117e-02	4.944383e-02	5.135955e-02	4.963898e-02
	max.	5.402336e-02	5.751294e-02	5.351393e-02	5.388395e-02	5.023794e-02	5.314023e-02	4.963972e-02	6.989750e-02	4.955471e-02	5.714042e-02	5.048806e-02
	std.	6.389898e-05	5.586253e-04	1.286842e-04	1.130609e-04	1.136804e-04	7.805034e-04	1.315083e-05	2.980237e-03	1.124351e-05	1.502387e-03	1.487457e-04

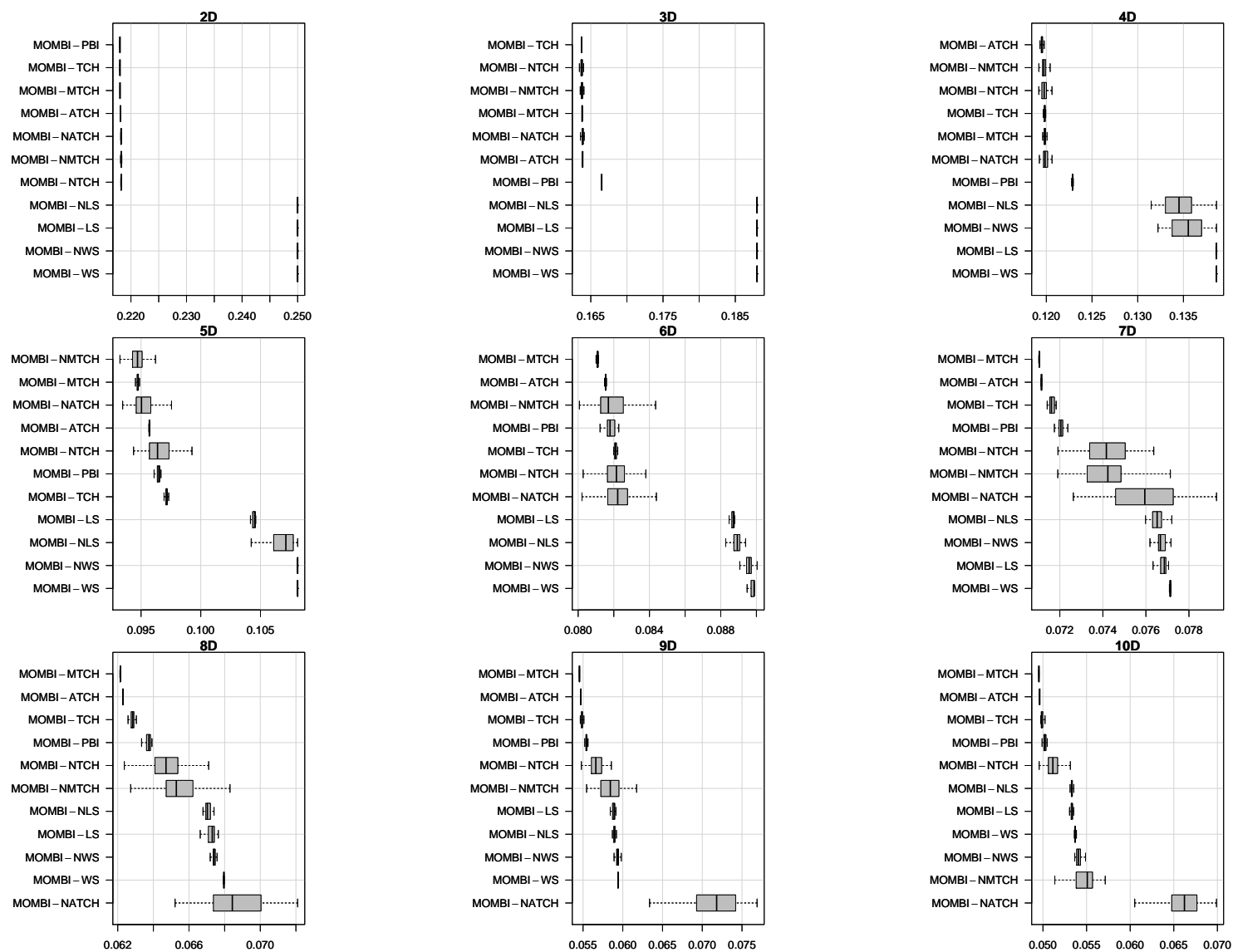


Figure A.298: Box-plot of the R2 indicator values for different optimizers on the DTLZ5 test problem.

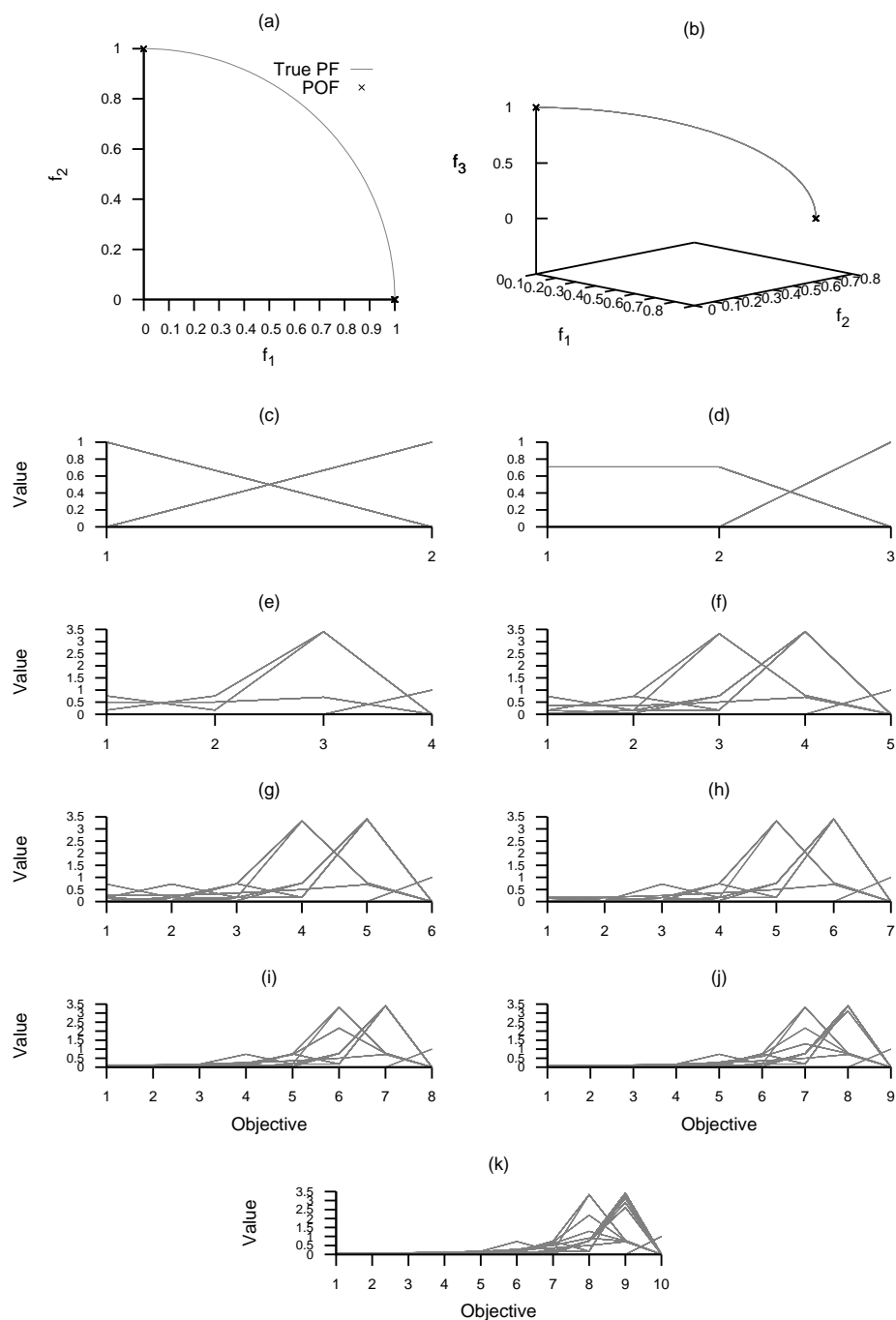


Figure A.299: Plots of the approximations obtained by MOMBI-WS from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

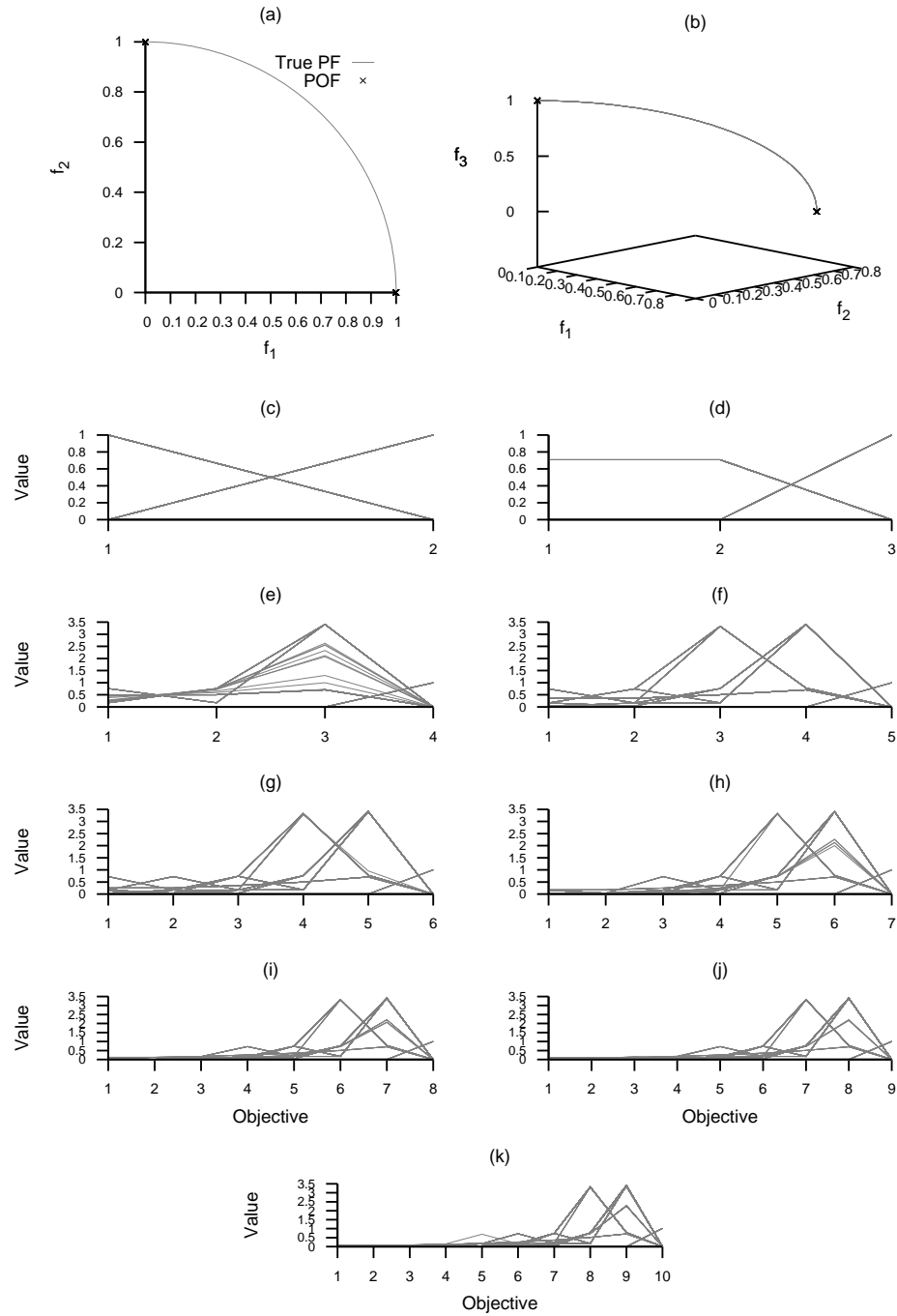


Figure A.300: Plots of the approximations obtained by MOMBI-NWS from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

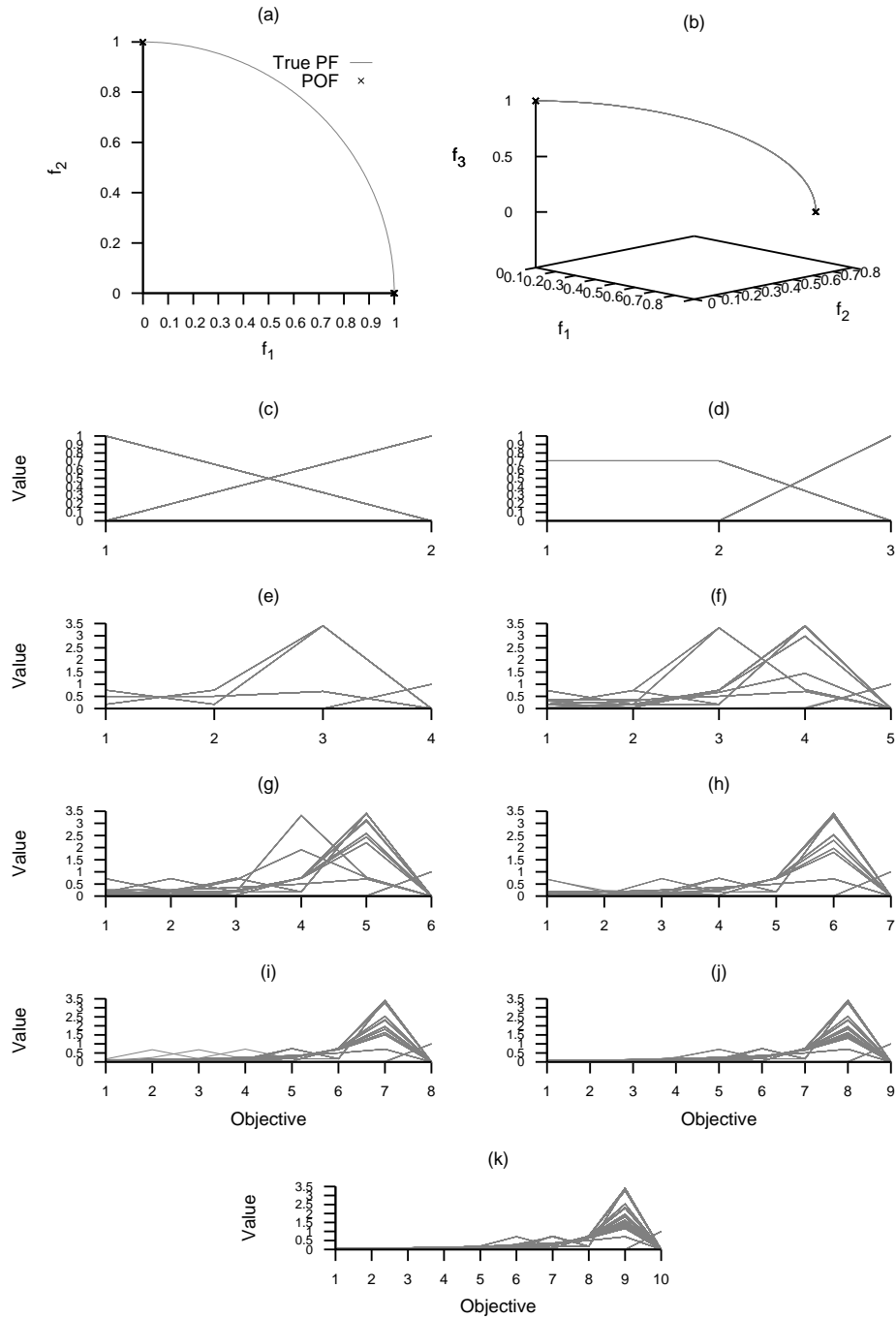


Figure A.301: Plots of the approximations obtained by MOMBI-LS from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

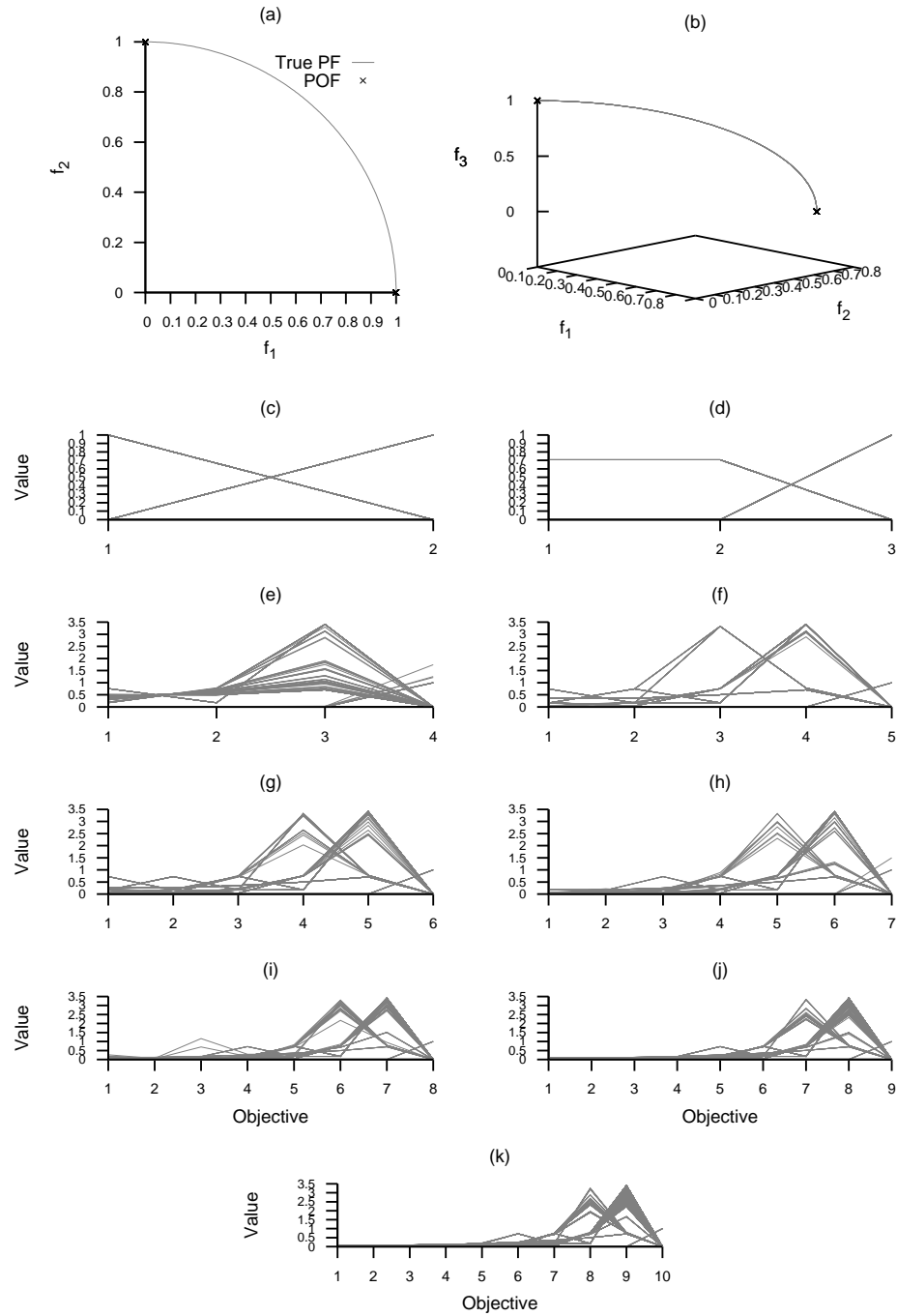


Figure A.302: Plots of the approximations obtained by MOMBI-NLS from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

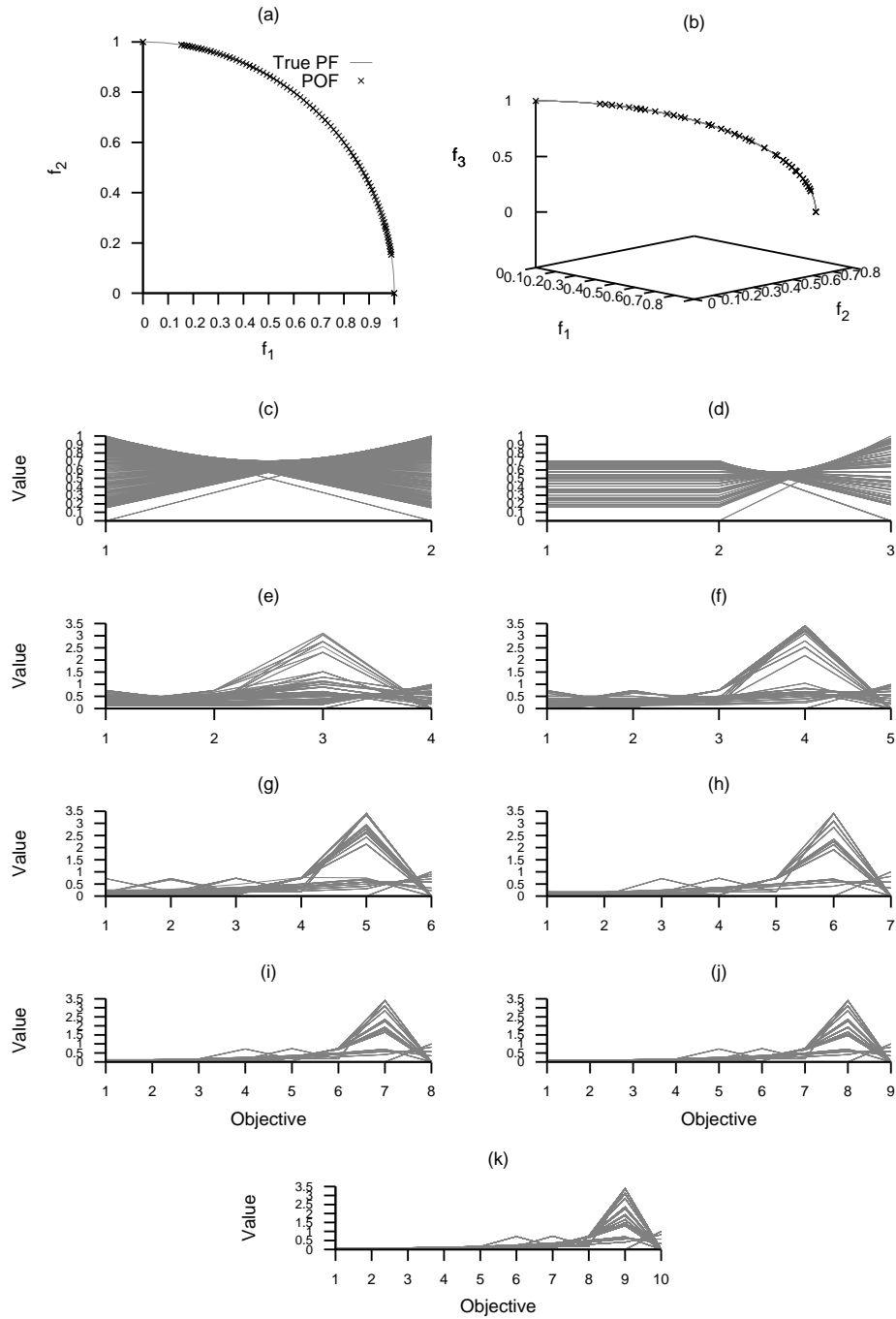


Figure A.303: Plots of the approximations obtained by MOMBI-ATCH from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



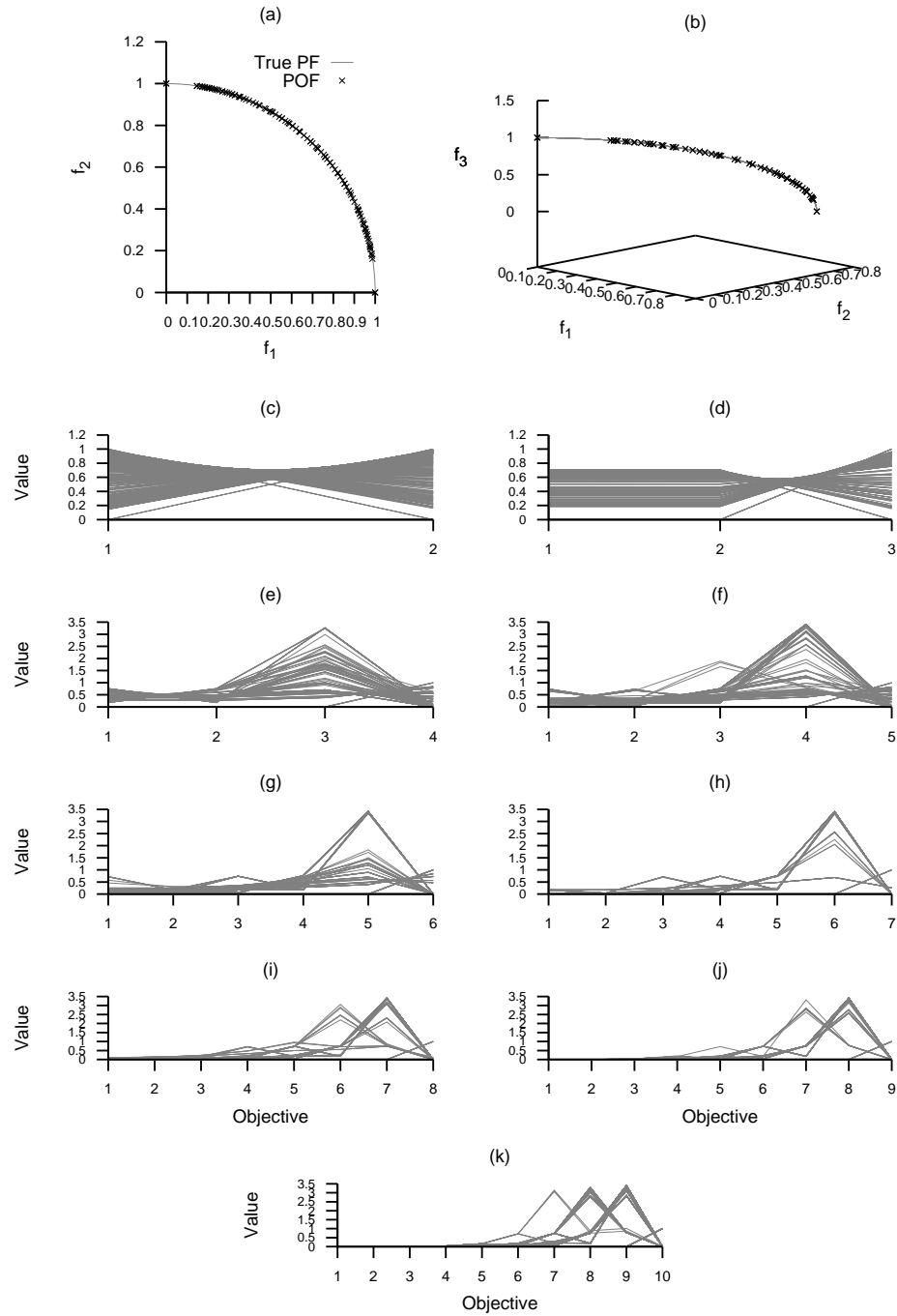


Figure A.304: Plots of the approximations obtained by MOMBI-NATCH from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

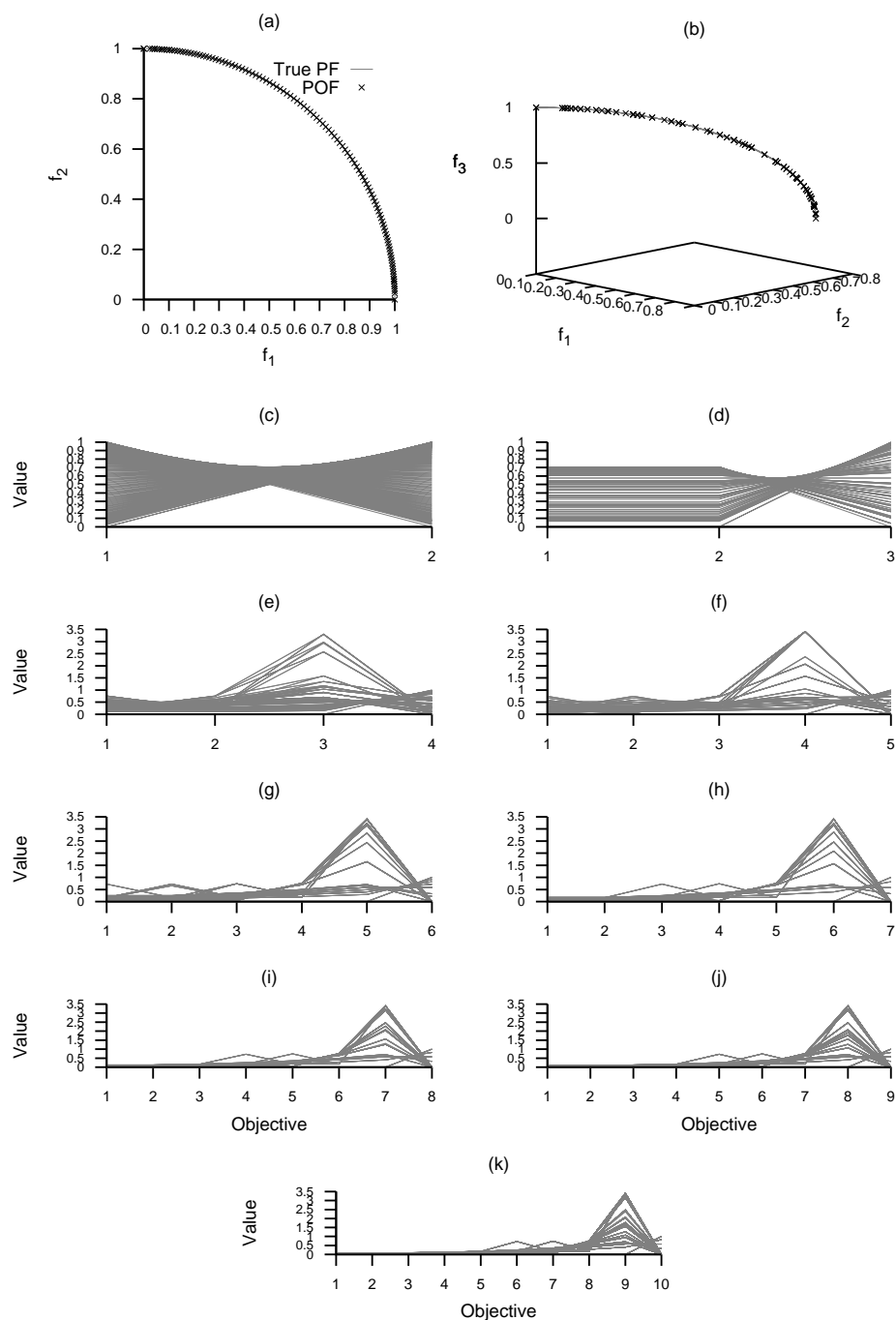


Figure A.305: Plots of the approximations obtained by MOMBI-MTCH from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

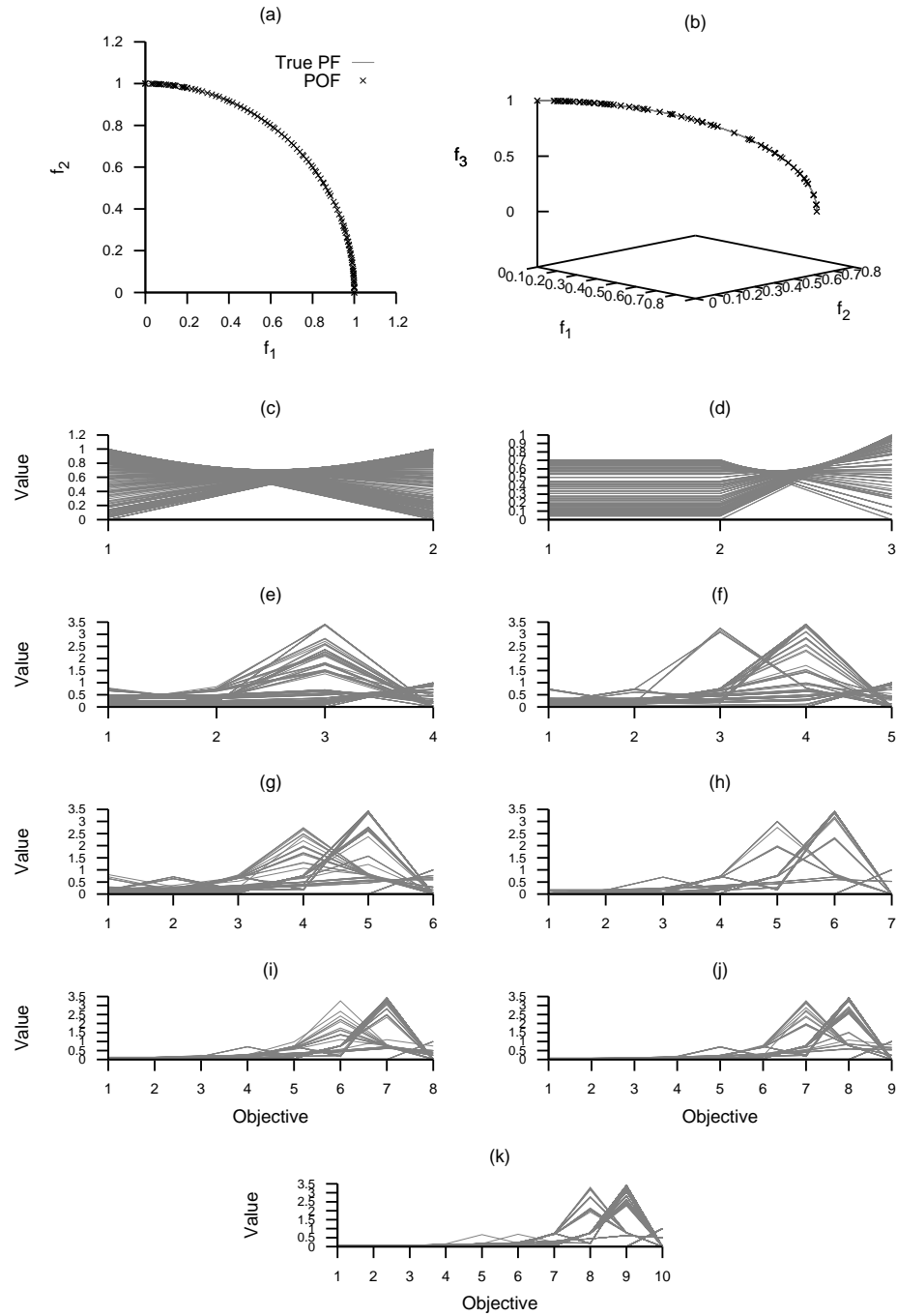


Figure A.306: Plots of the approximations obtained by MOMBI-NMTCH from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.123: Comparison of hypervolume indicator values for different optimizers on the DTLZ6 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	1.198660e+02	1.198673e+02	1.198607e+02	1.198611e+02	1.201018e+02	1.201010e+02	1.201027e+02	1.200982e+02	1.201049e+02	1.201048e+02	1.201308e+02
	avg.	1.198671e+02	1.198649e+02	1.198552e+02	1.198533e+02	1.201037e+02	1.201007e+02	1.201047e+02	1.200920e+02	1.201047e+02	1.201047e+02	1.201201e+02
	min.	1.197238e+02	1.197056e+02	1.197072e+02	1.197401e+02	1.199797e+02	1.199835e+02	1.200238e+02	1.199962e+02	1.199673e+02	1.199943e+02	1.200458e+02
	max.	1.200000e+02	1.200000e+02	1.200000e+02	1.199575e+02	1.201856e+02	1.202077e+02	1.202098e+02	1.202088e+02	1.202108e+02	1.201807e+02	1.202082e+02
	std.	5.939092e-02	6.155706e-02	6.546464e-02	5.524699e-02	4.544105e-02	4.604835e-02	4.083456e-02	4.242898e-02	4.253053e-02	4.222821e-02	3.521253e-02
3D	med.	1.314736e+03	1.314712e+03	1.314737e+03	1.314707e+03	1.317866e+03	1.317891e+03	1.317974e+03	1.317942e+03	1.317990e+03	1.317902e+03	1.313623e+03
	avg.	1.314691e+03	1.314649e+03	1.314630e+03	1.314686e+03	1.317882e+03	1.317896e+03	1.317973e+03	1.317961e+03	1.318020e+03	1.317897e+03	1.313550e+03
	min.	1.313517e+03	1.312931e+03	1.313103e+03	1.313571e+03	1.316945e+03	1.316914e+03	1.317252e+03	1.316979e+03	1.316959e+03	1.316962e+03	1.311621e+03
	max.	1.315944e+03	1.315944e+03	1.315654e+03	1.315944e+03	1.318945e+03	1.318789e+03	1.318938e+03	1.318956e+03	1.318970e+03	1.318978e+03	1.314842e+03
	std.	5.348564e-01	5.466559e-01	5.400088e-01	4.729212e-01	4.361856e-01	3.963302e-01	3.553542e-01	3.858357e-01	4.551044e-01	3.723188e-01	6.689099e-01
4D	med.	1.443470e+04	1.443606e+04	1.445589e+04	1.446475e+04	1.446264e+04	1.448228e+04	1.450879e+04	1.451378e+04	1.450113e+04	1.450996e+04	1.442441e+04
	avg.	1.443481e+04	1.443980e+04	1.445462e+04	1.446438e+04	1.446193e+04	1.448253e+04	1.450890e+04	1.451349e+04	1.450074e+04	1.451029e+04	1.442358e+04
	min.	1.442583e+04	1.442715e+04	1.443527e+04	1.444312e+04	1.444652e+04	1.445333e+04	1.449711e+04	1.450145e+04	1.448918e+04	1.449667e+04	1.439498e+04
	max.	1.444728e+04	1.446875e+04	1.447064e+04	1.448748e+04	1.447536e+04	1.450527e+04	1.452117e+04	1.452490e+04	1.451312e+04	1.452444e+04	1.444258e+04
	std.	4.631234e+00	8.982224e+00	7.324734e+00	8.186454e+00	6.950383e+00	9.868716e+00	4.323518e+00	4.588718e+00	4.934516e+00	5.225723e+00	9.205827e+00
5D	med.	1.569150e+05	1.567987e+05	1.580581e+05	1.578088e+05	1.566709e+05	1.573197e+05	1.591239e+05	1.590220e+05	1.589633e+05	1.590055e+05	1.585887e+05
	avg.	1.567321e+05	1.565538e+05	1.580396e+05	1.578125e+05	1.566210e+05	1.571640e+05	1.591255e+05	1.590195e+05	1.589519e+05	1.589933e+05	1.585913e+05
	min.	1.537187e+05	1.529072e+05	1.567747e+05	1.568542e+05	1.546064e+05	1.545502e+05	1.587265e+05	1.587265e+05	1.580401e+05	1.586927e+05	1.582254e+05
	max.	1.577857e+05	1.578338e+05	1.585540e+05	1.582527e+05	1.573363e+05	1.581430e+05	1.592973e+05	1.593056e+05	1.591559e+05	1.592349e+05	1.588528e+05
	std.	7.348929e+02	8.113086e+02	2.621391e+02	2.290291e+02	3.929044e+02	6.842785e+02	6.916541e+01	9.611291e+01	1.363810e+02	1.115975e+02	1.247353e+02
6D	med.	1.634664e+06	1.644069e+06	1.737765e+06	1.735176e+06	1.703865e+06	1.708620e+06	1.747852e+06	1.742412e+06	1.744051e+06	1.738623e+06	1.741482e+06
	avg.	1.633223e+06	1.642234e+06	1.737862e+06	1.735300e+06	1.702777e+06	1.705764e+06	1.747926e+06	1.742367e+06	1.743520e+06	1.738131e+06	1.741170e+06
	min.	1.556656e+06	1.573371e+06	1.734382e+06	1.728630e+06	1.669805e+06	1.670858e+06	1.745084e+06	1.736104e+06	1.730247e+06	1.728990e+06	1.734727e+06
	max.	1.683024e+06	1.691463e+06	1.741145e+06	1.741207e+06	1.719151e+06	1.724872e+06	1.749986e+06	1.746071e+06	1.747259e+06	1.742731e+06	1.744897e+06
	std.	2.369449e+04	2.150141e+04	1.466933e+03	2.284197e+03	8.622567e+03	1.149160e+04	9.071465e+02	1.852551e+03	2.242712e+03	2.924822e+03	1.991335e+03
7D	med.	1.857619e+07	1.822493e+07	1.924091e+07	1.909426e+07	1.896229e+07	1.872954e+07	1.926220e+07	1.869741e+07	1.923205e+07	1.863881e+07	1.921863e+07
	avg.	1.853272e+07	1.816211e+07	1.924028e+07	1.909242e+07	1.892731e+07	1.871174e+07	1.926335e+07	1.868314e+07	1.923190e+07	1.859905e+07	1.921738e+07
	min.	1.792738e+07	1.689091e+07	1.921780e+07	1.895748e+07	1.854869e+07	1.834694e+07	1.924096e+07	1.808159e+07	1.919600e+07	1.810630e+07	1.919881e+07
	max.	1.881389e+07	1.864302e+07	1.926680e+07	1.914834e+07	1.910874e+07	1.896455e+07	1.929327e+07	1.892858e+07	1.926357e+07	1.887265e+07	1.923289e+07
	std.	1.683388e+05	3.193881e+05	1.062725e+04	2.634031e+04	1.101932e+05	1.387722e+05	9.596532e+03	1.300736e+05	1.337954e+04	1.684492e+05	8.731965e+03
8D	med.	2.050409e+08	1.912499e+08	2.112993e+08	2.088919e+08	2.067603e+08	2.047120e+08	2.115156e+08	1.957056e+08	2.110115e+08	1.965838e+08	2.103815e+08
	avg.	2.049025e+08	1.910240e+08	2.113012e+08	2.088556e+08	2.065986e+08	2.045049e+08	2.115158e+08	1.958403e+08	2.110241e+08	1.968852e+08	2.103450e+08
	min.	2.017608e+08	1.762444e+08	2.110017e+08	2.073760e+08	2.002435e+08	1.995958e+08	2.111499e+08	1.842613e+08	2.106606e+08	1.880437e+08	2.089426e+08
	max.	2.069185e+08	2.014214e+08	2.116025e+08	2.095990e+08	2.086865e+08	2.079567e+08	2.118118e+08	2.041905e+08	2.114734e+08	2.032687e+08	2.109451e+08
	std.	1.191468e+06	4.818168e+06	1.028808e+05	3.563613e+05	1.169597e+06	1.714996e+06	1.305811e+05	4.675854e+06	1.593313e+05	2.934812e+06	3.231798e+05
9D	med.	2.253602e+09	2.000338e+09	2.321166e+09	2.287609e+09	2.261619e+09	2.233758e+09	2.323401e+09	2.027886e+09	2.315849e+09	2.094618e+09	2.290923e+09
	avg.	2.249937e+09	1.989691e+09	2.321353e+09	2.287446e+09	2.261184e+09	2.229845e+09	2.323546e+09	2.028852e+09	2.315846e+09	2.096814e+09	2.291057e+09
	min.	2.199467e+09	1.775165e+09	2.318388e+09	2.274766e+09	2.233329e+09	2.152185e+09	2.318954e+09	1.856967e+09	2.309400e+09	1.972752e+09	2.274904e+09
	max.	2.268326e+09	2.134839e+09	2.325446e+09	2.297681e+09	2.282361e+09	2.267321e+09	2.327487e+09	2.193422e+09	2.319966e+09	2.176353e+09	2.302524e+09
	std.	1.416765e+07	7.106880e+07	1.317393e+06	3.978881e+06	9.303722e+06	2.150211e+07	1.731199e+06	6.987326e+07	1.704984e+06	4.109419e+07	5.126465e+06
10D	med.	2.467704e+10	2.125104e+10	2.548807e+10	2.506792e+10	2.479181e+10	2.429910e+10	2.552860e+10	2.120423e+10	2.541815e+10	2.236668e+10	2.491322e+10
	avg.	2.464686e+10	2.113556e+10	2.548938e+10	2.506551e+10	2.477195e+10	2.429478e+10	2.552928e+10	2.138476e+10	2.541969e+10	2.237146e+10	2.491120e+10
	min.	2.427833e+10	1.901610e+10	2.545525e+10	2.495011e+10	2.452590e+10	2.369020e+10	2.545861e+10	1.968740e+10	2.537432e+10	2.116375e+10	2.473447e+10
	max.	2.485012e+10	2.298153e+10	2.554600e+10	2.522846e+10	2.507327e+10	2.486669e+10	2.558273e+10	2.308507e+10	2.549848e+10	2.331603e+10	2.507942e+10
	std.	1.332525e+08	7.858979e+08	1.545703e+07	4.952879e+07	1.091595e+08	2.179111e+08	2.254978e+07	8.294581e+08	2.082108e+07	4.817311e+08	6.080835e+07

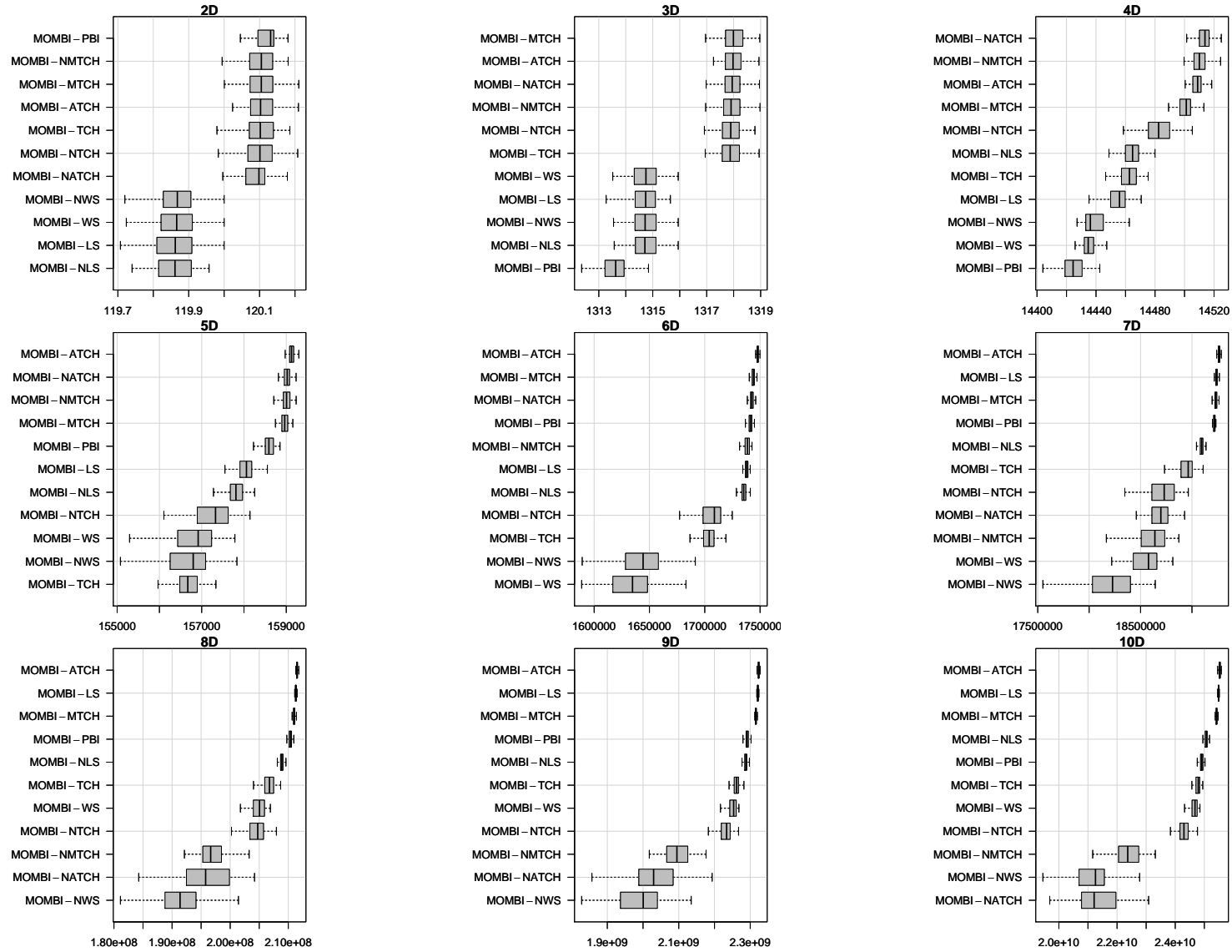


Figure A.307: Box-plot of the hypervolume indicator values for different optimizers on the DTLZ6 test problem.

Table A.124: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ6 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
2D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.53e-34$	$1.68e-34$	$1.68e-34$	$1.28e-34$	–	> 0.05	> 0.05	$1.94e-02$	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	$1.53e-34$	$1.68e-34$	$1.68e-34$	$1.28e-34$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	–	$2.15e-02$	> 0.05	> 0.05	> 0.05
MOMBI-NATCH	$1.36e-34$	$1.40e-34$	$1.40e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$1.44e-34$	$1.40e-34$	$1.44e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	$8.13e-03$	–	> 0.05	> 0.05
MOMBI-NMTCH	$1.44e-34$	$1.53e-34$	$1.53e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	$9.40e-03$	> 0.05	–	> 0.05
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.22e-02$	$2.43e-03$	$7.35e-03$	$7.18e-06$	$1.42e-02$	$1.85e-02$	–
3D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.30e-24$
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.92e-23$
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$9.53e-23$
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.02e-25$
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$3.44e-02$	$3.96e-02$	–	> 0.05	> 0.05	$1.52e-02$	$1.28e-34$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	$1.28e-34$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$7.32e-03$	$9.37e-03$	> 0.05	> 0.05	–	$2.94e-03$	$1.28e-34$
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	$1.28e-34$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
4D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.79e-19$
MOMBI-NWS	$4.77e-05$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.31e-24$
MOMBI-LS	$1.63e-32$	$2.58e-21$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.13e-34$
MOMBI-NLS	$1.44e-34$	$3.15e-30$	$5.71e-15$	–	$2.47e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-TCH	$1.32e-34$	$4.69e-29$	$8.26e-11$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NTCH	$1.28e-34$	$2.63e-34$	$8.85e-33$	$6.38e-25$	$5.84e-29$	–	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$3.19e-34$	–	> 0.05	$1.06e-22$	> 0.05	$1.28e-34$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.44e-34$	$9.92e-12$	–	$1.54e-30$	$2.93e-06$	$1.28e-34$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$7.91e-30$	> 0.05	> 0.05	–	> 0.05	$1.28e-34$
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.33e-34$	$2.92e-02$	> 0.05	$6.76e-24$	–	$1.28e-34$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–

Table A.125: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ6 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
5D											
MOMBI-WS	–	$3.06e-02$	$> 0.05$	$> 0.05$	$6.48e-04$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$6.05e-33$	$4.52e-33$	–	$1.75e-12$	$1.27e-33$	$1.34e-27$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NLS	$9.79e-32$	$2.06e-32$	$> 0.05$	–	$3.34e-34$	$4.43e-19$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-TCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NTCH	$2.68e-07$	$6.09e-11$	$> 0.05$	$> 0.05$	$3.30e-15$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	$8.65e-16$	$1.43e-24$	$2.54e-16$	$1.28e-34$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	–	$4.91e-05$	$> 0.05$	$2.20e-34$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$6.43e-34$	$2.20e-34$	$1.28e-34$	$1.44e-34$	$> 0.05$	$> 0.05$	–	$> 0.05$	$2.83e-32$
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$> 0.05$	$3.21e-03$	–	$1.48e-33$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$3.47e-33$	$1.36e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–
6D											
MOMBI-WS	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NWS	$4.14e-03$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$6.18e-16$	$1.28e-34$	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NLS	$1.28e-34$	$1.28e-34$	$> 0.05$	–	$1.28e-34$	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-TCH	$1.63e-34$	$2.55e-34$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NTCH	$1.73e-34$	$2.71e-34$	$> 0.05$	$> 0.05$	$9.19e-04$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	$1.63e-34$	$4.00e-34$	$1.28e-34$	$1.28e-34$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$4.60e-30$	$3.57e-33$	$1.28e-34$	$1.28e-34$	$> 0.05$	–	$> 0.05$	$1.58e-23$	$1.36e-05$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$7.83e-31$	$8.73e-32$	$1.28e-34$	$1.28e-34$	$> 0.05$	$2.87e-08$	–	$1.44e-29$	$7.80e-19$
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$3.75e-02$	$6.05e-13$	$1.28e-34$	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$8.97e-23$	$1.72e-30$	$1.28e-34$	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$6.11e-15$	–
7D											
MOMBI-WS	–	$2.09e-20$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$2.98e-06$	$1.28e-34$	$5.09e-29$
MOMBI-NLS	$1.28e-34$	$1.28e-34$	$> 0.05$	–	$8.98e-32$	$1.32e-34$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$
MOMBI-TCH	$4.77e-32$	$1.36e-34$	$> 0.05$	$> 0.05$	–	$2.76e-22$	$> 0.05$	$2.37e-25$	$> 0.05$	$6.17e-29$	$> 0.05$
MOMBI-NTCH	$1.76e-13$	$9.51e-32$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$1.43e-06$	$> 0.05$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$3.23e-28$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$1.30e-31$	$1.28e-34$	$1.28e-34$
MOMBI-NATCH	$4.81e-12$	$4.08e-31$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$1.99e-04$	$> 0.05$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	–	$1.28e-34$	$3.96e-15$
MOMBI-NMTCH	$1.29e-03$	$1.64e-24$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.28e-34$	–

Table A.126: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ6 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
8D											
MOMBI-WS	–	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$6.05e-33$	$> 0.05$	$3.15e-34$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$2.01e-25$	$1.28e-34$	$1.28e-34$
MOMBI-NLS	$1.28e-34$	$1.28e-34$	$> 0.05$	–	$2.37e-33$	$1.44e-34$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$
MOMBI-TCH	$2.31e-19$	$1.32e-34$	$> 0.05$	$> 0.05$	–	$4.43e-19$	$> 0.05$	$2.48e-34$	$> 0.05$	$1.89e-34$	$> 0.05$
MOMBI-NTCH	$> 0.05$	$1.58e-34$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$7.20e-31$	$> 0.05$	$3.18e-33$	$> 0.05$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$3.25e-23$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$2.66e-33$	$1.28e-34$	$1.28e-34$
MOMBI-NATCH	$> 0.05$	$9.69e-11$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	–	$1.28e-34$	$3.17e-33$
MOMBI-NMTCH	$> 0.05$	$5.75e-19$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$4.46e-02$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$> 0.05$	$6.24e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.28e-34$	–
9D											
MOMBI-WS	–	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$7.27e-14$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$2.88e-34$	$1.28e-34$	$1.28e-34$
MOMBI-NLS	$1.28e-34$	$1.28e-34$	$> 0.05$	–	$2.33e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$
MOMBI-TCH	$1.15e-09$	$1.28e-34$	$> 0.05$	$> 0.05$	–	$2.40e-27$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$
MOMBI-NTCH	$> 0.05$	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$1.73e-34$	$> 0.05$	$2.26e-34$	$> 0.05$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.21e-17$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$1.40e-34$	$1.28e-34$	$1.28e-34$
MOMBI-NATCH	$> 0.05$	$1.70e-04$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	–	$1.28e-34$	$1.28e-34$
MOMBI-NMTCH	$> 0.05$	$4.23e-24$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$3.88e-13$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.41e-08$	$2.13e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.28e-34$	–
10D											
MOMBI-WS	–	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$3.22e-24$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$2.51e-33$	$1.28e-34$	$1.28e-34$
MOMBI-NLS	$1.28e-34$	$1.28e-34$	$> 0.05$	–	$1.35e-33$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.28e-34$	$8.48e-32$
MOMBI-TCH	$1.94e-12$	$1.28e-34$	$> 0.05$	$> 0.05$	–	$4.57e-31$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$
MOMBI-NTCH	$> 0.05$	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$2.88e-26$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$1.68e-34$	$1.28e-34$	$1.28e-34$
MOMBI-NATCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	–	$1.28e-34$	$1.28e-34$
MOMBI-NMTCH	$> 0.05$	$8.32e-25$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.40e-16$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$2.58e-33$	$1.28e-34$	$> 0.05$	$> 0.05$	$6.10e-23$	$2.63e-34$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.28e-34$	–



Table A.127: Comparison of R2 indicator values for different optimizers on the DTLZ6 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	2.662130e-01	2.659298e-01	2.665117e-01	2.667673e-01	2.323030e-01	2.324871e-01	2.324057e-01	2.329754e-01	2.321628e-01	2.321406e-01	2.278902e-01
	avg.	2.660056e-01	2.662619e-01	2.673748e-01	2.676282e-01	2.320777e-01	2.324766e-01	2.320768e-01	2.337341e-01	2.321150e-01	2.321580e-01	2.294203e-01
	min.	2.500000e-01	2.500000e-01	2.500000e-01	2.552555e-01	2.214009e-01	2.182409e-01	2.181204e-01	2.182386e-01	2.180382e-01	2.221088e-01	2.180301e-01
	max.	2.824275e-01	2.844325e-01	2.842570e-01	2.806130e-01	2.478769e-01	2.471937e-01	2.424183e-01	2.457170e-01	2.494189e-01	2.461352e-01	2.389038e-01
	std.	6.965896e-03	7.200681e-03	7.669029e-03	6.457358e-03	5.853467e-03	5.961957e-03	5.311322e-03	5.479515e-03	5.515124e-03	5.462184e-03	4.626470e-03
3D	med.	1.952948e-01	1.955241e-01	1.954192e-01	1.955998e-01	1.702623e-01	1.706702e-01	1.702572e-01	1.706009e-01	1.702447e-01	1.709414e-01	1.969011e-01
	avg.	1.957084e-01	1.959653e-01	1.960763e-01	1.957370e-01	1.701575e-01	1.705680e-01	1.703303e-01	1.705227e-01	1.700234e-01	1.710165e-01	1.967296e-01
	min.	1.880164e-01	1.880164e-01	1.897770e-01	1.880164e-01	1.637182e-01	1.640080e-01	1.638744e-01	1.636630e-01	1.638024e-01	1.637535e-01	1.888050e-01
	max.	2.028969e-01	2.064415e-01	2.054084e-01	2.025701e-01	1.762086e-01	1.776957e-01	1.751661e-01	1.772094e-01	1.769177e-01	1.773610e-01	2.024398e-01
	std.	3.289305e-03	3.357768e-03	3.316151e-03	2.909109e-03	2.777870e-03	2.646381e-03	2.385903e-03	2.598489e-03	2.981647e-03	2.487611e-03	2.977667e-03
4D	med.	1.460611e-01	1.456285e-01	1.580772e-01	1.515963e-01	1.419547e-01	1.376939e-01	1.307527e-01	1.290003e-01	1.336757e-01	1.301650e-01	1.499260e-01
	avg.	1.461086e-01	1.452825e-01	1.580975e-01	1.515551e-01	1.425163e-01	1.377935e-01	1.308070e-01	1.291847e-01	1.338067e-01	1.301078e-01	1.497855e-01
	min.	1.425750e-01	1.392114e-01	1.515245e-01	1.421044e-01	1.369255e-01	1.306917e-01	1.243924e-01	1.238067e-01	1.283045e-01	1.238857e-01	1.448348e-01
	max.	1.493378e-01	1.488862e-01	1.653626e-01	1.588205e-01	1.479877e-01	1.482880e-01	1.359087e-01	1.351944e-01	1.395121e-01	1.356343e-01	1.548199e-01
	std.	1.589280e-03	1.876149e-03	2.763673e-03	3.111153e-03	2.493543e-03	3.642539e-03	2.002504e-03	2.160098e-03	2.090804e-03	2.505983e-03	2.178689e-03
5D	med.	1.530543e-01	1.543372e-01	1.531702e-01	1.590630e-01	1.552072e-01	1.528388e-01	1.204838e-01	1.236975e-01	1.233702e-01	1.249270e-01	1.137689e-01
	avg.	1.567439e-01	1.591749e-01	1.530011e-01	1.590759e-01	1.561298e-01	1.543821e-01	1.204834e-01	1.246518e-01	1.239367e-01	1.254860e-01	1.137131e-01
	min.	1.328403e-01	1.372603e-01	1.391133e-01	1.460027e-01	1.459536e-01	1.294958e-01	1.156771e-01	1.168571e-01	1.195199e-01	1.198756e-01	1.091463e-01
	max.	2.086479e-01	2.092640e-01	1.809910e-01	1.785837e-01	1.887882e-01	1.985798e-01	1.256486e-01	1.318133e-01	1.454975e-01	1.344118e-01	1.191137e-01
	std.	1.376022e-02	1.371605e-02	6.848530e-03	5.772391e-03	6.727796e-03	1.266717e-02	2.106073e-03	2.709975e-03	3.398449e-03	3.181604e-03	2.180917e-03
6D	med.	2.160410e-01	2.105836e-01	1.276918e-01	1.325864e-01	1.562838e-01	1.542910e-01	1.065268e-01	1.186865e-01	1.125814e-01	1.257661e-01	9.945379e-02
	avg.	2.164287e-01	2.111279e-01	1.278771e-01	1.322740e-01	1.569309e-01	1.572294e-01	1.065591e-01	1.187015e-01	1.128535e-01	1.264309e-01	1.001087e-01
	min.	1.842264e-01	1.824975e-01	1.210180e-01	1.223866e-01	1.408012e-01	1.389475e-01	1.019027e-01	1.110603e-01	1.067952e-01	1.179295e-01	9.528606e-02
	max.	2.501108e-01	2.442061e-01	1.344559e-01	1.450231e-01	1.893706e-01	1.891832e-01	1.111639e-01	1.289678e-01	1.357445e-01	1.440394e-01	1.068658e-01
	std.	1.331260e-02	1.292648e-02	2.748552e-03	4.004805e-03	9.019922e-03	1.231435e-02	1.938770e-03	3.703349e-03	4.004708e-03	4.972634e-03	2.634812e-03
7D	med.	1.561399e-01	1.766120e-01	8.964706e-02	1.131837e-01	1.158966e-01	1.404222e-01	8.340192e-02	1.632980e-01	8.634158e-02	1.680879e-01	7.462297e-02
	avg.	1.578046e-01	1.787691e-01	8.965044e-02	1.137849e-01	1.187538e-01	1.414890e-01	8.334006e-02	1.647574e-01	8.621482e-02	1.692566e-01	7.478994e-02
	min.	1.328960e-01	1.474008e-01	8.485929e-02	1.065156e-01	9.918806e-02	1.212897e-01	7.910562e-02	1.433175e-01	8.091981e-02	1.499643e-01	7.263488e-02
	max.	1.953188e-01	2.327051e-01	9.367537e-02	1.307955e-01	1.517133e-01	1.680706e-01	8.734445e-02	1.974943e-01	9.166429e-02	1.956117e-01	7.738500e-02
	std.	1.209582e-02	1.729220e-02	1.903158e-03	3.543993e-03	1.089723e-02	1.174512e-02	1.654968e-03	9.130229e-03	2.125215e-03	1.118823e-02	1.146835e-03
8D	med.	1.316829e-01	1.912293e-01	8.377574e-02	1.123460e-01	1.179791e-01	1.330467e-01	7.843502e-02	1.809595e-01	8.270472e-02	1.801582e-01	7.479205e-02
	avg.	1.325443e-01	1.910368e-01	8.365563e-02	1.127970e-01	1.188541e-01	1.340864e-01	7.854542e-02	1.802037e-01	8.252689e-02	1.798800e-01	7.515750e-02
	min.	1.191092e-01	1.540539e-01	7.970873e-02	1.071918e-01	1.041289e-01	1.145638e-01	7.385523e-02	1.530403e-01	7.732180e-02	1.569029e-01	7.037710e-02
	max.	1.504723e-01	2.262354e-01	8.809735e-02	1.264516e-01	1.579243e-01	1.585540e-01	8.232822e-02	2.095374e-01	8.657229e-02	2.035791e-01	8.069110e-02
	std.	7.620019e-03	1.389237e-02	1.330696e-03	3.377420e-03	8.162500e-03	1.044694e-02	1.500975e-03	1.371980e-02	1.891052e-03	9.100164e-03	2.148409e-03
9D	med.	1.172941e-01	1.912376e-01	7.649987e-02	1.079595e-01	1.125894e-01	1.280371e-01	7.321245e-02	1.849766e-01	7.775013e-02	1.759570e-01	8.479665e-02
	avg.	1.194536e-01	1.920337e-01	7.625757e-02	1.081373e-01	1.131387e-01	1.286584e-01	7.327350e-02	1.841388e-01	7.786848e-02	1.759683e-01	8.439052e-02
	min.	1.088141e-01	1.637035e-01	7.177144e-02	1.005390e-01	1.006280e-01	1.087135e-01	7.062208e-02	1.534261e-01	7.437740e-02	1.586226e-01	7.760236e-02
	max.	1.418902e-01	2.220349e-01	7.887287e-02	1.165660e-01	1.280376e-01	1.570184e-01	7.698171e-02	2.092156e-01	8.311632e-02	1.955616e-01	8.942109e-02
	std.	6.904752e-03	1.170031e-02	1.310754e-03	2.778598e-03	5.101881e-03	9.329564e-03	1.213115e-03	1.121174e-02	1.431044e-03	8.032520e-03	2.520816e-03
10D	med.	1.107789e-01	1.828609e-01	7.265056e-02	1.029382e-01	1.060291e-01	1.246103e-01	6.926829e-02	1.797320e-01	7.444003e-02	1.709244e-01	9.075629e-02
	avg.	1.119770e-01	1.835423e-01	7.256475e-02	1.030194e-01	1.067121e-01	1.246333e-01	6.931449e-02	1.787421e-01	7.441856e-02	1.704918e-01	9.068825e-02
	min.	1.035707e-01	1.578318e-01	6.847432e-02	9.410551e-02	9.325561e-02	1.060415e-01	6.592869e-02	1.527559e-01	7.060788e-02	1.565347e-01	8.481696e-02
	max.	1.255560e-01	2.044731e-01	7.527907e-02	1.083287e-01	1.235902e-01	1.428208e-01	7.354314e-02	1.952494e-01	7.746228e-02	1.861600e-01	9.509754e-02
	std.	5.187460e-03	9.136702e-03	1.172636e-03	2.501039e-03	4.517857e-03	7.625195e-03	1.183878e-03	9.375794e-03	1.302615e-03	6.241501e-03	2.064426e-03

Figure A.308: Box-plot of the R2 indicator values for different optimizers on the DTLZ6 test problem.

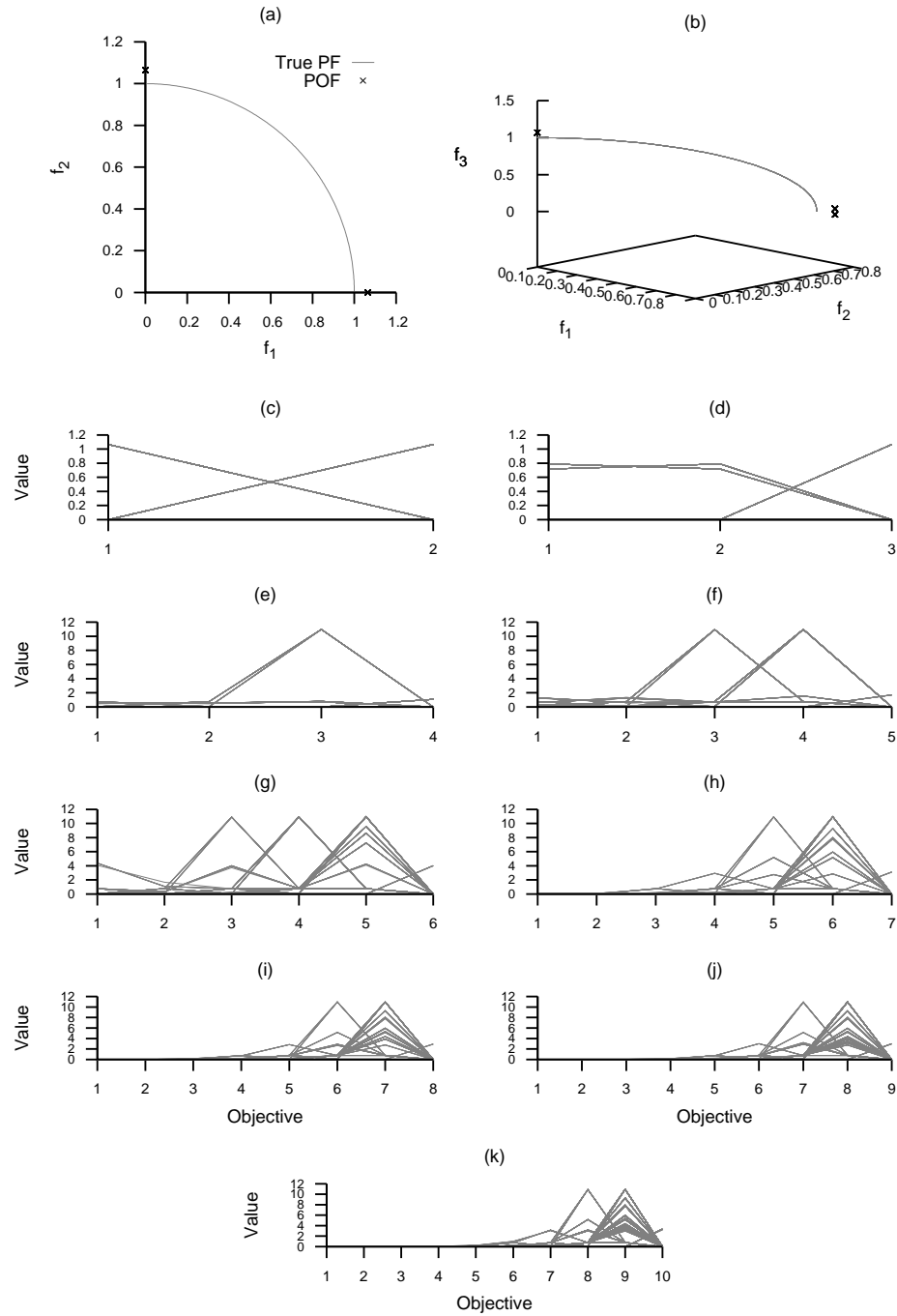


Figure A.309: Plots of the approximations obtained by MOMBI-WS from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

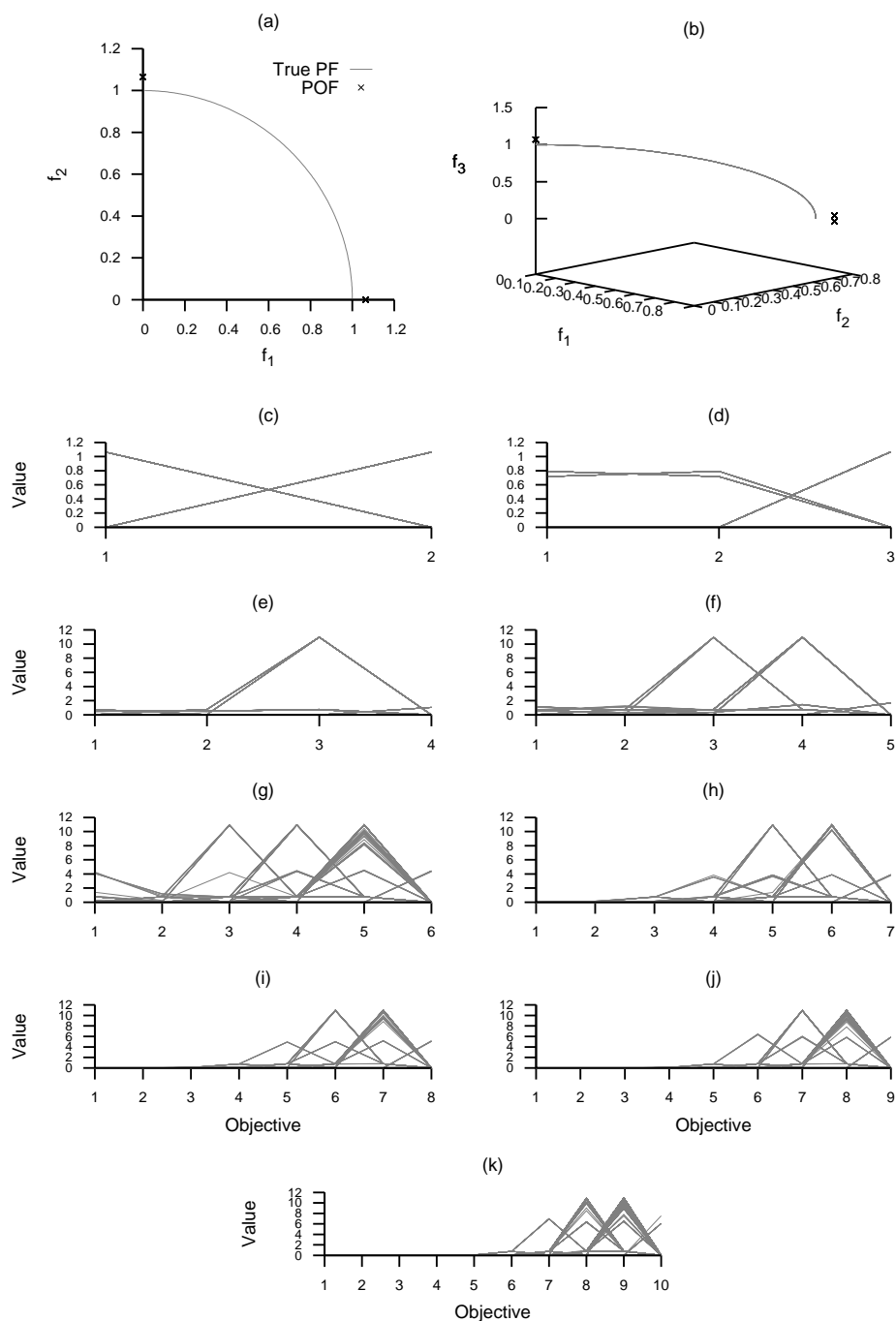


Figure A.310: Plots of the approximations obtained by MOMBI-NWS from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

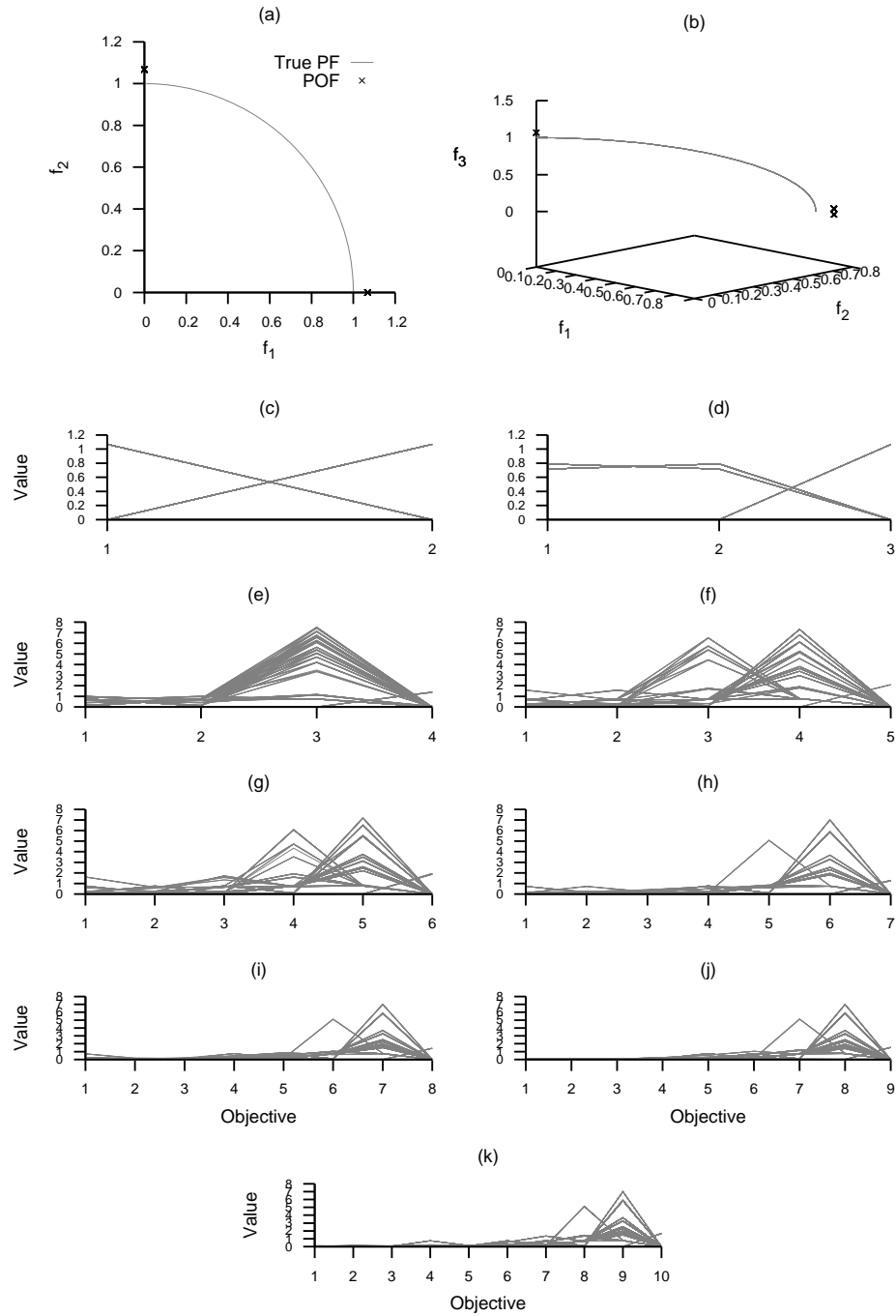


Figure A.311: Plots of the approximations obtained by MOMBI-LS from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

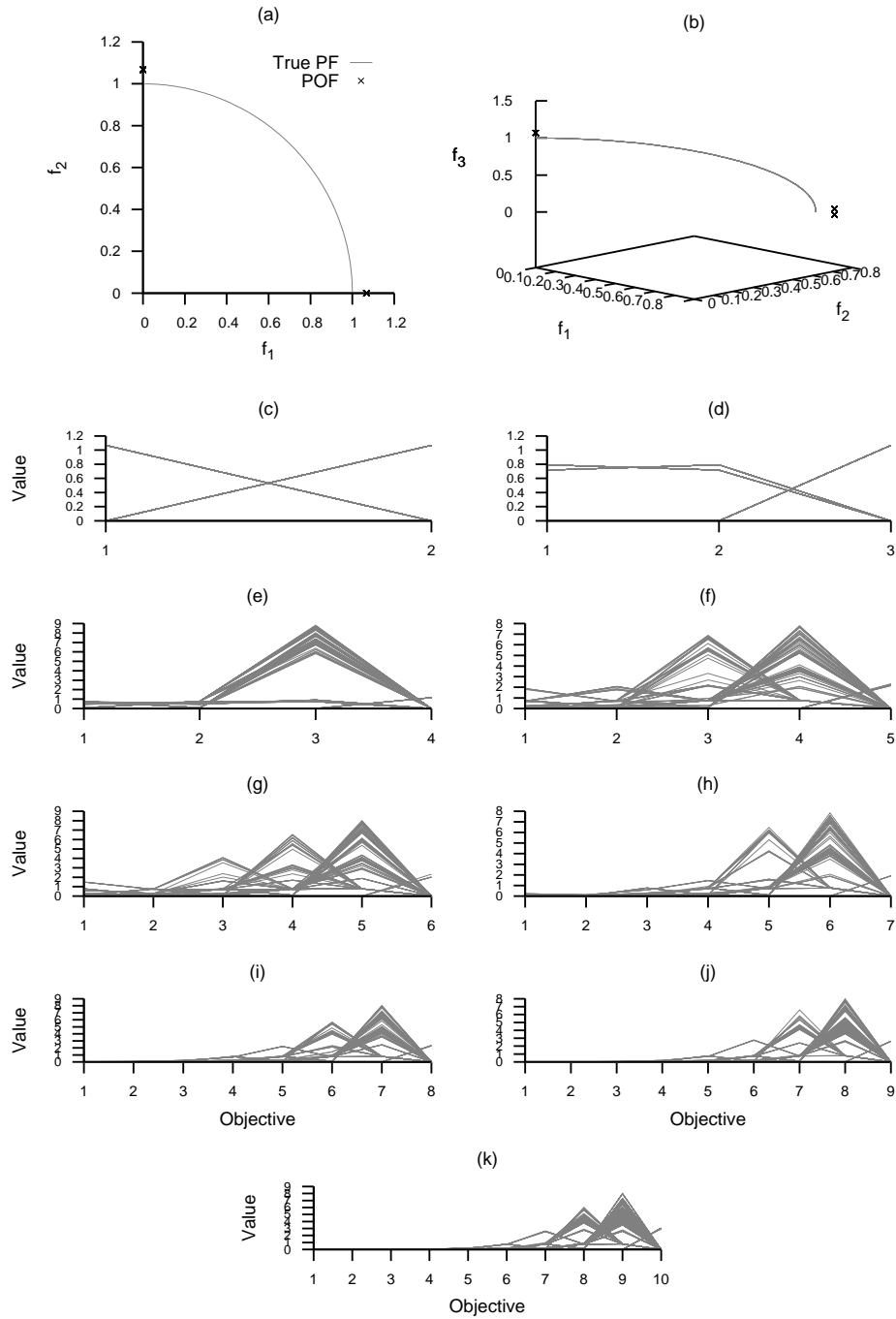


Figure A.312: Plots of the approximations obtained by MOMBI-NLS from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

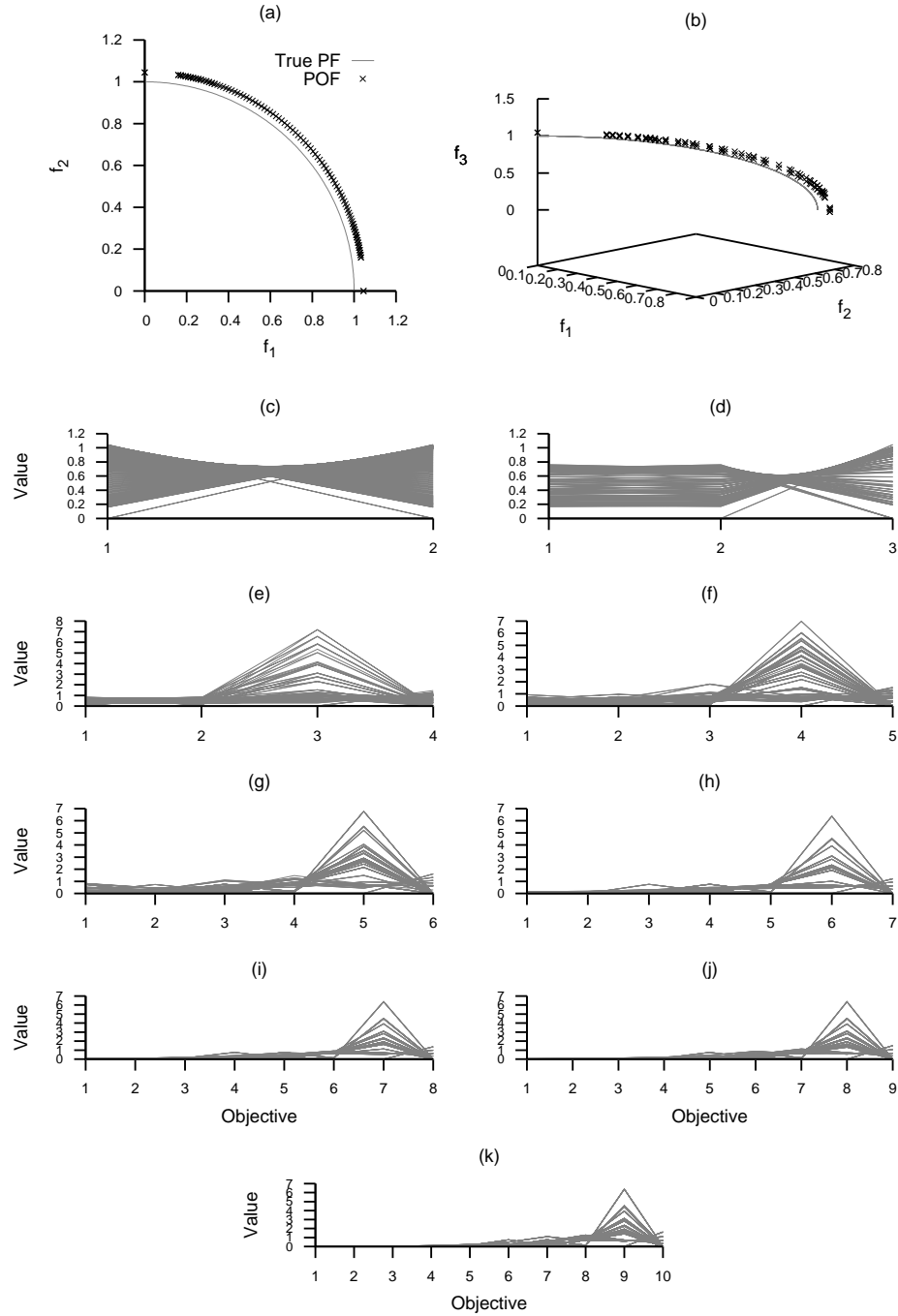


Figure A.313: Plots of the approximations obtained by MOMBI-ATCH from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

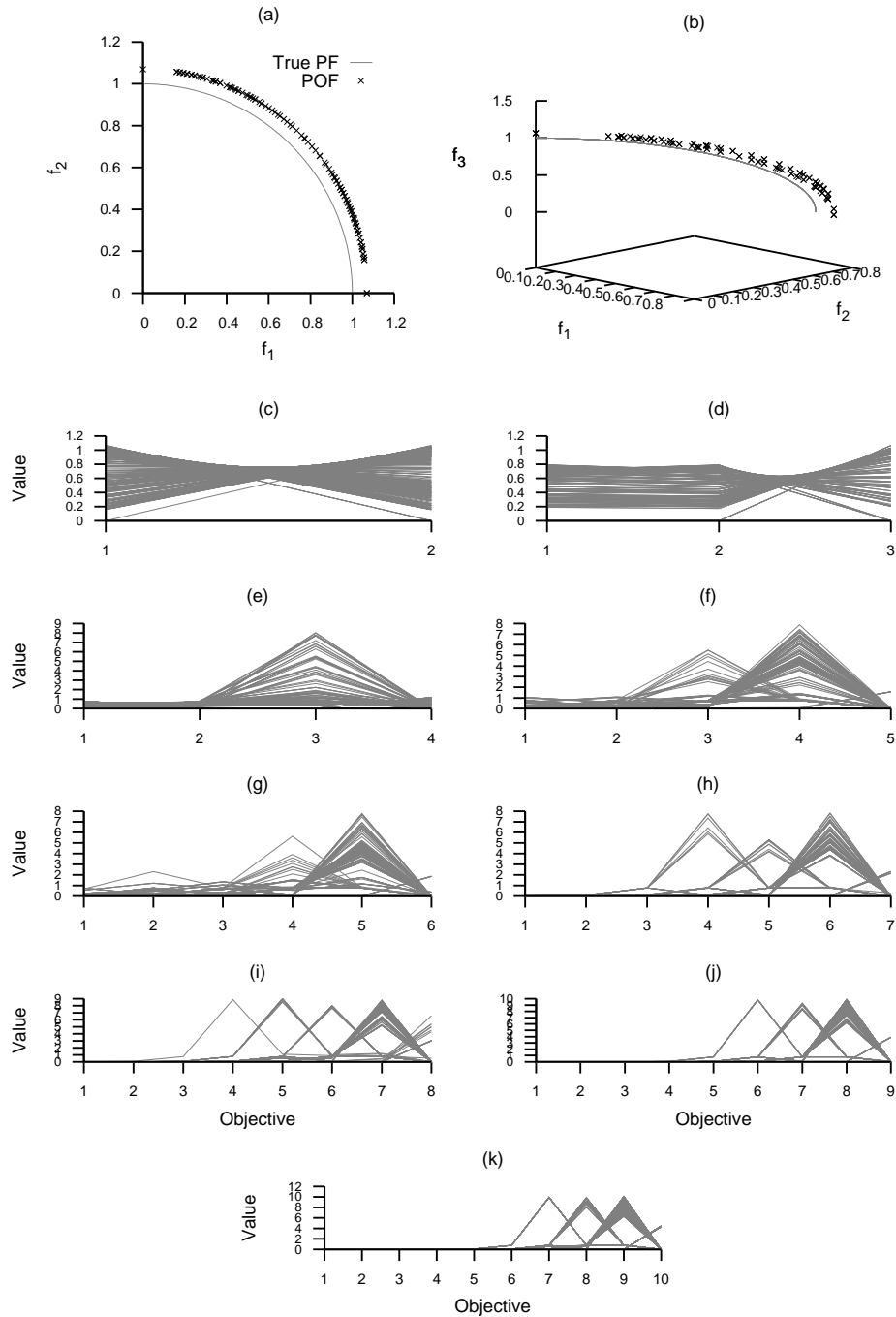


Figure A.314: Plots of the approximations obtained by MOMBI-NATCH from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



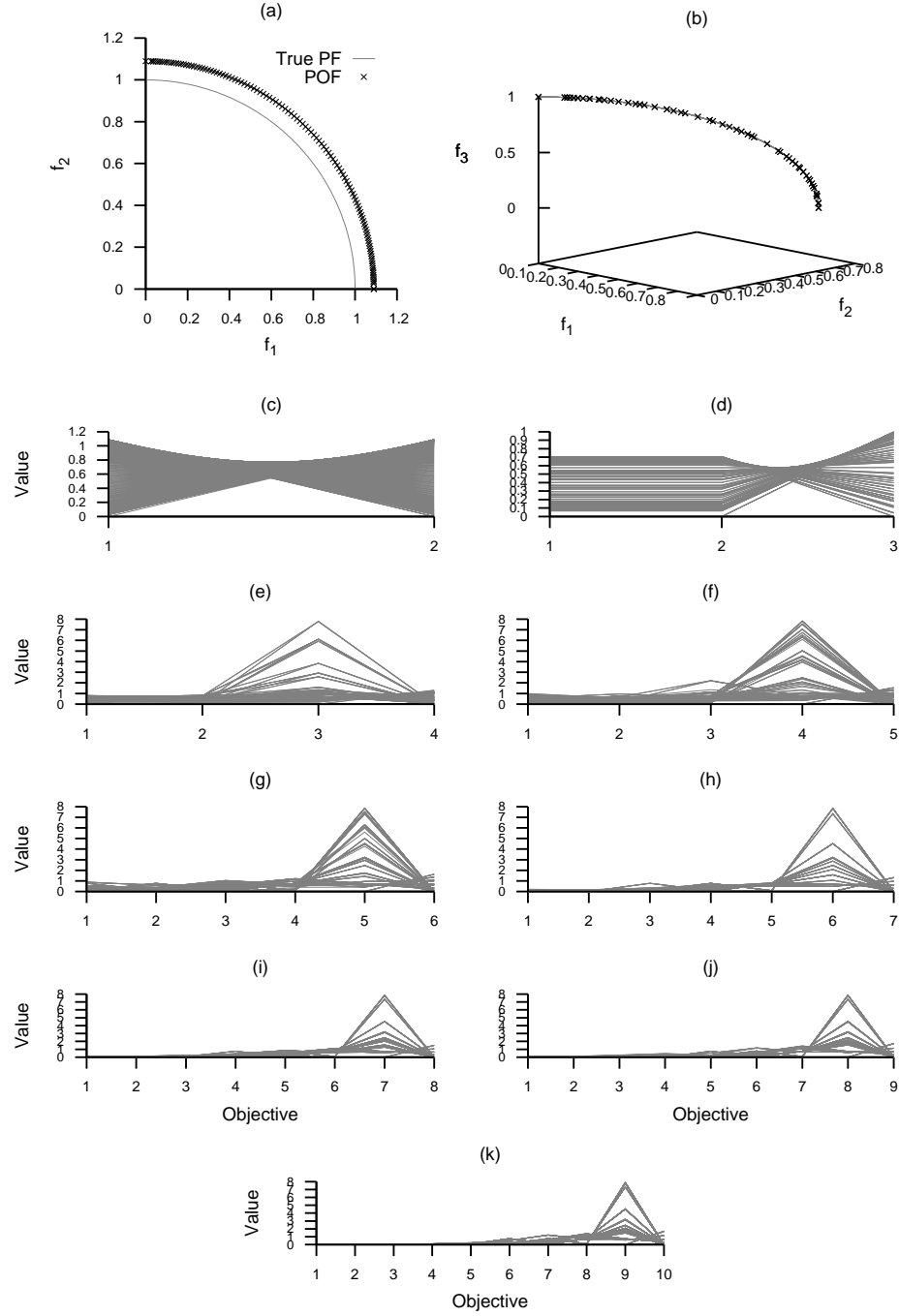


Figure A.315: Plots of the approximations obtained by MOMBI-MTCH from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

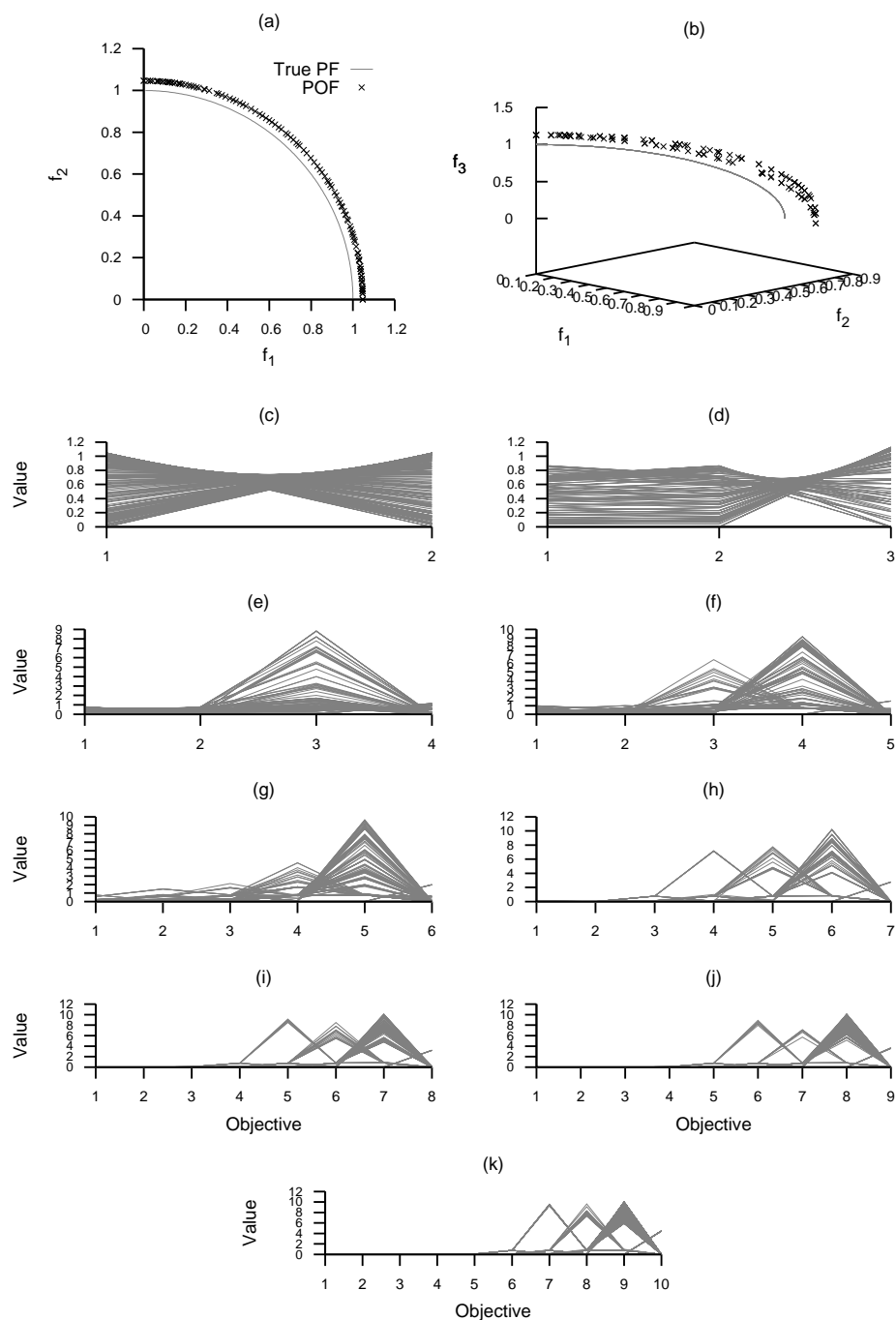


Figure A.316: Plots of the approximations obtained by MOMBI-NMTCH from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.128: Comparison of hypervolume indicator values for different optimizers on the DTLZ7 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	1.728137e + 01	1.728087e + 01	1.765618e + 01	1.766214e + 01	1.772197e + 01	1.772270e + 01	1.772262e + 01	1.772359e + 01	1.772403e + 01	1.772382e + 01	1.771595e + 01
	avg.	1.728137e + 01	1.728140e + 01	1.765568e + 01	1.766216e + 01	1.772189e + 01	1.772266e + 01	1.771904e + 01	1.772354e + 01	1.772393e + 01	1.772380e + 01	1.771570e + 01
	min.	1.728134e + 01	1.727979e + 01	1.764320e + 01	1.765739e + 01	1.771383e + 01	1.772021e + 01	1.737737e + 01	1.772183e + 01	1.772272e + 01	1.772320e + 01	1.770257e + 01
	max.	1.728140e + 01	1.731475e + 01	1.766147e + 01	1.766422e + 01	1.772415e + 01	1.772400e + 01	1.772417e + 01	1.772421e + 01	1.772408e + 01	1.772434e + 01	1.772224e + 01
	std.	1.202165e - 05	3.906016e - 03	4.114939e - 03	1.080536e - 03	1.714168e - 03	8.129700e - 04	3.453887e - 02	4.098344e - 04	2.467137e - 04	2.349450e - 04	4.186494e - 03
3D	med.	1.554964e + 01	1.555603e + 01	1.571400e + 01	1.620459e + 01	1.611744e + 01	1.630126e + 01	1.612840e + 01	1.628411e + 01	1.626309e + 01	1.629742e + 01	1.624668e + 01
	avg.	1.555262e + 01	1.555680e + 01	1.571428e + 01	1.620489e + 01	1.611784e + 01	1.629562e + 01	1.612695e + 01	1.626840e + 01	1.626117e + 01	1.628271e + 01	1.624647e + 01
	min.	1.554728e + 01	1.554943e + 01	1.570265e + 01	1.619236e + 01	1.600816e + 01	1.605722e + 01	1.574061e + 01	1.600392e + 01	1.604985e + 01	1.574389e + 01	1.622866e + 01
	max.	1.576529e + 01	1.562475e + 01	1.572736e + 01	1.622755e + 01	1.614431e + 01	1.633157e + 01	1.614122e + 01	1.631511e + 01	1.627533e + 01	1.631411e + 01	1.625886e + 01
	std.	2.161978e - 02	7.464927e - 03	4.233148e - 03	5.678730e - 03	1.261497e - 02	4.224310e - 02	1.271816e - 02	8.681034e - 02	2.141227e - 02	7.317055e - 02	6.739578e - 03
4D	med.	1.378171e + 01	1.375010e + 01	1.375358e + 01	1.455702e + 01	1.383638e + 01	1.449452e + 01	1.388706e + 01	1.433381e + 01	1.441730e + 01	1.433769e + 01	1.409819e + 01
	avg.	1.379110e + 01	1.376579e + 01	1.375627e + 01	1.454921e + 01	1.384391e + 01	1.449544e + 01	1.389229e + 01	1.433678e + 01	1.441598e + 01	1.434210e + 01	1.409434e + 01
	min.	1.377956e + 01	1.374042e + 01	1.373711e + 01	1.440608e + 01	1.382584e + 01	1.433679e + 01	1.387141e + 01	1.425858e + 01	1.433637e + 01	1.428082e + 01	1.398546e + 01
	max.	1.393783e + 01	1.386165e + 01	1.392856e + 01	1.459522e + 01	1.418933e + 01	1.458689e + 01	1.408361e + 01	1.440192e + 01	1.442746e + 01	1.443759e + 01	1.413562e + 01
	std.	2.821628e - 02	3.693785e - 02	2.449152e - 02	3.878623e - 02	4.296504e - 02	3.156033e - 02	2.909144e - 02	2.502639e - 02	1.168645e - 02	2.855315e - 02	2.903825e - 02
5D	med.	1.203787e + 01	1.195300e + 01	1.191506e + 01	1.279484e + 01	1.195475e + 01	1.246668e + 01	1.203692e + 01	1.214457e + 01	1.222867e + 01	1.194148e + 01	8.387793e + 00
	avg.	1.205676e + 01	1.197204e + 01	1.192255e + 01	1.277091e + 01	1.199139e + 01	1.249869e + 01	1.204700e + 01	1.215505e + 01	1.225791e + 01	1.199786e + 01	7.134455e + 00
	min.	1.202301e + 01	1.180161e + 01	1.188806e + 01	1.255296e + 01	1.192681e + 01	1.238050e + 01	1.201549e + 01	1.184717e + 01	1.219981e + 01	1.182795e + 01	6.226556e + 00
	max.	1.219759e + 01	1.212627e + 01	1.204733e + 01	1.282677e + 01	1.243209e + 01	1.268362e + 01	1.225689e + 01	1.243005e + 01	1.242246e + 01	1.243157e + 01	1.168835e + 01
	std.	3.867397e - 02	4.517848e - 02	3.108325e - 02	5.253857e - 02	7.384724e - 02	7.588989e - 02	3.502248e - 02	1.520737e - 01	6.201215e - 02	1.576678e - 01	3.594329e + 00
6D	med.	9.328466e + 00	9.541808e + 00	5.972829e + 00	1.059740e + 01	6.054436e + 00	8.127237e + 00	6.376859e + 00	9.657379e + 00	6.474191e + 00	9.627215e + 00	1.647893e - 01
	avg.	8.143230e + 00	9.643102e + 00	6.154378e + 00	1.063501e + 01	7.656988e + 00	8.839305e + 00	7.057919e + 00	9.765252e + 00	7.623748e + 00	9.594892e + 00	3.092605e - 01
	min.	6.257483e + 00	9.539104e + 00	5.924818e + 00	1.057433e + 01	5.888631e + 00	7.214579e + 00	6.295344e + 00	9.320762e + 00	6.227356e + 00	9.304300e + 00	3.461684e - 02
	max.	1.043638e + 01	1.022147e + 01	9.029235e + 00	1.071262e + 01	1.066771e + 01	1.069791e + 01	1.042491e + 01	1.054434e + 01	1.057309e + 01	1.047739e + 01	8.296355e + 00
	std.	1.635910e + 00	1.914883e - 01	7.255676e - 01	4.951636e - 02	1.730969e + 00	1.181205e + 00	1.315873e + 00	3.473767e - 01	1.598099e + 00	3.110484e - 01	8.549529e - 01
7D	med.	3.397868e + 00	7.097321e + 00	5.077203e - 01	8.426879e + 00	1.300288e + 00	4.674585e + 00	6.194194e - 01	7.464767e + 00	1.551522e + 00	7.181125e + 00	5.315226e - 03
	avg.	3.524910e + 00	7.234054e + 00	6.968789e - 01	8.403854e + 00	1.578923e + 00	5.008386e + 00	1.625507e + 00	7.508695e + 00	1.705373e + 00	7.376929e + 00	1.214401e - 02
	min.	6.086098e - 01	7.031653e + 00	5.028349e - 01	8.311160e + 00	4.869255e - 01	2.033824e + 00	6.058023e - 01	7.017161e + 00	6.000994e - 01	7.011218e + 00	3.801991e - 04
	max.	8.404071e + 00	8.437897e + 00	2.995332e + 00	8.577506e + 00	5.408080e + 00	8.640790e + 00	5.991186e + 00	8.259888e + 00	7.639375e + 00	8.536313e + 00	1.285612e - 01
	std.	2.305542e + 00	2.811889e - 01	4.040863e - 01	6.264966e - 02	1.299178e + 00	1.720668e + 00	1.461494e + 00	3.644529e - 01	1.397845e + 00	3.901113e - 01	2.203898e - 02
8D	med.	1.071695e + 00	5.025422e + 00	1.200417e - 01	6.543096e + 00	3.221806e - 01	2.937004e + 00	1.552877e - 01	5.164584e + 00	4.271039e - 01	5.054363e + 00	7.861898e - 04
	avg.	1.303577e + 00	5.100345e + 00	1.452392e - 01	6.475352e + 00	3.526923e - 01	3.194623e + 00	2.579946e - 01	5.369655e + 00	4.703431e - 01	5.190282e + 00	1.489169e - 03
	min.	1.487616e - 01	5.005945e + 00	1.174860e - 01	6.284063e + 00	1.098376e - 01	1.227866e + 00	1.489663e - 01	4.989504e + 00	1.492075e - 01	4.981085e + 00	1.299216e - 04
	max.	5.665065e + 00	6.478422e + 00	3.532738e - 01	6.620631e + 00	3.754249e + 00	6.973019e + 00	1.109031e + 00	6.632260e + 00	2.426726e + 00	6.674643e + 00	1.661220e - 02
	std.	1.259288e + 00	1.939773e - 01	7.205777e - 02	1.134149e - 01	4.776658e - 01	1.368911e + 00	1.932724e - 01	4.072633e - 01	4.955899e - 01	2.848281e - 01	2.317462e - 03
9D	med.	1.130010e - 01	3.001090e + 00	2.627568e - 02	4.641475e + 00	2.662758e - 02	1.618894e + 00	3.616213e - 02	3.116268e + 00	3.715046e - 02	2.990283e + 00	1.177491e - 04
	avg.	3.844836e - 01	3.036216e + 00	2.625884e - 02	4.582178e + 00	7.147486e - 02	1.773252e + 00	4.911565e - 02	3.334658e + 00	7.362050e - 02	3.081011e + 00	1.243267e - 04
	min.	3.343898e - 02	2.979259e + 00	2.534820e - 02	4.262889e + 00	2.285187e - 02	6.291010e - 01	3.456415e - 02	2.916372e + 00	3.428146e - 02	2.876414e + 00	1.694464e - 05
	max.	2.857265e + 00	3.950461e + 00	2.739753e - 02	4.737432e + 00	6.009220e - 01	4.300545e + 00	2.987088e - 01	4.729423e + 00	7.320156e - 01	4.217819e + 00	5.310767e - 04
	std.	5.226562e - 01	1.146445e - 01	3.834621e - 04	1.228942e - 01	9.746736e - 02	8.026425e - 01	4.219430e - 02	4.816532e - 01	8.496324e - 02	2.349127e - 01	9.057391e - 05
10D	med.	2.692160e - 02	9.733050e - 01	5.463280e - 03	2.678785e + 00	5.608767e - 03	9.039359e - 01	8.037155e - 03	1.058422e + 00	7.963922e - 03	9.473147e - 01	1.735360e - 05
	avg.	5.504205e - 02	9.879624e - 01	5.586464e - 03	2.655906e + 00	1.041312e - 02	1.080215e + 00	9.042640e - 03	1.200667e + 00	1.112772e - 02	9.738563e - 01	1.591826e - 05
	min.	7.319268e - 03	8.707262e - 01	5.198163e - 03	2.280860e + 00	3.294302e - 03	3.294302e - 01	7.404142e - 03	8.114409e - 01	7.321378e - 03	6.736326e - 01	2.005296e - 06
	max.	5.516946e - 01	1.865516e + 00	1.751762e - 02	2.850882e + 00	5.250525e - 02	2.891992e + 00	2.562818e - 02	2.754698e + 00	7.897572e - 02	1.612034e + 00	8.728074e - 05
	std.	8.417731e - 02	9.705028e - 02	1.213274e - 03	1.128560e - 01	1.005113e - 02	5.245388e - 01	4.125236e - 03	3.689185e - 01	9.195294e - 03	1.363388e - 01	1.143081e - 05

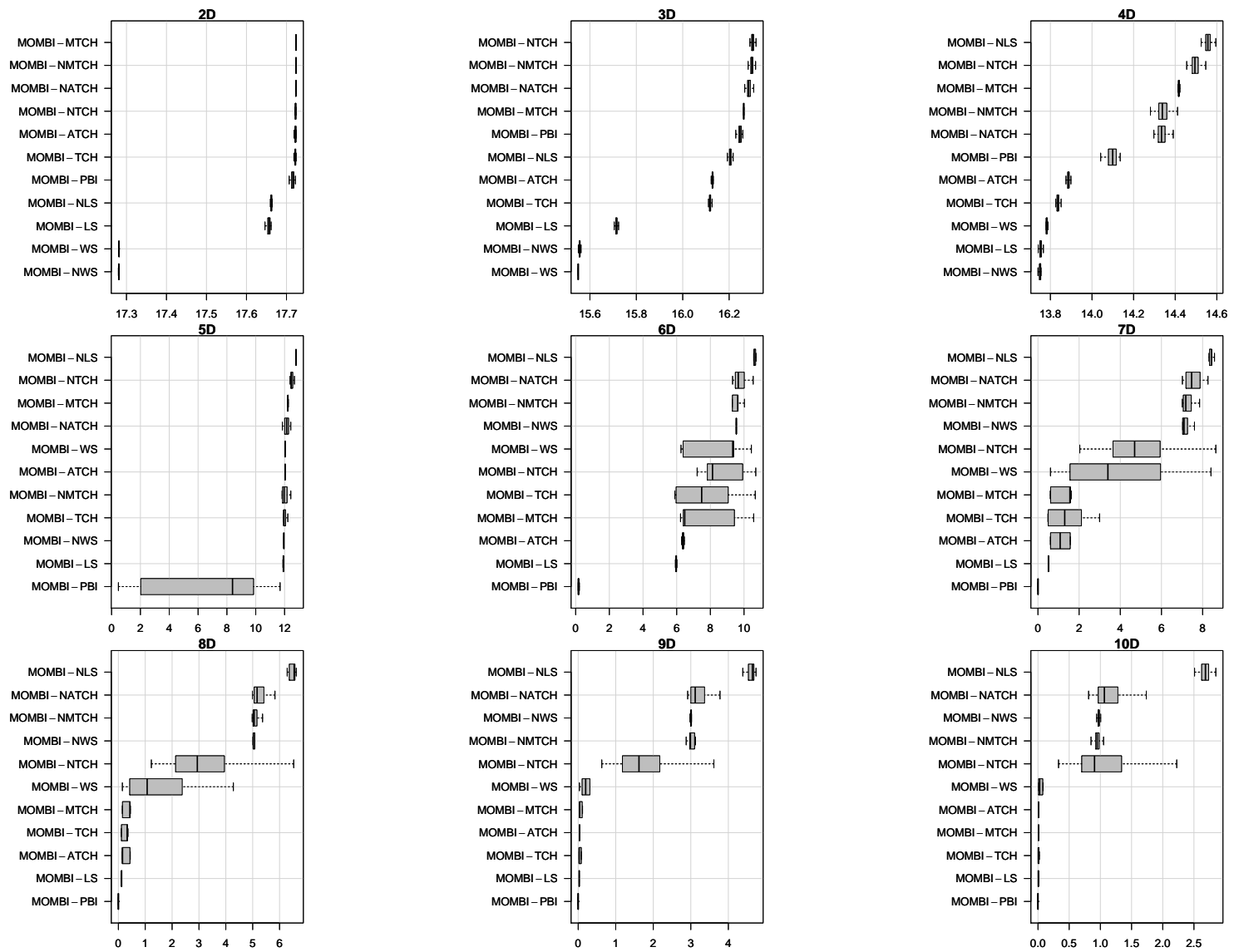


Figure A.317: Box-plot of the hypervolume indicator values for different optimizers on the DTLZ7 test problem.

Table A.129: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ7 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
2D											
MOMBI-WS	–	$7.80e-22$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$5.10e-35$	$1.28e-34$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NLS	$5.09e-35$	$1.28e-34$	$1.75e-32$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-TCH	$5.10e-35$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$2.52e-26$
MOMBI-NTCH	$5.09e-35$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$5.92e-04$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$2.24e-32$
MOMBI-ATCH	$5.09e-35$	$1.28e-34$	$2.51e-33$	$2.51e-33$	$5.78e-03$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$9.40e-29$
MOMBI-NATCH	$5.08e-35$	$1.27e-34$	$1.28e-34$	$1.28e-34$	$1.07e-17$	$4.04e-17$	$3.25e-09$	–	$> 0.05$	$> 0.05$	$1.48e-34$
MOMBI-MTCH	$4.75e-35$	$1.19e-34$	$1.20e-34$	$1.20e-34$	$1.74e-25$	$2.76e-30$	$6.21e-16$	$1.38e-16$	–	$1.61e-08$	$1.20e-34$
MOMBI-NMTCH	$5.05e-35$	$1.27e-34$	$1.27e-34$	$1.27e-34$	$5.24e-23$	$2.20e-26$	$9.53e-14$	$8.61e-07$	$> 0.05$	–	$1.27e-34$
MOMBI-PBI	$5.10e-35$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–
3D											
MOMBI-WS	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NWS	$5.93e-24$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$2.48e-33$	$1.28e-34$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NLS	$1.27e-34$	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-TCH	$1.26e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NTCH	$1.27e-34$	$1.28e-34$	$1.28e-34$	$8.06e-31$	$7.40e-31$	–	$7.40e-31$	$2.00e-22$	$7.39e-31$	$8.29e-06$	$8.06e-31$
MOMBI-ATCH	$1.26e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$7.78e-26$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NATCH	$1.34e-34$	$1.28e-34$	$1.28e-34$	$2.04e-28$	$1.88e-28$	$> 0.05$	$1.88e-28$	–	$2.90e-28$	$> 0.05$	$2.04e-28$
MOMBI-MTCH	$1.26e-34$	$1.28e-34$	$1.28e-34$	$2.50e-33$	$2.43e-33$	$> 0.05$	$2.43e-33$	$> 0.05$	–	$> 0.05$	$2.50e-33$
MOMBI-NMTCH	$1.30e-34$	$1.28e-34$	$1.28e-34$	$2.04e-28$	$1.83e-28$	$> 0.05$	$1.83e-28$	$5.19e-15$	$1.88e-28$	–	$2.04e-28$
MOMBI-PBI	$1.27e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$	$> 0.05$	$> 0.05$	–
4D											
MOMBI-WS	–	$1.36e-16$	$4.12e-32$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$> 0.05$	$1.00e-02$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NLS	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$1.38e-23$	$1.28e-34$	$1.28e-34$	$3.03e-31$	$2.07e-34$	$1.28e-34$
MOMBI-TCH	$1.72e-25$	$2.04e-21$	$4.13e-32$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	–	$1.28e-34$	$1.35e-33$	$4.18e-32$	$2.23e-33$	$1.28e-34$
MOMBI-ATCH	$4.01e-32$	$1.28e-34$	$3.57e-32$	$> 0.05$	$3.02e-29$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.28e-34$	–	$> 0.05$	$> 0.05$	$1.28e-34$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.28e-34$	$1.81e-33$	–	$1.01e-30$	$1.28e-34$
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.28e-34$	$> 0.05$	$> 0.05$	–	$1.28e-34$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$3.17e-33$	$> 0.05$	$6.63e-34$	$> 0.05$	$> 0.05$	$> 0.05$	–

Table A.130: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ7 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
5D											
MOMBI-WS	–	$5.98e-25$	$2.38e-31$	$> 0.05$	$4.63e-11$	$> 0.05$	$1.81e-02$	$> 0.05$	$> 0.05$	$1.90e-06$	$1.28e-34$
MOMBI-NWS	$> 0.05$	–	$7.88e-27$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$
MOMBI-LS	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$
MOMBI-NLS	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$6.24e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$> 0.05$	$> 0.05$	$9.54e-29$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$2.48e-34$	–	$1.28e-34$	$9.19e-34$	$8.16e-34$	$2.48e-34$	$1.28e-34$
MOMBI-ATCH	$> 0.05$	$4.65e-22$	$7.61e-31$	$> 0.05$	$9.69e-09$	$> 0.05$	–	$> 0.05$	$> 0.05$	$4.56e-06$	$1.28e-34$
MOMBI-NATCH	$3.37e-02$	$2.87e-23$	$2.94e-30$	$> 0.05$	$1.18e-18$	$> 0.05$	$3.00e-02$	–	$> 0.05$	$4.41e-10$	$1.28e-34$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$> 0.05$	$6.23e-33$	$> 0.05$	$1.31e-33$	$3.63e-09$	–	$1.06e-25$	$1.28e-34$
MOMBI-NMTCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$1.28e-34$
MOMBI-PBI	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–
6D											
MOMBI-WS	–	$> 0.05$	$3.44e-31$	$> 0.05$	$1.19e-07$	$> 0.05$	$3.97e-07$	$> 0.05$	$> 0.05$	$> 0.05$	$4.92e-34$
MOMBI-NWS	$3.97e-19$	–	$1.28e-34$	$> 0.05$	$2.50e-18$	$4.17e-03$	$1.12e-29$	$> 0.05$	$7.40e-23$	$> 0.05$	$1.28e-34$
MOMBI-LS	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$2.10e-33$
MOMBI-NLS	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	$6.43e-34$	$3.37e-33$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	$> 0.05$	$> 0.05$	$7.32e-08$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$5.71e-34$
MOMBI-NTCH	$9.96e-05$	$> 0.05$	$1.88e-30$	$> 0.05$	$2.93e-06$	–	$1.80e-19$	$> 0.05$	$4.55e-11$	$> 0.05$	$6.63e-34$
MOMBI-ATCH	$> 0.05$	$> 0.05$	$8.56e-29$	$> 0.05$	$2.37e-02$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$1.31e-33$
MOMBI-NATCH	$1.99e-12$	$4.42e-05$	$1.28e-34$	$> 0.05$	$5.63e-22$	$2.90e-05$	$8.19e-26$	–	$5.14e-17$	$2.91e-10$	$1.28e-34$
MOMBI-MTCH	$> 0.05$	$> 0.05$	$7.18e-30$	$> 0.05$	$2.28e-04$	$> 0.05$	$2.32e-08$	$> 0.05$	–	$> 0.05$	$8.40e-34$
MOMBI-NMTCH	$3.09e-07$	$> 0.05$	$1.28e-34$	$> 0.05$	$4.62e-19$	$3.12e-03$	$1.31e-21$	$> 0.05$	$6.73e-13$	–	$1.28e-34$
MOMBI-PBI	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–
7D											
MOMBI-WS	–	$> 0.05$	$4.57e-31$	$> 0.05$	$1.68e-16$	$> 0.05$	$3.49e-13$	$> 0.05$	$2.27e-10$	$> 0.05$	$1.28e-34$
MOMBI-NWS	$4.35e-23$	–	$1.28e-34$	$> 0.05$	$1.28e-34$	$2.78e-18$	$1.28e-34$	$> 0.05$	$2.10e-33$	$> 0.05$	$1.28e-34$
MOMBI-LS	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$
MOMBI-NLS	$3.18e-33$	$1.16e-33$	$1.28e-34$	–	$1.28e-34$	$1.20e-31$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$4.01e-32$	$1.28e-34$
MOMBI-TCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$
MOMBI-NTCH	$4.39e-07$	$> 0.05$	$1.68e-34$	$> 0.05$	$6.68e-28$	–	$7.58e-26$	$> 0.05$	$1.05e-26$	$> 0.05$	$1.28e-34$
MOMBI-ATCH	$> 0.05$	$> 0.05$	$4.88e-22$	$> 0.05$	$1.12e-02$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$1.28e-34$
MOMBI-NATCH	$1.60e-24$	$3.85e-08$	$1.28e-34$	$> 0.05$	$1.28e-34$	$4.71e-21$	$1.28e-34$	–	$9.75e-34$	$3.18e-06$	$1.28e-34$
MOMBI-MTCH	$> 0.05$	$> 0.05$	$8.97e-25$	$> 0.05$	$2.84e-04$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$1.28e-34$
MOMBI-NMTCH	$1.00e-23$	$8.24e-03$	$1.28e-34$	$> 0.05$	$1.28e-34$	$1.18e-19$	$1.28e-34$	$> 0.05$	$1.31e-33$	–	$1.28e-34$
MOMBI-PBI	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–

Table A.131: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ7 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
8D											
MOMBI-WS	–	> 0.05	$1.22e-32$	> 0.05	$1.63e-20$	> 0.05	$1.04e-13$	> 0.05	$2.77e-06$	> 0.05	$1.28e-34$
MOMBI-NWS	$2.37e-33$	–	$1.28e-34$	> 0.05	$1.28e-34$	$1.28e-21$	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NLS	$1.28e-34$	$5.07e-34$	$1.28e-34$	–	$1.28e-34$	$6.43e-31$	$1.28e-34$	$5.23e-33$	$1.28e-34$	$2.51e-33$	$1.28e-34$
MOMBI-TCH	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NTCH	$1.28e-18$	> 0.05	$1.28e-34$	> 0.05	$4.02e-33$	–	$1.28e-34$	> 0.05	$8.85e-33$	> 0.05	$1.28e-34$
MOMBI-ATCH	> 0.05	> 0.05	$2.46e-25$	> 0.05	$9.72e-03$	> 0.05	–	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NATCH	$1.35e-33$	$2.11e-09$	$1.28e-34$	> 0.05	$1.28e-34$	$4.55e-24$	$1.28e-34$	–	$1.28e-34$	$4.64e-06$	$1.28e-34$
MOMBI-MTCH	> 0.05	> 0.05	$1.32e-28$	> 0.05	$7.72e-08$	> 0.05	$1.23e-04$	> 0.05	–	> 0.05	$1.28e-34$
MOMBI-NMTCH	$1.87e-33$	$3.81e-02$	$1.28e-34$	> 0.05	$1.28e-34$	$1.45e-22$	$1.28e-34$	> 0.05	$1.28e-34$	–	$1.28e-34$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
9D											
MOMBI-WS	–	> 0.05	$1.28e-34$	> 0.05	$1.28e-21$	> 0.05	$8.70e-19$	> 0.05	$1.62e-12$	> 0.05	$1.28e-34$
MOMBI-NWS	$1.28e-34$	–	$1.28e-34$	> 0.05	$1.28e-34$	$3.52e-25$	$1.28e-34$	> 0.05	$1.28e-34$	$4.75e-03$	$1.28e-34$
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NLS	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$1.53e-34$	$1.28e-34$	$2.30e-29$	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NTCH	$4.97e-28$	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	–	$1.28e-34$	> 0.05	$1.53e-34$	> 0.05	$1.28e-34$
MOMBI-ATCH	> 0.05	> 0.05	$1.28e-34$	> 0.05	$7.07e-05$	> 0.05	–	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NATCH	$1.28e-34$	$7.28e-07$	$1.28e-34$	> 0.05	$1.28e-34$	$3.39e-27$	$1.28e-34$	–	$1.28e-34$	$1.89e-08$	$1.28e-34$
MOMBI-MTCH	> 0.05	> 0.05	$1.28e-34$	> 0.05	$7.32e-08$	> 0.05	$3.65e-06$	> 0.05	–	> 0.05	$1.28e-34$
MOMBI-NMTCH	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	$4.00e-25$	$1.28e-34$	> 0.05	$1.28e-34$	–	$1.28e-34$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
10D											
MOMBI-WS	–	> 0.05	$3.15e-34$	> 0.05	$1.26e-22$	> 0.05	$3.41e-21$	> 0.05	$1.15e-18$	> 0.05	$1.28e-34$
MOMBI-NWS	$1.28e-34$	–	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	$2.41e-06$	$1.28e-34$
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NLS	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$3.27e-33$	$1.28e-34$	$1.57e-33$	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-TCH	> 0.05	> 0.05	$5.95e-04$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NTCH	$2.01e-34$	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	–	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$
MOMBI-ATCH	> 0.05	> 0.05	$2.10e-33$	> 0.05	$1.94e-08$	> 0.05	–	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NATCH	$1.28e-34$	$3.55e-07$	$1.28e-34$	> 0.05	$1.28e-34$	$2.92e-04$	$1.28e-34$	–	$1.28e-34$	$5.80e-11$	$1.28e-34$
MOMBI-MTCH	> 0.05	> 0.05	$1.61e-33$	> 0.05	$6.98e-10$	> 0.05	> 0.05	> 0.05	–	> 0.05	$1.28e-34$
MOMBI-NMTCH	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	–	$1.28e-34$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–

Table A.132: Comparison of R2 indicator values for different optimizers on the DTLZ7 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	1.192281e + 00	1.192302e + 00	1.182371e + 00	1.182860e + 00	1.179869e + 00	1.179954e + 00	1.179876e + 00	1.179962e + 00	1.179835e + 00	1.179898e + 00	1.180624e + 00
	avg.	1.192281e + 00	1.192284e + 00	1.182364e + 00	1.182816e + 00	1.179869e + 00	1.179955e + 00	1.185944e + 00	1.179964e + 00	1.179835e + 00	1.179899e + 00	1.180621e + 00
	min.	1.192279e + 00	1.190875e + 00	1.182085e + 00	1.182130e + 00	1.179840e + 00	1.179859e + 00	1.179841e + 00	1.179866e + 00	1.179831e + 00	1.179857e + 00	1.180186e + 00
	max.	1.192282e + 00	1.192348e + 00	1.182726e + 00	1.183682e + 00	1.179925e + 00	1.180057e + 00	1.786873e + 00	1.180068e + 00	1.179840e + 00	1.179953e + 00	1.181139e + 00
	std.	5.700812e - 07	1.487844e - 04	1.328613e - 04	2.886080e - 04	1.921492e - 05	4.635247e - 05	6.069994e - 02	4.344786e - 05	1.148492e - 06	1.962076e - 05	2.037037e - 04
3D	med.	9.144594e - 01	9.144123e - 01	9.056795e - 01	9.061610e - 01	9.021321e - 01	9.026211e - 01	9.021283e - 01	9.026802e - 01	9.018698e - 01	9.025061e - 01	9.024264e - 01
	avg.	9.144442e - 01	9.183587e - 01	9.056796e - 01	9.060856e - 01	9.061571e - 01	9.146904e - 01	9.061543e - 01	9.310869e - 01	9.058955e - 01	9.267660e - 01	9.024425e - 01
	min.	9.139845e - 01	9.141557e - 01	9.054753e - 01	9.045491e - 01	9.020037e - 01	9.022213e - 01	9.020300e - 01	9.023789e - 01	9.017983e - 01	9.022260e - 01	9.021684e - 01
	max.	9.144659e - 01	1.309442e + 00	9.058859e - 01	9.072948e - 01	1.305042e + 00	1.305246e + 00	1.304993e + 00	1.719460e + 00	1.304879e + 00	1.719422e + 00	9.028608e - 01
	std.	5.451798e - 05	3.950347e - 02	7.964231e - 05	5.547674e - 04	4.029141e - 02	6.902463e - 02	4.028675e - 02	1.325648e - 01	4.030137e - 02	1.126723e - 01	1.588453e - 04
4D	med.	7.704201e - 01	7.705688e - 01	7.632379e - 01	7.671728e - 01	7.607825e - 01	7.614676e - 01	7.607767e - 01	7.615583e - 01	7.601676e - 01	7.613902e - 01	7.792398e - 01
	avg.	7.972651e - 01	8.060179e - 01	7.692134e - 01	7.878855e - 01	7.698894e - 01	7.946450e - 01	7.667736e - 01	8.338462e - 01	7.661923e - 01	8.157042e - 01	7.788931e - 01
	min.	7.701106e - 01	7.705039e - 01	7.630648e - 01	7.656511e - 01	7.606419e - 01	7.607812e - 01	7.609709e - 01	7.609709e - 01	7.600333e - 01	7.606466e - 01	7.687694e - 01
	max.	1.373422e + 00	1.066193e + 00	1.062665e + 00	1.067055e + 00	1.371470e + 00	1.371780e + 00	1.061056e + 00	1.372047e + 00	1.061462e + 00	1.371822e + 00	7.833252e - 01
	std.	1.049517e - 01	9.648444e - 02	4.211049e - 02	7.643798e - 02	6.778441e - 02	1.131063e - 01	4.225050e - 02	1.432250e - 01	4.239349e - 02	1.3872023e - 01	2.401189e - 03
5D	med.	7.231868e - 01	7.236952e - 01	7.191884e - 01	7.224997e - 01	7.181310e - 01	7.197122e - 01	7.178832e - 01	7.789185e - 01	7.181475e - 01	7.191348e - 01	7.499845e - 01
	avg.	7.956783e - 01	7.882841e - 01	7.348208e - 01	7.872401e - 01	8.040682e - 01	7.846378e - 01	7.491395e - 01	9.447563e - 01	7.834003e - 01	7.971192e - 01	7.502658e - 01
	min.	7.229617e - 01	7.228653e - 01	7.189918e - 01	7.201273e - 01	7.179326e - 01	7.190375e - 01	7.177755e - 01	7.191259e - 01	7.177314e - 01	7.185387e - 01	7.317907e - 01
	max.	1.247232e + 00	1.247436e + 00	9.797790e - 01	1.247220e + 00	1.512456e + 00	1.245222e + 00	1.243502e + 00	1.512977e + 00	1.244940e + 00	1.512553e + 00	7.638230e - 01
	std.	1.384992e - 01	1.297323e - 01	6.202598e - 02	1.249075e - 01	1.442380e - 01	1.355308e - 01	9.290338e - 02	2.303735e - 01	1.410718e - 01	1.413639e - 01	5.564723e - 03
6D	med.	8.236190e - 01	6.250209e - 01	6.221221e - 01	6.253583e - 01	6.205445e - 01	6.212941e - 01	6.200612e - 01	8.225383e - 01	6.206672e - 01	8.215692e - 01	6.656601e - 01
	avg.	7.710164e - 01	6.770064e - 01	6.339945e - 01	7.309987e - 01	7.516339e - 01	6.817912e - 01	6.642757e - 01	9.066418e - 01	7.174989e - 01	8.016062e - 01	6.637293e - 01
	min.	6.237362e - 01	6.249759e - 01	6.209466e - 01	6.222602e - 01	6.198650e - 01	6.206767e - 01	6.194042e - 01	6.208274e - 01	6.199608e - 01	6.208329e - 01	6.402896e - 01
	max.	1.237179e + 00	1.029091e + 00	8.222813e - 01	1.031587e + 00	1.446395e + 00	1.027973e + 00	1.027053e + 00	1.658192e + 00	1.236163e + 00	1.446805e + 00	6.716694e - 01
	std.	1.625469e - 01	1.012061e - 01	4.777456e - 02	1.266078e - 01	1.594750e - 01	1.168833e - 01	9.325937e - 02	2.468889e - 01	1.427620e - 01	2.079915e - 01	6.527901e - 03
7D	med.	9.185457e - 01	7.455022e - 01	5.749109e - 01	7.500422e - 01	7.432616e - 01	7.433110e - 01	5.747748e - 01	1.094836e + 00	7.436245e - 01	9.175421e - 01	6.227815e - 01
	avg.	9.068022e - 01	8.117977e - 01	6.121084e - 01	8.715325e - 01	7.382674e - 01	7.857269e - 01	7.060951e - 01	1.060966e + 00	7.249017e - 01	9.655146e - 01	6.266252e - 01
	min.	5.768376e - 01	5.777323e - 01	5.747679e - 01	5.776397e - 01	5.738568e - 01	5.739616e - 01	5.737249e - 01	5.747082e - 01	5.737442e - 01	5.742507e - 01	6.077754e - 01
	max.	1.451211e + 00	1.451111e + 00	9.176836e - 01	1.453017e + 00	1.094565e + 00	1.272491e + 00	1.094480e + 00	1.451728e + 00	1.272888e + 00	1.630378e + 00	7.805526e - 01
	std.	2.172673e - 01	2.081792e - 01	7.452862e - 02	2.277617e - 01	1.528561e - 01	2.047761e - 01	1.584511e - 01	2.319737e - 01	1.497785e - 01	2.546221e - 01	2.870184e - 02
8D	med.	8.388732e - 01	6.875245e - 01	5.381619e - 01	6.923218e - 01	6.875528e - 01	6.872912e - 01	5.394152e - 01	9.923584e - 01	6.877498e - 01	8.378221e - 01	5.831288e - 01
	avg.	8.073064e - 01	7.672813e - 01	5.544715e - 01	7.874674e - 01	6.428980e - 01	7.385432e - 01	5.884366e - 01	1.045721e + 00	6.533965e - 01	9.052131e - 01	5.859689e - 01
	min.	5.399775e - 01	5.397617e - 01	5.379831e - 01	5.402275e - 01	5.386930e - 01	5.377889e - 01	5.386501e - 01	5.376119e - 01	5.386146e - 01	5.371542e - 01	5.703586e - 01
	max.	1.301465e + 00	1.456836e + 00	6.868012e - 01	1.304794e + 00	1.146492e + 00	1.302567e + 00	8.403814e - 01	1.613342e + 00	9.936311e - 01	1.612925e + 00	8.750184e - 01
	std.	1.837337e - 01	1.771590e - 01	4.665749e - 02	1.901274e - 01	1.200164e - 01	1.855470e - 01	7.953570e - 02	2.391731e - 01	1.229720e - 01	2.329125e - 01	3.299774e - 02
9D	med.	6.652489e - 01	6.575769e - 01	5.226694e - 01	6.629776e - 01	5.325124e - 01	6.602453e - 01	5.258438e - 01	1.071037e + 00	5.275353e - 01	7.924555e - 01	5.622039e - 01
	avg.	7.511629e - 01	7.378918e - 01	5.226665e - 01	7.324532e - 01	6.003667e - 01	6.913190e - 01	5.457402e - 01	1.079929e + 00	5.804830e - 01	8.646666e - 01	5.620410e - 01
	min.	5.263183e - 01	5.243660e - 01	5.225202e - 01	5.252488e - 01	5.265550e - 01	5.249187e - 01	5.237226e - 01	5.221903e - 01	5.241242e - 01	5.218015e - 01	5.508404e - 01
	max.	1.214080e + 00	1.351389e + 00	5.231986e - 01	1.355297e + 00	9.385134e - 01	1.074816e + 00	7.956788e - 01	1.633752e + 00	9.350018e - 01	1.492109e + 00	5.663447e - 01
	std.	1.656780e - 01	1.793307e - 01	1.847260e - 04	1.809503e - 01	1.015891e - 01	1.429063e - 01	5.527399e - 02	2.699462e - 01	8.112472e - 02	2.360285e - 01	2.556063e - 03
10D	med.	6.401641e - 01	6.277377e - 01	5.046639e - 01	7.516155e - 01	5.270039e - 01	6.380796e - 01	5.102659e - 01	1.126902e + 00	5.123809e - 01	8.733758e - 01	5.410037e - 01
	avg.	6.573108e - 01	6.869517e - 01	5.058790e - 01	7.119009e - 01	5.637997e - 01	7.082042e - 01	5.174074e - 01	1.105130e + 00	5.314459e - 01	8.521785e - 01	5.424542e - 01
	min.	5.165579e - 01	5.048463e - 01	5.036841e - 01	5.094042e - 01	5.141943e - 01	5.105194e - 01	5.069658e - 01	6.254828e - 01	5.073739e - 01	5.023883e - 01	5.373630e - 01
	max.	1.012577e + 00	1.380180e + 00	6.258602e - 01	1.133505e + 00	7.659561e - 01	1.261974e + 00	6.323071e - 01	1.635159e + 00	7.552294e - 01	1.380083e + 00	6.606763e - 01
	std.	1.199077e - 01	1.807778e - 01	1.212847e - 02	1.424562e - 01	6.510892e - 02	1.462401e - 01	2.895385e - 02	2.220464e - 01	4.786036e - 02	2.147590e - 01	1.209602e - 02



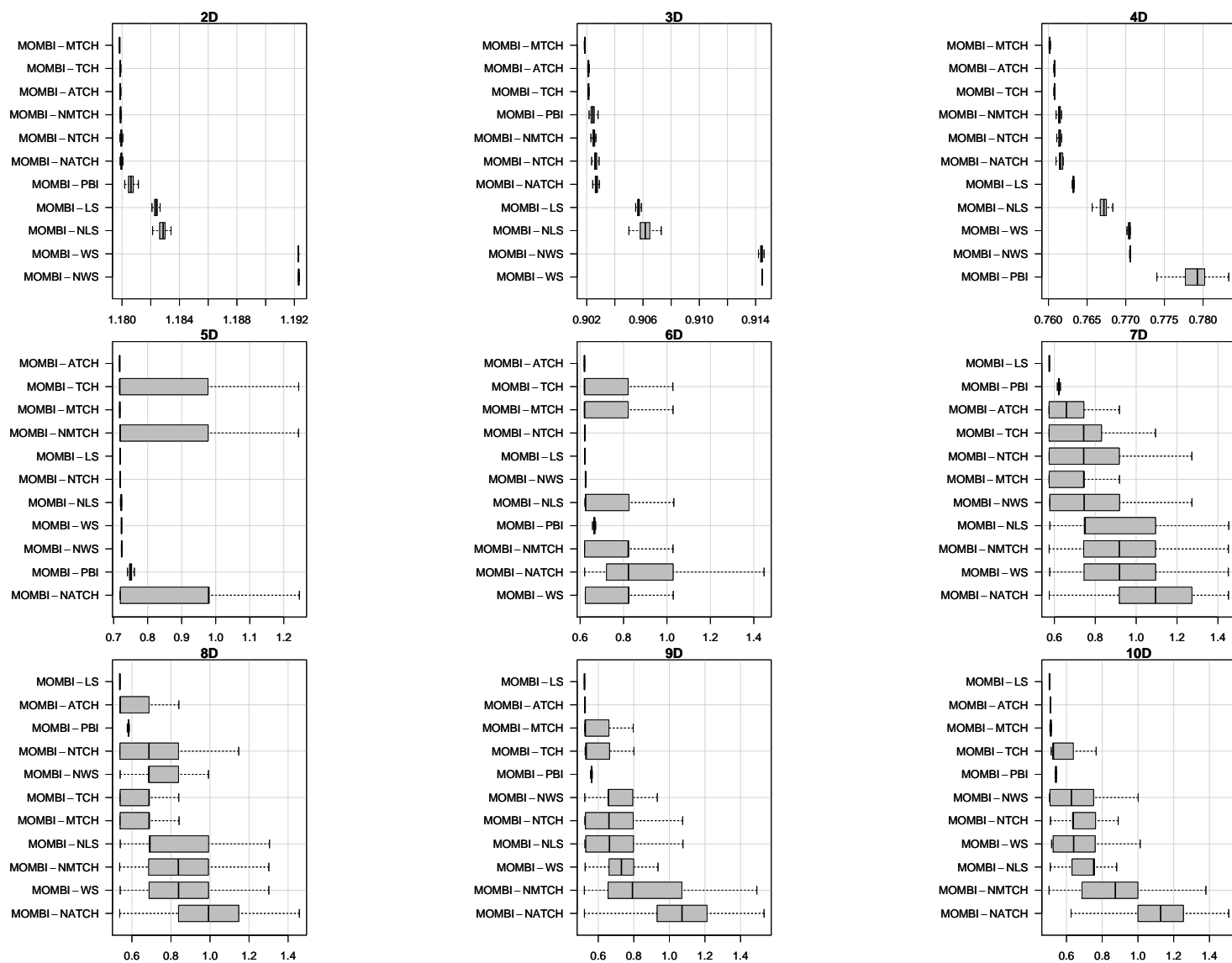


Figure A.318: Box-plot of the R2 indicator values for different optimizers on the DTLZ7 test problem.

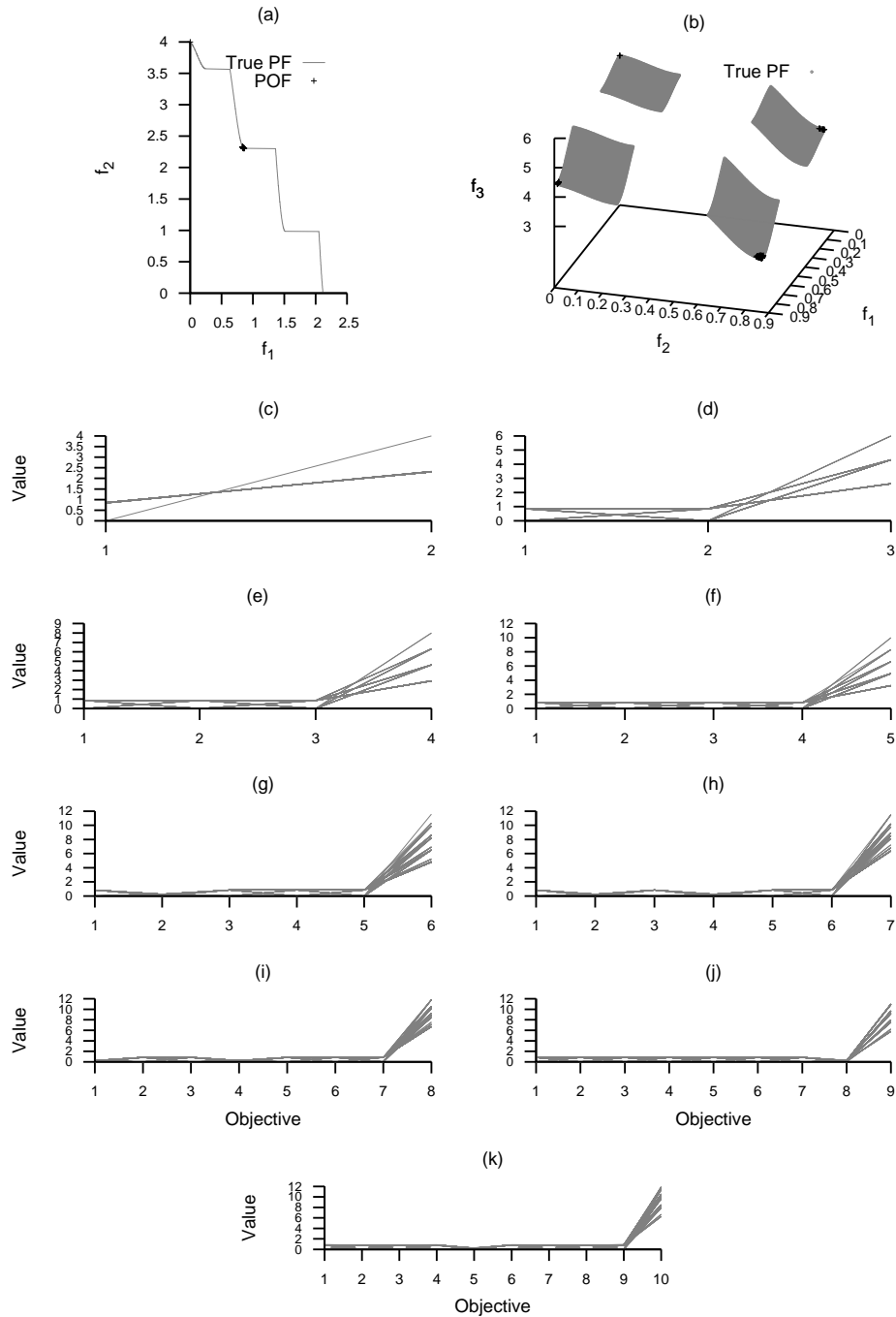


Figure A.319: Plots of the approximations obtained by MOMBI-WS from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

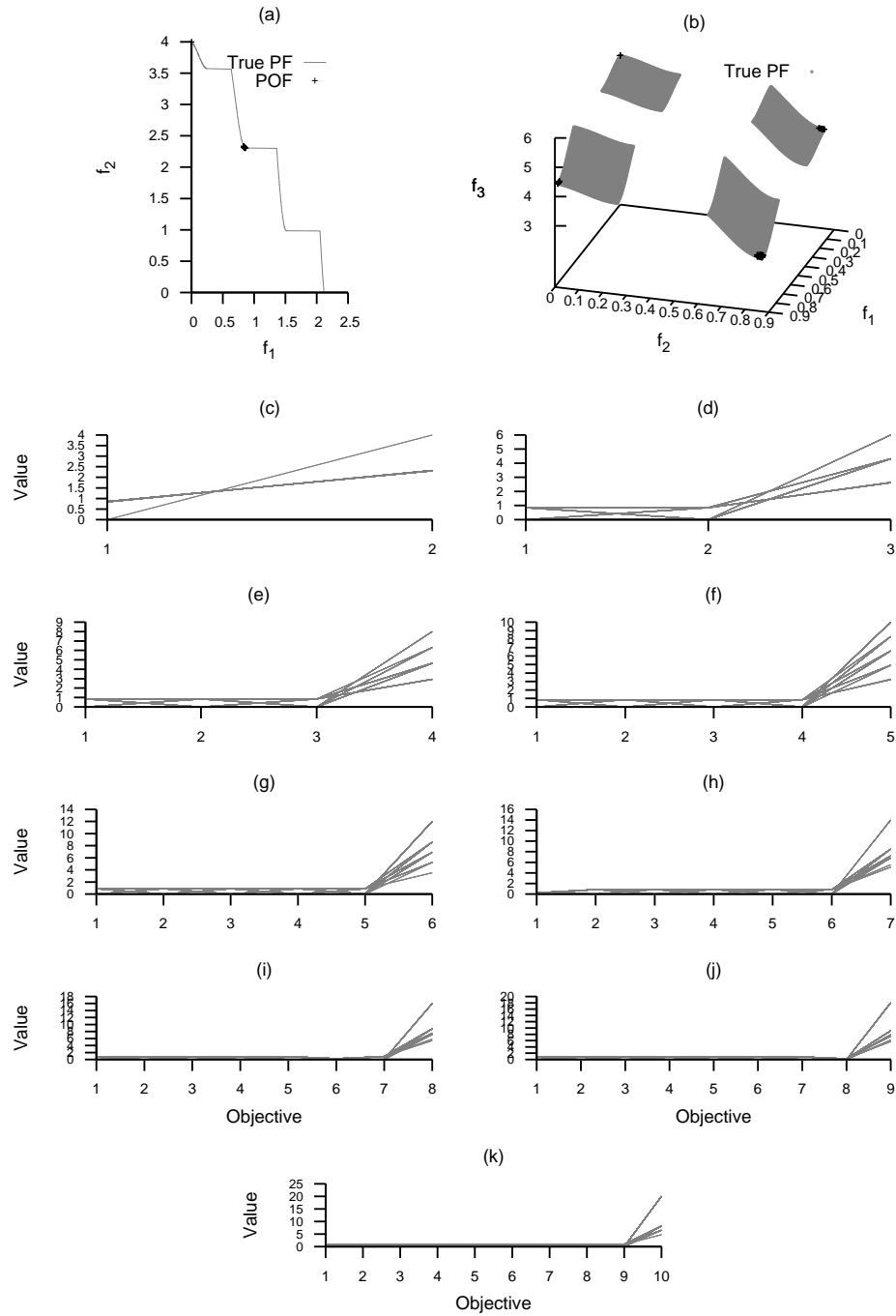


Figure A.320: Plots of the approximations obtained by MOMBI-NWS from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

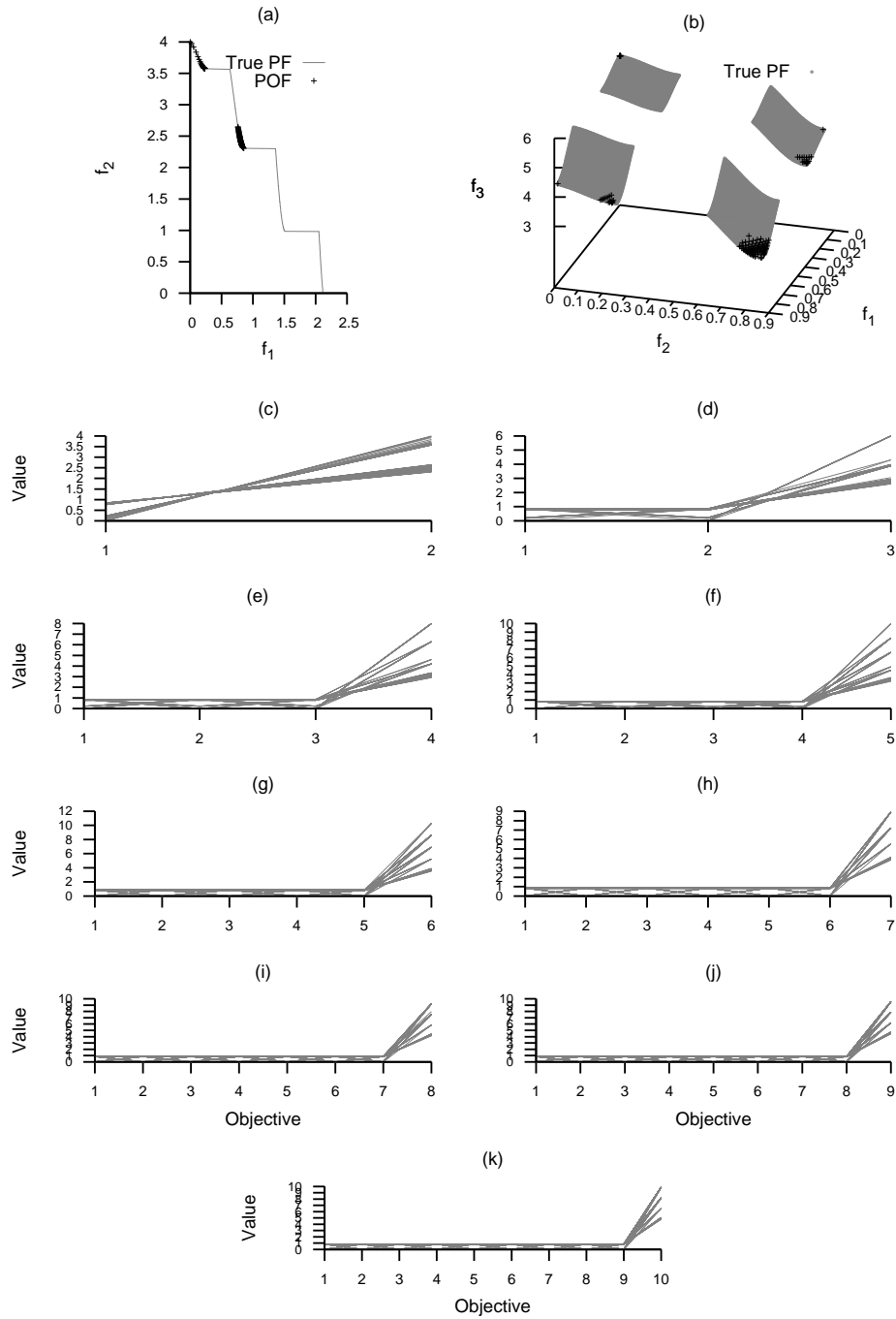


Figure A.321: Plots of the approximations obtained by MOMBI-LS from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

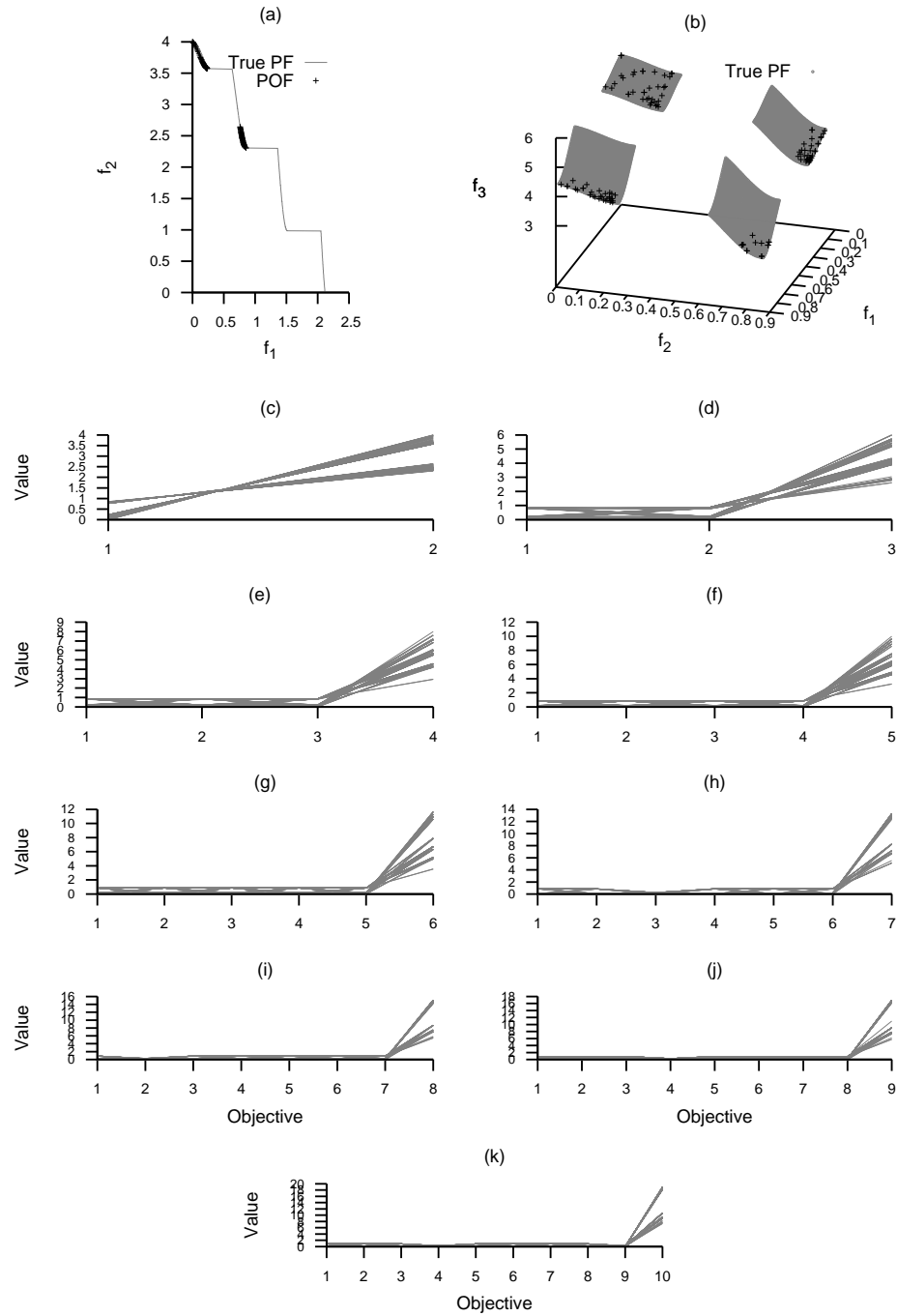


Figure A.322: Plots of the approximations obtained by MOMBI-NLS from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

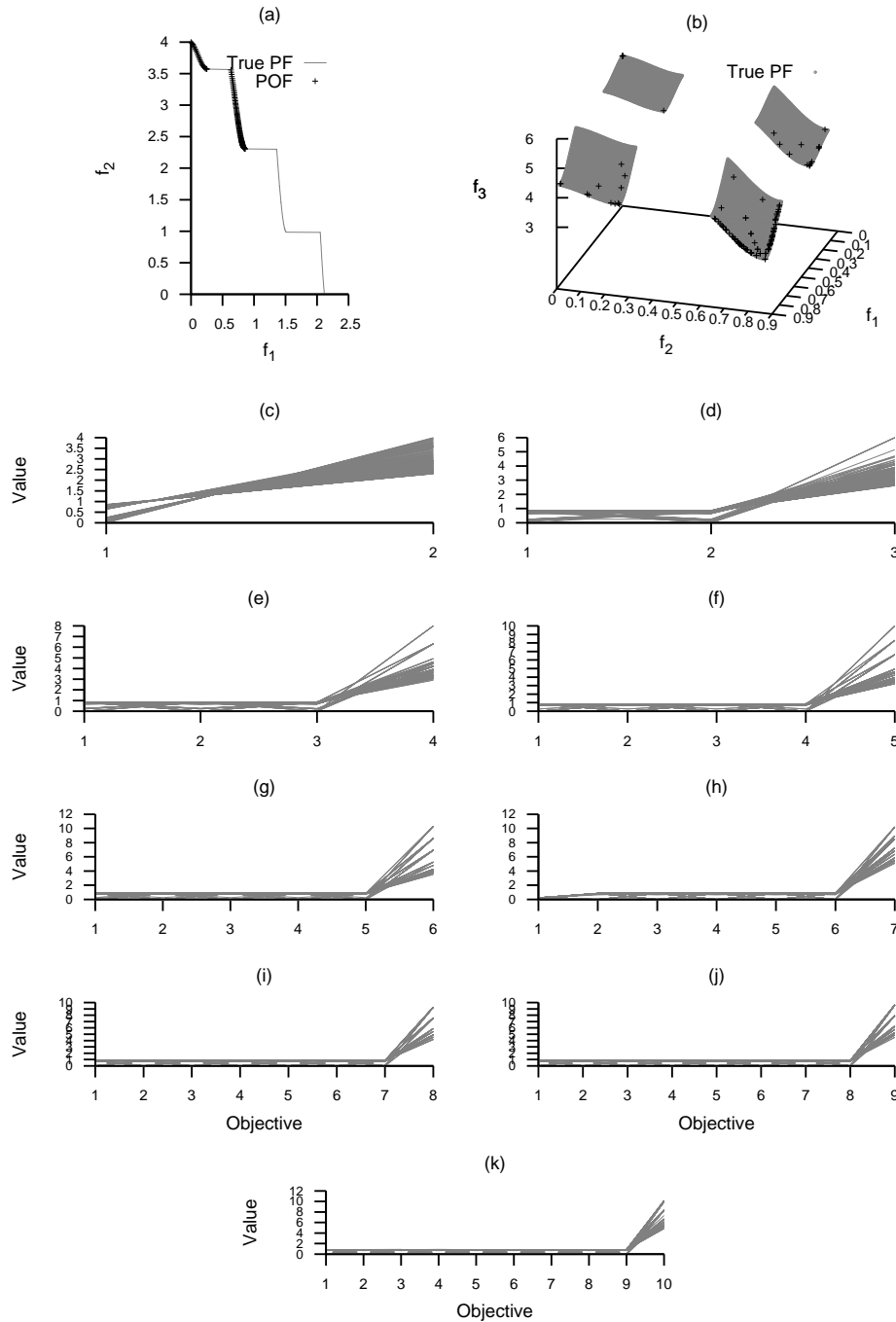


Figure A.323: Plots of the approximations obtained by MOMBI-ATCH from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

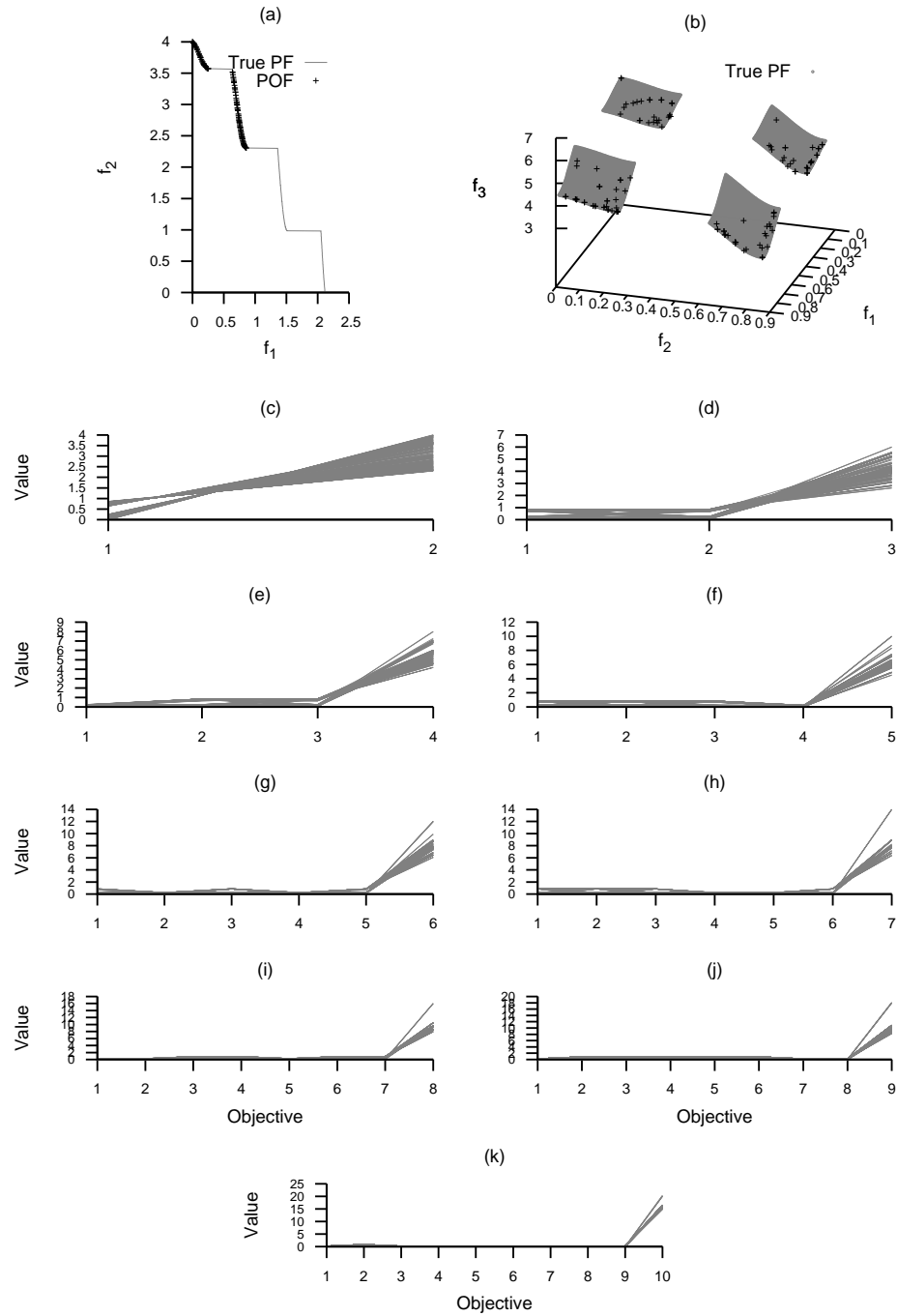


Figure A.324: Plots of the approximations obtained by MOMBI-NATCH from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

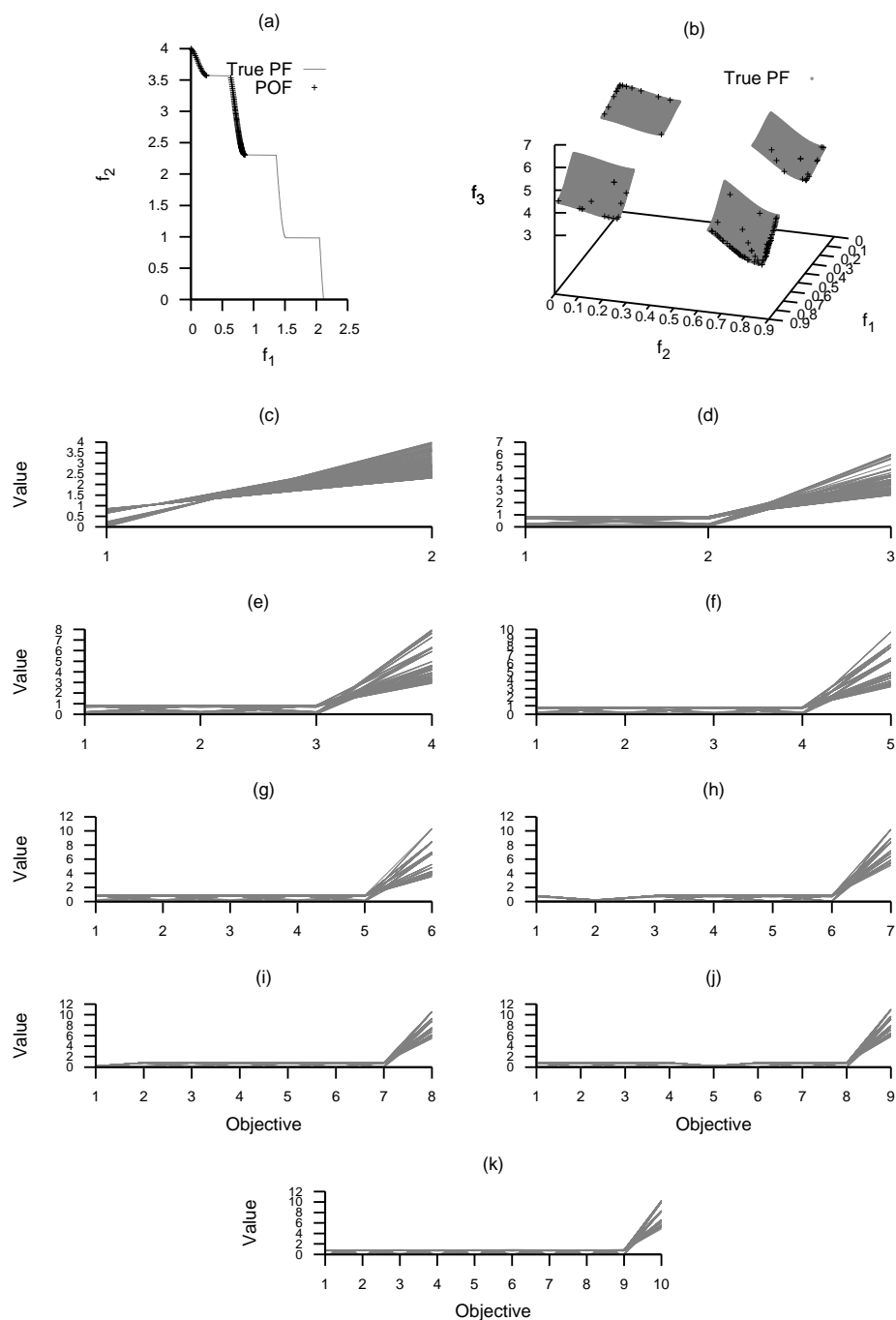


Figure A.325: Plots of the approximations obtained by MOMBI-MTCH from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



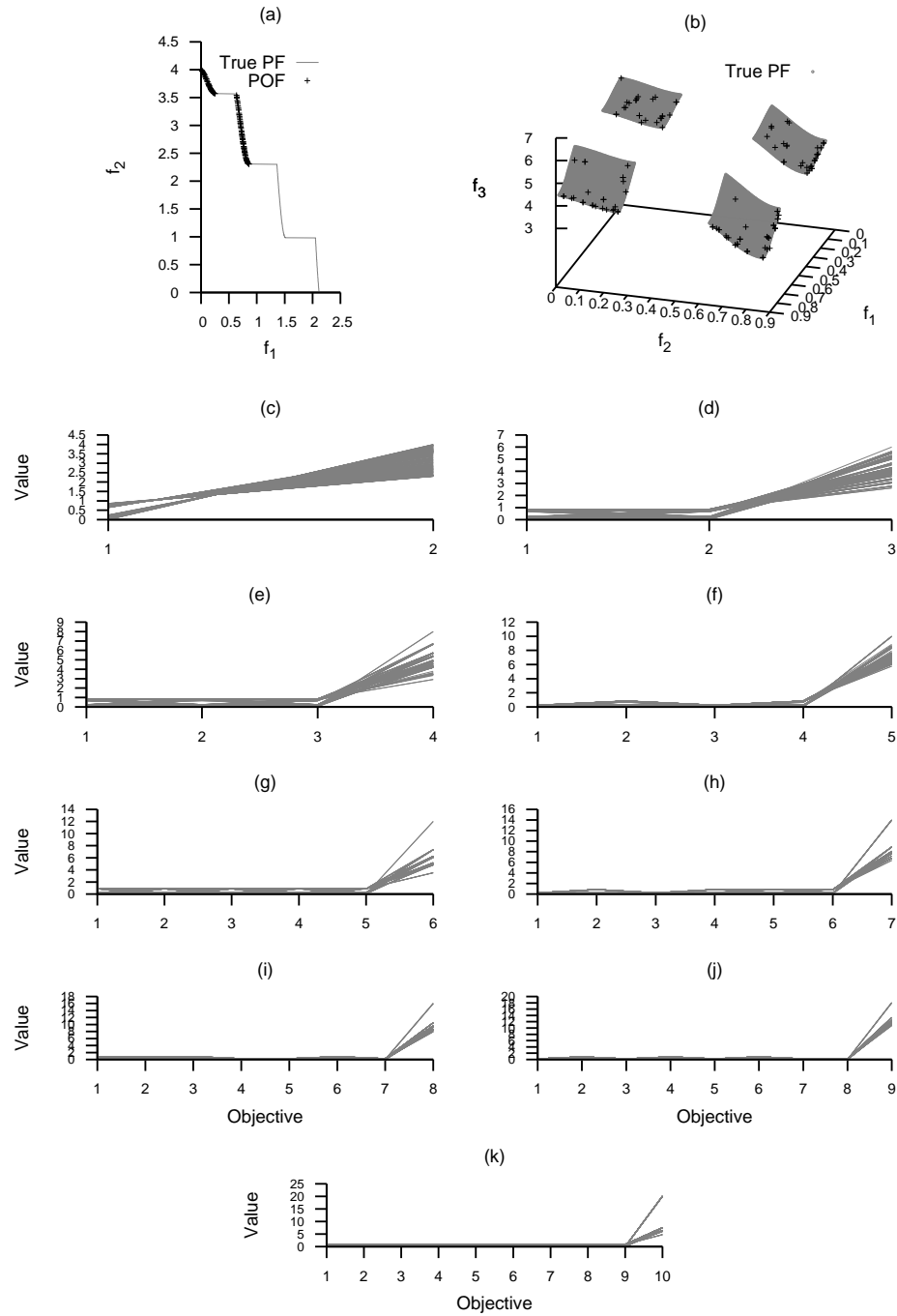


Figure A.326: Plots of the approximations obtained by MOMBI-NMTCH from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.133: Comparison of hypervolume indicator values for different optimizers on the WFG1 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	6.151281e+00	6.160308e+00	6.341512e+00	6.638726e+00	5.758975e+00	5.841737e+00	5.588084e+00	5.777796e+00	5.718700e+00	5.803737e+00	4.969062e+00
	avg.	6.099059e+00	6.083751e+00	6.341538e+00	6.626933e+00	5.795916e+00	5.897195e+00	5.661370e+00	5.802950e+00	5.687234e+00	5.858746e+00	5.004117e+00
	min.	5.060570e+00	5.129024e+00	5.062795e+00	5.521345e+00	4.688235e+00	5.005461e+00	4.781773e+00	4.864750e+00	4.840540e+00	4.750304e+00	4.499420e+00
	max.	6.929225e+00	7.408740e+00	7.417169e+00	8.010472e+00	7.768496e+00	7.648688e+00	7.490875e+00	7.576897e+00	7.320619e+00	7.568849e+00	6.248522e+00
	std.	4.583595e-01	5.144970e-01	5.726426e-01	4.946151e-01	5.261803e-01	4.676732e-01	4.844864e-01	4.819499e-01	4.287588e-01	4.948563e-01	2.652831e-01
3D	med.	6.175883e+01	6.259442e+01	6.091397e+01	6.102195e+01	5.396319e+01	5.450008e+01	5.485932e+01	5.507642e+01	5.469701e+01	5.510251e+01	4.941793e+01
	avg.	6.197842e+01	6.268176e+01	6.097226e+01	6.074131e+01	5.399566e+01	5.439503e+01	5.502945e+01	5.499948e+01	5.460439e+01	5.496145e+01	4.892257e+01
	min.	5.674274e+01	5.652055e+01	5.514291e+01	5.391405e+01	5.002276e+01	5.003974e+01	5.170367e+01	4.909388e+01	5.071139e+01	5.025763e+01	4.272444e+01
	max.	6.870272e+01	6.868824e+01	6.507235e+01	6.784623e+01	5.707353e+01	5.660851e+01	5.779902e+01	5.858952e+01	5.657121e+01	5.841229e+01	5.131585e+01
	std.	2.389597e+00	2.213587e+00	1.971739e+00	2.829315e+00	9.375607e-01	1.223374e+00	1.076520e+00	1.1713700e+00	1.017730e+00	1.130978e+00	1.823310e+00
4D	med.	4.950069e+02	4.958322e+02	4.784779e+02	4.717377e+02	4.318292e+02	4.328080e+02	4.455321e+02	4.396683e+02	4.409229e+02	4.376752e+02	3.758471e+02
	avg.	4.958155e+02	4.958885e+02	4.781221e+02	4.711571e+02	4.319723e+02	4.330509e+02	4.448836e+02	4.376677e+02	4.412457e+02	4.371469e+02	3.845531e+02
	min.	4.763291e+02	4.705793e+02	4.551859e+02	4.448341e+02	4.160464e+02	4.064273e+02	4.266100e+02	4.098125e+02	4.209424e+02	4.125412e+02	3.614928e+02
	max.	5.220068e+02	5.255670e+02	5.010057e+02	4.980847e+02	4.445835e+02	4.517398e+02	4.563954e+02	4.597295e+02	4.552905e+02	4.532322e+02	4.128433e+02
	std.	9.428340e+01	1.040598e+01	9.438897e+00	1.052341e+01	5.965569e+00	6.195477e+00	5.346725e+00	9.809911e+00	5.989080e+00	8.532753e+00	1.823350e+01
5D	med.	4.660013e+03	4.667317e+03	4.591597e+03	4.518384e+03	4.347508e+03	4.285412e+03	4.514267e+03	4.335842e+03	4.401676e+03	4.294416e+03	3.712312e+03
	avg.	4.668978e+03	4.665081e+03	4.589676e+03	4.528972e+03	4.357315e+03	4.302492e+03	4.513718e+03	4.348115e+03	4.399656e+03	4.310753e+03	3.752227e+03
	min.	4.507847e+03	4.495880e+03	4.402413e+03	4.382956e+03	4.232671e+03	4.158414e+03	4.286640e+03	4.186158e+03	4.260472e+03	4.180671e+03	3.581703e+03
	max.	4.867064e+03	4.880547e+03	4.784491e+03	4.716223e+03	4.586844e+03	4.508712e+03	4.667778e+03	4.625463e+03	4.595238e+03	4.458873e+03	4.125300e+03
	std.	6.980498e+01	7.354955e+01	7.683949e+01	7.259324e+01	7.228248e+01	7.270936e+01	6.803364e+01	9.232896e+01	6.433120e+01	7.639908e+01	1.252918e+02
6D	med.	5.470400e+04	5.431201e+04	5.347474e+04	5.313319e+04	5.233583e+04	5.161499e+04	5.394214e+04	5.235507e+04	5.300674e+04	5.215980e+04	4.547379e+04
	avg.	5.455658e+04	5.438750e+04	5.361142e+04	5.316056e+04	5.237113e+04	5.172517e+04	5.391570e+04	5.256733e+04	5.295641e+04	5.223690e+04	4.556192e+04
	min.	5.285666e+04	5.304952e+04	5.231309e+04	5.168926e+04	5.077657e+04	5.029308e+04	5.168420e+04	5.077029e+04	5.111081e+04	5.069978e+04	4.388694e+04
	max.	5.606806e+04	5.608532e+04	5.587694e+04	5.544746e+04	5.456358e+04	5.377037e+04	5.580852e+04	5.479360e+04	5.528060e+04	5.456938e+04	5.027479e+04
	std.	7.171712e+02	6.019606e+02	7.210420e+02	6.906866e+02	7.914407e+02	6.727627e+02	8.431889e+02	8.972865e+02	9.066253e+02	7.595383e+02	1.095637e+03
7D	med.	7.751922e+05	7.808695e+05	7.598394e+05	7.662224e+05	7.595214e+05	7.549936e+05	7.666134e+05	7.647123e+05	7.683215e+05	7.590228e+05	6.851338e+05
	avg.	7.712835e+05	7.820282e+05	7.601942e+05	7.666550e+05	7.579384e+05	7.546757e+05	7.679808e+05	7.657450e+05	7.695425e+05	7.584198e+05	6.874122e+05
	min.	6.940579e+05	7.586251e+05	7.328216e+05	7.453759e+05	6.603747e+05	7.269411e+05	7.333881e+05	7.301809e+05	7.414985e+05	7.172521e+05	6.483820e+05
	max.	8.185703e+05	7.939781e+05	7.941300e+05	7.998928e+05	8.003638e+05	7.803444e+05	8.137253e+05	8.174784e+05	8.008232e+05	8.042768e+05	7.754588e+05
	std.	2.251221e+04	1.091929e+04	1.162557e+04	1.058172e+04	1.596897e+04	9.681607e+03	1.397841e+04	1.503869e+04	1.238024e+04	1.221676e+04	1.988776e+04
8D	med.	1.205809e+07	1.228360e+07	1.205911e+07	1.206943e+07	1.188580e+07	1.189977e+07	1.205271e+07	1.211402e+07	1.205485e+07	1.196935e+07	1.133253e+07
	avg.	1.207328e+07	1.227706e+07	1.205593e+07	1.205941e+07	1.189976e+07	1.192068e+07	1.206728e+07	1.211788e+07	1.206653e+07	1.199684e+07	1.135540e+07
	min.	1.172360e+07	1.197308e+07	1.159058e+07	1.170152e+07	1.136431e+07	1.161395e+07	1.177250e+07	1.166383e+07	1.171416e+07	1.166127e+07	1.081080e+07
	max.	1.250961e+07	1.275084e+07	1.242714e+07	1.247230e+07	1.230131e+07	1.237599e+07	1.256344e+07	1.271333e+07	1.254951e+07	1.241085e+07	1.211626e+07
	std.	1.685472e+05	1.478104e+05	1.517629e+05	1.371195e+05	1.557615e+05	1.470849e+05	1.617534e+05	1.964008e+05	1.709931e+05	1.631173e+05	2.596615e+05
9D	med.	2.127064e+08	2.174669e+08	2.170735e+08	2.148224e+08	2.113979e+08	2.119995e+08	2.155212e+08	2.142202e+08	2.132380e+08	2.133687e+08	2.041949e+08
	avg.	2.130766e+08	2.174690e+08	2.172198e+08	2.146971e+08	2.113231e+08	2.122041e+08	2.155838e+08	2.143902e+08	2.135373e+08	2.133332e+08	2.046877e+08
	min.	2.080778e+08	2.130817e+08	2.119853e+08	2.098950e+08	2.064246e+08	2.072555e+08	2.113855e+08	2.061835e+08	2.090494e+08	2.077769e+08	1.955390e+08
	max.	2.195357e+08	2.221252e+08	2.220689e+08	2.193147e+08	2.171946e+08	2.185090e+08	2.203403e+08	2.224721e+08	2.184636e+08	2.205260e+08	2.119586e+08
	std.	2.318061e+06	1.708648e+06	2.113790e+06	2.029530e+06	2.102008e+06	1.996992e+06	1.806720e+06	3.394905e+06	2.051022e+06	2.291119e+06	3.240587e+06
10D	med.	4.173831e+09	4.286424e+09	4.311241e+09	4.223055e+09	4.164088e+09	4.179608e+09	4.276177e+09	4.220245e+09	4.218501e+09	4.202627e+09	4.051979e+09
	avg.	4.184176e+09	4.282551e+09	4.312843e+09	4.224223e+09	4.162638e+09	4.177991e+09	4.282071e+09	4.222508e+09	4.199912e+09	4.199912e+09	4.058091e+09
	min.	4.098823e+09	4.216033e+09	4.209919e+09	4.126477e+09	4.079302e+09	4.087870e+09	4.201788e+09	4.126748e+09	4.151614e+09	4.091493e+09	3.951448e+09
	max.	4.331304e+09	4.338403e+09	4.400580e+09	4.341061e+09	4.286667e+09	4.352640e+09	4.471146e+09	4.346269e+09	4.318195e+09	4.318195e+09	4.179472e+09
	std.	3.745827e+07	2.802228e+07	3.957929e+07	3.468156e+07	3.933105e+07	4.121285e+07	3.944287e+07	4.431251e+07	3.279025e+07	3.553572e+07	4.746199e+07

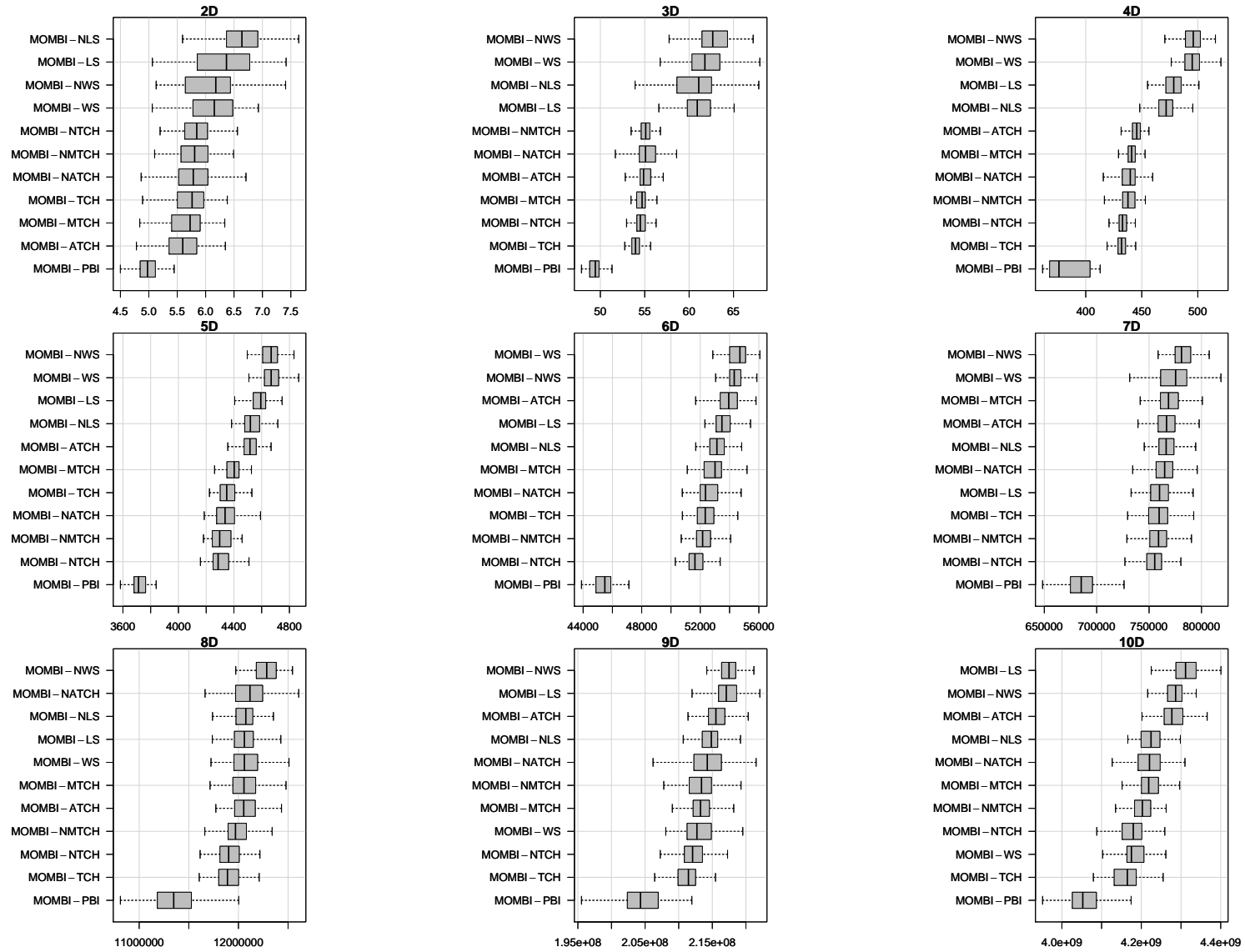


Figure A.327: Box-plot of the hypervolume indicator values for different optimizers on the WFG1 test problem.

Table A.134: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG1 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
2D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	$2.97e-07$	$4.51e-05$	$1.79e-11$	$7.74e-07$	$2.07e-10$	$8.02e-06$	$7.56e-32$
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	$6.01e-06$	$5.01e-04$	$1.20e-09$	$8.67e-06$	$1.24e-08$	$1.37e-04$	$2.90e-31$
MOMBI-LS	$9.30e-04$	$6.88e-04$	–	> 0.05	$7.15e-11$	$1.09e-08$	$1.72e-15$	$1.08e-10$	$2.20e-14$	$2.56e-09$	$1.73e-32$
MOMBI-NLS	$1.86e-12$	$3.33e-12$	$8.49e-04$	–	$1.86e-20$	$3.12e-19$	$3.22e-24$	$3.33e-21$	$6.14e-25$	$2.10e-19$	$3.88e-34$
MOMBI-TCH	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	$4.75e-03$	> 0.05	> 0.05	> 0.05	$2.13e-26$
MOMBI-NTCH	> 0.05	> 0.05	> 0.05	> 0.05	$2.94e-02$	–	$3.82e-06$	> 0.05	$9.30e-04$	> 0.05	$5.27e-31$
MOMBI-ATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$4.77e-26$
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.24e-03$	–	$4.26e-02$	> 0.05	$2.60e-27$
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	$7.71e-25$
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.01e-04$	> 0.05	$1.25e-02$	–	$7.59e-30$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
3D											
MOMBI-WS	–	> 0.05	$1.34e-03$	$1.81e-03$	$1.40e-34$	$1.28e-34$	$1.89e-34$	$4.24e-34$	$1.28e-34$	$1.73e-34$	$1.28e-34$
MOMBI-NWS	$8.63e-03$	–	$9.90e-09$	$3.13e-07$	$1.40e-34$	$1.32e-34$	$2.33e-34$	$4.64e-34$	$1.36e-34$	$1.73e-34$	$1.28e-34$
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	$1.84e-34$	$3.34e-34$	$1.27e-33$	$3.47e-33$	$3.44e-34$	$1.10e-33$	$1.28e-34$
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	$2.91e-33$	$4.77e-32$	$2.94e-30$	$4.56e-29$	$1.13e-31$	$2.28e-30$	$1.28e-34$
MOMBI-TCH	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.20e-34$
MOMBI-NTCH	> 0.05	> 0.05	> 0.05	> 0.05	$5.55e-06$	–	> 0.05	> 0.05	> 0.05	> 0.05	$2.88e-34$
MOMBI-ATCH	> 0.05	> 0.05	> 0.05	> 0.05	$1.23e-13$	$8.95e-05$	–	> 0.05	$2.87e-03$	> 0.05	$1.28e-34$
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	$1.74e-10$	$6.41e-05$	> 0.05	–	$1.09e-03$	> 0.05	$1.87e-33$
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	$2.55e-08$	> 0.05	> 0.05	> 0.05	–	> 0.05	$1.53e-34$
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	$6.45e-14$	$7.85e-06$	> 0.05	> 0.05	$4.59e-04$	–	$1.95e-34$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
4D											
MOMBI-WS	–	> 0.05	$1.24e-24$	$3.38e-30$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-NWS	> 0.05	–	$8.15e-23$	$2.11e-29$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-LS	> 0.05	> 0.05	–	$1.43e-06$	$1.28e-34$	$1.28e-34$	$1.32e-34$	$1.49e-34$	$1.36e-34$	$1.28e-34$	$1.28e-34$
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	$1.28e-34$	$1.73e-34$	$1.26e-32$	$9.19e-34$	$1.43e-33$	$4.92e-34$	$1.28e-34$
MOMBI-TCH	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	$2.07e-34$
MOMBI-ATCH	> 0.05	> 0.05	> 0.05	> 0.05	$4.47e-28$	$5.42e-26$	–	$1.67e-09$	$9.07e-07$	$3.22e-12$	$1.28e-34$
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	$3.31e-08$	$8.23e-07$	> 0.05	–	> 0.05	> 0.05	$1.53e-34$
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	$4.60e-20$	$2.30e-18$	> 0.05	$1.40e-02$	–	$2.41e-04$	$1.28e-34$
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	$8.47e-08$	$4.53e-06$	> 0.05	> 0.05	> 0.05	–	$1.36e-34$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–

Table A.135: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG1 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
5D											
MOMBI-WS	–	> 0.05	$3.95e-12$	$2.70e-24$	$1.89e-34$	$1.32e-34$	$1.56e-28$	$6.06e-34$	$3.15e-34$	$1.28e-34$	$1.28e-34$
MOMBI-NWS	> 0.05	–	$1.79e-10$	$4.14e-23$	$2.33e-34$	$1.32e-34$	$1.66e-27$	$9.19e-34$	$4.50e-34$	$1.28e-34$	$1.28e-34$
MOMBI-LS	> 0.05	> 0.05	–	$3.74e-08$	$2.25e-32$	$4.24e-34$	$1.16e-11$	$1.23e-30$	$1.54e-30$	$4.12e-34$	$1.28e-34$
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	$1.04e-28$	$4.93e-33$	> 0.05	$4.41e-26$	$2.91e-24$	$7.43e-33$	$1.28e-34$
MOMBI-TCH	> 0.05	> 0.05	> 0.05	> 0.05	–	$3.29e-07$	> 0.05	> 0.05	> 0.05	$2.46e-05$	$1.28e-34$
MOMBI-NTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-ATCH	> 0.05	> 0.05	> 0.05	> 0.05	$1.03e-26$	$8.24e-32$	–	$2.84e-24$	$5.37e-22$	$2.00e-31$	$1.28e-34$
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$3.28e-04$	> 0.05	–	> 0.05	$4.06e-03$	$1.28e-34$
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	$9.46e-06$	$7.14e-17$	> 0.05	$6.85e-07$	–	$1.25e-13$	$1.28e-34$
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	$1.28e-34$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
6D											
MOMBI-WS	–	$1.36e-02$	$3.02e-15$	$2.21e-24$	$1.20e-31$	$5.07e-34$	$9.05e-08$	$4.97e-28$	$1.49e-23$	$7.78e-32$	$1.28e-34$
MOMBI-NWS	> 0.05	–	$5.17e-14$	$2.05e-24$	$6.00e-32$	$3.44e-34$	$3.64e-05$	$4.54e-27$	$3.17e-23$	$2.06e-31$	$1.28e-34$
MOMBI-LS	> 0.05	> 0.05	–	$1.95e-05$	$3.92e-21$	$5.74e-31$	> 0.05	$2.26e-15$	$1.59e-07$	$1.18e-24$	$1.28e-34$
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	$1.68e-11$	$4.13e-26$	> 0.05	$7.62e-08$	$3.24e-02$	$6.06e-16$	$1.28e-34$
MOMBI-TCH	> 0.05	> 0.05	> 0.05	> 0.05	–	$2.71e-09$	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-ATCH	> 0.05	> 0.05	$1.60e-03$	$3.29e-10$	$1.25e-23$	$2.31e-31$	–	$1.18e-18$	$2.24e-12$	$1.17e-25$	$1.28e-34$
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$4.38e-12$	> 0.05	–	> 0.05	$4.39e-03$	$1.28e-34$
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	$5.19e-06$	$1.61e-19$	> 0.05	$1.10e-03$	–	$4.38e-09$	$1.28e-34$
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$4.17e-07$	> 0.05	> 0.05	> 0.05	–	$1.28e-34$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
7D											
MOMBI-WS	–	> 0.05	$7.78e-09$	$1.75e-04$	$9.66e-10$	$6.69e-14$	$1.89e-03$	$2.02e-04$	$1.19e-02$	$2.74e-10$	$5.55e-33$
MOMBI-NWS	$1.41e-04$	–	$9.67e-25$	$5.87e-18$	$1.37e-25$	$1.84e-31$	$2.46e-14$	$9.73e-16$	$2.40e-12$	$6.74e-27$	$2.96e-34$
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	> 0.05	$5.14e-04$	> 0.05	> 0.05	> 0.05	> 0.05	$2.23e-33$
MOMBI-NLS	> 0.05	> 0.05	$4.38e-05$	–	$5.74e-06$	$1.18e-13$	> 0.05	> 0.05	> 0.05	$3.73e-07$	$1.43e-33$
MOMBI-TCH	> 0.05	> 0.05	> 0.05	> 0.05	–	$8.63e-03$	> 0.05	> 0.05	> 0.05	> 0.05	$4.50e-32$
MOMBI-NTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	$3.18e-33$
MOMBI-ATCH	> 0.05	> 0.05	$1.64e-05$	> 0.05	$2.17e-06$	$4.56e-13$	–	> 0.05	> 0.05	$2.08e-07$	$1.43e-33$
MOMBI-NATCH	> 0.05	> 0.05	$2.98e-03$	> 0.05	$6.32e-04$	$3.14e-09$	> 0.05	–	> 0.05	$1.67e-04$	$1.61e-33$
MOMBI-MTCH	> 0.05	> 0.05	$1.61e-07$	$4.65e-02$	$1.76e-08$	$1.84e-16$	> 0.05	$1.19e-02$	–	$8.12e-10$	$1.10e-33$
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$8.29e-03$	> 0.05	> 0.05	> 0.05	–	$2.91e-33$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–

Table A.136: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG1 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
8D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	$1.13e-11$	$1.97e-10$	> 0.05	> 0.05	> 0.05	$7.61e-04$	$1.22e-32$
MOMBI-NWS	$4.54e-15$	–	$2.06e-18$	$2.50e-19$	$7.38e-30$	$1.51e-28$	$1.29e-16$	$2.74e-10$	$1.16e-15$	$2.73e-22$	$2.26e-34$
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	$2.57e-11$	$1.94e-10$	> 0.05	> 0.05	> 0.05	$9.00e-04$	$1.37e-32$
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	$2.32e-12$	$1.58e-11$	> 0.05	> 0.05	> 0.05	$3.11e-04$	$9.95e-33$
MOMBI-TCH	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.79e-29$
MOMBI-NTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	$1.72e-30$
MOMBI-ATCH	> 0.05	> 0.05	> 0.05	> 0.05	$1.02e-11$	$9.84e-11$	–	> 0.05	> 0.05	$6.71e-04$	$1.09e-32$
MOMBI-NATCH	$4.83e-02$	> 0.05	$1.19e-02$	$1.28e-02$	$1.35e-14$	$3.37e-13$	$1.96e-02$	–	$2.50e-02$	$3.43e-06$	$6.05e-33$
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	$5.62e-11$	$8.18e-10$	> 0.05	> 0.05	–	$1.69e-03$	$1.37e-32$
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	$4.20e-05$	$2.97e-04$	> 0.05	> 0.05	> 0.05	–	$1.30e-31$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
9D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	$3.37e-07$	$5.02e-03$	> 0.05	> 0.05	> 0.05	> 0.05	$1.09e-32$
MOMBI-NWS	$4.19e-26$	–	> 0.05	$4.27e-18$	$3.08e-33$	$2.82e-31$	$2.06e-12$	$1.04e-12$	$3.52e-25$	$7.02e-26$	$1.28e-34$
MOMBI-LS	$1.85e-23$	> 0.05	–	$2.75e-14$	$6.36e-32$	$4.95e-29$	$1.62e-08$	$7.04e-11$	$4.66e-22$	$8.97e-23$	$1.28e-34$
MOMBI-NLS	$2.07e-07$	> 0.05	> 0.05	–	$1.57e-20$	$3.20e-15$	> 0.05	> 0.05	$2.38e-05$	$5.43e-06$	$3.05e-34$
MOMBI-TCH	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$5.84e-29$
MOMBI-NTCH	> 0.05	> 0.05	> 0.05	> 0.05	$2.85e-03$	–	> 0.05	> 0.05	> 0.05	> 0.05	$1.69e-31$
MOMBI-ATCH	$2.32e-14$	> 0.05	> 0.05	$7.24e-04$	$1.32e-27$	$6.40e-23$	–	$6.82e-04$	$3.89e-12$	$8.54e-13$	$1.36e-34$
MOMBI-NATCH	$5.95e-04$	> 0.05	> 0.05	> 0.05	$1.33e-12$	$3.95e-08$	> 0.05	–	$1.12e-02$	$5.17e-03$	$2.75e-32$
MOMBI-MTCH	$4.66e-02$	> 0.05	> 0.05	> 0.05	$7.78e-12$	$5.55e-06$	> 0.05	> 0.05	–	> 0.05	$1.27e-33$
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	$2.52e-09$	$1.16e-04$	> 0.05	> 0.05	> 0.05	–	$8.60e-33$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
10D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	$1.17e-04$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.27e-31$
MOMBI-NWS	$2.12e-31$	–	> 0.05	$4.04e-23$	$1.52e-33$	$6.36e-32$	> 0.05	$2.29e-19$	$1.38e-24$	$1.63e-30$	$1.28e-34$
MOMBI-LS	$9.66e-33$	$5.07e-09$	–	$2.04e-28$	$4.92e-34$	$8.35e-33$	$4.02e-09$	$1.69e-26$	$1.44e-29$	$1.45e-32$	$1.28e-34$
MOMBI-NLS	$7.76e-14$	> 0.05	> 0.05	–	$2.70e-21$	$1.93e-15$	> 0.05	> 0.05	> 0.05	$4.09e-06$	$2.26e-34$
MOMBI-TCH	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.12e-28$
MOMBI-NTCH	> 0.05	> 0.05	> 0.05	> 0.05	$6.03e-03$	–	> 0.05	> 0.05	> 0.05	> 0.05	$5.90e-31$
MOMBI-ATCH	$6.25e-31$	> 0.05	> 0.05	$4.50e-21$	$4.39e-33$	$2.18e-31$	–	$1.33e-17$	$1.20e-22$	$1.56e-29$	$1.28e-34$
MOMBI-NATCH	$3.05e-10$	> 0.05	> 0.05	> 0.05	$1.20e-17$	$6.25e-12$	> 0.05	–	> 0.05	$2.46e-04$	$6.83e-34$
MOMBI-MTCH	$8.61e-13$	> 0.05	> 0.05	> 0.05	$1.45e-20$	$1.05e-14$	> 0.05	> 0.05	–	$1.63e-05$	$2.88e-34$
MOMBI-NMTCH	$1.57e-04$	> 0.05	> 0.05	> 0.05	$2.28e-11$	$6.01e-06$	> 0.05	> 0.05	> 0.05	–	$4.79e-33$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–

Table A.137: Comparison of R2 indicator values for different optimizers on the WFG1 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	8.310092e-01	8.297446e-01	8.201490e-01	7.966364e-01	9.023724e-01	8.936055e-01	9.211309e-01	9.003371e-01	9.067201e-01	8.983442e-01	9.936766e-01
	avg.	8.392058e-01	8.413860e-01	8.330322e-01	8.012578e-01	8.995622e-01	8.886859e-01	9.145798e-01	8.989549e-01	9.114516e-01	8.930184e-01	9.912587e-01
	min.	7.384883e-01	6.956982e-01	7.191121e-01	6.633153e-01	6.908346e-01	7.030471e-01	7.166670e-01	7.105344e-01	7.311611e-01	7.124680e-01	8.481836e-01
	max.	9.493971e-01	9.593571e-01	9.810609e-01	9.270747e-01	1.025674e+00	9.891340e-01	1.015532e+00	1.005751e+00	1.008629e+00	1.020145e+00	1.050239e+00
	std.	5.565749e-02	6.143401e-02	6.421294e-02	5.328159e-02	5.828637e-02	5.109278e-02	5.354232e-02	5.325825e-02	4.778206e-02	5.431376e-02	3.099518e-02
3D	med.	4.096318e-01	4.065350e-01	4.208741e-01	4.239355e-01	5.086526e-01	5.027982e-01	4.961016e-01	4.969756e-01	5.017024e-01	4.974231e-01	5.593882e-01
	avg.	4.112007e-01	4.069328e-01	4.209701e-01	4.262807e-01	5.078034e-01	5.040339e-01	4.945756e-01	4.960608e-01	5.012709e-01	4.986221e-01	5.644177e-01
	min.	3.454318e-01	3.521709e-01	3.754843e-01	3.623171e-01	4.726235e-01	4.742127e-01	4.634396e-01	4.592622e-01	4.785872e-01	4.652024e-01	5.291478e-01
	max.	4.679330e-01	4.645548e-01	4.821809e-01	4.982759e-01	5.454573e-01	5.447505e-01	5.269684e-01	5.523872e-01	5.372851e-01	5.434679e-01	6.220584e-01
	std.	2.383304e-02	2.207780e-02	2.086423e-02	2.608828e-02	1.038940e-02	1.220383e-02	1.200857e-02	1.682449e-02	1.129848e-02	1.162115e-02	1.707565e-02
4D	med.	3.775434e-01	3.786267e-01	3.975557e-01	4.032358e-01	4.477314e-01	4.481530e-01	4.352991e-01	4.408785e-01	4.408162e-01	4.438006e-01	4.942595e-01
	avg.	3.767941e-01	3.785401e-01	3.969825e-01	4.034446e-01	4.479611e-01	4.486049e-01	4.354550e-01	4.422905e-01	4.401807e-01	4.438115e-01	4.896273e-01
	min.	3.483713e-01	3.414945e-01	3.757923e-01	3.764957e-01	4.322944e-01	4.325254e-01	4.215237e-01	4.239988e-01	4.230988e-01	4.247233e-01	4.644733e-01
	max.	4.000316e-01	3.988004e-01	4.187883e-01	4.239098e-01	4.620713e-01	4.710592e-01	4.534286e-01	4.651222e-01	4.601460e-01	4.644876e-01	5.100315e-01
	std.	1.024234e-02	1.127973e-02	9.944548e-03	9.847239e-03	5.734274e-03	6.161299e-03	5.791824e-03	8.664009e-03	6.380346e-03	7.982287e-03	1.383074e-02
5D	med.	3.698487e-01	3.678689e-01	3.757729e-01	3.785204e-01	3.821833e-01	4.011880e-01	3.821833e-01	3.943620e-01	3.929133e-01	3.948333e-01	4.349331e-01
	avg.	3.690641e-01	3.682827e-01	3.762732e-01	3.788008e-01	3.972069e-01	4.004118e-01	3.828827e-01	3.943620e-01	3.932949e-01	3.981433e-01	4.338609e-01
	min.	3.529197e-01	3.536482e-01	3.573437e-01	3.656562e-01	3.766803e-01	3.845810e-01	3.711926e-01	3.723015e-01	3.729796e-01	3.871409e-01	4.123889e-01
	max.	3.848243e-01	3.816465e-01	3.910157e-01	3.901776e-01	4.105606e-01	4.113415e-01	3.988055e-01	4.103933e-01	4.048858e-01	4.118653e-01	4.490335e-01
	std.	5.508269e-03	5.385801e-03	6.213237e-03	5.020477e-03	6.884930e-03	5.444773e-03	5.425472e-03	7.271262e-03	5.222000e-03	5.434940e-03	7.188133e-03
6D	med.	3.476446e-01	3.463922e-01	3.489187e-01	3.499569e-01	3.604690e-01	3.611079e-01	3.479609e-01	3.538283e-01	3.558860e-01	3.571640e-01	3.838064e-01
	avg.	3.481963e-01	3.462472e-01	3.488892e-01	3.496911e-01	3.601289e-01	3.610206e-01	3.482145e-01	3.526912e-01	3.560424e-01	3.568386e-01	3.837040e-01
	min.	3.365001e-01	3.361102e-01	3.402736e-01	3.405634e-01	3.421449e-01	3.478924e-01	3.391908e-01	3.314240e-01	3.423975e-01	3.429210e-01	3.673223e-01
	max.	3.584179e-01	3.538745e-01	3.576446e-01	3.574597e-01	3.703894e-01	3.677199e-01	3.583816e-01	3.637251e-01	3.651971e-01	3.661463e-01	3.950498e-01
	std.	4.345734e-03	3.524368e-03	3.709226e-03	3.461225e-03	5.660184e-03	4.158771e-03	4.145120e-03	5.909499e-03	5.104114e-03	4.238947e-03	5.561090e-03
7D	med.	3.170665e-01	3.109014e-01	3.116030e-01	3.085693e-01	3.217887e-01	3.184947e-01	3.099289e-01	3.117973e-01	3.139947e-01	3.156954e-01	3.256926e-01
	avg.	3.170998e-01	3.106341e-01	3.113472e-01	3.085294e-01	3.214827e-01	3.186668e-01	3.095768e-01	3.108446e-01	3.132493e-01	3.156175e-01	3.249379e-01
	min.	3.004895e-01	2.924796e-01	3.001639e-01	2.949448e-01	3.048468e-01	3.104983e-01	2.935804e-01	2.936007e-01	3.008409e-01	2.946647e-01	3.064713e-01
	max.	3.384221e-01	3.185497e-01	3.188593e-01	3.155956e-01	3.359325e-01	3.287056e-01	3.196869e-01	3.220076e-01	3.231701e-01	3.268236e-01	3.393524e-01
	std.	6.697924e-03	4.671301e-03	3.798244e-03	4.167287e-03	5.614260e-03	3.396173e-03	4.373416e-03	6.070985e-03	4.610442e-03	4.629665e-03	7.370183e-03
8D	med.	2.992626e-01	2.905157e-01	2.879445e-01	2.887938e-01	3.010396e-01	2.955629e-01	2.887452e-01	2.885189e-01	2.937315e-01	2.928783e-01	2.930060e-01
	avg.	2.992478e-01	2.904446e-01	2.881055e-01	2.887358e-01	3.012293e-01	2.954094e-01	2.887877e-01	2.884817e-01	2.934400e-01	2.925924e-01	2.927997e-01
	min.	2.882176e-01	2.808643e-01	2.808351e-01	2.830534e-01	2.884310e-01	2.868205e-01	2.797102e-01	2.749094e-01	2.847880e-01	2.823705e-01	2.785930e-01
	max.	3.082604e-01	2.994341e-01	2.962858e-01	2.967332e-01	3.089473e-01	3.014685e-01	2.952979e-01	3.001378e-01	3.012136e-01	2.997343e-01	3.057954e-01
	std.	4.181607e-03	3.167180e-03	3.266666e-03	2.624006e-03	4.118613e-03	3.195061e-03	2.485489e-03	3.796963e-03	3.298744e-03	3.459026e-03	5.572694e-03
9D	med.	2.830504e-01	2.728218e-01	2.661712e-01	2.713345e-01	2.841809e-01	2.764445e-01	2.690031e-01	2.715039e-01	2.767055e-01	2.742535e-01	2.739276e-01
	avg.	2.824862e-01	2.728299e-01	2.664315e-01	2.713366e-01	2.838310e-01	2.763589e-01	2.690686e-01	2.717277e-01	2.765034e-01	2.742732e-01	2.732834e-01
	min.	2.709841e-01	2.682120e-01	2.610022e-01	2.667944e-01	2.699136e-01	2.698873e-01	2.649899e-01	2.644396e-01	2.720737e-01	2.677016e-01	2.660516e-01
	max.	2.892250e-01	2.776338e-01	2.732742e-01	2.752056e-01	2.896638e-01	2.805642e-01	2.730897e-01	2.811551e-01	2.811975e-01	2.797544e-01	2.820174e-01
	std.	3.257393e-03	2.023365e-03	2.364595e-03	1.932052e-03	2.854377e-03	1.906691e-03	1.761481e-03	3.358628e-03	2.279791e-03	3.459026e-03	3.352132e-03
10D	med.	2.680091e-01	2.582396e-01	2.499436e-01	2.576960e-01	2.687816e-01	2.607953e-01	2.533127e-01	2.568589e-01	2.610329e-01	2.596144e-01	2.582638e-01
	avg.	2.680181e-01	2.583946e-01	2.500536e-01	2.576090e-01	2.686796e-01	2.610191e-01	2.531049e-01	2.569084e-01	2.610433e-01	2.595871e-01	2.583639e-01
	min.	2.609408e-01	2.554522e-01	2.456556e-01	2.544780e-01	2.639109e-01	2.571423e-01	2.468100e-01	2.519701e-01	2.563760e-01	2.570101e-01	2.533472e-01
	max.	2.729493e-01	2.614692e-01	2.538443e-01	2.603598e-01	2.728716e-01	2.656657e-01	2.556174e-01	2.608537e-01	2.648032e-01	2.634897e-01	2.638558e-01
	std.	1.980912e-03	1.222441e-03	1.582796e-03	1.307862e-03	2.134968e-03	1.638986e-03	1.390989e-03	1.914591e-03	1.525338e-03	1.399916e-03	2.092622e-03

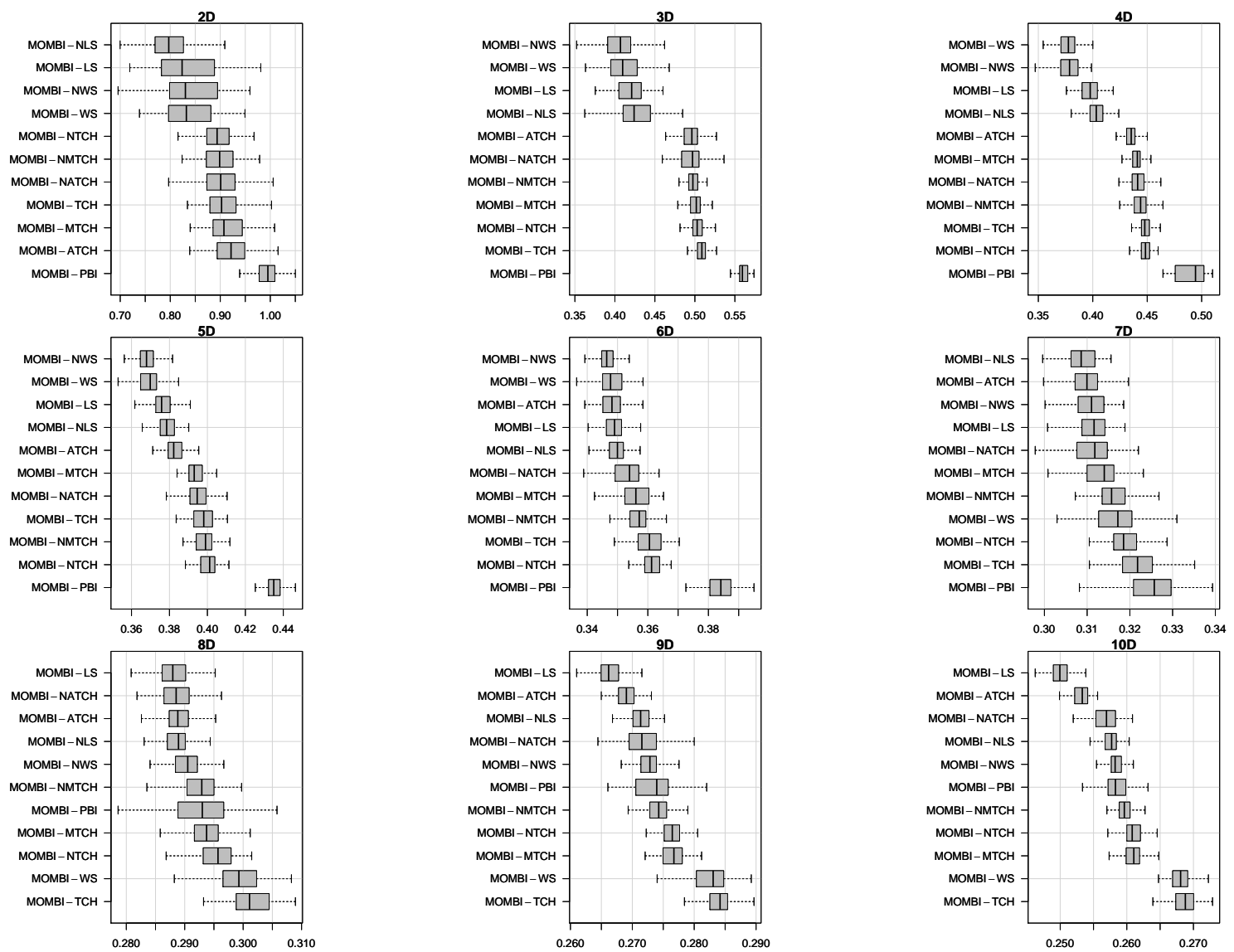


Figure A.328: Box-plot of the R<sup>2</sup> indicator values for different optimizers on the WFG1 test problem.



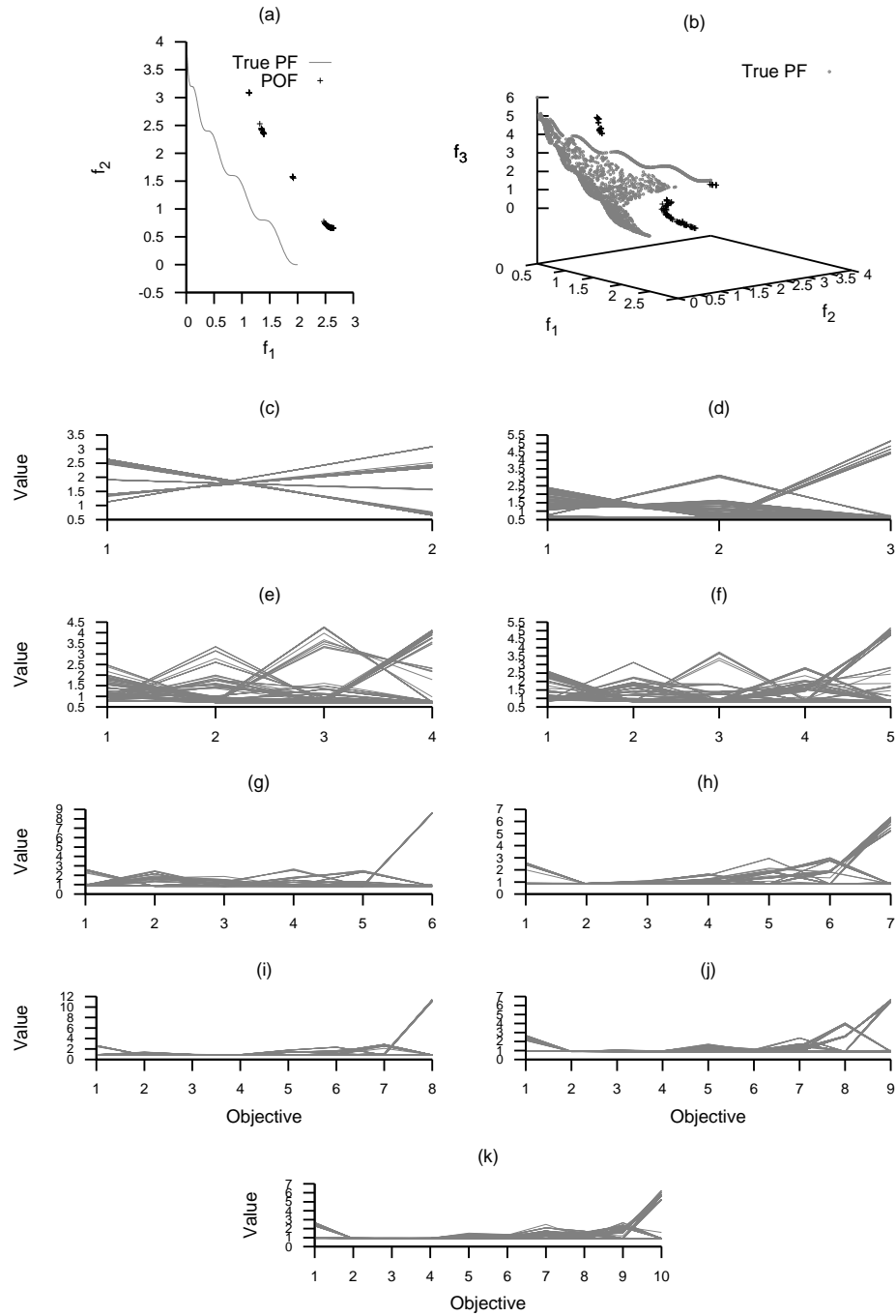


Figure A.329: Plots of the approximations obtained by MOMBI-WS from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

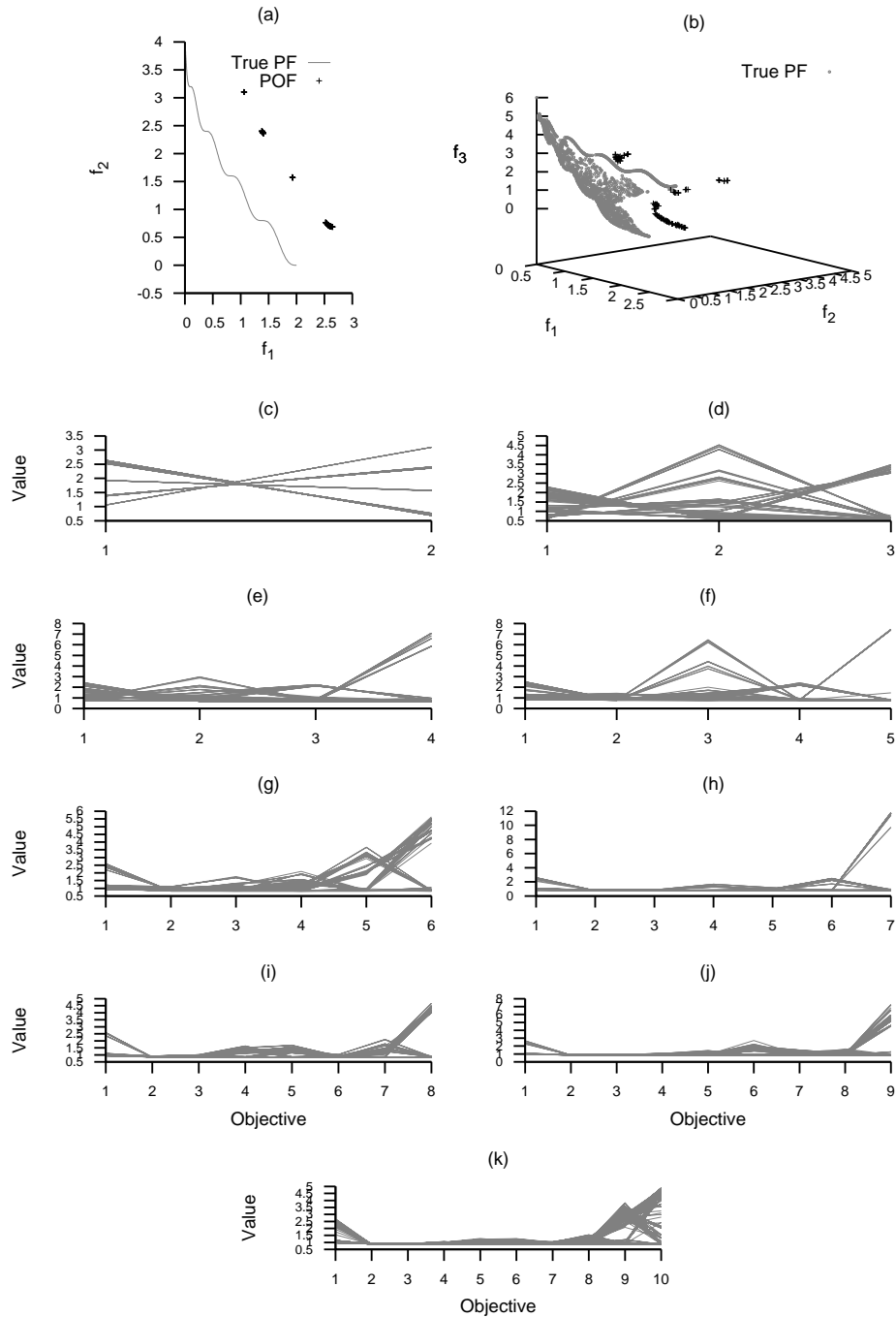


Figure A.330: Plots of the approximations obtained by MOMBI-NWS from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

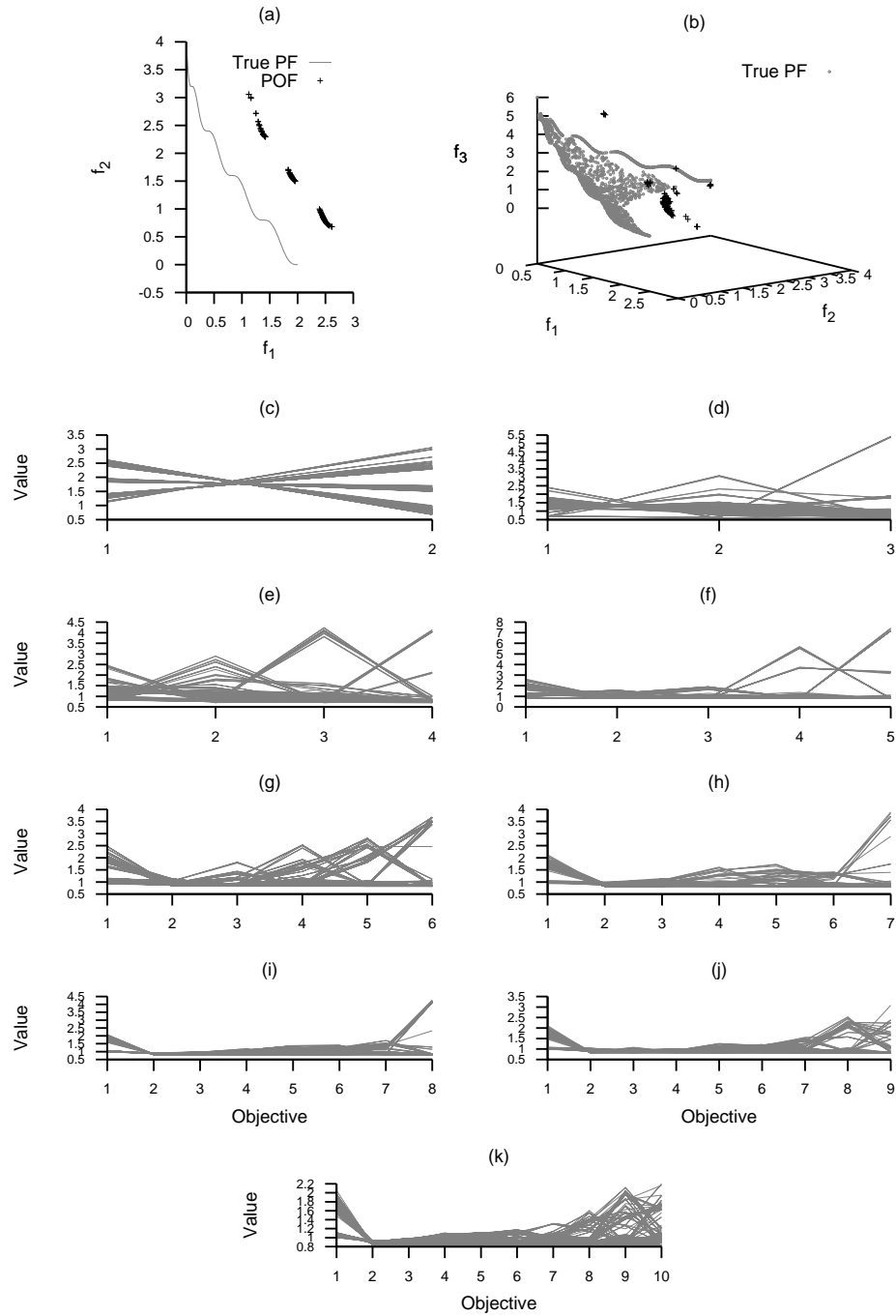


Figure A.331: Plots of the approximations obtained by MOMBI-LS from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

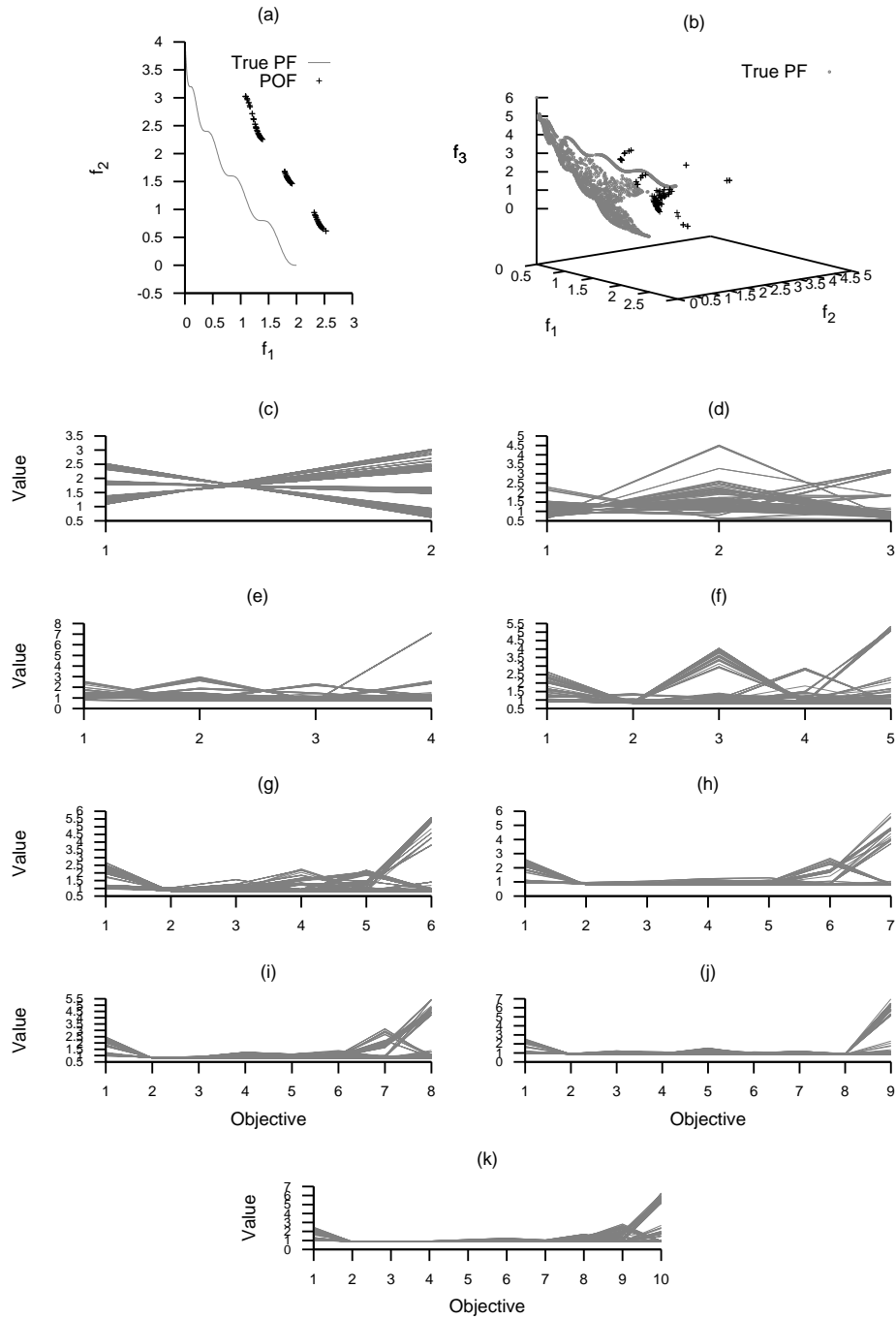


Figure A.332: Plots of the approximations obtained by MOMBI-NLS from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

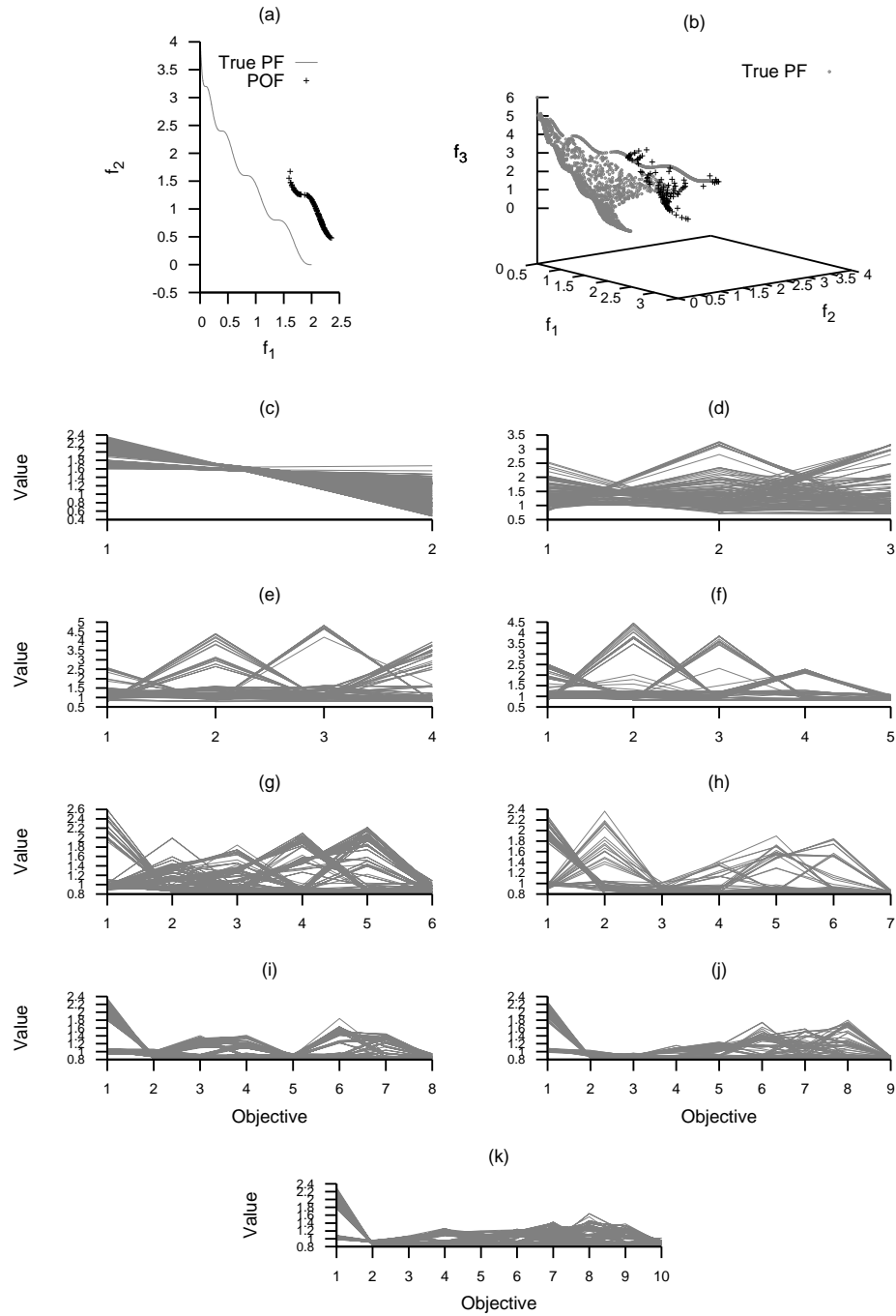


Figure A.333: Plots of the approximations obtained by MOMBI-ATCH from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

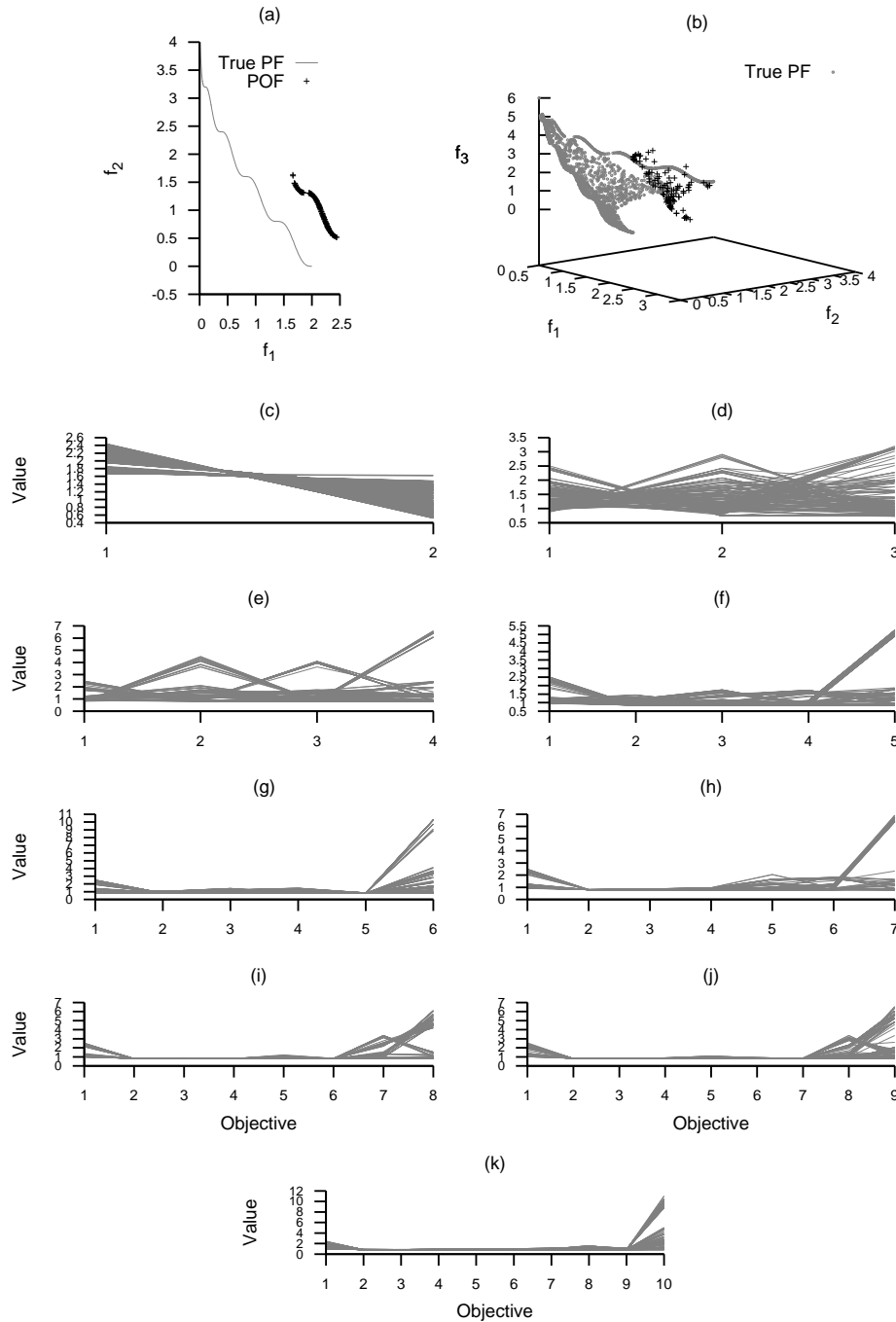


Figure A.334: Plots of the approximations obtained by MOMBI-NATCH from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

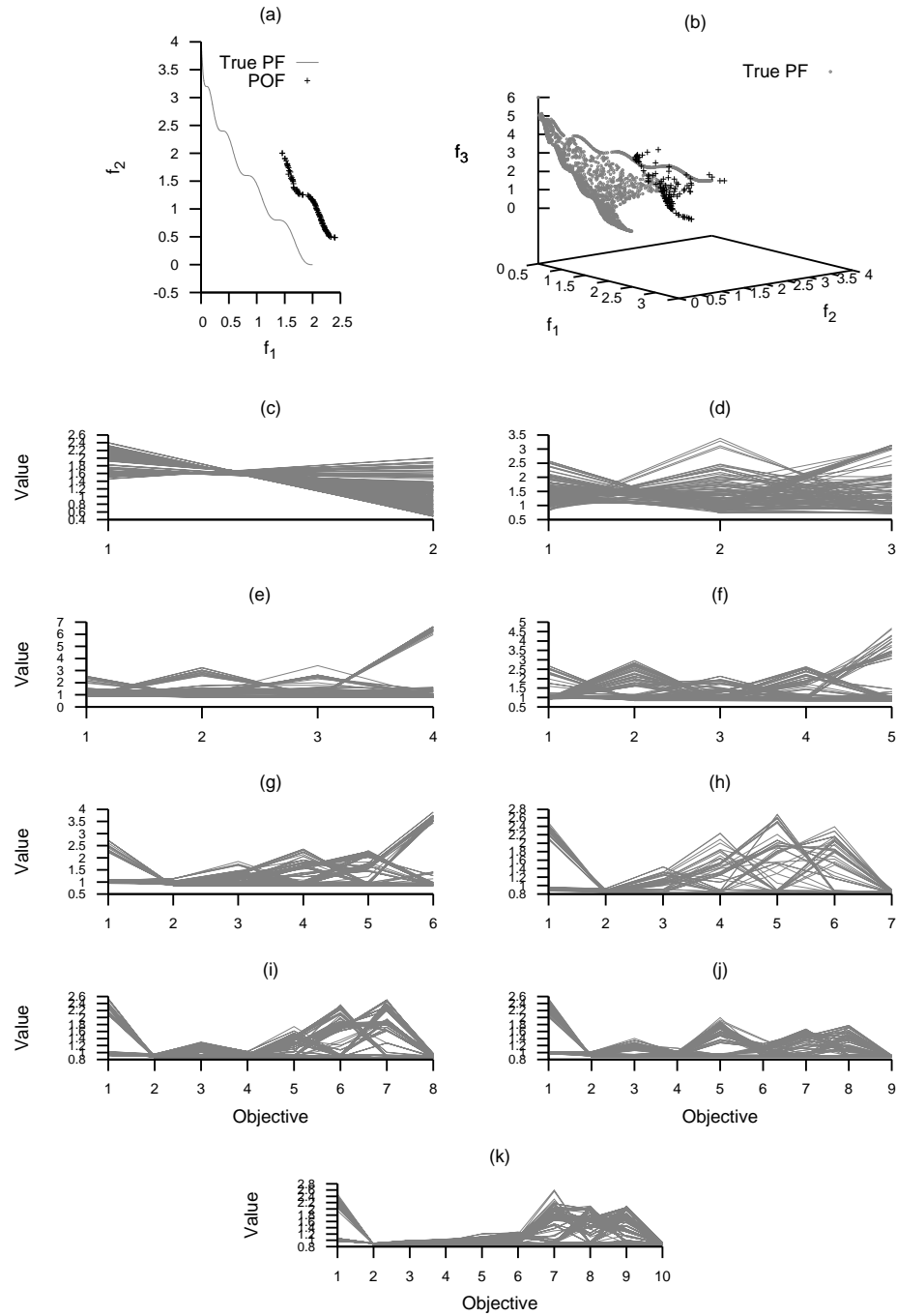


Figure A.335: Plots of the approximations obtained by MOMBI-MTCH from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

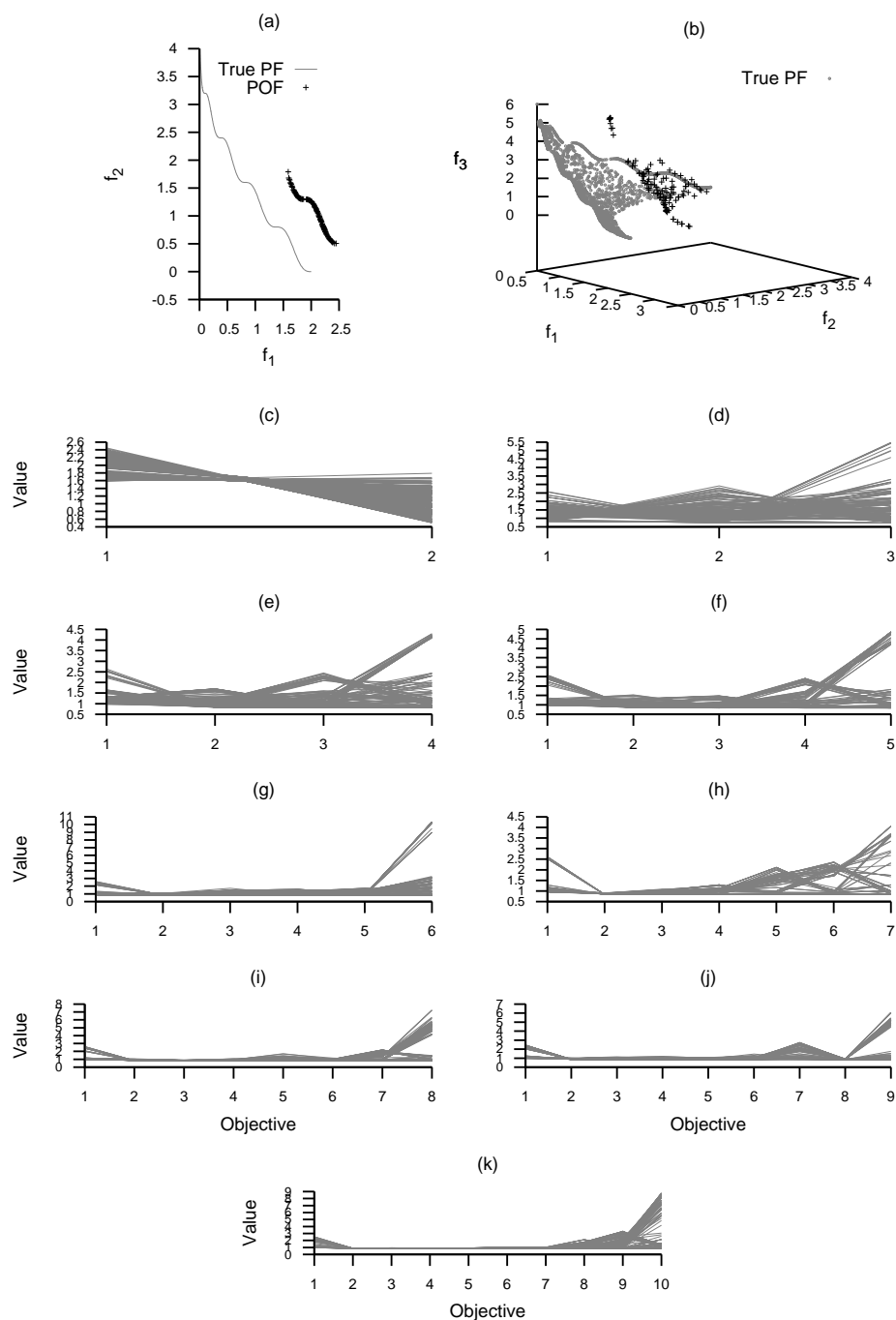


Figure A.336: Plots of the approximations obtained by MOMBI-NMTCH from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



Table A.138: Comparison of hypervolume indicator values for different optimizers on the WFG2 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	1.043617e + 01	1.044193e + 01	1.047761e + 01	1.048404e + 01	1.057505e + 01	1.058318e + 01	1.058125e + 01	1.058584e + 01	1.057628e + 01	1.058244e + 01	1.036368e + 01
	avg.	1.042884e + 01	1.043555e + 01	1.048521e + 01	1.049275e + 01	1.058348e + 01	1.058084e + 01	1.058614e + 01	1.059524e + 01	1.057329e + 01	1.056653e + 01	1.025591e + 01
	min.	1.021325e + 01	1.030672e + 01	1.027159e + 01	1.038319e + 01	1.045229e + 01	1.054234e + 01	1.052764e + 01	1.050403e + 01	1.047415e + 01	9.251765e + 00	9.753026e + 00
	max.	1.046807e + 01	1.105613e + 01	1.126131e + 01	1.126533e + 01	1.138252e + 01	1.060247e + 01	1.142235e + 01	1.139389e + 01	1.060691e + 01	1.141011e + 01	1.117480e + 01
	std.	4.030792e - 02	7.113381e - 02	1.146733e - 01	1.131600e - 01	1.163311e - 01	1.253502e - 02	8.569363e - 02	1.153302e - 01	2.034007e - 02	2.228577e - 01	2.403303e - 01
3D	med.	9.895035e + 01	9.914899e + 01	9.675910e + 01	8.484788e + 01	9.720952e + 01	9.847743e + 01	9.802945e + 01	9.923597e + 01	9.878809e + 01	9.984745e + 01	9.724340e + 01
	avg.	9.373728e + 01	9.240049e + 01	9.161420e + 01	9.153799e + 01	9.231611e + 01	9.343381e + 01	9.327176e + 01	9.349076e + 01	9.401193e + 01	9.432362e + 01	9.384548e + 01
	min.	8.399360e + 01	8.444497e + 01	8.344654e + 01	8.435926e + 01	8.265241e + 01	8.390227e + 01	8.293282e + 01	8.389742e + 01	8.396154e + 01	8.397918e + 01	8.180573e + 01
	max.	9.956666e + 01	1.000822e + 02	9.836282e + 01	9.929976e + 01	9.928450e + 01	9.976555e + 01	9.918817e + 01	1.000837e + 02	9.987998e + 01	1.005109e + 02	9.852999e + 01
	std.	7.008737e + 00	7.435528e + 00	6.692504e + 00	7.173694e + 00	6.827412e + 00	7.020947e + 00	6.882446e + 00	7.288692e + 00	6.989696e + 00	7.344106e + 00	6.277119e + 00
4D	med.	8.941689e + 02	9.161353e + 02	7.652530e + 02	9.136484e + 02	8.809826e + 02	9.015148e + 02	9.002045e + 02	9.141345e + 02	9.100725e + 02	9.187628e + 02	8.611440e + 02
	avg.	8.450985e + 02	8.577074e + 02	8.315923e + 02	8.552436e + 02	8.371013e + 02	8.655679e + 02	8.460398e + 02	8.686149e + 02	8.515231e + 02	8.765453e + 02	8.157000e + 02
	min.	7.528800e + 02	7.491333e + 02	7.472725e + 02	7.508672e + 02	7.366924e + 02	7.478995e + 02	7.516608e + 02	7.535559e + 02	7.419694e + 02	7.544804e + 02	7.068432e + 02
	max.	9.120839e + 02	9.263447e + 02	9.162543e + 02	9.266290e + 02	9.172491e + 02	9.22648e + 02	9.204145e + 02	9.256022e + 02	9.243411e + 02	9.316572e + 02	8.909977e + 02
	std.	6.815644e + 01	7.598545e + 01	7.413845e + 01	7.721359e + 01	6.851132e + 01	6.697292e + 01	7.336510e + 01	7.203666e + 01	7.605104e + 01	7.139933e + 01	7.021216e + 01
5D	med.	9.914404e + 03	1.002335e + 04	9.969147e + 03	1.002143e + 04	9.935222e + 03	1.004574e + 04	9.931186e + 03	9.937300e + 03	9.964531e + 03	1.007145e + 04	9.142975e + 03
	avg.	9.458791e + 03	9.466924e + 03	9.321094e + 03	9.422659e + 03	9.312919e + 03	9.403126e + 03	9.241071e + 03	9.314395e + 03	9.319404e + 03	9.494239e + 03	8.676754e + 03
	min.	8.151649e + 03	8.187991e + 03	7.893854e + 03	8.086481e + 03	8.109498e + 03	8.185300e + 03	8.085590e + 03	8.156477e + 03	8.159161e + 03	8.139784e + 03	7.412510e + 03
	max.	1.015924e + 04	1.023784e + 04	1.019846e + 04	1.025093e + 04	1.020384e + 04	1.026795e + 04	1.023310e + 04	1.026810e + 04	1.026636e + 04	1.027154e + 04	9.613852e + 03
	std.	7.888383e + 02	8.540163e + 02	8.855214e + 02	8.835655e + 02	8.615909e + 02	8.878493e + 02	8.927026e + 02	8.958009e + 02	8.945427e + 02	8.776838e + 02	7.657932e + 02
6D	med.	1.255448e + 05	1.258579e + 05	1.276650e + 05	1.281358e + 05	1.260610e + 05	1.290947e + 05	1.266860e + 05	1.285057e + 05	1.273265e + 05	1.289966e + 05	1.141864e + 05
	avg.	1.185759e + 05	1.182683e + 05	1.187702e + 05	1.204193e + 05	1.184894e + 05	1.234035e + 05	1.186711e + 05	1.216622e + 05	1.183835e + 05	1.220783e + 05	1.095864e + 05
	min.	1.042420e + 05	1.034917e + 05	1.018882e + 05	1.037899e + 05	1.025844e + 05	1.030385e + 05	1.027331e + 05	1.050038e + 05	1.008096e + 05	1.047706e + 05	9.361428e + 04
	max.	1.318282e + 05	1.318914e + 05	1.313216e + 05	1.324851e + 05	1.322685e + 05	1.323748e + 05	1.318433e + 05	1.326622e + 05	1.323933e + 05	1.325830e + 05	1.222521e + 05
	std.	1.086424e + 04	1.128675e + 04	1.185043e + 04	1.148941e + 04	1.157930e + 04	1.056351e + 04	1.174919e + 04	1.099270e + 04	1.207403e + 04	1.103283e + 04	9.656042e + 03
7D	med.	1.542477e + 06	1.538193e + 06	1.551689e + 06	1.563895e + 06	1.559474e + 06	1.573850e + 06	1.569866e + 06	1.554965e + 06	1.569894e + 06	1.560319e + 06	1.417267e + 06
	avg.	1.634344e + 06	1.601756e + 06	1.653705e + 06	1.672965e + 06	1.654831e + 06	1.684877e + 06	1.691879e + 06	1.655078e + 06	1.681157e + 06	1.670666e + 06	1.493680e + 06
	min.	1.440454e + 06	1.454040e + 06	1.462462e + 06	1.460920e + 06	1.441777e + 06	1.490430e + 06	1.466378e + 06	1.465321e + 06	1.460694e + 06	1.487459e + 06	1.319628e + 06
	max.	1.900204e + 06	1.834495e + 06	1.932382e + 06	1.918172e + 06	1.905909e + 06	1.917907e + 06	1.942255e + 06	1.893088e + 06	1.953619e + 06	1.889973e + 06	1.763452e + 06
	std.	1.443907e + 05	1.178558e + 05	1.682647e + 05	1.657197e + 05	1.556274e + 05	1.577743e + 05	1.751854e + 05	1.518777e + 05	1.730824e + 05	1.504027e + 05	1.375027e + 05
8D	med.	2.951091e + 07	2.876643e + 07	2.667050e + 07	3.107118e + 07	3.040309e + 07	3.040328e + 07	3.099699e + 07	3.055068e + 07	2.649152e + 07	3.024220e + 07	2.694871e + 07
	avg.	2.875133e + 07	2.817516e + 07	2.890476e + 07	2.943240e + 07	2.890260e + 07	2.900532e + 07	2.902932e + 07	2.920181e + 07	2.851338e + 07	2.904767e + 07	2.603993e + 07
	min.	2.512794e + 07	2.501271e + 07	2.480276e + 07	2.492482e + 07	2.495142e + 07	2.500367e + 07	2.501639e + 07	2.518296e + 07	2.523475e + 07	2.491253e + 07	2.194803e + 07
	max.	3.243086e + 07	3.126920e + 07	3.300298e + 07	3.253665e + 07	3.282022e + 07	3.243436e + 07	3.277463e + 07	3.231412e + 07	3.271416e + 07	3.208936e + 07	2.906035e + 07
	std.	2.572940e + 06	2.056547e + 06	3.077816e + 06	2.623912e + 06	2.882961e + 06	2.679838e + 06	2.999143e + 06	2.465957e + 06	2.883599e + 06	2.487804e + 06	2.256528e + 06
9D	med.	5.826752e + 08	5.623497e + 08	5.995733e + 08	5.940486e + 08	5.926027e + 08	5.936675e + 08	6.014039e + 08	5.821321e + 08	6.005357e + 08	5.803059e + 08	5.158276e + 08
	avg.	5.574582e + 08	5.421621e + 08	5.681638e + 08	5.698736e + 08	5.636382e + 08	5.728778e + 08	5.642131e + 08	5.605870e + 08	5.644362e + 08	5.609838e + 08	4.977574e + 08
	min.	4.680772e + 08	4.745094e + 08	4.737327e + 08	4.783112e + 08	4.697757e + 08	4.830166e + 08	4.768196e + 08	4.722041e + 08	4.821468e + 08	4.825388e + 08	4.253888e + 08
	max.	6.162937e + 08	6.063150e + 08	6.263780e + 08	6.227673e + 08	6.214600e + 08	6.202466e + 08	6.246131e + 08	6.185408e + 08	6.281434e + 08	6.053556e + 08	5.483502e + 08
	std.	5.288588e + 07	4.082651e + 07	5.407611e + 07	4.773647e + 07	5.130548e + 07	4.601026e + 07	5.628418e + 07	4.742099e + 07	5.641826e + 07	4.195269e + 07	3.881664e + 07
10D	med.	1.253054e + 10	1.210445e + 10	1.260775e + 10	1.255091e + 10	1.249567e + 10	1.251362e + 10	1.270460e + 10	1.245592e + 10	1.276365e + 10	1.233890e + 10	1.092349e + 10
	avg.	1.203348e + 10	1.191130e + 10	1.223790e + 10	1.212394e + 10	1.206292e + 10	1.221615e + 10	1.228442e + 10	1.207918e + 10	1.200393e + 10	1.198039e + 10	1.067170e + 10
	min.	1.011582e + 10	1.020083e + 10	1.004547e + 10	1.011781e + 10	1.005914e + 10	1.027051e + 10	1.013687e + 10	1.019585e + 10	9.992927e + 09	1.010308e + 10	9.001203e + 09
	max.	1.297086e + 10	1.287557e + 10	1.295149e + 10	1.300170e + 10	1.295376e + 10	1.309091e + 10	1.307751e + 10	1.294093e + 10	1.313041e + 10	1.297800e + 10	1.163393e + 10
	std.	9.777509e + 08	6.936074e + 08	9.047129e + 08	9.475028e + 08	9.531141e + 08	8.473727e + 08	9.547449e + 08	8.861471e + 08	1.178333e + 09	8.637800e + 08	7.252807e + 08

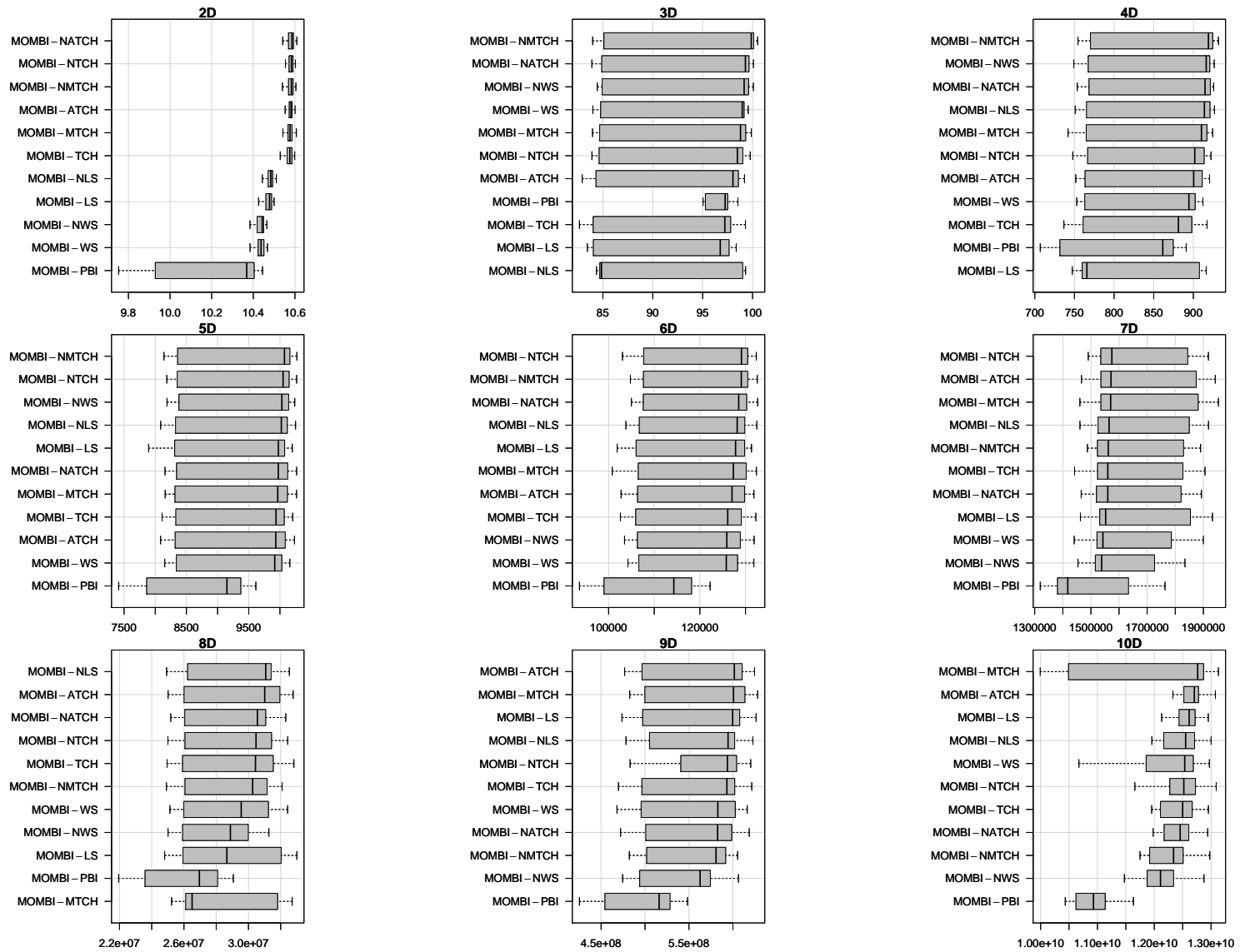


Figure A.337: Box-plot of the hypervolume indicator values for different optimizers on the WFG2 test problem.

Table A.139: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG2 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
2D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$3.90e-23$
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.89e-22$
MOMBI-LS	$1.68e-21$	$7.47e-21$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$5.27e-31$
MOMBI-NLS	$6.30e-25$	$3.02e-24$	$7.35e-03$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$8.73e-32$
MOMBI-TCH	$4.12e-34$	$6.42e-33$	$3.02e-29$	$9.94e-29$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.37e-33$
MOMBI-NTCH	$1.28e-34$	$2.51e-33$	$4.63e-32$	$4.63e-32$	$7.32e-05$	–	> 0.05	> 0.05	$2.53e-03$	> 0.05	$2.51e-33$
MOMBI-ATCH	$1.28e-34$	$2.44e-33$	$4.37e-32$	$4.37e-32$	$3.54e-03$	> 0.05	–	> 0.05	$4.65e-02$	> 0.05	$2.44e-33$
MOMBI-NATCH	$1.28e-34$	$2.37e-33$	$4.13e-32$	$4.37e-32$	$4.40e-05$	> 0.05	$2.05e-02$	–	$1.17e-03$	> 0.05	$2.37e-33$
MOMBI-MTCH	$1.28e-34$	$2.51e-33$	$2.59e-31$	$5.12e-31$	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	$2.51e-33$
MOMBI-NMTCH	$4.63e-32$	$7.19e-31$	$5.23e-29$	$1.04e-28$	$2.27e-03$	> 0.05	> 0.05	> 0.05	$2.66e-02$	–	$7.20e-31$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
3D											
MOMBI-WS	–	> 0.05	$1.70e-12$	$9.86e-04$	$6.81e-11$	$1.74e-02$	$1.43e-08$	> 0.05	> 0.05	> 0.05	$4.90e-08$
MOMBI-NWS	$2.50e-04$	–	$3.50e-08$	$2.75e-07$	$4.67e-07$	$8.04e-05$	$3.04e-06$	> 0.05	$4.39e-03$	> 0.05	$9.15e-04$
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	$3.37e-07$	–	$1.15e-05$	> 0.05	$6.54e-04$	> 0.05	> 0.05	> 0.05	$5.70e-03$
MOMBI-TCH	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	> 0.05	> 0.05	$1.02e-11$	> 0.05	$1.69e-09$	–	$3.39e-05$	> 0.05	> 0.05	> 0.05	$2.55e-07$
MOMBI-ATCH	> 0.05	> 0.05	$7.09e-10$	> 0.05	$1.69e-06$	> 0.05	–	> 0.05	> 0.05	> 0.05	$5.69e-07$
MOMBI-NATCH	$2.84e-05$	> 0.05	$1.25e-10$	$6.70e-09$	$1.91e-09$	$3.73e-06$	$4.71e-08$	–	$5.75e-04$	> 0.05	$3.33e-06$
MOMBI-MTCH	> 0.05	> 0.05	$2.62e-13$	$3.50e-02$	$2.01e-11$	$3.94e-02$	$1.10e-07$	> 0.05	–	> 0.05	$8.36e-09$
MOMBI-NMTCH	$6.00e-10$	$2.55e-08$	$3.27e-12$	$8.19e-14$	$3.75e-11$	$2.91e-10$	$3.24e-10$	$5.53e-08$	$3.85e-09$	–	$1.26e-07$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
4D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	$1.53e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.17e-10$
MOMBI-NWS	$2.89e-08$	–	$6.29e-11$	> 0.05	$3.28e-09$	$4.55e-04$	$5.76e-07$	> 0.05	$3.16e-03$	> 0.05	$2.00e-10$
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.00e-06$
MOMBI-NLS	$2.93e-06$	> 0.05	$2.66e-08$	–	$1.05e-07$	$5.32e-03$	$6.34e-05$	> 0.05	$3.16e-02$	> 0.05	$5.15e-10$
MOMBI-TCH	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.97e-08$
MOMBI-NTCH	$1.70e-05$	> 0.05	$6.01e-05$	> 0.05	$5.39e-08$	–	> 0.05	> 0.05	> 0.05	> 0.05	$1.79e-15$
MOMBI-ATCH	$2.13e-03$	> 0.05	$2.20e-03$	> 0.05	$9.04e-05$	> 0.05	–	> 0.05	> 0.05	> 0.05	$3.28e-09$
MOMBI-NATCH	$1.66e-10$	> 0.05	$9.68e-12$	> 0.05	$1.02e-11$	$7.43e-05$	$1.47e-07$	–	$2.10e-03$	> 0.05	$4.80e-14$
MOMBI-MTCH	$7.18e-06$	> 0.05	$3.29e-07$	> 0.05	$6.52e-07$	> 0.05	$3.23e-03$	> 0.05	–	> 0.05	$1.29e-09$
MOMBI-NMTCH	$2.48e-13$	$2.45e-03$	$9.73e-16$	$8.14e-04$	$1.85e-14$	$9.49e-09$	$7.40e-12$	$4.82e-03$	$3.01e-07$	–	$1.47e-15$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–

Table A.140: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG2 test problem (continuation).

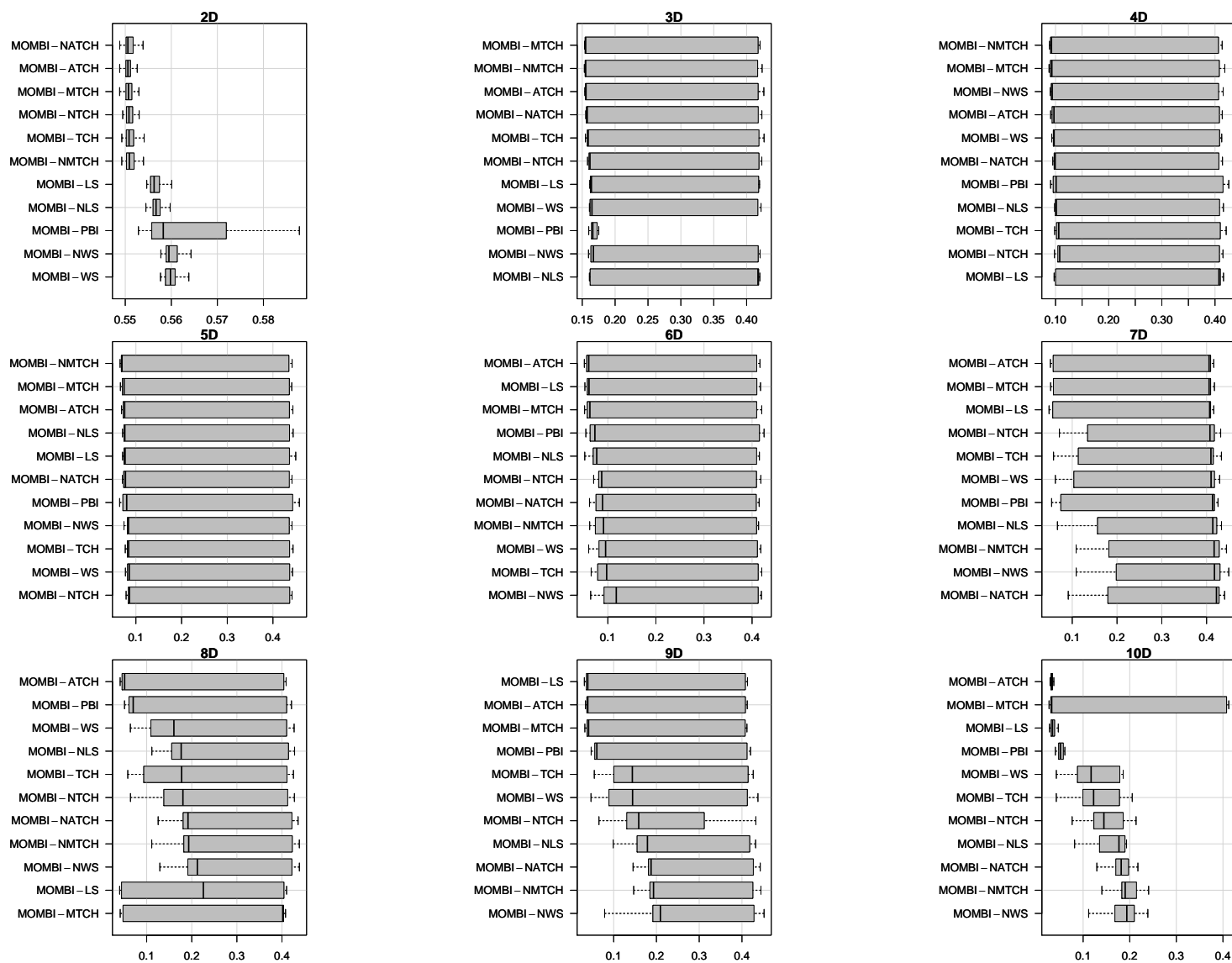
Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
5D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.41e-15$
MOMBI-NWS	$2.10e-04$	–	$1.98e-03$	> 0.05	$1.73e-03$	> 0.05	$1.38e-03$	> 0.05	> 0.05	> 0.05	$2.97e-13$
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$4.63e-11$
MOMBI-NLS	$6.59e-03$	> 0.05	$4.67e-02$	–	$4.70e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$8.17e-13$
MOMBI-TCH	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$9.69e-11$
MOMBI-NTCH	$1.49e-03$	> 0.05	$4.45e-03$	> 0.05	$6.87e-03$	–	$5.24e-03$	> 0.05	> 0.05	> 0.05	$1.52e-11$
MOMBI-ATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$7.14e-09$
MOMBI-NATCH	$4.49e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	$5.73e-10$
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	$2.38e-10$
MOMBI-NMTCH	$3.09e-05$	> 0.05	$1.79e-04$	$1.99e-02$	$2.67e-04$	> 0.05	$3.16e-04$	$2.38e-02$	$1.47e-02$	–	$1.06e-13$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
6D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$3.23e-09$
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$4.17e-08$
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.81e-08$
MOMBI-NLS	$6.46e-03$	$1.37e-02$	> 0.05	–	$1.10e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$7.62e-11$
MOMBI-TCH	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$7.67e-09$
MOMBI-NTCH	$5.29e-07$	$2.41e-06$	$3.31e-04$	$1.85e-02$	$5.58e-06$	–	$1.40e-03$	> 0.05	$2.48e-02$	> 0.05	$1.26e-16$
MOMBI-ATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$4.23e-08$
MOMBI-NATCH	$4.27e-05$	$1.41e-04$	$4.01e-03$	> 0.05	$2.04e-04$	> 0.05	$9.72e-03$	–	> 0.05	> 0.05	$1.29e-12$
MOMBI-MTCH	$4.73e-02$	$4.65e-02$	> 0.05	> 0.05	$3.62e-02$	> 0.05	> 0.05	> 0.05	–	> 0.05	$4.55e-07$
MOMBI-NMTCH	$3.78e-06$	$7.34e-06$	$3.31e-04$	$2.52e-02$	$2.05e-05$	> 0.05	$1.11e-03$	> 0.05	$1.34e-02$	–	$4.32e-13$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
7D											
MOMBI-WS	–	$4.13e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.49e-10$
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$7.56e-09$
MOMBI-LS	$3.54e-02$	$1.28e-03$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.18e-09$
MOMBI-NLS	$1.90e-02$	$1.21e-04$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$7.52e-12$
MOMBI-TCH	> 0.05	$9.69e-04$	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$5.62e-11$
MOMBI-NTCH	$7.18e-04$	$3.13e-07$	> 0.05	> 0.05	$2.41e-02$	–	> 0.05	$1.28e-02$	> 0.05	$4.97e-02$	$5.74e-13$
MOMBI-ATCH	$2.20e-04$	$1.43e-06$	$4.11e-02$	$3.33e-02$	$4.61e-03$	> 0.05	–	$2.25e-03$	> 0.05	$9.04e-03$	$3.57e-12$
MOMBI-NATCH	> 0.05	$1.30e-03$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	$1.65e-11$
MOMBI-MTCH	$3.22e-04$	$3.33e-06$	$2.76e-02$	$2.78e-02$	$5.17e-03$	> 0.05	> 0.05	$3.09e-03$	–	$1.11e-02$	$8.26e-11$
MOMBI-NMTCH	$2.67e-02$	$2.26e-05$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	$6.27e-13$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–

Table A.141: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG2 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
8D											
MOMBI-WS	–	$2.58e-03$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$3.38e-09$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$8.50e-10$
MOMBI-LS	$> 0.05$	$3.48e-03$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$4.12e-07$
MOMBI-NLS	$1.07e-02$	$2.16e-08$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$3.28e-03$	$> 0.05$	$4.55e-03$	$3.76e-12$
MOMBI-TCH	$> 0.05$	$1.11e-03$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.73e-08$
MOMBI-NTCH	$> 0.05$	$3.83e-05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.24e-09$
MOMBI-ATCH	$3.28e-02$	$5.37e-04$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$2.17e-02$	$> 0.05$	$1.19e-02$	$1.55e-07$
MOMBI-NATCH	$> 0.05$	$1.65e-06$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$2.11e-13$
MOMBI-MTCH	$> 0.05$	$2.55e-02$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$1.10e-04$
MOMBI-NMTCH	$> 0.05$	$8.69e-05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$7.10e-13$
MOMBI-PBI	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–
9D											
MOMBI-WS	–	$3.09e-05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$5.99e-11$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$6.69e-12$
MOMBI-LS	$1.47e-02$	$7.89e-09$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$4.67e-04$	$> 0.05$	$4.29e-05$	$2.57e-13$
MOMBI-NLS	$> 0.05$	$3.18e-11$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$3.51e-03$	$> 0.05$	$5.14e-05$	$2.01e-15$
MOMBI-TCH	$> 0.05$	$4.65e-08$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$4.04e-02$	$> 0.05$	$5.10e-03$	$8.61e-13$
MOMBI-NTCH	$1.84e-02$	$1.33e-12$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$5.32e-04$	$> 0.05$	$1.49e-05$	$4.74e-17$
MOMBI-ATCH	$6.07e-03$	$2.55e-07$	$> 0.05$	$4.84e-02$	$1.72e-02$	$> 0.05$	–	$3.59e-04$	$> 0.05$	$1.57e-04$	$1.77e-10$
MOMBI-NATCH	$> 0.05$	$5.35e-07$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$8.50e-14$
MOMBI-MTCH	$7.18e-04$	$2.62e-07$	$> 0.05$	$2.25e-02$	$3.26e-03$	$> 0.05$	$> 0.05$	$2.50e-04$	–	$1.70e-04$	$2.96e-09$
MOMBI-NMTCH	$> 0.05$	$8.43e-07$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$6.43e-16$
MOMBI-PBI	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–
10D											
MOMBI-WS	–	$1.23e-05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$4.30e-03$	$1.42e-14$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.91e-20$
MOMBI-LS	$3.32e-02$	$1.13e-11$	–	$> 0.05$	$1.32e-02$	$> 0.05$	$> 0.05$	$3.95e-04$	$> 0.05$	$3.64e-07$	$1.92e-19$
MOMBI-NLS	$> 0.05$	$8.36e-08$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$2.78e-02$	$> 0.05$	$1.73e-04$	$1.49e-16$
MOMBI-TCH	$> 0.05$	$1.40e-06$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$3.83e-03$	$7.24e-16$
MOMBI-NTCH	$> 0.05$	$1.11e-09$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$1.57e-02$	$> 0.05$	$4.07e-05$	$1.72e-19$
MOMBI-ATCH	$2.67e-05$	$5.16e-13$	$2.74e-03$	$2.74e-04$	$2.67e-06$	$2.02e-03$	–	$5.92e-08$	$> 0.05$	$6.19e-11$	$5.62e-18$
MOMBI-NATCH	$> 0.05$	$3.07e-06$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$2.52e-02$	$2.83e-17$
MOMBI-MTCH	$4.51e-04$	$1.28e-05$	$4.78e-03$	$1.74e-03$	$1.14e-04$	$1.19e-02$	$> 0.05$	$1.24e-04$	–	$8.57e-06$	$5.76e-09$
MOMBI-NMTCH	$> 0.05$	$2.81e-03$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$2.89e-17$
MOMBI-PBI	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–

Table A.142: Comparison of R2 indicator values for different optimizers on the WFG2 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	5.597752e-01	5.594553e-01	5.562142e-01	5.566809e-01	5.508194e-01	5.508069e-01	5.505662e-01	5.505677e-01	5.506811e-01	5.508606e-01	5.581808e-01
	avg.	5.604587e-01	5.594324e-01	5.545955e-01	5.548862e-01	5.491406e-01	5.509875e-01	5.495752e-01	5.488069e-01	5.510050e-01	5.546744e-01	5.615547e-01
	min.	5.576567e-01	4.531987e-01	4.429397e-01	4.432244e-01	4.351592e-01	5.494509e-01	4.335691e-01	4.348702e-01	5.487807e-01	4.342155e-01	4.407468e-01
	max.	5.770167e-01	5.701949e-01	5.721800e-01	5.644515e-01	5.604169e-01	5.540000e-01	5.547768e-01	5.569300e-01	5.586795e-01	8.367697e-01	5.878058e-01
	std.	3.100437e-03	1.105002e-02	1.616365e-02	1.612755e-02	1.641522e-02	9.511204e-04	1.176864e-02	1.638363e-02	1.442585e-03	4.372026e-02	1.500200e-02
3D	med.	1.647366e-01	1.669628e-01	1.639286e-01	4.171633e-01	1.588951e-01	1.617152e-01	1.557420e-01	1.575184e-01	1.551990e-01	1.552607e-01	1.660604e-01
	avg.	2.574649e-01	2.880275e-01	2.727163e-01	2.954299e-01	2.601076e-01	2.588652e-01	2.498494e-01	2.641200e-01	2.469002e-01	2.545523e-01	2.275237e-01
	min.	1.609409e-01	1.595741e-01	1.614969e-01	1.609168e-01	1.548608e-01	1.580826e-01	1.539586e-01	1.553501e-01	1.535144e-01	1.532046e-01	1.599237e-01
	max.	4.219273e-01	4.204768e-01	4.196611e-01	4.201067e-01	4.265448e-01	4.231356e-01	4.262103e-01	4.232201e-01	4.204415e-01	4.235999e-01	4.294335e-01
	std.	1.236085e-01	1.280554e-01	1.271473e-01	1.286031e-01	1.281068e-01	1.260974e-01	1.268181e-01	1.291101e-01	1.261482e-01	1.280942e-01	1.113380e-01
4D	med.	9.725896e-02	9.370591e-02	4.072918e-01	1.015313e-01	1.058222e-01	1.078761e-01	9.691944e-02	9.930795e-02	9.308596e-02	9.242639e-02	1.008240e-01
	avg.	2.204973e-01	2.186395e-01	2.583060e-01	2.270149e-01	2.321060e-01	1.940182e-01	2.295901e-01	1.978099e-01	2.245161e-01	1.834578e-01	2.213545e-01
	min.	9.291462e-02	8.991522e-02	9.736464e-02	9.832724e-02	9.847879e-02	9.808405e-02	9.084427e-02	9.485027e-02	8.780979e-02	8.880582e-02	9.079385e-02
	max.	4.126325e-01	4.153489e-01	4.159750e-01	4.158323e-01	4.210175e-01	4.149973e-01	4.135888e-01	4.140565e-01	4.184181e-01	4.134335e-01	4.258729e-01
	std.	1.532593e-01	1.552121e-01	1.557332e-01	1.528644e-01	1.528639e-01	1.387109e-01	1.563484e-01	1.456736e-01	1.575931e-01	1.445329e-01	1.565173e-01
5D	med.	8.544696e-02	8.369522e-02	7.626998e-02	7.568491e-02	8.411297e-02	8.594393e-02	7.536467e-02	7.703127e-02	7.392848e-02	6.958118e-02	7.966078e-02
	avg.	1.933247e-01	2.095932e-01	2.234312e-01	2.089443e-01	2.310867e-01	2.252008e-01	2.435315e-01	2.336712e-01	2.283026e-01	1.974301e-01	2.334746e-01
	min.	7.706783e-02	7.392687e-02	7.083084e-02	7.059895e-02	7.673107e-02	7.856058e-02	6.861458e-02	7.029095e-02	6.606008e-02	6.535941e-02	6.442048e-02
	max.	4.429676e-01	4.417941e-01	4.498882e-01	4.441988e-01	4.439761e-01	4.418691e-01	4.436980e-01	4.417167e-01	4.412091e-01	4.419550e-01	4.578487e-01
	std.	1.648877e-01	1.713079e-01	1.793267e-01	1.762786e-01	1.763197e-01	1.741042e-01	1.826662e-01	1.809757e-01	1.818401e-01	1.763612e-01	1.847262e-01
6D	med.	9.484984e-02	1.143951e-01	6.070440e-02	7.683532e-02	9.731537e-02	8.708886e-02	6.017196e-02	8.876619e-02	6.153844e-02	9.073103e-02	7.303826e-02
	avg.	2.243653e-01	2.419490e-01	2.163848e-01	2.041292e-01	2.284200e-01	1.732718e-01	2.192586e-01	1.977280e-01	2.302379e-01	1.936553e-01	2.036521e-01
	min.	6.019110e-02	6.428197e-02	5.225422e-02	5.200691e-02	6.532150e-02	7.050188e-02	5.112450e-02	6.183957e-02	5.167281e-02	6.225365e-02	5.424402e-02
	max.	4.181588e-01	4.193483e-01	4.175737e-01	4.152447e-01	4.201818e-01	4.182222e-01	4.163820e-01	4.147121e-01	4.200905e-01	4.135644e-01	4.249539e-01
	std.	1.641200e-01	1.591704e-01	1.763175e-01	1.661676e-01	1.648258e-01	1.460949e-01	1.771398e-01	1.575965e-01	1.772499e-01	1.567567e-01	1.722321e-01
7D	med.	4.097077e-01	4.174560e-01	4.068314e-01	4.136471e-01	4.096299e-01	4.065295e-01	4.055364e-01	4.219497e-01	4.054743e-01	4.133384e-01	4.131052e-01
	avg.	2.906907e-01	3.367882e-01	2.846980e-01	2.989177e-01	2.903231e-01	2.807688e-01	2.508035e-01	3.167685e-01	2.680217e-01	3.085547e-01	2.788823e-01
	min.	6.231093e-02	1.089021e-01	4.820275e-02	6.695689e-02	5.828561e-02	7.130872e-02	5.121945e-02	9.112249e-02	5.177764e-02	1.087521e-01	5.398093e-02
	max.	4.291303e-01	4.495213e-01	4.160360e-01	4.332438e-01	4.331287e-01	4.315044e-01	4.159781e-01	4.402170e-01	4.175363e-01	4.443700e-01	4.254946e-01
	std.	1.550569e-01	1.176173e-01	1.701518e-01	1.401491e-01	1.521758e-01	1.442866e-01	1.757486e-01	1.274490e-01	1.729916e-01	1.255022e-01	1.706997e-01
8D	med.	1.563710e-01	2.118882e-01	5.084789e-02	1.764237e-01	1.767233e-01	1.794550e-01	5.073845e-02	1.915863e-01	4.016453e-01	1.928894e-01	6.983420e-02
	avg.	2.470219e-01	2.758006e-01	2.244719e-01	2.529155e-01	2.479712e-01	2.584957e-01	2.213141e-01	2.657768e-01	2.573638e-01	2.698197e-01	2.025879e-01
	min.	6.387070e-02	1.291866e-01	4.007905e-02	1.115302e-01	5.811039e-02	6.409734e-02	4.120008e-02	1.254482e-01	4.131900e-02	1.112715e-01	5.090758e-02
	max.	4.269108e-01	4.385763e-01	4.102315e-01	4.279482e-01	4.254226e-01	4.275075e-01	4.091792e-01	4.357426e-01	4.079246e-01	4.384941e-01	4.210895e-01
	std.	1.490051e-01	1.127431e-01	1.810138e-01	1.282885e-01	1.560941e-01	1.382308e-01	1.799520e-01	1.154465e-01	1.761724e-01	1.162271e-01	1.715662e-01
9D	med.	1.441389e-01	2.083891e-01	3.948817e-02	1.789197e-01	1.436354e-01	1.580530e-01	3.953692e-02	1.864194e-01	4.134394e-02	1.923518e-01	6.056911e-02
	avg.	2.223116e-01	2.739742e-01	1.567186e-01	2.331075e-01	2.162817e-01	2.148350e-01	1.788515e-01	2.595316e-01	1.898224e-01	2.554119e-01	1.850861e-01
	min.	4.697629e-02	7.888760e-02	3.142667e-02	9.899176e-02	5.470087e-02	6.563277e-02	3.415906e-02	1.453538e-01	3.263223e-02	1.468141e-01	4.736457e-02
	max.	4.373767e-01	4.516869e-01	4.129643e-01	4.319234e-01	4.263123e-01	4.324652e-01	4.123933e-01	4.427173e-01	4.116730e-01	4.443459e-01	4.200361e-01
	std.	1.534183e-01	1.161721e-01	1.740765e-01	1.207311e-01	1.445022e-01	1.231322e-01	1.808694e-01	1.152875e-01	1.827108e-01	1.074250e-01	1.722737e-01
10D	med.	1.168543e-01	1.929609e-01	3.415843e-02	1.763686e-01	1.216232e-01	1.445288e-01	3.237523e-02	1.812338e-01	3.254871e-02	1.897513e-01	5.139198e-02
	avg.	1.794780e-01	2.165697e-01	9.781174e-02	2.135295e-01	1.834168e-01	1.881732e-01	1.036517e-01	2.261371e-01	1.597139e-01	2.364733e-01	1.263057e-01
	min.	4.251023e-02	1.005574e-01	2.775429e-02	8.169052e-02	4.242214e-02	7.642986e-02	2.895170e-02	1.267884e-01	2.683795e-02	1.401645e-01	4.045679e-02
	max.	4.259394e-01	4.323195e-01	4.119331e-01	4.330818e-01	4.330018e-01	4.22982e-01	4.109113e-01	4.385420e-01	4.124812e-01	4.436392e-01	4.184235e-01
	std.	1.364018e-01	9.182935e-02	1.419570e-01	1.123573e-01	1.290951e-01	1.089990e-01	1.486793e-01	1.022543e-01	1.797081e-01	9.870262e-02	1.489140e-01

Figure A.338: Box-plot of the  $R^2$  indicator values for different optimizers on the WFG2 test problem.

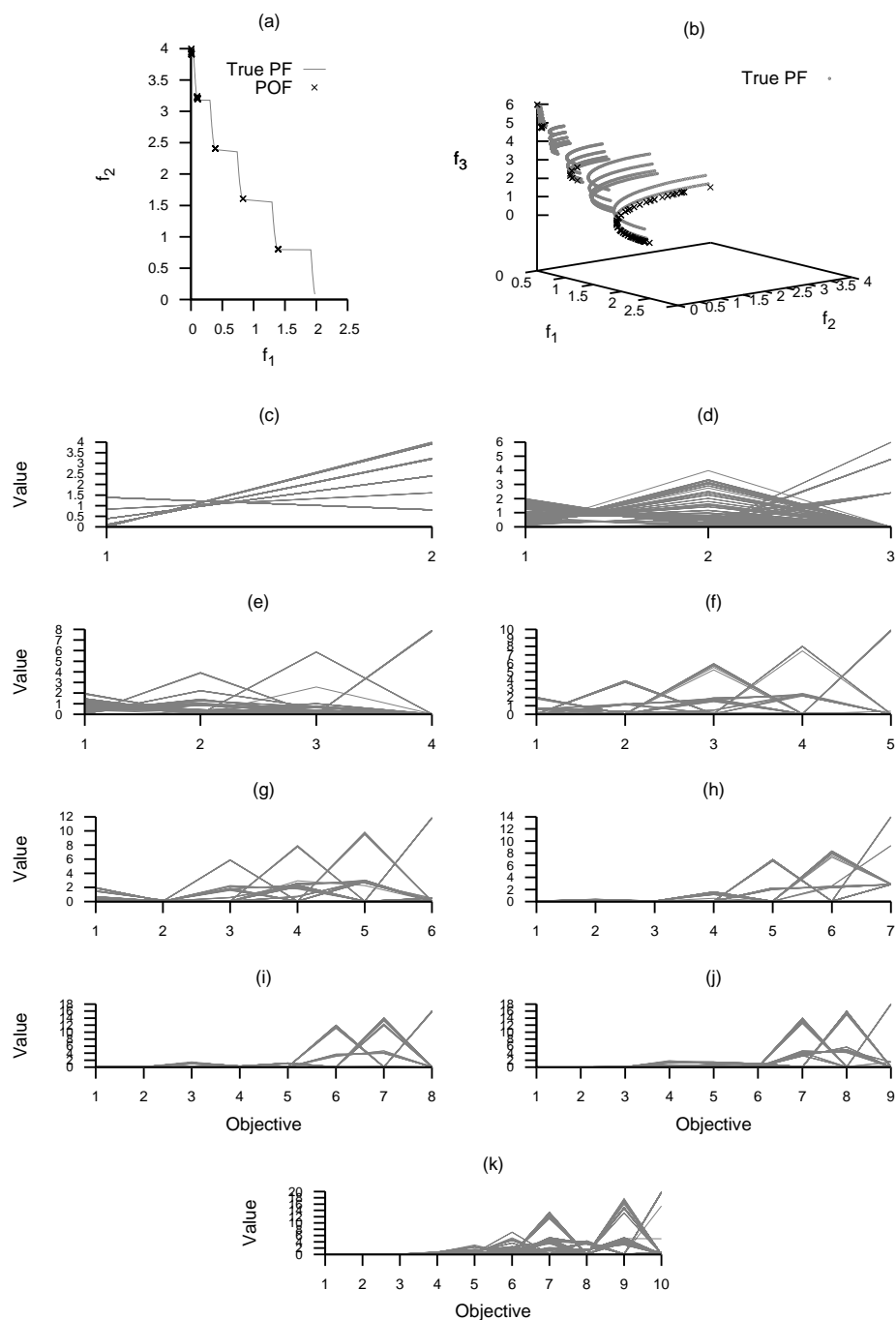


Figure A.339: Plots of the approximations obtained by MOMBI-WS from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



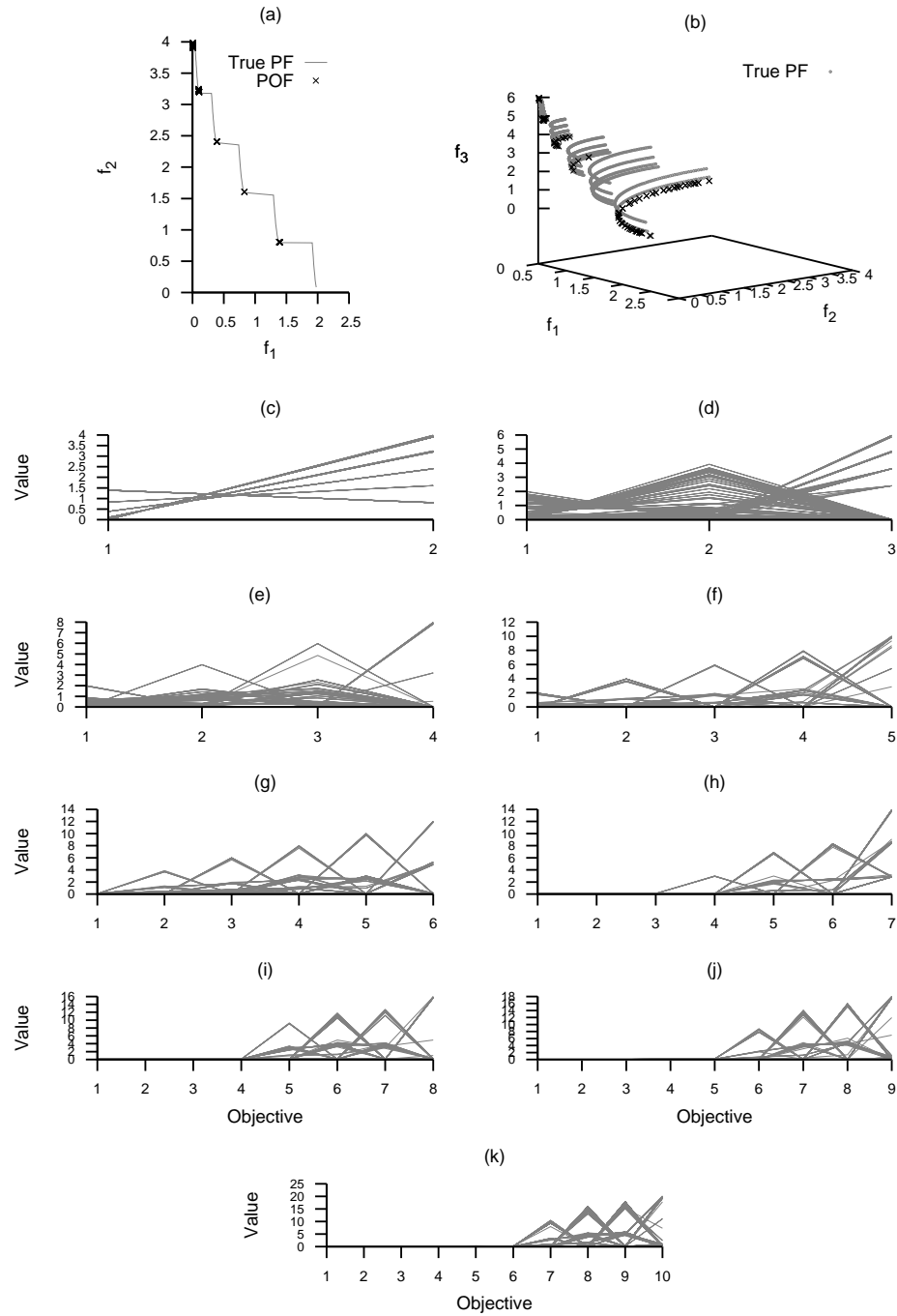


Figure A.340: Plots of the approximations obtained by MOMBI-NWS from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

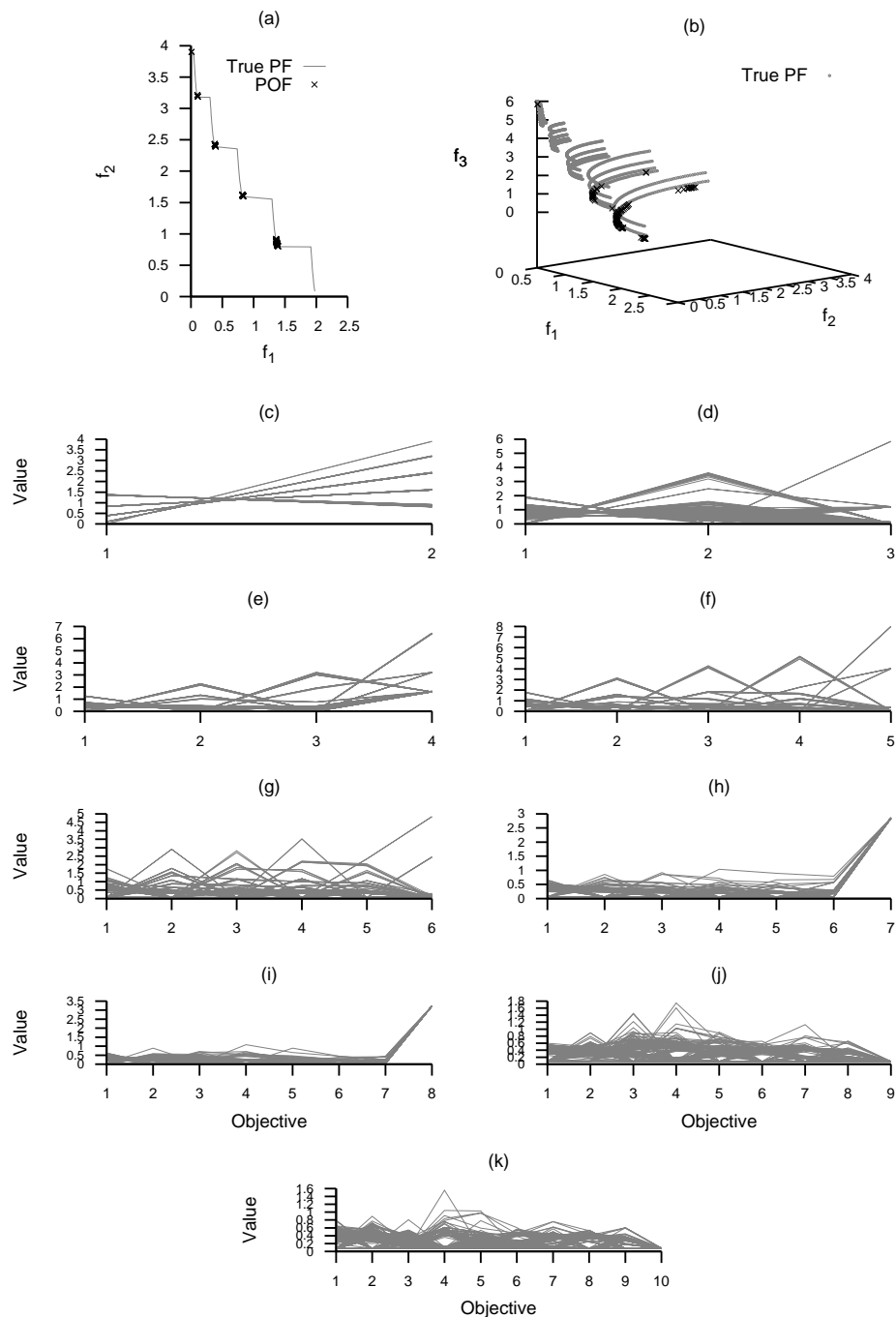


Figure A.341: Plots of the approximations obtained by MOMBI-LS from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

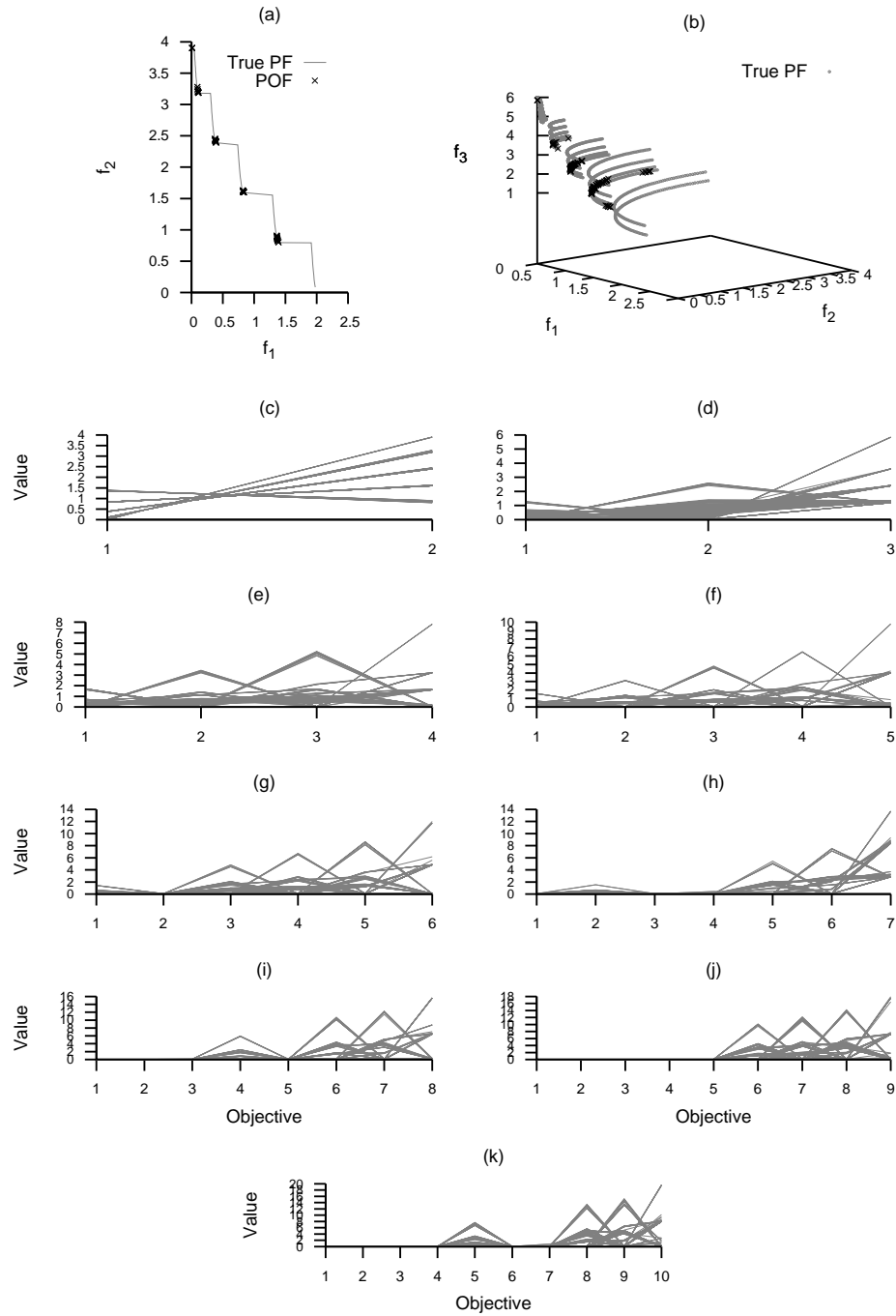


Figure A.342: Plots of the approximations obtained by MOMBI-NLS from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

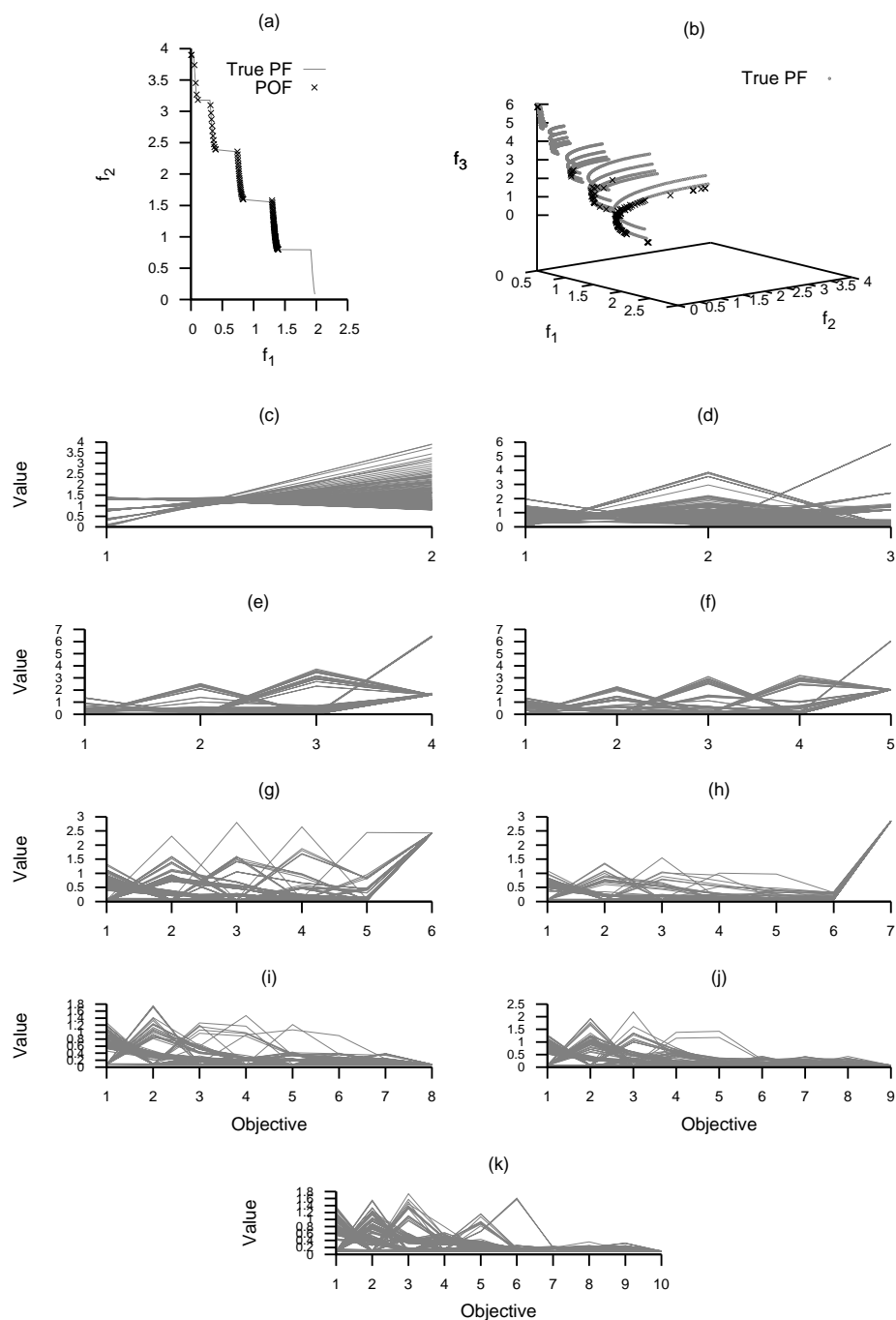


Figure A.343: Plots of the approximations obtained by MOMBI-ATCH from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

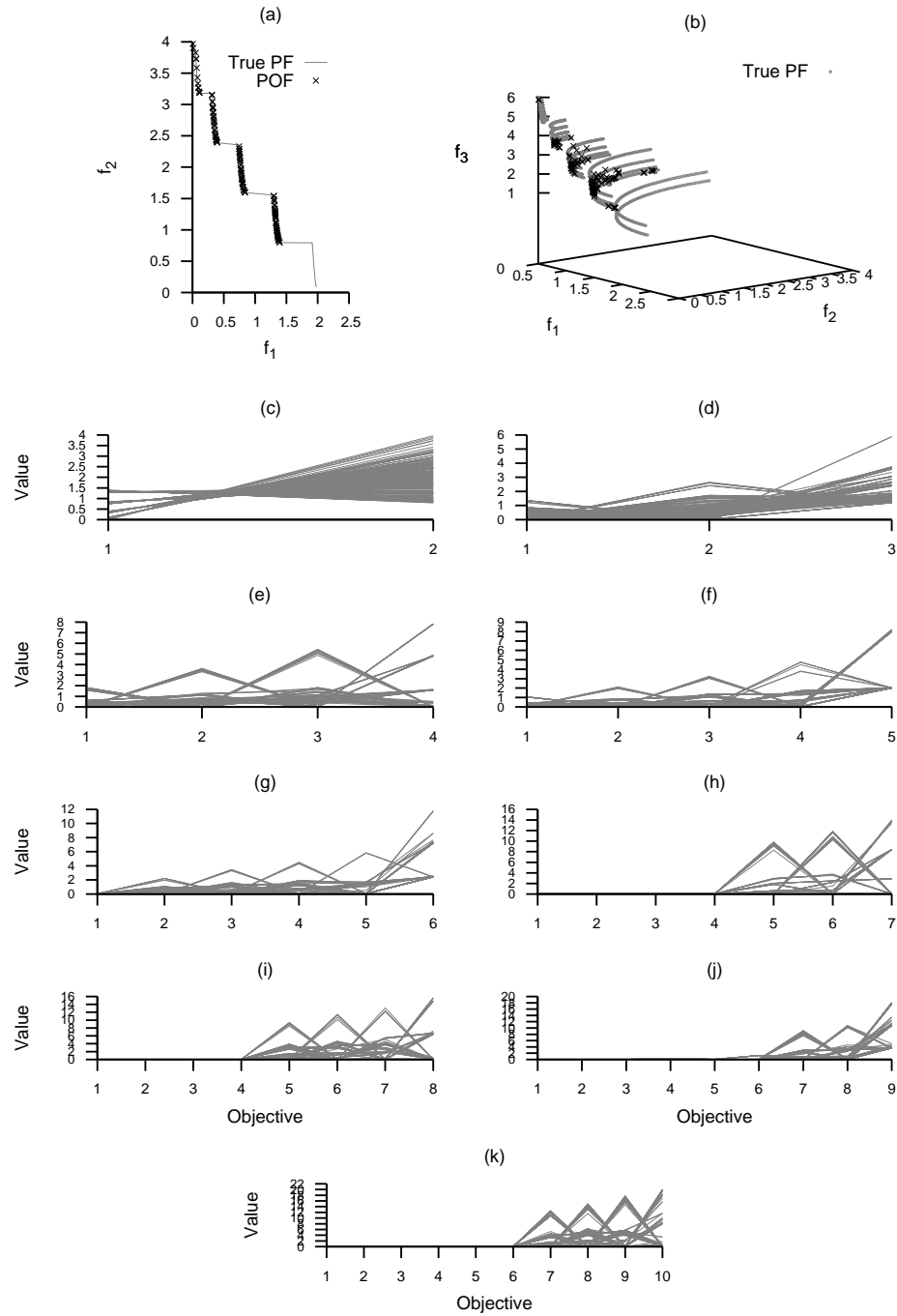


Figure A.344: Plots of the approximations obtained by MOMBI-NATCH from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

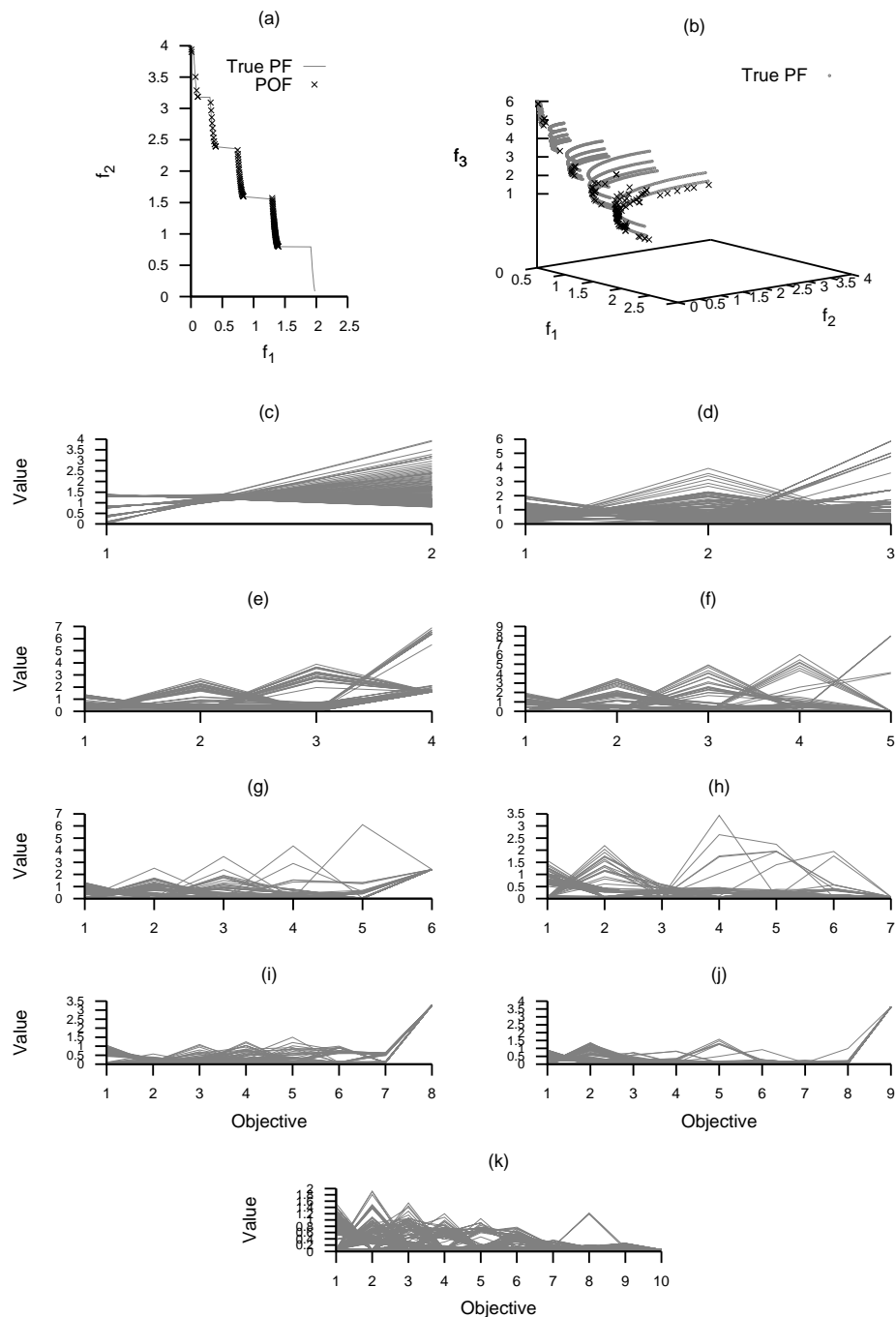


Figure A.345: Plots of the approximations obtained by MOMBI-MTCH from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

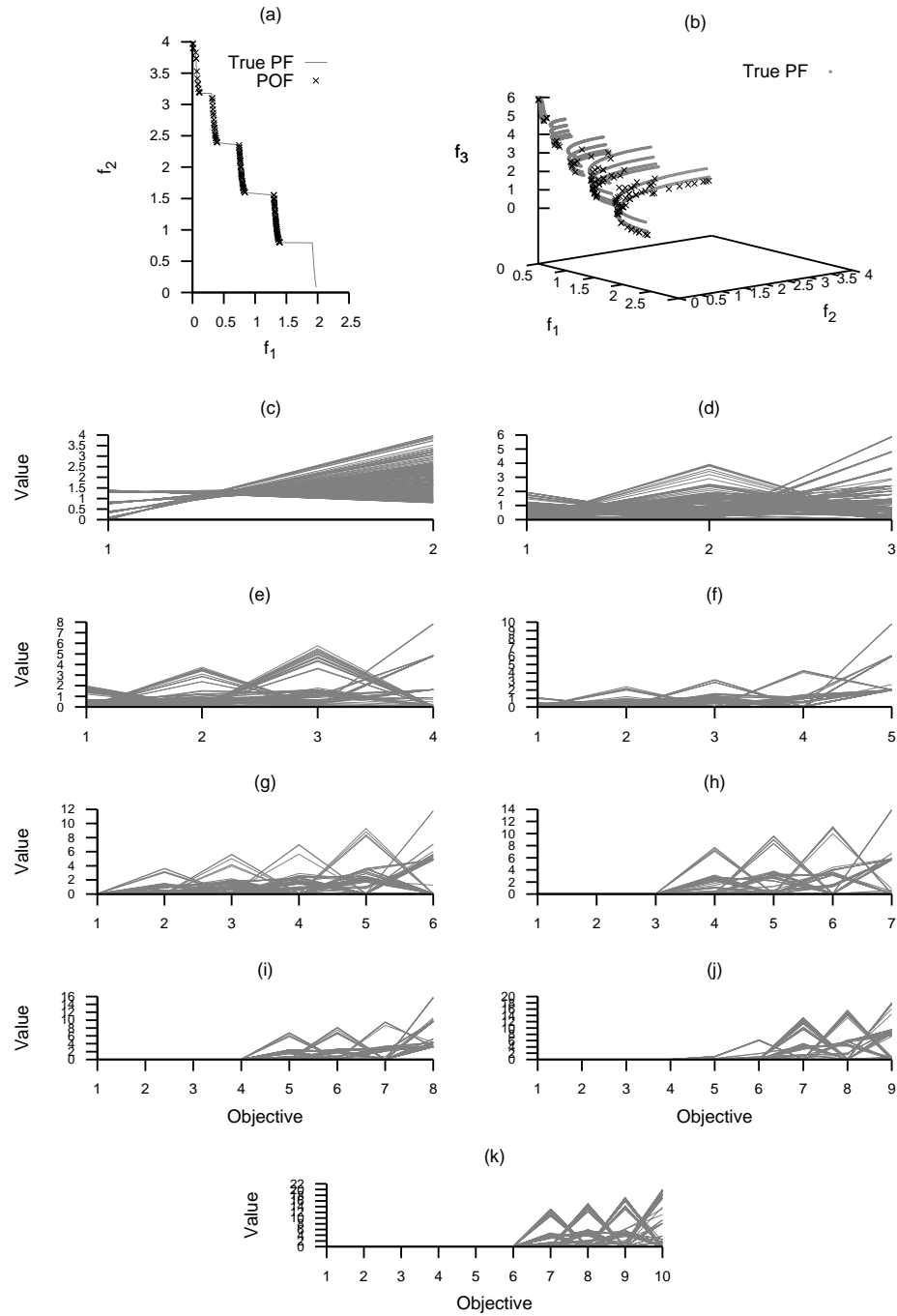


Figure A.346: Plots of the approximations obtained by MOMBI-NMTCH from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.143: Comparison of hypervolume indicator values for different optimizers on the WFG3 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	6.970247e+00	6.971384e+00	1.089540e+01	1.091793e+01	1.088980e+01	1.089360e+01	1.089331e+01	1.089532e+01	1.088989e+01	1.089325e+01	1.081841e+01
	avg.	6.979149e+00	7.000967e+00	1.089318e+01	1.091627e+01	1.088524e+01	1.089090e+01	1.089041e+01	1.089398e+01	1.088736e+01	1.089058e+01	1.080738e+01
	min.	6.885889e+00	6.887374e+00	1.081493e+01	1.087492e+01	1.071567e+01	1.075608e+01	1.081724e+01	1.078515e+01	1.080954e+01	1.082467e+01	1.047683e+01
	max.	7.255585e+00	8.612807e+00	1.092119e+01	1.093762e+01	1.092116e+01	1.091974e+01	1.092135e+01	1.092316e+01	1.092132e+01	1.092323e+01	1.087238e+01
	std.	5.094574e-02	1.720178e-01	1.505150e-02	1.401974e-02	2.486657e-02	2.140151e-02	1.812682e-02	1.942716e-02	1.678779e-02	1.762921e-02	5.062692e-02
3D	med.	5.105878e+01	5.098965e+01	7.073028e+01	7.519042e+01	7.411451e+01	7.526634e+01	7.430477e+01	7.532193e+01	7.439001e+01	7.530663e+01	7.299249e+01
	avg.	5.112210e+01	5.110577e+01	7.070865e+01	7.508941e+01	7.408124e+01	7.522820e+01	7.423985e+01	7.528672e+01	7.436567e+01	7.528093e+01	7.282192e+01
	min.	5.083001e+01	5.077782e+01	7.000198e+01	7.399130e+01	7.304570e+01	7.473235e+01	7.327785e+01	7.469302e+01	7.332560e+01	7.449106e+01	6.959113e+01
	max.	5.207910e+01	5.262907e+01	7.106126e+01	7.563916e+01	7.462066e+01	7.567428e+01	7.472038e+01	7.572674e+01	7.508470e+01	7.565783e+01	7.367372e+01
	std.	2.061387e-01	3.355935e-01	2.139513e-01	3.547286e-01	3.205914e-01	1.932683e-01	3.184802e-01	1.886559e-01	3.450136e-01	2.101410e-01	6.779435e-01
4D	med.	4.694192e+02	4.714610e+02	5.382936e+02	6.024989e+02	5.722443e+02	6.345222e+02	5.936928e+02	6.413649e+02	6.006538e+02	6.390634e+02	5.985129e+02
	avg.	4.628040e+02	4.669676e+02	5.386819e+02	6.032358e+02	5.741271e+02	6.349053e+02	5.898562e+02	6.403785e+02	5.980441e+02	6.390560e+02	5.981122e+02
	min.	4.429692e+02	4.425311e+02	5.254847e+02	5.864074e+02	5.527301e+02	6.210114e+02	5.573405e+02	6.170004e+02	5.666477e+02	6.236831e+02	5.545040e+02
	max.	4.915787e+02	4.864409e+02	5.517058e+02	6.351071e+02	6.127123e+02	6.483249e+02	6.110499e+02	6.513212e+02	6.129477e+02	6.534285e+02	6.175652e+02
	std.	1.274820e+01	1.237341e+01	5.382308e+00	8.509613e+00	1.204015e+01	5.069671e+00	1.391468e+01	6.001551e+00	1.032568e+01	5.739153e+00	9.263141e+00
5D	med.	5.324453e+03	5.286201e+03	5.586300e+03	5.862344e+03	5.579525e+03	6.308030e+03	5.659212e+03	6.622069e+03	5.692147e+03	6.447966e+03	5.809780e+03
	avg.	5.322810e+03	5.283972e+03	5.593345e+03	5.855369e+03	5.586117e+03	6.306428e+03	5.663506e+03	6.607882e+03	5.706460e+03	6.465885e+03	5.824558e+03
	min.	5.103029e+03	4.848680e+03	5.224399e+03	5.611128e+03	5.400584e+03	6.164653e+03	5.415577e+03	6.326105e+03	5.362841e+03	6.261650e+03	5.546812e+03
	max.	5.946204e+03	5.996461e+03	5.921159e+03	6.008231e+03	5.787423e+03	6.450021e+03	6.053716e+03	6.793033e+03	6.097230e+03	6.784421e+03	6.057912e+03
	std.	1.139776e+02	1.960849e+02	1.481900e+02	6.983915e+01	8.308132e+01	5.069640e+01	1.079257e+02	1.074186e+02	1.181978e+02	1.054451e+02	1.104783e+02
6D	med.	6.206874e+04	6.887732e+04	7.943131e+04	7.412013e+04	6.510934e+04	7.774458e+04	6.584108e+04	8.099620e+04	6.551004e+04	7.955271e+04	6.759501e+04
	avg.	6.192563e+04	6.912568e+04	7.928507e+04	7.417649e+04	6.498220e+04	7.794846e+04	6.595815e+04	8.085054e+04	6.569771e+04	7.946553e+04	6.761730e+04
	min.	5.529353e+04	6.477412e+04	7.518488e+04	7.106397e+04	6.090417e+04	7.506293e+04	6.140547e+04	7.740659e+04	6.130193e+04	7.634331e+04	6.515817e+04
	max.	6.570862e+04	7.256537e+04	8.281259e+04	8.053217e+04	6.768832e+04	8.095279e+04	7.214239e+04	8.477602e+04	7.380012e+04	8.243749e+04	7.112268e+04
	std.	2.042941e+03	2.018242e+03	1.722940e+03	1.561185e+03	1.493638e+03	1.112927e+03	1.614443e+03	1.310499e+03	1.964073e+03	1.146842e+03	1.244244e+03
7D	med.	8.438209e+05	9.957373e+05	1.180752e+06	1.045293e+06	8.954888e+05	1.062717e+06	9.021953e+05	1.121405e+06	8.921268e+05	1.098228e+06	9.167028e+05
	avg.	8.407856e+05	9.939616e+05	1.182682e+06	1.043285e+06	8.962556e+05	1.063823e+06	9.030190e+05	1.123105e+06	8.998201e+05	1.097183e+06	8.890790e+05
	min.	7.179156e+05	9.325973e+05	1.126885e+06	9.804649e+05	8.539183e+05	9.958999e+05	8.394981e+05	1.067499e+06	8.432903e+05	1.031755e+06	5.354910e+05
	max.	1.020297e+06	1.049200e+06	1.242905e+06	1.100853e+06	9.513333e+05	1.160632e+06	1.035173e+06	1.163458e+06	1.064430e+06	1.176312e+06	9.802554e+05
	std.	4.273224e+04	2.118203e+04	2.427998e+04	2.421006e+04	2.030052e+04	2.394368e+04	3.139049e+04	1.935589e+04	3.975476e+04	2.527409e+04	8.517975e+04
8D	med.	1.397285e+07	1.668490e+07	2.016966e+07	1.767961e+07	1.493913e+07	1.780645e+07	1.518354e+07	1.900328e+07	1.782742e+07	1.852383e+07	1.203149e+07
	avg.	1.401347e+07	1.662724e+07	2.020112e+07	1.769183e+07	1.498363e+07	1.777980e+07	1.541872e+07	1.897427e+07	1.775953e+07	1.852245e+07	1.178342e+07
	min.	1.271252e+07	1.584057e+07	1.923637e+07	1.687865e+07	1.392815e+07	1.709042e+07	1.403360e+07	1.815028e+07	1.493251e+07	1.765596e+07	7.877099e+06
	max.	1.515287e+07	1.757343e+07	2.122104e+07	1.904948e+07	1.625886e+07	1.848919e+07	1.785691e+07	1.965516e+07	1.864848e+07	1.956241e+07	1.387171e+07
	std.	6.013349e+05	3.826242e+05	3.813282e+05	4.449571e+05	3.742424e+05	3.293823e+05	9.161382e+05	3.075036e+05	5.197356e+05	3.980961e+05	1.250191e+06
9D	med.	2.631264e+08	3.157233e+08	3.826580e+08	3.544835e+08	2.835329e+08	3.349791e+08	2.848453e+08	3.581556e+08	3.326492e+08	3.470076e+08	2.068661e+08
	avg.	2.640466e+08	3.156705e+08	3.824989e+08	3.504331e+08	2.844417e+08	3.352725e+08	2.920202e+08	3.588692e+08	3.283750e+08	3.474677e+08	2.044578e+08
	min.	2.390019e+08	2.947329e+08	3.639239e+08	3.183150e+08	2.694996e+08	3.182389e+08	2.637599e+08	3.486800e+08	2.806599e+08	3.302467e+08	1.211687e+08
	max.	3.144541e+08	3.367804e+08	3.994792e+08	3.768973e+08	3.069435e+08	3.523938e+08	3.426882e+08	3.741501e+08	3.522896e+08	3.645310e+08	2.424188e+08
	std.	1.113816e+07	8.051723e+06	5.884172e+06	1.434377e+07	8.035404e+06	6.199772e+06	2.066867e+07	5.203331e+06	1.651072e+07	7.210559e+06	1.946046e+07
10D	med.	5.620161e+09	6.607577e+09	7.932648e+09	7.339175e+09	5.949703e+09	7.064959e+09	5.863587e+09	7.581964e+09	6.790208e+09	7.259666e+09	4.050459e+09
	avg.	5.873790e+09	6.699780e+09	7.913712e+09	7.362969e+09	6.068470e+09	7.098739e+09	5.954717e+09	7.581306e+09	6.621161e+09	7.241152e+09	3.919545e+09
	min.	4.754411e+09	6.177903e+09	7.521696e+09	6.811567e+09	5.551105e+09	6.673112e+09	5.341718e+09	7.283548e+09	5.578727e+09	6.804463e+09	2.189048e+09
	max.	6.830063e+09	7.506610e+09	8.184002e+09	7.718515e+09	7.053473e+09	7.586001e+09	7.040715e+09	7.850457e+09	7.412207e+09	7.587001e+09	4.525077e+09
	std.	5.859032e+08	2.948654e+08	1.299263e+08	1.681112e+08	3.674212e+08	2.159556e+08	3.277599e+08	1.096972e+08	4.824305e+08	1.515130e+08	4.788430e+08



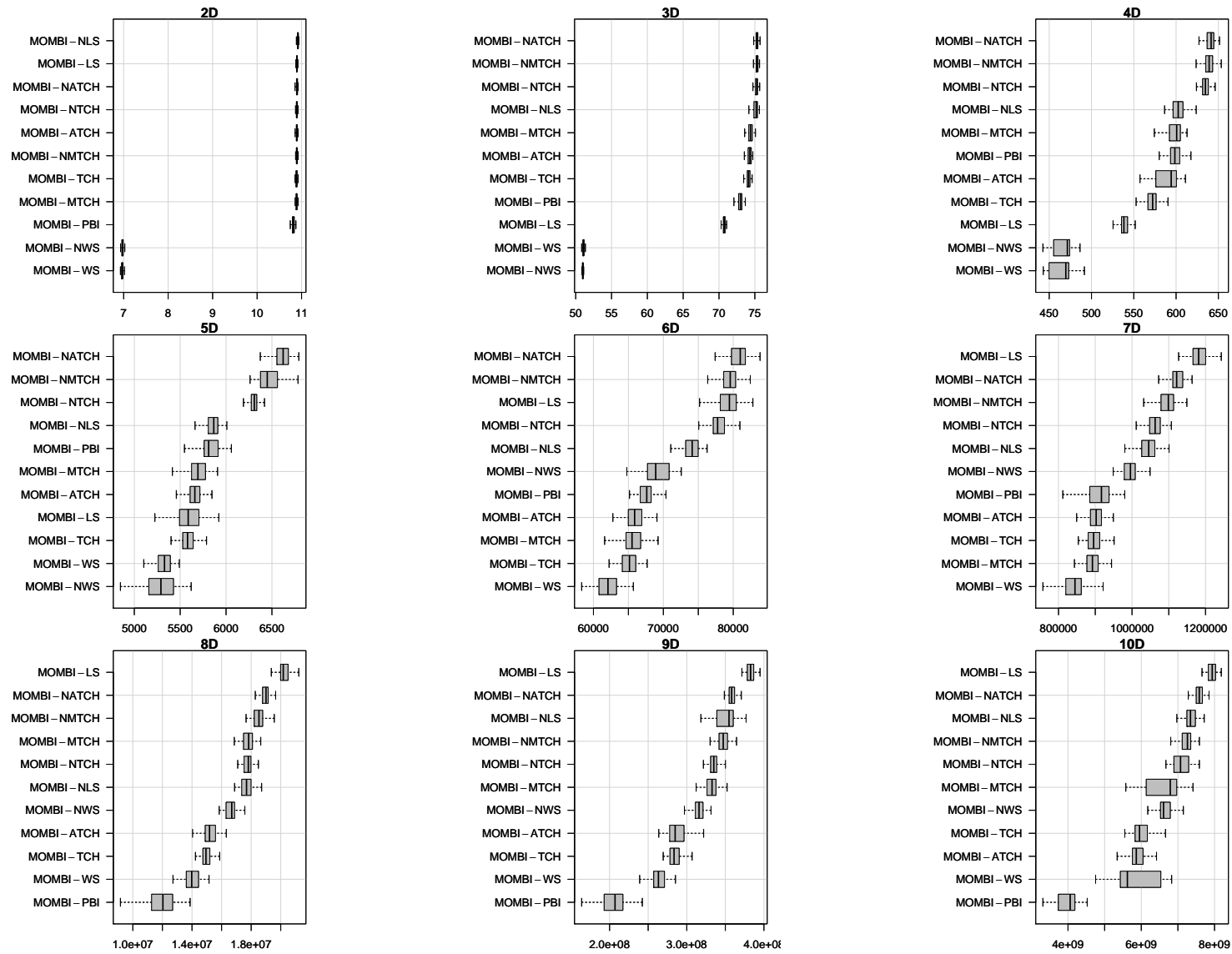


Figure A.347: Box-plot of the hypervolume indicator values for different optimizers on the WFG3 test problem.

Table A.144: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG3 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
2D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	> 0.05	$8.25e-04$	> 0.05	> 0.05	> 0.05	$1.73e-03$	> 0.05	$2.17e-33$
MOMBI-NLS	$1.28e-34$	$1.28e-34$	$1.06e-21$	–	$1.34e-24$	$3.61e-21$	$1.05e-21$	$2.11e-17$	$1.58e-24$	$5.12e-22$	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.27e-31$
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$8.98e-03$	–	> 0.05	> 0.05	$1.68e-02$	> 0.05	$3.00e-32$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$2.40e-02$	> 0.05	–	> 0.05	$3.39e-02$	> 0.05	$7.01e-33$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$9.77e-04$	> 0.05	> 0.05	–	$1.19e-03$	> 0.05	$2.66e-33$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	$3.37e-33$
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$2.21e-02$	> 0.05	> 0.05	> 0.05	$3.33e-02$	–	$6.81e-33$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
3D											
MOMBI-WS	–	$4.33e-07$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	$1.30e-31$	> 0.05	$5.53e-29$	> 0.05	$4.77e-25$	> 0.05	$1.28e-34$
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$4.93e-33$
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.06e-02$	$1.28e-34$	–	$1.28e-34$	> 0.05	$1.10e-33$	> 0.05	$1.28e-34$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	$7.21e-05$	> 0.05	–	> 0.05	> 0.05	> 0.05	$4.50e-34$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$3.39e-05$	$1.28e-34$	$1.36e-02$	$1.32e-34$	–	$5.22e-34$	> 0.05	$1.28e-34$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	$2.45e-09$	> 0.05	$4.99e-03$	> 0.05	–	> 0.05	$3.76e-34$
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$3.42e-05$	$1.49e-34$	$1.46e-02$	$2.96e-34$	> 0.05	$2.58e-33$	–	$1.28e-34$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$5.74e-31$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
4D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	$1.27e-02$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	$2.16e-28$	> 0.05	$9.05e-12$	> 0.05	$2.54e-03$	> 0.05	$3.85e-04$
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$6.63e-34$	$1.28e-34$	–	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	$1.70e-12$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.55e-34$	$1.28e-34$	$1.98e-11$	$1.28e-34$	–	$1.28e-34$	$3.65e-02$	$1.32e-34$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	$4.78e-24$	> 0.05	$6.91e-06$	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.79e-34$	$1.28e-34$	$1.67e-07$	$1.28e-34$	> 0.05	$1.28e-34$	–	$1.28e-34$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	$1.04e-24$	> 0.05	$2.99e-05$	> 0.05	> 0.05	> 0.05	–

Table A.145: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG3 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
5D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$6.56e-27$	$2.37e-23$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$2.04e-33$	$2.44e-33$	$7.24e-28$	–	$1.92e-33$	> 0.05	$3.49e-26$	> 0.05	$1.44e-19$	> 0.05	$1.52e-02$
MOMBI-TCH	$2.75e-32$	$2.67e-27$	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$
MOMBI-ATCH	$8.60e-33$	$6.43e-31$	$2.18e-04$	> 0.05	$1.28e-08$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$3.90e-33$	$1.28e-34$	–	$1.28e-34$	$2.41e-16$	$1.28e-34$
MOMBI-MTCH	$1.05e-32$	$1.01e-31$	$2.81e-08$	> 0.05	$2.91e-14$	> 0.05	$2.96e-03$	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$4.89e-25$	$1.28e-34$	> 0.05	$1.28e-34$	–	$1.28e-34$
MOMBI-PBI	$1.61e-33$	$3.08e-33$	$3.42e-22$	> 0.05	$5.09e-29$	> 0.05	$5.27e-19$	> 0.05	$6.47e-12$	> 0.05	–
6D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	$1.78e-34$	–	> 0.05	> 0.05	$3.77e-29$	> 0.05	$1.06e-21$	> 0.05	$5.15e-23$	> 0.05	$1.26e-08$
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$1.42e-31$	$1.28e-34$	$1.37e-09$	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$
MOMBI-NLS	$1.28e-34$	$1.92e-33$	> 0.05	–	$1.28e-34$	> 0.05	$1.49e-34$	> 0.05	$1.43e-33$	> 0.05	$1.32e-34$
MOMBI-TCH	$2.28e-22$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	> 0.05	$4.35e-30$	$1.28e-34$	–	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$
MOMBI-ATCH	$3.77e-29$	> 0.05	> 0.05	> 0.05	$2.73e-05$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.58e-10$	$1.61e-33$	$1.28e-34$	$1.64e-28$	$1.28e-34$	–	$1.28e-34$	$1.31e-12$	$1.28e-34$
MOMBI-MTCH	$3.59e-26$	> 0.05	> 0.05	> 0.05	$5.99e-03$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	> 0.05	$5.05e-32$	$1.28e-34$	$6.30e-16$	$1.28e-34$	> 0.05	$1.28e-34$	–	$1.28e-34$
MOMBI-PBI	$1.58e-34$	> 0.05	> 0.05	> 0.05	$3.34e-25$	> 0.05	$3.19e-14$	> 0.05	$3.91e-16$	> 0.05	–
7D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	$1.92e-33$	–	> 0.05	> 0.05	$1.49e-34$	> 0.05	$5.58e-31$	> 0.05	$2.34e-28$	> 0.05	$1.05e-32$
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$1.28e-34$	$2.55e-34$	$1.28e-34$	$1.10e-31$	$1.28e-34$	$1.10e-33$	$1.28e-34$
MOMBI-NLS	$2.33e-34$	$4.25e-27$	> 0.05	–	$1.28e-34$	> 0.05	$1.48e-33$	> 0.05	$8.24e-32$	> 0.05	$1.28e-34$
MOMBI-TCH	$1.56e-23$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	$1.36e-34$	$3.08e-33$	> 0.05	$1.39e-08$	$1.28e-34$	–	$2.20e-34$	> 0.05	$2.91e-33$	> 0.05	$1.28e-34$
MOMBI-ATCH	$4.02e-24$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	$2.30e-02$	> 0.05	> 0.05
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	> 0.05	$5.07e-34$	$1.28e-34$	$2.70e-30$	$1.28e-34$	–	$1.28e-34$	$2.81e-13$	$1.28e-34$
MOMBI-MTCH	$1.06e-21$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$1.28e-34$	$1.36e-34$	> 0.05	$4.01e-28$	$1.28e-34$	$2.25e-17$	$1.36e-34$	> 0.05	$1.78e-34$	–	$1.28e-34$
MOMBI-PBI	$2.04e-13$	> 0.05	> 0.05	> 0.05	$1.76e-04$	> 0.05	$7.91e-03$	> 0.05	$5.85e-04$	> 0.05	–

Table A.146: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG3 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
8D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.69e-31$
MOMBI-NWS	$1.28e-34$	–	> 0.05	> 0.05	$2.71e-34$	> 0.05	$3.19e-19$	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$7.69e-34$	$1.28e-34$	$1.89e-34$	$1.28e-34$
MOMBI-NLS	$1.28e-34$	$4.70e-31$	> 0.05	–	$1.28e-34$	> 0.05	$2.71e-29$	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-TCH	$1.81e-24$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NTCH	$1.28e-34$	$3.57e-33$	> 0.05	$4.60e-02$	$1.28e-34$	–	$5.42e-31$	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-ATCH	$7.29e-27$	> 0.05	> 0.05	> 0.05	$2.26e-04$	> 0.05	–	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	> 0.05	$5.39e-33$	$1.28e-34$	$5.38e-34$	$1.28e-34$	–	$2.66e-33$	$6.17e-15$	$1.28e-34$
MOMBI-MTCH	$1.53e-34$	$2.86e-30$	> 0.05	$4.83e-02$	$6.24e-34$	> 0.05	$8.96e-30$	> 0.05	–	> 0.05	$1.28e-34$
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	> 0.05	$7.91e-25$	$1.28e-34$	$1.68e-25$	$1.53e-34$	> 0.05	$8.97e-23$	–	$1.28e-34$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
9D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.44e-34$
MOMBI-NWS	$4.37e-34$	–	> 0.05	> 0.05	$3.88e-34$	> 0.05	$3.27e-15$	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$4.24e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.79e-34$	$1.28e-34$	$1.32e-34$	$1.28e-34$
MOMBI-NLS	$1.28e-34$	$9.24e-32$	> 0.05	–	$1.28e-34$	$2.70e-14$	$3.25e-31$	> 0.05	$1.75e-17$	$1.64e-03$	$1.28e-34$
MOMBI-TCH	$3.41e-28$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NTCH	$1.28e-34$	$1.13e-31$	> 0.05	> 0.05	$1.28e-34$	–	$7.85e-28$	> 0.05	$4.58e-03$	> 0.05	$1.28e-34$
MOMBI-ATCH	$9.24e-27$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	> 0.05	$2.38e-05$	$1.28e-34$	$1.89e-34$	$1.28e-34$	–	$2.20e-34$	$9.65e-23$	$1.28e-34$
MOMBI-MTCH	$2.88e-34$	$1.53e-15$	> 0.05	> 0.05	$1.27e-30$	> 0.05	$4.66e-22$	> 0.05	–	> 0.05	$1.28e-34$
MOMBI-NMTCH	$1.28e-34$	$1.68e-34$	> 0.05	> 0.05	$1.28e-34$	$5.15e-23$	$1.00e-33$	> 0.05	$3.92e-24$	–	$1.28e-34$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
10D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NWS	$3.76e-18$	–	> 0.05	> 0.05	$3.74e-21$	> 0.05	$7.69e-27$	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$3.05e-34$	$1.28e-34$	$1.36e-34$	$1.28e-34$	$5.74e-31$	$1.28e-34$	$1.49e-34$	$1.28e-34$
MOMBI-NLS	$1.36e-34$	$6.00e-28$	> 0.05	–	$1.73e-34$	$4.12e-15$	$1.73e-34$	> 0.05	$2.48e-30$	$3.33e-07$	$1.28e-34$
MOMBI-TCH	$1.17e-04$	> 0.05	> 0.05	> 0.05	–	> 0.05	$7.50e-03$	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NTCH	$2.63e-34$	$2.98e-19$	> 0.05	> 0.05	$3.68e-32$	–	$1.54e-32$	> 0.05	$2.32e-14$	> 0.05	$1.28e-34$
MOMBI-ATCH	$3.83e-03$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NATCH	$1.28e-34$	$4.37e-34$	> 0.05	$2.61e-18$	$1.28e-34$	$4.50e-32$	$1.28e-34$	–	$1.63e-34$	$1.20e-30$	$1.28e-34$
MOMBI-MTCH	$3.84e-18$	> 0.05	> 0.05	> 0.05	$3.50e-14$	> 0.05	$2.30e-18$	> 0.05	–	> 0.05	$1.28e-34$
MOMBI-NMTCH	$1.36e-34$	$8.45e-24$	> 0.05	> 0.05	$2.48e-34$	$9.99e-07$	$2.48e-34$	> 0.05	$1.30e-25$	–	$1.28e-34$
MOMBI-PBI	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–

Table A.147: Comparison of R2 indicator values for different optimizers on the WFG3 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	6.676476e-01	6.676722e-01	4.587235e-01	4.582991e-01	4.591212e-01	4.591982e-01	4.588833e-01	4.591208e-01	4.591118e-01	4.592624e-01	4.619121e-01
	avg.	6.671923e-01	6.664575e-01	4.588475e-01	4.583959e-01	4.592408e-01	4.593375e-01	4.590132e-01	4.592273e-01	4.592454e-01	4.593990e-01	4.621818e-01
	min.	6.543027e-01	6.017109e-01	4.578143e-01	4.574806e-01	4.580598e-01	4.581273e-01	4.580782e-01	4.579733e-01	4.579390e-01	4.581936e-01	4.599101e-01
	max.	6.706484e-01	6.706640e-01	4.626426e-01	4.602465e-01	4.668922e-01	4.655664e-01	4.615620e-01	4.627373e-01	4.624352e-01	4.621127e-01	4.735162e-01
	std.	2.382734e-03	6.910468e-03	6.906247e-04	5.587570e-04	9.900968e-04	8.553352e-04	6.237240e-04	6.864157e-04	6.667537e-04	6.794058e-04	1.707719e-03
3D	med.	4.972236e-01	4.975803e-01	4.021997e-01	3.930533e-01	3.882033e-01	3.898306e-01	3.881898e-01	3.910385e-01	3.882618e-01	3.903345e-01	3.916800e-01
	avg.	4.969786e-01	4.972953e-01	4.023604e-01	3.933458e-01	3.883952e-01	3.899781e-01	3.883233e-01	3.909325e-01	3.883267e-01	3.905032e-01	3.926319e-01
	min.	4.925596e-01	4.927363e-01	3.996116e-01	3.892611e-01	3.866702e-01	3.874455e-01	3.864336e-01	3.880151e-01	3.863845e-01	3.876508e-01	3.894663e-01
	max.	4.977286e-01	4.978168e-01	4.078485e-01	3.977731e-01	3.911785e-01	3.933687e-01	3.910202e-01	3.952289e-01	3.913369e-01	3.947057e-01	4.129715e-01
	std.	7.887530e-04	8.776996e-04	1.435497e-03	1.883489e-03	9.267157e-04	1.147520e-03	9.751463e-04	1.434499e-03	9.702063e-04	1.431883e-03	3.415464e-03
4D	med.	4.073518e-01	4.061116e-01	3.803904e-01	3.764256e-01	3.667881e-01	3.579645e-01	3.631126e-01	3.584470e-01	3.612761e-01	3.576172e-01	3.627543e-01
	avg.	4.081573e-01	4.069594e-01	3.807629e-01	3.770064e-01	3.665611e-01	3.576248e-01	3.639387e-01	3.591209e-01	3.617150e-01	3.578889e-01	3.637996e-01
	min.	4.006137e-01	4.017445e-01	3.758642e-01	3.678385e-01	3.572999e-01	3.528923e-01	3.589223e-01	3.521412e-01	3.572427e-01	3.541489e-01	3.567969e-01
	max.	4.130470e-01	4.116678e-01	3.854040e-01	3.864697e-01	3.732302e-01	3.644682e-01	3.736364e-01	3.898289e-01	3.695186e-01	3.649278e-01	4.188515e-01
	std.	2.618015e-03	2.147755e-03	2.044097e-03	3.592871e-03	2.922082e-03	2.333914e-03	3.133200e-03	5.769400e-03	2.443320e-03	2.364883e-03	6.682516e-03
5D	med.	3.426331e-01	3.637253e-01	3.445770e-01	3.422053e-01	3.436094e-01	3.466239e-01	3.410270e-01	3.585279e-01	3.406034e-01	3.558277e-01	3.406397e-01
	avg.	3.428066e-01	3.618148e-01	3.445710e-01	3.427631e-01	3.435047e-01	3.468216e-01	3.411316e-01	3.611794e-01	3.412493e-01	3.558924e-01	3.406725e-01
	min.	3.375116e-01	3.422834e-01	3.362461e-01	3.349198e-01	3.331966e-01	3.361312e-01	3.353625e-01	3.380661e-01	3.361996e-01	3.394598e-01	3.307409e-01
	max.	3.489459e-01	3.808039e-01	3.541063e-01	3.600793e-01	3.504879e-01	3.598148e-01	3.469616e-01	3.963628e-01	3.520163e-01	3.746558e-01	3.543785e-01
	std.	2.314264e-03	1.025766e-02	2.653085e-03	4.356217e-03	3.144153e-03	5.931901e-03	2.441873e-03	1.208953e-02	3.039496e-03	6.850111e-03	5.272469e-03
6D	med.	3.610611e-01	3.637596e-01	3.536486e-01	3.609562e-01	3.587024e-01	3.730608e-01	3.546986e-01	4.231321e-01	3.565230e-01	3.848500e-01	3.391376e-01
	avg.	3.607069e-01	3.656526e-01	3.538461e-01	3.614486e-01	3.584836e-01	3.728731e-01	3.546339e-01	4.195583e-01	3.564309e-01	3.857328e-01	3.395094e-01
	min.	3.457236e-01	3.464677e-01	3.393240e-01	3.429828e-01	3.456057e-01	3.562330e-01	3.407951e-01	3.845277e-01	3.415812e-01	3.628748e-01	3.303237e-01
	max.	3.753970e-01	3.915716e-01	3.651305e-01	3.867000e-01	3.688185e-01	3.866025e-01	3.673271e-01	4.495104e-01	3.687201e-01	4.095048e-01	3.558782e-01
	std.	5.974853e-03	8.959700e-03	4.837184e-03	7.177911e-03	4.592302e-03	6.291690e-03	5.027933e-03	1.677406e-02	4.965912e-03	9.142938e-03	4.156572e-03
7D	med.	3.894488e-01	3.748312e-01	3.736147e-01	3.732455e-01	3.795245e-01	3.989292e-01	3.737849e-01	4.766867e-01	3.714095e-01	4.380477e-01	3.238635e-01
	avg.	3.898187e-01	3.756262e-01	3.737546e-01	3.735046e-01	3.805073e-01	4.003109e-01	3.738227e-01	4.775138e-01	3.717433e-01	4.401929e-01	3.366803e-01
	min.	3.497373e-01	3.520470e-01	3.555945e-01	3.545852e-01	3.619287e-01	3.693028e-01	3.487407e-01	4.254865e-01	3.504291e-01	4.124409e-01	3.173493e-01
	max.	4.282485e-01	4.033437e-01	3.929833e-01	4.040329e-01	4.057608e-01	4.306050e-01	3.993084e-01	5.409695e-01	3.946124e-01	4.700591e-01	4.452982e-01
	std.	1.867029e-02	1.052225e-02	7.641136e-03	9.166512e-03	1.018539e-02	1.345244e-02	8.874863e-03	1.952346e-02	8.776750e-03	1.441593e-02	2.798389e-02
8D	med.	3.967420e-01	3.797730e-01	3.739467e-01	3.764665e-01	3.786519e-01	4.002327e-01	3.753911e-01	4.933791e-01	3.668963e-01	4.382004e-01	3.717563e-01
	avg.	3.941190e-01	3.800161e-01	3.743479e-01	3.758979e-01	3.796338e-01	3.997689e-01	3.739966e-01	4.993652e-01	3.664795e-01	4.402337e-01	3.755839e-01
	min.	3.501085e-01	3.547754e-01	3.550967e-01	3.505242e-01	3.568604e-01	3.748131e-01	3.486062e-01	4.586507e-01	3.477123e-01	4.116366e-01	3.460318e-01
	max.	4.271520e-01	4.068818e-01	3.942398e-01	4.012333e-01	4.084501e-01	4.399145e-01	3.939299e-01	5.548505e-01	3.816346e-01	4.739894e-01	4.292478e-01
	std.	1.894159e-02	1.288067e-02	9.067267e-03	1.086477e-02	9.939004e-03	1.287841e-02	9.610904e-03	2.484136e-02	7.080882e-03	1.382320e-02	1.795801e-02
9D	med.	4.015420e-01	3.873178e-01	3.748568e-01	3.765312e-01	3.773151e-01	3.992521e-01	3.713393e-01	5.228590e-01	3.664218e-01	4.369517e-01	3.692804e-01
	avg.	3.981680e-01	3.858788e-01	3.744001e-01	3.763194e-01	3.778551e-01	3.988029e-01	3.719197e-01	5.173680e-01	3.667623e-01	4.377543e-01	3.701119e-01
	min.	3.542338e-01	3.575155e-01	3.498905e-01	3.505370e-01	3.567431e-01	3.702509e-01	3.535732e-01	4.672863e-01	3.386777e-01	4.127309e-01	3.403356e-01
	max.	4.312783e-01	4.104980e-01	3.916149e-01	4.053331e-01	4.014168e-01	4.259031e-01	3.917042e-01	5.363270e-01	3.932736e-01	4.636504e-01	4.059133e-01
	std.	1.910540e-02	1.288067e-02	8.786803e-03	1.171453e-02	1.063248e-02	1.133013e-02	8.293043e-03	1.473882e-02	8.551091e-03	1.068610e-02	1.588733e-02
10D	med.	3.914555e-01	3.859075e-01	3.744608e-01	3.786293e-01	3.720141e-01	3.938740e-01	3.718522e-01	5.263940e-01	3.692858e-01	4.422041e-01	3.580729e-01
	avg.	3.892836e-01	3.863224e-01	3.752870e-01	3.779288e-01	3.723443e-01	3.934945e-01	3.714919e-01	5.262107e-01	3.700183e-01	4.406103e-01	3.605561e-01
	min.	3.519878e-01	3.573849e-01	3.521226e-01	3.490716e-01	3.438797e-01	3.673169e-01	3.530980e-01	5.123372e-01	3.470204e-01	3.982402e-01	3.416184e-01
	max.	4.285307e-01	4.154142e-01	3.910310e-01	3.978395e-01	3.990315e-01	4.195039e-01	3.922179e-01	5.346568e-01	3.965502e-01	4.737928e-01	3.961750e-01
	std.	1.874112e-02	1.395513e-02	8.343062e-03	1.139727e-02	1.038353e-02	1.258148e-02	7.706418e-03	3.747567e-03	1.065798e-02	1.166494e-02	1.187466e-02

Figure A.348: Box-plot of the R2 indicator values for different optimizers on the WFG3 test problem.

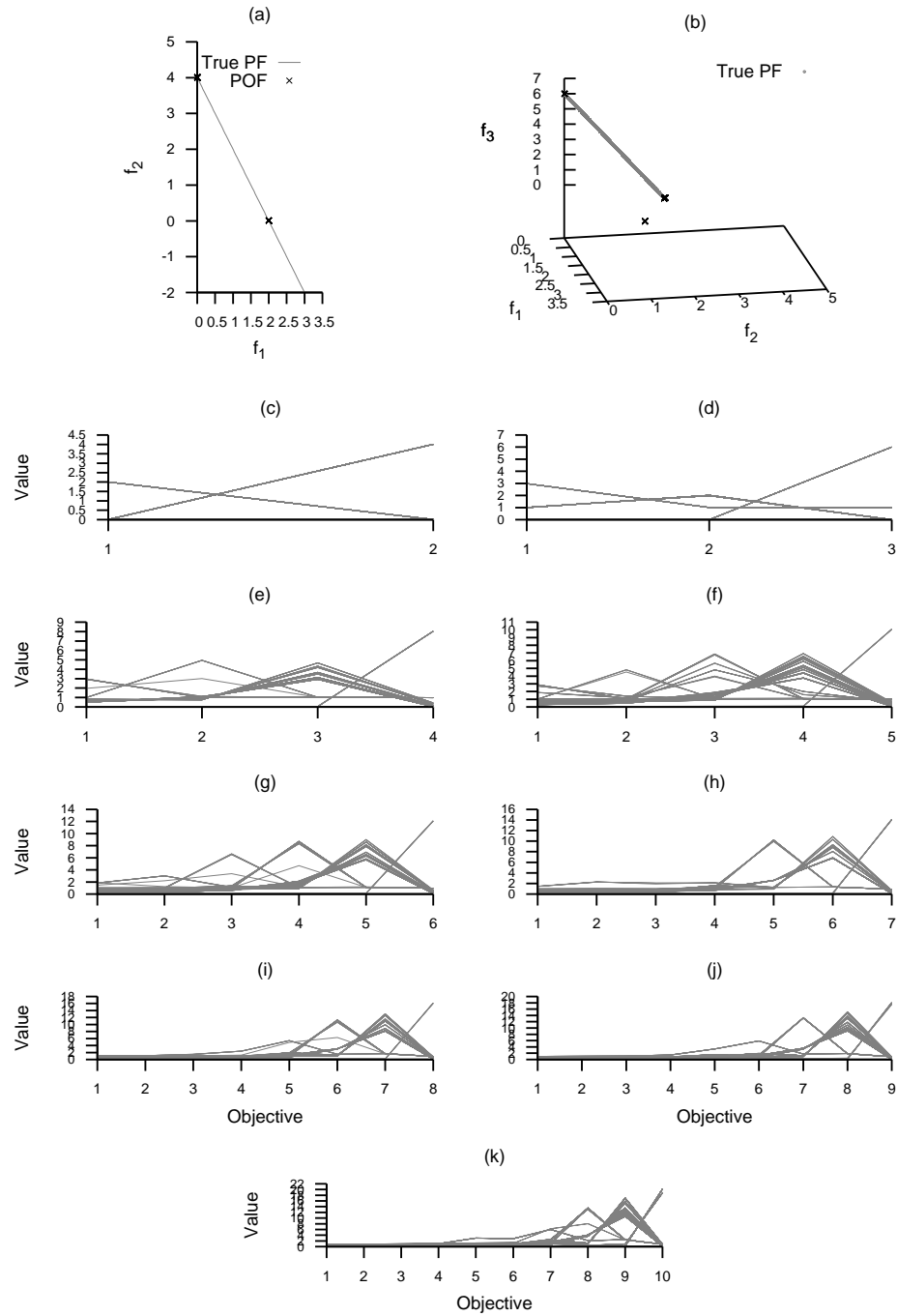


Figure A.349: Plots of the approximations obtained by MOMBI-WS from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

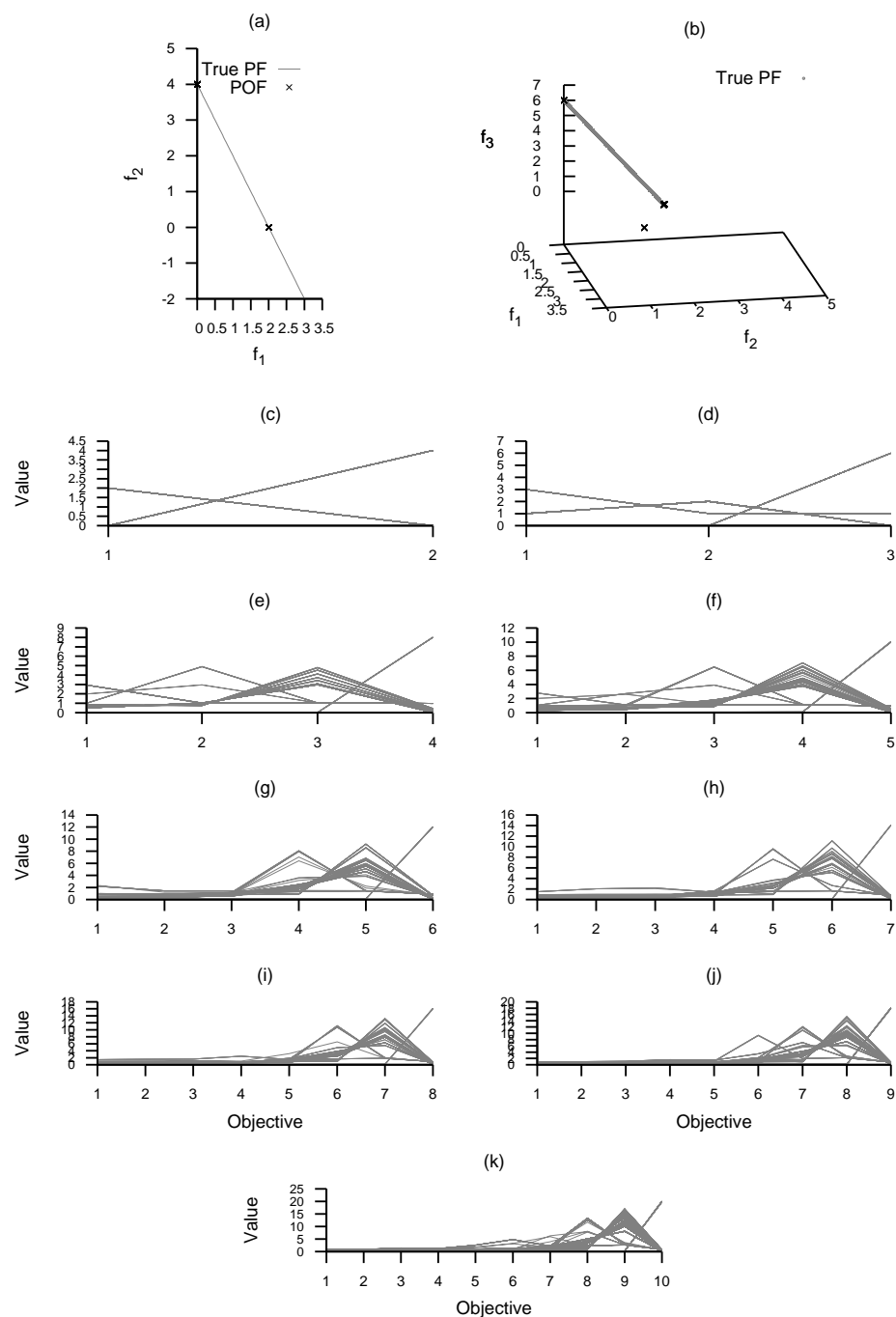


Figure A.350: Plots of the approximations obtained by MOMBI-NWS from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



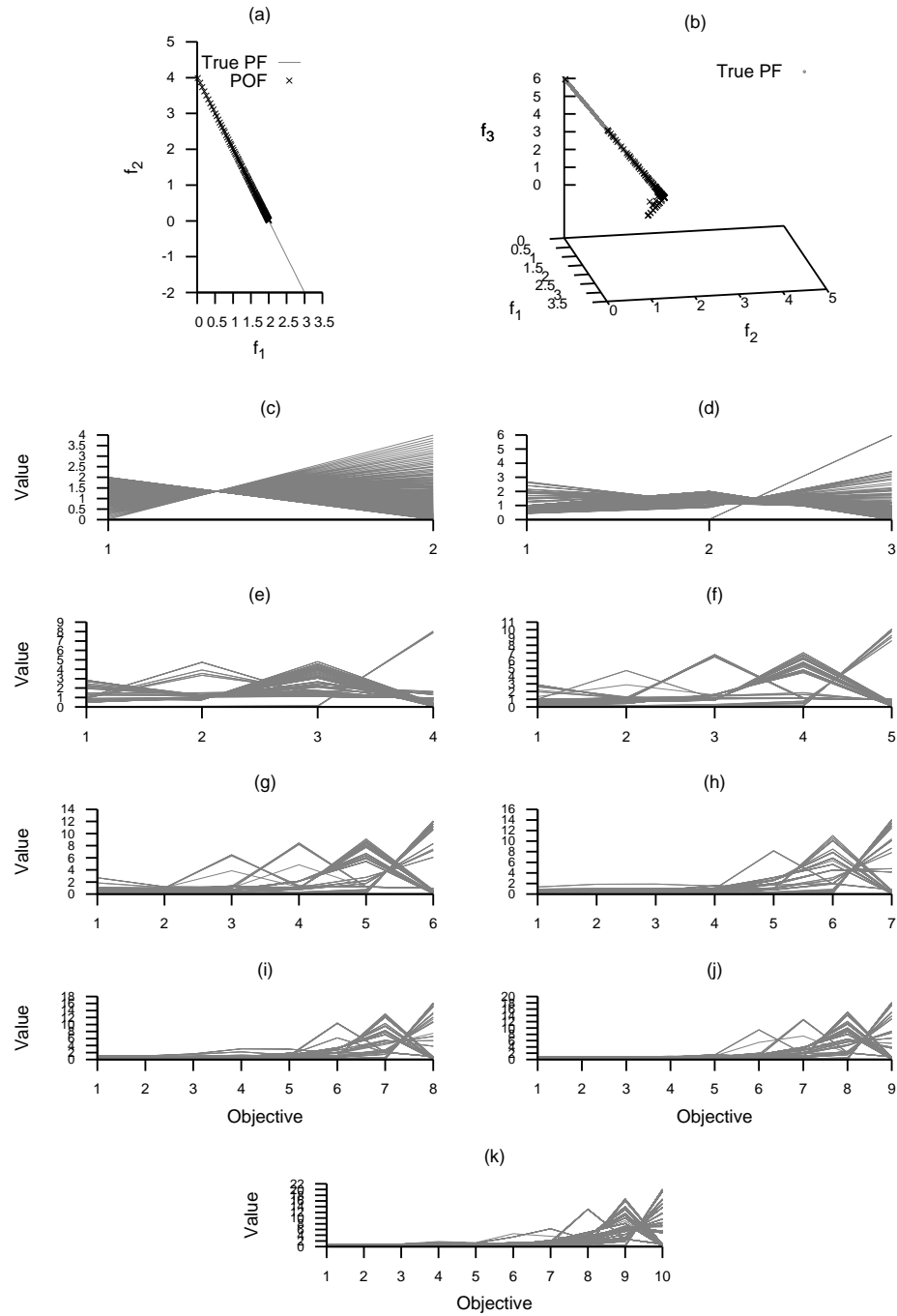


Figure A.351: Plots of the approximations obtained by MOMBI-LS from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

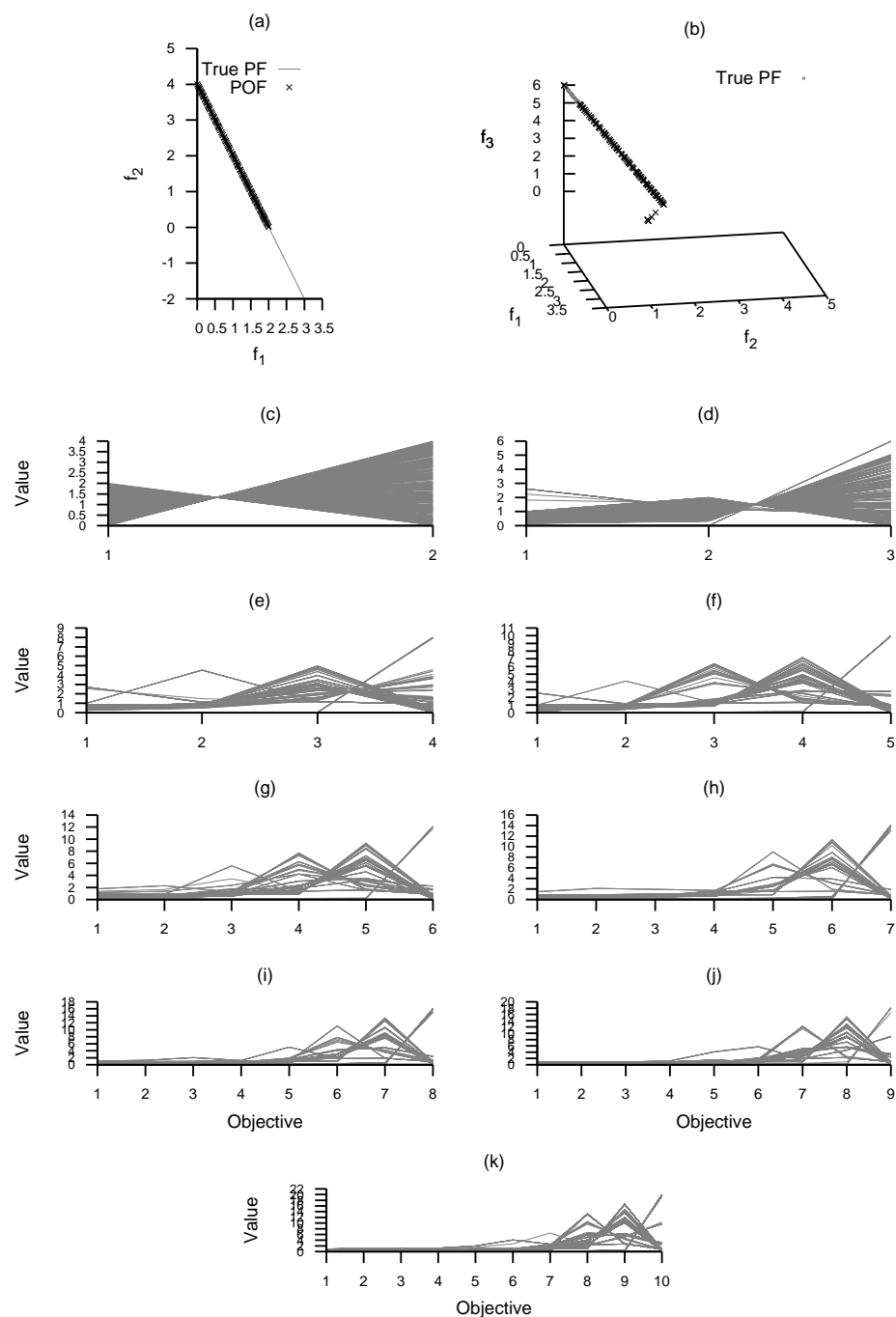


Figure A.352: Plots of the approximations obtained by MOMBI-NLS from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

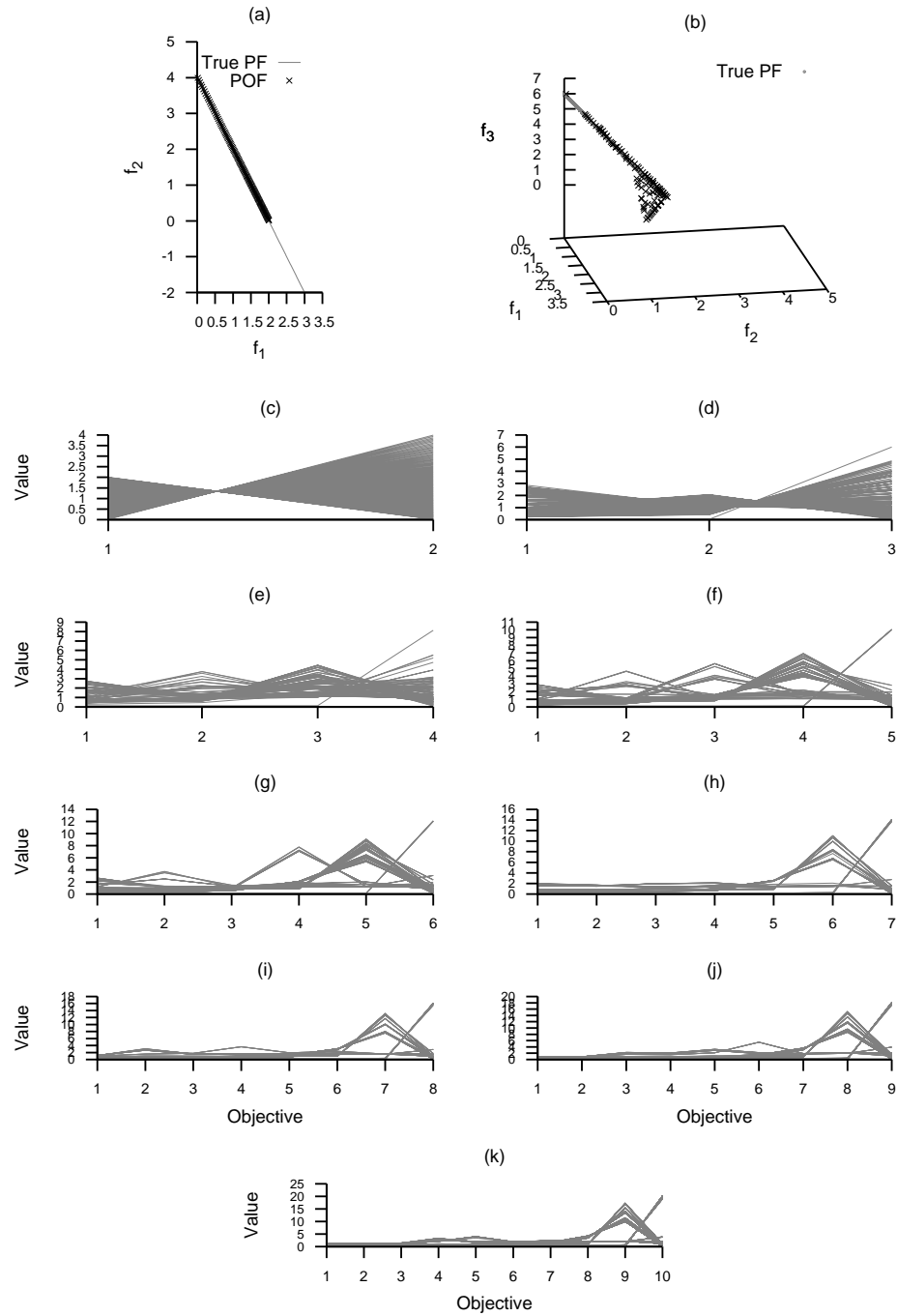


Figure A.353: Plots of the approximations obtained by MOMBI-ATCH from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

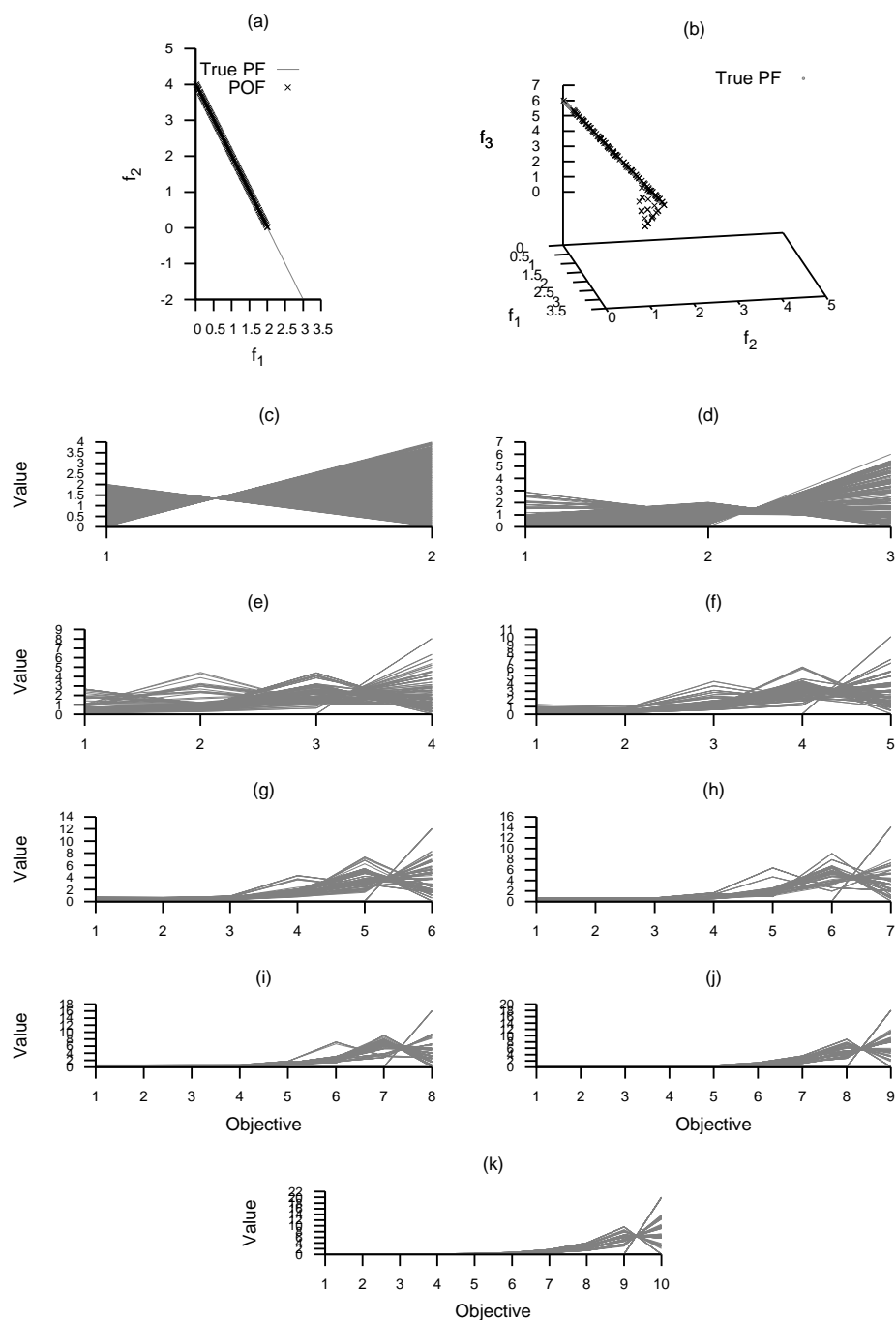


Figure A.354: Plots of the approximations obtained by MOMBI-NATCH from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

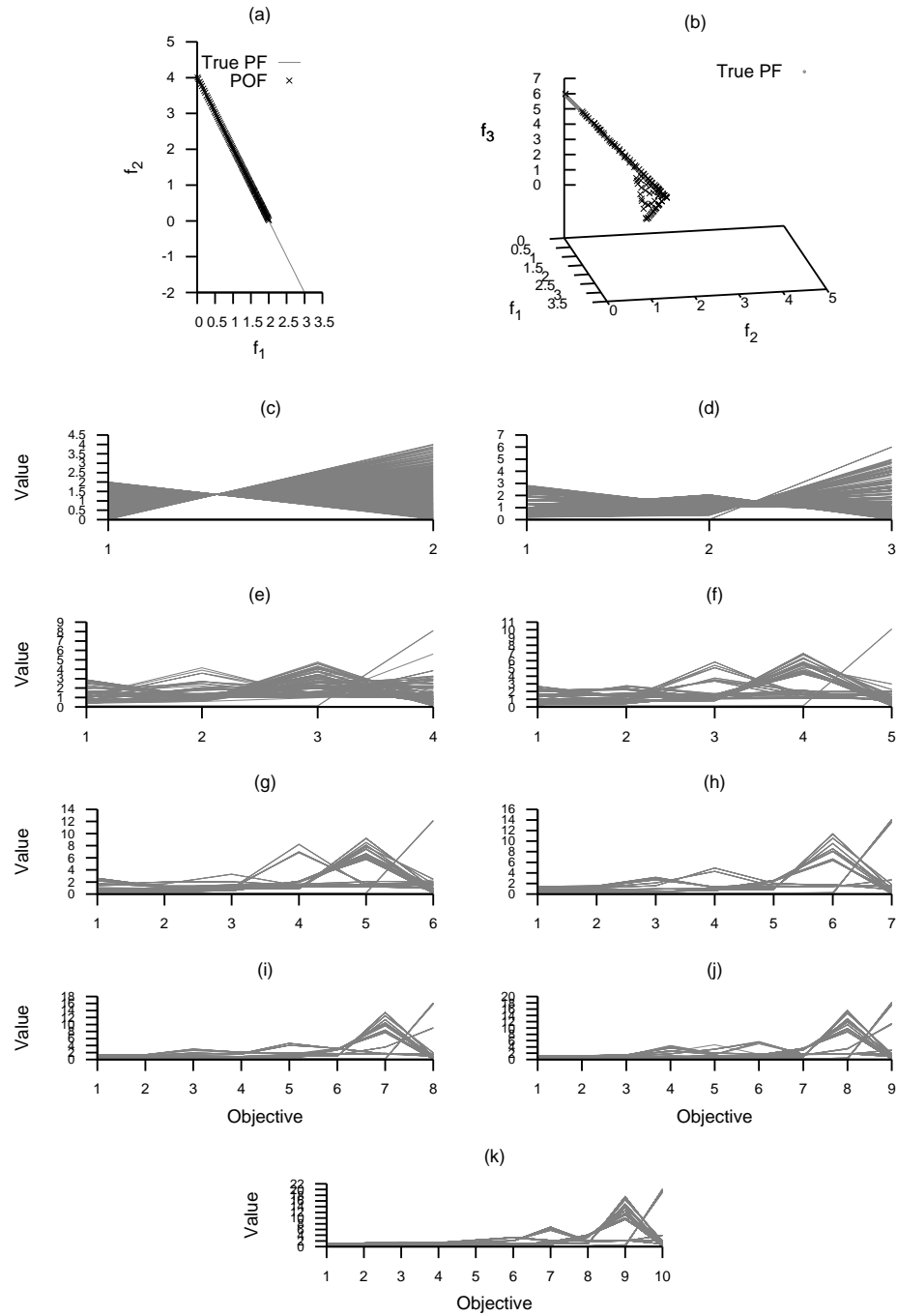


Figure A.355: Plots of the approximations obtained by MOMBI-MTCH from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

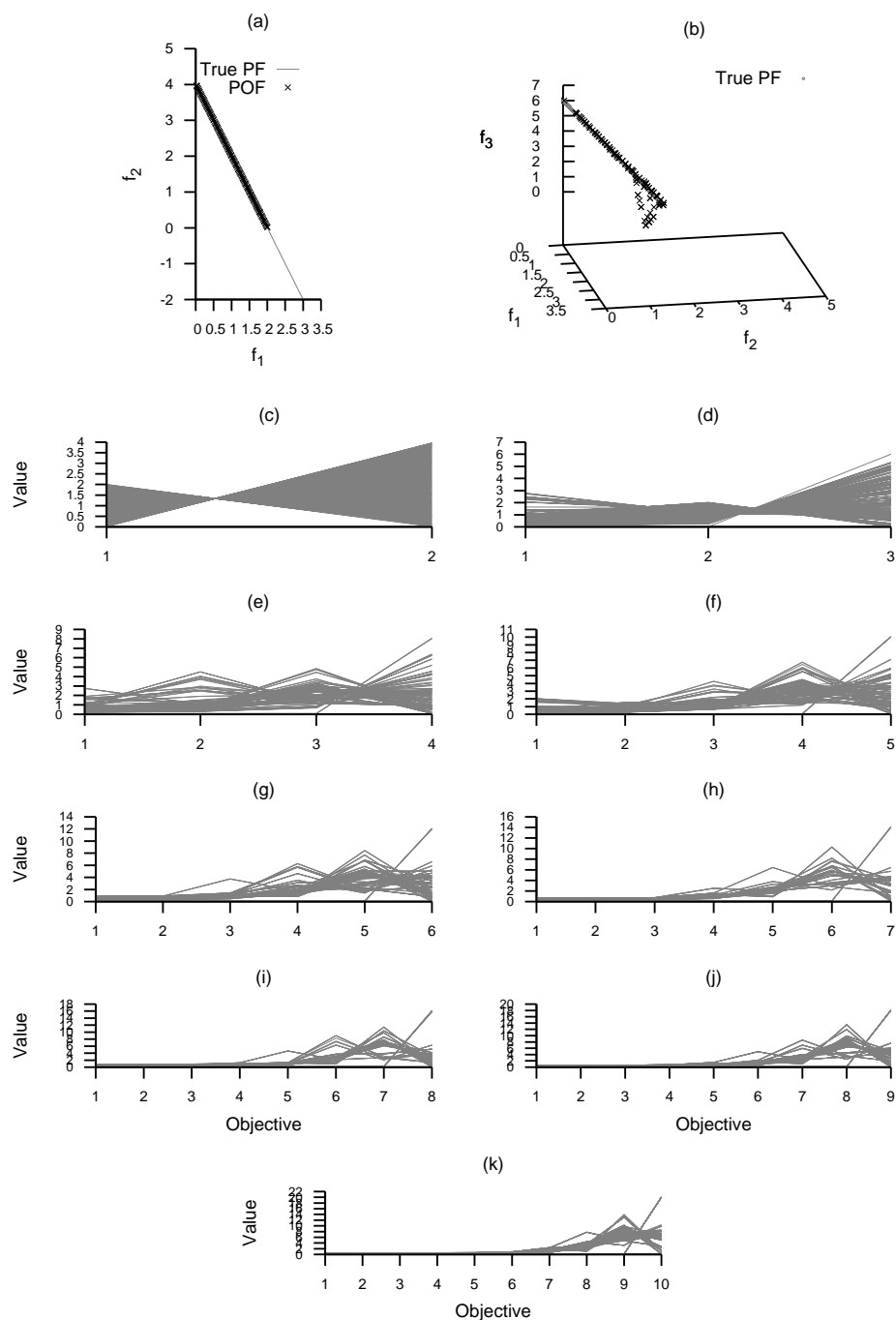


Figure A.356: Plots of the approximations obtained by MOMBI-NMTCH from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.148: Comparison of hypervolume indicator values for different optimizers on the WFG4 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	6.994347e+00	6.994658e+00	7.000076e+00	7.005384e+00	8.624837e+00	8.625435e+00	8.609963e+00	8.612946e+00	8.628731e+00	8.625526e+00	8.517025e+00
	avg.	6.994280e+00	6.994335e+00	7.001894e+00	7.013140e+00	8.620591e+00	8.622215e+00	8.599482e+00	8.600279e+00	8.624632e+00	8.622394e+00	8.513769e+00
	min.	6.988960e+00	6.988757e+00	6.995793e+00	6.996259e+00	8.538900e+00	8.503966e+00	8.515959e+00	8.488439e+00	8.556513e+00	8.549568e+00	8.451533e+00
	max.	6.997108e+00	6.997767e+00	7.027278e+00	7.414195e+00	8.649107e+00	8.650490e+00	8.642241e+00	8.648243e+00	8.649208e+00	8.652032e+00	8.562199e+00
	std.	1.619880e-03	1.868875e-03	5.396365e-03	4.250431e-02	1.907472e-02	1.878395e-02	3.138507e-02	3.864497e-02	1.670053e-02	1.957464e-02	2.754873e-02
3D	med.	5.697561e+01	5.697351e+01	6.173449e+01	5.697636e+01	7.331470e+01	7.422285e+01	7.402377e+01	7.499888e+01	7.381469e+01	7.439002e+01	7.330311e+01
	avg.	5.716172e+01	5.713201e+01	6.121015e+01	5.707802e+01	7.337723e+01	7.422038e+01	7.401990e+01	7.498466e+01	7.381667e+01	7.439942e+01	7.332993e+01
	min.	5.689889e+01	5.693561e+01	5.697001e+01	5.694146e+01	7.287265e+01	7.354261e+01	7.349724e+01	7.424156e+01	7.325536e+01	7.371102e+01	7.237142e+01
	max.	5.952362e+01	6.083458e+01	6.810014e+01	6.098501e+01	7.435962e+01	7.514611e+01	7.443067e+01	7.533603e+01	7.439348e+01	7.493471e+01	7.402936e+01
	std.	4.577224e-01	4.717884e-01	2.974629e+00	4.315084e-01	2.675850e-01	2.730128e-01	1.970985e-01	1.986597e-01	2.060081e-01	2.361849e-01	3.225044e-01
4D	med.	5.676721e+02	5.704280e+02	5.660347e+02	5.626806e+02	6.552191e+02	6.728447e+02	6.418752e+02	6.739587e+02	6.407294e+02	6.582364e+02	7.117651e+02
	avg.	5.778351e+02	5.791265e+02	5.770559e+02	5.678918e+02	6.573782e+02	6.755515e+02	6.420900e+02	6.732130e+02	6.408302e+02	6.587856e+02	7.113301e+02
	min.	5.590655e+02	5.580519e+02	5.590098e+02	5.585815e+02	6.346218e+02	6.552581e+02	6.330483e+02	6.606215e+02	6.343590e+02	6.497688e+02	6.982258e+02
	max.	6.335499e+02	6.410053e+02	6.143846e+02	6.265654e+02	6.956430e+02	7.157398e+02	6.494783e+02	6.874653e+02	6.479006e+02	6.741601e+02	7.211089e+02
	std.	2.140322e+01	1.976606e+01	1.855561e+01	1.303130e+01	1.471682e+01	1.118078e+01	2.508706e+00	5.804204e+00	2.477392e+00	4.608084e+00	4.740455e+00
5D	med.	7.233993e+03	7.048901e+03	6.497208e+03	6.782563e+03	7.799347e+03	6.65136e+03	6.697153e+03	6.639397e+03	6.653988e+03	6.651175e+03	6.653988e+03
	avg.	7.188180e+03	6.971462e+03	6.496685e+03	6.808170e+03	7.762192e+03	6.760477e+03	6.696073e+03	6.602191e+03	6.650868e+03	6.646440e+03	7.756692e+03
	min.	6.539833e+03	5.422222e+03	6.462822e+03	6.490725e+03	7.095589e+03	7.063768e+03	6.647222e+03	5.383290e+03	6.568727e+03	6.579670e+03	7.493820e+03
	max.	7.800468e+03	7.686280e+03	6.526231e+03	7.467117e+03	8.133148e+03	8.263419e+03	6.728607e+03	6.761068e+03	6.715908e+03	6.722283e+03	7.966459e+03
	std.	2.726448e+02	4.083691e+02	1.260453e+01	2.494207e+02	2.035908e+02	2.447645e+02	1.632105e+01	2.052480e+02	2.453533e+01	2.559662e+01	1.007820e+02
6D	med.	7.876108e+04	7.475984e+04	8.362204e+04	7.520878e+04	8.943976e+04	8.596384e+04	7.629148e+04	6.357112e+04	7.592251e+04	6.424734e+04	9.394986e+04
	avg.	7.924267e+04	7.443714e+04	8.120078e+04	7.554941e+04	8.915431e+04	8.524693e+04	7.589742e+04	6.067858e+04	7.385690e+04	6.319537e+04	9.381779e+04
	min.	6.547409e+04	5.539296e+04	7.530972e+04	6.384293e+04	7.937242e+04	6.384048e+04	6.438960e+04	5.245268e+04	5.722285e+04	5.231910e+04	8.299117e+04
	max.	9.542524e+04	9.057263e+04	8.533885e+04	9.572592e+04	9.875701e+04	9.886051e+04	8.431012e+04	6.525120e+04	8.741527e+04	7.659132e+04	1.000348e+05
	std.	6.760567e+03	6.565022e+03	3.518064e+03	5.932203e+03	4.020386e+03	5.993057e+03	7.599085e+03	4.719877e+03	7.667297e+03	5.311012e+03	3.320790e+03
7D	med.	6.470483e+05	7.426437e+05	1.081222e+06	7.860375e+05	1.066079e+06	9.910364e+05	6.546154e+05	6.198860e+05	6.078216e+05	6.413268e+05	1.266859e+06
	avg.	6.812714e+05	7.500551e+05	1.098086e+06	7.936291e+05	1.063730e+06	9.903152e+05	7.281901e+05	6.561425e+05	7.036821e+05	6.834286e+05	1.173295e+06
	min.	5.984309e+05	5.991867e+05	8.053696e+05	5.971284e+05	8.917491e+05	8.250412e+05	5.968623e+05	5.452479e+05	5.983622e+05	5.992637e+05	6.646442e+05
	max.	9.269019e+05	9.435098e+05	1.294164e+06	9.977990e+05	1.193118e+06	1.168868e+06	1.179989e+06	9.019322e+05	1.091756e+06	9.148052e+05	1.443983e+06
	std.	7.470759e+04	8.128124e+04	7.565407e+04	9.616661e+04	5.578447e+04	7.305225e+04	1.628196e+05	7.698726e+04	1.439051e+05	8.276924e+04	2.583438e+05
8D	med.	9.992576e+06	1.307045e+07	1.746523e+07	1.379408e+07	1.715461e+07	1.609159e+07	8.811452e+06	1.154990e+07	8.807474e+06	1.214156e+07	1.965851e+07
	avg.	1.053931e+07	1.289455e+07	1.747951e+07	1.385608e+07	1.728921e+07	1.610300e+07	9.531548e+06	1.157168e+07	9.318665e+06	1.226164e+07	1.735853e+07
	min.	8.711828e+06	8.872861e+06	1.289035e+07	1.124246e+07	1.425796e+07	1.309651e+07	8.651900e+06	8.717015e+06	8.633665e+06	8.606639e+06	1.099428e+07
	max.	1.597465e+07	1.564330e+07	2.474614e+07	1.655598e+07	2.007035e+07	1.868268e+07	1.691895e+07	1.511104e+07	1.372070e+07	1.566931e+07	2.349603e+07
	std.	1.433302e+06	1.554135e+06	1.956826e+06	1.428914e+06	1.106818e+06	1.044973e+06	1.619996e+06	1.865115e+06	9.988182e+05	1.799709e+06	5.037639e+06
9D	med.	2.128702e+08	2.395530e+08	3.137633e+08	2.754483e+08	3.140284e+08	3.039049e+08	1.454002e+08	2.381416e+08	1.451128e+08	2.539998e+08	3.427051e+08
	avg.	2.133996e+08	2.396214e+08	3.116323e+08	2.702596e+08	3.144210e+08	3.003458e+08	1.555853e+08	2.362578e+08	1.535787e+08	2.489775e+08	3.195271e+08
	min.	1.532603e+08	1.515880e+08	2.150570e+08	2.054114e+08	2.654946e+08	2.498860e+08	1.417101e+08	1.475923e+08	1.407632e+08	1.558523e+08	2.043257e+08
	max.	2.839742e+08	3.301583e+08	4.084874e+08	3.226256e+08	3.615811e+08	3.441377e+08	3.149062e+08	2.883891e+08	2.133666e+08	3.042073e+08	4.173660e+08
	std.	3.356615e+07	3.619872e+07	4.488891e+07	2.336970e+07	2.096124e+07	1.948595e+07	2.626434e+07	3.006911e+07	1.760861e+07	2.894299e+07	8.150470e+07
10D	med.	4.993661e+09	5.576170e+09	5.997497e+09	5.803494e+09	6.305079e+09	6.142738e+09	2.664229e+09	5.232517e+09	2.663815e+09	5.605663e+09	6.723745e+09
	avg.	4.987637e+09	5.446565e+09	6.086288e+09	5.735583e+09	6.334450e+09	6.082889e+09	2.748967e+09	5.199044e+09	2.740640e+09	5.494017e+09	6.063999e+09
	min.	3.225198e+09	4.130586e+09	4.053518e+09	4.133352e+09	5.383757e+09	4.691073e+09	2.568361e+09	4.006542e+09	2.609488e+09	4.368102e+09	4.008528e+09
	max.	6.324206e+09	6.451205e+09	9.031740e+09	6.845824e+09	7.368427e+09	7.430875e+09	3.801694e+09	6.211505e+09	3.997967e+09	6.302400e+09	8.428165e+09
	std.	6.695752e+08	5.498352e+08	1.029400e+09	4.945086e+08	3.833560e+08	4.757956e+08	2.320321e+08	4.993445e+08	2.408773e+08	4.421937e+08	1.637680e+09

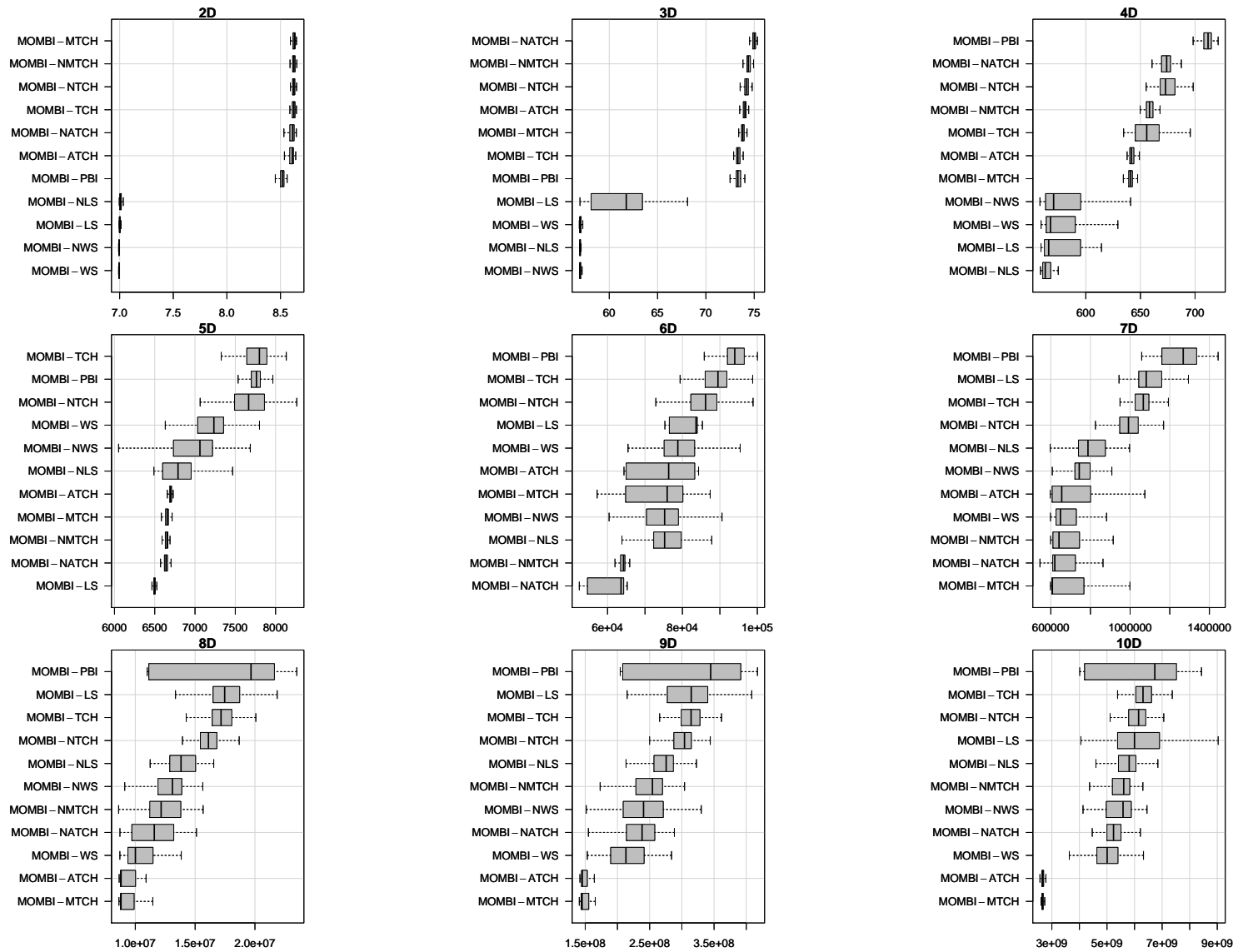


Figure A.357: Box-plot of the hypervolume indicator values for different optimizers on the WFG4 test problem.



Table A.149: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG4 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
2D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.57e-33$	$4.26e-33$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$2.26e-34$	$2.33e-34$	$1.86e-08$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	> 0.05	$9.23e-09$	$2.21e-05$	> 0.05	> 0.05	$2.26e-34$
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	–	$2.45e-10$	$2.77e-06$	> 0.05	> 0.05	$1.00e-33$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$3.78e-30$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	$1.03e-26$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$1.33e-12$	$5.76e-08$	–	> 0.05	$1.40e-34$
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$8.63e-10$	$2.34e-06$	> 0.05	–	$1.84e-34$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
3D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$3.02e-30$	$6.43e-31$	–	$2.18e-32$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$8.48e-32$	–	$1.80e-08$	> 0.05	$5.05e-21$	> 0.05	$7.22e-33$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.22e-29$	> 0.05	–	> 0.05	$3.51e-11$	> 0.05	$4.57e-31$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.32e-34$	$2.52e-32$	$1.89e-34$	–	$1.40e-34$	$6.43e-31$	$1.28e-34$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$6.40e-23$	> 0.05	> 0.05	> 0.05	–	> 0.05	$2.55e-23$
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.27e-33$	$6.93e-07$	$7.13e-22$	> 0.05	$6.07e-30$	–	$2.79e-34$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
4D											
MOMBI-WS	–	> 0.05	> 0.05	$5.55e-07$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	$9.54e-08$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	> 0.05	> 0.05	–	$1.33e-04$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.95e-34$	$1.28e-34$	$1.28e-34$	–	> 0.05	$6.58e-17$	> 0.05	$1.29e-19$	> 0.05	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.32e-16$	–	$1.28e-34$	> 0.05	$1.28e-34$	$1.20e-27$	> 0.05
MOMBI-ATCH	$1.32e-34$	$3.65e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	–	> 0.05	$1.47e-04$	> 0.05	> 0.05
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.33e-15$	> 0.05	$1.28e-34$	–	$1.28e-34$	$6.61e-31$	> 0.05
MOMBI-MTCH	$1.28e-34$	$6.24e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	–	> 0.05
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.45e-32$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–

Table A.150: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG4 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
5D											
MOMBI-WS	–	$2.54e-05$	$1.28e-34$	$2.83e-17$	> 0.05	> 0.05	$1.01e-25$	$1.95e-29$	$1.83e-28$	$1.04e-28$	> 0.05
MOMBI-NWS	> 0.05	–	$5.55e-25$	$9.36e-06$	> 0.05	> 0.05	$3.81e-11$	$8.53e-15$	$1.61e-13$	$7.48e-14$	> 0.05
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	$4.45e-33$	–	> 0.05	> 0.05	> 0.05	$1.83e-04$	$4.27e-03$	$2.01e-03$	> 0.05
MOMBI-TCH	$4.56e-29$	$1.30e-31$	$1.28e-34$	$5.71e-34$	–	$1.56e-03$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05
MOMBI-NTCH	$5.03e-24$	$1.25e-28$	$1.28e-34$	$3.18e-33$	> 0.05	–	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05
MOMBI-ATCH	> 0.05	> 0.05	$1.28e-34$	> 0.05	> 0.05	> 0.05	–	$8.56e-29$	$4.97e-28$	$4.32e-29$	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	$1.32e-29$	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	> 0.05	> 0.05	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	$9.58e-05$	–	> 0.05	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	$6.03e-03$	> 0.05	–	> 0.05
MOMBI-PBI	$1.68e-32$	$4.92e-34$	$1.28e-34$	$1.28e-34$	> 0.05	$3.61e-03$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–
6D											
MOMBI-WS	–	$2.67e-06$	> 0.05	$4.12e-05$	> 0.05	> 0.05	$1.34e-02$	$1.28e-34$	$4.14e-06$	$3.78e-32$	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$8.96e-30$	> 0.05	$1.41e-24$	> 0.05
MOMBI-LS	$9.69e-04$	$5.67e-14$	–	$7.76e-15$	> 0.05	> 0.05	$2.61e-10$	$1.28e-34$	$1.24e-16$	$2.96e-34$	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$4.26e-33$	> 0.05	$4.84e-28$	> 0.05
MOMBI-TCH	$4.34e-22$	$1.59e-31$	$5.54e-28$	$8.06e-31$	–	$5.03e-07$	$5.12e-31$	$1.28e-34$	$8.48e-32$	$1.28e-34$	> 0.05
MOMBI-NTCH	$8.53e-11$	$6.64e-22$	$1.50e-09$	$1.03e-20$	> 0.05	–	$1.17e-16$	$4.37e-34$	$2.62e-20$	$2.51e-33$	> 0.05
MOMBI-ATCH	> 0.05	> 0.05	> 0.05	$4.28e-02$	> 0.05	> 0.05	–	$6.18e-32$	$9.97e-03$	$1.64e-26$	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.47e-28$	–	$1.28e-21$	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.45e-04$	> 0.05	–	> 0.05
MOMBI-PBI	$4.47e-30$	$3.65e-34$	$1.33e-32$	$2.30e-33$	$1.93e-15$	$3.14e-24$	$8.16e-34$	$1.28e-34$	$4.24e-34$	$1.28e-34$	–
7D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$7.07e-05$	$7.60e-03$	> 0.05	> 0.05
MOMBI-NWS	$6.85e-09$	–	> 0.05	> 0.05	> 0.05	> 0.05	$1.68e-05$	$2.39e-13$	$2.08e-07$	$9.29e-08$	> 0.05
MOMBI-LS	$1.68e-34$	$2.48e-34$	–	$6.83e-34$	$1.29e-04$	$2.15e-18$	$7.46e-29$	$1.53e-34$	$2.06e-32$	$1.68e-34$	> 0.05
MOMBI-NLS	$1.27e-14$	$2.18e-04$	> 0.05	–	> 0.05	> 0.05	$1.29e-07$	$9.91e-19$	$1.35e-09$	$8.05e-14$	> 0.05
MOMBI-TCH	$1.36e-34$	$1.58e-34$	> 0.05	$4.64e-34$	–	$4.72e-13$	$7.69e-27$	$1.32e-34$	$1.07e-31$	$1.36e-34$	> 0.05
MOMBI-NTCH	$4.12e-34$	$8.35e-33$	> 0.05	$1.78e-28$	> 0.05	–	$1.15e-20$	$2.63e-34$	$2.34e-25$	$5.22e-34$	> 0.05
MOMBI-ATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	$1.45e-02$	> 0.05	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.16e-02$	$1.29e-02$	–	> 0.05
MOMBI-PBI	$1.91e-25$	$1.39e-17$	$1.98e-11$	$2.06e-16$	$3.68e-13$	$4.14e-14$	$6.60e-24$	$4.84e-28$	$4.77e-26$	$1.07e-24$	–

Table A.151: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG4 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
8D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$5.16e-13$	> 0.05	$1.07e-14$	> 0.05	> 0.05
MOMBI-NWS	$8.70e-19$	–	> 0.05	> 0.05	> 0.05	> 0.05	$1.01e-25$	$4.12e-07$	$1.12e-29$	$5.06e-03$	> 0.05
MOMBI-LS	$4.50e-34$	$1.23e-30$	–	$1.11e-26$	> 0.05	$3.50e-10$	$2.17e-33$	$1.05e-32$	$1.78e-34$	$9.51e-32$	> 0.05
MOMBI-NLS	$2.67e-27$	$3.91e-05$	> 0.05	–	> 0.05	> 0.05	$7.07e-29$	$2.01e-15$	$4.79e-33$	$1.22e-09$	> 0.05
MOMBI-TCH	$1.68e-34$	$4.12e-34$	> 0.05	$7.56e-32$	–	$2.98e-12$	$1.13e-33$	$1.89e-34$	$1.28e-34$	$3.34e-34$	> 0.05
MOMBI-NTCH	$5.88e-34$	$1.23e-30$	> 0.05	$4.66e-22$	> 0.05	–	$5.20e-32$	$3.27e-33$	$1.40e-34$	$1.74e-31$	> 0.05
MOMBI-ATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NATCH	$8.95e-05$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$4.95e-18$	–	$2.45e-20$	> 0.05	> 0.05
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$4.86e-11$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$5.76e-22$	$7.65e-03$	$3.34e-25$	–	> 0.05
MOMBI-PBI	$8.77e-21$	$8.04e-05$	$2.66e-02$	$1.62e-03$	$5.43e-03$	$1.84e-03$	$3.07e-31$	$7.78e-12$	$5.83e-32$	$1.88e-07$	–
9D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$6.23e-27$	> 0.05	$1.60e-28$	> 0.05	> 0.05
MOMBI-NWS	$8.96e-07$	–	> 0.05	> 0.05	> 0.05	> 0.05	$3.68e-30$	> 0.05	$2.25e-32$	> 0.05	> 0.05
MOMBI-LS	$4.00e-30$	$7.47e-22$	–	$4.77e-12$	> 0.05	$1.65e-02$	$1.13e-33$	$7.52e-25$	$1.28e-34$	$5.04e-20$	> 0.05
MOMBI-NLS	$1.21e-24$	$2.07e-10$	> 0.05	–	> 0.05	> 0.05	$1.26e-32$	$1.50e-15$	$1.65e-34$	$4.34e-08$	> 0.05
MOMBI-TCH	$3.05e-34$	$2.52e-31$	> 0.05	$1.73e-26$	–	$3.53e-06$	$6.63e-34$	$1.13e-33$	$1.28e-34$	$4.91e-32$	> 0.05
MOMBI-NTCH	$1.92e-33$	$3.31e-27$	> 0.05	$1.02e-17$	> 0.05	–	$1.57e-33$	$3.64e-31$	$1.28e-34$	$3.13e-27$	> 0.05
MOMBI-ATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NATCH	$1.40e-06$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$3.20e-30$	–	$1.45e-32$	> 0.05	> 0.05
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$1.29e-12$	$3.41e-02$	> 0.05	> 0.05	> 0.05	> 0.05	$2.59e-31$	$1.47e-03$	$1.27e-33$	–	> 0.05
MOMBI-PBI	$2.26e-15$	$3.61e-10$	$1.51e-02$	$4.14e-06$	$4.21e-04$	$2.33e-05$	$3.00e-32$	$4.08e-09$	$2.17e-33$	$2.52e-07$	–
10D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.71e-34$	> 0.05	$2.88e-34$	> 0.05	> 0.05
MOMBI-NWS	$6.13e-07$	–	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$	$2.95e-04$	$1.28e-34$	> 0.05	> 0.05
MOMBI-LS	$1.38e-14$	$3.37e-06$	–	$1.93e-02$	> 0.05	> 0.05	$1.28e-34$	$7.52e-12$	$1.28e-34$	$1.14e-05$	> 0.05
MOMBI-NLS	$9.93e-15$	$1.52e-04$	> 0.05	–	> 0.05	> 0.05	$1.28e-34$	$7.61e-13$	$1.28e-34$	$8.95e-05$	> 0.05
MOMBI-TCH	$6.98e-30$	$7.15e-25$	$1.61e-03$	$1.49e-16$	–	$1.63e-04$	$1.28e-34$	$8.24e-32$	$1.28e-34$	$6.39e-27$	> 0.05
MOMBI-NTCH	$6.13e-24$	$1.13e-14$	> 0.05	$5.55e-07$	> 0.05	–	$1.28e-34$	$1.81e-23$	$1.28e-34$	$1.62e-15$	> 0.05
MOMBI-ATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NATCH	$2.12e-03$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.68e-34$	–	$1.68e-34$	> 0.05	> 0.05
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$8.24e-09$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$	$5.31e-06$	$1.28e-34$	–	> 0.05
MOMBI-PBI	$5.37e-04$	$7.30e-03$	> 0.05	$1.40e-02$	> 0.05	> 0.05	$1.28e-34$	$2.15e-03$	$1.28e-34$	$8.98e-03$	–

Table A.152: Comparison of R2 indicator values for different optimizers on the WFG4 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	6.668318e-01	6.668261e-01	6.666055e-01	6.664405e-01	5.890592e-01	5.892449e-01	5.894240e-01	5.893745e-01	5.890160e-01	5.892364e-01	5.931737e-01
	avg.	6.668380e-01	6.668361e-01	6.664940e-01	6.661922e-01	5.891134e-01	5.892917e-01	5.896617e-01	5.896796e-01	5.890538e-01	5.892920e-01	5.932159e-01
	min.	6.667399e-01	6.667429e-01	6.652842e-01	6.519485e-01	5.886655e-01	5.887249e-01	5.889463e-01	5.888220e-01	5.886336e-01	5.887162e-01	5.911402e-01
	max.	6.670709e-01	6.670347e-01	6.667961e-01	6.668021e-01	5.905894e-01	5.928699e-01	5.916998e-01	5.931757e-01	5.901334e-01	5.911648e-01	5.952664e-01
	std.	5.027178e-05	5.617415e-05	2.960323e-04	1.500021e-03	3.027693e-04	4.516549e-04	6.686937e-04	8.616929e-04	2.529694e-04	3.669928e-04	8.483117e-04
3D	med.	3.638550e-01	3.638582e-01	3.506772e-01	3.638540e-01	3.169083e-01	3.176563e-01	3.166589e-01	3.178911e-01	3.164370e-01	3.176459e-01	3.211756e-01
	avg.	3.632245e-01	3.634053e-01	3.513517e-01	3.636315e-01	3.169535e-01	3.176455e-01	3.166595e-01	3.179141e-01	3.164840e-01	3.176900e-01	3.211972e-01
	min.	3.545994e-01	3.517922e-01	3.334355e-01	3.583864e-01	3.162423e-01	3.160844e-01	3.161148e-01	3.169749e-01	3.155385e-01	3.166655e-01	3.192156e-01
	max.	3.639898e-01	3.639238e-01	3.638593e-01	3.639219e-01	3.180499e-01	3.191855e-01	3.178262e-01	3.197540e-01	3.180761e-01	3.190567e-01	3.236093e-01
	std.	1.460697e-03	1.418488e-03	8.310901e-03	6.683846e-04	3.498146e-04	4.989628e-04	2.867865e-04	4.359336e-04	3.824620e-04	4.763526e-04	8.584808e-04
4D	med.	2.384549e-01	2.386169e-01	2.383290e-01	2.395609e-01	2.241270e-01	2.243388e-01	2.269120e-01	2.276899e-01	2.263236e-01	2.285222e-01	2.182358e-01
	avg.	2.367024e-01	2.376625e-01	2.365747e-01	2.391291e-01	2.240056e-01	2.242957e-01	2.268672e-01	2.276522e-01	2.263437e-01	2.284149e-01	2.183418e-01
	min.	2.268148e-01	2.279986e-01	2.293113e-01	2.309732e-01	2.197421e-01	2.176228e-01	2.252398e-01	2.236670e-01	2.251803e-01	2.252812e-01	2.161974e-01
	max.	2.402491e-01	2.404058e-01	2.402564e-01	2.402867e-01	2.274424e-01	2.289705e-01	2.278943e-01	2.292415e-01	2.275080e-01	2.296331e-01	2.221160e-01
	std.	3.661142e-03	2.622953e-03	3.491765e-03	1.485633e-03	1.271748e-03	2.136946e-03	3.541616e-04	2.607742e-04	4.288869e-04	7.740975e-04	1.134520e-03
5D	med.	1.745340e-01	1.781886e-01	1.823787e-01	1.803966e-01	1.723934e-01	1.715109e-01	1.791531e-01	1.822767e-01	1.810294e-01	1.821183e-01	1.844267e-01
	avg.	1.754823e-01	1.851651e-01	1.823876e-01	1.806614e-01	1.723863e-01	1.718654e-01	1.791602e-01	1.865871e-01	1.809823e-01	1.821727e-01	1.846141e-01
	min.	1.686293e-01	1.696030e-01	1.821868e-01	1.762167e-01	1.667643e-01	1.644933e-01	1.789033e-01	1.811289e-01	1.800842e-01	1.815286e-01	1.798258e-01
	max.	1.962317e-01	2.939589e-01	1.826370e-01	1.824240e-01	1.786676e-01	1.794153e-01	1.795870e-01	3.110896e-01	1.825726e-01	1.828128e-01	1.900985e-01
	std.	4.150030e-03	2.460972e-02	8.607759e-05	1.491990e-03	2.405964e-03	2.846278e-03	1.319901e-04	2.232124e-02	3.640832e-04	2.290871e-04	1.990150e-03
6D	med.	2.242223e-01	2.299871e-01	1.435859e-01	2.289403e-01	2.223848e-01	2.238154e-01	1.694475e-01	2.350576e-01	1.696615e-01	2.330007e-01	1.613565e-01
	avg.	2.202526e-01	2.440837e-01	1.534647e-01	2.233115e-01	2.203829e-01	2.340802e-01	1.779595e-01	2.749181e-01	1.871849e-01	2.508111e-01	1.592827e-01
	min.	1.566196e-01	1.657061e-01	1.428429e-01	1.538002e-01	1.644760e-01	1.919119e-01	1.433055e-01	2.315695e-01	1.324689e-01	1.679962e-01	1.457533e-01
	max.	2.979261e-01	3.385666e-01	1.909675e-01	2.344070e-01	3.014938e-01	3.300416e-01	2.328025e-01	3.477271e-01	3.143152e-01	3.475779e-01	1.915657e-01
	std.	2.545695e-02	3.502170e-02	1.270221e-02	1.767369e-02	1.834967e-02	3.213161e-02	3.611837e-02	5.266646e-02	3.883933e-02	4.791999e-02	9.625138e-03
7D	med.	3.703201e-01	3.663509e-01	1.616184e-01	3.637295e-01	3.477590e-01	3.531237e-01	3.392912e-01	3.709827e-01	3.714617e-01	3.698682e-01	1.625834e-01
	avg.	3.672973e-01	3.691182e-01	1.635732e-01	3.633192e-01	3.499191e-01	3.661138e-01	3.088675e-01	3.878483e-01	3.211978e-01	3.715222e-01	1.773953e-01
	min.	2.691731e-01	3.544727e-01	1.241417e-01	3.521953e-01	2.902041e-01	3.389440e-01	1.387686e-01	3.577021e-01	1.620142e-01	3.539473e-01	1.295915e-01
	max.	4.009223e-01	4.673077e-01	2.910734e-01	3.773890e-01	4.798845e-01	5.070937e-01	3.728567e-01	5.466745e-01	4.179790e-01	5.347681e-01	2.870567e-01
	std.	1.783344e-02	1.505493e-02	2.700482e-02	5.264035e-03	2.207314e-02	4.109345e-02	7.551287e-02	5.244588e-02	7.141053e-02	2.365842e-02	5.458291e-02
8D	med.	3.915967e-01	3.827201e-01	1.574957e-01	3.784465e-01	3.669960e-01	3.732207e-01	3.925395e-01	3.857169e-01	3.925436e-01	3.836232e-01	1.475500e-01
	avg.	3.886206e-01	3.855855e-01	1.751833e-01	3.786196e-01	3.683802e-01	3.773510e-01	3.631792e-01	3.979866e-01	3.687589e-01	3.910933e-01	1.783948e-01
	min.	3.052615e-01	3.685535e-01	1.090108e-01	3.678600e-01	3.217859e-01	3.460284e-01	1.584159e-01	3.740333e-01	2.197343e-01	3.705230e-01	1.156903e-01
	max.	4.418476e-01	4.917825e-01	3.086632e-01	3.996105e-01	4.346571e-01	4.954027e-01	3.934958e-01	5.541718e-01	4.004379e-01	5.300952e-01	2.531704e-01
	std.	1.736533e-02	1.666247e-02	4.639508e-02	5.262896e-03	1.441838e-02	1.909130e-02	5.199648e-02	4.395966e-02	3.858542e-02	3.317905e-02	5.880292e-02
9D	med.	3.999509e-01	3.956229e-01	1.845365e-01	3.875784e-01	3.853096e-01	3.845056e-01	4.070790e-01	3.938034e-01	4.071239e-01	3.906366e-01	1.374498e-01
	avg.	3.996508e-01	3.969895e-01	1.911905e-01	3.880558e-01	3.869014e-01	3.880289e-01	3.833115e-01	3.996057e-01	3.839565e-01	3.939709e-01	1.579245e-01
	min.	3.056520e-01	3.715474e-01	1.069950e-01	3.779797e-01	3.470331e-01	3.210518e-01	1.856336e-01	3.832125e-01	2.518613e-01	3.769804e-01	1.064162e-01
	max.	4.440138e-01	4.800304e-01	3.152718e-01	3.990451e-01	4.285665e-01	5.095054e-01	4.083084e-01	5.601809e-01	4.109699e-01	5.259042e-01	2.264003e-01
	std.	2.034679e-02	1.271886e-02	5.499756e-02	4.618562e-03	1.391898e-02	1.975010e-02	4.704072e-02	2.945590e-02	4.254862e-02	1.871448e-02	4.754973e-02
10D	med.	4.004063e-01	3.962630e-01	2.057258e-01	3.934099e-01	3.921357e-01	3.944197e-01	4.162196e-01	3.981266e-01	4.162477e-01	3.955339e-01	1.304714e-01
	avg.	3.983006e-01	3.970919e-01	2.057028e-01	3.945406e-01	3.955938e-01	3.965561e-01	4.030575e-01	3.985220e-01	4.054770e-01	3.961457e-01	1.530825e-01
	min.	3.189450e-01	3.835386e-01	8.998132e-02	3.791005e-01	3.189666e-01	3.752381e-01	2.649943e-01	3.880172e-01	2.629737e-01	3.869308e-01	1.009510e-01
	max.	4.419504e-01	4.157834e-01	3.228086e-01	4.206335e-01	4.305123e-01	4.335538e-01	4.239557e-01	4.129837e-01	4.337530e-01	4.083376e-01	2.080895e-01
	std.	1.774458e-02	6.608679e-03	5.193551e-02	6.304747e-03	1.553290e-02	1.117266e-02	3.199901e-02	4.553152e-03	3.135725e-02	4.215707e-03	4.391644e-02

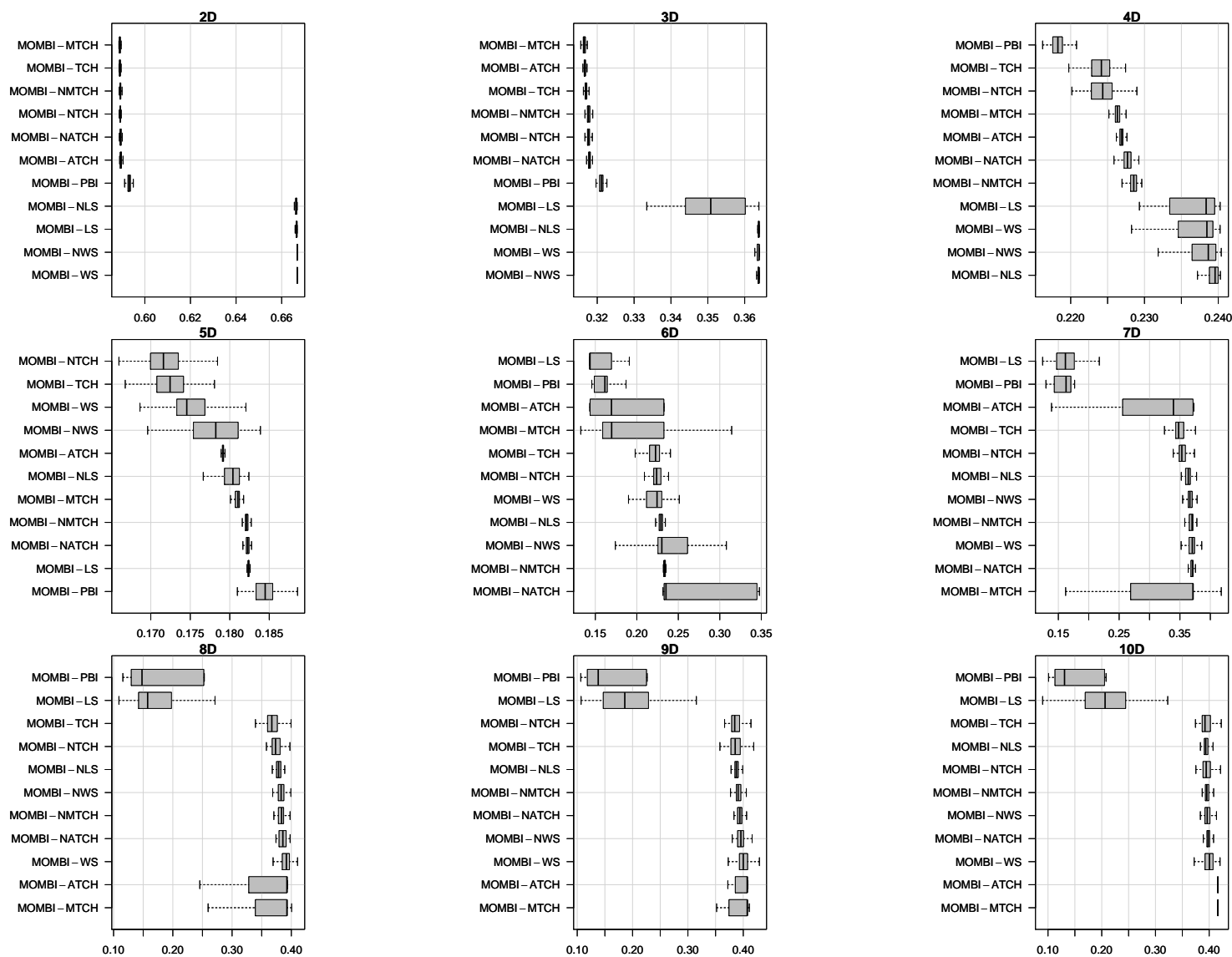


Figure A.358: Box-plot of the R2 indicator values for different optimizers on the WFG4 test problem.

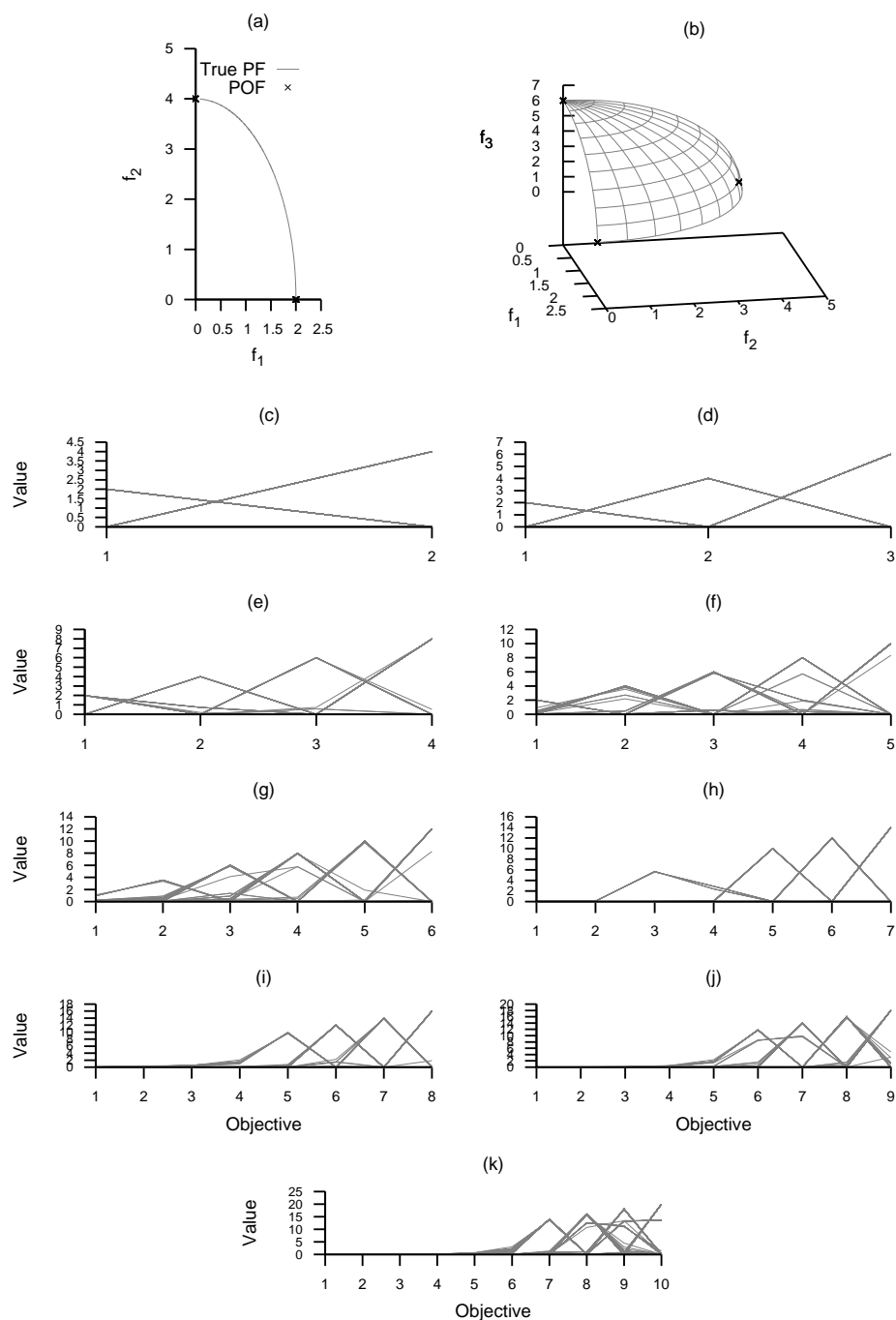


Figure A.359: Plots of the approximations obtained by MOMBI-WS from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

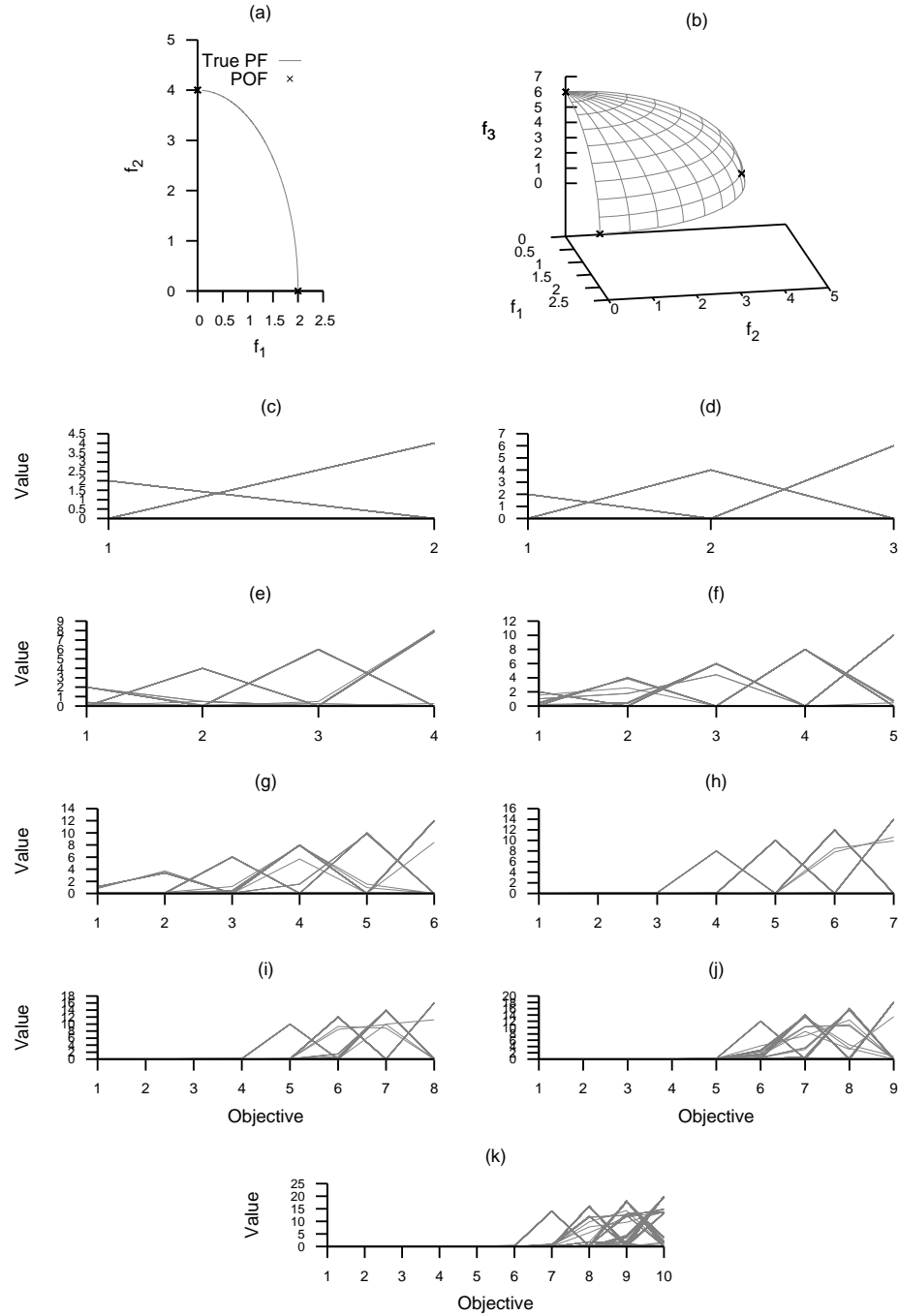


Figure A.360: Plots of the approximations obtained by MOMBI-NWS from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

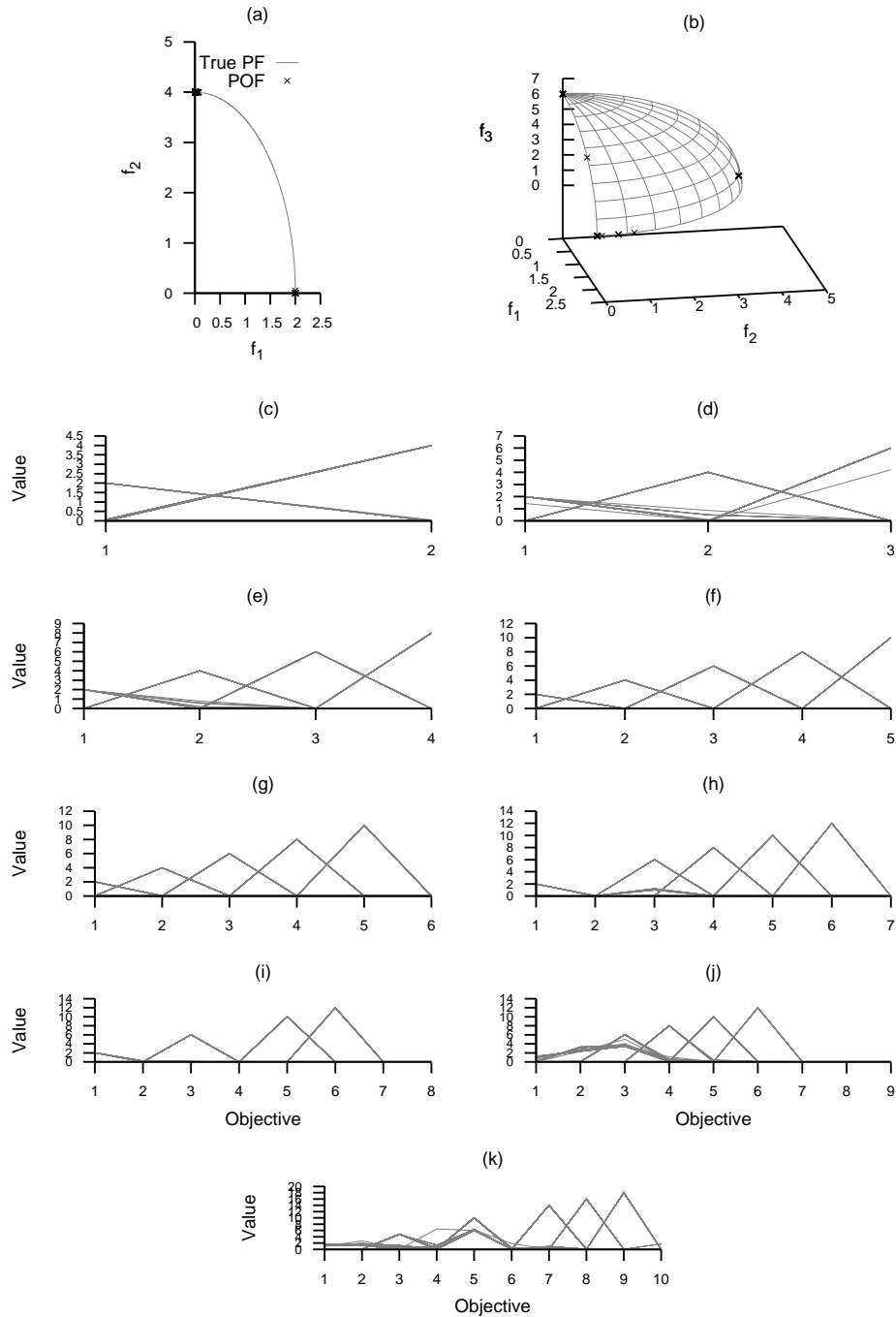


Figure A.361: Plots of the approximations obtained by MOMBI-LS from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



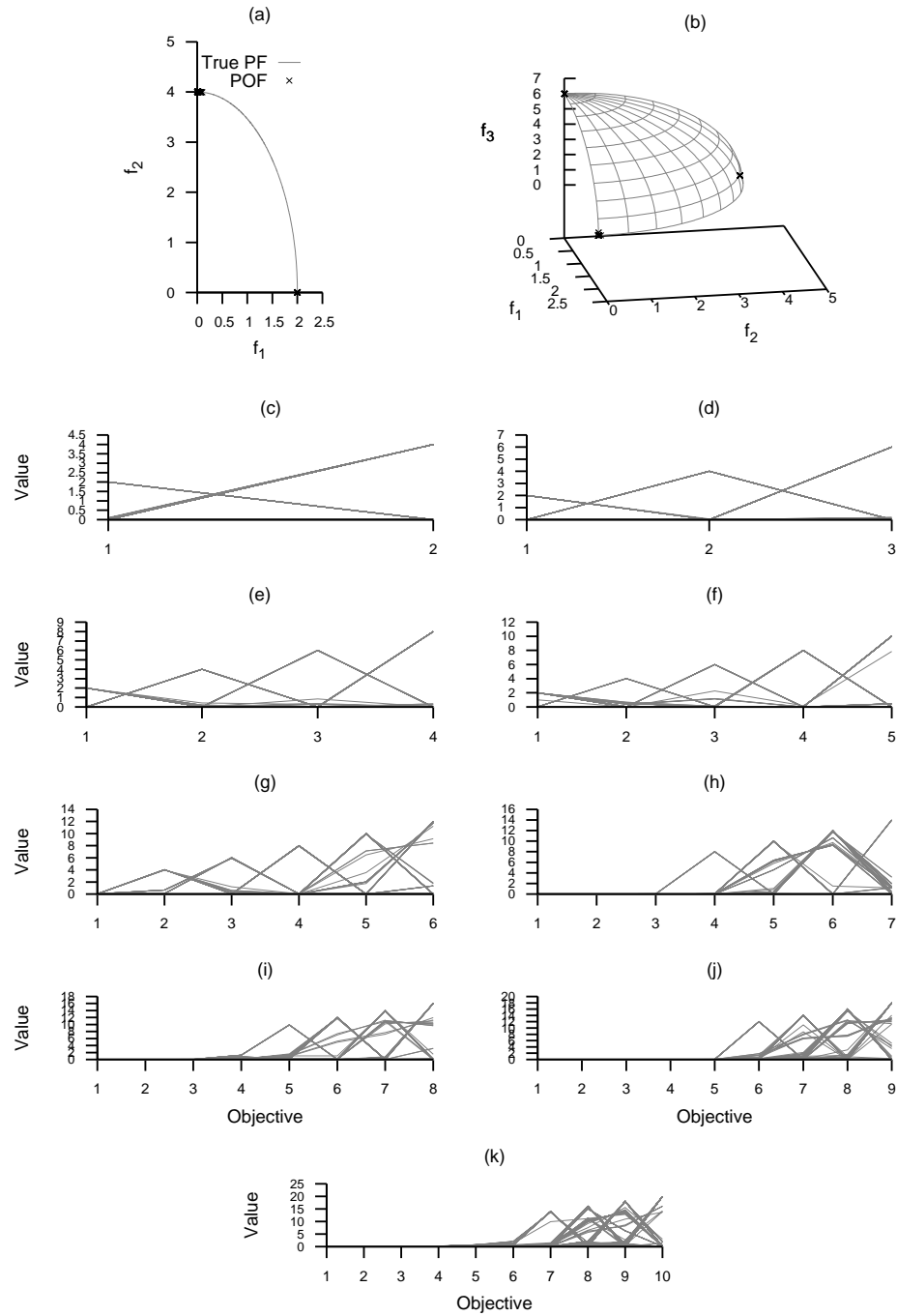


Figure A.362: Plots of the approximations obtained by MOMBI-NLS from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

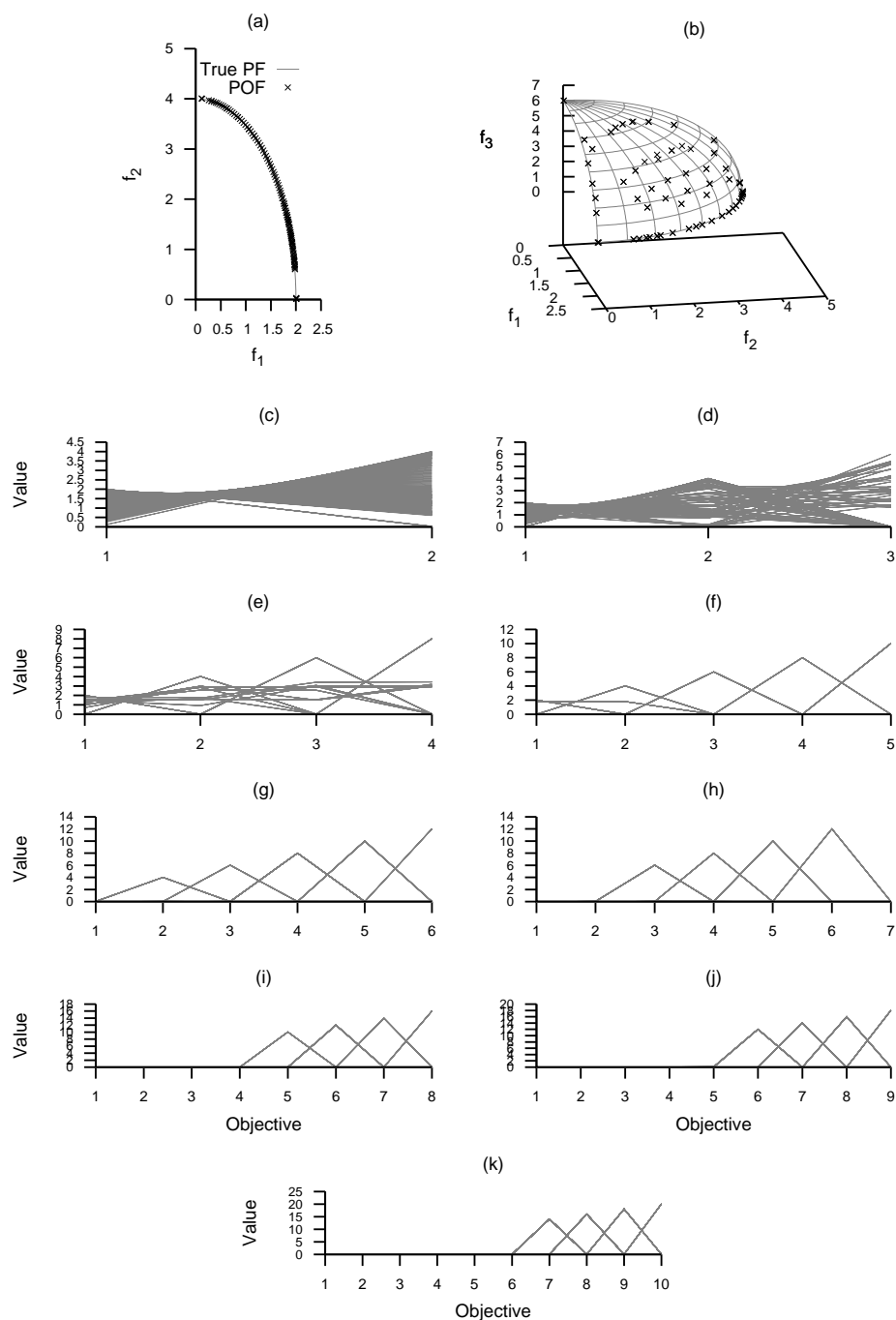


Figure A.363: Plots of the approximations obtained by MOMBI-ATCH from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

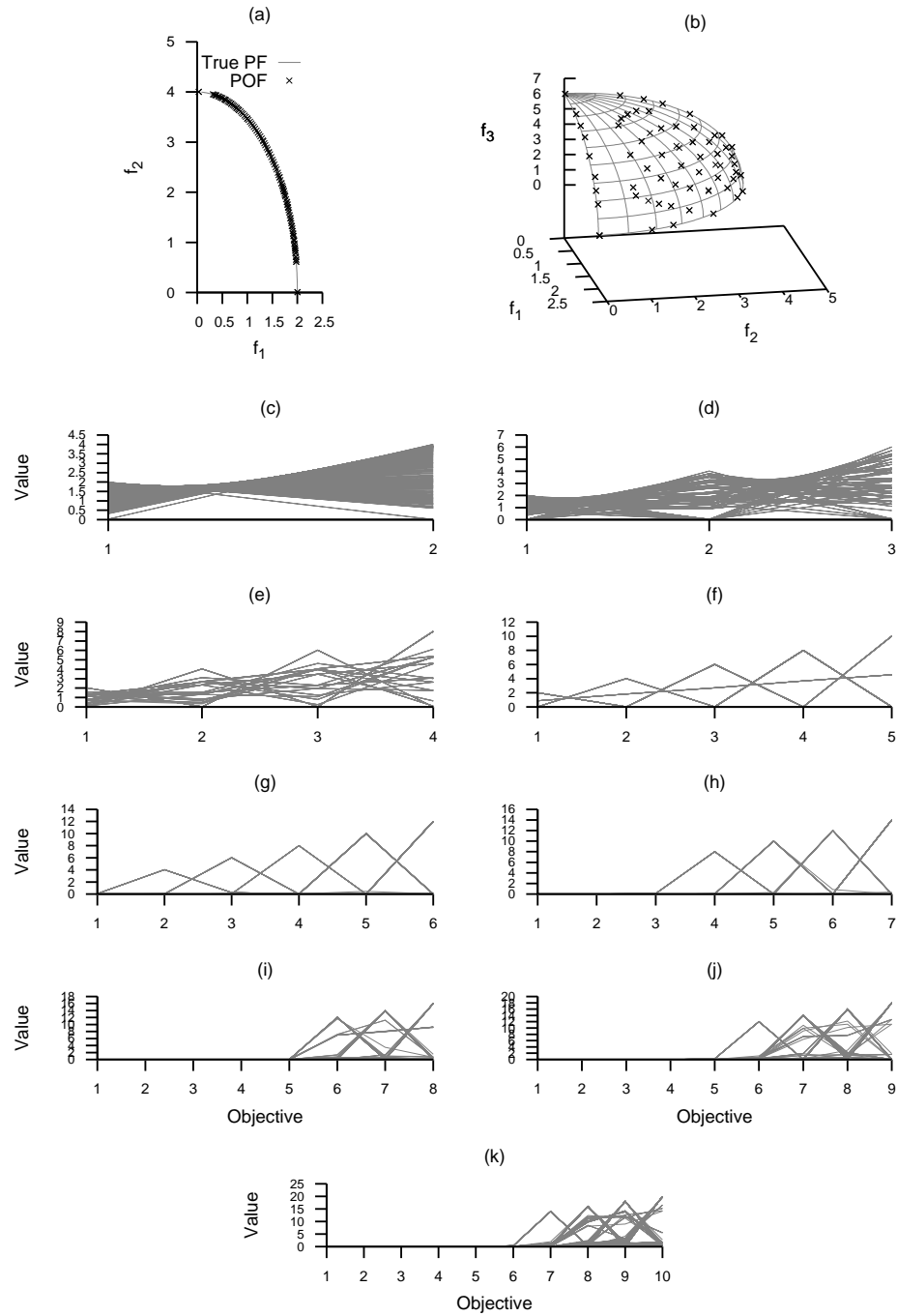


Figure A.364: Plots of the approximations obtained by MOMBI-NATCH from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

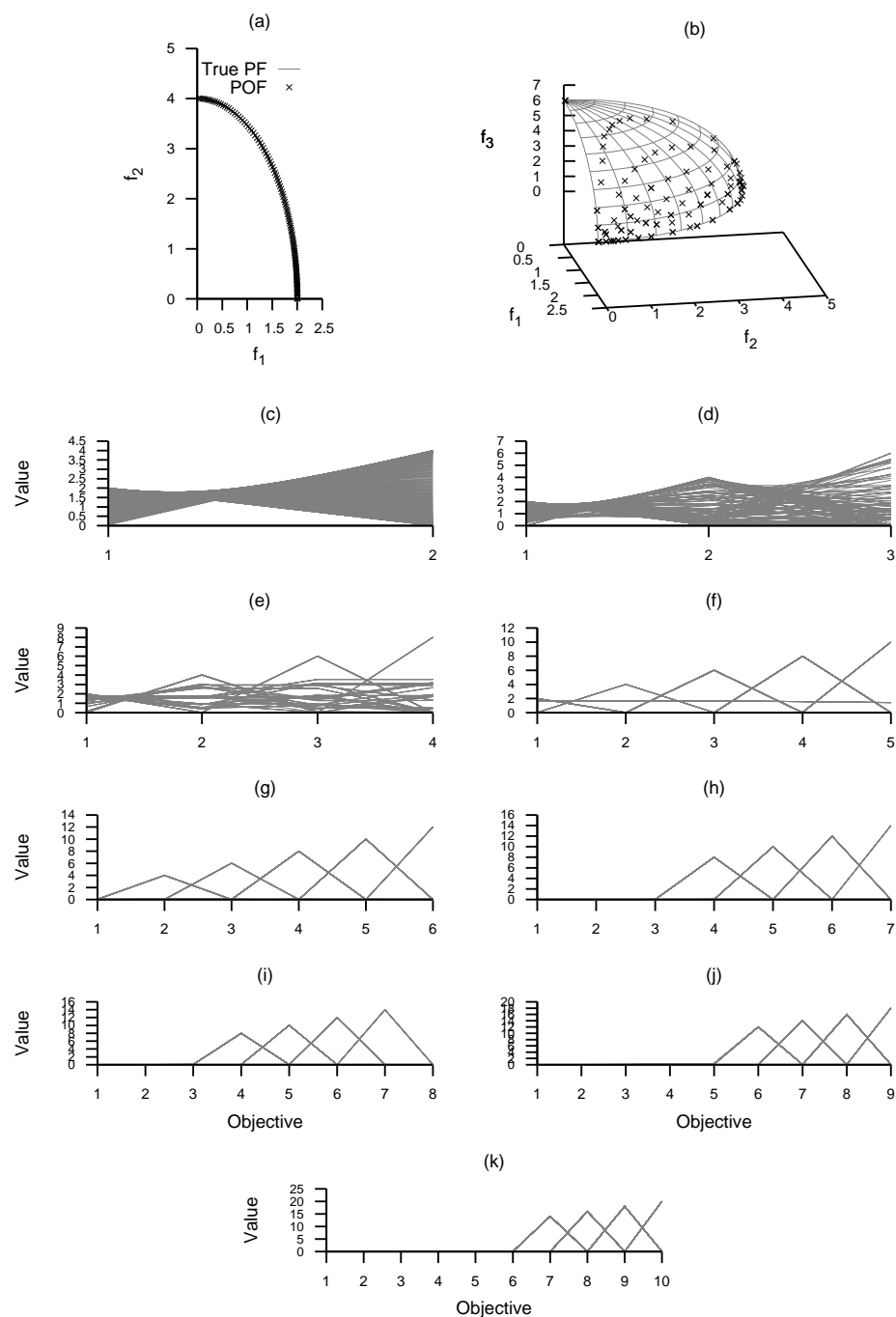


Figure A.365: Plots of the approximations obtained by MOMBI-MTCH from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

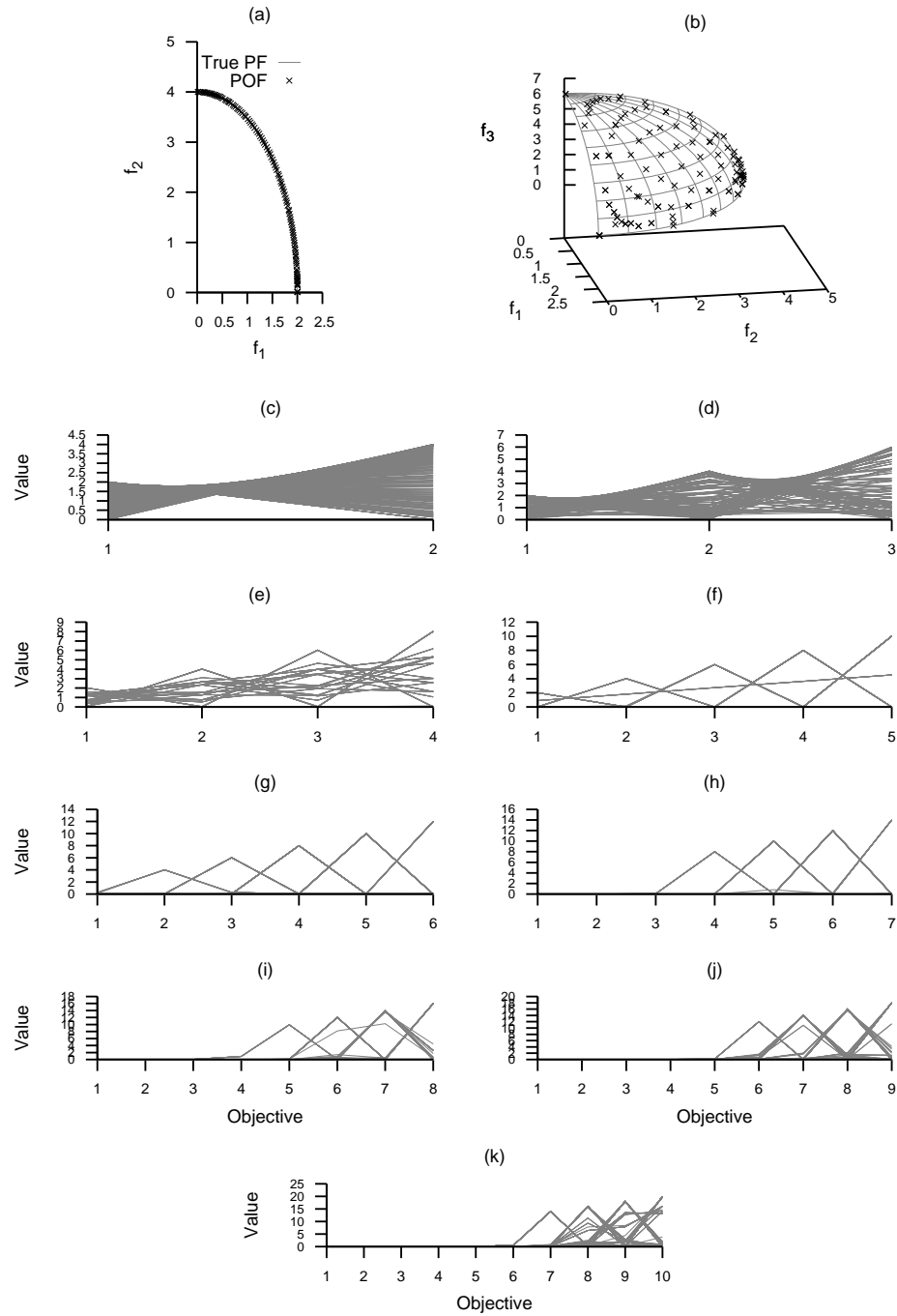


Figure A.366: Plots of the approximations obtained by MOMBI-NMTCH from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.153: Comparison of hypervolume indicator values for different optimizers on the WFG5 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	6.568678e+00	6.566457e+00	7.388654e+00	7.607138e+00	8.125500e+00	8.129355e+00	8.121611e+00	8.124547e+00	8.127097e+00	8.128985e+00	8.093644e+00
	avg.	6.579417e+00	6.575917e+00	7.390197e+00	7.609968e+00	8.133667e+00	8.133499e+00	8.124147e+00	8.125414e+00	8.129969e+00	8.134792e+00	8.095784e+00
	min.	6.503390e+00	6.531901e+00	6.610281e+00	7.040728e+00	8.109335e+00	8.089862e+00	8.101566e+00	8.108858e+00	8.114145e+00	8.113748e+00	8.064608e+00
	max.	6.602480e+00	6.602428e+00	7.597993e+00	7.805624e+00	8.198402e+00	8.167940e+00	8.157779e+00	8.157520e+00	8.184364e+00	8.202842e+00	8.149014e+00
	std.	2.419933e-02	2.294137e-02	1.562972e-01	1.174490e-01	1.741494e-02	1.484110e-02	1.115430e-02	8.334502e-03	1.240880e-02	1.672177e-02	1.565664e-02
3D	med.	5.348728e+01	5.348716e+01	5.598714e+01	5.348707e+01	6.972794e+01	7.097946e+01	7.050682e+01	7.175193e+01	7.020635e+01	7.117266e+01	7.073992e+01
	avg.	5.345233e+01	5.343760e+01	5.671882e+01	5.350540e+01	6.970426e+01	7.095419e+01	7.046725e+01	7.174221e+01	7.019436e+01	7.112621e+01	7.075624e+01
	min.	5.272926e+01	5.307449e+01	5.307541e+01	5.307139e+01	6.924826e+01	7.029598e+01	6.990316e+01	7.111399e+01	6.966036e+01	7.039449e+01	7.003869e+01
	max.	5.361557e+01	5.351246e+01	6.330045e+01	5.824250e+01	7.090126e+01	7.137271e+01	7.125780e+01	7.235050e+01	7.075793e+01	7.185570e+01	7.197908e+01
	std.	1.278410e-01	1.345387e-01	3.083779e+00	6.421632e-01	2.639675e-01	2.555284e-01	2.735393e-01	2.679299e-01	2.450939e-01	2.870813e-01	3.212496e-01
4D	med.	5.216130e+02	5.241565e+02	5.245735e+02	5.214105e+02	6.315117e+02	6.575341e+02	6.100561e+02	6.515195e+02	6.100373e+02	6.334710e+02	6.893034e+02
	avg.	5.224930e+02	5.228370e+02	5.278396e+02	5.235512e+02	6.330228e+02	6.583645e+02	6.101920e+02	6.516785e+02	6.102813e+02	6.339533e+02	6.893383e+02
	min.	5.182307e+02	5.182246e+02	5.200753e+02	5.182000e+02	6.061454e+02	6.405795e+02	6.030848e+02	6.396162e+02	6.037184e+02	6.264798e+02	6.811589e+02
	max.	5.254372e+02	5.251748e+02	5.661887e+02	5.606408e+02	6.802827e+02	6.848058e+02	6.189281e+02	6.647053e+02	6.211751e+02	6.528182e+02	6.997767e+02
	std.	1.947218e+00	1.917338e+00	9.561949e+00	6.658536e+00	1.434957e+01	1.494417e+00	3.087203e+00	4.939359e+00	3.272220e+00	4.621994e+00	3.383602e+00
5D	med.	6.101135e+03	6.100196e+03	6.093194e+03	6.078970e+03	7.240220e+03	7.690376e+03	6.294846e+03	6.259859e+03	6.277747e+03	6.253840e+03	7.502574e+03
	avg.	6.155343e+03	6.124412e+03	6.098614e+03	6.132370e+03	7.265915e+03	7.678372e+03	6.307443e+03	6.259807e+03	6.281326e+03	6.254259e+03	7.502481e+03
	min.	6.044221e+03	6.041721e+03	6.050829e+03	6.043685e+03	6.688947e+03	6.988771e+03	6.252281e+03	6.213245e+03	6.231579e+03	6.214465e+03	7.124650e+03
	max.	6.706713e+03	6.620495e+03	6.434363e+03	7.118467e+03	7.828443e+03	8.002569e+03	6.457376e+03	6.315600e+03	6.327208e+03	6.334157e+03	7.713789e+03
	std.	1.303717e+02	9.551235e+01	5.032476e+01	1.449448e+02	2.867155e+02	1.778095e+02	1.512737e+01	2.165628e+01	2.154193e+01	2.049913e+01	1.209082e+02
6D	med.	8.122378e+04	7.250768e+04	7.885878e+04	7.652947e+04	9.408159e+04	8.843183e+04	7.852884e+04	7.102501e+04	8.188431e+04	7.148639e+04	9.387380e+04
	avg.	8.131772e+04	7.326064e+04	7.886563e+04	7.620896e+04	9.379539e+04	8.893164e+04	7.858692e+04	7.074637e+04	8.071998e+04	7.162315e+04	9.340320e+04
	min.	6.913380e+04	6.130294e+04	7.762944e+04	6.113330e+04	8.329197e+04	7.561283e+04	7.764083e+04	5.956678e+04	7.687931e+04	5.999789e+04	4.514629e+04
	max.	9.078709e+04	8.654230e+04	8.452018e+04	9.262887e+04	1.023326e+05	1.012047e+05	8.196691e+04	8.061628e+04	8.600106e+04	8.188629e+04	1.014639e+05
	std.	4.631172e+03	6.148070e+03	8.967332e+02	5.841206e+03	4.305630e+03	5.639692e+03	6.604480e+02	2.488650e+03	2.077339e+03	2.976665e+03	6.326311e+03
7D	med.	9.175551e+05	5.816589e+05	1.110747e+06	5.722663e+05	1.268138e+06	1.004266e+06	1.105074e+06	5.565294e+05	1.090969e+06	5.611265e+05	1.337172e+06
	avg.	9.090867e+05	6.058541e+05	1.108343e+06	6.080598e+05	1.262356e+06	1.014250e+06	1.097878e+06	5.566719e+05	1.087412e+06	5.709680e+05	1.296323e+06
	min.	5.714621e+05	5.615334e+05	1.082578e+06	5.570160e+05	1.004008e+06	8.566273e+05	7.436222e+05	5.464143e+05	6.536376e+05	5.531363e+05	6.347856e+05
	max.	1.156099e+06	8.734215e+05	1.176425e+06	1.043756e+06	1.472058e+06	1.193855e+06	1.178414e+06	6.038188e+05	1.217729e+06	9.813459e+05	1.450018e+06
	std.	1.437729e+05	5.865048e+04	1.358142e+04	8.279409e+04	1.210119e+05	7.121227e+04	4.268794e+04	7.982909e+03	7.991177e+04	5.336570e+04	1.420874e+05
8D	med.	1.246322e+07	9.213563e+06	1.873993e+07	1.032801e+07	1.717930e+07	1.518398e+07	1.843801e+07	8.180055e+06	1.800733e+07	8.333660e+06	2.094185e+07
	avg.	1.248322e+07	9.640493e+06	1.867909e+07	9.952564e+06	1.734334e+07	1.517259e+07	1.790583e+07	8.340592e+06	1.778268e+07	8.478512e+06	1.898352e+07
	min.	8.411235e+06	8.328703e+06	1.696634e+07	8.258904e+06	1.492940e+07	1.203087e+07	9.110778e+06	8.011301e+06	1.075679e+07	8.114210e+06	1.029725e+07
	max.	1.847728e+07	1.447200e+07	2.014038e+07	1.370663e+07	2.144081e+07	1.758792e+07	1.977887e+07	1.033612e+07	2.194529e+07	1.179829e+07	2.413317e+07
	std.	2.830474e+06	1.136533e+06	4.362727e+05	1.385331e+06	1.324158e+06	1.055815e+06	1.791182e+06	5.414670e+05	1.985558e+06	5.629063e+05	4.344009e+06
9D	med.	1.922477e+08	1.816531e+08	3.533319e+08	2.079959e+08	2.927297e+08	2.602578e+08	3.461181e+08	1.753937e+08	3.339115e+08	1.776852e+08	3.160117e+08
	avg.	1.989461e+08	1.799469e+08	3.545006e+08	2.052087e+08	2.951045e+08	2.625677e+08	3.359881e+08	1.667034e+08	3.139134e+08	1.761410e+08	2.903635e+08
	min.	1.434119e+08	1.431295e+08	3.188961e+08	1.435726e+08	2.400622e+08	2.068944e+08	1.424086e+08	1.361429e+08	1.722802e+08	1.373252e+08	1.933040e+08
	max.	2.991607e+08	2.637516e+08	3.964790e+08	2.967000e+08	3.654624e+08	3.375619e+08	3.732496e+08	2.362357e+08	3.809397e+08	2.718483e+08	4.223762e+08
	std.	3.277632e+07	2.829355e+07	1.416110e+07	3.355870e+07	2.104485e+07	2.675795e+07	3.622163e+07	2.523010e+07	5.249437e+07	3.252775e+07	8.133381e+07
10D	med.	4.155736e+09	4.159852e+09	7.278687e+09	4.868818e+09	5.555625e+09	5.184389e+09	6.884765e+09	4.353575e+09	6.271534e+09	4.415054e+09	4.120829e+09
	avg.	4.117336e+09	4.184942e+09	7.351385e+09	4.823090e+09	5.514981e+09	5.190267e+09	6.653427e+09	4.320596e+09	6.031433e+09	4.415914e+09	5.005058e+09
	min.	2.697426e+09	2.766009e+09	6.456259e+09	2.785251e+09	4.453675e+09	3.592761e+09	3.569805e+09	2.881147e+09	3.449496e+09	2.726957e+09	3.796002e+09
	max.	6.485068e+09	5.666169e+09	8.448639e+09	6.069204e+09	6.631803e+09	6.365398e+09	7.556336e+09	5.618361e+09	8.409555e+09	5.822111e+09	8.408899e+09
	std.	8.078422e+08	6.550299e+08	4.916042e+08	6.420973e+08	4.690451e+08	5.552914e+08	7.909852e+08	5.435154e+08	1.228986e+09	6.766391e+08	1.366123e+09

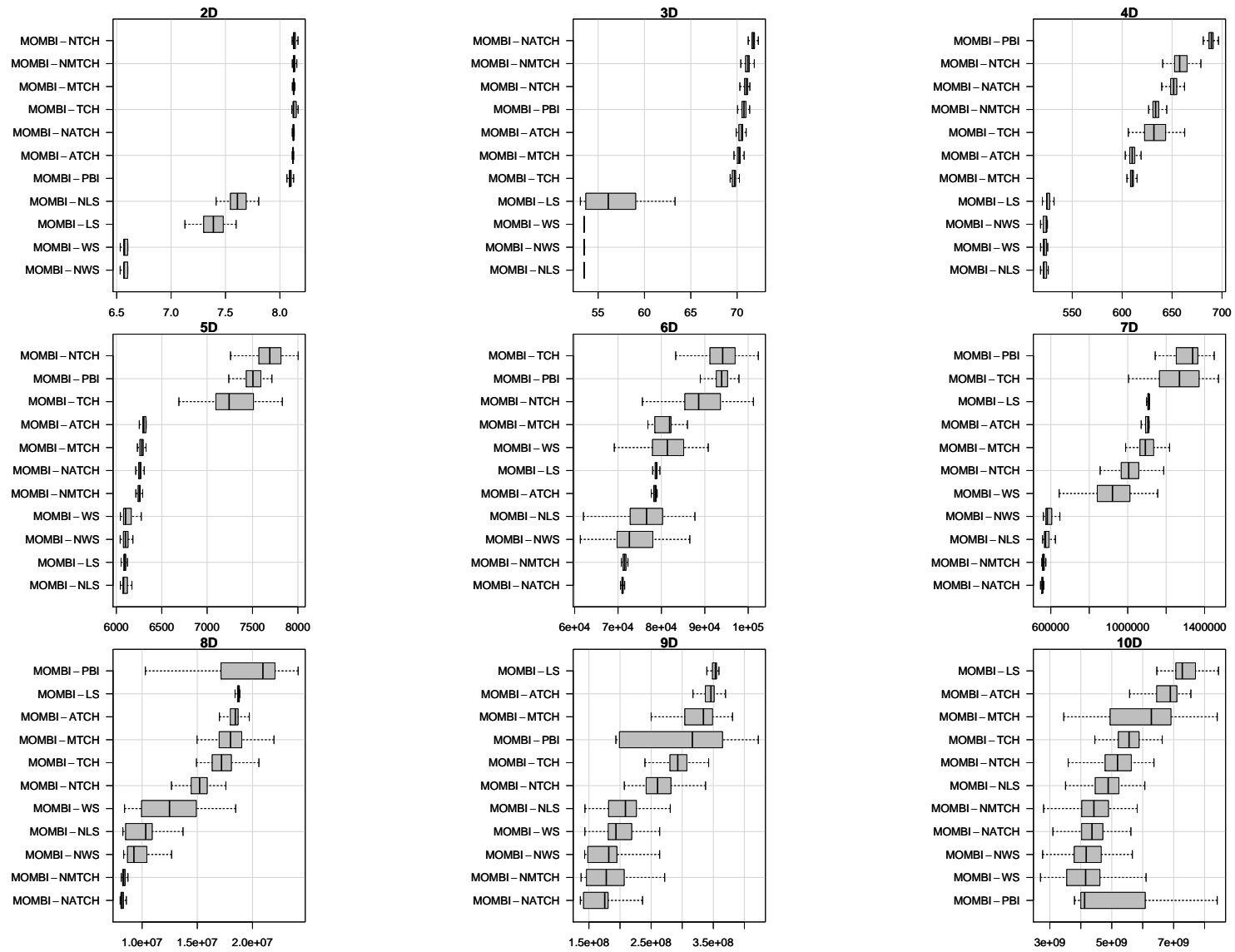


Figure A.367: Box-plot of the hypervolume indicator values for different optimizers on the WFG5 test problem.

Table A.154: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG5 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
2D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$1.28e-34$	$1.28e-34$	$1.41e-21$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	> 0.05	$2.26e-08$	$6.96e-03$	> 0.05	> 0.05	$9.80e-29$
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	–	$1.15e-10$	$5.39e-07$	$8.80e-03$	> 0.05	$2.63e-29$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$6.80e-25$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$6.60e-05$	–	> 0.05	> 0.05	$4.08e-27$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$5.29e-09$	$2.79e-03$	–	> 0.05	$7.46e-29$
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$4.53e-02$	> 0.05	$3.95e-12$	$8.23e-07$	$1.04e-02$	–	$5.14e-30$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
3D											
MOMBI-WS	–	$4.59e-03$	> 0.05	$7.50e-04$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.41e-28$	$4.41e-29$	–	$9.17e-28$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.22e-34$	$1.27e-34$	$1.28e-34$	$1.27e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	$1.22e-34$	$1.27e-34$	$1.28e-34$	$1.27e-34$	$4.00e-34$	–	$1.20e-22$	> 0.05	$2.75e-32$	> 0.05	$4.50e-07$
MOMBI-ATCH	$1.22e-34$	$1.27e-34$	$1.28e-34$	$1.27e-34$	$1.01e-31$	> 0.05	–	> 0.05	$1.82e-10$	> 0.05	> 0.05
MOMBI-NATCH	$1.22e-34$	$1.27e-34$	$1.28e-34$	$1.27e-34$	$1.28e-34$	$6.05e-33$	$1.53e-34$	–	$1.28e-34$	$7.24e-28$	$1.33e-32$
MOMBI-MTCH	$1.22e-34$	$1.27e-34$	$1.28e-34$	$1.27e-34$	$8.53e-25$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$1.22e-34$	$1.27e-34$	$1.28e-34$	$1.27e-34$	$2.48e-34$	$5.02e-06$	$4.35e-28$	> 0.05	$1.20e-33$	–	$3.74e-15$
MOMBI-PBI	$1.22e-34$	$1.27e-34$	$1.28e-34$	$1.27e-34$	$1.20e-33$	> 0.05	$1.62e-10$	> 0.05	$2.59e-25$	> 0.05	–
4D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$2.01e-16$	$3.36e-16$	–	$5.49e-13$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	> 0.05	$3.41e-28$	> 0.05	$4.71e-28$	> 0.05	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$5.02e-26$	–	$1.28e-34$	$2.42e-08$	$1.28e-34$	$8.40e-34$	> 0.05
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$7.30e-22$	> 0.05	$1.28e-34$	–	$1.28e-34$	$3.47e-33$	> 0.05
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	–	> 0.05
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.53e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–



Table A.155: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG5 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
5D											
MOMBI-WS	–	> 0.05	$7.06e-03$	$2.27e-03$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	$3.30e-02$	$2.45e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.36e-34$	$1.28e-34$	$1.28e-34$	$3.05e-34$	–	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.32e-34$	$2.05e-21$	–	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.28e-14$
MOMBI-ATCH	$4.10e-17$	$3.59e-26$	$4.37e-32$	$6.36e-21$	> 0.05	> 0.05	–	$3.59e-26$	$3.92e-14$	$2.23e-29$	> 0.05
MOMBI-NATCH	$2.62e-16$	$8.19e-26$	$4.63e-32$	$8.19e-21$	> 0.05	> 0.05	> 0.05	–	> 0.05	$1.76e-02$	> 0.05
MOMBI-MTCH	$1.05e-16$	$4.41e-26$	$4.63e-32$	$8.19e-21$	> 0.05	> 0.05	> 0.05	$1.19e-10$	–	$5.19e-15$	> 0.05
MOMBI-NMTCH	$2.89e-16$	$9.81e-26$	$4.63e-32$	$8.19e-21$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.30e-10$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–
6D											
MOMBI-WS	–	$6.39e-18$	$9.46e-06$	$1.51e-10$	> 0.05	> 0.05	$1.09e-06$	$1.02e-32$	> 0.05	$2.16e-30$	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.30e-07$	> 0.05	$9.04e-05$	> 0.05
MOMBI-LS	> 0.05	$2.67e-13$	–	$6.87e-05$	> 0.05	> 0.05	$1.48e-06$	$2.30e-33$	> 0.05	$1.36e-28$	> 0.05
MOMBI-NLS	> 0.05	$7.29e-05$	> 0.05	–	> 0.05	> 0.05	> 0.05	$3.73e-24$	> 0.05	$7.30e-21$	> 0.05
MOMBI-TCH	$2.52e-31$	$2.20e-34$	$1.49e-34$	$1.10e-33$	–	$9.59e-10$	$1.28e-34$	$1.28e-34$	$1.58e-34$	$1.28e-34$	> 0.05
MOMBI-NTCH	$7.10e-18$	$6.61e-31$	$1.22e-28$	$1.03e-27$	> 0.05	–	$1.52e-29$	$1.53e-34$	$1.42e-22$	$6.06e-34$	> 0.05
MOMBI-ATCH	> 0.05	$2.09e-12$	> 0.05	$3.75e-04$	> 0.05	> 0.05	–	$2.30e-33$	> 0.05	$1.51e-28$	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	> 0.05	$1.06e-18$	$3.64e-07$	$5.65e-12$	> 0.05	> 0.05	$3.79e-10$	$3.76e-34$	–	$4.50e-32$	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.50e-13$	> 0.05	–	> 0.05
MOMBI-PBI	$7.00e-31$	$4.13e-32$	$5.05e-32$	$1.74e-31$	> 0.05	$5.35e-11$	$5.05e-32$	$4.25e-32$	$2.99e-31$	$4.91e-32$	–
7D											
MOMBI-WS	–	$5.54e-28$	> 0.05	$2.98e-28$	> 0.05	> 0.05	> 0.05	$1.84e-34$	> 0.05	$9.12e-33$	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	$2.20e-04$	> 0.05	> 0.05	> 0.05	$1.89e-32$	> 0.05	$7.52e-25$	> 0.05
MOMBI-LS	$3.32e-28$	$1.28e-34$	–	$1.28e-34$	> 0.05	$8.81e-22$	$1.02e-06$	$1.28e-34$	$3.64e-05$	$1.28e-34$	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$1.84e-29$	> 0.05	$2.72e-16$	> 0.05
MOMBI-TCH	$1.68e-32$	$1.28e-34$	$5.39e-20$	$1.40e-34$	–	$9.21e-30$	$1.41e-21$	$1.28e-34$	$1.11e-21$	$1.28e-34$	> 0.05
MOMBI-NTCH	$1.11e-08$	$1.32e-34$	> 0.05	$1.16e-33$	> 0.05	–	> 0.05	$1.28e-34$	> 0.05	$3.76e-34$	> 0.05
MOMBI-ATCH	$4.19e-26$	$1.40e-34$	> 0.05	$1.58e-34$	> 0.05	$2.92e-19$	–	$1.28e-34$	$3.37e-02$	$1.36e-34$	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$3.42e-22$	$2.33e-34$	> 0.05	$4.24e-34$	> 0.05	$3.84e-14$	> 0.05	$1.28e-34$	–	$1.53e-34$	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.61e-13$	> 0.05	–	> 0.05
MOMBI-PBI	$1.59e-30$	$6.06e-34$	$2.47e-28$	$7.92e-34$	$3.09e-03$	$1.25e-29$	$2.21e-28$	$1.28e-34$	$5.25e-28$	$1.53e-34$	–

Table A.156: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG5 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
8D											
MOMBI-WS	–	$3.02e-13$	> 0.05	$7.03e-12$	> 0.05	> 0.05	> 0.05	$1.01e-31$	> 0.05	$5.42e-31$	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$6.00e-28$	> 0.05	$1.30e-25$	> 0.05
MOMBI-LS	$3.15e-34$	$1.28e-34$	–	$1.28e-34$	$1.65e-16$	$2.01e-34$	$4.03e-10$	$1.28e-34$	$3.61e-06$	$1.28e-34$	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$1.17e-27$	> 0.05	$1.45e-20$	> 0.05
MOMBI-TCH	$4.59e-28$	$1.28e-34$	> 0.05	$1.28e-34$	–	$4.12e-24$	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05
MOMBI-NTCH	$8.46e-12$	$3.15e-34$	> 0.05	$1.84e-34$	> 0.05	–	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05
MOMBI-ATCH	$1.36e-28$	$1.10e-33$	> 0.05	$2.66e-33$	$3.45e-08$	$2.87e-25$	–	$1.58e-34$	> 0.05	$1.63e-34$	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$3.22e-27$	$2.96e-34$	> 0.05	$6.83e-34$	$1.83e-04$	$1.98e-22$	> 0.05	$1.28e-34$	–	$1.36e-34$	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$6.28e-10$	> 0.05	–	> 0.05
MOMBI-PBI	$1.92e-19$	$2.83e-32$	$1.27e-07$	$1.51e-28$	$3.91e-09$	$6.25e-12$	$9.10e-09$	$1.32e-34$	$3.40e-08$	$2.26e-34$	–
9D											
MOMBI-WS	–	$1.12e-05$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.44e-13$	> 0.05	$8.96e-07$	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$6.57e-06$	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$1.02e-32$	$1.84e-34$	$3.69e-11$	$1.28e-34$	$3.44e-14$	$1.28e-34$	$2.55e-06$
MOMBI-NLS	> 0.05	$3.69e-08$	> 0.05	–	> 0.05	> 0.05	> 0.05	$2.16e-17$	> 0.05	$3.14e-09$	> 0.05
MOMBI-TCH	$3.47e-33$	$1.84e-34$	> 0.05	$5.39e-33$	–	$3.75e-16$	> 0.05	$1.28e-34$	> 0.05	$2.01e-34$	> 0.05
MOMBI-NTCH	$3.78e-26$	$7.56e-32$	> 0.05	$1.16e-23$	> 0.05	–	> 0.05	$2.96e-34$	> 0.05	$7.14e-32$	> 0.05
MOMBI-ATCH	$1.04e-31$	$2.52e-32$	> 0.05	$1.79e-31$	$6.33e-22$	$2.05e-27$	–	$5.55e-33$	$4.20e-05$	$1.12e-32$	$2.81e-03$
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$1.14e-26$	$3.20e-30$	> 0.05	$2.87e-25$	$1.22e-09$	$8.82e-16$	> 0.05	$1.01e-31$	–	$9.74e-30$	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.10e-03$	> 0.05	–	> 0.05
MOMBI-PBI	$2.91e-15$	$1.60e-24$	> 0.05	$3.33e-12$	> 0.05	$4.35e-02$	> 0.05	$1.93e-28$	> 0.05	$1.20e-21$	–
10D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.40e-34$	$1.28e-34$	–	$1.28e-34$	$1.84e-34$	$1.28e-34$	$1.29e-13$	$1.28e-34$	$2.16e-17$	$1.28e-34$	$9.55e-24$
MOMBI-NLS	$3.51e-11$	$1.23e-10$	> 0.05	–	> 0.05	> 0.05	> 0.05	$3.58e-09$	> 0.05	$1.07e-05$	> 0.05
MOMBI-TCH	$1.06e-25$	$5.38e-29$	> 0.05	$2.41e-14$	–	$1.99e-05$	> 0.05	$4.69e-29$	> 0.05	$5.55e-24$	$2.04e-06$
MOMBI-NTCH	$2.67e-19$	$8.00e-21$	> 0.05	$4.56e-05$	> 0.05	–	> 0.05	$8.62e-20$	> 0.05	$1.63e-14$	$1.92e-04$
MOMBI-ATCH	$2.16e-30$	$1.68e-30$	> 0.05	$5.11e-28$	$6.44e-24$	$4.77e-26$	–	$5.91e-30$	$4.99e-05$	$1.22e-29$	$5.57e-14$
MOMBI-NATCH	$7.50e-03$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$2.30e-21$	$5.67e-21$	> 0.05	$4.53e-12$	$3.03e-05$	$3.22e-08$	> 0.05	$2.00e-19$	–	$9.34e-18$	$8.47e-08$
MOMBI-NMTCH	$1.05e-03$	$8.24e-03$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	$9.86e-05$	$1.23e-03$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–

Table A.157: Comparison of R2 indicator values for different optimizers on the WFG5 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	6.819138e-01	6.820496e-01	6.496706e-01	6.386485e-01	6.090640e-01	6.090236e-01	6.092733e-01	6.091779e-01	6.090292e-01	6.090311e-01	6.105429e-01
	avg.	6.818650e-01	6.819228e-01	6.487767e-01	6.383011e-01	6.086890e-01	6.087894e-01	6.091413e-01	6.091397e-01	6.088806e-01	6.087073e-01	6.104793e-01
	min.	6.814253e-01	6.814259e-01	6.376423e-01	6.277115e-01	6.060164e-01	6.072105e-01	6.076272e-01	6.076423e-01	6.064855e-01	6.059925e-01	6.081199e-01
	max.	6.840904e-01	6.829893e-01	6.811256e-01	6.653723e-01	6.097931e-01	6.105419e-01	6.099401e-01	6.098475e-01	6.095771e-01	6.095016e-01	6.119251e-01
	std.	5.232765e-04	4.804436e-04	7.314483e-03	6.143733e-03	7.836244e-04	6.786908e-04	5.024156e-04	3.820869e-04	5.648947e-04	7.331974e-04	7.424841e-04
3D	med.	3.715638e-01	3.715640e-01	3.642799e-01	3.715642e-01	3.260716e-01	3.261454e-01	3.254061e-01	3.255379e-01	3.255765e-01	3.257593e-01	3.286664e-01
	avg.	3.716532e-01	3.716798e-01	3.631468e-01	3.716219e-01	3.264597e-01	3.263191e-01	3.257430e-01	3.256430e-01	3.259228e-01	3.260398e-01	3.287842e-01
	min.	3.712292e-01	3.715637e-01	3.447278e-01	3.658611e-01	3.246853e-01	3.242939e-01	3.240278e-01	3.242188e-01	3.243458e-01	3.243046e-01	3.252195e-01
	max.	3.741156e-01	3.725118e-01	3.725084e-01	3.725208e-01	3.284567e-01	3.281150e-01	3.275992e-01	3.276802e-01	3.280616e-01	3.286549e-01	3.315981e-01
	std.	3.460922e-04	3.085198e-04	8.130167e-03	8.241369e-04	9.376177e-04	9.252194e-04	9.745943e-04	7.700954e-04	9.255336e-04	9.355836e-04	1.177950e-03
4D	med.	2.459222e-01	2.453731e-01	2.452517e-01	2.459443e-01	2.293700e-01	2.275777e-01	2.323431e-01	2.323402e-01	2.319164e-01	2.333363e-01	2.240797e-01
	avg.	2.457699e-01	2.456950e-01	2.446418e-01	2.457220e-01	2.292113e-01	2.278926e-01	2.323360e-01	2.322476e-01	2.319210e-01	2.331038e-01	2.243092e-01
	min.	2.452217e-01	2.452969e-01	2.376430e-01	2.394558e-01	2.227190e-01	2.235856e-01	2.311161e-01	2.285412e-01	2.297520e-01	2.301473e-01	2.206623e-01
	max.	2.471955e-01	2.471970e-01	2.466671e-01	2.472771e-01	2.323204e-01	2.336107e-01	2.338575e-01	2.346118e-01	2.334848e-01	2.347321e-01	2.311285e-01
	std.	4.840737e-04	4.920376e-04	1.686594e-03	9.036273e-04	1.912400e-03	2.090447e-03	6.662603e-04	1.163291e-03	7.740687e-04	9.134723e-04	1.715806e-03
5D	med.	1.866977e-01	1.868105e-01	1.864977e-01	1.868313e-01	1.774474e-01	1.736854e-01	1.832535e-01	1.865737e-01	1.849765e-01	1.866006e-01	1.88782e-01
	avg.	1.862205e-01	1.866184e-01	1.865757e-01	1.865670e-01	1.776359e-01	1.742010e-01	1.834177e-01	1.866364e-01	1.849917e-01	1.867409e-01	1.889966e-01
	min.	1.795136e-01	1.818398e-01	1.854927e-01	1.770419e-01	1.706319e-01	1.707077e-01	1.826212e-01	1.857968e-01	1.858122e-01	1.851222e-01	1.826601e-01
	max.	1.878354e-01	1.878557e-01	1.874356e-01	1.877960e-01	1.828675e-01	1.825199e-01	1.846510e-01	1.875170e-01	1.861751e-01	1.879206e-01	1.981752e-01
	std.	1.530935e-03	1.118055e-03	2.967360e-04	1.310396e-03	2.791764e-03	2.597068e-03	3.788748e-04	4.149291e-04	4.458337e-04	4.530346e-04	3.129578e-03
6D	med.	1.679438e-01	1.756607e-01	1.468645e-01	1.730762e-01	1.706817e-01	1.808716e-01	1.472027e-01	1.758581e-01	1.380563e-01	1.742149e-01	1.618440e-01
	avg.	1.665332e-01	1.945269e-01	1.472335e-01	1.773104e-01	1.820221e-01	1.906907e-01	1.472484e-01	1.776075e-01	1.418162e-01	1.748826e-01	1.628061e-01
	min.	1.342920e-01	1.377185e-01	1.399203e-01	1.381836e-01	1.361413e-01	1.387858e-01	1.372587e-01	1.406379e-01	1.357140e-01	1.376454e-01	1.428884e-01
	max.	2.303438e-01	2.409161e-01	1.508954e-01	2.385675e-01	2.323570e-01	2.327130e-01	1.508736e-01	2.372130e-01	1.571014e-01	2.372028e-01	3.263237e-01
	std.	1.807295e-02	3.197906e-02	1.232860e-03	1.786638e-02	2.675996e-02	2.821933e-02	1.802969e-03	1.276381e-02	6.366015e-03	1.347916e-02	1.886741e-02
7D	med.	1.969912e-01	3.741413e-01	1.418331e-01	3.732178e-01	2.378364e-01	3.490601e-01	1.421229e-01	3.767278e-01	1.442619e-01	3.765069e-01	1.437619e-01
	avg.	2.231293e-01	3.688608e-01	1.422825e-01	3.670126e-01	2.552109e-01	3.390195e-01	1.444612e-01	3.757431e-01	1.481685e-01	3.705950e-01	1.530980e-01
	min.	1.439838e-01	1.999902e-01	1.307853e-01	1.892352e-01	1.565767e-01	1.896933e-01	1.293304e-01	3.418735e-01	1.230665e-01	1.784603e-01	1.287426e-01
	max.	3.795476e-01	3.982767e-01	1.476184e-01	3.783873e-01	3.645934e-01	3.886929e-01	2.650724e-01	3.818221e-01	3.154878e-01	3.807308e-01	2.900602e-01
	std.	6.151421e-02	2.466662e-02	2.025557e-03	3.295005e-02	6.331412e-02	3.690124e-02	1.295142e-02	5.207319e-03	2.422638e-02	2.746628e-02	2.728540e-02
8D	med.	2.930299e-01	3.952845e-01	1.240045e-01	3.937135e-01	3.653328e-01	3.735831e-01	1.261871e-01	3.991820e-01	1.316162e-01	3.983787e-01	1.424705e-01
	avg.	2.989641e-01	3.920184e-01	1.252397e-01	3.938230e-01	3.545694e-01	3.658290e-01	1.365809e-01	3.993344e-01	1.401095e-01	3.979966e-01	1.592308e-01
	min.	1.647152e-01	1.824154e-01	1.160367e-01	3.746668e-01	1.736159e-01	2.485806e-01	1.143888e-01	3.917845e-01	1.139681e-01	3.886262e-01	1.130859e-01
	max.	4.263341e-01	4.293896e-01	1.458319e-01	4.007213e-01	4.052761e-01	4.092574e-01	3.510316e-01	4.038660e-01	2.878582e-01	4.014582e-01	2.583830e-01
	std.	8.463316e-02	2.690025e-02	4.099354e-03	5.051216e-03	3.619902e-02	3.054498e-02	3.447898e-02	2.496393e-03	3.344230e-02	1.842351e-03	4.675565e-02
9D	med.	3.819499e-01	4.104170e-01	1.100824e-01	4.053107e-01	3.860013e-01	3.923500e-01	1.127347e-01	4.125017e-01	1.214694e-01	4.097988e-01	1.434931e-01
	avg.	3.666053e-01	4.066270e-01	1.114765e-01	4.048422e-01	3.794938e-01	3.899904e-01	1.234793e-01	4.118837e-01	1.440968e-01	4.090738e-01	1.724610e-01
	min.	1.990671e-01	2.849838e-01	1.012177e-01	3.746444e-01	2.395117e-01	2.850287e-01	1.012639e-01	4.000608e-01	9.746789e-02	3.897626e-01	1.135536e-01
	max.	4.515904e-01	4.466594e-01	1.295806e-01	4.163909e-01	4.357346e-01	4.394711e-01	3.917097e-01	4.187659e-01	3.059661e-01	4.156287e-01	2.297496e-01
	std.	5.377501e-02	2.114537e-02	5.366247e-03	6.143935e-03	3.673358e-02	3.152679e-02	3.932940e-02	4.001224e-03	5.131233e-02	5.104476e-03	4.647012e-02
10D	med.	4.006414e-01	4.109828e-01	1.022956e-01	4.061477e-01	4.013810e-01	3.992023e-01	1.068617e-01	4.130808e-01	1.352984e-01	4.091477e-01	2.046148e-01
	avg.	3.886333e-01	4.070918e-01	1.045444e-01	4.060098e-01	3.982019e-01	3.956632e-01	1.255788e-01	4.130358e-01	1.532019e-01	4.091072e-01	1.785847e-01
	min.	2.191301e-01	3.196883e-02	9.424389e-02	3.908634e-01	2.921304e-01	2.710638e-01	9.527239e-02	3.992051e-01	9.323098e-02	3.940249e-01	1.048091e-01
	max.	4.726678e-01	4.617043e-01	1.194838e-01	4.245019e-01	4.478907e-01	4.587942e-01	3.356394e-01	4.260094e-01	2.810718e-01	4.229892e-01	2.112232e-01
	std.	4.262445e-02	2.626553e-02	7.350728e-03	6.959164e-03	2.975912e-02	2.679618e-02	4.578847e-02	5.122026e-03	4.895872e-02	6.454280e-03	3.668464e-02

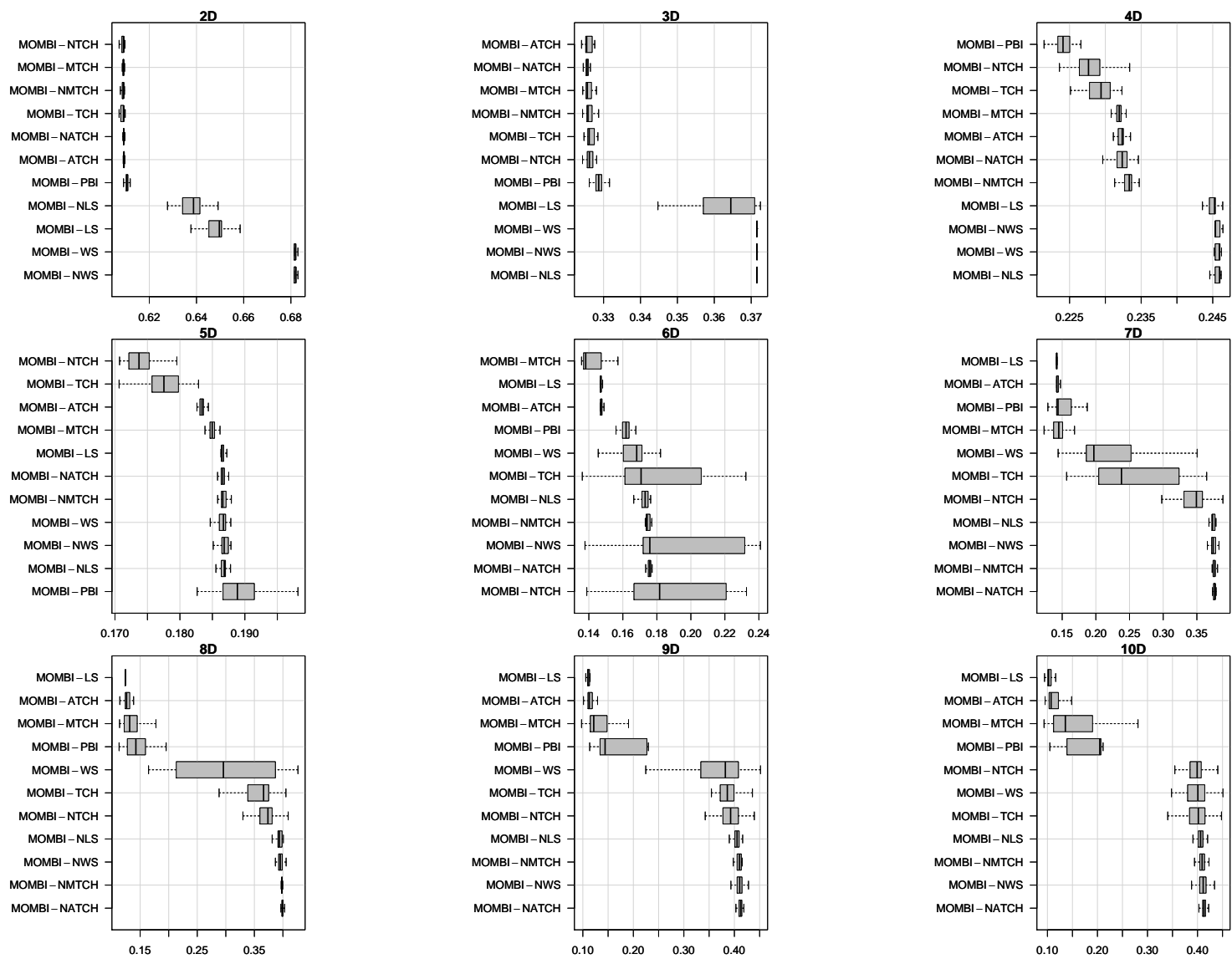


Figure A.368: Box-plot of the R2 indicator values for different optimizers on the WFG5 test problem.

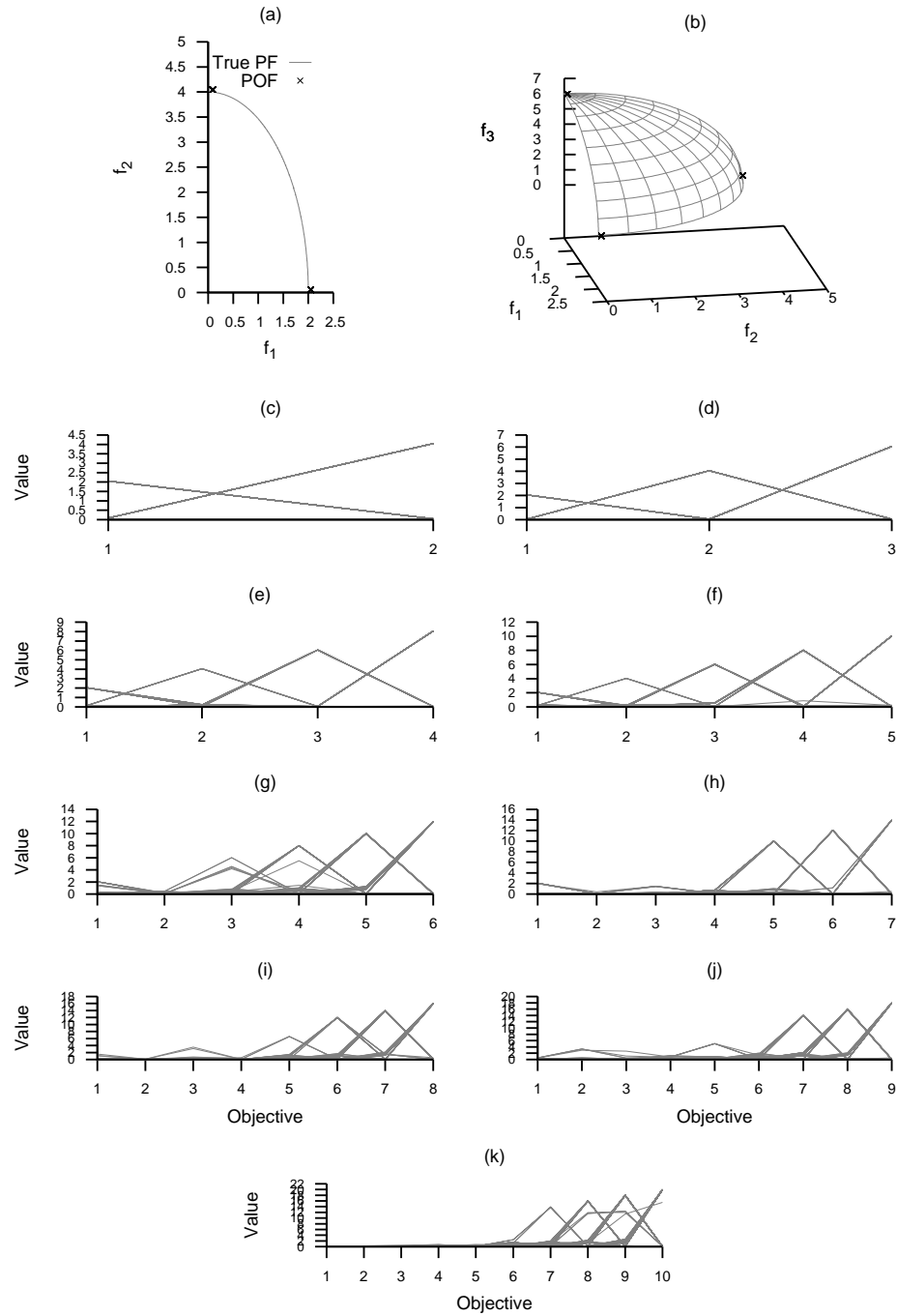


Figure A.369: Plots of the approximations obtained by MOMBI-WS from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

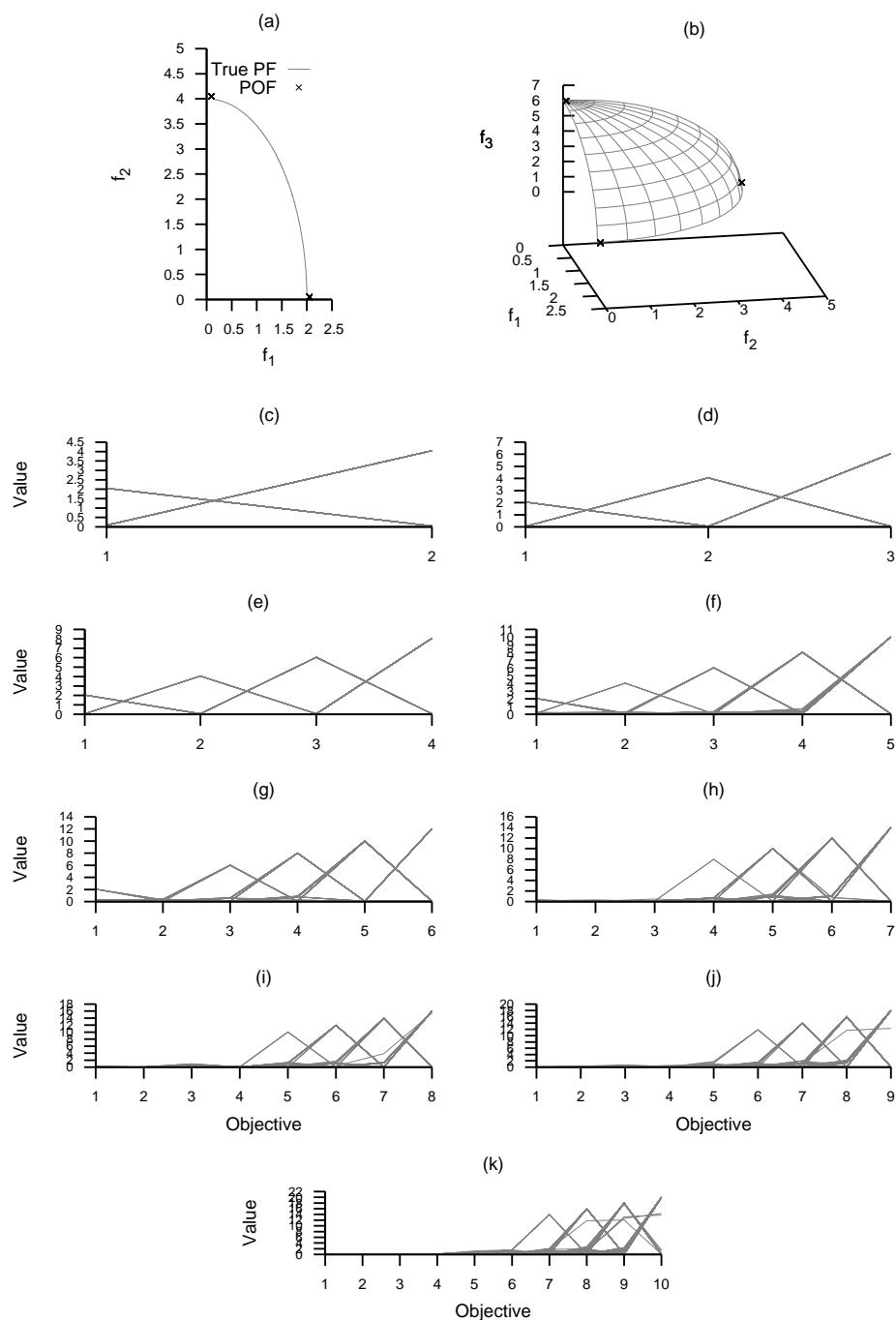


Figure A.370: Plots of the approximations obtained by MOMBI-NWS from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

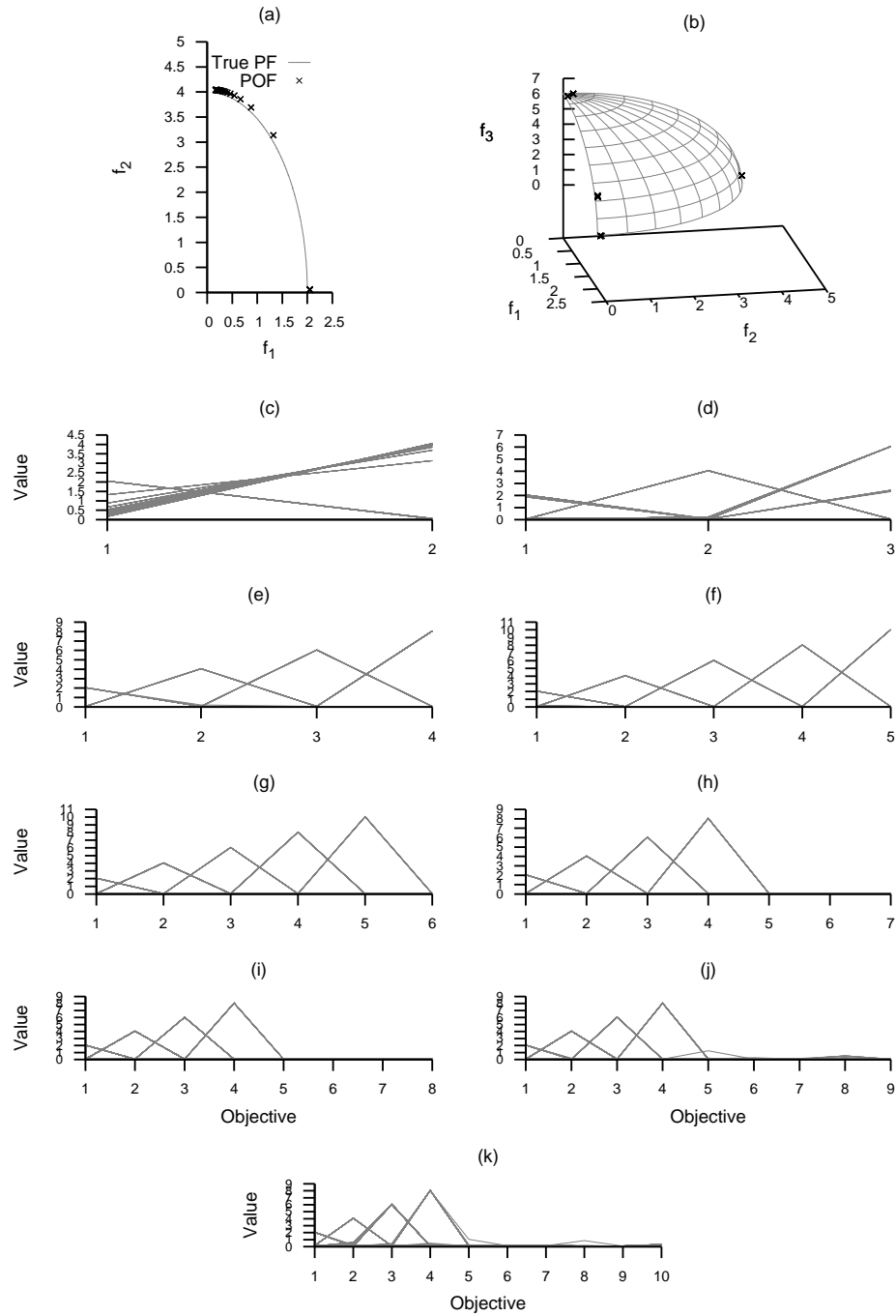


Figure A.371: Plots of the approximations obtained by MOMBI-LS from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

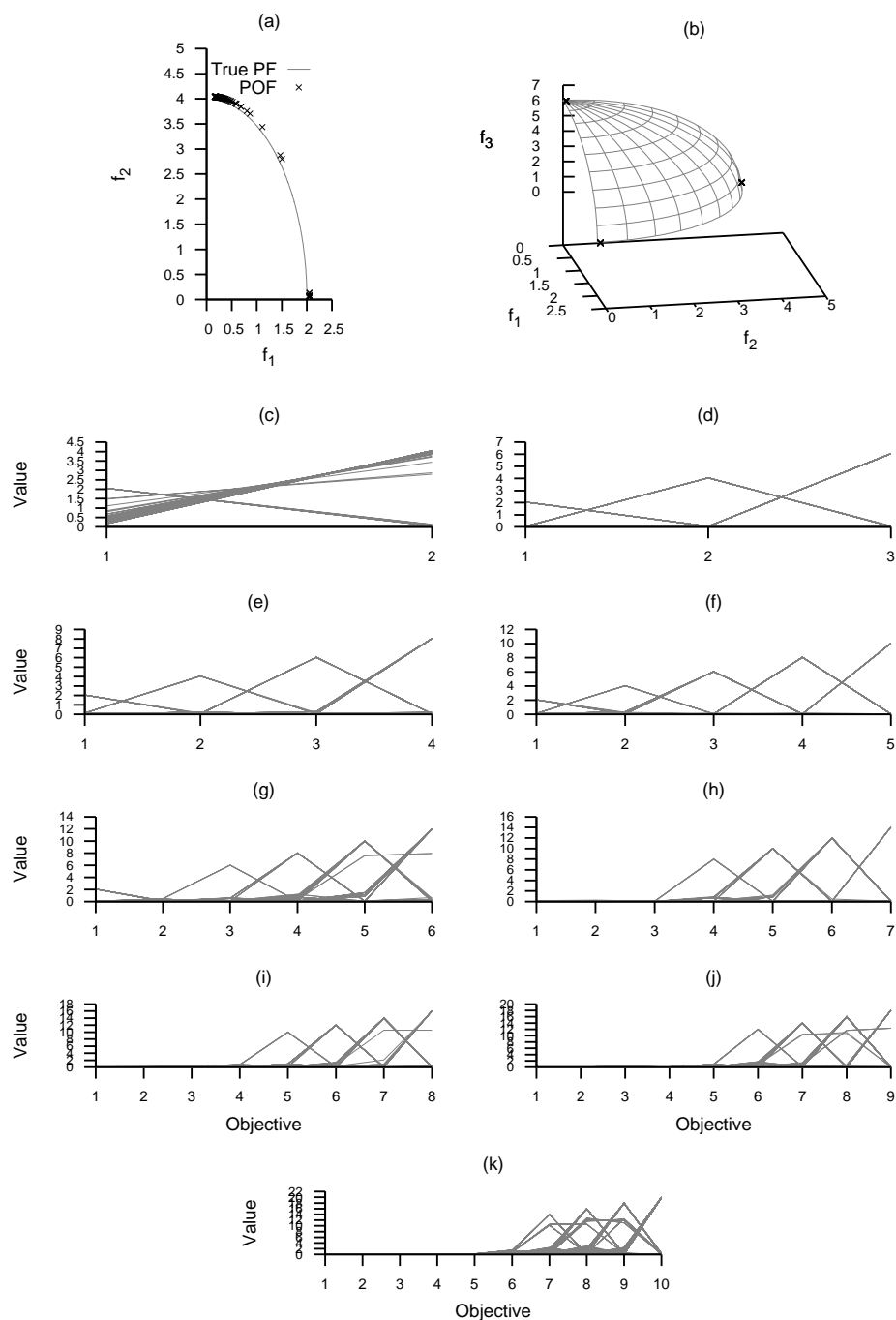


Figure A.372: Plots of the approximations obtained by MOMBI-NLS from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



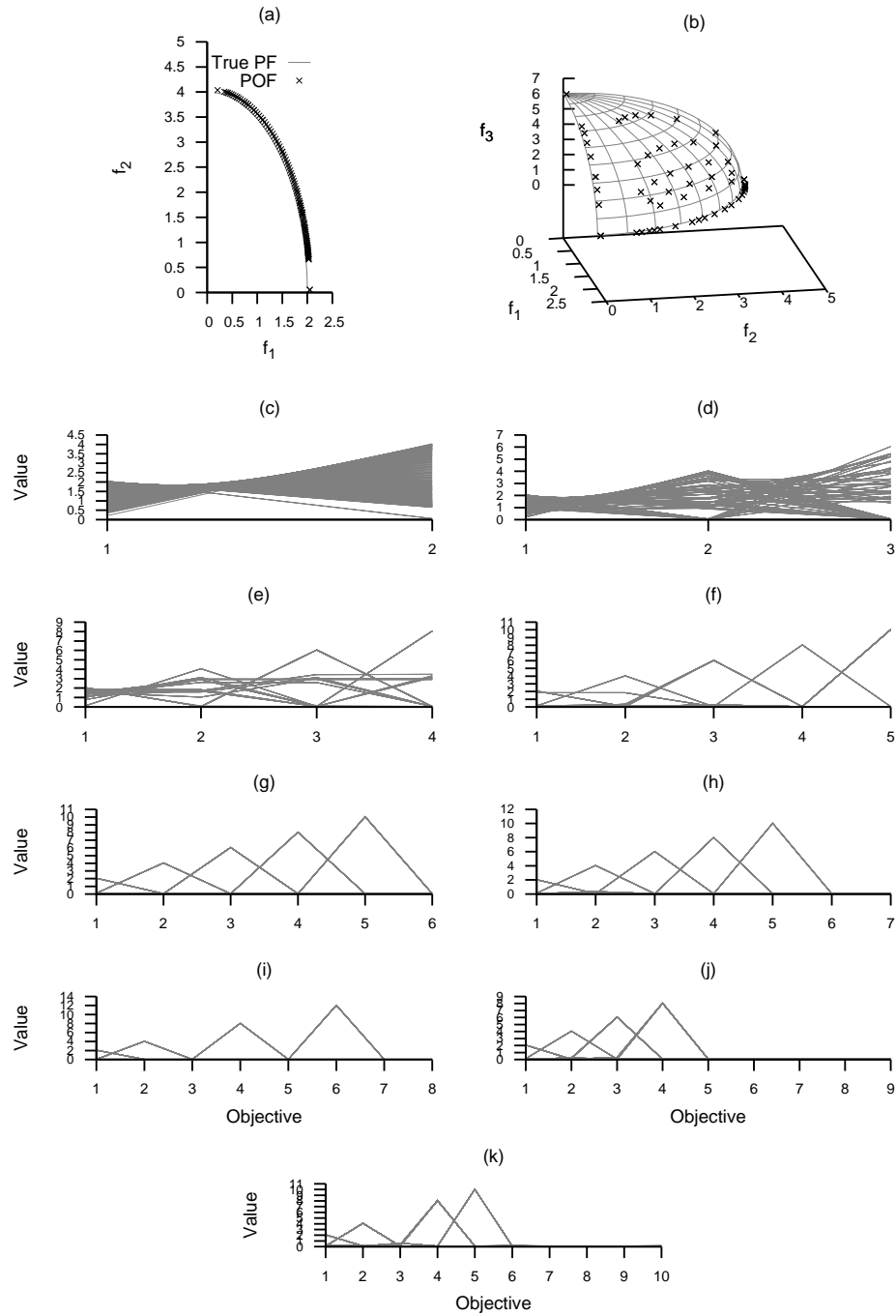


Figure A.373: Plots of the approximations obtained by MOMBI-ATCH from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

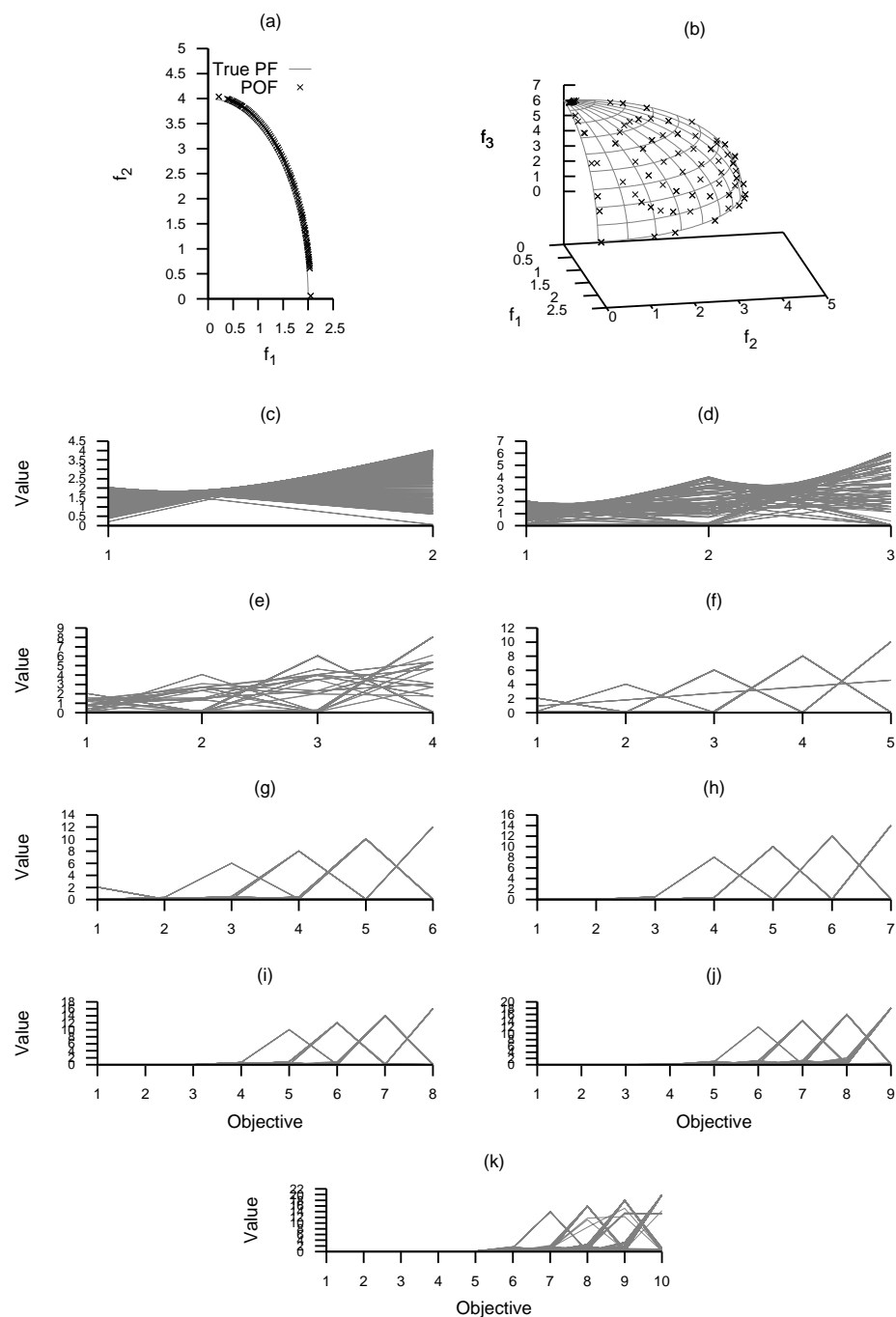


Figure A.374: Plots of the approximations obtained by MOMBI-NATCH from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

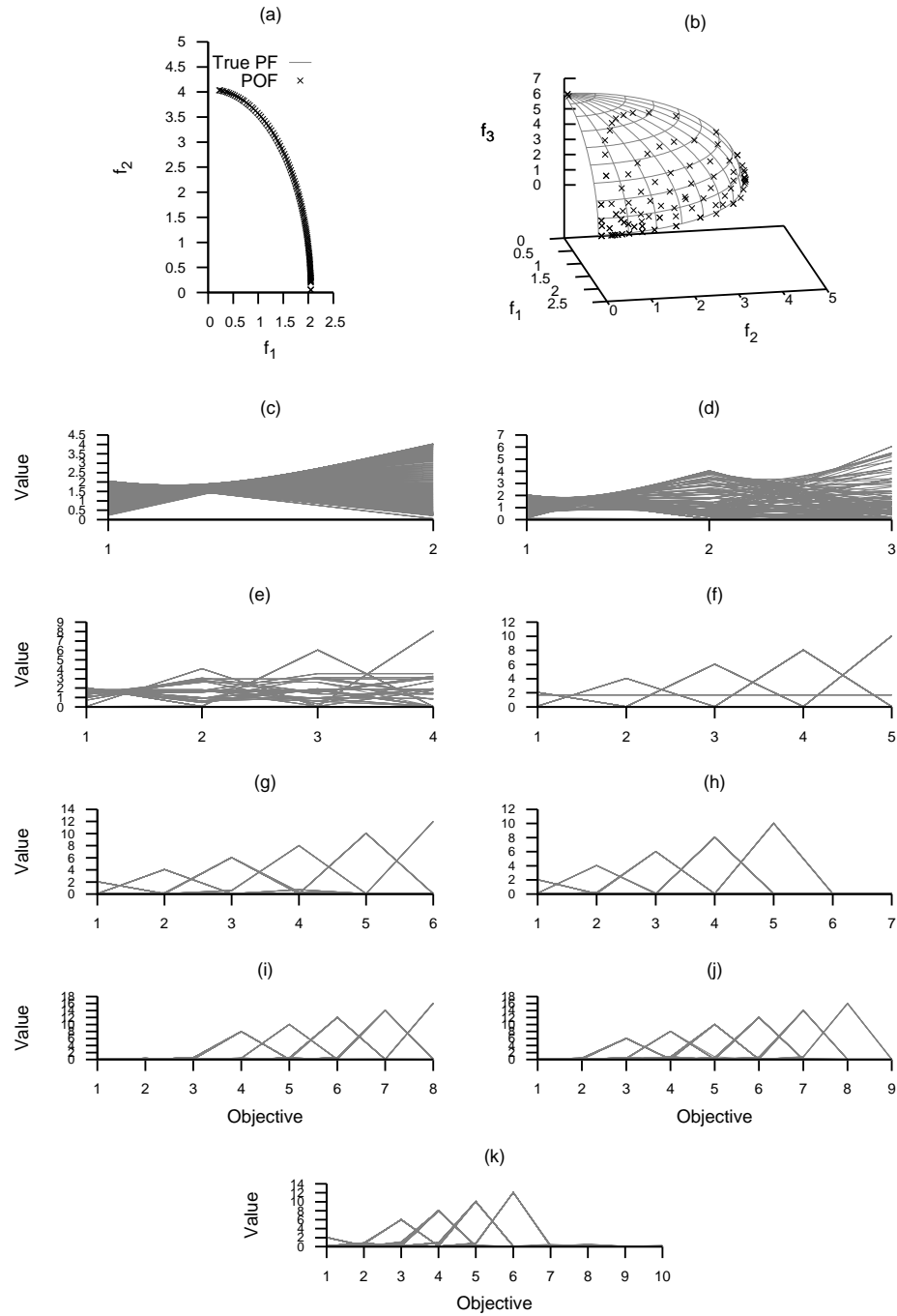


Figure A.375: Plots of the approximations obtained by MOMBI-MTCH from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

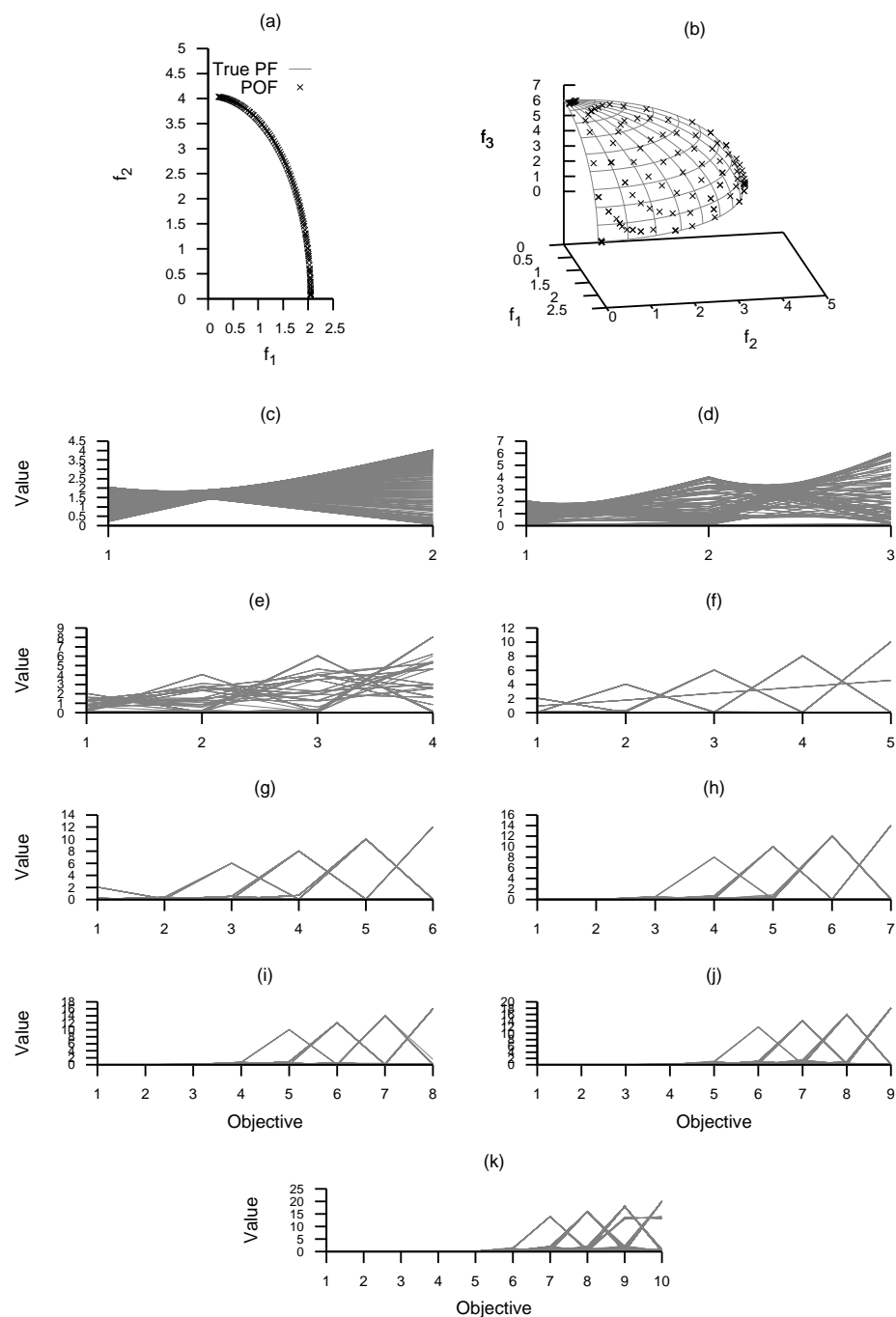


Figure A.376: Plots of the approximations obtained by MOMBI-NMTCH from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.158: Comparison of hypervolume indicator values for different optimizers on the WFG6 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	6.641920e+00	6.632984e+00	6.673995e+00	6.677249e+00	8.301332e+00	8.318069e+00	8.266517e+00	8.277923e+00	8.307104e+00	8.303954e+00	8.212515e+00
	avg.	6.623076e+00	6.619926e+00	6.725257e+00	6.713478e+00	8.289670e+00	8.316271e+00	8.262230e+00	8.276638e+00	8.297516e+00	8.306925e+00	8.207277e+00
	min.	6.391142e+00	6.277325e+00	6.512258e+00	6.439885e+00	7.885024e+00	8.154695e+00	8.063359e+00	8.082916e+00	7.956692e+00	8.161349e+00	8.071502e+00
	max.	6.803539e+00	6.814208e+00	7.673431e+00	7.844979e+00	8.488363e+00	8.421164e+00	8.423843e+00	8.478078e+00	8.430996e+00	8.458041e+00	8.345035e+00
	std.	9.986660e-02	1.003432e-01	2.174790e-01	2.195559e-01	9.327206e-02	5.637110e-02	7.251537e-02	6.550023e-02	7.269464e-02	6.101993e-02	5.560570e-02
3D	med.	5.405741e+01	5.401862e+01	5.447550e+01	5.404031e+01	7.064654e+01	7.176267e+01	7.167759e+01	7.248907e+01	7.118561e+01	7.193904e+01	7.063206e+01
	avg.	5.409106e+01	5.409628e+01	5.456735e+01	5.402257e+01	7.063211e+01	7.181231e+01	7.165826e+01	7.254701e+01	7.117864e+01	7.193216e+01	7.071295e+01
	min.	5.274364e+01	5.230334e+01	5.318219e+01	5.283804e+01	6.946511e+01	7.014474e+01	7.023858e+01	7.143911e+01	7.007598e+01	7.068065e+01	6.916781e+01
	max.	5.521469e+01	5.547494e+01	5.722668e+01	5.512992e+01	7.172144e+01	7.294742e+01	7.298119e+01	7.341573e+01	7.271000e+01	7.293843e+01	7.268921e+01
	std.	5.873146e-01	6.424450e-01	7.087896e-01	5.167387e-01	4.608997e-01	5.147862e-01	5.076903e-01	4.074942e-01	4.944282e-01	4.841629e-01	6.103820e-01
4D	med.	5.390133e+02	5.383151e+02	5.402580e+02	5.386297e+02	6.420895e+02	6.623484e+02	6.243200e+02	6.729670e+02	6.263961e+02	6.470957e+02	6.995967e+02
	avg.	5.392732e+02	5.390258e+02	5.398931e+02	5.385238e+02	6.418738e+02	6.633264e+02	6.246849e+02	6.729419e+02	6.264732e+02	6.475683e+02	7.000336e+02
	min.	5.315296e+02	5.321835e+02	5.306021e+02	5.298545e+02	6.151431e+02	6.434419e+02	6.125651e+02	6.583774e+02	6.172924e+02	6.336869e+02	6.865728e+02
	max.	5.489070e+02	5.520477e+02	5.494776e+02	5.455085e+02	6.718295e+02	7.042729e+02	6.345375e+02	6.870747e+02	6.383056e+02	6.576768e+02	7.131561e+02
	std.	3.828370e+00	3.736931e+00	4.209565e+00	3.365704e+00	1.107041e+01	1.209414e+01	4.519664e+00	6.355254e+00	4.133398e+00	4.579513e+00	5.301032e+00
5D	med.	6.342332e+03	6.344989e+03	6.319601e+03	6.333394e+03	7.515014e+03	8.131741e+03	6.546516e+03	6.521883e+03	6.520549e+03	6.511903e+03	7.734932e+03
	avg.	6.341540e+03	6.342414e+03	6.320425e+03	6.331570e+03	7.504929e+03	8.115189e+03	6.543870e+03	6.520965e+03	6.519872e+03	6.512670e+03	7.740659e+03
	min.	6.256962e+03	6.278230e+03	6.243510e+03	6.252246e+03	7.073158e+03	7.558812e+03	6.463257e+03	6.448471e+03	6.455430e+03	6.440978e+03	7.324446e+03
	max.	6.441202e+03	6.425352e+03	6.382085e+03	6.430906e+03	7.785954e+03	8.462129e+03	6.615064e+03	6.609376e+03	6.610415e+03	6.581057e+03	8.009799e+03
	std.	3.394692e+01	3.186458e+01	2.652534e+01	3.216311e+01	1.085360e+02	1.801280e+02	3.110363e+01	3.085298e+01	2.917466e+01	3.150346e+01	1.148623e+02
6D	med.	7.513319e+04	7.503885e+04	8.155164e+04	7.518642e+04	9.500774e+04	9.498454e+04	8.176283e+04	7.472259e+04	7.541672e+04	7.472468e+04	9.567930e+04
	avg.	7.498154e+04	7.620015e+04	7.938935e+04	7.525730e+04	9.581964e+04	9.617764e+04	7.973969e+04	7.362157e+04	7.801406e+04	7.263816e+04	9.527503e+04
	min.	6.385769e+04	6.276691e+04	7.428340e+04	6.269833e+04	8.614461e+04	8.900976e+04	7.413770e+04	6.246527e+04	7.392502e+04	6.285205e+04	8.260152e+04
	max.	8.656519e+04	8.679869e+04	8.309293e+04	8.797698e+04	1.062303e+05	1.054532e+05	8.331860e+04	7.867949e+04	8.274939e+04	8.573040e+04	1.003623e+05
	std.	3.996956e+03	7.632526e+03	3.403107e+03	3.905776e+03	4.392397e+03	3.779025e+03	3.184466e+03	3.730606e+03	3.425463e+03	5.117729e+03	3.165960e+03
7D	med.	8.146960e+05	6.004078e+05	1.058325e+06	6.038189e+05	1.126872e+06	1.092089e+06	9.924554e+05	5.948065e+05	9.877856e+05	5.943398e+05	1.237019e+06
	avg.	8.088594e+05	6.134422e+05	1.069816e+06	6.172381e+05	1.129197e+06	1.100298e+06	1.014373e+06	6.002268e+05	9.860022e+05	6.014589e+05	1.190128e+06
	min.	5.808848e+05	5.831365e+05	9.322636e+05	5.869519e+05	9.440217e+05	9.861570e+05	7.080363e+05	5.837761e+05	6.859268e+05	5.830709e+05	6.487416e+05
	max.	1.078721e+06	9.217589e+05	1.171821e+06	9.478551e+05	1.450102e+06	1.313773e+06	1.171989e+06	9.652078e+05	1.168883e+06	9.280402e+05	1.467676e+06
	std.	1.021064e+05	4.028819e+04	7.025267e+04	4.190536e+04	1.070308e+05	5.558262e+04	8.194855e+04	3.861095e+04	9.145964e+04	3.662095e+04	2.303698e+05
8D	med.	1.174624e+07	8.851375e+06	1.839901e+07	9.048280e+06	1.835743e+07	1.737286e+07	1.716696e+07	8.822388e+06	1.626160e+07	8.797882e+06	2.005944e+07
	avg.	1.173660e+07	9.010406e+06	1.836329e+07	9.102053e+06	1.845954e+07	1.742937e+07	1.703535e+07	8.866090e+06	1.638439e+07	8.852375e+06	1.874467e+07
	min.	8.726238e+06	8.590312e+06	1.585914e+07	8.663233e+06	1.512820e+07	1.535507e+07	1.088817e+07	8.555931e+06	8.825819e+06	8.599589e+06	1.105536e+07
	max.	1.792560e+07	1.052195e+07	1.990935e+07	1.024300e+07	2.336928e+07	1.944598e+07	1.993784e+07	9.797968e+06	1.983328e+07	1.123460e+07	2.454414e+07
	std.	1.694883e+06	3.932601e+05	1.141138e+06	2.816549e+05	2.030387e+06	7.864744e+05	1.671819e+06	1.855925e+06	2.089226e+06	2.831463e+05	4.178028e+06
9D	med.	2.109499e+08	1.526719e+08	3.399569e+08	1.559866e+08	3.591841e+08	3.057945e+08	3.134459e+08	1.503624e+08	3.004665e+08	1.504523e+08	3.478203e+08
	avg.	2.136359e+08	1.553288e+08	3.446198e+08	1.574164e+08	3.574551e+08	3.032697e+08	3.038125e+08	1.513993e+08	2.868203e+08	1.515398e+08	3.241904e+08
	min.	1.483699e+08	1.454070e+08	2.892337e+08	1.475334e+08	2.758753e+08	2.406976e+08	1.515605e+08	1.452718e+08	1.441978e+08	1.452711e+08	2.085142e+08
	max.	2.932411e+08	1.988456e+08	3.762623e+08	2.159344e+08	4.301564e+08	3.679387e+08	3.737189e+08	1.723618e+08	3.886951e+08	1.652313e+08	4.400577e+08
	std.	3.163937e+07	9.390991e+06	2.186601e+07	8.871180e+06	3.442881e+07	2.390778e+07	5.534670e+07	4.901520e+06	5.593053e+07	4.295522e+06	7.571624e+07
10D	med.	3.991511e+09	2.985743e+09	7.364395e+09	3.054133e+09	6.996172e+09	4.689442e+09	6.245937e+09	2.871063e+09	5.890599e+09	2.906164e+09	6.666949e+09
	avg.	4.089510e+09	3.106139e+09	7.246517e+09	3.084661e+09	6.944732e+09	4.764419e+09	5.918375e+09	2.877635e+09	5.569833e+09	2.943378e+09	6.314757e+09
	min.	2.806422e+09	2.794236e+09	6.110463e+09	2.754015e+09	5.002917e+09	3.476942e+09	2.678382e+09	2.716994e+09	2.749606e+09	2.744965e+09	4.293993e+09
	max.	5.799673e+09	4.532943e+09	7.816725e+09	4.151322e+09	8.667519e+09	6.781654e+09	7.705741e+09	3.319625e+09	8.087944e+09	3.444303e+09	8.322964e+09
	std.	6.776100e+08	3.533511e+08	4.464396e+08	2.253288e+08	7.321306e+08	7.243040e+08	1.413529e+09	9.691457e+07	1.377664e+09	1.418754e+08	1.314644e+09

Figure A.377: Box-plot of the hypervolume indicator values for different optimizers on the WFG6 test problem.

Table A.159: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG6 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
2D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$4.24e-05$	$4.23e-06$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$2.14e-04$	$5.04e-05$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	> 0.05	$4.43e-04$	$8.92e-03$	> 0.05	> 0.05	$1.97e-16$
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.58e-02$	–	$9.76e-09$	$8.08e-07$	$4.08e-02$	> 0.05	$6.28e-24$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$7.35e-09$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	$1.44e-13$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$7.36e-05$	$1.62e-03$	–	> 0.05	$1.85e-18$
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$9.57e-06$	$5.75e-04$	> 0.05	–	$1.22e-21$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
3D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$3.48e-06$	$6.01e-06$	–	$2.23e-08$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.09e-29$	–	$2.63e-02$	> 0.05	$1.62e-15$	> 0.05	$9.67e-25$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.77e-26$	> 0.05	–	> 0.05	$1.51e-10$	> 0.05	$2.14e-21$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.53e-34$	$1.41e-19$	$8.32e-25$	–	$2.31e-32$	$3.61e-16$	$1.37e-32$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$4.32e-13$	> 0.05	> 0.05	> 0.05	–	> 0.05	$1.78e-09$
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$6.25e-31$	$3.30e-02$	$9.77e-05$	> 0.05	$3.33e-19$	–	$9.99e-27$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
4D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	> 0.05	$3.43e-02$	–	$1.03e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	> 0.05	$3.49e-26$	> 0.05	$1.00e-23$	> 0.05	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.10e-24$	–	$1.28e-34$	> 0.05	$1.28e-34$	$8.45e-24$	> 0.05
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$5.71e-33$	$1.16e-13$	$1.28e-34$	–	$1.28e-34$	$1.28e-34$	> 0.05
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$3.33e-03$	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$6.36e-07$	> 0.05	$1.32e-34$	> 0.05	$1.53e-34$	–	> 0.05
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.38e-32$	$1.28e-34$	$1.32e-34$	$1.28e-34$	$1.28e-34$	–

Table A.160: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG6 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
5D											
MOMBI-WS	–	> 0.05	$2.12e-06$	$2.40e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	$7.69e-07$	$8.40e-03$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	$6.28e-03$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$8.40e-34$	–	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$3.90e-28$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	–	$2.75e-07$	$1.97e-08$	$9.53e-11$	> 0.05
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	–	> 0.05	$4.22e-02$	> 0.05
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$1.32e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$4.54e-27$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–
6D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.70e-03$	> 0.05	$1.24e-04$	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.76e-02$	> 0.05	$1.28e-03$	> 0.05
MOMBI-LS	$1.60e-11$	$1.60e-02$	–	$1.28e-09$	> 0.05	> 0.05	> 0.05	$1.50e-19$	$5.82e-03$	$1.26e-19$	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$7.52e-08$	> 0.05	$9.16e-09$	> 0.05
MOMBI-TCH	$1.32e-34$	$1.49e-34$	$1.28e-34$	$1.36e-34$	–	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	–	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05
MOMBI-ATCH	$3.99e-14$	$6.87e-03$	> 0.05	$1.64e-12$	> 0.05	> 0.05	–	$3.76e-22$	$9.86e-04$	$1.09e-21$	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$6.50e-06$	> 0.05	> 0.05	$5.51e-04$	> 0.05	> 0.05	> 0.05	$4.89e-14$	–	$9.03e-15$	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	$2.01e-34$	$2.04e-33$	$1.84e-34$	$1.68e-34$	> 0.05	> 0.05	$1.89e-34$	$1.28e-34$	$1.44e-34$	$1.40e-34$	–
7D											
MOMBI-WS	–	$1.26e-26$	> 0.05	$2.84e-26$	> 0.05	> 0.05	> 0.05	$4.59e-28$	> 0.05	$3.70e-28$	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.55e-07$	> 0.05	$3.09e-07$	> 0.05
MOMBI-LS	$2.04e-33$	$1.28e-34$	–	$1.36e-34$	> 0.05	> 0.05	$5.62e-07$	$1.40e-34$	$1.82e-11$	$1.28e-34$	> 0.05
MOMBI-NLS	> 0.05	$4.17e-02$	> 0.05	–	> 0.05	> 0.05	> 0.05	$4.56e-13$	> 0.05	$3.76e-12$	> 0.05
MOMBI-TCH	$6.06e-34$	$1.28e-34$	$2.36e-05$	$1.32e-34$	–	> 0.05	$9.49e-14$	$1.36e-34$	$4.09e-18$	$1.28e-34$	> 0.05
MOMBI-NTCH	$3.88e-34$	$1.28e-34$	$3.91e-03$	$1.28e-34$	> 0.05	–	$9.75e-15$	$1.28e-34$	$1.70e-21$	$1.28e-34$	> 0.05
MOMBI-ATCH	$1.06e-29$	$1.44e-34$	> 0.05	$2.33e-34$	> 0.05	> 0.05	–	$2.88e-34$	$2.29e-02$	$1.40e-34$	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$6.46e-25$	$2.01e-34$	> 0.05	$3.15e-34$	> 0.05	> 0.05	> 0.05	$3.44e-34$	–	$1.73e-34$	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	$6.03e-20$	$2.37e-33$	$1.08e-17$	$1.59e-32$	$4.92e-09$	$4.58e-16$	$1.77e-18$	$2.96e-34$	$1.08e-18$	$4.50e-34$	–



Table A.161: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG6 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
8D											
MOMBI-WS	–	$1.45e-27$	> 0.05	$1.86e-25$	> 0.05	> 0.05	> 0.05	$8.96e-30$	> 0.05	$4.23e-30$	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.42e-02$	> 0.05	$1.16e-03$	> 0.05
MOMBI-LS	$3.44e-34$	$1.28e-34$	–	$1.28e-34$	> 0.05	$1.31e-09$	$4.92e-10$	$1.28e-34$	$9.00e-16$	$1.28e-34$	> 0.05
MOMBI-NLS	> 0.05	$5.86e-05$	> 0.05	–	> 0.05	> 0.05	> 0.05	$3.08e-13$	> 0.05	$3.08e-15$	> 0.05
MOMBI-TCH	$4.78e-34$	$1.28e-34$	> 0.05	$1.28e-34$	–	$2.08e-04$	$4.58e-06$	$1.28e-34$	$2.17e-10$	$1.28e-34$	> 0.05
MOMBI-NTCH	$1.24e-33$	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05	–	$3.46e-02$	$1.28e-34$	$1.78e-07$	$1.28e-34$	> 0.05
MOMBI-ATCH	$2.74e-31$	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05	> 0.05	–	$1.28e-34$	$2.76e-03$	$1.32e-34$	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$6.86e-28$	$1.92e-33$	> 0.05	$4.02e-33$	> 0.05	> 0.05	> 0.05	$6.63e-34$	–	$6.06e-34$	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	$2.50e-19$	$1.28e-34$	$1.88e-07$	$1.28e-34$	$3.34e-04$	$6.25e-12$	$7.26e-11$	$1.28e-34$	$4.93e-12$	$2.33e-34$	–
9D											
MOMBI-WS	–	$1.40e-29$	> 0.05	$3.06e-28$	> 0.05	> 0.05	> 0.05	$1.46e-31$	> 0.05	$1.27e-31$	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.60e-04$	> 0.05	$5.37e-04$	> 0.05
MOMBI-LS	$1.32e-34$	$1.28e-34$	–	$1.28e-34$	> 0.05	$3.67e-23$	$5.02e-11$	$1.28e-34$	$2.50e-20$	$1.28e-34$	> 0.05
MOMBI-NLS	> 0.05	$7.42e-04$	> 0.05	–	> 0.05	> 0.05	> 0.05	$1.33e-11$	> 0.05	$3.32e-11$	> 0.05
MOMBI-TCH	$1.49e-34$	$1.28e-34$	$9.77e-04$	$1.28e-34$	–	$4.44e-22$	$1.85e-14$	$1.28e-34$	$4.94e-21$	$1.28e-34$	$1.83e-02$
MOMBI-NTCH	$7.01e-33$	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05	–	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05
MOMBI-ATCH	$3.42e-22$	$1.89e-32$	> 0.05	$6.00e-32$	> 0.05	$8.98e-03$	–	$1.52e-33$	$2.74e-03$	$1.76e-33$	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$4.36e-18$	$9.24e-32$	> 0.05	$1.64e-31$	> 0.05	> 0.05	> 0.05	$7.88e-33$	–	$8.85e-33$	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	$2.11e-18$	$1.28e-34$	> 0.05	$2.96e-34$	> 0.05	$3.97e-07$	$1.91e-05$	$1.28e-34$	$1.94e-08$	$1.28e-34$	–
10D											
MOMBI-WS	–	$1.14e-21$	> 0.05	$3.76e-23$	> 0.05	> 0.05	> 0.05	$1.59e-30$	> 0.05	$4.12e-28$	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.18e-13$	> 0.05	$1.58e-05$	> 0.05
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$2.02e-04$	$3.65e-34$	$5.99e-18$	$1.28e-34$	$7.30e-22$	$1.28e-34$	$1.32e-07$
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$1.71e-17$	> 0.05	$7.67e-09$	> 0.05
MOMBI-TCH	$2.79e-34$	$1.28e-34$	> 0.05	$1.28e-34$	–	$1.04e-31$	$7.14e-09$	$1.28e-34$	$6.69e-14$	$1.28e-34$	$3.19e-03$
MOMBI-NTCH	$4.92e-10$	$2.92e-32$	> 0.05	$5.22e-34$	> 0.05	–	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	> 0.05
MOMBI-ATCH	$1.17e-16$	$4.97e-28$	> 0.05	$1.84e-27$	> 0.05	$3.95e-12$	–	$1.54e-32$	$9.10e-03$	$6.61e-31$	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$1.52e-13$	$2.23e-29$	> 0.05	$2.78e-30$	> 0.05	$3.64e-07$	> 0.05	$3.68e-33$	–	$2.06e-32$	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$3.75e-04$	> 0.05	–	> 0.05
MOMBI-PBI	$2.42e-23$	$4.50e-34$	> 0.05	$1.28e-34$	> 0.05	$1.35e-14$	$2.24e-03$	$1.28e-34$	$1.00e-05$	$1.28e-34$	–

Table A.162: Comparison of R2 indicator values for different optimizers on the WFG6 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	6.795301e-01	6.801368e-01	6.784883e-01	6.784899e-01	6.026044e-01	6.021734e-01	6.035460e-01	6.035572e-01	6.022800e-01	6.028259e-01	6.064450e-01
	avg.	6.806925e-01	6.809297e-01	6.765995e-01	6.769208e-01	6.032876e-01	6.023080e-01	6.040157e-01	6.034281e-01	6.027755e-01	6.027292e-01	6.066293e-01
	min.	6.736915e-01	6.733294e-01	6.395007e-01	6.285254e-01	5.953457e-01	5.974073e-01	5.972072e-01	5.957672e-01	5.972382e-01	5.965092e-01	6.009068e-01
	max.	6.947750e-01	7.001605e-01	6.850499e-01	6.879638e-01	6.318784e-01	6.082590e-01	6.130777e-01	6.120121e-01	6.210644e-01	6.088985e-01	6.127358e-01
	std.	4.325263e-03	4.531746e-03	8.628924e-03	9.406419e-03	4.753316e-03	2.306046e-03	2.870594e-03	2.605271e-03	3.221346e-03	2.580401e-03	2.412405e-03
3D	med.	3.699836e-01	3.701654e-01	3.689079e-01	3.701853e-01	3.239703e-01	3.239855e-01	3.232561e-01	3.241744e-01	3.232887e-01	3.238760e-01	3.296128e-01
	avg.	3.700828e-01	3.700975e-01	3.687139e-01	3.702630e-01	3.240601e-01	3.239337e-01	3.233169e-01	3.240408e-01	3.234644e-01	3.240687e-01	3.294686e-01
	min.	3.674187e-01	3.668603e-01	3.597594e-01	3.676469e-01	3.212622e-01	3.208480e-01	3.197916e-01	3.216644e-01	3.194150e-01	3.215762e-01	3.232282e-01
	max.	3.733269e-01	3.748133e-01	3.724445e-01	3.733152e-01	3.272024e-01	3.287611e-01	3.266292e-01	3.271472e-01	3.264428e-01	3.282352e-01	3.348194e-01
	std.	1.424769e-03	1.574989e-03	1.901040e-03	1.266974e-03	1.206304e-03	1.406654e-03	1.308723e-03	1.162431e-03	1.344663e-03	1.388065e-03	2.011531e-03
4D	med.	2.427901e-01	2.429642e-01	2.425407e-01	2.428643e-01	2.275685e-01	2.266385e-01	2.291700e-01	2.288143e-01	2.286696e-01	2.304744e-01	2.216790e-01
	avg.	2.427712e-01	2.428927e-01	2.426069e-01	2.429373e-01	2.274802e-01	2.265172e-01	2.291290e-01	2.288012e-01	2.286886e-01	2.304438e-01	2.216804e-01
	min.	2.410392e-01	2.414719e-01	2.406360e-01	2.418617e-01	2.231467e-01	2.209704e-01	2.274973e-01	2.252798e-01	2.269873e-01	2.274791e-01	2.218954e-01
	max.	2.440472e-01	2.439309e-01	2.442051e-01	2.443421e-01	2.303079e-01	2.306867e-01	2.304759e-01	2.315078e-01	2.303818e-01	2.323075e-01	2.255828e-01
	std.	6.221833e-04	5.253622e-04	7.315875e-04	5.153792e-04	1.203791e-03	2.153570e-03	6.776180e-04	1.051678e-03	7.246614e-04	7.298320e-04	1.428991e-03
5D	med.	1.836394e-01	1.836780e-01	1.839113e-01	1.837658e-01	1.760157e-01	1.695100e-01	1.805245e-01	1.832776e-01	1.819830e-01	1.833736e-01	1.850790e-01
	avg.	1.836235e-01	1.837298e-01	1.839325e-01	1.838029e-01	1.761652e-01	1.697467e-01	1.805614e-01	1.833217e-01	1.819936e-01	1.833973e-01	1.852552e-01
	min.	1.826418e-01	1.830063e-01	1.833509e-01	1.829390e-01	1.734490e-01	1.667381e-01	1.798757e-01	1.822522e-01	1.806060e-01	1.827375e-01	1.797897e-01
	max.	1.843662e-01	1.843110e-01	1.847243e-01	1.846132e-01	1.806228e-01	1.761207e-01	1.814174e-01	1.841001e-01	1.828656e-01	1.840876e-01	1.925876e-01
	std.	3.646891e-04	3.213637e-04	2.625443e-04	3.204959e-04	1.225891e-03	1.891600e-03	3.149768e-04	3.153858e-04	4.285713e-04	2.939276e-04	2.370560e-03
6D	med.	1.705862e-01	1.708921e-01	1.447439e-01	1.701276e-01	1.645602e-01	1.894324e-01	1.446357e-01	1.704489e-01	1.699852e-01	1.704922e-01	1.583320e-01
	avg.	1.770119e-01	1.738765e-01	1.549480e-01	1.720440e-01	1.662620e-01	1.901277e-01	1.538667e-01	1.770624e-01	1.597242e-01	1.834830e-01	1.564667e-01
	min.	1.329799e-01	1.328755e-01	1.438878e-01	1.338193e-01	1.293021e-01	1.308863e-01	1.437727e-01	1.596774e-01	1.440700e-01	1.333715e-01	1.428497e-01
	max.	2.352614e-01	2.372013e-01	1.707182e-01	2.384164e-01	2.262917e-01	2.258958e-01	1.708232e-01	2.342403e-01	1.709748e-01	2.357120e-01	1.915464e-01
	std.	2.145011e-02	3.434242e-02	1.189161e-02	1.791820e-02	2.443595e-02	2.618974e-02	1.114329e-02	1.999876e-02	1.186814e-02	2.715003e-02	1.042255e-02
7D	med.	2.580254e-01	3.716356e-01	1.622432e-01	3.714274e-01	2.795005e-01	3.455755e-01	1.779644e-01	3.719223e-01	1.782517e-01	3.719123e-01	1.648572e-01
	avg.	2.634886e-01	3.658565e-01	1.596511e-01	3.642581e-01	2.808770e-01	3.332207e-01	1.737043e-01	3.700419e-01	1.824746e-01	3.683287e-01	1.739992e-01
	min.	1.842244e-01	1.933189e-01	1.391334e-01	1.906775e-01	1.518258e-01	1.787779e-01	1.391286e-01	1.894070e-01	1.392345e-01	1.911749e-01	1.280622e-01
	max.	3.875244e-01	3.991292e-01	1.909753e-01	3.926315e-01	3.727696e-01	3.723614e-01	3.028746e-01	3.878895e-01	3.065186e-01	4.007042e-01	2.892569e-01
	std.	5.563427e-02	2.268836e-02	1.561638e-02	2.406290e-02	4.467469e-02	3.825674e-02	2.436003e-02	1.986723e-02	3.176119e-02	2.105880e-02	4.813142e-02
8D	med.	2.949779e-01	3.924767e-01	1.325381e-01	3.917951e-01	3.042004e-01	3.666314e-01	1.504233e-01	3.925488e-01	1.620405e-01	3.926727e-01	1.473515e-01
	avg.	3.029523e-01	3.919080e-01	1.367169e-01	3.913417e-01	3.021909e-01	3.633999e-01	1.547155e-01	3.941110e-01	1.673363e-01	3.932064e-01	1.626493e-01
	min.	1.689514e-01	3.532985e-01	1.214681e-01	3.397556e-01	1.696229e-01	2.742966e-01	1.214177e-01	3.510967e-01	1.216015e-01	2.982041e-01	1.123342e-01
	max.	4.015093e-01	4.261769e-01	1.666787e-01	4.072437e-01	4.032801e-01	3.966933e-01	2.971482e-01	4.199488e-01	3.923259e-01	4.200885e-01	2.527952e-01
	std.	5.445728e-02	1.072784e-02	1.301398e-02	7.452114e-03	5.807804e-02	2.037326e-02	2.860303e-02	9.011836e-03	4.652082e-02	1.043850e-02	4.764742e-02
9D	med.	3.003241e-01	4.062000e-01	1.260966e-01	4.057053e-01	2.895613e-01	3.808956e-01	1.391062e-01	4.066035e-01	1.464784e-01	4.064962e-01	1.371346e-01
	avg.	3.033756e-01	4.062085e-01	1.231276e-01	4.064781e-01	2.937660e-01	3.678296e-01	1.595952e-01	4.083627e-01	1.764761e-01	4.082351e-01	1.556076e-01
	min.	1.867369e-01	3.051164e-01	1.074634e-01	3.237411e-01	1.684922e-01	2.532813e-01	1.076716e-01	3.861287e-01	1.041525e-01	3.930854e-01	1.030979e-01
	max.	4.336792e-01	4.420602e-01	1.567656e-01	4.377017e-01	4.199977e-01	4.186730e-01	3.858872e-01	4.350362e-01	4.072272e-01	4.403329e-01	2.251147e-01
	std.	5.706567e-02	1.570159e-02	1.185278e-02	1.135885e-02	4.717075e-02	3.597261e-02	6.669996e-02	6.706811e-03	7.441764e-02	6.334626e-03	4.434413e-02
10D	med.	3.199373e-01	4.139640e-01	1.053944e-01	4.139508e-01	3.071391e-01	3.999941e-01	1.314139e-01	4.153478e-01	1.621415e-01	4.149298e-01	1.293227e-01
	avg.	3.241196e-01	4.076565e-01	1.096290e-01	4.137601e-01	3.071011e-01	3.694906e-01	1.668467e-01	4.182579e-01	1.892731e-01	4.169552e-01	1.450393e-01
	min.	2.001107e-01	3.120324e-01	9.904429e-02	3.372289e-01	2.178660e-01	2.510576e-01	9.953364e-02	4.108842e-01	9.281072e-02	4.082661e-01	1.005790e-01
	max.	4.431555e-01	4.347318e-01	1.354885e-01	4.393115e-01	4.143904e-01	4.471469e-01	4.161475e-01	4.418451e-01	4.159744e-01	4.563566e-01	2.036832e-01
	std.	5.243860e-02	2.446192e-02	1.002076e-02	1.026640e-02	4.186218e-02	4.738194e-02	8.288681e-02	6.431591e-03	8.154535e-02	6.393290e-03	3.722362e-02

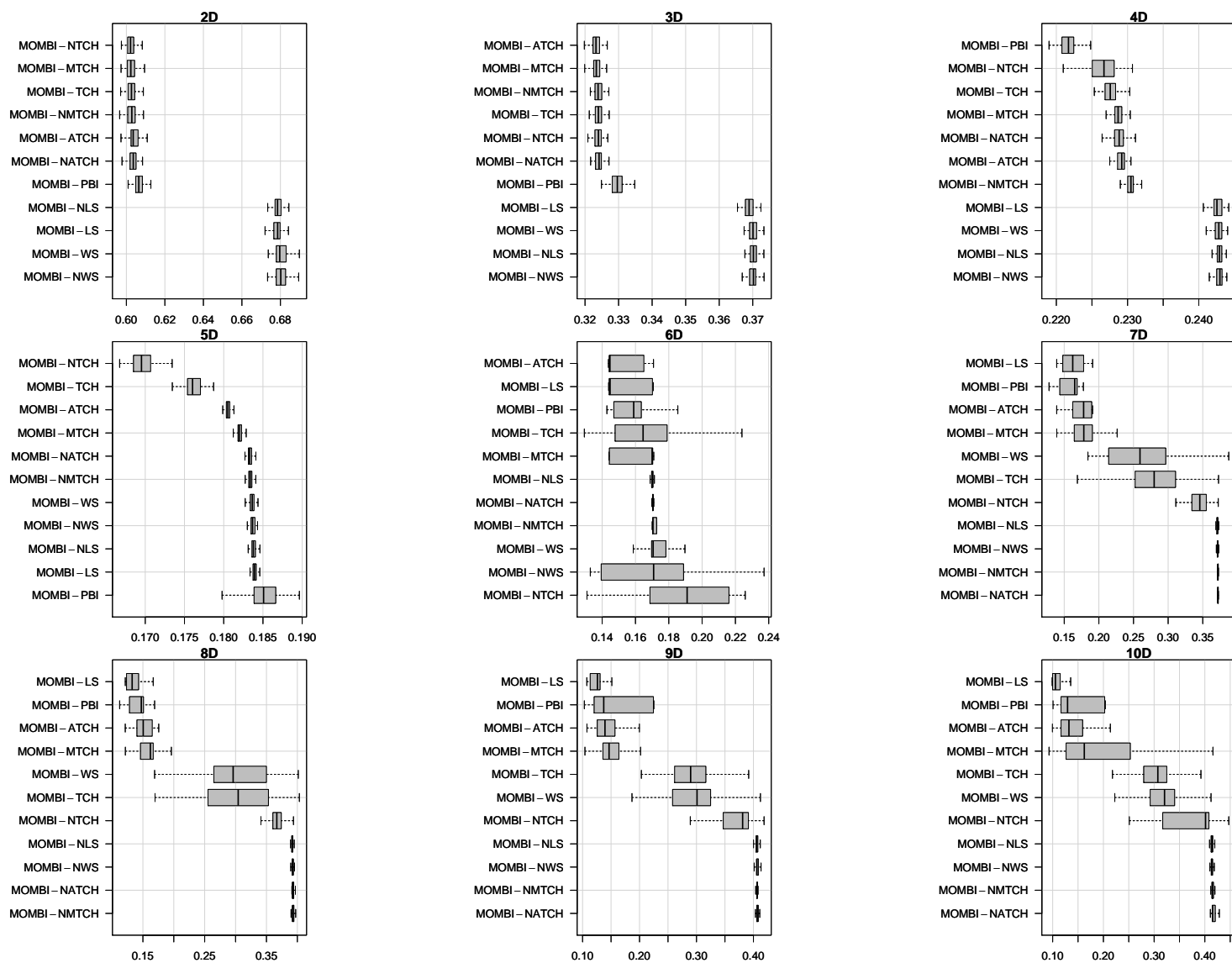


Figure A.378: Box-plot of the R2 indicator values for different optimizers on the WFG6 test problem.

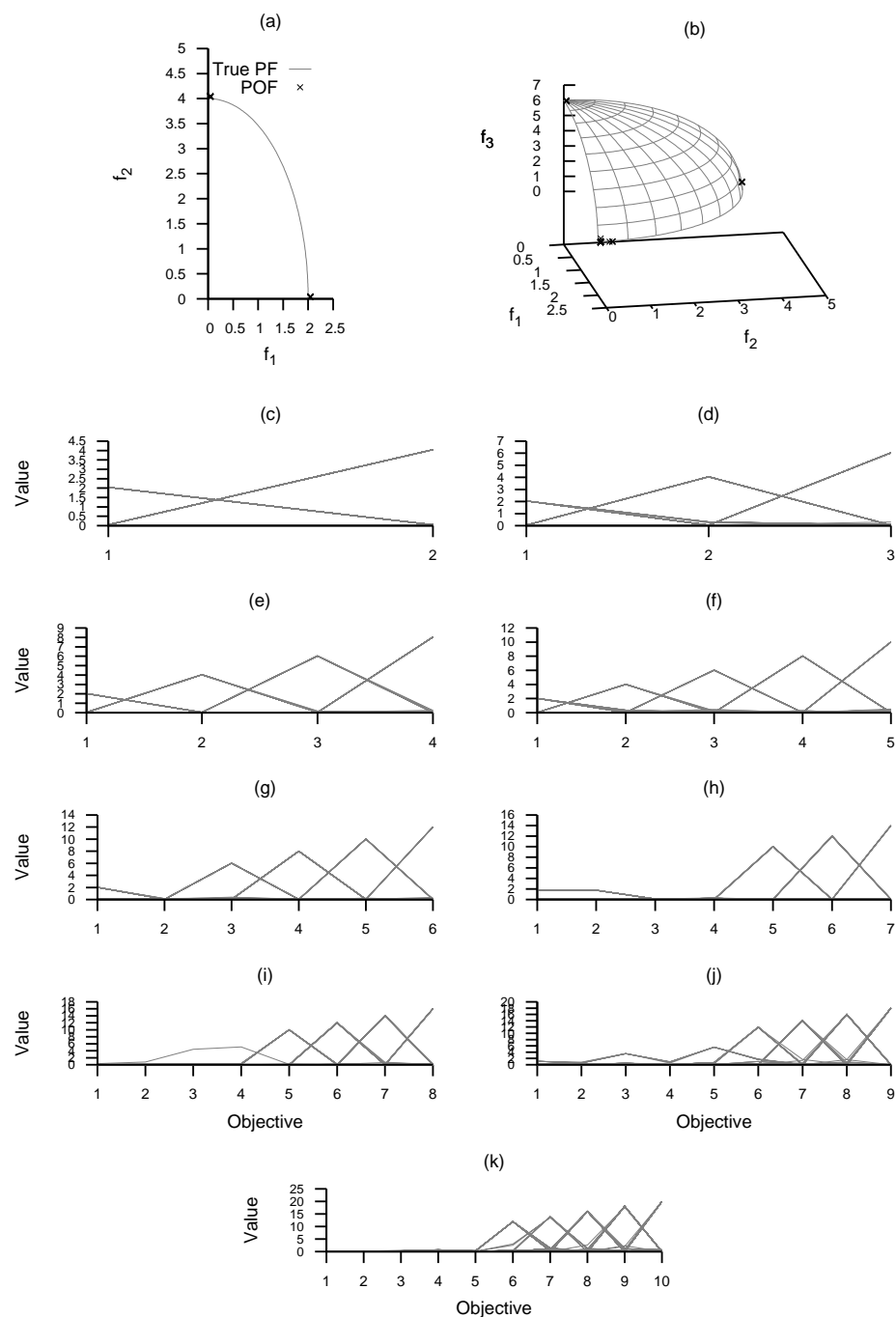


Figure A.379: Plots of the approximations obtained by MOMBI-WS from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

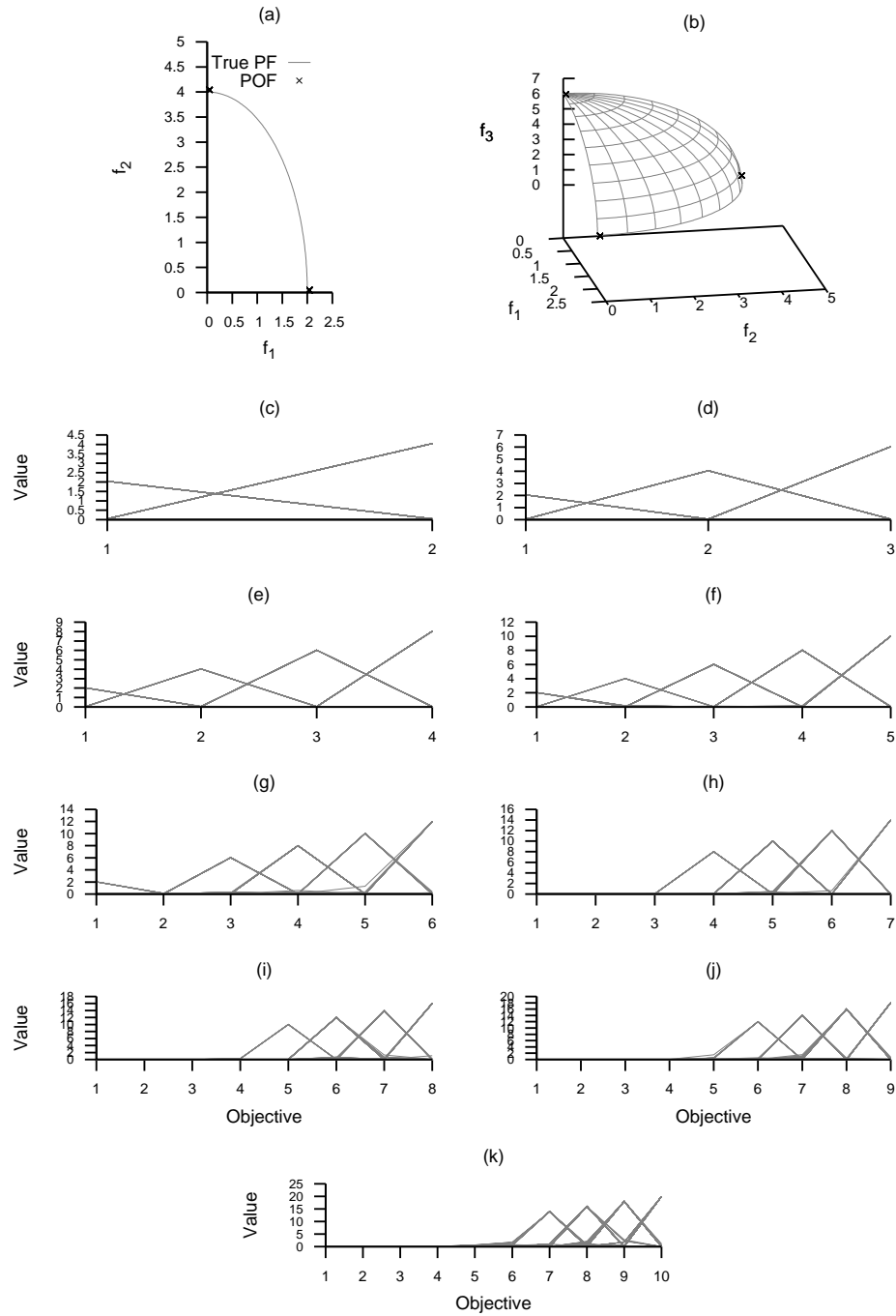


Figure A.380: Plots of the approximations obtained by MOMBI-NWS from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

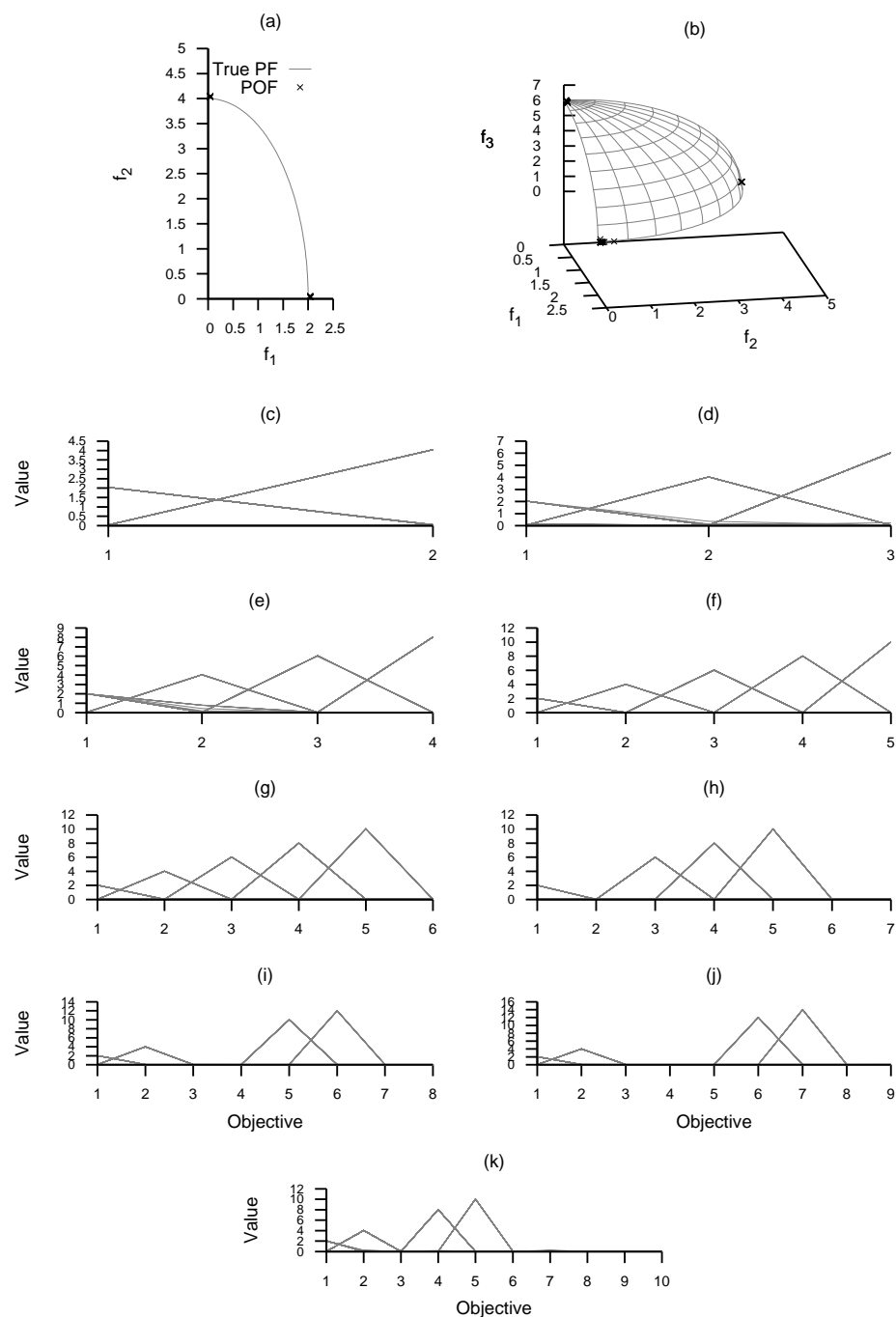


Figure A.381: Plots of the approximations obtained by MOMBI-LS from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

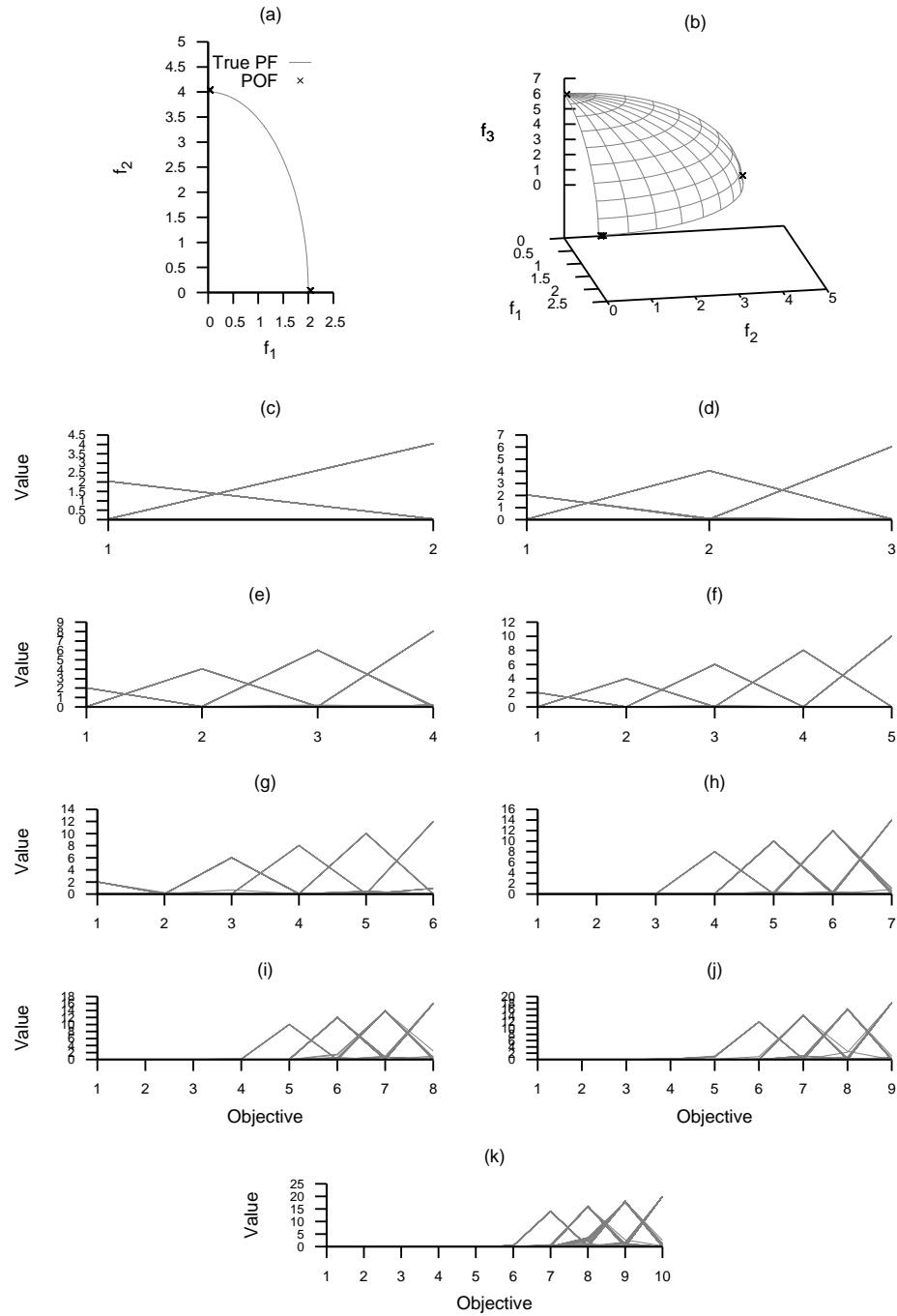


Figure A.382: Plots of the approximations obtained by MOMBI-NLS from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

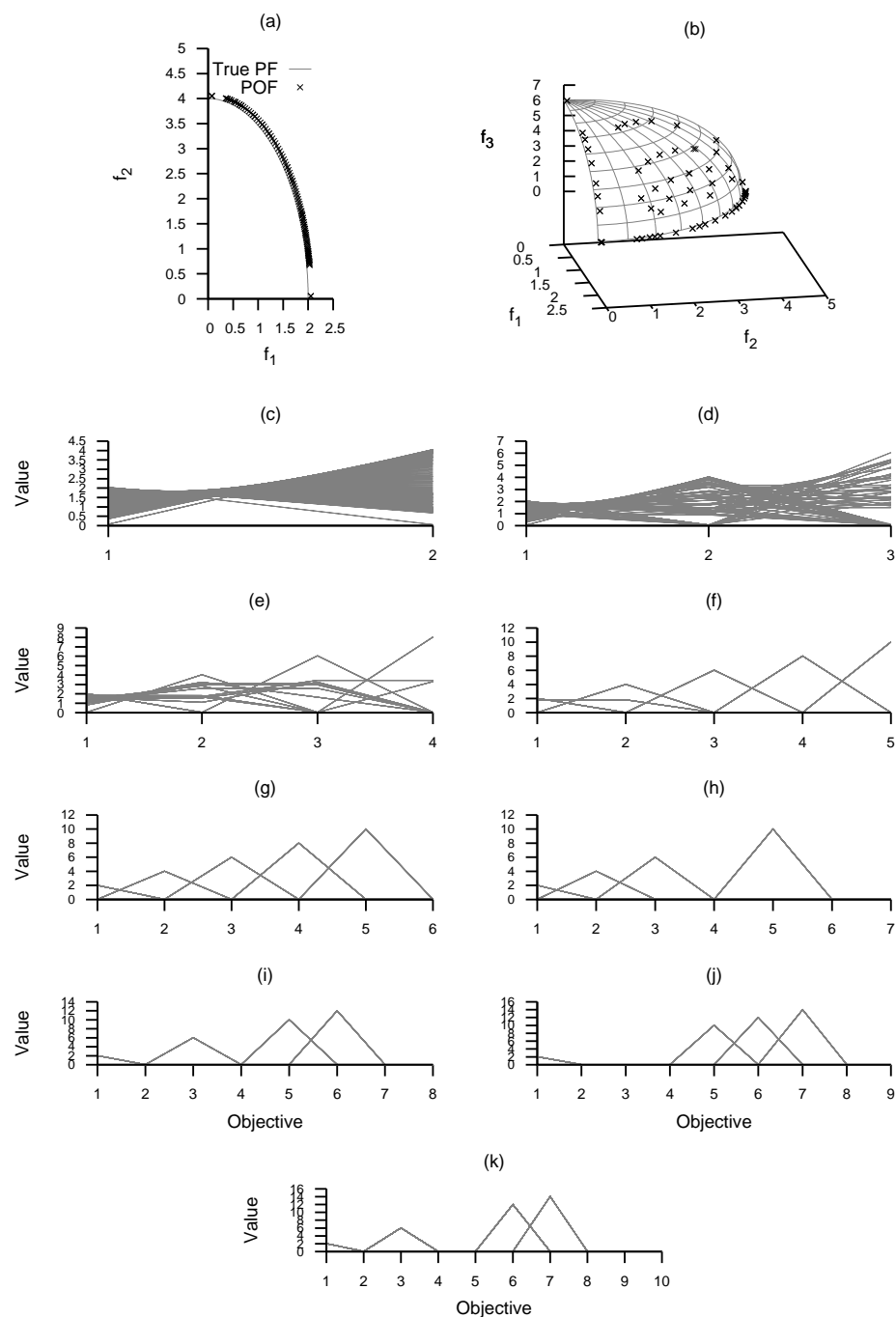


Figure A.383: Plots of the approximations obtained by MOMBI-ATCH from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



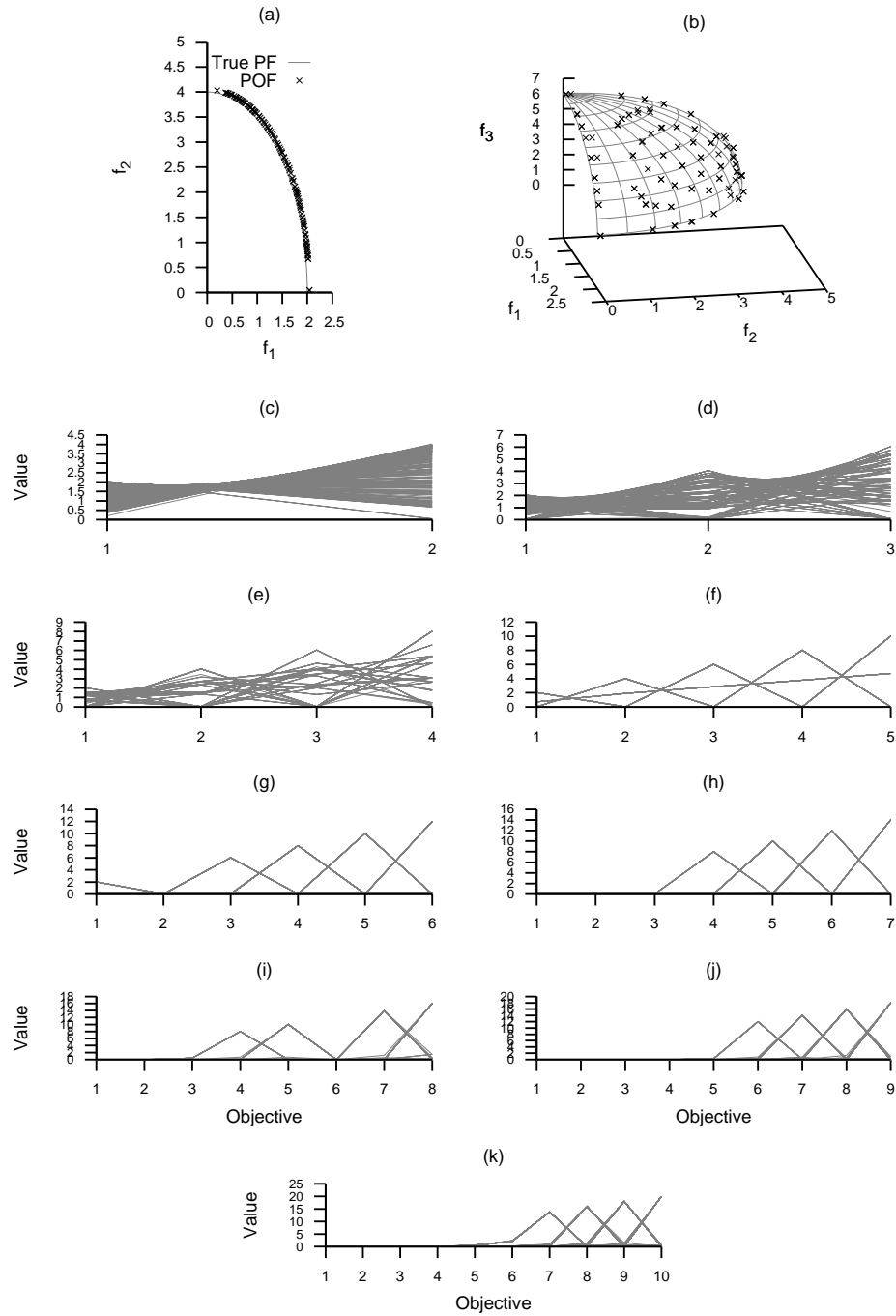


Figure A.384: Plots of the approximations obtained by MOMBI-NATCH from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

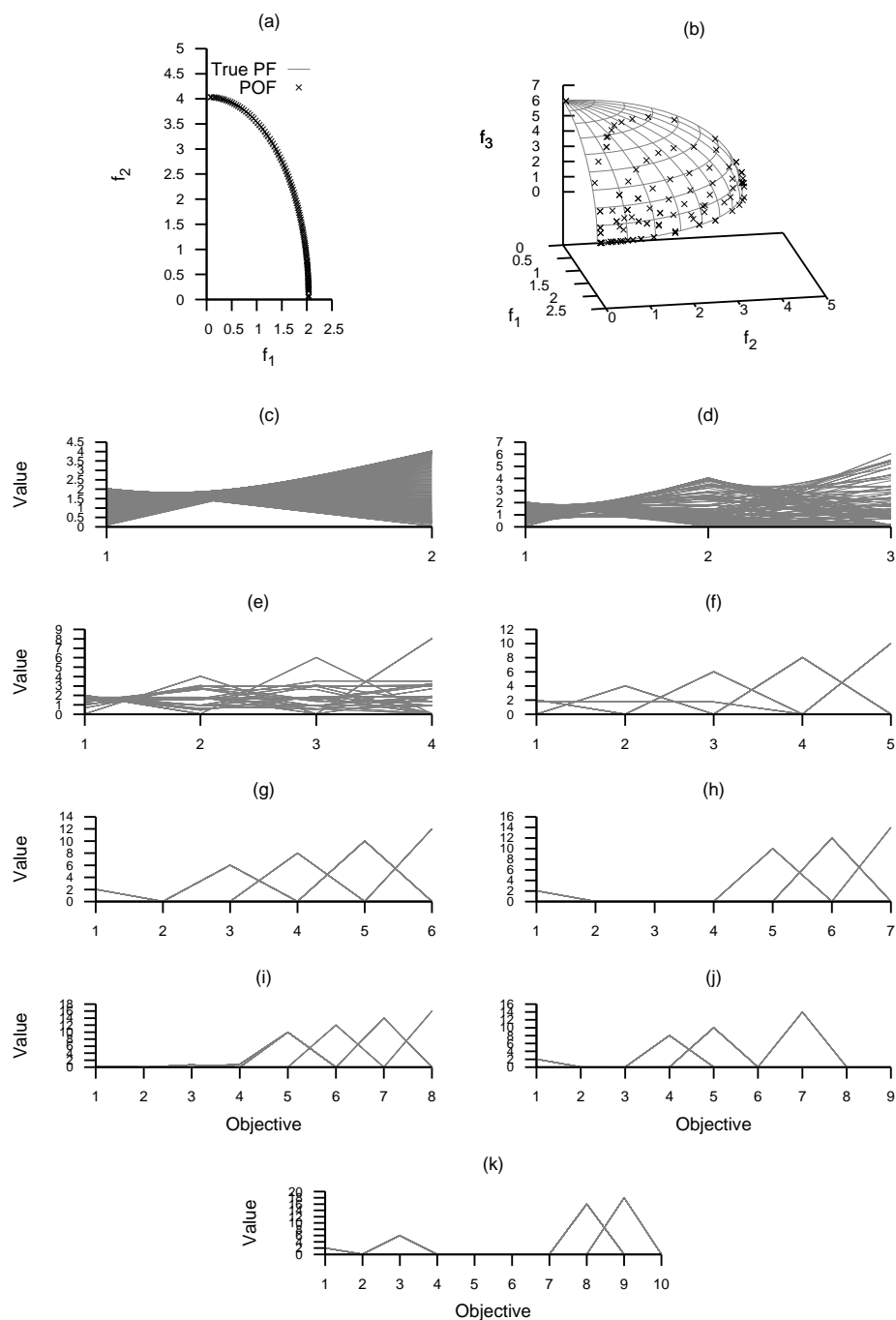


Figure A.385: Plots of the approximations obtained by MOMBI-MTCH from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

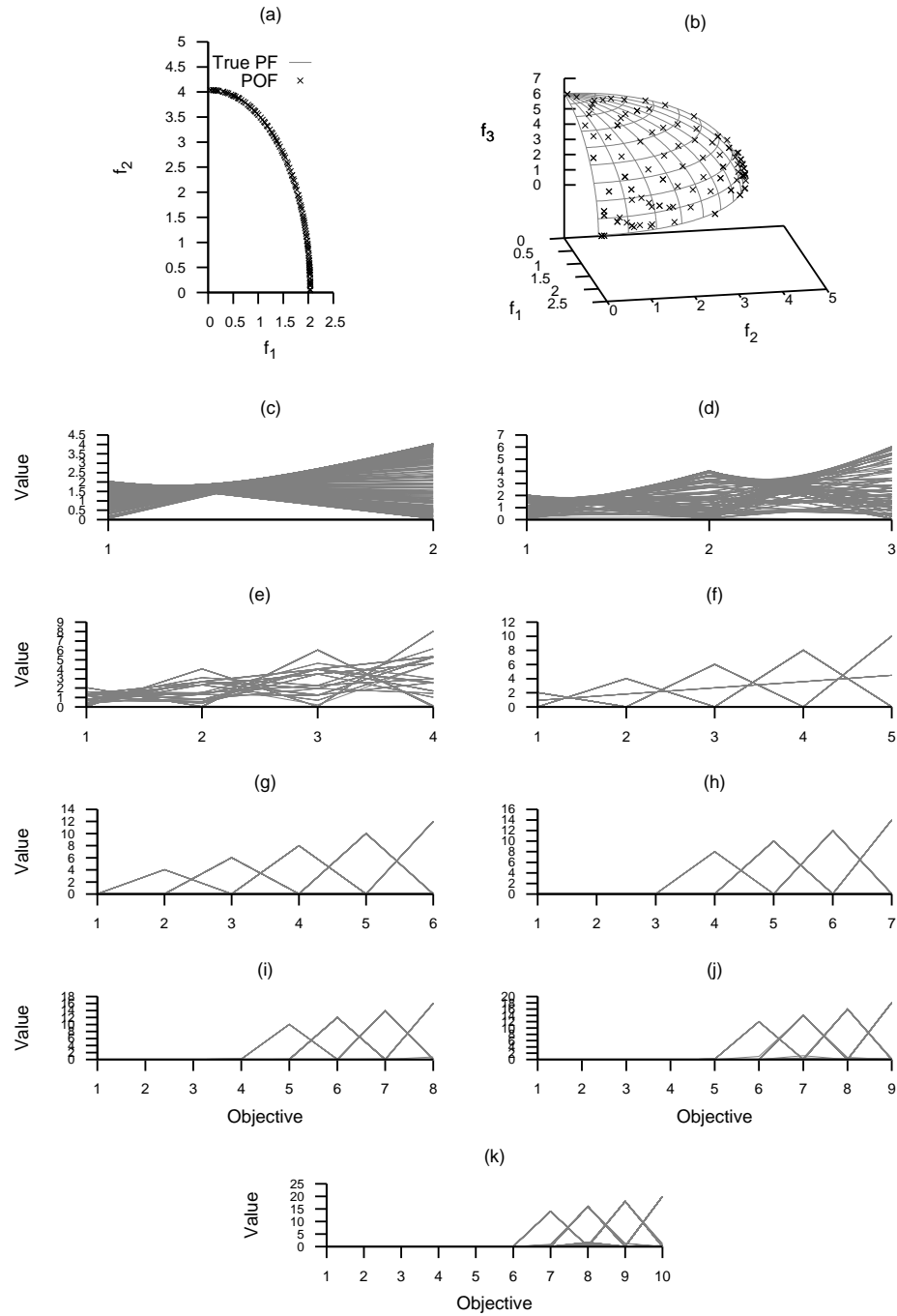


Figure A.386: Plots of the approximations obtained by MOMBI-NMTCH from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.163: Comparison of hypervolume indicator values for different optimizers on the WFG7 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	6.997308e+00	6.997402e+00	6.998788e+00	6.999057e+00	8.642920e+00	8.647603e+00	8.640464e+00	8.644278e+00	8.647469e+00	8.649000e+00	8.570478e+00
	avg.	6.997306e+00	6.997273e+00	6.998943e+00	7.003729e+00	8.639193e+00	8.647108e+00	8.639459e+00	8.643600e+00	8.646743e+00	8.648085e+00	8.568626e+00
	min.	6.995048e+00	6.995720e+00	6.997403e+00	6.997745e+00	8.564228e+00	8.609262e+00	8.612816e+00	8.603550e+00	8.613723e+00	8.617886e+00	8.488860e+00
	max.	6.998802e+00	6.998638e+00	7.005077e+00	7.374088e+00	8.655886e+00	8.663564e+00	8.658678e+00	8.657859e+00	8.665995e+00	8.663637e+00	8.611838e+00
	std.	6.851233e-04	6.420880e-04	9.041127e-04	3.755079e-02	1.315758e-02	9.255785e-03	9.842195e-03	8.999898e-03	9.962710e-03	9.628049e-03	1.947859e-02
3D	med.	5.698621e+01	5.698580e+01	5.711095e+01	5.698838e+01	7.374157e+01	7.493855e+01	7.442926e+01	7.573459e+01	7.422047e+01	7.496646e+01	7.411601e+01
	avg.	5.698963e+01	5.699236e+01	5.892216e+01	5.699037e+01	7.382261e+01	7.492971e+01	7.452201e+01	7.573565e+01	7.423769e+01	7.498286e+01	7.409673e+01
	min.	5.696859e+01	5.696535e+01	5.698651e+01	5.697541e+01	7.345018e+01	7.430012e+01	7.423086e+01	7.551365e+01	7.402481e+01	7.472090e+01	7.268522e+01
	max.	5.711172e+01	5.712630e+01	6.530176e+01	5.703034e+01	7.470797e+01	7.543947e+01	7.509719e+01	7.594085e+01	7.463468e+01	7.536475e+01	7.492238e+01
	std.	1.659854e-02	2.624412e-02	2.716242e+00	8.809099e-03	2.890893e-01	2.210690e-01	2.005572e-01	8.907803e-02	1.115948e-01	1.366901e-01	3.387509e-01
4D	med.	5.607113e+02	5.608786e+02	5.608151e+02	5.607400e+02	6.825203e+02	7.043270e+02	6.480238e+02	6.923664e+02	6.485770e+02	6.683610e+02	7.263648e+02
	avg.	5.608271e+02	5.616662e+02	5.704974e+02	5.612789e+02	6.825784e+02	7.044881e+02	6.470581e+02	6.922816e+02	6.488967e+02	6.691189e+02	7.260723e+02
	min.	5.604471e+02	5.603619e+02	5.604144e+02	5.604445e+02	6.594813e+02	6.837622e+02	6.417316e+02	6.813293e+02	6.460569e+02	6.631961e+02	7.108984e+02
	max.	5.621930e+02	6.021457e+02	6.087383e+02	5.978511e+02	7.174516e+02	7.270158e+02	6.546784e+02	7.074012e+02	6.545977e+02	6.798230e+02	7.381467e+02
	std.	3.357084e-01	4.249976e+00	1.576295e+01	3.740920e+00	1.765719e+01	9.063756e+00	3.224915e+00	5.035906e+00	1.703990e+00	3.562786e+00	5.633826e+00
5D	med.	6.548962e+03	6.561205e+03	6.541823e+03	6.549252e+03	7.640031e+03	8.294396e+03	6.761179e+03	6.728859e+03	6.756623e+03	6.730851e+03	7.847136e+03
	avg.	6.561551e+03	6.592156e+03	6.541644e+03	6.605537e+03	7.637208e+03	8.225433e+03	6.754656e+03	6.723767e+03	6.753172e+03	6.728907e+03	7.805739e+03
	min.	6.537404e+03	6.539720e+03	6.533662e+03	6.539920e+03	7.074836e+03	7.743376e+03	6.534359e+03	6.626133e+03	6.730826e+03	6.628110e+03	7.395362e+03
	max.	6.963038e+03	6.943278e+03	6.549672e+03	7.125183e+03	8.102770e+03	8.645989e+03	6.768867e+03	6.776088e+03	6.773384e+03	6.778233e+03	8.096231e+03
	std.	5.791428e+01	8.611487e+01	2.409864e+00	1.330335e+02	1.895131e+02	2.173222e+02	3.806414e+01	3.156453e+01	9.692258e+00	1.798882e+01	1.895594e+02
6D	med.	7.746171e+04	7.768856e+04	7.732794e+04	7.749226e+04	9.726836e+04	1.004994e+05	7.727950e+04	7.707855e+04	7.727069e+04	7.702108e+04	9.403985e+04
	avg.	7.806766e+04	7.901290e+04	7.939995e+04	8.019429e+04	9.701349e+04	1.001940e+05	7.975275e+04	7.538598e+04	7.905823e+04	7.603543e+04	9.114068e+04
	min.	6.672905e+04	6.522831e+04	7.602940e+04	6.472615e+04	8.721859e+04	9.066124e+04	7.281550e+04	6.530258e+04	7.429533e+04	6.456288e+04	4.498656e+04
	max.	8.836095e+04	9.471423e+04	8.502452e+04	9.804151e+04	1.080276e+05	1.069559e+05	8.496520e+04	8.575771e+04	8.496625e+04	8.499384e+04	1.030294e+05
	std.	2.760135e+03	6.368170e+03	3.019714e+03	7.455020e+03	4.536846e+03	3.118228e+03	3.163099e+03	5.956832e+03	2.986719e+03	4.256413e+03	1.149279e+04
7D	med.	8.665944e+05	7.351512e+05	1.051614e+06	7.570269e+05	1.116411e+06	1.165885e+06	9.884837e+05	6.144525e+05	9.876145e+05	6.162745e+05	6.750858e+05
	avg.	8.452526e+05	7.556500e+05	1.054272e+06	7.950359e+05	1.138403e+06	1.172022e+06	9.946070e+05	6.388249e+05	9.824324e+05	6.487505e+05	9.765471e+05
	min.	5.561892e+05	5.470547e+05	9.654646e+05	6.115260e+05	9.274674e+05	9.611057e+05	6.124551e+05	5.449880e+05	5.600697e+05	5.551829e+05	6.744960e+05
	max.	1.082158e+06	1.123576e+06	1.200031e+06	1.171667e+06	1.465564e+06	1.322579e+06	1.200414e+06	9.684462e+05	1.177195e+06	9.642604e+05	1.490148e+06
	std.	1.187098e+05	1.358017e+05	6.086009e+04	1.507589e+05	1.214841e+05	6.969629e+04	8.134990e+04	7.601779e+04	8.891268e+04	7.632429e+04	3.127678e+05
8D	med.	1.309568e+07	1.114546e+07	1.738761e+07	1.375221e+07	1.833802e+07	1.835531e+07	1.573946e+07	9.152114e+06	1.574264e+07	9.379098e+06	1.146030e+07
	avg.	1.280286e+07	1.111359e+07	1.763272e+07	1.349103e+07	1.878741e+07	1.831950e+07	1.549959e+07	9.830721e+06	1.579697e+07	1.018581e+07	1.568883e+07
	min.	8.978467e+06	8.816573e+06	1.603701e+07	8.991990e+06	1.480212e+07	1.567497e+07	8.677331e+06	8.384178e+06	8.982457e+06	8.564929e+06	1.144129e+07
	max.	1.873898e+07	1.683702e+07	2.035227e+07	1.942026e+07	2.625739e+07	2.151594e+07	1.979977e+07	1.441522e+07	1.871482e+07	1.538359e+07	2.424961e+07
	std.	1.764207e+06	1.985263e+06	1.077253e+06	2.155606e+06	2.280824e+06	1.017997e+06	2.026226e+06	1.285708e+06	1.490840e+06	1.637815e+06	5.043111e+06
9D	med.	2.233023e+08	2.050795e+08	3.255743e+08	2.674850e+08	3.522461e+08	3.233205e+08	2.940456e+08	1.986125e+08	2.882618e+08	1.981265e+08	2.172102e+08
	avg.	2.244879e+08	2.120746e+08	3.317167e+08	2.549213e+08	3.566500e+08	3.263401e+08	2.813057e+08	1.980080e+08	2.666891e+08	1.988766e+08	2.566865e+08
	min.	1.442389e+08	1.489511e+08	2.981787e+08	1.510323e+08	2.841685e+08	2.660174e+08	1.480564e+08	1.400428e+08	1.353626e+08	1.371875e+08	2.168030e+08
	max.	3.175971e+08	2.998067e+08	3.851269e+08	3.196483e+08	4.433051e+08	3.984883e+08	3.658991e+08	2.705408e+08	3.479304e+08	2.932968e+08	4.235579e+08
	std.	4.207369e+07	3.780625e+07	2.245247e+07	3.307169e+07	3.265161e+07	2.317448e+07	4.568361e+07	3.629386e+07	5.480002e+07	4.551265e+07	6.645530e+07
10D	med.	5.020578e+09	4.632117e+09	6.793466e+09	5.657407e+09	6.714911e+09	6.441574e+09	5.794088e+09	4.952443e+09	5.617004e+09	5.253163e+09	4.536020e+09
	avg.	5.049776e+09	4.627409e+09	6.905895e+09	5.605975e+09	6.785805e+09	6.515523e+09	5.250256e+09	4.837977e+09	5.492606e+09	4.965421e+09	5.277456e+09
	min.	3.229341e+09	2.860636e+09	6.035290e+09	3.916558e+09	5.333681e+09	5.708789e+09	2.509782e+09	2.659224e+09	2.582195e+09	2.601750e+09	4.520939e+09
	max.	7.466136e+09	6.747255e+09	8.618618e+09	6.648715e+09	8.669152e+09	7.751784e+09	6.982521e+09	5.849199e+09	6.980622e+09	6.103850e+09	8.430382e+09
	std.	8.852408e+08	8.168356e+08	5.137782e+08	4.205796e+08	5.568829e+08	4.324194e+08	1.217396e+09	7.265327e+08	1.308713e+09	8.204003e+08	1.291158e+09

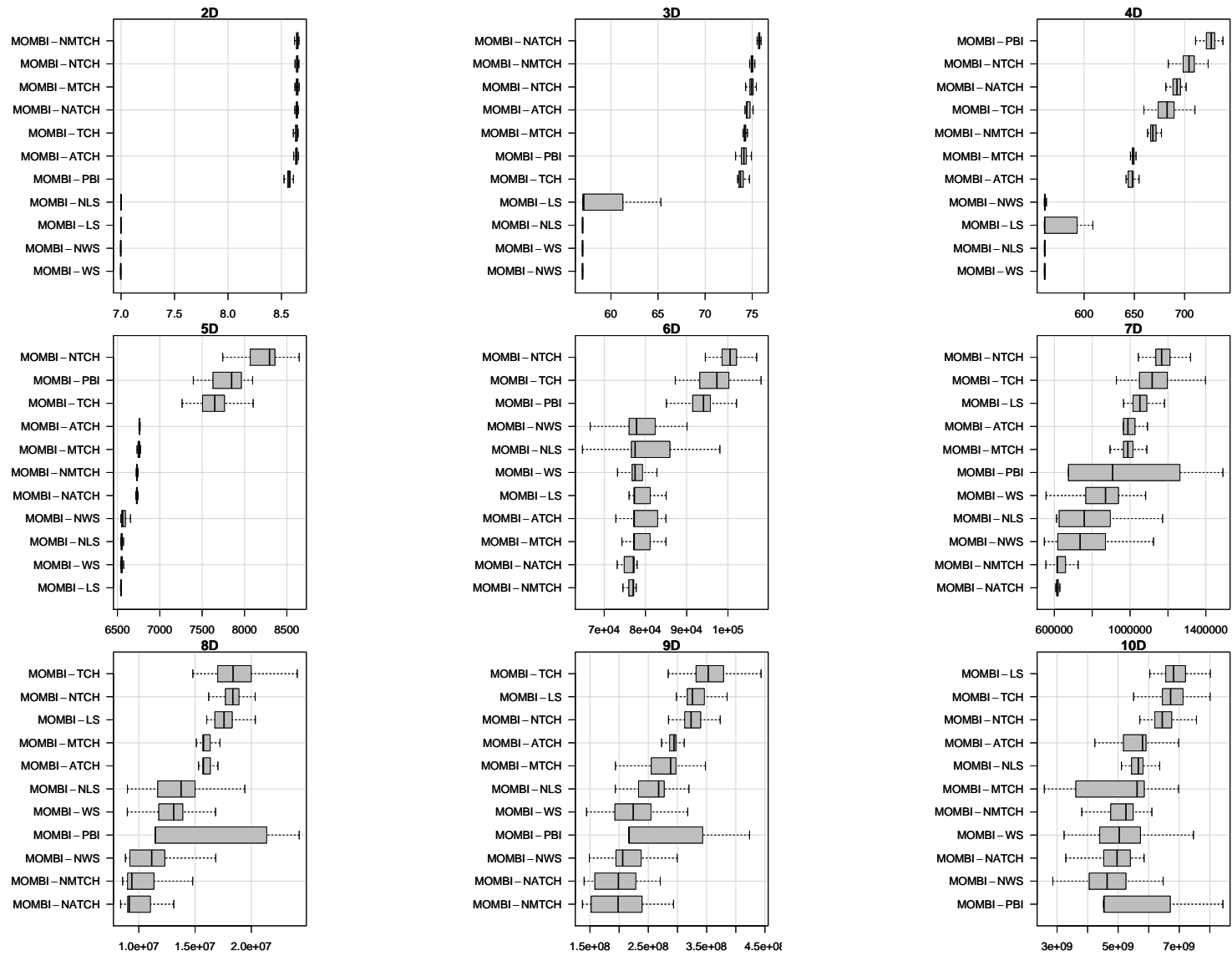


Figure A.387: Box-plot of the hypervolume indicator values for different optimizers on the WFG7 test problem.

Table A.164: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG7 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
2D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.97e-31$	$2.42e-32$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$1.39e-32$	$2.95e-33$	$1.21e-03$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$8.92e-34$
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$8.43e-07$	–	$2.81e-08$	$2.29e-03$	> 0.05	> 0.05	$1.32e-34$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.19e-02$	> 0.05	$1.04e-03$	–	> 0.05	> 0.05	$1.36e-34$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.00e-06$	> 0.05	$1.21e-07$	$4.11e-03$	–	> 0.05	$1.28e-34$
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$5.53e-08$	> 0.05	$2.45e-09$	$3.14e-04$	> 0.05	–	$1.28e-34$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
3D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$3.11e-30$	$7.07e-29$	–	$1.46e-30$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$1.87e-03$	$8.18e-04$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.63e-34$	–	$2.28e-22$	> 0.05	$4.64e-34$	> 0.05	$3.90e-33$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.20e-31$	> 0.05	–	> 0.05	$1.73e-26$	> 0.05	$3.52e-18$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	$1.28e-34$	$1.28e-34$	$1.28e-34$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.35e-20$	> 0.05	> 0.05	> 0.05	–	> 0.05	$4.88e-04$
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	$5.53e-29$	> 0.05	$1.28e-34$	–	$4.00e-34$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$3.05e-10$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
4D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	$5.74e-03$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.39e-02$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	$7.75e-17$	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.60e-24$	–	$1.28e-34$	$9.40e-21$	$1.28e-34$	$1.28e-34$	> 0.05
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.85e-11$	> 0.05	$1.28e-34$	–	$1.28e-34$	$1.28e-34$	> 0.05
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$3.64e-05$	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$1.28e-34$	> 0.05	$1.28e-34$	–	> 0.05
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.73e-34$	$2.12e-31$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–

Table A.165: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG7 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
5D											
MOMBI-WS	–	> 0.05	$8.21e-22$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	$3.53e-09$	–	$4.44e-31$	$1.81e-05$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	> 0.05	> 0.05	$1.05e-26$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.36e-34$	–	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$3.35e-31$	–	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$4.33e-24$
MOMBI-ATCH	$9.03e-29$	$3.67e-23$	$1.78e-32$	$8.82e-16$	> 0.05	> 0.05	–	$6.33e-22$	$8.06e-10$	$2.10e-28$	> 0.05
MOMBI-NATCH	$4.91e-32$	$7.99e-26$	$1.28e-34$	$6.12e-18$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$4.63e-32$	$4.08e-26$	$1.28e-34$	$6.12e-18$	> 0.05	> 0.05	> 0.05	$1.90e-23$	–	$1.60e-29$	> 0.05
MOMBI-NMTCH	$4.77e-32$	$4.89e-26$	$1.28e-34$	$6.12e-18$	> 0.05	> 0.05	> 0.05	$9.65e-03$	> 0.05	–	> 0.05
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$6.02e-09$	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–
6D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.59e-05$	> 0.05	$1.24e-07$	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.97e-06$	> 0.05	$2.33e-07$	> 0.05
MOMBI-LS	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$2.36e-02$	$1.41e-15$	$4.22e-04$	$5.50e-22$	> 0.05
MOMBI-NLS	$1.63e-02$	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$1.85e-07$	$2.91e-02$	$4.40e-08$	> 0.05
MOMBI-TCH	$1.40e-34$	$7.65e-33$	$1.28e-34$	$1.23e-31$	–	> 0.05	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.36e-07$
MOMBI-NTCH	$1.28e-34$	$2.26e-34$	$1.28e-34$	$5.88e-34$	$5.53e-08$	–	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$6.10e-23$
MOMBI-ATCH	$1.27e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	$1.35e-14$	$3.91e-02$	$1.03e-20$	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.11e-09$	–	$9.84e-14$	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	$4.23e-28$	$1.52e-24$	$3.05e-27$	$1.20e-21$	> 0.05	> 0.05	$4.19e-27$	$8.06e-28$	$1.41e-27$	$3.90e-28$	–
7D											
MOMBI-WS	–	$5.98e-07$	> 0.05	$9.77e-04$	> 0.05	> 0.05	> 0.05	$1.90e-23$	> 0.05	$1.93e-22$	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.53e-10$	> 0.05	$1.35e-08$	> 0.05
MOMBI-LS	$7.61e-31$	$1.55e-31$	–	$3.15e-26$	> 0.05	> 0.05	$3.89e-12$	$1.89e-34$	$1.77e-14$	$1.28e-34$	> 0.05
MOMBI-NLS	> 0.05	$3.05e-02$	> 0.05	–	> 0.05	> 0.05	> 0.05	$1.63e-14$	> 0.05	$2.67e-13$	> 0.05
MOMBI-TCH	$1.89e-32$	$9.66e-33$	$8.70e-08$	$2.63e-29$	–	> 0.05	$7.54e-20$	$1.68e-34$	$1.20e-21$	$1.44e-34$	$9.59e-03$
MOMBI-NTCH	$3.44e-34$	$3.76e-34$	$1.89e-22$	$2.92e-32$	$2.24e-04$	–	$1.36e-29$	$1.40e-34$	$3.48e-30$	$1.32e-34$	$2.27e-03$
MOMBI-ATCH	$6.13e-24$	$3.58e-27$	> 0.05	$6.97e-21$	> 0.05	> 0.05	–	$9.79e-32$	> 0.05	$2.92e-32$	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$2.77e-21$	$1.20e-25$	> 0.05	$1.32e-19$	> 0.05	> 0.05	> 0.05	$3.00e-32$	–	$1.83e-32$	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05
MOMBI-PBI	> 0.05	$1.73e-06$	> 0.05	$4.02e-04$	> 0.05	> 0.05	> 0.05	$3.29e-30$	> 0.05	$1.11e-26$	–

Table A.166: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG7 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
8D											
MOMBI-WS	–	$3.45e-10$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.64e-23$	$> 0.05$	$1.59e-18$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$5.42e-07$	$> 0.05$	$2.28e-04$	$> 0.05$
MOMBI-LS	$4.65e-33$	$3.88e-34$	–	$1.82e-30$	$> 0.05$	$> 0.05$	$4.65e-25$	$1.28e-34$	$3.18e-25$	$1.28e-34$	$1.63e-02$
MOMBI-NLS	$3.51e-03$	$3.25e-13$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$3.43e-25$	$> 0.05$	$4.82e-21$	$> 0.05$
MOMBI-TCH	$2.58e-33$	$5.22e-34$	$2.79e-04$	$4.32e-31$	–	$> 0.05$	$1.90e-23$	$1.28e-34$	$2.37e-23$	$1.32e-34$	$6.87e-05$
MOMBI-NTCH	$1.48e-33$	$1.73e-34$	$1.13e-06$	$7.14e-32$	$> 0.05$	–	$3.98e-29$	$1.28e-34$	$9.54e-29$	$1.28e-34$	$6.07e-03$
MOMBI-ATCH	$4.02e-20$	$5.69e-24$	$> 0.05$	$5.47e-17$	$> 0.05$	$> 0.05$	–	$6.69e-29$	$> 0.05$	$4.23e-28$	$> 0.05$
MOMBI-NATCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$2.87e-25$	$3.41e-28$	$> 0.05$	$6.36e-21$	$> 0.05$	$> 0.05$	$> 0.05$	$3.27e-32$	–	$8.01e-32$	$> 0.05$
MOMBI-NMTCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$> 0.05$	$8.17e-13$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$6.16e-29$	$> 0.05$	$2.05e-21$	–
9D											
MOMBI-WS	–	$1.55e-02$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$7.02e-06$	$> 0.05$	$3.91e-05$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$7.20e-03$	$> 0.05$	$1.19e-02$	$> 0.05$
MOMBI-LS	$6.63e-34$	$1.36e-34$	–	$1.81e-33$	$> 0.05$	$> 0.05$	$3.80e-28$	$1.28e-34$	$3.41e-28$	$1.28e-34$	$2.41e-11$
MOMBI-NLS	$1.05e-07$	$5.67e-14$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$2.34e-20$	$> 0.05$	$7.38e-16$	$9.00e-04$
MOMBI-TCH	$3.15e-34$	$1.58e-34$	$4.85e-09$	$4.92e-34$	–	$5.19e-12$	$3.78e-30$	$1.28e-34$	$5.90e-31$	$1.36e-34$	$1.58e-17$
MOMBI-NTCH	$3.90e-33$	$2.79e-34$	$> 0.05$	$4.91e-32$	$> 0.05$	–	$8.76e-23$	$1.32e-34$	$3.64e-24$	$2.01e-34$	$1.21e-10$
MOMBI-ATCH	$1.89e-18$	$6.21e-21$	$> 0.05$	$1.23e-15$	$> 0.05$	$> 0.05$	–	$3.25e-23$	$> 0.05$	$2.31e-23$	$1.02e-04$
MOMBI-NATCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$4.55e-11$	$8.35e-14$	$> 0.05$	$1.10e-06$	$> 0.05$	$> 0.05$	$> 0.05$	$1.07e-16$	–	$1.97e-16$	$1.92e-02$
MOMBI-NMTCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$3.07e-02$	$4.79e-07$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$3.84e-10$	$> 0.05$	$9.76e-09$	–
10D											
MOMBI-WS	–	$1.20e-03$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$7.18e-30$	$3.65e-34$	–	$3.08e-33$	$> 0.05$	$3.22e-08$	$7.78e-32$	$1.28e-34$	$4.57e-31$	$1.40e-34$	$4.71e-14$
MOMBI-NLS	$1.03e-07$	$1.53e-18$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$5.39e-18$	$2.83e-02$	$7.76e-14$	$1.60e-08$
MOMBI-TCH	$4.47e-28$	$2.51e-33$	$> 0.05$	$1.09e-29$	–	$3.22e-05$	$4.97e-28$	$8.40e-34$	$4.84e-28$	$2.37e-33$	$2.62e-13$
MOMBI-NTCH	$9.56e-26$	$4.52e-33$	$> 0.05$	$5.68e-29$	$> 0.05$	–	$2.28e-25$	$1.95e-34$	$8.19e-26$	$3.65e-34$	$5.35e-11$
MOMBI-ATCH	$5.70e-04$	$2.92e-09$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$3.40e-10$	$2.05e-02$	$3.79e-09$	$1.87e-02$
MOMBI-NATCH	$> 0.05$	$1.59e-02$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$> 0.05$	$1.20e-03$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$2.04e-03$	–	$6.15e-03$	$> 0.05$
MOMBI-NMTCH	$> 0.05$	$1.38e-04$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$2.22e-02$	$> 0.05$	–	$1.09e-02$
MOMBI-PBI	$> 0.05$	$7.10e-03$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–



Table A.167: Comparison of R2 indicator values for different optimizers on the WFG7 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	6.667668e-01	6.667629e-01	6.667119e-01	6.667050e-01	5.884837e-01	5.886485e-01	5.887891e-01	5.887307e-01	5.884694e-01	5.886116e-01	5.910825e-01
	avg.	6.667678e-01	6.667689e-01	6.667037e-01	6.664725e-01	5.885111e-01	5.886547e-01	5.888118e-01	5.887351e-01	5.884706e-01	5.886247e-01	5.911528e-01
	min.	6.667164e-01	6.667217e-01	6.667362e-01	6.469667e-01	5.882729e-01	5.883349e-01	5.885654e-01	5.885254e-01	5.882908e-01	5.883165e-01	5.901807e-01
	max.	6.668460e-01	6.668213e-01	6.667635e-01	6.667521e-01	5.896481e-01	5.890223e-01	5.890509e-01	5.890462e-01	5.887797e-01	5.891413e-01	5.931627e-01
	std.	2.369114e-05	2.198604e-05	4.493702e-05	1.972939e-03	1.543833e-04	1.467205e-04	1.024789e-04	1.056822e-04	9.345267e-05	1.356388e-04	4.235313e-04
3D	med.	3.638423e-01	3.638455e-01	3.633237e-01	3.638385e-01	3.159984e-01	3.159284e-01	3.157431e-01	3.164292e-01	3.154165e-01	3.161174e-01	3.193188e-01
	avg.	3.638278e-01	3.638276e-01	3.584902e-01	3.638303e-01	3.159900e-01	3.159070e-01	3.157783e-01	3.164231e-01	3.154150e-01	3.161011e-01	3.194835e-01
	min.	3.634435e-01	3.634733e-01	3.392976e-01	3.636726e-01	3.152859e-01	3.151725e-01	3.152922e-01	3.158387e-01	3.149313e-01	3.154106e-01	3.172809e-01
	max.	3.638790e-01	3.638845e-01	3.638432e-01	3.638655e-01	3.164544e-01	3.167023e-01	3.167446e-01	3.171778e-01	3.158315e-01	3.166150e-01	3.248227e-01
	std.	5.624813e-05	6.289615e-05	7.470917e-03	2.997401e-05	2.189445e-04	3.560636e-04	1.796507e-04	2.391503e-04	1.912813e-04	2.304587e-04	1.20602e-03
4D	med.	2.400684e-01	2.400479e-01	2.400460e-01	2.400723e-01	2.232310e-01	2.202451e-01	2.262345e-01	2.255576e-01	2.253836e-01	2.270726e-01	2.152846e-01
	avg.	2.400447e-01	2.399944e-01	2.383294e-01	2.399870e-01	2.229210e-01	2.202847e-01	2.258611e-01	2.254221e-01	2.253248e-01	2.268450e-01	2.159968e-01
	min.	2.396335e-01	2.378361e-01	2.307683e-01	2.335315e-01	2.178779e-01	2.169514e-01	2.244795e-01	2.233932e-01	2.237555e-01	2.235715e-01	2.121994e-01
	max.	2.401173e-01	2.401297e-01	2.401239e-01	2.401154e-01	2.258565e-01	2.234185e-01	2.264834e-01	2.268221e-01	2.259675e-01	2.275340e-01	2.262710e-01
	std.	8.688086e-05	2.469184e-04	2.789080e-03	6.553000e-04	1.842752e-03	1.422408e-03	5.736494e-04	7.480688e-04	4.230244e-04	6.492072e-04	2.815928e-03
5D	med.	1.819976e-01	1.819261e-01	1.821000e-01	1.820035e-01	1.754247e-01	1.786961e-01	1.883369e-01	1.815452e-01	1.793456e-01	1.815292e-01	1.839597e-01
	avg.	1.819174e-01	1.817927e-01	1.821020e-01	1.817207e-01	1.754938e-01	1.683121e-01	1.787999e-01	1.816379e-01	1.795339e-01	1.815348e-01	1.839476e-01
	min.	1.793073e-01	1.775522e-01	1.820510e-01	1.776479e-01	1.688927e-01	1.651622e-01	1.786446e-01	1.806901e-01	1.785821e-01	1.806466e-01	1.782477e-01
	max.	1.821068e-01	1.821086e-01	1.821534e-01	1.821084e-01	1.787281e-01	1.719923e-01	1.821531e-01	1.834028e-01	1.801992e-01	1.832798e-01	1.904683e-01
	std.	3.824527e-04	5.206766e-04	1.538870e-05	7.552294e-04	1.759050e-03	1.659027e-03	5.863798e-04	5.082857e-04	4.004630e-04	2.223403e-04	3.449197e-03
6D	med.	1.688737e-01	1.690053e-01	1.689531e-01	1.687893e-01	1.642958e-01	1.635323e-01	1.689715e-01	1.690875e-01	1.689806e-01	1.691248e-01	1.608545e-01
	avg.	1.719610e-01	1.780665e-01	1.620672e-01	1.782226e-01	1.704335e-01	1.741260e-01	1.606890e-01	1.812150e-01	1.626269e-01	1.754742e-01	1.714320e-01
	min.	1.565133e-01	1.290506e-01	1.430295e-01	1.373321e-01	1.311817e-01	1.357275e-01	1.430517e-01	1.430435e-01	1.430555e-01	1.430254e-01	1.403943e-01
	max.	2.310467e-01	2.372113e-01	1.699344e-01	2.360488e-01	2.204467e-01	2.248897e-01	1.762523e-01	2.323492e-01	1.772076e-01	2.347861e-01	3.334193e-01
	std.	1.285174e-02	2.388796e-02	1.001455e-02	2.504722e-02	1.817013e-02	2.225385e-02	1.043096e-02	2.912480e-02	9.579656e-03	2.134894e-02	3.905799e-02
7D	med.	2.465550e-01	3.623876e-01	1.677448e-01	3.620104e-01	2.608159e-01	3.086377e-01	1.883369e-01	3.711934e-01	1.883444e-01	3.710945e-01	1.704314e-01
	avg.	2.583000e-01	3.158752e-01	1.686749e-01	3.373703e-01	2.660393e-01	2.923160e-01	1.856078e-01	3.591685e-01	1.906780e-01	3.530747e-01	2.210465e-01
	min.	1.665250e-01	1.695368e-01	1.382889e-01	1.864989e-01	1.491986e-01	1.792561e-01	1.382799e-01	1.893011e-01	1.506934e-01	1.893878e-01	1.265960e-01
	max.	3.767491e-01	4.000189e-01	1.893501e-01	3.712720e-01	3.623620e-01	3.746822e-01	3.713050e-01	3.797650e-01	3.759553e-01	3.946778e-01	2.858235e-01
	std.	6.626688e-02	7.128086e-02	1.402554e-02	5.862663e-02	5.182142e-02	5.753083e-02	3.324882e-02	3.903735e-02	3.652653e-02	4.018119e-02	6.577638e-02
8D	med.	2.722796e-01	3.855318e-01	1.454690e-01	3.774257e-01	2.755048e-01	3.572954e-01	1.741806e-01	3.917634e-01	1.741737e-01	3.909415e-01	2.501164e-01
	avg.	2.778032e-01	3.696230e-01	1.497273e-01	3.657374e-01	2.738484e-01	3.370754e-01	1.874816e-01	3.886699e-01	1.800328e-01	3.819131e-01	2.005989e-01
	min.	1.598865e-01	1.939212e-01	1.207855e-01	1.695959e-01	1.386384e-01	1.842207e-01	1.255096e-01	3.383130e-01	1.349962e-01	1.753328e-01	1.129247e-01
	max.	4.031486e-01	4.142801e-01	1.699603e-01	4.151625e-01	3.796571e-01	3.910363e-01	3.923243e-01	4.213477e-01	3.922145e-01	4.172077e-01	2.502422e-01
	std.	6.226400e-02	4.325255e-02	1.288818e-02	4.324038e-02	6.494509e-02	4.612499e-02	5.606543e-02	1.074383e-02	4.239902e-02	3.096753e-02	5.785028e-02
9D	med.	3.065848e-01	3.998289e-01	1.383058e-01	3.895172e-01	2.979613e-01	3.766106e-01	1.561502e-01	4.000370e-01	1.623829e-01	4.001600e-01	2.224754e-01
	avg.	3.085314e-01	3.937681e-01	1.344705e-01	3.906280e-01	2.978110e-01	3.631502e-01	1.809481e-01	4.002664e-01	2.051432e-01	4.008264e-01	1.978154e-01
	min.	1.596512e-01	2.672308e-01	1.065454e-01	3.685687e-01	1.752122e-01	1.940618e-01	1.131914e-01	3.711073e-01	1.242491e-01	3.544287e-01	1.121304e-01
	max.	4.308586e-01	4.437271e-01	1.556423e-01	4.114350e-01	4.040777e-01	4.187111e-01	4.067852e-01	4.353721e-01	4.098837e-01	4.496310e-01	2.225917e-01
	std.	7.318374e-02	2.596748e-02	1.226247e-02	8.045524e-03	6.666365e-02	4.286057e-02	7.233361e-02	9.298694e-03	8.794740e-02	1.314608e-02	4.099585e-02
10D	med.	3.147928e-01	4.036208e-01	1.248974e-01	3.958301e-01	3.744044e-01	3.855567e-01	1.504075e-01	4.021714e-01	1.605233e-01	3.998341e-01	2.005054e-01
	avg.	3.228140e-01	4.033079e-01	1.224787e-01	3.964301e-01	3.576190e-01	3.793955e-01	2.007942e-01	4.043053e-01	2.267918e-01	4.030036e-01	1.795932e-01
	min.	1.913946e-01	3.706882e-01	9.685424e-02	3.636820e-01	2.032021e-01	2.816180e-01	1.178646e-01	3.916107e-01	1.174222e-01	3.901669e-01	1.047579e-01
	max.	4.410680e-01	4.437083e-01	1.589066e-01	4.326701e-01	4.227538e-01	4.334593e-01	4.173218e-01	4.455957e-01	4.171132e-01	4.585953e-01	2.006888e-01
	std.	6.571752e-02	1.251186e-02	1.198311e-02	7.792921e-03	4.768531e-02	2.699228e-02	9.771140e-02	1.048567e-02	1.027357e-01	9.992665e-03	3.578327e-02

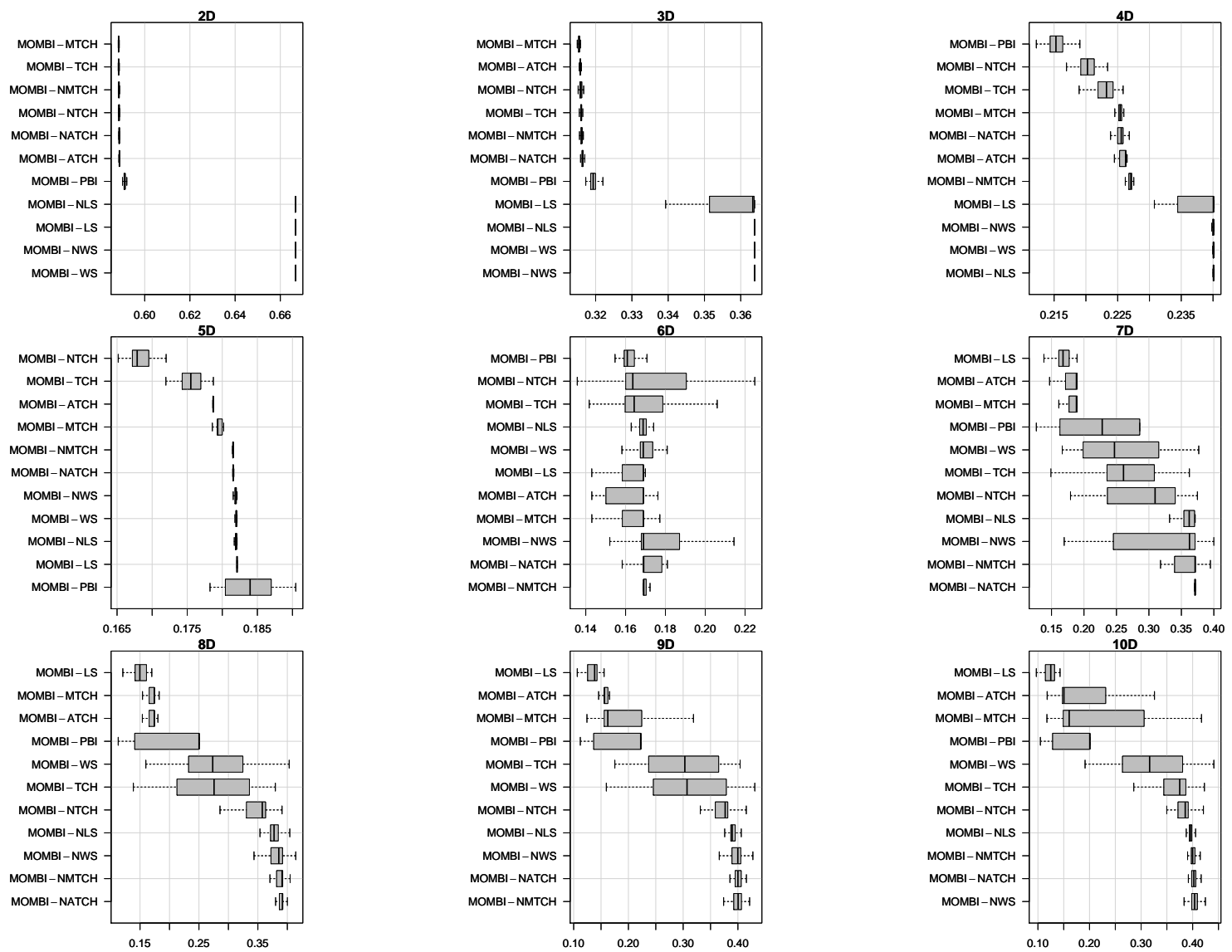


Figure A.388: Box-plot of the R2 indicator values for different optimizers on the WFG7 test problem.

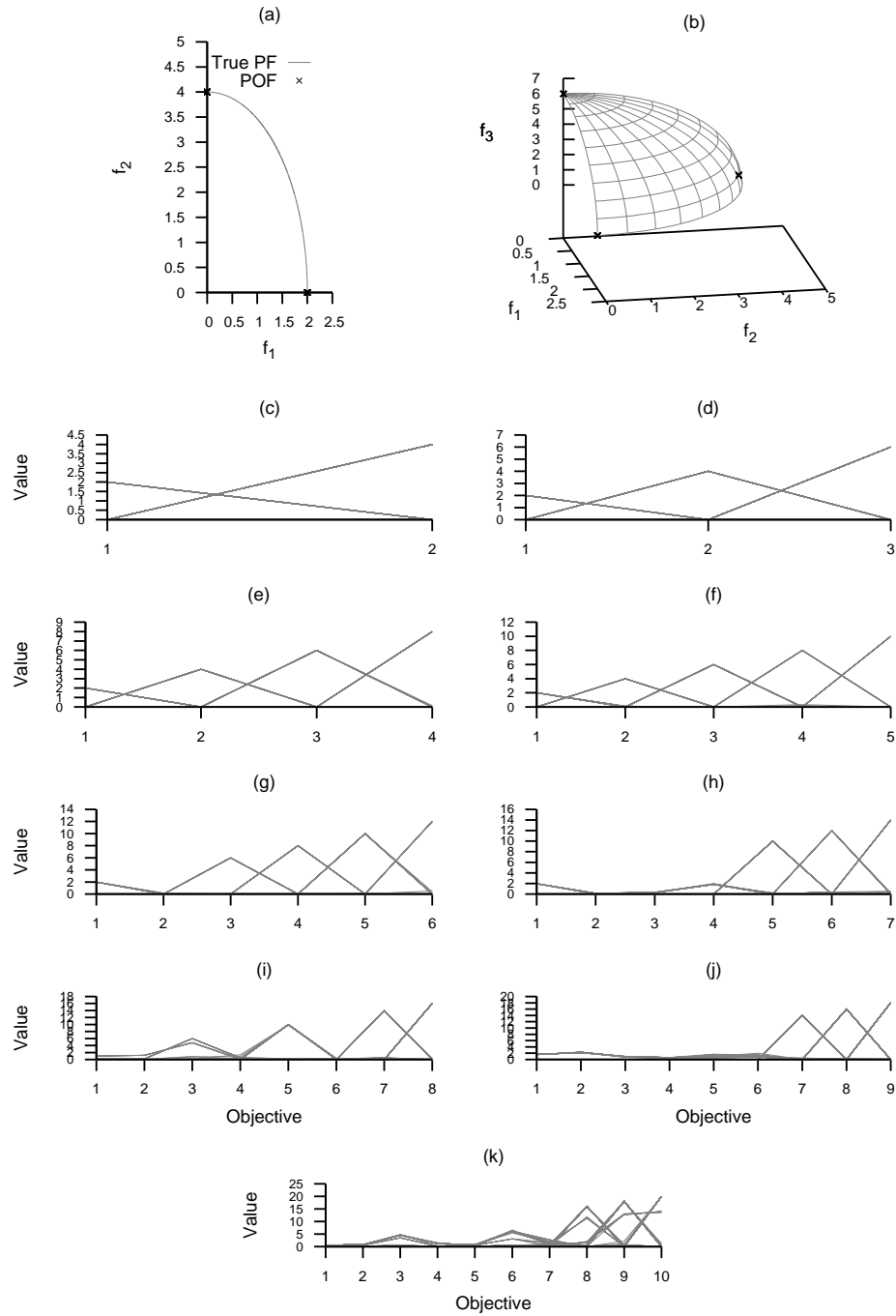


Figure A.389: Plots of the approximations obtained by MOMBI-WS from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

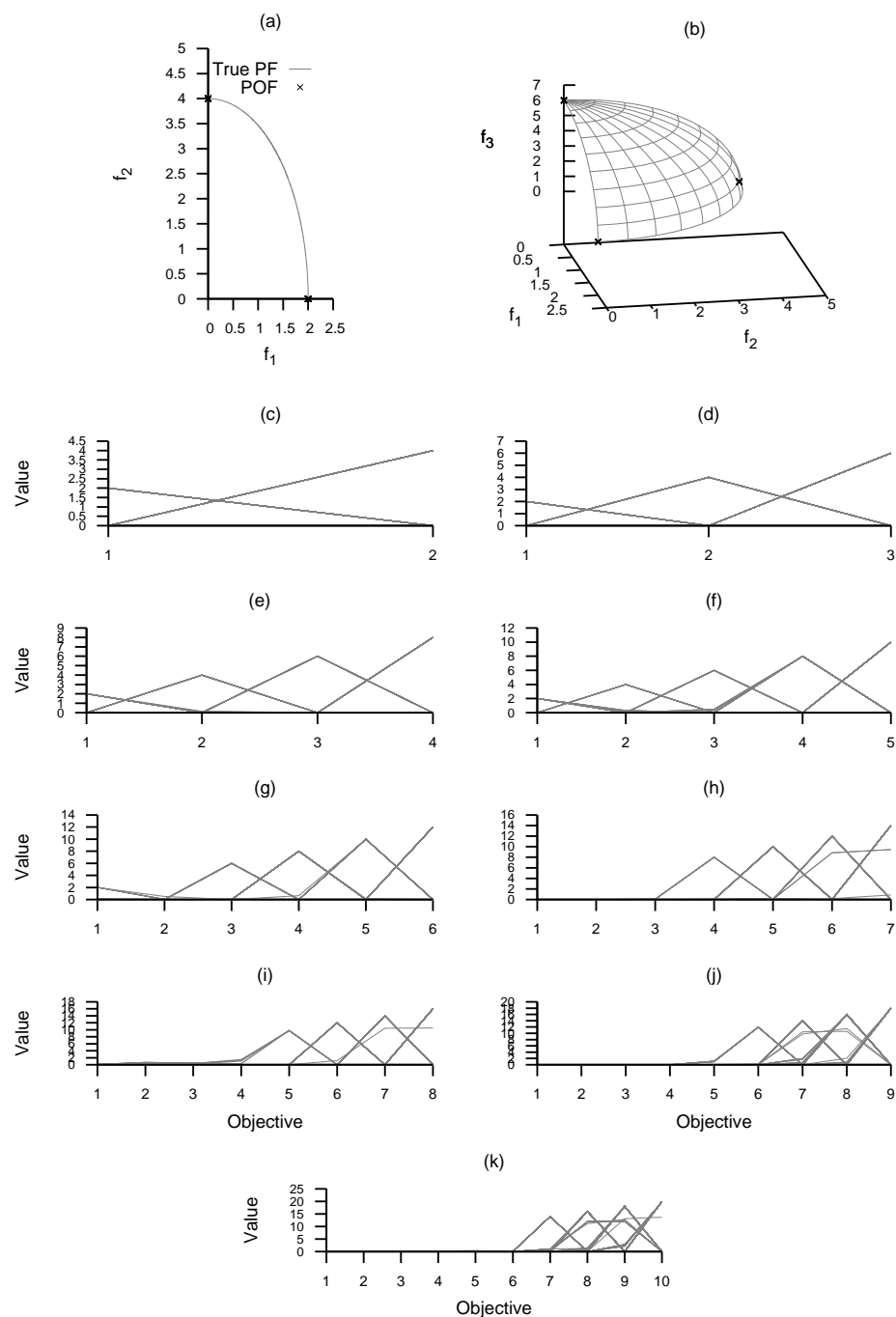


Figure A.390: Plots of the approximations obtained by MOMBI-NWS from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

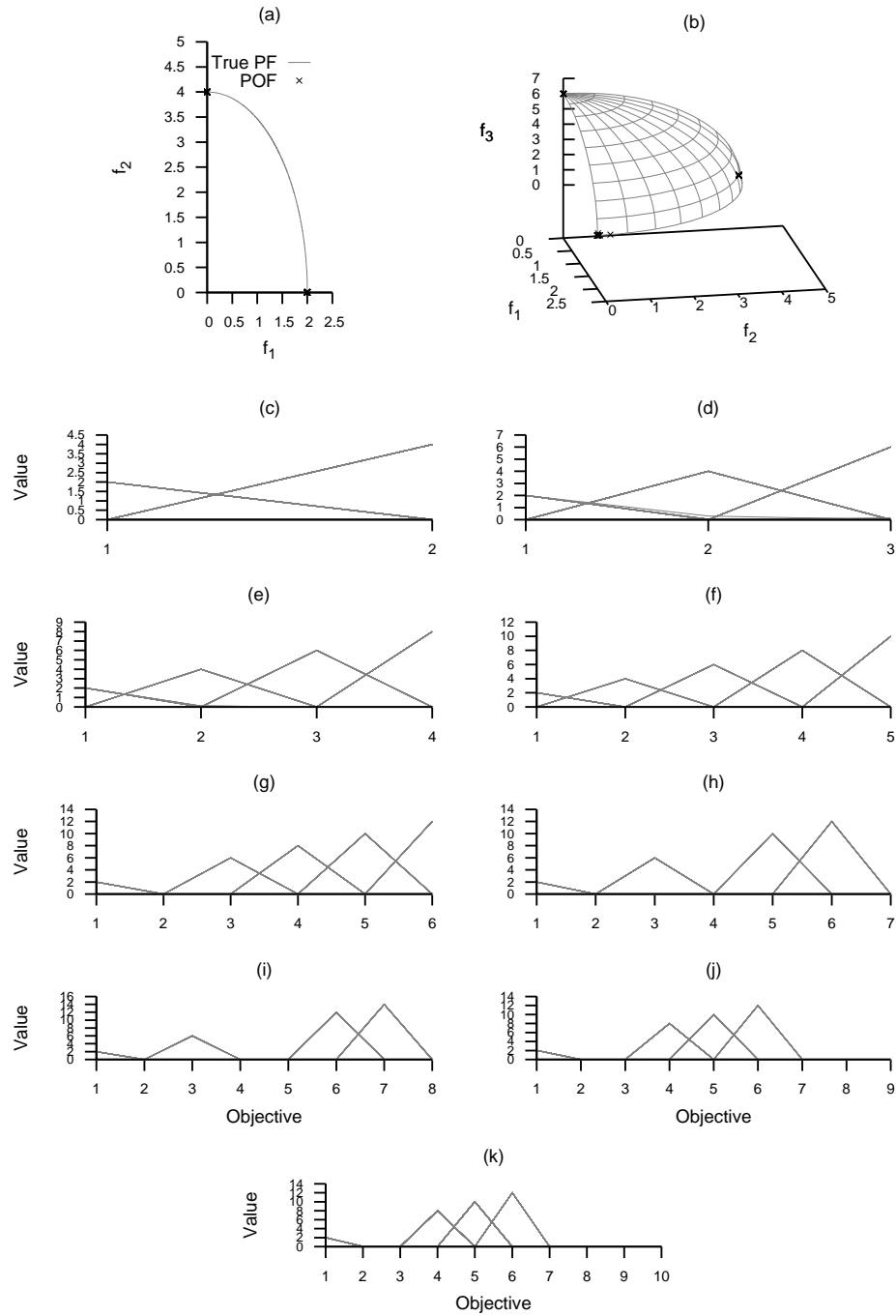


Figure A.391: Plots of the approximations obtained by MOMBI-LS from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

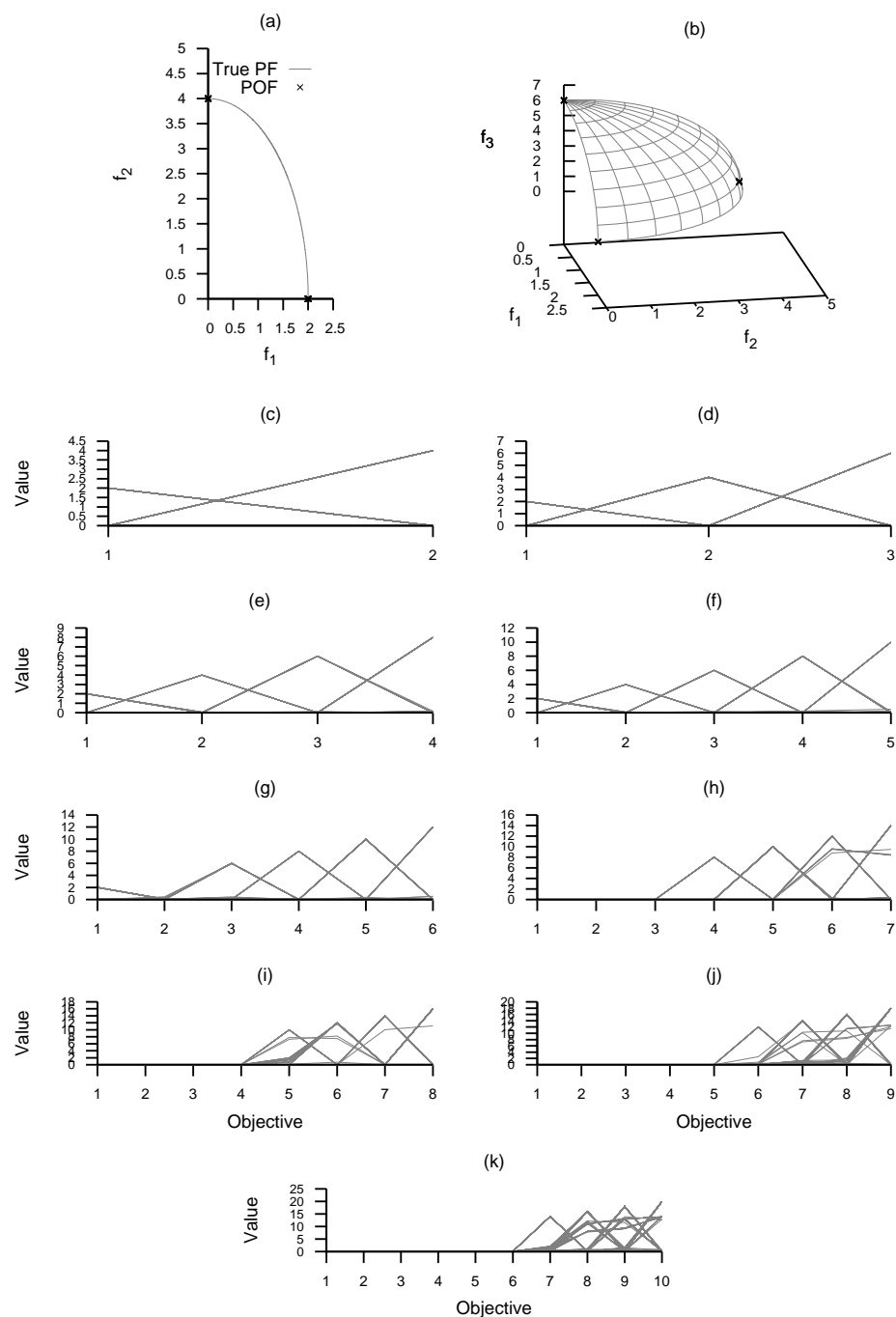


Figure A.392: Plots of the approximations obtained by MOMBI-NLS from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

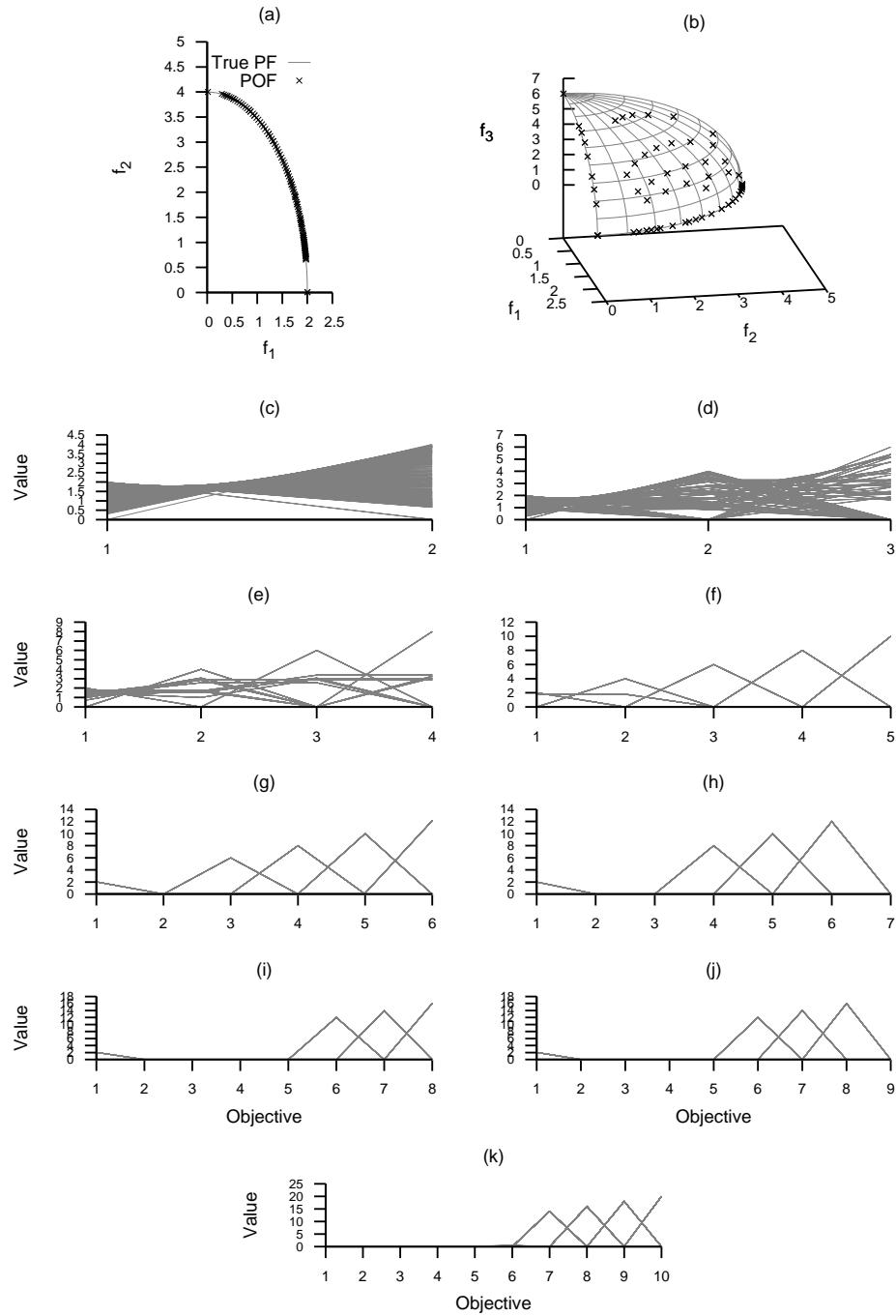


Figure A.393: Plots of the approximations obtained by MOMBI-ATCH from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

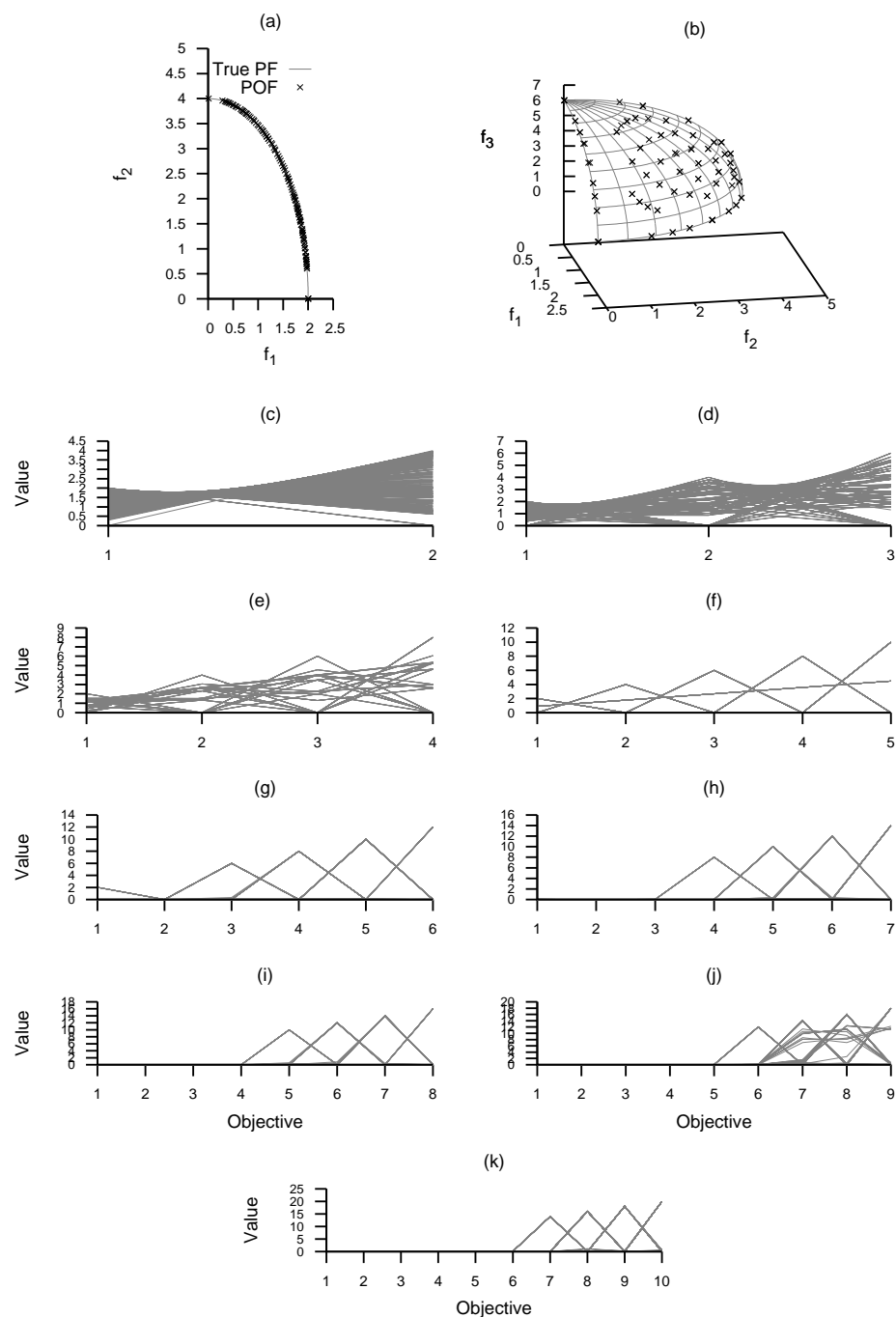


Figure A.394: Plots of the approximations obtained by MOMBI-NATCH from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



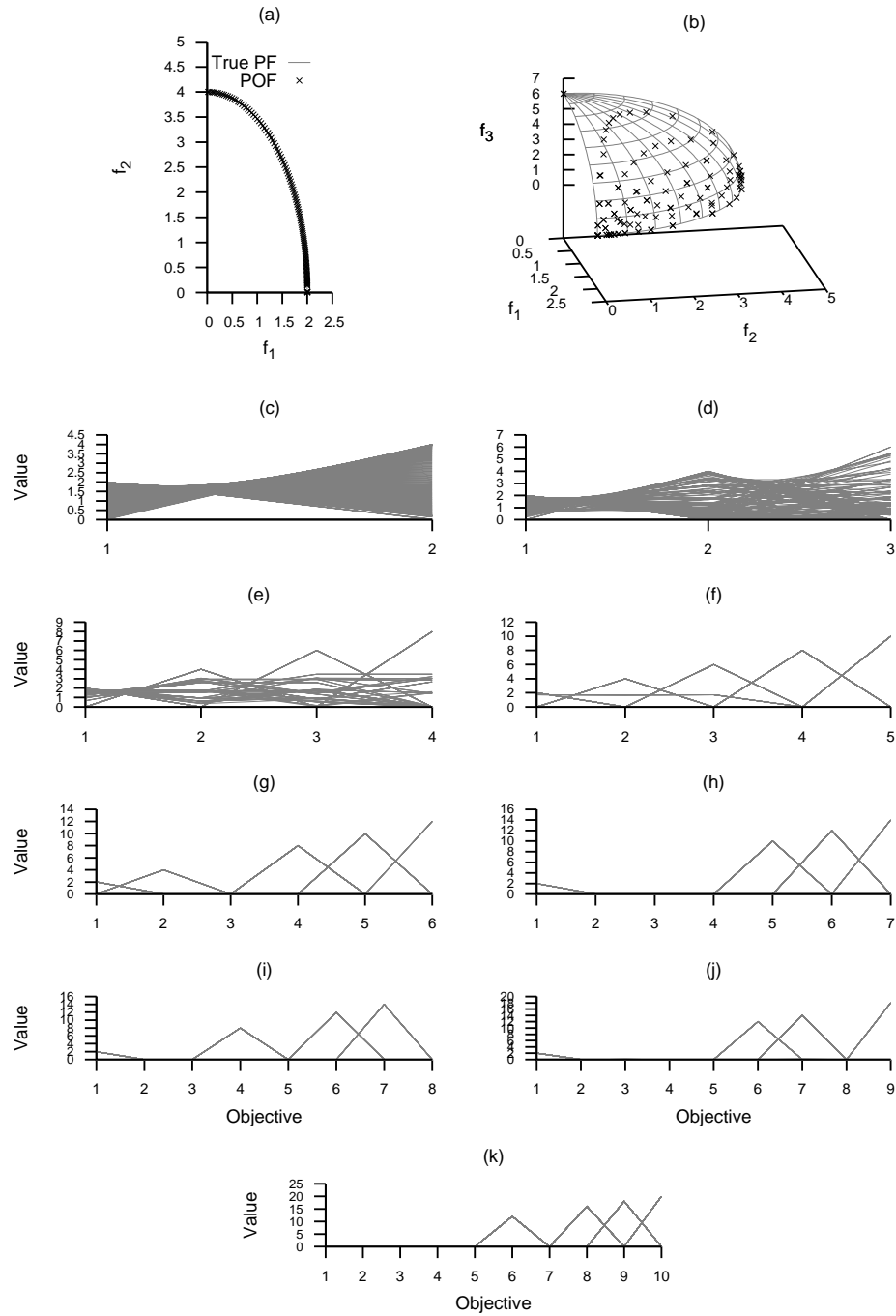


Figure A.395: Plots of the approximations obtained by MOMBI-MTCH from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

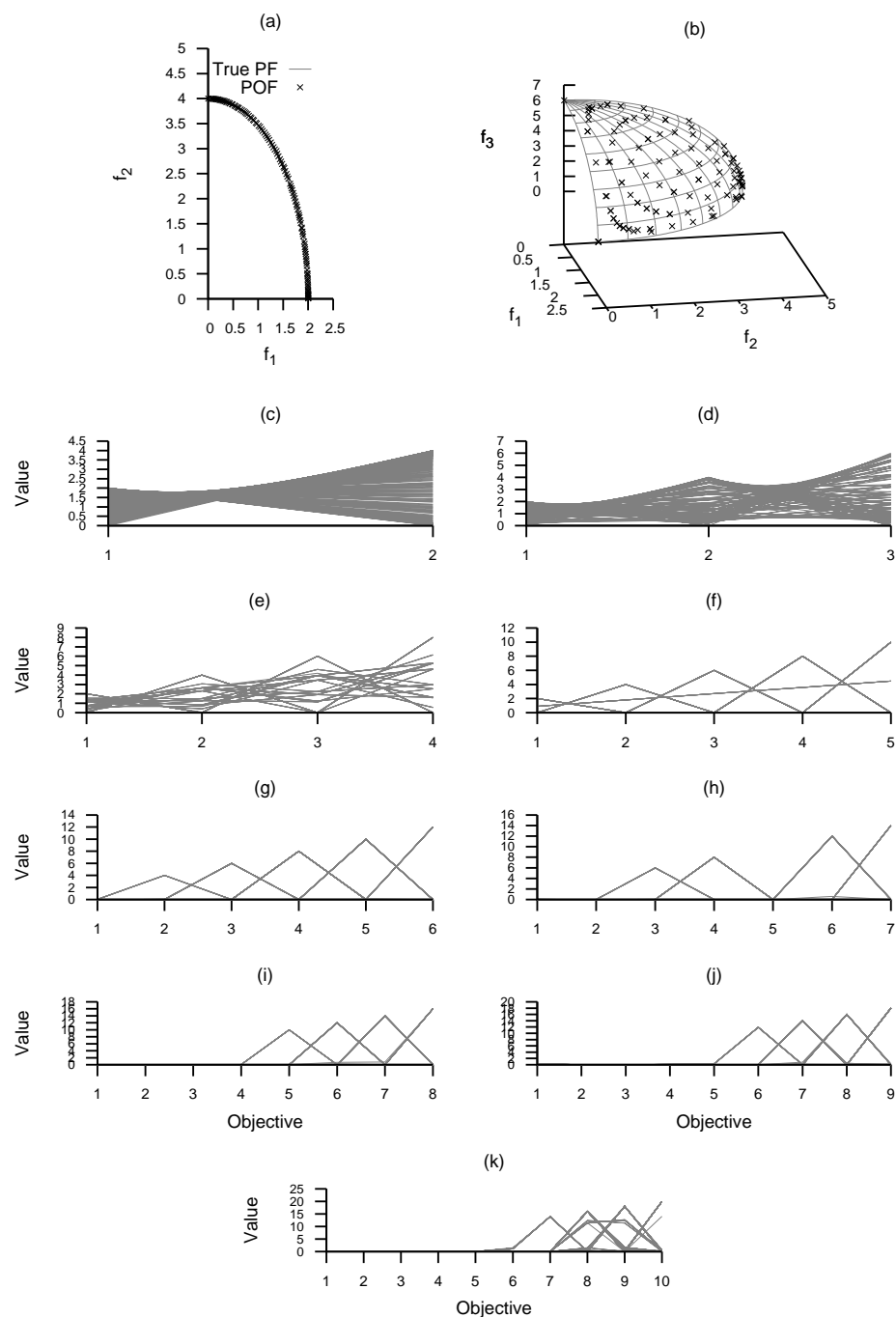


Figure A.396: Plots of the approximations obtained by MOMBI-NMTCH from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.168: Comparison of hypervolume indicator values for different optimizers on the WFG8 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	5.403588e+00	5.389641e+00	7.214370e+00	7.203570e+00	7.609085e+00	7.624457e+00	7.600113e+00	7.614079e+00	7.612038e+00	7.631366e+00	7.415266e+00
	avg.	5.442186e+00	5.425298e+00	7.207942e+00	7.192758e+00	7.604423e+00	7.622388e+00	7.600241e+00	7.613877e+00	7.610099e+00	7.628201e+00	7.414898e+00
	min.	5.097081e+00	4.941256e+00	6.943008e+00	6.889956e+00	7.508699e+00	7.525578e+00	7.431792e+00	7.528814e+00	7.522335e+00	7.501081e+00	7.269843e+00
	max.	5.700094e+00	5.681582e+00	7.467035e+00	7.489247e+00	7.667519e+00	7.688959e+00	7.675442e+00	7.676024e+00	7.675036e+00	7.679836e+00	7.499497e+00
	std.	1.288413e-01	1.440666e-01	9.776034e-02	1.220574e-01	2.778367e-02	2.845679e-02	3.048231e-02	2.734717e-02	2.692064e-02	2.668873e-02	4.121885e-02
3D	med.	4.897569e+01	4.895757e+01	5.029487e+01	5.036715e+01	6.732509e+01	6.816540e+01	6.794015e+01	6.820359e+01	6.775208e+01	6.843922e+01	6.727678e+01
	avg.	4.907312e+01	4.905555e+01	5.012790e+01	5.074974e+01	6.731089e+01	6.814704e+01	6.792765e+01	6.820221e+01	6.774309e+01	6.843049e+01	6.723880e+01
	min.	4.620304e+01	4.618600e+01	4.797639e+01	4.765017e+01	6.670845e+01	6.770151e+01	6.727574e+01	6.765590e+01	6.716497e+01	6.788887e+01	6.630210e+01
	max.	5.052106e+01	5.050861e+01	5.717249e+01	5.507015e+01	6.791372e+01	6.862492e+01	6.828990e+01	6.859865e+01	6.837453e+01	6.891208e+01	6.815403e+01
	std.	8.789653e-01	8.842672e-01	1.603988e+00	1.918352e+00	2.215034e-01	1.707991e-01	2.048586e-01	1.932749e-01	2.259560e-01	2.006246e-01	3.596731e-01
4D	med.	4.756710e+02	4.768800e+02	5.516589e+02	5.497251e+02	5.734627e+02	6.037960e+02	5.762500e+02	6.143927e+02	5.743428e+02	5.993977e+02	6.597493e+02
	avg.	4.764579e+02	4.795140e+02	5.511215e+02	5.488842e+02	5.781552e+02	6.032799e+02	5.759601e+02	6.153819e+02	5.739433e+02	5.996276e+02	6.598811e+02
	min.	4.510548e+02	4.591092e+02	5.324067e+02	5.230541e+02	5.562036e+02	5.834862e+02	5.660168e+02	5.950000e+02	5.593171e+02	5.854887e+02	6.463316e+02
	max.	5.537646e+02	5.466175e+02	6.621684e+02	6.06586e+02	6.056842e+02	6.321033e+02	5.858256e+02	6.434410e+02	5.84220e+02	6.112667e+02	6.731411e+02
	std.	1.115112e+01	1.200245e+01	5.898741e+00	6.996264e+00	1.324467e+01	1.081194e+01	4.230109e+00	9.521640e+00	4.578713e+00	5.261089e+00	5.284769e+00
5D	med.	5.515556e+03	5.483235e+03	5.511007e+03	5.852120e+03	5.773034e+03	5.832085e+03	5.773409e+03	5.782318e+03	5.764875e+03	5.784364e+03	5.7316973e+03
	avg.	5.498997e+03	5.447464e+03	5.550440e+03	5.821717e+03	5.800117e+03	5.894233e+03	5.773651e+03	5.777178e+03	5.769970e+03	5.780587e+03	7.290833e+03
	min.	4.261343e+03	4.265816e+03	5.294867e+03	5.245086e+03	5.599457e+03	5.658690e+03	5.585638e+03	5.618749e+03	5.636169e+03	5.606842e+03	6.948792e+03
	max.	5.781562e+03	5.891695e+03	6.224267e+03	6.372162e+03	6.170199e+03	6.306293e+03	5.903679e+03	6.082667e+03	5.885841e+03	5.911023e+03	7.465138e+03
	std.	1.401387e+02	2.529456e+02	1.707693e+02	2.706055e+02	1.283883e+02	1.555903e+02	6.974446e+01	6.785052e+01	5.840198e+01	6.365151e+01	1.096125e+02
6D	med.	5.800665e+04	5.794250e+04	6.568200e+04	6.430597e+04	5.878897e+04	6.249328e+04	5.843468e+04	5.818962e+04	5.808198e+04	5.791440e+04	8.663068e+04
	avg.	5.805681e+04	5.786383e+04	6.560048e+04	6.464275e+04	6.010661e+04	6.247097e+04	5.834565e+04	5.813698e+04	5.797073e+04	5.788367e+04	8.608720e+04
	min.	5.598908e+04	5.559473e+04	5.799093e+04	5.561712e+04	5.526178e+04	5.487817e+04	5.593509e+04	5.187059e+04	5.556204e+04	5.265873e+04	7.848818e+04
	max.	6.274651e+04	5.931468e+04	7.188086e+04	7.411355e+04	7.125925e+04	7.413161e+04	5.942385e+04	5.962583e+04	5.995588e+04	6.395362e+04	9.119549e+04
	std.	1.054620e+03	7.001883e+02	2.677354e+03	6.666258e+03	3.323979e+03	4.125350e+03	7.351077e+02	8.896511e+02	7.734032e+02	1.134813e+03	2.558988e+03
7D	med.	5.859915e+05	5.919568e+05	6.030481e+05	6.065844e+05	6.976539e+05	7.095892e+05	5.915746e+05	5.931351e+05	5.950932e+05	5.942128e+05	1.183367e+06
	avg.	5.917620e+05	5.915332e+05	6.239241e+05	6.788085e+05	6.989644e+05	7.126282e+05	5.914111e+05	5.915746e+05	5.923101e+05	5.934648e+05	1.166356e+06
	min.	5.493478e+05	5.473813e+05	5.799316e+05	5.753872e+05	5.713853e+05	5.666266e+05	5.644459e+05	5.643453e+05	5.629780e+05	5.544889e+05	8.906768e+05
	max.	7.595316e+05	7.089215e+05	9.953616e+05	8.768097e+05	8.737499e+05	8.972318e+05	6.147459e+05	6.105775e+05	6.294169e+05	6.657504e+05	1.320610e+06
	std.	2.840456e+04	1.826296e+04	6.199341e+04	1.011872e+05	7.441732e+04	7.211921e+04	1.009658e+04	9.543620e+03	1.178640e+04	1.237380e+04	8.674241e+04
8D	med.	8.647523e+06	8.651300e+06	1.123923e+07	8.716715e+06	1.112850e+07	1.060354e+07	8.639528e+06	8.681996e+06	8.628415e+06	8.699272e+06	1.812283e+07
	avg.	8.750478e+06	8.723093e+06	1.081836e+07	8.925996e+06	1.132586e+07	1.069699e+07	8.643805e+06	8.679194e+06	8.674824e+06	8.679054e+06	1.758172e+07
	min.	8.326286e+06	8.208370e+06	8.601765e+06	8.386965e+06	8.422552e+06	8.355769e+06	8.324896e+06	8.229955e+06	8.115498e+06	8.289118e+06	3.888926e+06
	max.	1.094872e+07	1.067987e+07	1.340102e+07	1.359506e+07	1.554525e+07	1.688408e+07	9.308775e+06	8.947761e+06	9.891038e+06	9.535604e+06	2.062126e+07
	std.	4.736656e+05	3.372402e+05	1.178544e+06	8.267262e+05	1.569218e+06	1.529550e+06	1.544846e+05	1.296331e+05	2.772909e+05	1.530144e+05	2.538966e+06
9D	med.	1.434823e+08	1.439431e+08	1.632579e+08	1.440652e+08	1.937291e+08	1.853019e+08	1.419625e+08	1.433391e+08	1.420570e+08	1.436746e+08	3.177429e+08
	avg.	1.466707e+08	1.488805e+08	1.712171e+08	1.506532e+08	1.984018e+08	1.842723e+08	1.424095e+08	1.438414e+08	1.425905e+08	1.437845e+08	3.107687e+08
	min.	1.365797e+08	1.382291e+08	1.418580e+08	1.383337e+08	1.408550e+08	1.384740e+08	1.370740e+08	1.394455e+08	1.375083e+08	1.380735e+08	1.405438e+08
	max.	1.861734e+08	1.988373e+08	2.236815e+08	2.197678e+08	2.704520e+08	2.635946e+08	1.623395e+08	1.728499e+08	1.563227e+08	1.538713e+08	3.683227e+08
	std.	9.487726e+06	1.278828e+07	1.929564e+07	1.719798e+07	3.052486e+07	3.453164e+07	3.064155e+06	3.814712e+06	2.664782e+06	2.150478e+06	3.620601e+07
10D	med.	2.763812e+09	2.703838e+09	3.934171e+09	2.686542e+09	4.187950e+09	3.590280e+09	2.582939e+09	2.648117e+09	2.598304e+09	2.652363e+09	6.095114e+09
	avg.	2.989066e+09	3.011765e+09	3.859656e+09	2.965107e+09	4.150554e+09	3.563114e+09	2.590261e+09	2.691883e+09	2.605899e+09	2.704342e+09	5.859459e+09
	min.	2.556044e+09	2.541454e+09	2.937808e+09	2.54974e+09	2.652625e+09	2.564343e+09	2.421570e+09	2.558444e+09	2.443862e+09	2.536120e+09	2.368188e+09
	max.	4.459100e+09	4.640225e+09	5.123381e+09	4.238682e+09	5.660756e+09	5.085340e+09	2.862449e+09	3.600448e+09	2.983528e+09	3.607615e+09	7.230739e+09
	std.	4.554016e+08	5.221914e+08	4.241289e+08	4.698974e+08	6.239485e+08	6.530327e+08	5.407409e+07	2.018302e+08	6.896670e+07	2.063691e+08	9.459379e+08

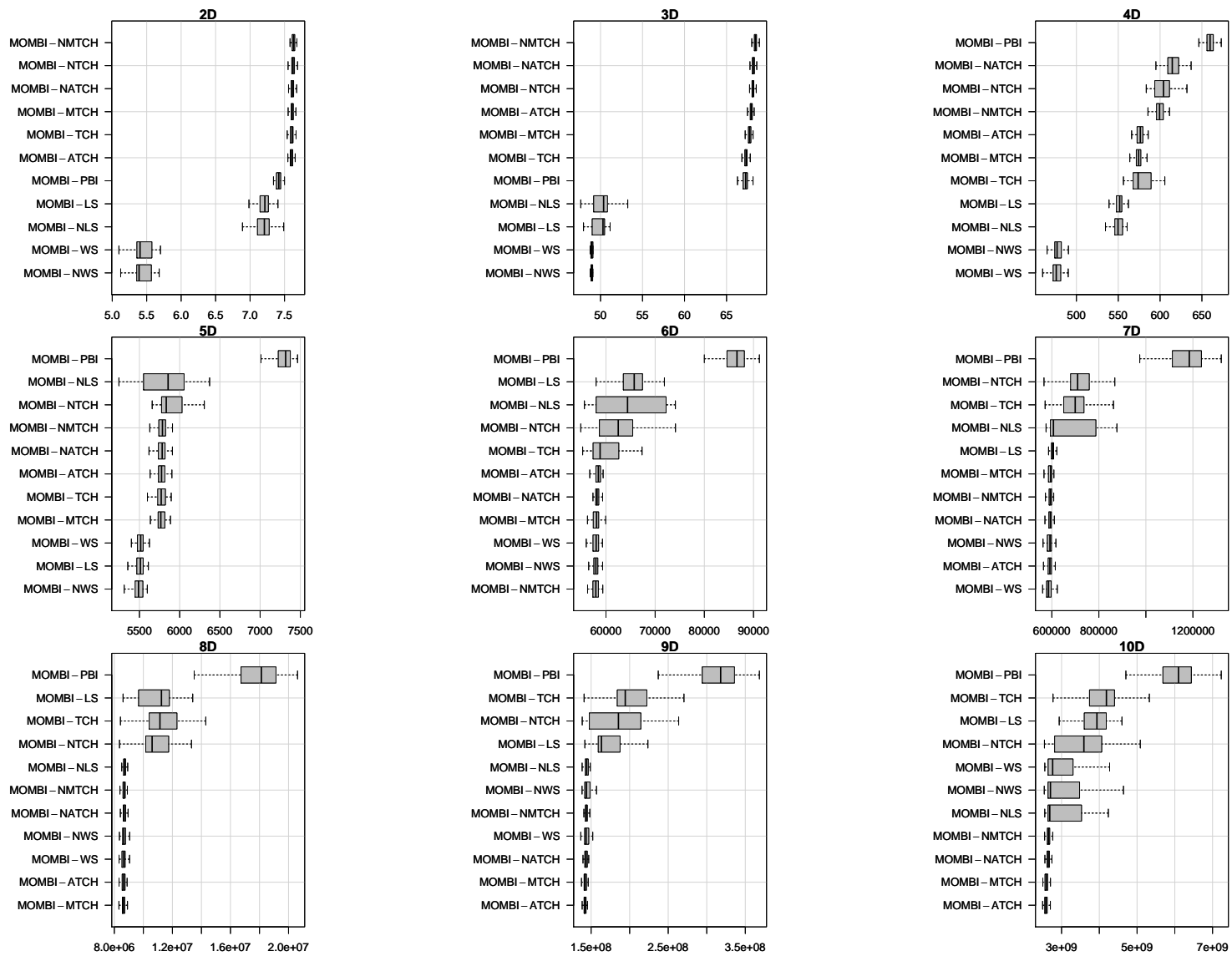


Figure A.397: Box-plot of the hypervolume indicator values for different optimizers on the WFG8 test problem.

Table A.169: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG8 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
2D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$1.28e-34$	$1.28e-34$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.28e-34$
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$6.95e-06$	–	$3.40e-08$	$1.66e-02$	$7.18e-04$	> 0.05	$1.28e-34$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.36e-34$	$1.44e-34$	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$3.76e-34$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.09e-02$	> 0.05	$2.58e-04$	–	> 0.05	> 0.05	$1.28e-34$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	$4.30e-03$	> 0.05	–	> 0.05	$1.28e-34$
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$8.63e-10$	$4.13e-02$	$4.09e-12$	$4.60e-05$	$2.90e-07$	–	$1.28e-34$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$6.80e-31$	$1.15e-29$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
3D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$9.69e-11$	$6.09e-11$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$2.69e-15$	$2.22e-15$	$8.18e-03$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.89e-34$	–	$1.08e-13$	> 0.05	$1.30e-25$	> 0.05	$9.19e-34$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$5.42e-31$	> 0.05	–	> 0.05	$1.05e-09$	> 0.05	$1.52e-29$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.95e-34$	$1.17e-02$	$2.49e-17$	–	$4.91e-27$	> 0.05	$6.63e-34$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$9.20e-25$	> 0.05	> 0.05	> 0.05	–	> 0.05	$7.65e-22$
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.32e-34$	$5.63e-19$	$5.74e-30$	$1.42e-13$	$9.66e-33$	–	$1.73e-34$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
4D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	$4.22e-02$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.00e-33$	$2.55e-34$	–	$3.58e-03$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$1.16e-33$	$2.88e-34$	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$5.71e-34$	$2.40e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$5.84e-25$	–	$1.40e-34$	> 0.05	$1.32e-34$	$1.81e-02$	> 0.05
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	–	> 0.05	$1.04e-03$	> 0.05	> 0.05
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.44e-33$	$2.67e-13$	$1.28e-34$	–	$1.28e-34$	$3.23e-26$	> 0.05
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.78e-34$	$1.32e-34$	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$3.14e-24$	> 0.05	$1.32e-34$	> 0.05	$1.28e-34$	–	> 0.05
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–

Table A.170: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG8 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
5D											
MOMBI-WS	–	$1.98e-02$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$> 0.05$	$1.97e-02$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NLS	$2.67e-16$	$2.15e-18$	$1.64e-12$	–	$> 0.05$	$> 0.05$	$9.22e-03$	$8.18e-03$	$5.06e-03$	$1.09e-02$	$> 0.05$
MOMBI-TCH	$1.92e-33$	$2.83e-32$	$9.68e-22$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NTCH	$4.24e-34$	$4.65e-33$	$2.70e-24$	$> 0.05$	$5.55e-07$	–	$1.31e-08$	$4.29e-08$	$1.26e-09$	$2.36e-07$	$> 0.05$
MOMBI-ATCH	$8.85e-33$	$1.79e-31$	$1.47e-19$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NATCH	$1.16e-33$	$5.51e-32$	$4.71e-20$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$1.61e-33$	$6.00e-32$	$6.74e-20$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$
MOMBI-NMTCH	$1.16e-33$	$4.37e-32$	$6.03e-20$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–
6D											
MOMBI-WS	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$1.92e-33$	$5.38e-34$	–	$> 0.05$	$5.97e-22$	$3.23e-09$	$1.13e-33$	$1.10e-33$	$6.06e-34$	$1.27e-33$	$> 0.05$
MOMBI-NLS	$7.42e-10$	$1.15e-10$	$> 0.05$	–	$1.26e-06$	$1.65e-02$	$4.80e-06$	$3.17e-07$	$1.69e-09$	$1.39e-10$	$> 0.05$
MOMBI-TCH	$1.25e-03$	$3.05e-04$	$> 0.05$	$> 0.05$	–	$> 0.05$	$4.35e-02$	$2.00e-02$	$8.28e-04$	$2.95e-04$	$> 0.05$
MOMBI-NTCH	$1.00e-13$	$1.16e-14$	$> 0.05$	$> 0.05$	$2.10e-05$	–	$6.25e-12$	$8.92e-13$	$4.14e-14$	$2.41e-14$	$> 0.05$
MOMBI-ATCH	$6.94e-05$	$3.97e-07$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$1.17e-02$	$1.29e-04$	$8.29e-06$	$> 0.05$
MOMBI-NATCH	$9.91e-03$	$1.68e-04$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$3.07e-02$	$2.02e-03$	$> 0.05$
MOMBI-MTCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$
MOMBI-NMTCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–
7D											
MOMBI-WS	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$2.84e-16$	$6.30e-16$	–	$> 0.05$	$> 0.05$	$> 0.05$	$1.72e-15$	$9.91e-17$	$1.04e-13$	$8.06e-15$	$> 0.05$
MOMBI-NLS	$8.35e-14$	$6.92e-12$	$> 0.05$	–	$> 0.05$	$> 0.05$	$1.07e-10$	$1.56e-10$	$8.37e-10$	$4.08e-09$	$> 0.05$
MOMBI-TCH	$1.48e-25$	$1.14e-25$	$3.19e-13$	$1.17e-02$	–	$> 0.05$	$6.67e-26$	$6.67e-26$	$3.43e-25$	$5.15e-25$	$> 0.05$
MOMBI-NTCH	$1.05e-23$	$1.03e-23$	$2.69e-15$	$5.54e-03$	$> 0.05$	–	$1.00e-23$	$1.49e-23$	$1.22e-23$	$5.28e-23$	$> 0.05$
MOMBI-ATCH	$9.10e-03$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NATCH	$4.06e-03$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$3.12e-03$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$
MOMBI-NMTCH	$2.48e-04$	$2.36e-02$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.58e-34$	$1.28e-34$	$1.28e-34$	$1.32e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–

Table A.171: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG8 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
8D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$3.47e-29$	$7.80e-30$	–	$4.89e-25$	> 0.05	> 0.05	$1.46e-31$	$1.13e-30$	$1.10e-30$	$1.07e-30$	> 0.05
MOMBI-NLS	$1.28e-04$	$1.09e-03$	> 0.05	–	> 0.05	> 0.05	$1.37e-05$	$4.00e-02$	$1.75e-06$	$1.28e-02$	> 0.05
MOMBI-TCH	$4.66e-27$	$8.28e-28$	$3.09e-02$	$5.98e-24$	–	$1.61e-03$	$3.37e-29$	$1.40e-28$	$9.54e-29$	$1.47e-28$	> 0.05
MOMBI-NTCH	$6.12e-18$	$3.10e-18$	> 0.05	$3.62e-13$	> 0.05	–	$1.32e-19$	$1.28e-17$	$9.85e-20$	$8.40e-18$	> 0.05
MOMBI-ATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NATCH	$4.70e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$6.87e-03$	–	$3.94e-03$	> 0.05	> 0.05
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.15e-02$	> 0.05	$6.28e-03$	–	> 0.05
MOMBI-PBI	$4.63e-32$	$4.63e-32$	$1.10e-31$	$5.83e-32$	$9.02e-31$	$2.99e-31$	$4.63e-32$	$4.63e-32$	$4.63e-32$	$4.63e-32$	–
9D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$6.57e-06$	> 0.05	$1.22e-04$	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	$9.29e-07$	$4.15e-02$	$1.45e-05$	> 0.05	> 0.05
MOMBI-LS	$8.32e-25$	$2.87e-20$	–	$1.80e-19$	> 0.05	> 0.05	$9.95e-33$	$2.45e-31$	$7.65e-33$	$5.35e-32$	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	$7.64e-10$	$7.63e-03$	$5.68e-08$	$3.97e-02$	> 0.05
MOMBI-TCH	$1.53e-27$	$5.57e-26$	$1.56e-10$	$6.93e-24$	–	$9.30e-04$	$1.30e-31$	$2.63e-29$	$3.75e-31$	$3.87e-29$	> 0.05
MOMBI-NTCH	$3.07e-17$	$1.82e-15$	$3.44e-02$	$1.06e-13$	> 0.05	–	$4.90e-24$	$6.89e-20$	$3.25e-23$	$3.48e-19$	> 0.05
MOMBI-ATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.93e-06$	–	$1.28e-04$	> 0.05	> 0.05
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.10e-08$	> 0.05	$3.00e-06$	–	> 0.05
MOMBI-PBI	$1.81e-33$	$1.61e-33$	$3.27e-33$	$2.23e-33$	$5.83e-32$	$2.12e-32$	$1.43e-33$	$2.23e-33$	$1.48e-33$	$2.44e-33$	–
10D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$4.04e-22$	$6.24e-08$	$3.40e-19$	$4.33e-07$	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	$1.35e-22$	$1.42e-06$	$5.63e-19$	$1.93e-05$	> 0.05
MOMBI-LS	$2.20e-23$	$2.58e-21$	–	$1.85e-23$	> 0.05	$2.79e-04$	$1.28e-34$	$3.68e-33$	$1.36e-34$	$5.08e-33$	> 0.05
MOMBI-NLS	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	$4.04e-23$	$6.01e-06$	$2.98e-19$	$9.04e-05$	> 0.05
MOMBI-TCH	$4.19e-26$	$3.26e-25$	$2.14e-04$	$1.57e-27$	–	$6.37e-10$	$1.78e-34$	$2.51e-33$	$2.48e-34$	$4.39e-33$	> 0.05
MOMBI-NTCH	$7.64e-10$	$1.03e-08$	> 0.05	$2.02e-09$	> 0.05	–	$8.97e-28$	$4.65e-18$	$7.58e-26$	$1.98e-17$	> 0.05
MOMBI-ATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$9.13e-17$	–	$6.36e-12$	> 0.05	> 0.05
MOMBI-MTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$4.72e-02$	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$5.14e-17$	> 0.05	$4.10e-13$	–	> 0.05
MOMBI-PBI	$6.98e-30$	$2.78e-30$	$4.91e-27$	$7.40e-31$	$6.67e-26$	$2.98e-28$	$3.08e-33$	$1.50e-32$	$4.79e-33$	$2.75e-32$	–

Table A.172: Comparison of R2 indicator values for different optimizers on the WFG8 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	7.429313e-01	7.433299e-01	6.593917e-01	6.595366e-01	6.398409e-01	6.397775e-01	6.408916e-01	6.412932e-01	6.395313e-01	6.397659e-01	6.493808e-01
	avg.	7.396846e-01	7.409250e-01	6.597217e-01	6.607450e-01	6.401154e-01	6.407164e-01	6.412184e-01	6.415508e-01	6.398640e-01	6.401559e-01	6.495803e-01
	min.	7.263097e-01	7.268851e-01	6.455970e-01	6.446543e-01	6.370252e-01	6.367837e-01	6.368865e-01	6.373088e-01	6.369148e-01	6.369205e-01	6.449271e-01
	max.	7.605636e-01	7.739856e-01	6.772279e-01	6.807102e-01	6.460712e-01	6.528726e-01	6.640203e-01	6.532785e-01	6.458602e-01	6.546850e-01	6.590612e-01
	std.	7.925043e-03	9.114725e-03	5.578826e-03	6.792433e-03	1.828504e-03	2.784134e-03	3.162089e-03	2.770162e-03	1.832296e-03	2.343959e-03	2.412341e-03
3D	med.	4.002906e-01	4.003761e-01	3.921426e-01	3.921523e-01	3.443883e-01	3.457630e-01	3.442236e-01	3.460856e-01	3.443291e-01	3.451527e-01	3.494428e-01
	avg.	3.996476e-01	3.998565e-01	3.949466e-01	3.944280e-01	3.444437e-01	3.457475e-01	3.442790e-01	3.462064e-01	3.443596e-01	3.452848e-01	3.495586e-01
	min.	3.916142e-01	3.917118e-01	3.796198e-01	3.827291e-01	3.433668e-01	3.443573e-01	3.430558e-01	3.447946e-01	3.430601e-01	3.438008e-01	3.469132e-01
	max.	4.185946e-01	4.185118e-01	4.085509e-01	4.093051e-01	3.458058e-01	3.473570e-01	3.468784e-01	3.491714e-01	3.460604e-01	3.494448e-01	3.552899e-01
	std.	5.319199e-03	5.436308e-03	5.300496e-03	5.607010e-03	4.793220e-04	5.192117e-04	5.448050e-04	9.056877e-04	5.551510e-04	7.690171e-04	1.423550e-03
4D	med.	2.658728e-01	2.656810e-01	2.609539e-01	2.613649e-01	2.482860e-01	2.504609e-01	2.475579e-01	2.496013e-01	2.475980e-01	2.506303e-01	2.422000e-01
	avg.	2.664270e-01	2.656250e-01	2.614270e-01	2.613897e-01	2.484686e-01	2.509346e-01	2.481400e-01	2.503849e-01	2.479556e-01	2.511445e-01	2.422746e-01
	min.	2.576249e-01	2.607760e-01	2.566971e-01	2.569881e-01	2.456395e-01	2.460410e-01	2.461194e-01	2.465082e-01	2.461198e-01	2.476478e-01	2.382727e-01
	max.	2.782407e-01	2.743204e-01	2.73382e-01	2.702988e-01	2.523696e-01	2.555227e-01	2.523096e-01	2.557793e-01	2.521504e-01	2.607433e-01	2.474552e-01
	std.	3.599493e-03	2.858742e-03	3.114653e-03	3.038013e-03	1.419307e-03	1.897813e-03	1.583437e-03	2.266359e-03	1.212908e-03	2.299332e-03	1.799383e-03
5D	med.	2.067276e-01	2.076269e-01	2.065186e-01	2.055125e-01	2.029873e-01	2.038059e-01	2.011783e-01	2.045170e-01	2.023393e-01	2.039263e-01	1.966200e-01
	avg.	2.081861e-01	2.125396e-01	2.070041e-01	2.058526e-01	2.031829e-01	2.039082e-01	2.010384e-01	2.048203e-01	2.023785e-01	2.041104e-01	1.991179e-01
	min.	2.027433e-01	2.043312e-01	2.011609e-01	2.002977e-01	1.990342e-01	2.000977e-01	1.976615e-01	2.006230e-01	1.987342e-01	2.003489e-01	1.925871e-01
	max.	3.269170e-01	3.265933e-01	2.157514e-01	2.172733e-01	2.085356e-01	2.099640e-01	2.060061e-01	2.102881e-01	2.080342e-01	2.119518e-01	2.150669e-01
	std.	1.215162e-02	2.344689e-02	2.500254e-03	2.677271e-03	1.637280e-03	1.942791e-03	1.905828e-03	2.043291e-03	1.968954e-03	2.134912e-03	5.648823e-03
6D	med.	2.420976e-01	2.421595e-01	2.391576e-01	2.399208e-01	2.423048e-01	2.421542e-01	2.403286e-01	2.413080e-01	2.415759e-01	2.420594e-01	1.749027e-01
	avg.	2.422721e-01	2.423761e-01	2.393445e-01	2.395689e-01	2.429301e-01	2.439100e-01	2.407950e-01	2.424904e-01	2.419242e-01	2.432221e-01	1.742950e-01
	min.	2.392906e-01	2.396702e-01	2.348499e-01	2.328072e-01	2.372821e-01	2.351402e-01	2.384981e-01	2.387109e-01	2.394600e-01	2.393963e-01	1.632321e-01
	max.	2.475587e-01	2.494129e-01	2.445269e-01	2.499102e-01	2.521683e-01	3.377606e-01	2.465265e-01	3.485933e-01	2.486429e-01	3.369265e-01	1.928382e-01
	std.	1.651453e-03	1.640822e-03	1.781365e-03	3.770732e-03	2.763346e-03	1.369185e-02	1.414805e-03	1.077265e-02	1.693007e-03	9.634853e-03	6.074812e-03
7D	med.	3.746426e-01	3.736219e-01	3.716914e-01	3.715129e-01	3.698785e-01	3.669635e-01	3.721520e-01	3.721172e-01	3.720144e-01	3.720789e-01	1.589027e-01
	avg.	3.780843e-01	3.772800e-01	3.727296e-01	3.679301e-01	3.731502e-01	3.693094e-01	3.726842e-01	3.722390e-01	3.740603e-01	3.726196e-01	1.614215e-01
	min.	3.605095e-01	3.693315e-01	3.472604e-01	3.541465e-01	3.567537e-01	3.522448e-01	3.714988e-01	3.713710e-01	3.711100e-01	3.667731e-01	1.441052e-01
	max.	4.054380e-01	4.074121e-01	4.052065e-01	4.031777e-01	4.069790e-01	4.024507e-01	4.086309e-01	3.738519e-01	4.049108e-01	3.979870e-01	2.029330e-01
	std.	8.798349e-03	8.245157e-03	7.768151e-03	8.521449e-03	1.124688e-02	9.119343e-03	3.839914e-03	5.330734e-04	6.251275e-03	3.400345e-03	1.210143e-02
8D	med.	4.000403e-01	3.963250e-01	3.456642e-01	3.927845e-01	3.908338e-01	3.879585e-01	3.929170e-01	3.928251e-01	3.929772e-01	3.928432e-01	1.498899e-01
	avg.	4.023268e-01	3.993793e-01	3.562104e-01	3.937179e-01	3.928184e-01	3.910542e-01	3.919643e-01	3.931621e-01	3.955467e-01	3.937166e-01	1.579727e-01
	min.	3.863738e-01	3.853697e-01	3.147852e-01	3.743396e-01	3.712150e-01	3.606845e-01	3.507860e-01	3.923650e-01	3.642916e-01	3.923700e-01	1.352093e-01
	max.	4.272457e-01	4.283144e-01	4.240224e-01	4.210545e-01	4.191823e-01	4.248196e-01	3.939415e-01	4.103050e-01	4.296619e-01	4.296229e-01	3.536715e-01
	std.	9.039749e-03	8.065328e-03	2.943207e-02	6.172841e-03	1.089941e-02	1.051151e-02	5.826371e-03	1.990592e-03	9.195047e-03	4.461083e-03	2.769930e-02
9D	med.	4.149755e-01	4.115820e-01	3.522677e-01	4.072741e-01	4.070222e-01	4.085849e-01	4.075072e-01	4.073793e-01	4.074991e-01	4.073196e-01	1.426328e-01
	avg.	4.193043e-01	4.173032e-01	3.551160e-01	4.073620e-01	4.102326e-01	4.106413e-01	4.057066e-01	4.082534e-01	4.077925e-01	4.087635e-01	1.478927e-01
	min.	3.991813e-01	3.994161e-01	2.970562e-01	3.930464e-01	3.886643e-01	3.809876e-01	3.522886e-01	4.069169e-01	3.860456e-01	4.068845e-01	1.318081e-01
	max.	4.513436e-01	4.519313e-01	4.160227e-01	4.358281e-01	4.444233e-01	4.516393e-01	4.081739e-01	4.445023e-01	4.512536e-01	4.364250e-01	2.575059e-01
	std.	1.264334e-02	1.296362e-02	2.309826e-02	5.829153e-03	1.445168e-02	1.428040e-02	7.403395e-03	5.139481e-03	5.012531e-03	5.516865e-03	1.627974e-02
10D	med.	4.210948e-01	4.183221e-01	3.237461e-01	4.162276e-01	4.113576e-01	4.151131e-01	4.166679e-01	4.163161e-01	4.166004e-01	4.164216e-01	1.415611e-01
	avg.	4.272499e-01	4.221860e-01	3.279806e-01	4.164520e-01	4.152835e-01	4.187741e-01	4.153350e-01	4.166505e-01	4.178059e-01	4.178059e-01	1.483641e-01
	min.	4.001781e-01	4.010248e-01	2.776787e-01	4.055571e-01	3.861102e-01	3.994166e-01	3.669577e-01	4.107881e-01	3.663475e-01	4.104955e-01	1.277287e-01
	max.	4.664515e-01	4.593642e-01	3.982697e-01	4.588117e-01	4.571221e-01	4.582675e-01	4.177747e-01	4.454926e-01	4.554178e-01	4.500109e-01	2.494620e-01
	std.	1.607144e-02	1.159249e-02	2.709769e-02	7.156888e-03	1.366058e-02	1.285155e-02	6.184748e-03	3.431601e-03	7.481602e-03	5.882115e-03	2.409169e-02



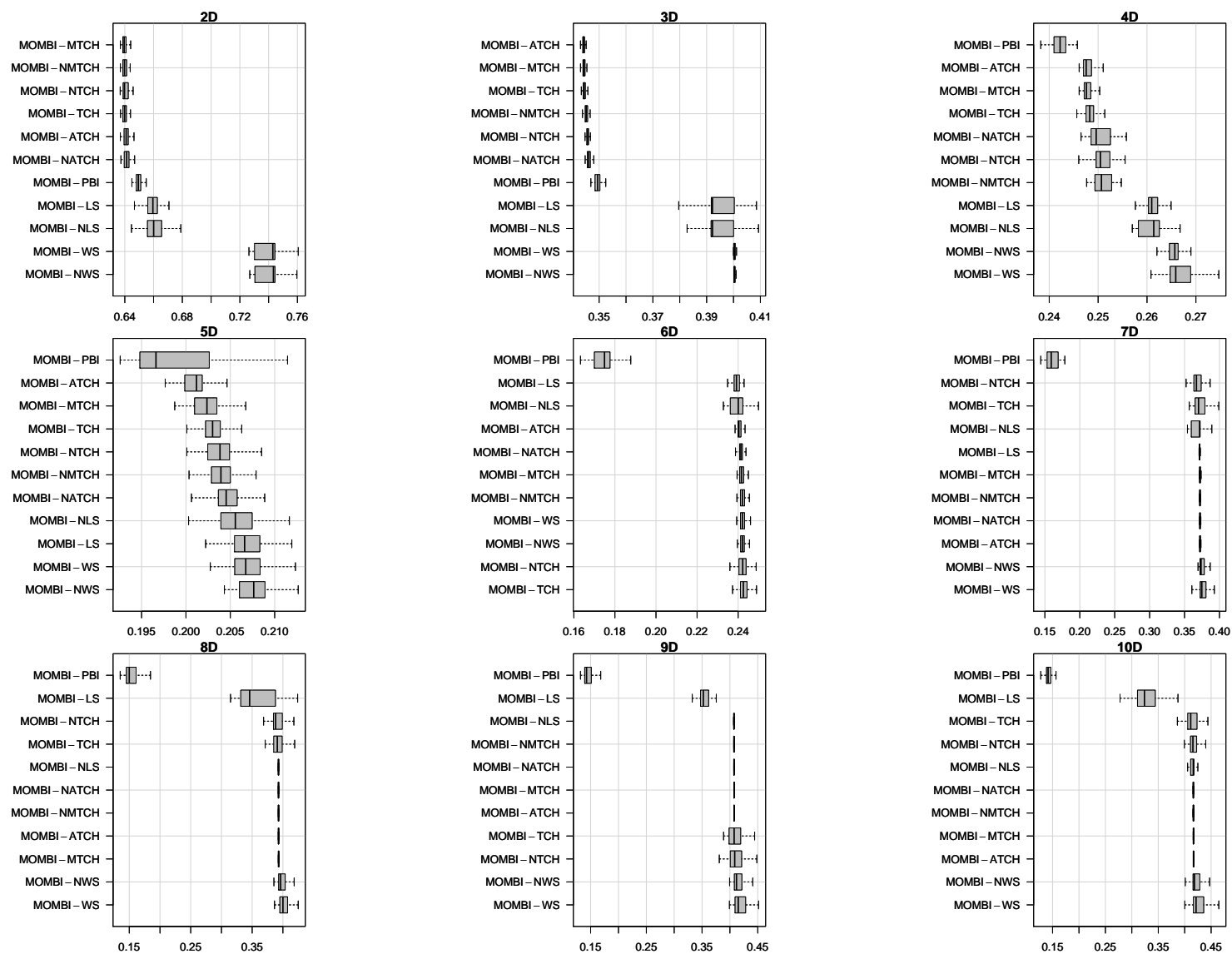


Figure A.398: Box-plot of the R2 indicator values for different optimizers on the WFG8 test problem.

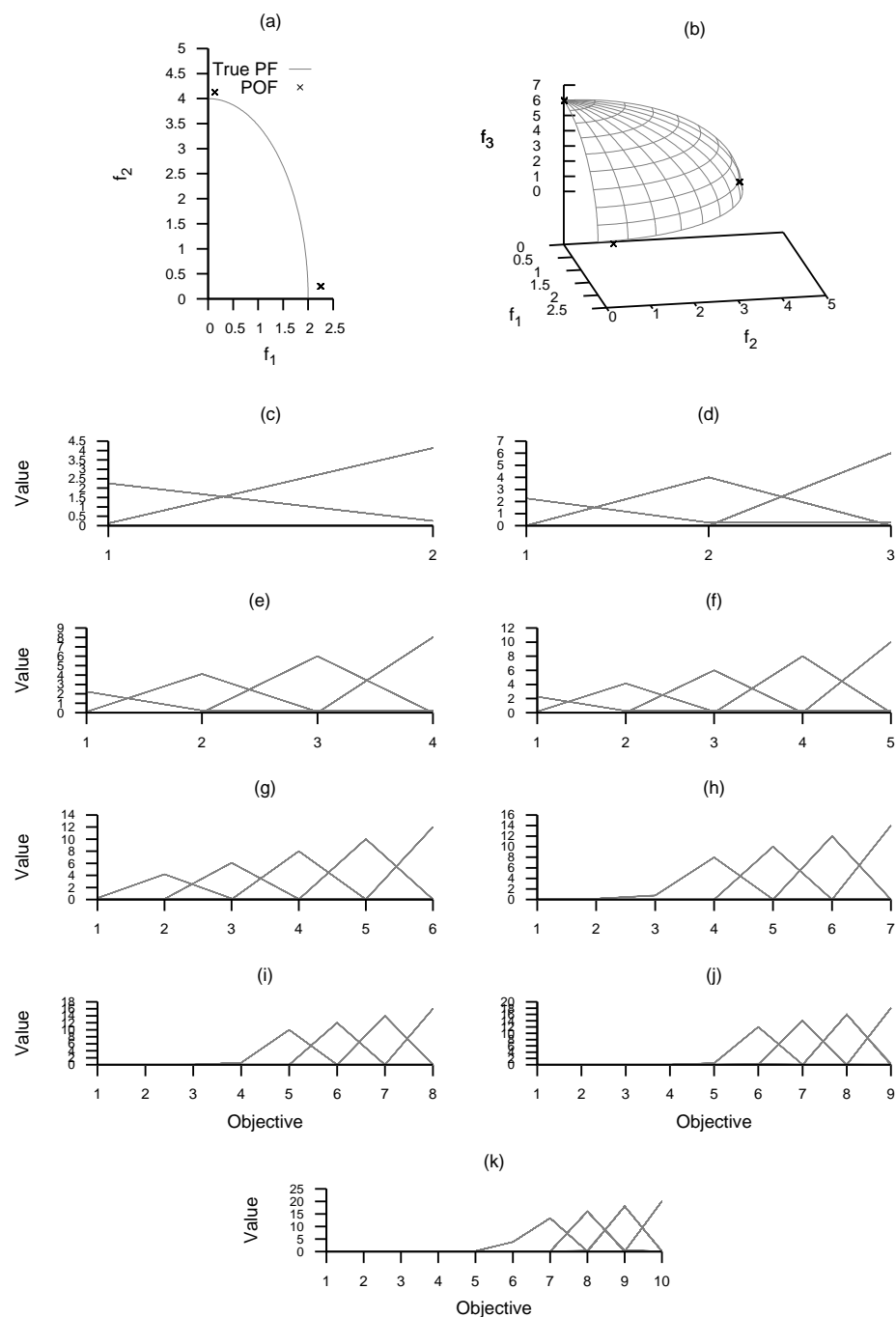


Figure A.399: Plots of the approximations obtained by MOMBI-WS from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

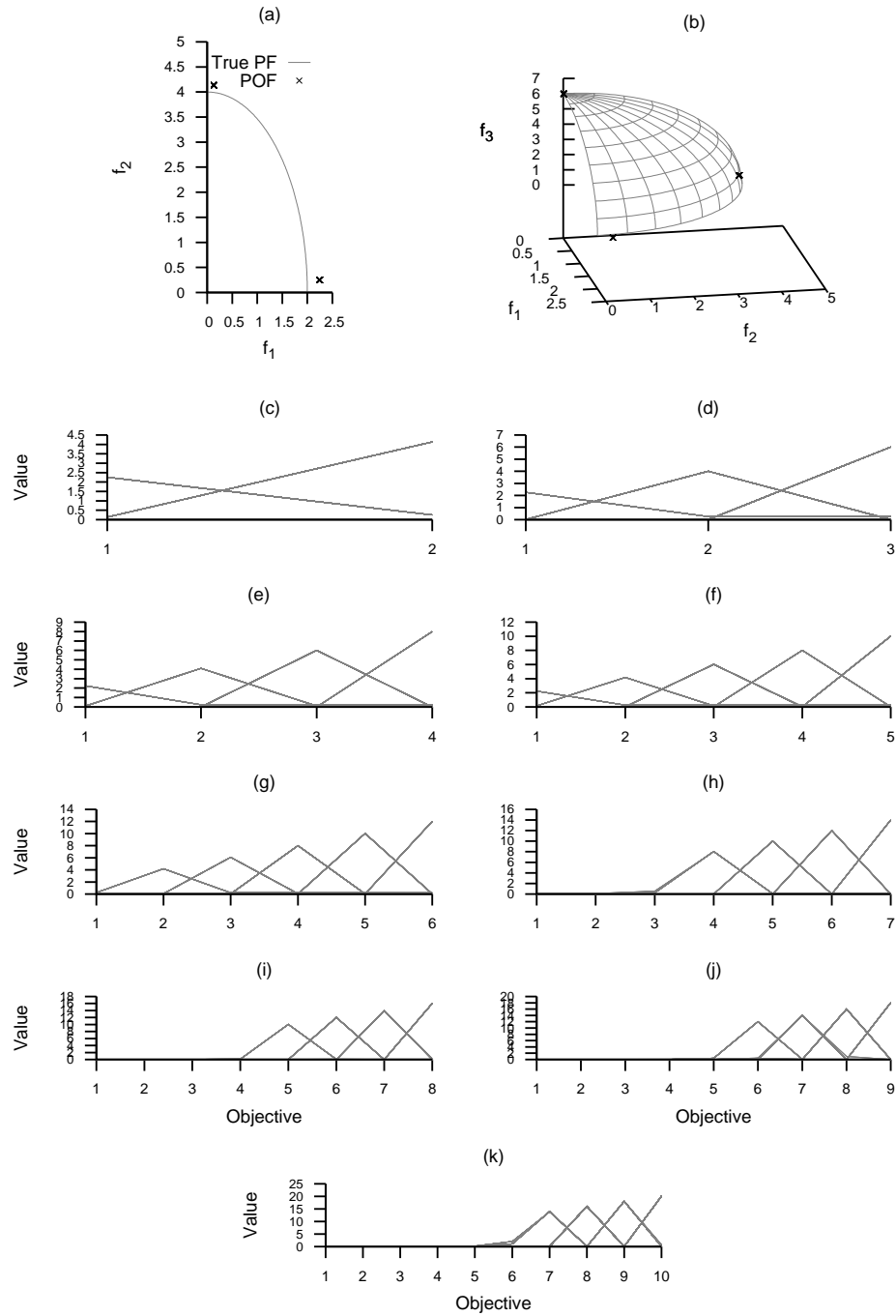


Figure A.400: Plots of the approximations obtained by MOMBI-NWS from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

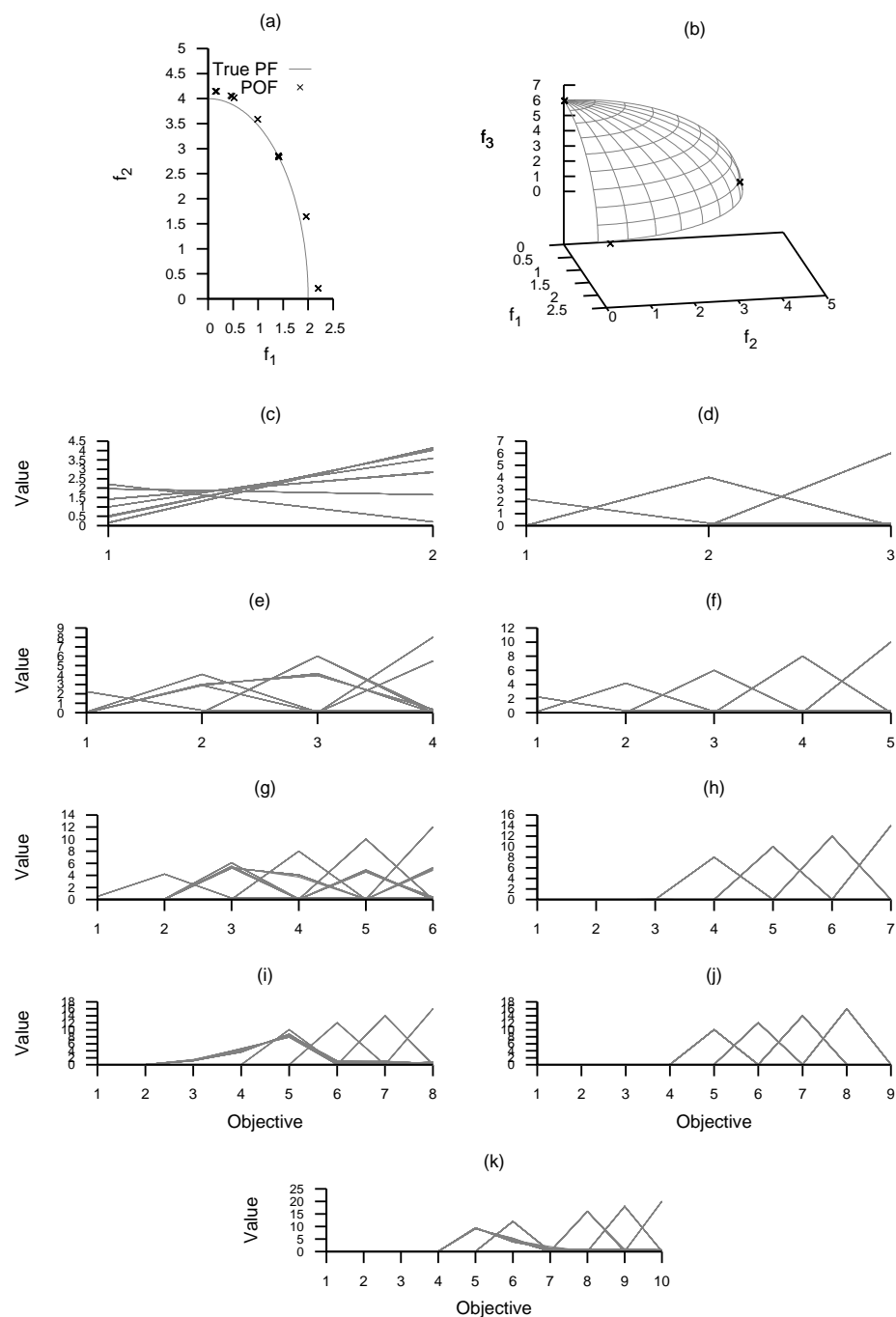


Figure A.401: Plots of the approximations obtained by MOMBI-LS from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

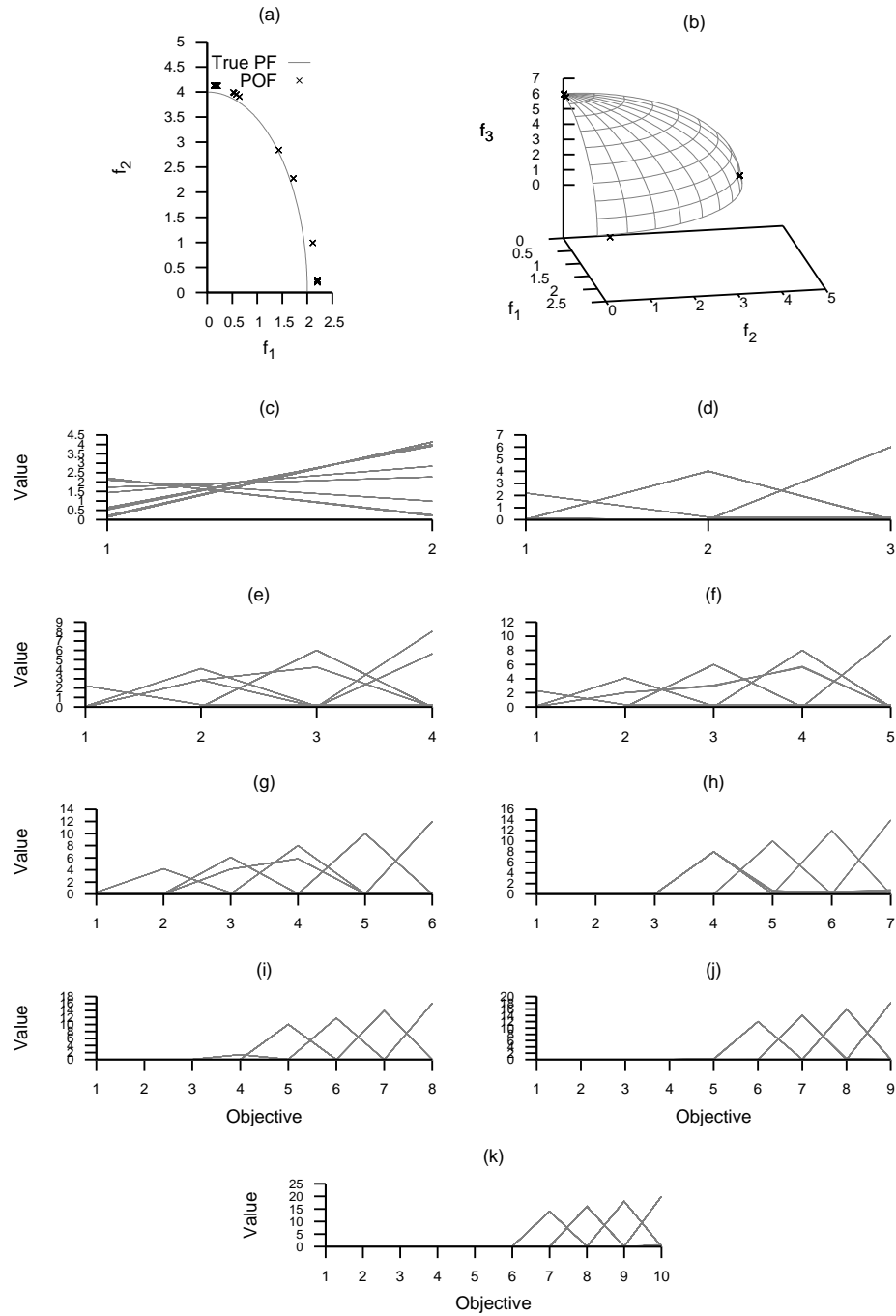


Figure A.402: Plots of the approximations obtained by MOMBI-NLS from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

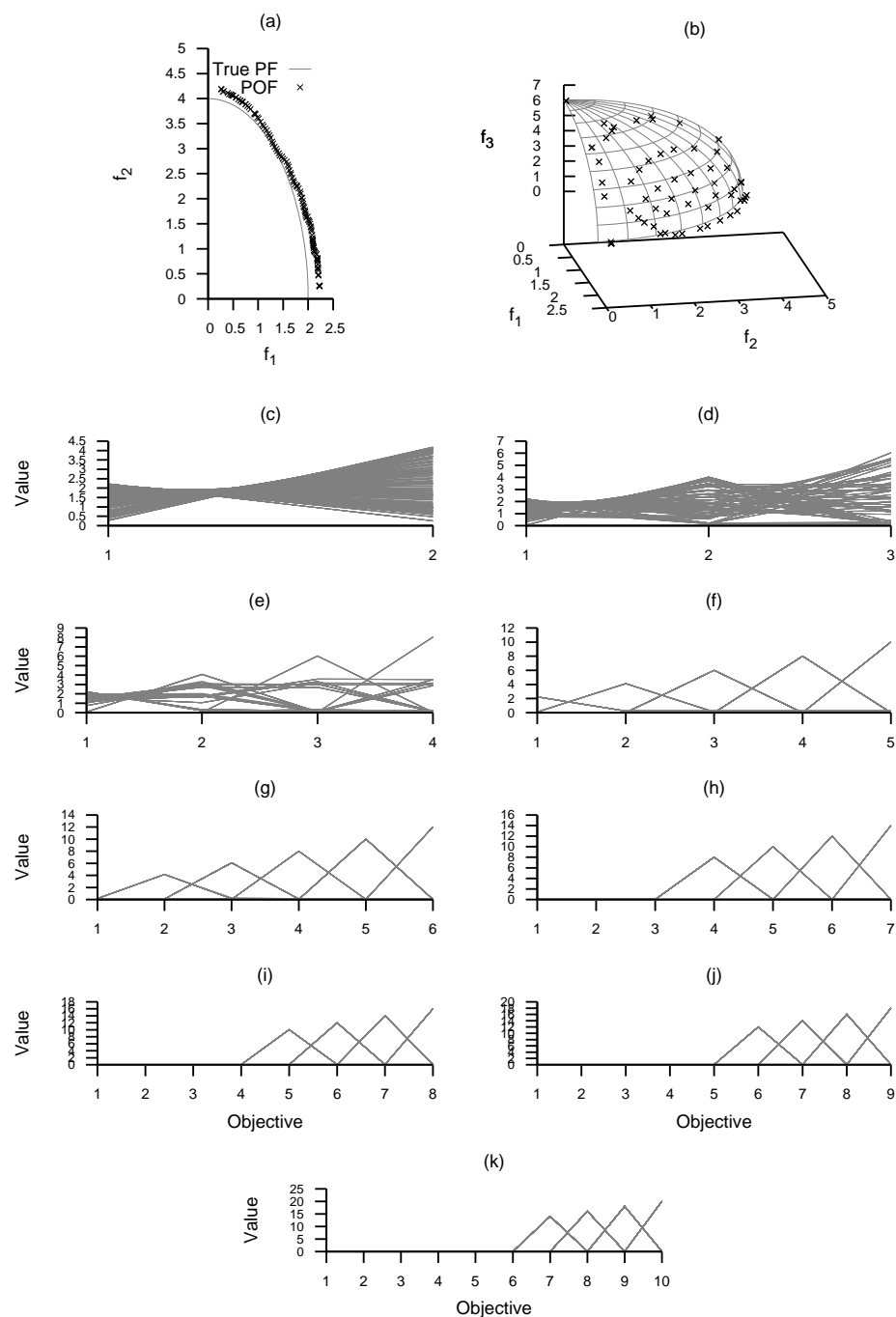


Figure A.403: Plots of the approximations obtained by MOMBI-ATCH from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

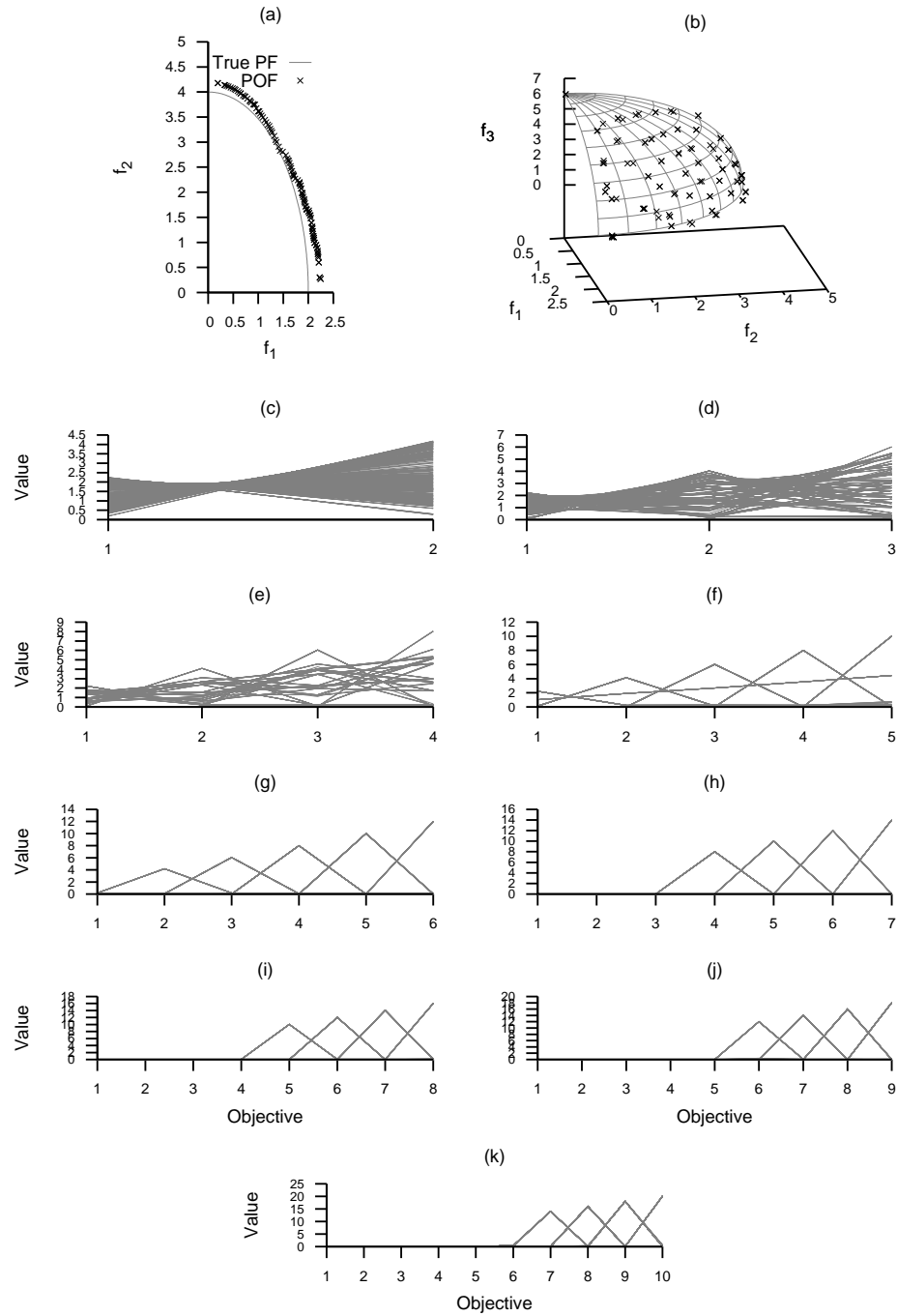


Figure A.404: Plots of the approximations obtained by MOMBI-NATCH from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

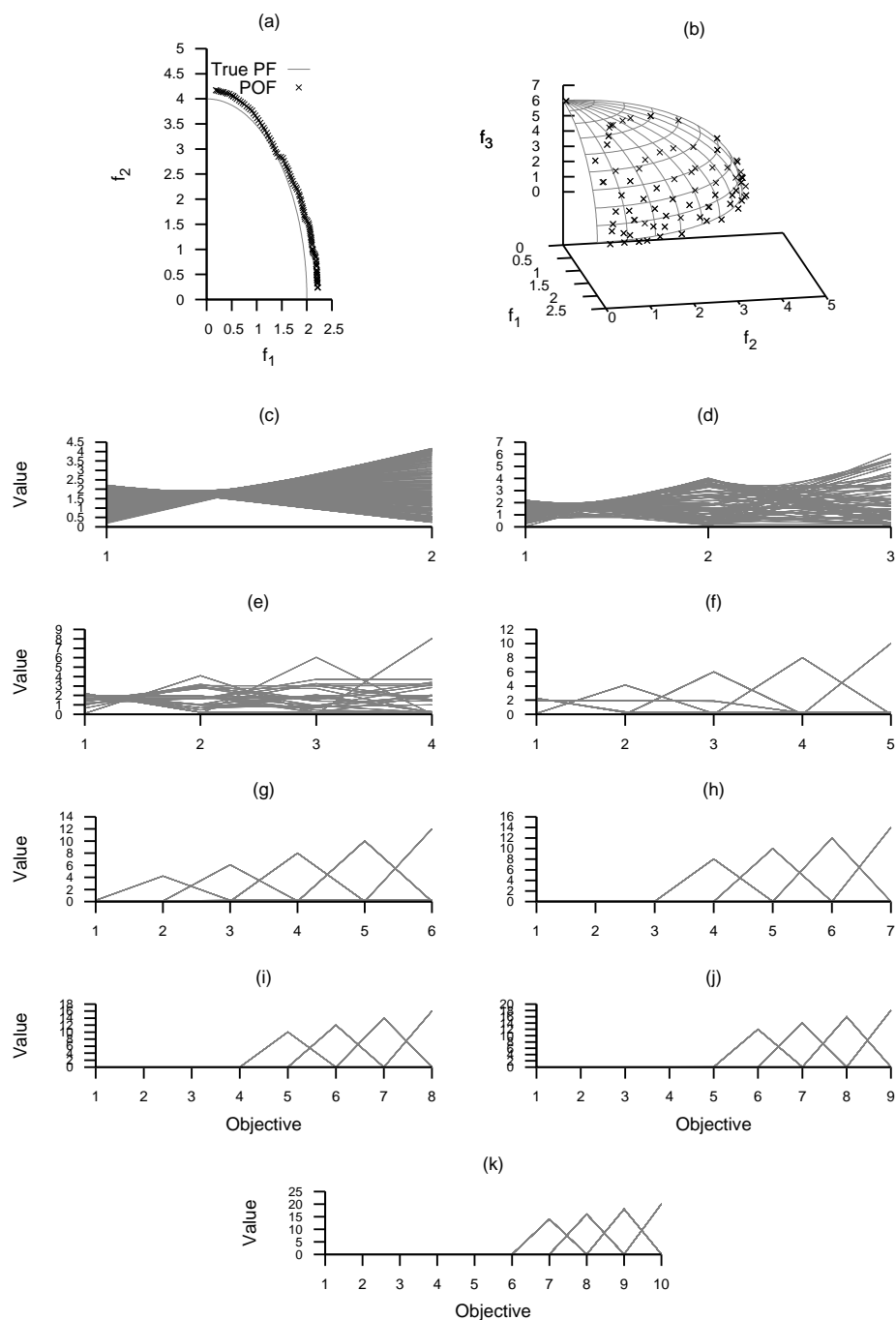


Figure A.405: Plots of the approximations obtained by MOMBI-MTCH from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



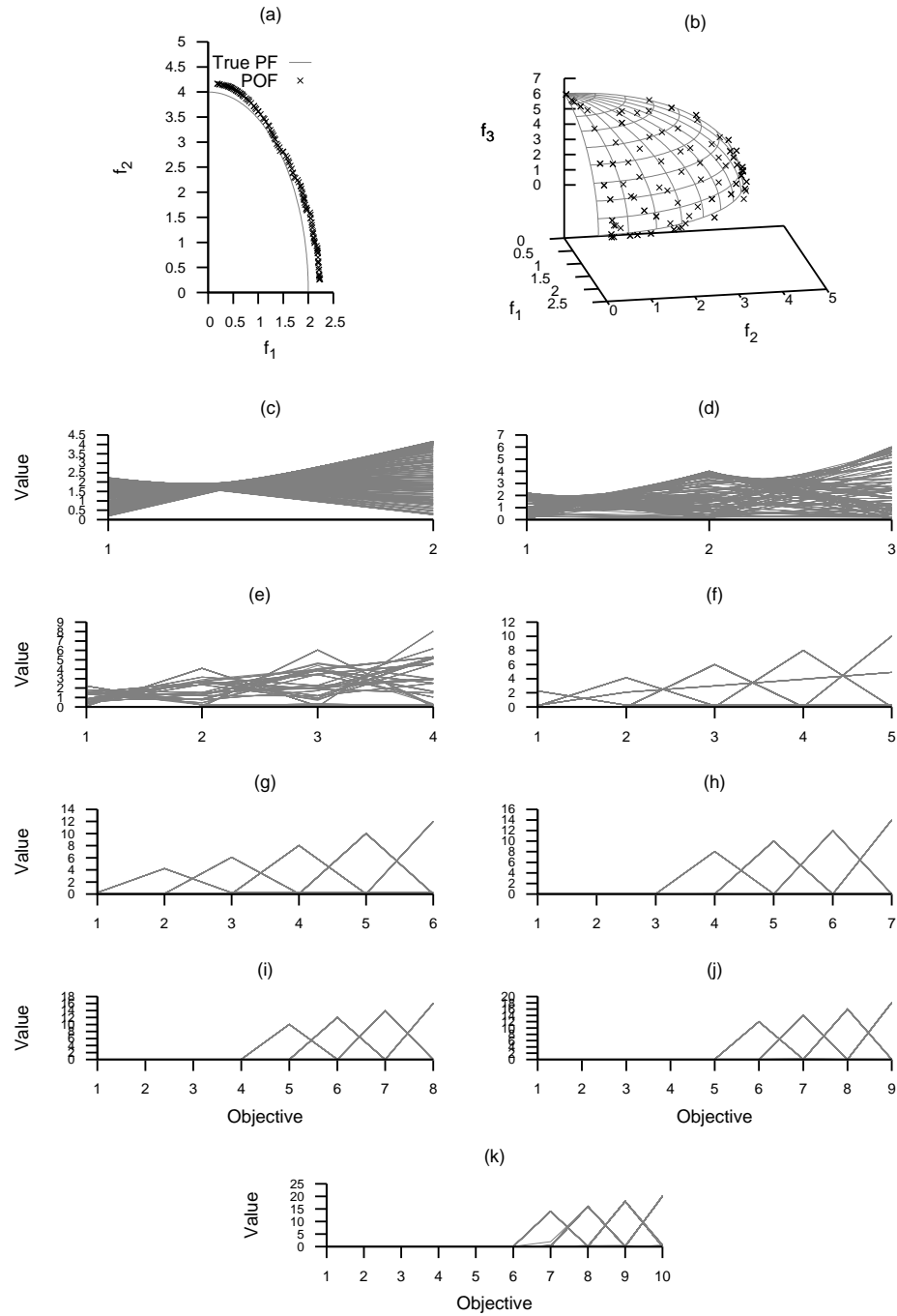


Figure A.406: Plots of the approximations obtained by MOMBI-NMTCH from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.173: Comparison of hypervolume indicator values for different optimizers on the WFG9 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	6.027818e+00	6.027598e+00	7.277967e+00	7.417969e+00	7.700486e+00	7.701148e+00	7.694716e+00	7.696532e+00	7.699452e+00	7.700494e+00	7.692248e+00
	avg.	6.207726e+00	6.179483e+00	7.446121e+00	7.560936e+00	7.966292e+00	7.972487e+00	7.989469e+00	7.970661e+00	7.872326e+00	7.917140e+00	7.917278e+00
	min.	5.861403e+00	5.785498e+00	7.052648e+00	7.316366e+00	7.686714e+00	7.685986e+00	7.669291e+00	7.676436e+00	7.688383e+00	7.685991e+00	7.673312e+00
	max.	6.660553e+00	6.611574e+00	8.078517e+00	8.216841e+00	8.410597e+00	8.423378e+00	8.350212e+00	8.360912e+00	8.380749e+00	8.372656e+00	8.285155e+00
	std.	2.240624e-01	2.067855e-01	3.008084e-01	2.904834e-01	3.124217e-01	3.183506e-01	3.051939e-01	3.114766e-01	2.829405e-01	3.031810e-01	2.711205e-01
3D	med.	4.836378e+01	4.835712e+01	5.286418e+01	5.741221e+01	6.523567e+01	6.657399e+01	6.591357e+01	6.698367e+01	6.578944e+01	6.665892e+01	6.698151e+01
	avg.	4.844718e+01	4.838074e+01	5.331269e+01	5.707435e+01	6.709243e+01	6.827780e+01	6.797611e+01	6.874602e+01	6.792567e+01	6.810733e+01	6.752785e+01
	min.	4.798131e+01	4.822388e+01	5.202738e+01	4.897630e+01	6.491361e+01	6.608424e+01	6.523898e+01	6.668138e+01	6.512366e+01	6.640449e+01	6.623870e+01
	max.	5.365633e+01	5.141186e+01	5.857855e+01	6.201491e+01	7.076204e+01	7.197612e+01	7.158562e+01	7.256954e+01	7.100377e+01	7.195120e+01	7.071966e+01
	std.	5.543145e-01	3.130063e-01	1.485794e+00	2.866211e+00	2.447262e+00	2.410179e+00	2.542608e+00	2.461312e+00	2.521097e+00	2.291024e+00	1.160807e+00
4D	med.	4.866030e+02	4.865102e+02	4.922861e+02	5.792567e+02	5.770974e+02	6.085994e+02	5.716648e+02	6.116398e+02	5.744462e+02	6.094865e+02	6.462445e+02
	avg.	4.907322e+02	4.883090e+02	4.961203e+02	5.756748e+02	5.804719e+02	6.097673e+02	5.787818e+02	6.141410e+02	5.806014e+02	6.111643e+02	6.427612e+02
	min.	4.822252e+02	4.819466e+02	4.892044e+02	4.968305e+02	5.696797e+02	5.994044e+02	5.654142e+02	6.012874e+02	5.702265e+02	5.987865e+02	6.045526e+02
	max.	5.462574e+02	5.294767e+02	5.667240e+02	6.165712e+02	6.129156e+02	6.479022e+02	6.139263e+02	6.589249e+02	6.112784e+02	6.404083e+02	6.673277e+02
	std.	1.001258e+01	7.345347e+00	1.057488e+01	2.358396e+01	1.029064e+01	7.797199e+00	1.399931e+01	1.044375e+01	1.239962e+01	7.185752e+00	1.557233e+01
5D	med.	5.701308e+03	5.686784e+03	5.837099e+03	6.333220e+03	6.521352e+03	6.139728e+03	5.894433e+03	5.883653e+03	5.892258e+03	5.897404e+03	6.729522e+03
	avg.	5.734725e+03	5.623332e+03	5.881497e+03	6.299400e+03	6.513576e+03	6.162646e+03	5.901545e+03	5.870895e+03	5.906865e+03	5.896755e+03	6.724401e+03
	min.	4.018648e+03	4.031778e+03	5.729435e+03	4.800183e+03	5.778884e+03	5.424955e+03	5.471021e+03	5.166847e+03	5.139387e+03	4.989259e+03	6.089398e+03
	max.	6.263482e+03	5.984654e+03	6.232512e+03	6.709276e+03	6.844224e+03	6.686132e+03	6.194400e+03	6.103373e+03	6.211220e+03	6.182181e+03	7.388576e+03
	std.	2.590391e+02	3.679461e+02	1.233688e+02	2.387690e+02	1.786935e+02	1.766256e+02	8.229927e+01	1.041426e+02	1.028182e+02	1.055757e+02	2.453772e+02
6D	med.	6.221323e+04	5.888687e+04	7.240146e+04	6.826427e+04	7.567265e+04	7.071589e+04	7.692052e+04	5.720340e+04	7.735003e+04	5.752243e+04	7.624184e+04
	avg.	6.315105e+04	6.145372e+04	7.193315e+04	7.002755e+04	7.620588e+04	7.186581e+04	7.406903e+04	5.812071e+04	7.459847e+04	5.924643e+04	7.641641e+04
	min.	4.367661e+04	4.762323e+04	4.855353e+04	5.655244e+04	6.582123e+04	5.762931e+04	4.802468e+04	4.498910e+04	4.576289e+04	5.189091e+04	6.486926e+04
	max.	8.077477e+04	7.764862e+04	8.428328e+04	8.839155e+04	8.795840e+04	8.833794e+04	7.895396e+04	7.580069e+04	7.986917e+04	7.574027e+04	8.528999e+04
	std.	7.423781e+03	5.588651e+03	6.573215e+03	5.254016e+03	6.231699e+03	6.609513e+03	7.049525e+03	3.695702e+03	7.048157e+03	4.221148e+03	3.542387e+03
7D	med.	5.595112e+05	5.538045e+05	9.427511e+05	7.132993e+05	8.444668e+05	6.798409e+05	9.114062e+05	5.477679e+05	7.796823e+05	5.521359e+05	9.733499e+05
	avg.	5.809366e+05	5.683958e+05	9.232091e+05	7.086165e+05	8.504087e+05	6.893743e+05	9.013268e+05	5.479460e+05	8.420956e+05	5.537948e+05	9.749190e+05
	min.	3.915558e+05	4.403937e+05	5.664914e+05	4.268304e+05	5.782732e+05	5.554360e+05	5.420515e+05	4.792703e+05	4.595221e+05	5.148376e+05	8.286191e+05
	max.	9.802970e+05	7.254869e+05	1.198657e+06	9.002714e+05	1.125180e+06	8.910753e+05	1.202724e+06	5.760425e+05	1.180397e+06	6.866590e+05	1.124369e+06
	std.	8.442074e+04	4.681553e+04	1.558654e+05	5.683434e+04	8.984863e+04	7.139835e+04	1.946028e+05	8.993958e+03	1.843900e+05	1.615190e+04	6.176064e+04
8D	med.	8.410920e+06	8.291550e+06	1.641208e+07	1.147877e+07	1.352537e+07	1.111837e+07	1.321799e+07	8.146169e+06	1.324681e+07	8.224628e+06	1.430434e+07
	avg.	8.859498e+06	8.601105e+06	1.541155e+07	1.139032e+07	1.372705e+07	1.116640e+07	1.350323e+07	8.176225e+06	1.262122e+07	8.285074e+06	1.421906e+07
	min.	6.218946e+06	6.384710e+06	8.428085e+06	8.244396e+06	7.320319e+06	8.388377e+06	7.581726e+06	8.086201e+06	8.203105e+06	8.109216e+06	8.326687e+06
	max.	1.201430e+07	1.131115e+07	2.009263e+07	1.454100e+07	1.687871e+07	1.537566e+07	2.028448e+07	8.729697e+06	1.985820e+07	1.027195e+07	1.623556e+07
	std.	1.140838e+06	7.879524e+05	2.967640e+06	8.252847e+05	1.364100e+06	1.533899e+06	3.038480e+06	1.058422e+05	2.323233e+06	2.712969e+05	1.192671e+06
9D	med.	1.442050e+08	1.440924e+08	3.027757e+08	2.092071e+08	2.489392e+08	1.966403e+08	2.206668e+08	1.378568e+08	2.191278e+08	1.396148e+08	2.409593e+08
	avg.	1.554178e+08	1.557608e+08	2.898293e+08	2.080447e+08	2.494557e+08	2.014232e+08	2.289640e+08	1.388141e+08	2.122078e+08	1.407679e+08	2.398037e+08
	min.	1.111009e+08	1.219948e+08	1.437190e+08	1.270102e+08	2.018146e+08	1.416298e+08	1.391172e+08	1.371222e+08	1.346331e+08	1.203860e+08	1.809651e+08
	max.	2.718038e+08	2.247623e+08	3.830706e+08	2.741452e+08	3.109213e+08	2.683853e+08	3.712839e+08	1.539010e+08	3.854470e+08	1.682538e+08	2.962918e+08
	std.	2.585265e+07	2.267086e+07	6.205981e+07	2.105952e+07	2.107616e+07	2.818304e+07	5.005964e+07	2.753905e+06	3.942506e+07	5.653315e+06	1.804105e+07
10D	med.	2.854342e+09	3.128882e+09	6.770255e+09	4.382275e+09	5.180438e+09	4.124591e+09	4.068378e+09	2.626938e+09	4.136590e+09	2.651488e+09	4.166818e+09
	avg.	3.023499e+09	3.202096e+09	6.421781e+09	4.387632e+09	5.168349e+09	4.075246e+09	3.974098e+09	2.658948e+09	3.929842e+09	2.723963e+09	4.174219e+09
	min.	1.948344e+09	2.550532e+09	3.048621e+09	2.485210e+09	2.305831e+09	2.654392e+09	1.946569e+09	2.597119e+09	2.065999e+09	2.291649e+09	2.737692e+09
	max.	4.717718e+09	4.587369e+09	8.063392e+09	5.772932e+09	6.657141e+09	5.625604e+09	7.116602e+09	3.163716e+09	5.774850e+09	3.878700e+09	5.071384e+09
	std.	5.167134e+08	4.844050e+08	1.194428e+09	4.456799e+08	7.224238e+08	7.730208e+08	8.706726e+08	1.027202e+08	7.083141e+08	2.188045e+08	4.209723e+08

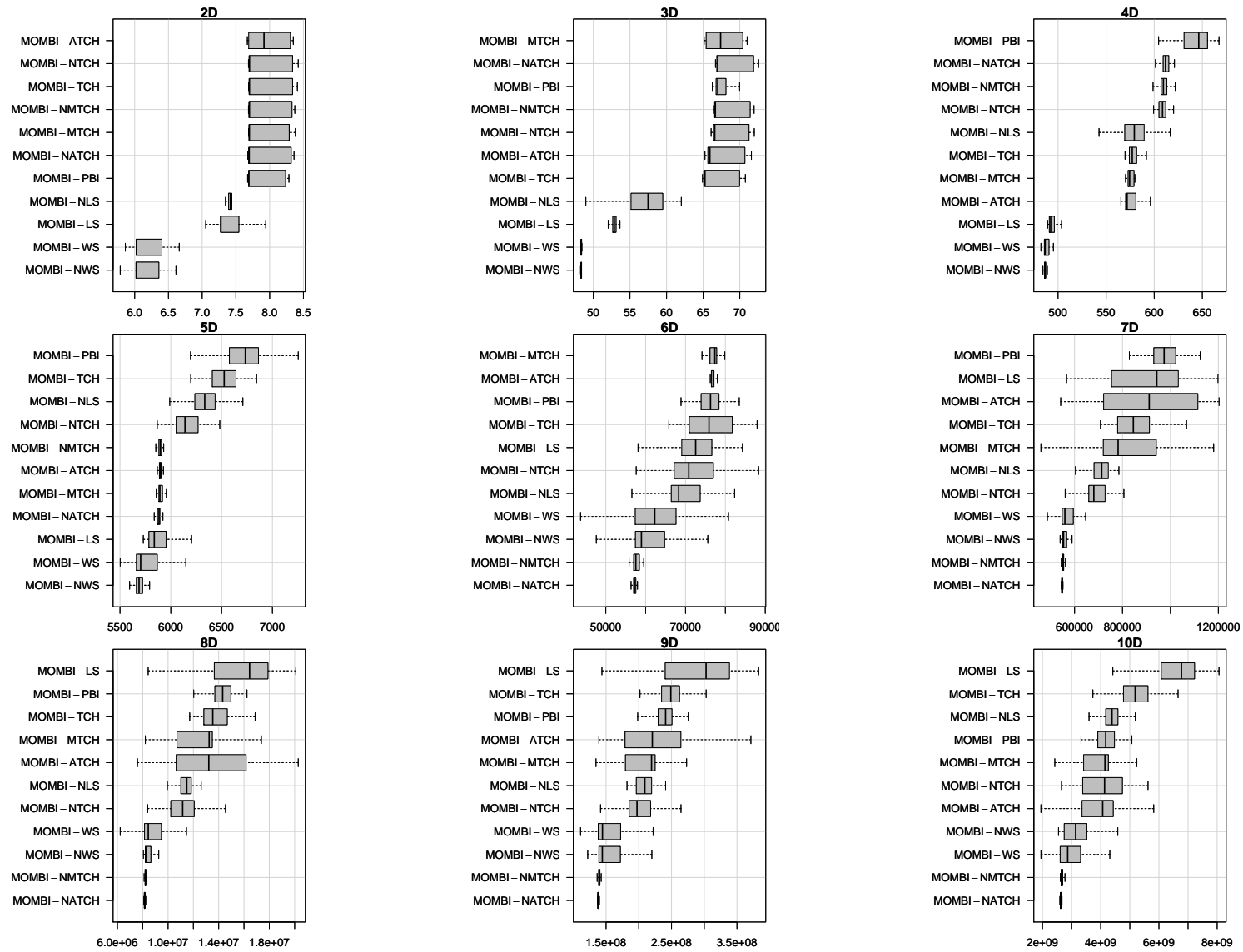


Figure A.407: Box-plot of the hypervolume indicator values for different optimizers on the WFG9 test problem.

Table A.174: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG9 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $p$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
2D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.28e-34$	$1.28e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$1.28e-34$	$1.28e-34$	$2.60e-14$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.23e-18$	$3.88e-19$	–	> 0.05	$2.22e-04$	$3.88e-04$	$1.02e-02$	$3.96e-02$	$2.48e-08$
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.23e-18$	$3.55e-19$	> 0.05	–	$4.70e-05$	$1.60e-04$	$2.30e-02$	$4.22e-02$	$1.99e-09$
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$2.56e-20$	$9.84e-21$	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	$5.17e-03$
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$4.14e-19$	$1.32e-19$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	$2.02e-04$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$2.74e-15$	$8.31e-16$	> 0.05	> 0.05	$1.54e-02$	$2.72e-02$	–	> 0.05	$1.34e-05$
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$8.24e-17$	$2.39e-17$	> 0.05	> 0.05	$5.99e-03$	$1.93e-02$	> 0.05	–	$3.65e-06$
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$2.11e-18$	$1.06e-18$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–
3D											
MOMBI-WS	–	$4.07e-02$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$1.76e-33$	$1.28e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$2.07e-34$	$1.44e-34$	$2.35e-21$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.03e-09$	–	$7.28e-07$	> 0.05	$6.35e-06$	> 0.05	> 0.05
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$3.75e-11$	> 0.05	–	> 0.05	$1.50e-03$	> 0.05	> 0.05
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.26e-09$	$3.56e-10$	$2.62e-07$	–	$5.49e-06$	$1.29e-12$	$9.38e-04$
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.10e-09$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$3.85e-08$	$1.48e-03$	$8.57e-06$	> 0.05	$1.24e-04$	–	> 0.05
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.28e-34$	$1.60e-03$	$4.39e-03$	> 0.05	> 0.05	> 0.05	$5.70e-04$	–
4D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-LS	$6.57e-14$	$1.52e-25$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NLS	$8.40e-34$	$2.48e-34$	$2.17e-33$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-TCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	–	> 0.05	$1.59e-07$	> 0.05	$6.15e-03$	> 0.05	> 0.05
MOMBI-NTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$9.21e-30$	$1.00e-29$	–	$9.24e-27$	> 0.05	$2.36e-29$	> 0.05	> 0.05
MOMBI-ATCH	$1.28e-34$	$1.28e-34$	$1.36e-34$	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05
MOMBI-NATCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$6.55e-32$	$6.93e-32$	$9.16e-06$	$4.60e-30$	–	$3.00e-32$	$2.21e-03$	> 0.05
MOMBI-MTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	> 0.05	> 0.05	> 0.05	$2.70e-06$	> 0.05	–	> 0.05	> 0.05
MOMBI-NMTCH	$1.28e-34$	$1.28e-34$	$1.28e-34$	$5.66e-31$	$7.00e-31$	$2.08e-02$	$7.64e-28$	> 0.05	$2.90e-31$	–	> 0.05
MOMBI-PBI	$1.28e-34$	$1.28e-34$	$1.28e-34$	$2.33e-34$	$2.26e-34$	$2.56e-29$	$3.34e-34$	$1.15e-24$	$2.07e-34$	$8.56e-29$	–

Table A.175: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG9 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
5D											
MOMBI-WS	–	$1.51e-02$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-LS	$6.18e-10$	$4.35e-23$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NLS	$4.23e-30$	$4.84e-31$	$1.52e-29$	–	$> 0.05$	$8.90e-12$	$1.88e-30$	$9.82e-31$	$3.29e-30$	$1.93e-30$	$> 0.05$
MOMBI-TCH	$1.07e-33$	$2.79e-34$	$8.35e-33$	$1.69e-14$	–	$4.10e-25$	$3.00e-32$	$8.60e-33$	$2.18e-32$	$2.18e-32$	$> 0.05$
MOMBI-NTCH	$1.60e-28$	$8.85e-33$	$2.33e-24$	$> 0.05$	$> 0.05$	–	$3.23e-28$	$2.59e-31$	$2.90e-27$	$1.12e-28$	$> 0.05$
MOMBI-ATCH	$5.84e-12$	$8.87e-24$	$1.75e-04$	$> 0.05$	$> 0.05$	$> 0.05$	–	$6.77e-10$	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-NATCH	$3.19e-10$	$2.90e-22$	$5.39e-03$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$7.35e-13$	$5.02e-25$	$4.46e-05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$7.28e-07$	–	$> 0.05$	$> 0.05$
MOMBI-NMTCH	$3.82e-12$	$1.48e-24$	$1.45e-04$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$4.44e-07$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$1.49e-34$	$1.28e-34$	$1.78e-34$	$1.28e-24$	$5.90e-11$	$9.02e-31$	$1.40e-34$	$1.32e-34$	$1.53e-34$	$1.40e-34$	–
6D											
MOMBI-WS	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$2.14e-09$	$> 0.05$	$4.64e-06$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.46e-10$	$> 0.05$	$5.57e-05$	$> 0.05$
MOMBI-LS	$2.49e-15$	$2.30e-21$	–	$3.79e-05$	$> 0.05$	$> 0.05$	$> 0.05$	$6.42e-30$	$> 0.05$	$1.24e-27$	$> 0.05$
MOMBI-NLS	$3.69e-12$	$2.62e-20$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$2.17e-29$	$> 0.05$	$9.24e-27$	$> 0.05$
MOMBI-TCH	$1.68e-23$	$7.26e-29$	$8.95e-05$	$2.09e-12$	–	$2.80e-06$	$> 0.05$	$1.57e-33$	$> 0.05$	$9.95e-33$	$> 0.05$
MOMBI-NTCH	$1.05e-14$	$5.50e-22$	$> 0.05$	$1.65e-02$	$> 0.05$	–	$> 0.05$	$6.25e-31$	$> 0.05$	$2.75e-28$	$> 0.05$
MOMBI-ATCH	$1.51e-17$	$9.02e-22$	$6.97e-05$	$1.02e-10$	$> 0.05$	$2.22e-04$	–	$1.09e-28$	$> 0.05$	$1.30e-25$	$> 0.05$
MOMBI-NATCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$2.61e-19$	$1.64e-24$	$4.48e-06$	$1.04e-11$	$> 0.05$	$5.30e-05$	$7.00e-05$	$1.79e-29$	–	$7.44e-28$	$> 0.05$
MOMBI-NMTCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$5.39e-03$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$1.14e-25$	$5.58e-31$	$5.84e-08$	$6.32e-17$	$> 0.05$	$3.18e-08$	$> 0.05$	$6.06e-34$	$> 0.05$	$1.48e-33$	–
7D											
MOMBI-WS	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$3.05e-10$	$> 0.05$	$3.03e-04$	$> 0.05$
MOMBI-NWS	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$1.29e-12$	$> 0.05$	$4.95e-03$	$> 0.05$
MOMBI-LS	$1.95e-29$	$8.35e-33$	–	$5.37e-22$	$1.10e-05$	$2.61e-23$	$4.42e-02$	$2.01e-34$	$4.51e-05$	$2.96e-34$	$> 0.05$
MOMBI-NLS	$5.41e-23$	$5.54e-28$	$> 0.05$	–	$> 0.05$	$1.87e-03$	$> 0.05$	$5.67e-32$	$> 0.05$	$4.44e-31$	$> 0.05$
MOMBI-TCH	$7.20e-31$	$3.55e-34$	$> 0.05$	$1.26e-26$	–	$2.63e-26$	$> 0.05$	$1.28e-34$	$> 0.05$	$1.40e-34$	$> 0.05$
MOMBI-NTCH	$1.78e-20$	$1.14e-26$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$2.79e-34$	$> 0.05$	$4.26e-33$	$> 0.05$
MOMBI-ATCH	$1.92e-26$	$4.44e-29$	$> 0.05$	$4.06e-14$	$> 0.05$	$2.60e-17$	–	$3.27e-32$	$> 0.05$	$1.82e-30$	$> 0.05$
MOMBI-NATCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-MTCH	$2.28e-22$	$1.77e-25$	$> 0.05$	$1.46e-09$	$> 0.05$	$2.86e-12$	$> 0.05$	$1.74e-29$	–	$4.19e-27$	$> 0.05$
MOMBI-NMTCH	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$> 0.05$	$7.34e-14$	$> 0.05$	–	$> 0.05$
MOMBI-PBI	$8.40e-34$	$1.28e-34$	$> 0.05$	$1.84e-34$	$5.63e-20$	$2.55e-34$	$3.19e-04$	$1.28e-34$	$3.85e-09$	$1.28e-34$	–

Table A.176: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG9 test problem (continuation).

Optimizer	MOMBI WS	MOMBI NWS	MOMBI LS	MOMBI NLS	MOMBI TCH	MOMBI NTCH	MOMBI ATCH	MOMBI NATCH	MOMBI MTCH	MOMBI NMTCH	MOMBI PBI
8D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$8.58e-08$	> 0.05	$1.20e-03$	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$3.61e-16$	> 0.05	$1.39e-04$	> 0.05
MOMBI-LS	$3.07e-31$	$7.22e-33$	–	$4.55e-18$	$2.93e-07$	$2.93e-20$	$1.34e-07$	$1.78e-34$	$2.00e-13$	$3.05e-34$	$1.17e-04$
MOMBI-NLS	$1.51e-28$	$7.14e-32$	> 0.05	–	> 0.05	> 0.05	> 0.05	$1.78e-34$	> 0.05	$5.07e-34$	> 0.05
MOMBI-TCH	$2.51e-33$	$2.44e-33$	> 0.05	$3.68e-30$	–	$3.02e-23$	> 0.05	$2.51e-33$	$3.60e-05$	$2.51e-33$	> 0.05
MOMBI-NTCH	$9.68e-22$	$1.70e-27$	> 0.05	> 0.05	> 0.05	–	> 0.05	$2.48e-34$	> 0.05	$2.37e-33$	> 0.05
MOMBI-ATCH	$1.38e-27$	$1.48e-29$	> 0.05	$5.87e-06$	> 0.05	$7.89e-09$	–	$3.47e-32$	> 0.05	$7.14e-32$	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$1.82e-26$	$1.09e-28$	> 0.05	$1.35e-04$	> 0.05	$6.28e-07$	> 0.05	$5.22e-34$	–	$3.57e-32$	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.56e-14$	> 0.05	–	> 0.05
MOMBI-PBI	$1.35e-33$	$7.03e-34$	> 0.05	$6.80e-31$	$1.88e-04$	$7.49e-27$	$7.42e-04$	$1.53e-34$	$1.57e-11$	$2.07e-34$	–
9D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$2.02e-08$	> 0.05	$8.70e-04$	> 0.05
MOMBI-NWS	> 0.05	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.10e-15$	> 0.05	$9.07e-07$	> 0.05
MOMBI-LS	$2.70e-30$	$1.72e-30$	–	$2.13e-20$	$3.23e-09$	$2.74e-20$	$1.62e-12$	$1.73e-34$	$3.84e-18$	$3.76e-34$	$2.57e-11$
MOMBI-NLS	$1.91e-25$	$1.03e-26$	> 0.05	–	> 0.05	$7.81e-03$	> 0.05	$5.83e-32$	> 0.05	$6.00e-32$	> 0.05
MOMBI-TCH	$3.27e-33$	$2.79e-34$	> 0.05	$2.91e-26$	–	$6.93e-24$	$6.42e-06$	$1.28e-34$	$1.58e-16$	$1.28e-34$	$6.65e-04$
MOMBI-NTCH	$6.64e-22$	$2.90e-22$	> 0.05	> 0.05	> 0.05	–	> 0.05	$2.40e-34$	> 0.05	$7.69e-34$	> 0.05
MOMBI-ATCH	$2.73e-25$	$5.99e-25$	> 0.05	$3.12e-03$	> 0.05	$5.41e-05$	–	$3.65e-34$	> 0.05	$6.23e-33$	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$4.56e-23$	$1.11e-22$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$5.39e-33$	–	$3.27e-32$	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$1.29e-13$	> 0.05	–	> 0.05
MOMBI-PBI	$1.19e-32$	$7.46e-34$	> 0.05	$4.24e-22$	> 0.05	$2.67e-19$	$1.42e-03$	$1.28e-34$	$1.12e-12$	$1.28e-34$	–
10D											
MOMBI-WS	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$6.28e-07$	> 0.05	$1.56e-04$	> 0.05
MOMBI-NWS	$8.07e-04$	–	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$5.81e-23$	> 0.05	$1.69e-15$	> 0.05
MOMBI-LS	$8.60e-33$	$2.25e-32$	–	$2.51e-22$	$1.50e-15$	$6.63e-25$	$2.34e-25$	$1.40e-34$	$1.23e-26$	$2.40e-34$	$1.45e-24$
MOMBI-NLS	$6.60e-30$	$5.38e-29$	> 0.05	–	> 0.05	$5.13e-03$	$1.10e-06$	$2.74e-33$	$1.11e-08$	$3.68e-33$	$1.34e-04$
MOMBI-TCH	$2.99e-31$	$6.43e-31$	> 0.05	$3.21e-20$	–	$7.00e-19$	$4.81e-20$	$5.20e-32$	$3.09e-23$	$5.83e-32$	$1.52e-24$
MOMBI-NTCH	$1.29e-19$	$3.96e-15$	> 0.05	> 0.05	> 0.05	–	> 0.05	$1.87e-33$	> 0.05	$4.08e-31$	> 0.05
MOMBI-ATCH	$2.44e-17$	$1.76e-12$	> 0.05	> 0.05	> 0.05	> 0.05	–	$5.04e-27$	> 0.05	$1.64e-25$	> 0.05
MOMBI-NATCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	–	> 0.05	> 0.05	> 0.05
MOMBI-MTCH	$2.25e-18$	$1.39e-13$	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$5.00e-30$	–	$3.06e-28$	> 0.05
MOMBI-NMTCH	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	> 0.05	$3.32e-11$	> 0.05	–	> 0.05
MOMBI-PBI	$1.30e-27$	$8.63e-26$	> 0.05	> 0.05	> 0.05	> 0.05	$8.29e-03$	$1.68e-34$	$1.73e-03$	$4.50e-34$	–

Table A.177: Comparison of R2 indicator values for different optimizers on the WFG9 test problem.

Dim.	Stat.	MOMBI-WS	MOMBI-NWS	MOMBI-LS	MOMBI-NLS	MOMBI-TCH	MOMBI-NTCH	MOMBI-ATCH	MOMBI-NATCH	MOMBI-MTCH	MOMBI-NMTCH	MOMBI-PBI
2D	med.	7.125511e-01	7.126412e-01	6.567840e-01	6.502234e-01	6.367608e-01	6.367940e-01	6.133633e-01	6.369401e-01	6.368197e-01	6.369500e-01	6.372877e-01
	avg.	7.020528e-01	7.035626e-01	6.475982e-01	6.421750e-01	6.219095e-01	6.219224e-01	6.203711e-01	6.217568e-01	6.271828e-01	6.250255e-01	6.246589e-01
	min.	6.794115e-01	6.805986e-01	6.142002e-01	6.064423e-01	5.968791e-01	5.959370e-01	6.002231e-01	6.001493e-01	5.972956e-01	5.998381e-01	6.037940e-01
	max.	7.243721e-01	7.303096e-01	6.634954e-01	6.545267e-01	6.384084e-01	6.387389e-01	6.402631e-01	6.395793e-01	6.383111e-01	6.386691e-01	6.399245e-01
	std.	1.293505e-02	1.232565e-02	1.628564e-02	1.588733e-02	1.759988e-02	1.769177e-02	1.759526e-02	1.767823e-02	1.598551e-02	1.693659e-02	1.544647e-02
3D	med.	3.960976e-01	3.961204e-01	3.828321e-01	3.767905e-01	3.508882e-01	3.501741e-01	3.499849e-01	3.492273e-01	3.334116e-01	3.496851e-01	3.501822e-01
	avg.	3.958013e-01	3.960703e-01	3.809216e-01	3.765586e-01	3.424089e-01	3.427139e-01	3.401250e-01	3.418110e-01	3.395013e-01	3.435251e-01	3.477200e-01
	min.	3.748070e-01	3.837756e-01	3.595080e-01	3.604546e-01	3.271170e-01	3.262882e-01	3.257314e-01	3.247144e-01	3.262304e-01	3.276322e-01	3.336679e-01
	max.	3.987250e-01	3.969496e-01	3.853096e-01	3.945204e-01	3.516538e-01	3.517928e-01	3.510136e-01	3.510980e-01	3.513202e-01	3.510952e-01	3.542531e-01
	std.	2.243946e-03	1.268808e-03	6.098284e-03	7.964704e-03	1.074645e-02	1.028469e-02	1.115573e-02	1.051136e-02	1.133945e-02	9.675349e-03	5.474474e-03
4D	med.	2.658705e-01	2.659179e-01	2.650559e-01	2.585970e-01	2.517687e-01	2.528189e-01	2.527532e-01	2.528031e-01	2.520555e-01	2.522097e-01	2.516943e-01
	avg.	2.652853e-01	2.655845e-01	2.641923e-01	2.581799e-01	2.508928e-01	2.522526e-01	2.505541e-01	2.521675e-01	2.499615e-01	2.517210e-01	2.532985e-01
	min.	2.555639e-01	2.524570e-01	2.513113e-01	2.470652e-01	2.416984e-01	2.398566e-01	2.395381e-01	2.406998e-01	2.401260e-01	2.406129e-01	2.418909e-01
	max.	2.681322e-01	2.682494e-01	2.654785e-01	2.651624e-01	2.545077e-01	2.558082e-01	2.552613e-01	2.552665e-01	2.531374e-01	2.541763e-01	2.711742e-01
	std.	1.968233e-03	2.042351e-03	2.187175e-03	3.391795e-03	2.701382e-03	2.498653e-03	4.389898e-03	2.904570e-03	4.059538e-03	2.517906e-03	7.168924e-03
5D	med.	2.105043e-01	2.108404e-01	2.091427e-01	2.096448e-01	2.070235e-01	2.097756e-01	2.071001e-01	2.112570e-01	2.082024e-01	2.107414e-01	2.309316e-01
	avg.	2.132398e-01	2.173283e-01	2.086642e-01	2.114270e-01	2.077797e-01	2.106324e-01	2.072255e-01	2.127683e-01	2.083240e-01	2.118978e-01	2.314546e-01
	min.	2.044029e-01	2.069862e-01	2.032986e-01	2.038598e-01	2.009522e-01	2.033981e-01	1.995888e-01	2.091088e-01	2.007060e-01	2.021313e-01	2.038343e-01
	max.	3.371795e-01	3.370232e-01	2.123910e-01	3.002465e-01	2.551757e-01	2.751761e-01	2.218792e-01	2.615518e-01	2.462185e-01	2.813472e-01	2.523129e-01
	std.	1.463994e-02	2.558408e-02	1.769558e-03	1.137050e-02	5.530340e-03	7.087026e-03	2.734650e-03	7.106395e-03	4.126577e-03	7.383379e-03	8.439566e-03
6D	med.	2.514292e-01	2.532384e-01	1.955146e-01	2.531370e-01	2.261525e-01	2.501342e-01	1.661160e-01	2.574434e-01	1.666807e-01	2.573686e-01	2.124234e-01
	avg.	2.368666e-01	2.400797e-01	1.981940e-01	2.386302e-01	2.238424e-01	2.355143e-01	1.793174e-01	2.534278e-01	1.811139e-01	2.494106e-01	2.115727e-01
	min.	1.685835e-01	1.739439e-01	1.661530e-01	1.693530e-01	1.689500e-01	1.738741e-01	1.626911e-01	1.737051e-01	1.612579e-01	1.724188e-01	1.900618e-01
	max.	3.660312e-01	3.092235e-01	3.085435e-01	2.617557e-01	2.918345e-01	2.670396e-01	3.094620e-01	3.374879e-01	3.318056e-01	3.046578e-01	2.353276e-01
	std.	3.268476e-02	2.588080e-02	2.788500e-02	2.541665e-02	3.099494e-02	2.697248e-02	3.245991e-02	2.040555e-02	3.340872e-02	2.244256e-02	9.280173e-03
7D	med.	3.829711e-01	3.828661e-01	2.122586e-01	3.772996e-01	3.625678e-01	3.793505e-01	2.066378e-01	3.863830e-01	2.472089e-01	3.855080e-01	2.097358e-01
	avg.	3.720367e-01	3.789509e-01	2.284980e-01	3.750185e-01	3.428143e-01	3.745961e-01	2.251113e-01	3.868951e-01	2.470161e-01	3.849666e-01	2.104251e-01
	min.	2.092319e-01	2.800301e-01	1.608624e-01	2.885576e-01	2.160032e-01	2.754462e-01	1.455265e-01	3.774357e-01	1.505223e-01	3.174278e-01	1.872890e-01
	max.	4.217902e-01	4.473095e-01	3.836220e-01	4.075088e-01	4.034324e-01	4.165614e-01	3.844666e-01	4.324546e-01	4.319945e-01	3.924041e-01	2.457789e-01
	std.	3.809582e-02	1.897478e-02	5.460314e-02	1.584991e-02	4.506151e-02	2.185862e-02	6.313836e-02	4.964431e-03	6.623750e-02	7.177190e-03	1.132125e-02
8D	med.	4.026208e-01	4.031819e-01	2.003330e-01	3.949540e-01	3.846491e-01	3.975434e-01	2.278562e-01	4.061312e-01	2.314227e-01	4.047438e-01	2.111288e-01
	avg.	3.865608e-01	3.971675e-01	2.198550e-01	3.944463e-01	3.638719e-01	3.892415e-01	2.366206e-01	4.061926e-01	2.541964e-01	4.050365e-01	2.129131e-01
	min.	2.744961e-01	3.032222e-01	1.583418e-01	3.676451e-01	2.219511e-01	3.059889e-01	1.393075e-01	4.020870e-01	1.484573e-01	3.965947e-01	1.933765e-01
	max.	4.287815e-01	4.443612e-01	4.031781e-01	4.046487e-01	4.116454e-01	4.287200e-01	4.047213e-01	4.115295e-01	4.036993e-01	4.241933e-01	2.799285e-01
	std.	3.953010e-02	2.268361e-02	5.557322e-02	5.664846e-03	4.016030e-02	2.479356e-02	5.821311e-02	1.395027e-03	5.280588e-02	2.850475e-03	1.326854e-02
9D	med.	4.176050e-01	4.176834e-01	1.937598e-01	4.094506e-01	4.005136e-01	4.111546e-01	2.450326e-01	4.210533e-01	2.516758e-01	4.199992e-01	2.059684e-01
	avg.	4.004861e-01	4.112181e-01	2.156834e-01	4.087321e-01	3.918195e-01	4.042179e-01	2.490226e-01	4.213282e-01	2.711669e-01	4.203306e-01	2.074464e-01
	min.	2.876839e-01	3.065302e-01	1.475415e-01	3.299543e-01	3.076181e-01	3.080999e-01	1.424915e-01	4.173023e-01	1.394306e-01	4.144308e-01	1.828746e-01
	max.	4.611613e-01	4.551982e-01	4.182834e-01	4.511988e-01	4.516133e-01	4.543710e-01	4.165202e-01	4.264785e-01	4.179279e-01	4.385643e-01	2.426309e-01
	std.	3.953010e-02	2.433176e-02	6.688760e-02	1.349311e-02	3.195105e-02	2.518379e-02	5.714747e-02	1.440777e-03	5.004601e-02	3.024337e-03	1.014240e-02
10D	med.	4.255634e-01	4.197335e-01	1.775253e-01	4.146999e-01	4.026970e-01	4.171852e-01	2.539202e-01	4.287678e-01	2.582768e-01	4.270607e-01	2.116536e-01
	avg.	4.097263e-01	4.032681e-01	1.949219e-01	4.122502e-01	3.893524e-01	4.056518e-01	2.799284e-01	4.290770e-01	2.838996e-01	4.264636e-01	2.127832e-01
	min.	3.091930e-01	2.999756e-01	1.479410e-01	3.212074e-01	2.930946e-01	3.206919e-01	4.227225e-01	4.227225e-01	1.783019e-01	3.471796e-01	1.913249e-01
	max.	4.701003e-01	4.677323e-01	4.050958e-01	4.560720e-01	4.488164e-01	4.574781e-01	4.839965e-01	4.365168e-01	4.257546e-01	4.372143e-01	2.678085e-01
	std.	3.841280e-02	3.731203e-02	5.545262e-02	1.693383e-02	3.716931e-02	2.883778e-02	6.428723e-02	1.759055e-03	5.601888e-02	8.334932e-03	1.220697e-02

Figure A.408: Box-plot of the R2 indicator values for different optimizers on the WFG9 test problem.



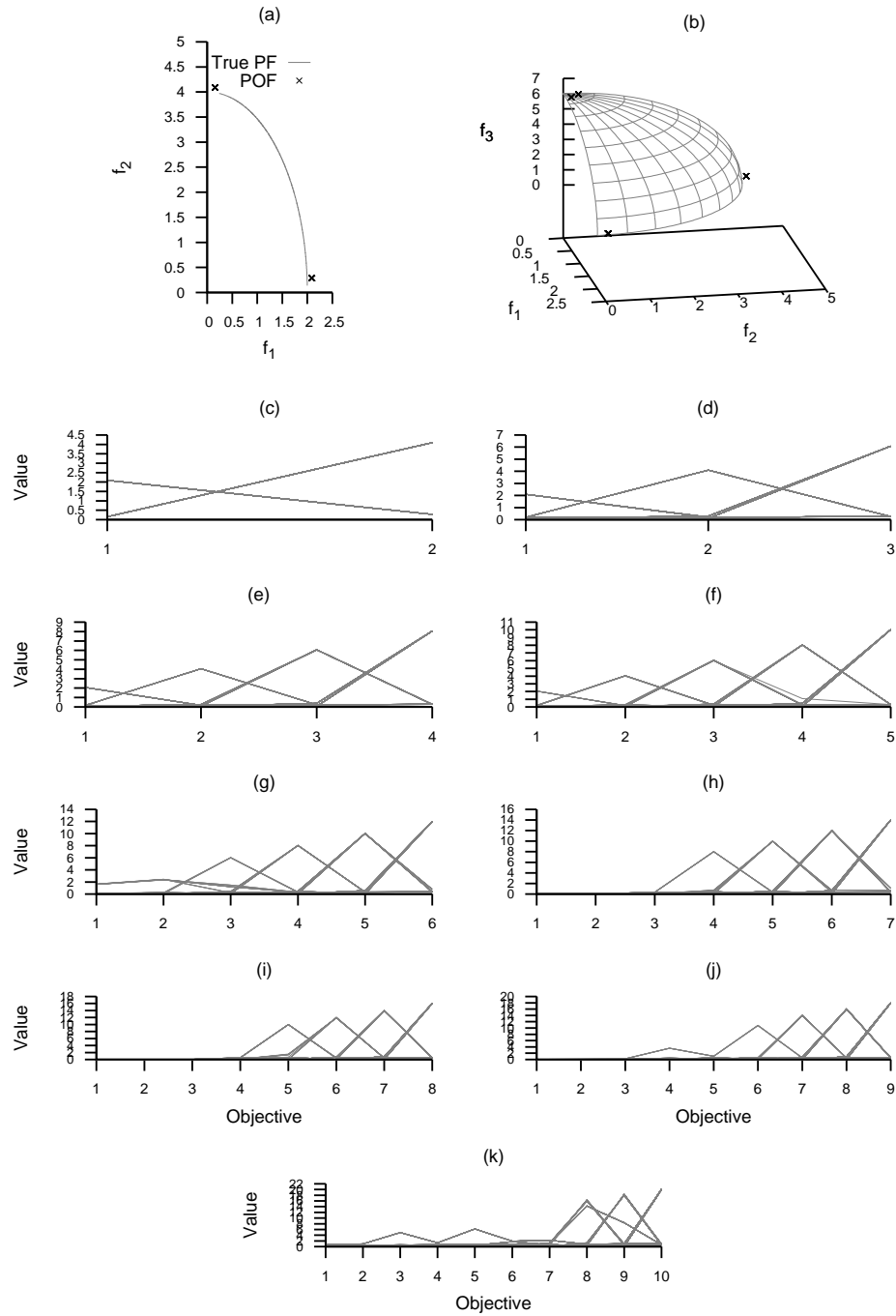


Figure A.409: Plots of the approximations obtained by MOMBI-WS from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

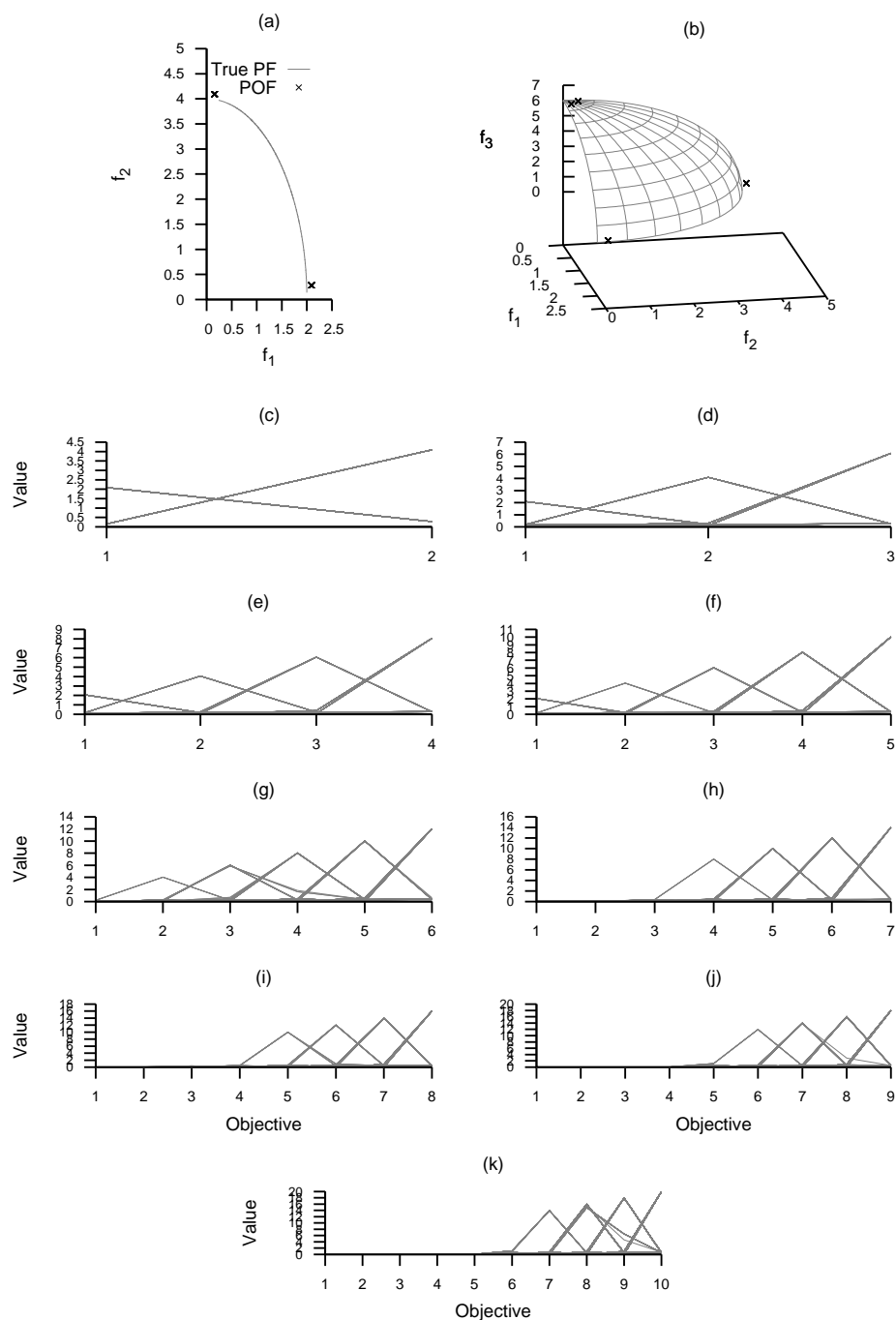


Figure A.410: Plots of the approximations obtained by MOMBI-NWS from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

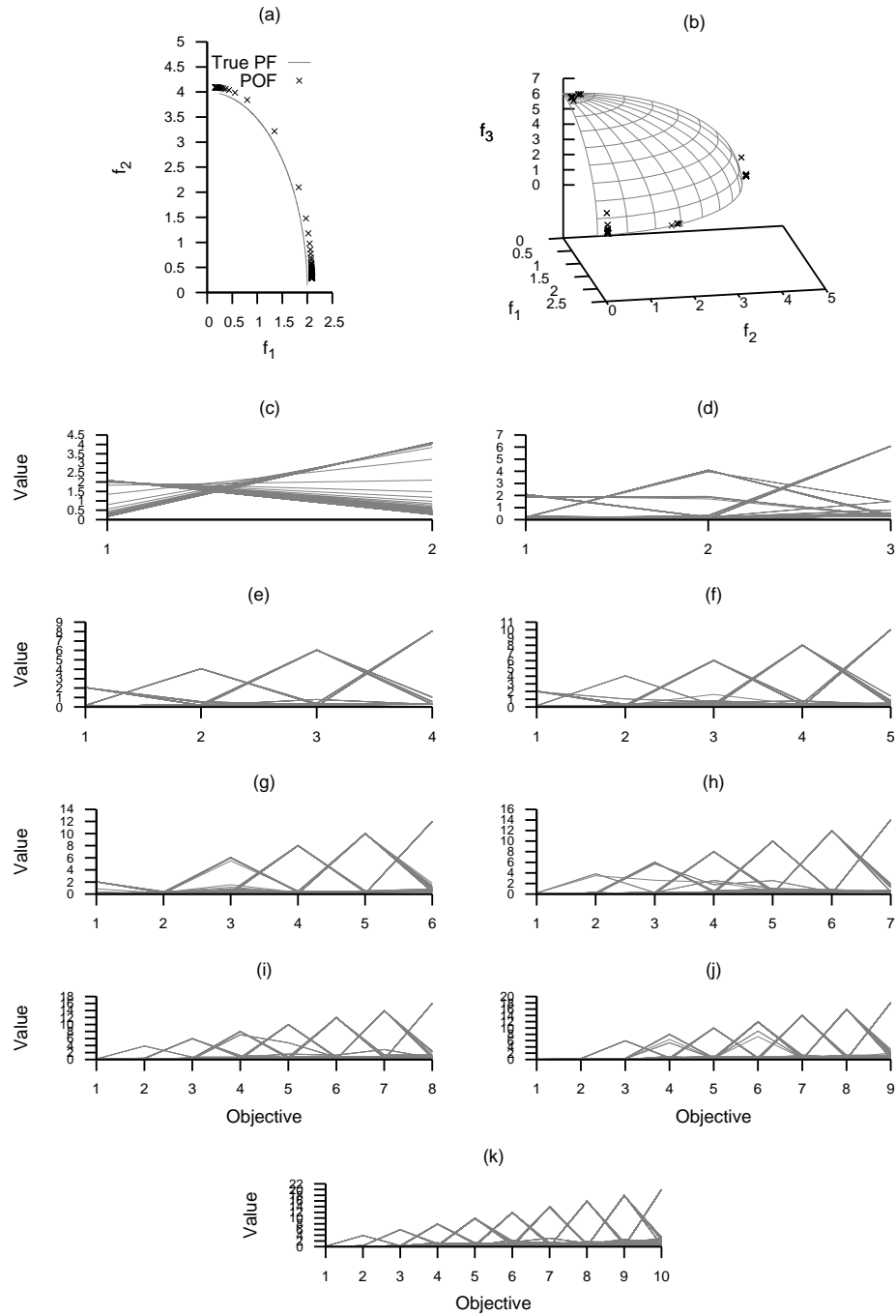


Figure A.411: Plots of the approximations obtained by MOMBI-LS from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

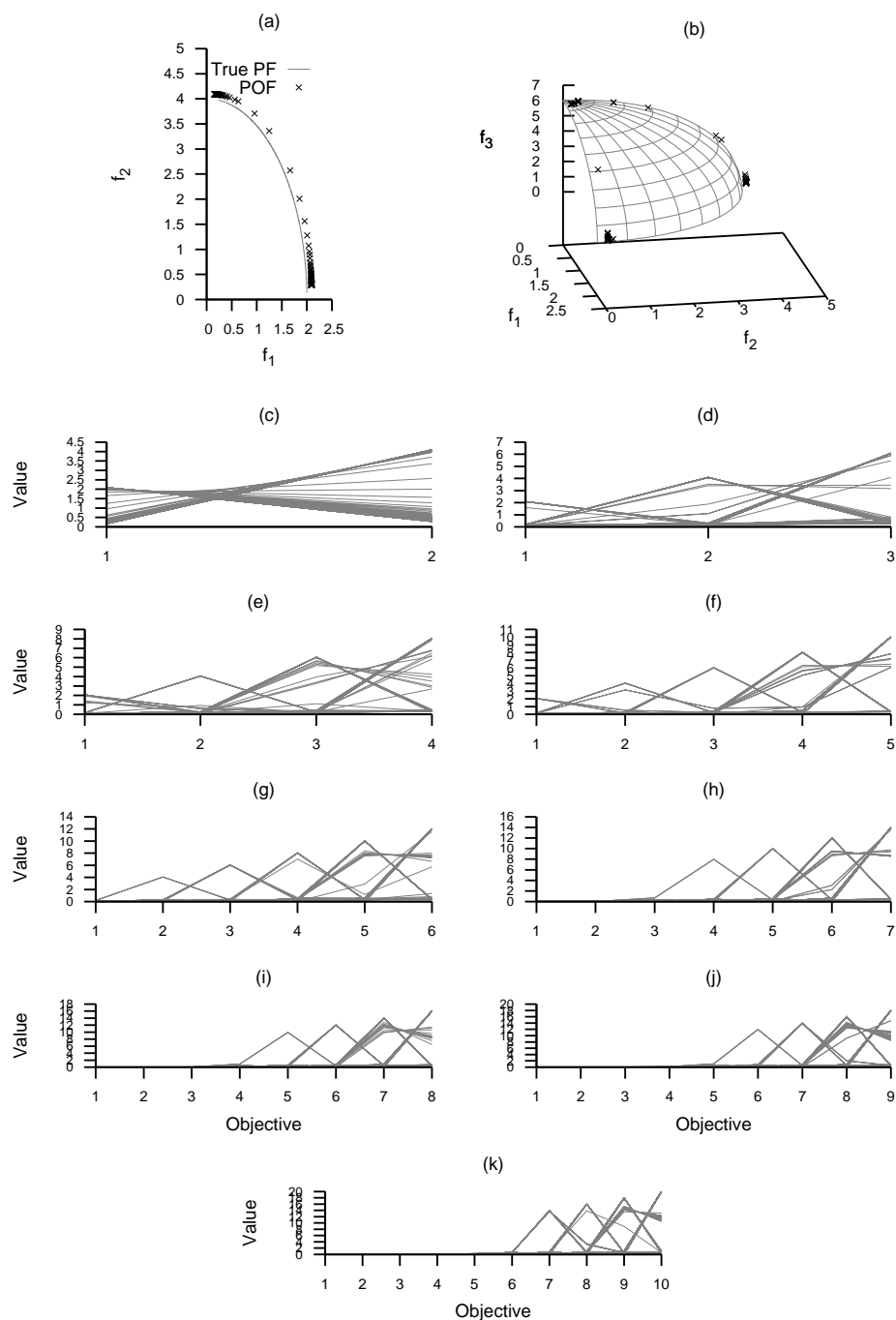


Figure A.412: Plots of the approximations obtained by MOMBI-NLS from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

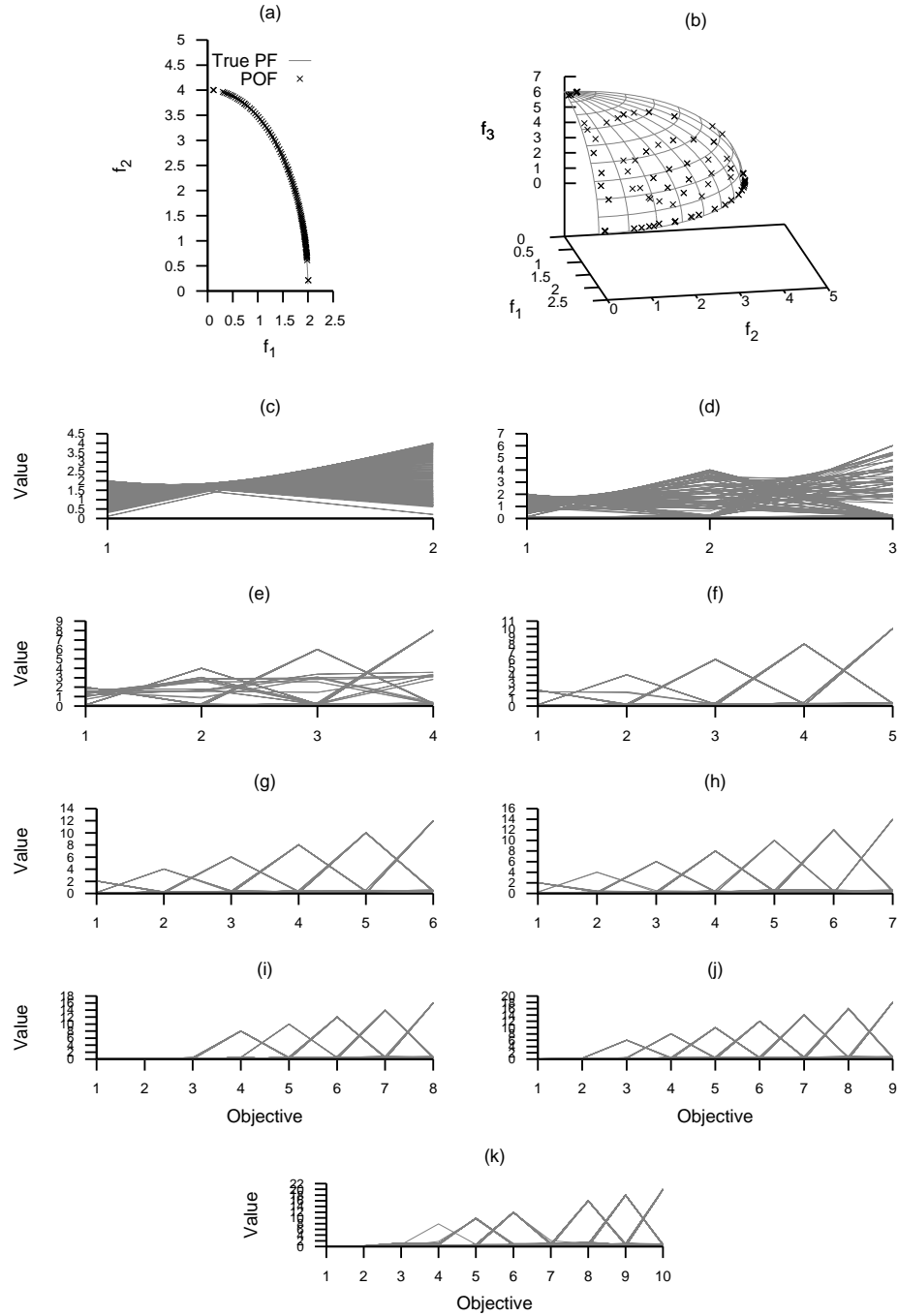


Figure A.413: Plots of the approximations obtained by MOMBI-ATCH from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

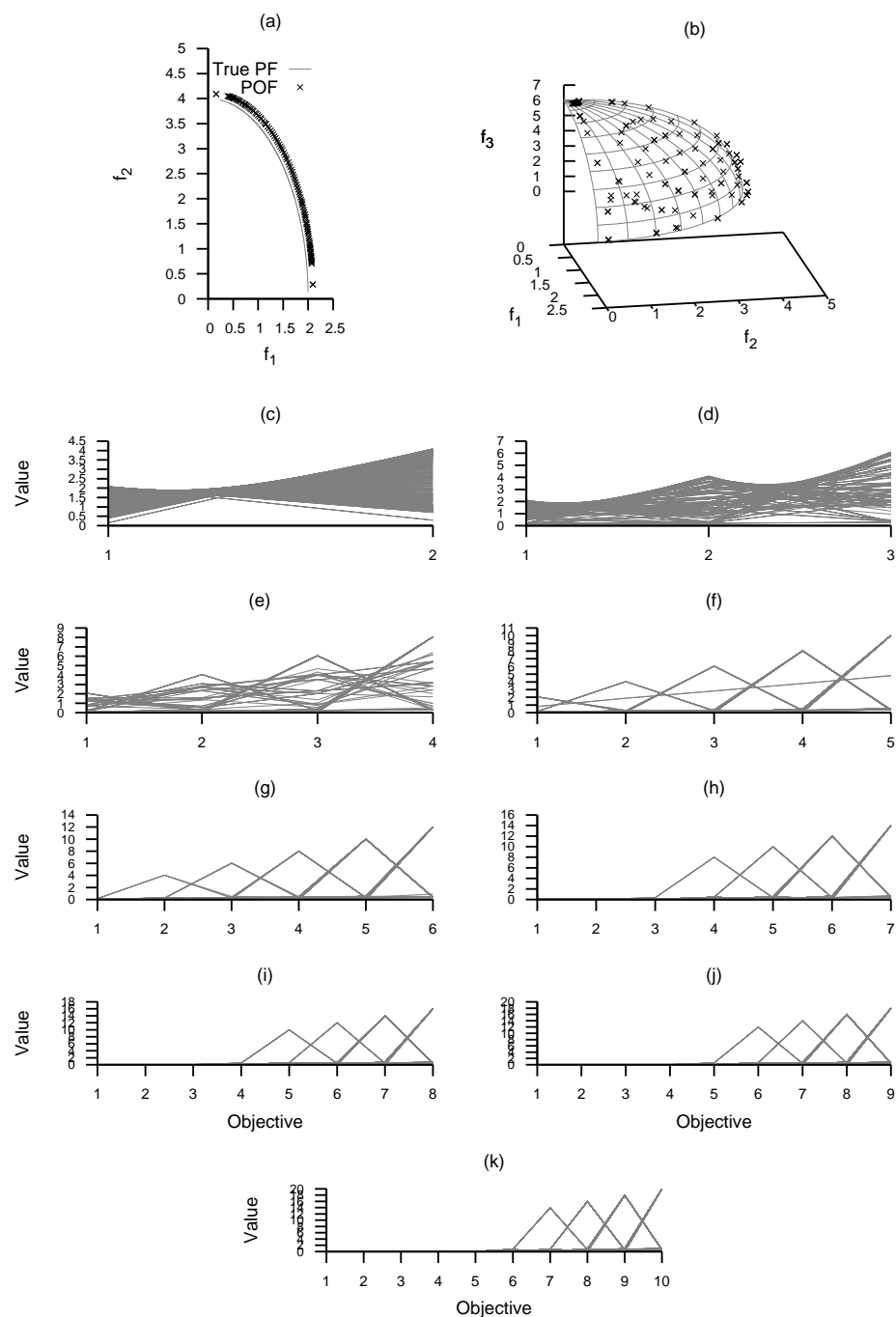


Figure A.414: Plots of the approximations obtained by MOMBI-NATCH from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

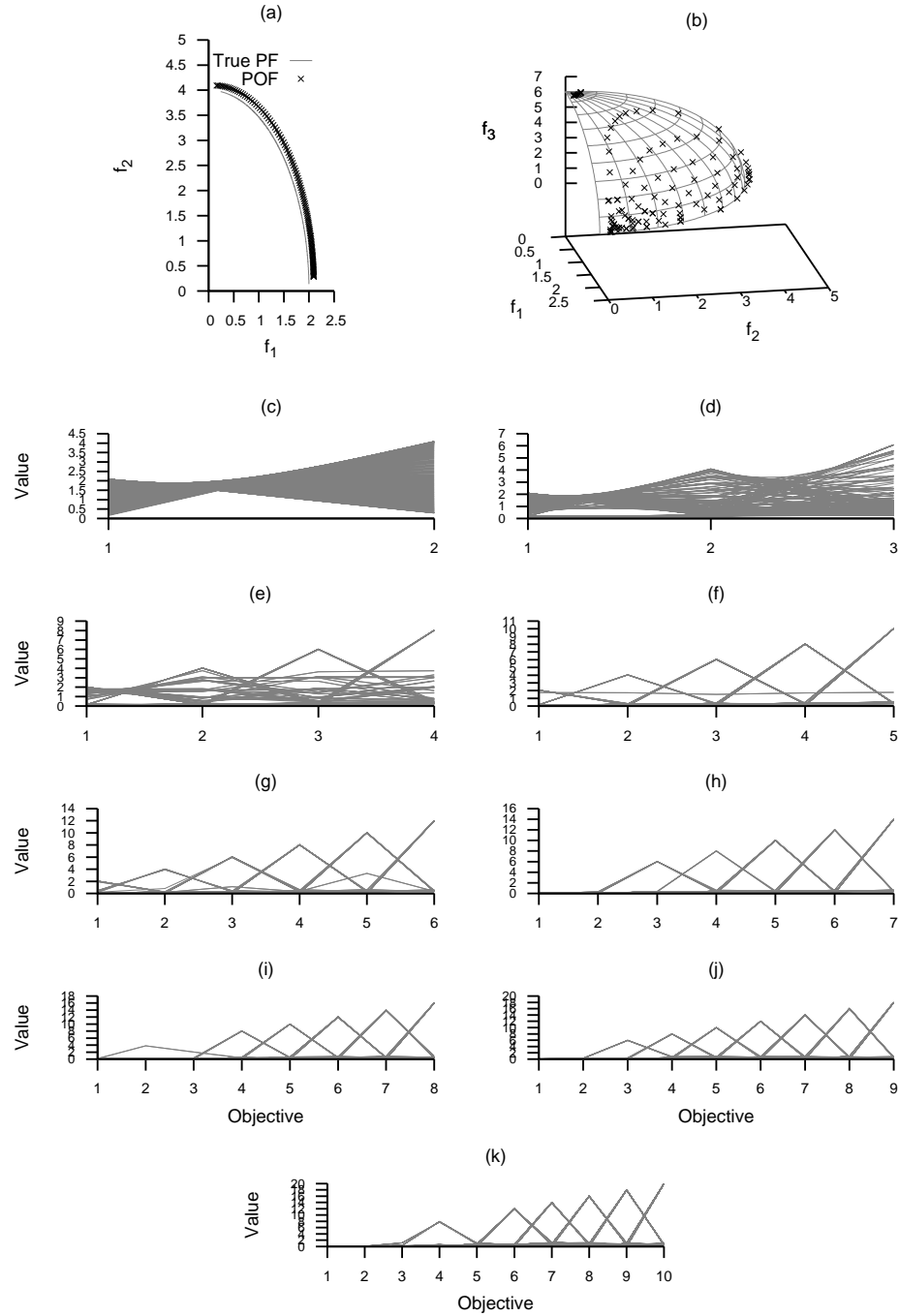


Figure A.415: Plots of the approximations obtained by MOMBI-MTCH from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

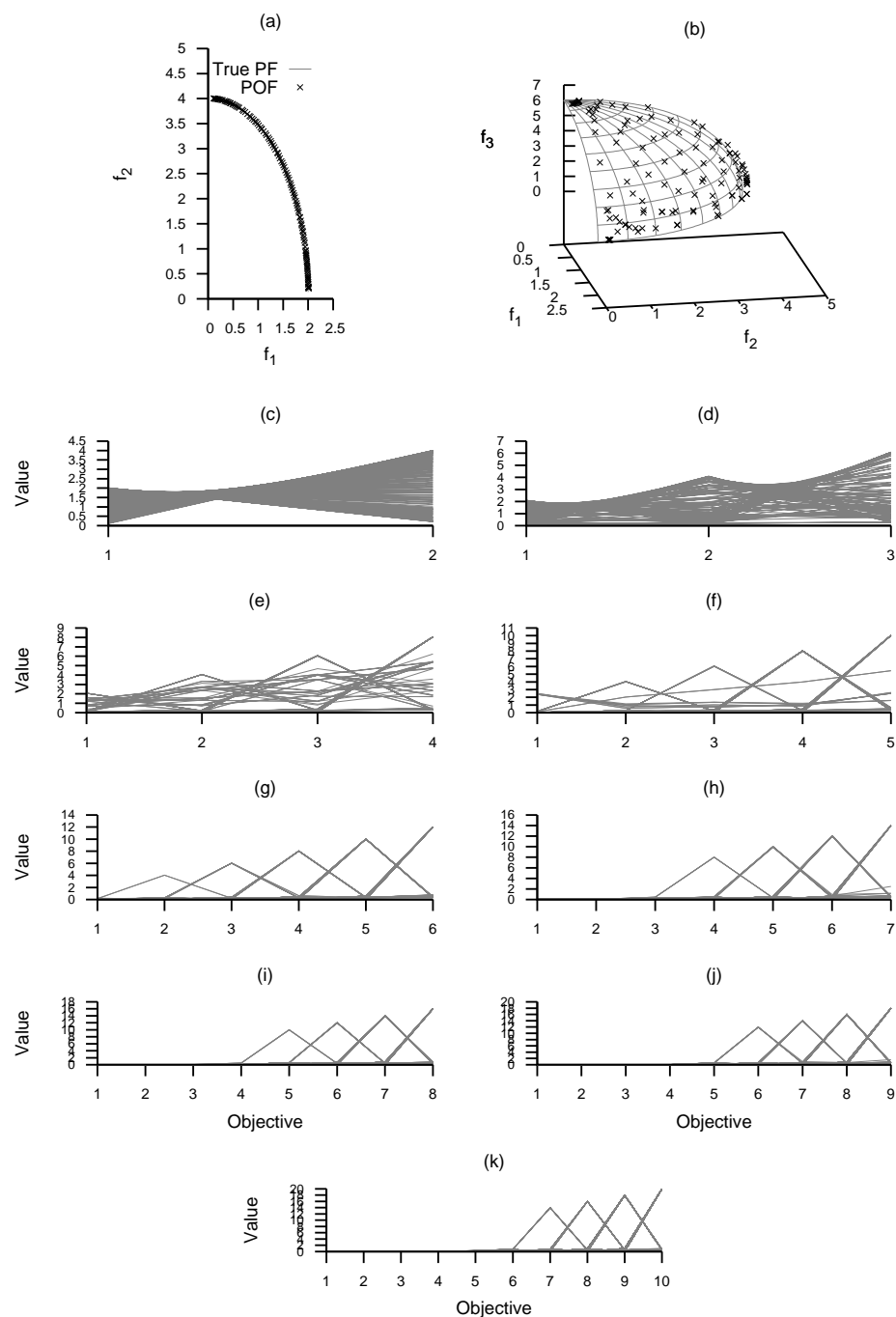


Figure A.416: Plots of the approximations obtained by MOMBI-NMTCH from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



Table A.178: Comparison of hypervolume indicator values for different optimizers on the DTLZ1 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	$7.941975e-01$	$8.736560e-01$	$8.729893e-01$	$8.639786e-01$
	avg.	$7.970731e-01$	$8.736123e-01$	$8.729844e-01$	$8.630434e-01$
	min.	$7.064293e-01$	$8.730033e-01$	$8.713337e-01$	$8.466370e-01$
	max.	$8.667165e-01$	$8.739318e-01$	$8.738589e-01$	$8.726288e-01$
	std.	$3.975122e-02$	$2.102350e-04$	$5.084015e-04$	$5.556512e-03$
<b>3D</b>	med.	$9.549635e-01$	$9.742762e-01$	$9.737024e-01$	$9.581150e-01$
	avg.	$9.545705e-01$	$9.742244e-01$	$9.736727e-01$	$9.581217e-01$
	min.	$9.454199e-01$	$9.736574e-01$	$9.725216e-01$	$9.521109e-01$
	max.	$9.621652e-01$	$9.744316e-01$	$9.741196e-01$	$9.626495e-01$
	std.	$3.421354e-03$	$1.694123e-04$	$2.604854e-04$	$2.289158e-03$
<b>4D</b>	med.	$9.152828e-01$	$9.943698e-01$	$9.925501e-01$	$9.727775e-01$
	avg.	$9.174246e-01$	$9.943577e-01$	$9.925385e-01$	$9.721083e-01$
	min.	$8.872999e-01$	$9.939941e-01$	$9.915878e-01$	$9.589191e-01$
	max.	$9.512724e-01$	$9.944724e-01$	$9.930727e-01$	$9.777571e-01$
	std.	$1.364201e-02$	$8.638177e-05$	$2.771975e-04$	$3.275618e-03$
<b>5D</b>	med.	$9.063328e-01$	$9.986248e-01$	$9.962342e-01$	$9.683278e-01$
	avg.	$9.090906e-01$	$9.986165e-01$	$9.961721e-01$	$9.672337e-01$
	min.	$8.800625e-01$	$9.984848e-01$	$9.949164e-01$	$9.451148e-01$
	max.	$9.480697e-01$	$9.986870e-01$	$9.969650e-01$	$9.774994e-01$
	std.	$1.386315e-02$	$3.848923e-05$	$4.314809e-04$	$5.669479e-03$
<b>6D</b>	med.	$8.131476e-01$	$9.995878e-01$	$9.901449e-01$	$9.684925e-01$
	avg.	$8.178699e-01$	$9.995849e-01$	$9.899192e-01$	$9.648366e-01$
	min.	$7.426839e-01$	$9.995116e-01$	$9.854869e-01$	$9.375843e-01$
	max.	$9.240026e-01$	$9.996239e-01$	$9.929567e-01$	$9.790311e-01$
	std.	$4.188738e-02$	$2.518113e-05$	$1.751526e-03$	$1.080428e-02$
<b>7D</b>	med.	$7.669481e-01$	$9.997988e-01$	$9.810383e-01$	$9.636069e-01$
	avg.	$7.657894e-01$	$9.997942e-01$	$9.808369e-01$	$9.608087e-01$
	min.	$6.960850e-01$	$9.996799e-01$	$9.694402e-01$	$9.066435e-01$
	max.	$8.417527e-01$	$9.998433e-01$	$9.869573e-01$	$9.740295e-01$
	std.	$3.239629e-02$	$2.914848e-05$	$3.273768e-03$	$9.502783e-03$
<b>8D</b>	med.	$8.297691e-01$	$9.998393e-01$	—	$9.604908e-01$
	avg.	$8.331822e-01$	$9.998380e-01$	—	$9.558016e-01$
	min.	$7.687257e-01$	$9.996419e-01$	—	$9.140396e-01$
	max.	$9.088854e-01$	$9.999259e-01$	—	$9.790480e-01$
	std.	$3.006733e-02$	$4.921365e-05$	—	$1.502431e-02$
<b>9D</b>	med.	$8.005794e-01$	$9.997084e-01$	—	$9.640018e-01$
	avg.	$7.984545e-01$	$9.996705e-01$	—	$9.561641e-01$
	min.	$6.998545e-01$	$9.990596e-01$	—	$9.191141e-01$
	max.	$8.772461e-01$	$9.999133e-01$	—	$9.777108e-01$
	std.	$3.822258e-02$	$1.627796e-04$	—	$1.722135e-02$
<b>10D</b>	med.	$8.250815e-01$	$9.993257e-01$	—	$9.671174e-01$
	avg.	$8.237003e-01$	$9.992019e-01$	—	$9.584804e-01$
	min.	$7.604792e-01$	$9.970679e-01$	—	$9.098820e-01$
	max.	$8.996962e-01$	$9.997918e-01$	—	$9.815430e-01$
	std.	$2.434437e-02$	$4.533906e-04$	—	$1.704458e-02$

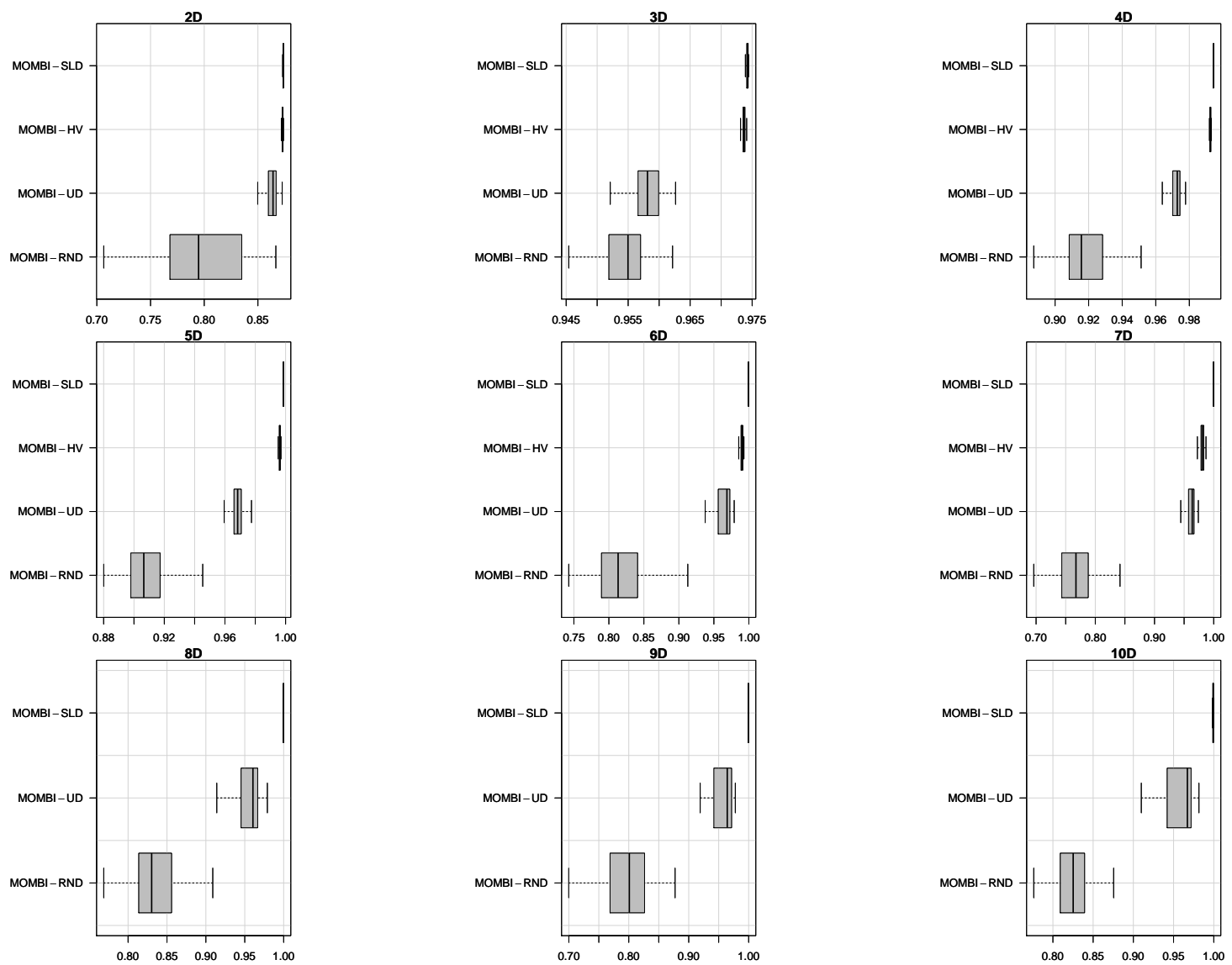


Figure A.417: Box-plot of the hypervolume indicator values for different optimizers on the DTLZ1 test problem.

Table A.179: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ1 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $ph$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
2D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e - 34$	—	$5.33e - 20$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	$> 0.05$	—	$4.50e - 34$
MOMBI-UD	$1.77e - 30$	$> 0.05$	$> 0.05$	—
3D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e - 34$	—	$6.43e - 31$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	$> 0.05$	—	$1.28e - 34$
MOMBI-UD	$8.82e - 14$	$> 0.05$	$> 0.05$	—
4D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e - 34$	—	$1.28e - 34$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	$> 0.05$	—	$1.28e - 34$
MOMBI-UD	$1.28e - 34$	$> 0.05$	$> 0.05$	—
5D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e - 34$	—	$1.28e - 34$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	$> 0.05$	—	$1.28e - 34$
MOMBI-UD	$1.36e - 34$	$> 0.05$	$> 0.05$	—
6D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e - 34$	—	$1.28e - 34$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	$> 0.05$	—	$1.28e - 34$
MOMBI-UD	$1.28e - 34$	$> 0.05$	$> 0.05$	—
7D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e - 34$	—	$1.28e - 34$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	$> 0.05$	—	$1.78e - 34$
MOMBI-UD	$1.28e - 34$	$> 0.05$	$> 0.05$	—
8D				
MOMBI-RND	—	$> 0.05$		$> 0.05$
MOMBI-SLD	$1.28e - 34$	—		$1.28e - 34$
MOMBI-HV			—	
MOMBI-UD	$1.28e - 34$	$> 0.05$		—
9D				
MOMBI-RND	—	$> 0.05$		$> 0.05$
MOMBI-SLD	$1.28e - 34$	—		$1.28e - 34$
MOMBI-HV			—	
MOMBI-UD	$1.28e - 34$	$> 0.05$		—
10D				
MOMBI-RND	—	$> 0.05$		$> 0.05$
MOMBI-SLD	$1.28e - 34$	—		$1.28e - 34$
MOMBI-HV			—	
MOMBI-UD	$1.28e - 34$	$> 0.05$		—

Table A.180: Comparison of R2 indicator values for different optimizers on the DTLZ1 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	1.010180e-01	8.373204e-02	8.373791e-02	8.405096e-02
	avg.	1.026478e-01	8.375566e-02	8.375806e-02	8.414599e-02
	min.	8.430195e-02	8.367867e-02	8.368256e-02	8.373661e-02
	max.	1.414499e-01	8.396698e-02	8.410722e-02	8.538105e-02
	std.	1.415208e-02	6.765192e-05	6.708311e-05	3.512658e-04
<b>3D</b>	med.	3.404730e-02	3.228705e-02	3.230546e-02	3.322912e-02
	avg.	3.406548e-02	3.230978e-02	3.232711e-02	3.325584e-02
	min.	3.375427e-02	3.222558e-02	3.223226e-02	3.306870e-02
	max.	3.450511e-02	3.254316e-02	3.276341e-02	3.366364e-02
	std.	1.509706e-04	7.065382e-05	8.428664e-05	1.165907e-04
<b>4D</b>	med.	2.593078e-02	1.813452e-02	1.813519e-02	1.925884e-02
	avg.	2.583096e-02	1.814745e-02	1.813991e-02	1.927842e-02
	min.	2.221402e-02	1.806313e-02	1.802870e-02	1.908301e-02
	max.	3.023398e-02	1.844347e-02	1.834275e-02	1.969619e-02
	std.	1.647730e-03	6.658049e-05	6.919026e-05	1.153949e-04
<b>5D</b>	med.	2.287347e-02	1.220317e-02	1.227872e-02	1.492158e-02
	avg.	2.275988e-02	1.222377e-02	1.230064e-02	1.497886e-02
	min.	1.915115e-02	1.216001e-02	1.218379e-02	1.472842e-02
	max.	2.651347e-02	1.244737e-02	1.249573e-02	1.615752e-02
	std.	1.494654e-03	5.529191e-05	7.281496e-05	2.374915e-04
<b>6D</b>	med.	3.084655e-02	8.633714e-03	9.616852e-03	1.141616e-02
	avg.	3.066830e-02	8.646896e-03	9.627725e-03	1.149576e-02
	min.	1.837616e-02	8.590536e-03	9.470320e-03	1.104735e-02
	max.	4.216396e-02	8.801789e-03	9.860168e-03	1.238694e-02
	std.	5.821572e-03	4.160145e-05	7.549020e-05	3.447134e-04
<b>7D</b>	med.	3.258535e-02	6.837556e-03	8.816618e-03	9.447218e-03
	avg.	3.284846e-02	6.846084e-03	8.845151e-03	9.506842e-03
	min.	2.332838e-02	6.805422e-03	8.679844e-03	9.247228e-03
	max.	4.272198e-02	6.983270e-03	1.000814e-02	1.068422e-02
	std.	4.377601e-03	2.872386e-05	1.469833e-04	2.257313e-04
<b>8D</b>	med.	2.165815e-02	5.222890e-03	—	7.736443e-03
	avg.	2.164487e-02	5.230699e-03	—	7.798986e-03
	min.	1.684277e-02	5.163810e-03	—	7.379155e-03
	max.	2.834540e-02	5.307099e-03	—	8.684587e-03
	std.	2.735941e-03	3.020193e-05	—	3.169051e-04
<b>9D</b>	med.	2.204794e-02	4.043053e-03	—	6.491613e-03
	avg.	2.243198e-02	4.055103e-03	—	6.621079e-03
	min.	1.572103e-02	3.977924e-03	—	6.163899e-03
	max.	3.275930e-02	4.221991e-03	—	7.499337e-03
	std.	3.725480e-03	5.130121e-05	—	3.473754e-04
<b>10D</b>	med.	1.769226e-02	3.291748e-03	—	5.495671e-03
	avg.	1.790186e-02	3.299901e-03	—	5.650498e-03
	min.	1.300887e-02	3.202420e-03	—	5.268867e-03
	max.	2.338112e-02	3.454175e-03	—	6.424647e-03
	std.	1.913300e-03	5.649567e-05	—	2.970884e-04

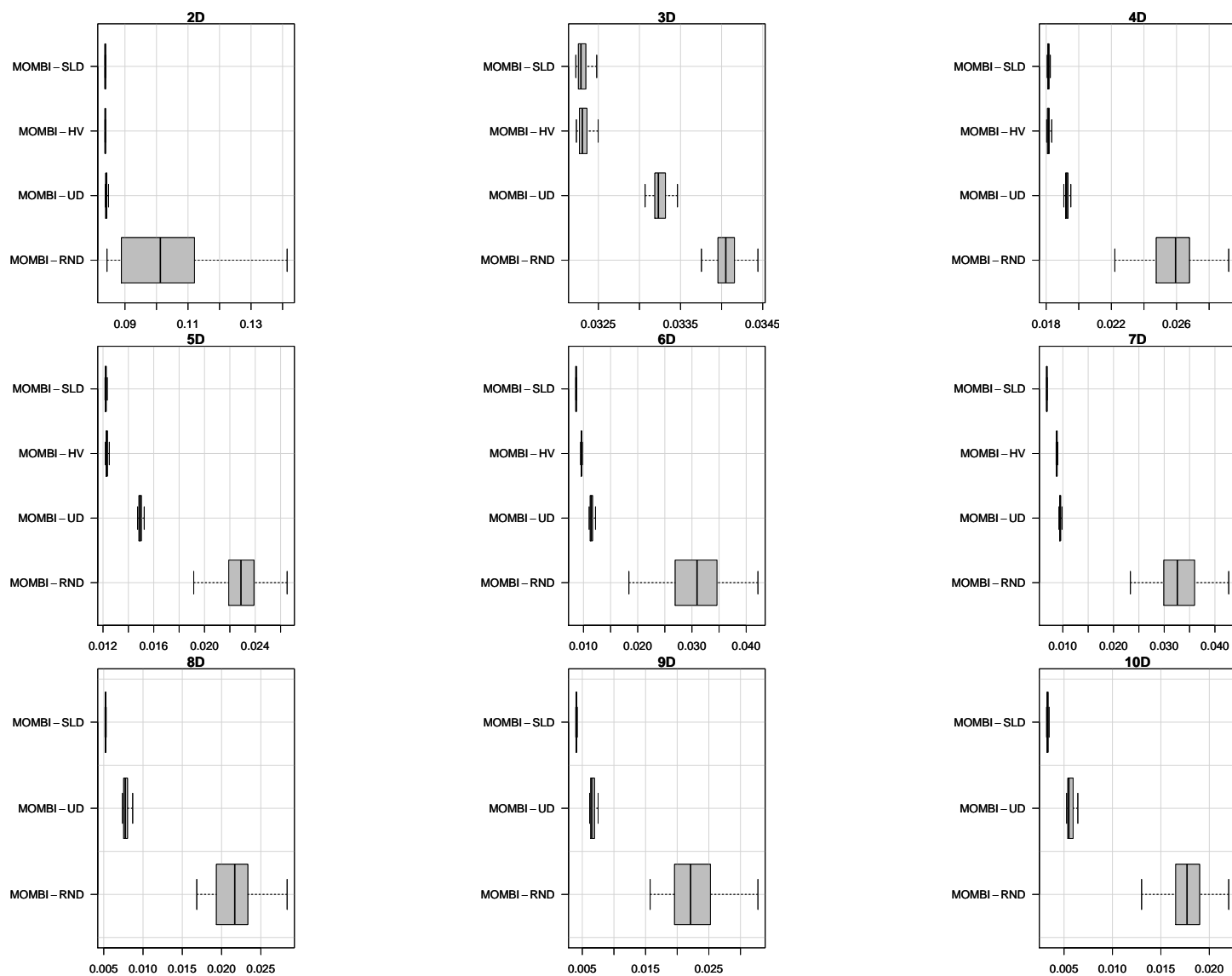


Figure A.418: Box-plot of the R2 indicator values for different optimizers on the DTLZ1 test problem.

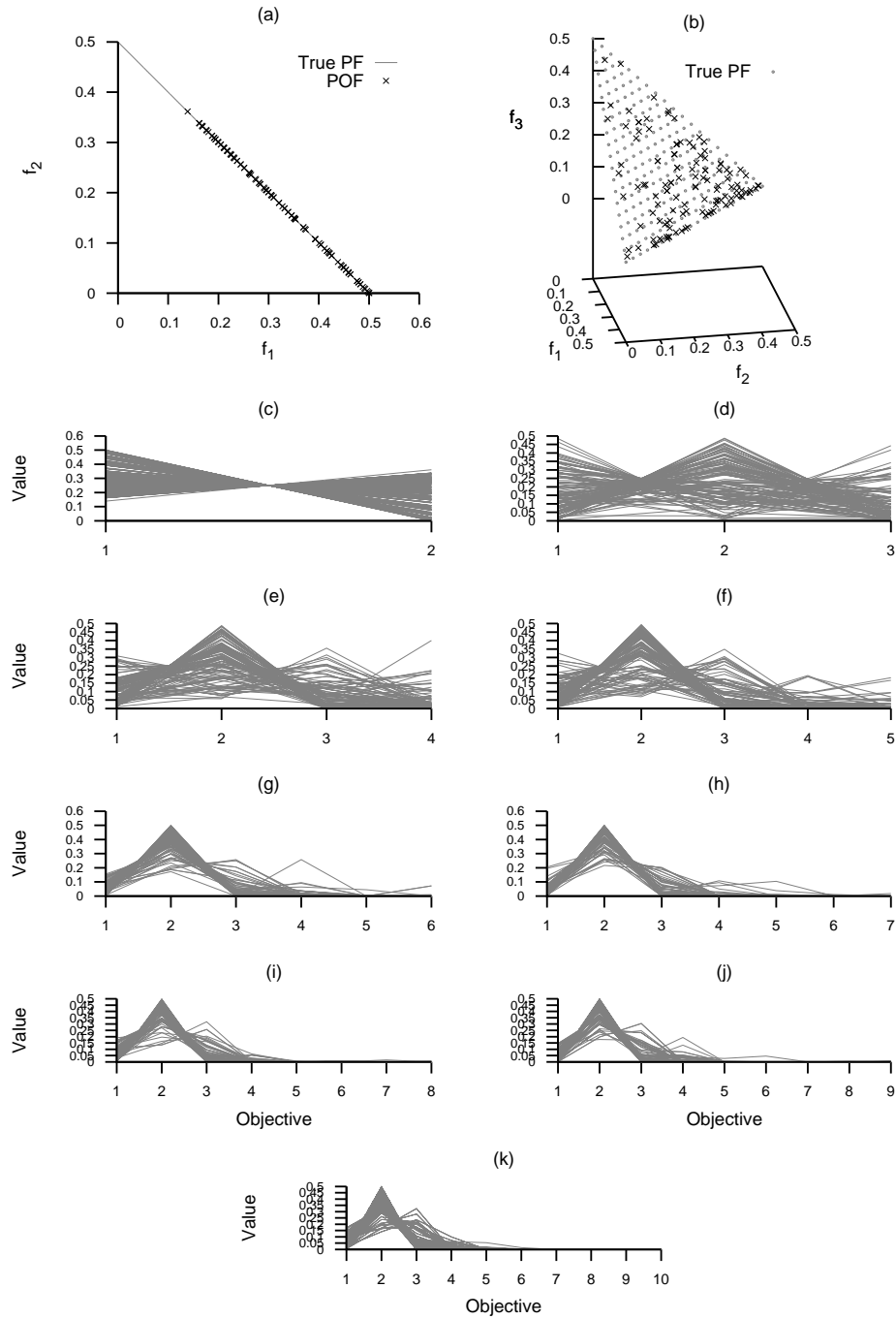


Figure A.419: Plots of the approximations obtained by MOMBI-RND from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

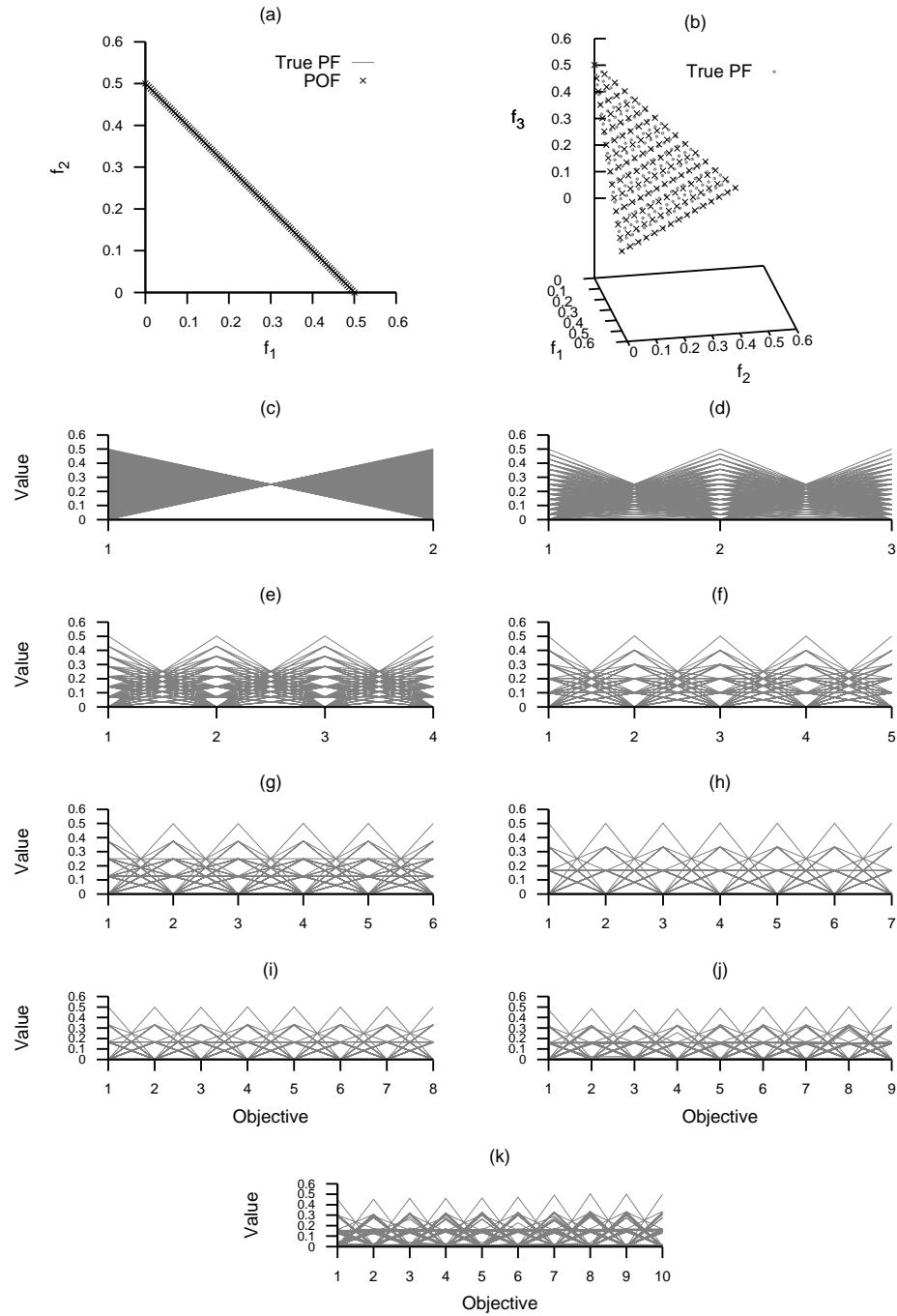


Figure A.420: Plots of the approximations obtained by MOMBI-SLD from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

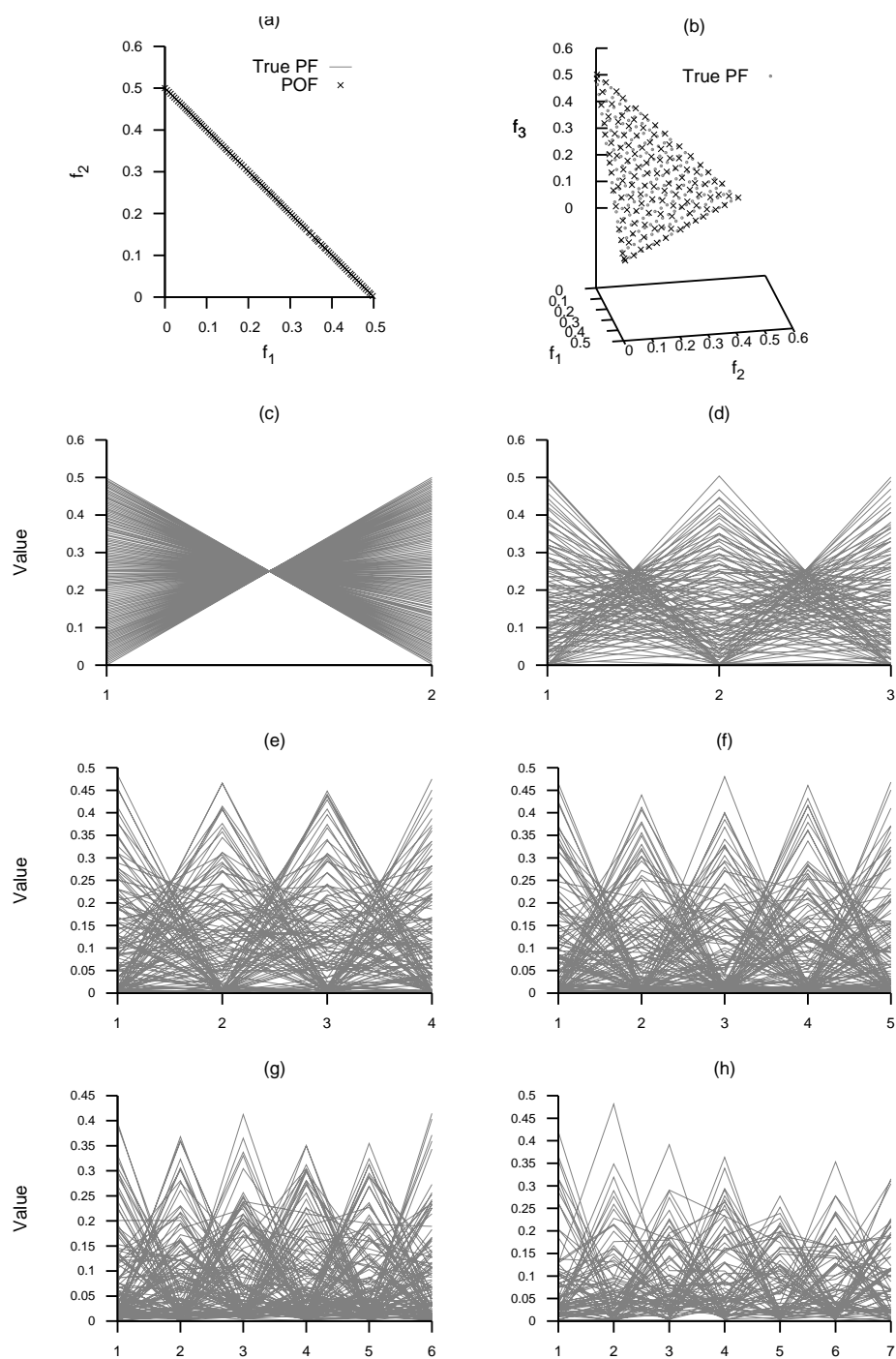


Figure A.421: Plots of the approximations obtained by MOMBI-HV from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



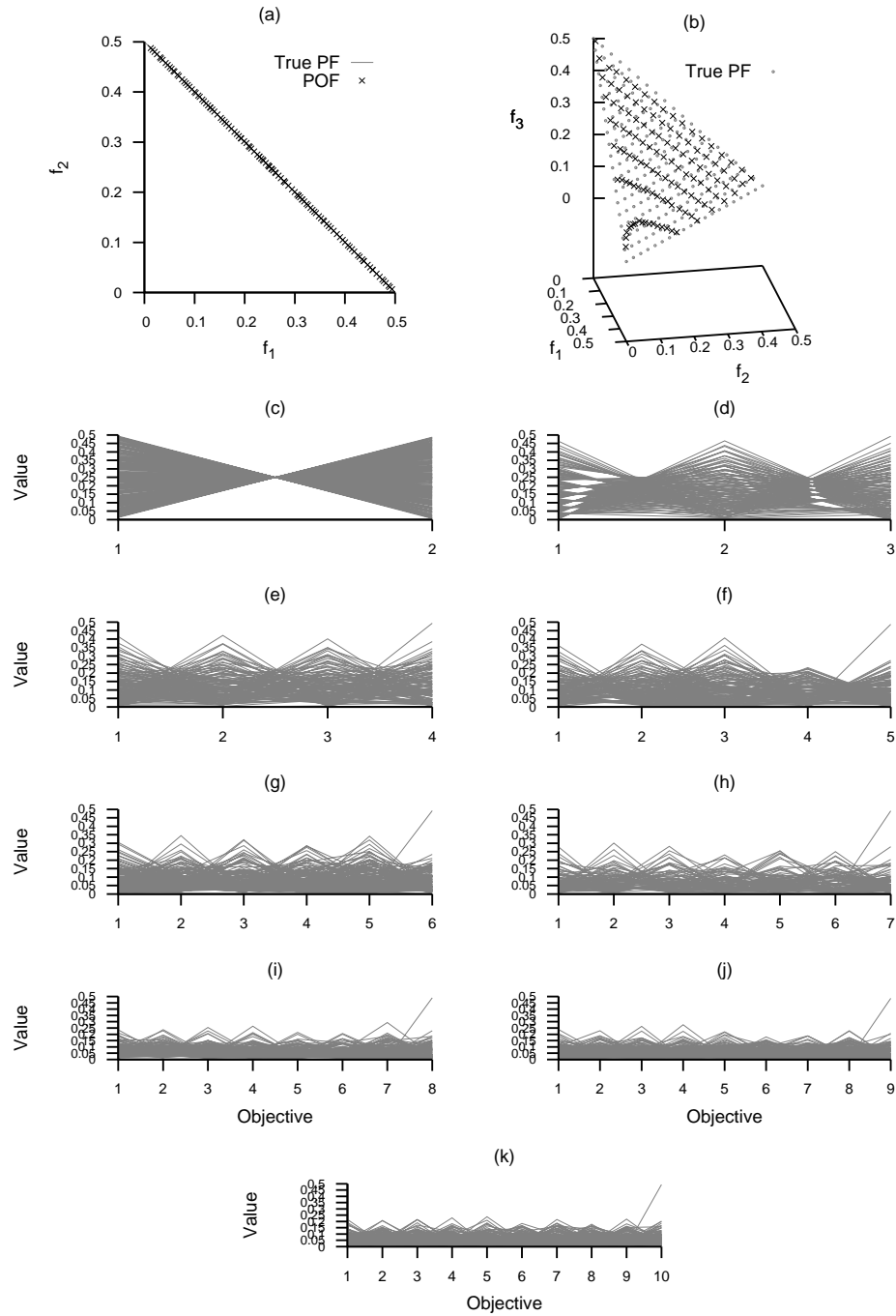


Figure A.422: Plots of the approximations obtained by MOMBI-UD from 2 to 10 objectives on the DTLZ1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.181: Comparison of hypervolume indicator values for different optimizers on the DTLZ2 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	3.106917e + 00	3.210541e + 00	3.208822e + 00	3.184305e + 00
	avg.	3.081609e + 00	3.210516e + 00	3.208646e + 00	3.181731e + 00
	min.	2.704845e + 00	3.210146e + 00	3.205303e + 00	3.135427e + 00
	max.	3.203559e + 00	3.210836e + 00	3.210413e + 00	3.205340e + 00
	std.	1.000512e - 01	1.568672e - 04	1.185428e - 03	1.460879e - 02
<b>3D</b>	med.	7.256699e + 00	7.421721e + 00	7.415775e + 00	7.289428e + 00
	avg.	7.257026e + 00	7.421717e + 00	7.415729e + 00	7.290867e + 00
	min.	7.207545e + 00	7.421361e + 00	7.414192e + 00	7.268878e + 00
	max.	7.298497e + 00	7.422041e + 00	7.417085e + 00	7.321747e + 00
	std.	2.102976e - 02	1.375916e - 04	6.396889e - 04	1.206509e - 02
<b>4D</b>	med.	1.441773e + 01	1.556709e + 01	1.552434e + 01	1.511818e + 01
	avg.	1.442393e + 01	1.556707e + 01	1.552414e + 01	1.511401e + 01
	min.	1.388546e + 01	1.556635e + 01	1.551742e + 01	1.503275e + 01
	max.	1.489787e + 01	1.556743e + 01	1.552871e + 01	1.518033e + 01
	std.	1.887859e - 01	2.065519e - 04	2.022403e - 03	2.816901e - 02
<b>5D</b>	med.	2.761002e + 01	3.166721e + 01	3.156341e + 01	3.023226e + 01
	avg.	2.758599e + 01	3.166718e + 01	3.156372e + 01	3.023941e + 01
	min.	2.623361e + 01	3.166595e + 01	3.154883e + 01	3.010080e + 01
	max.	2.902664e + 01	3.166772e + 01	3.157485e + 01	3.036597e + 01
	std.	6.114549e - 01	3.206453e - 04	4.927639e - 03	5.170721e - 02
<b>6D</b>	med.	4.822637e + 01	6.373818e + 01	6.292404e + 01	6.113891e + 01
	avg.	4.837323e + 01	6.373806e + 01	6.292416e + 01	6.113082e + 01
	min.	4.229493e + 01	6.373655e + 01	6.279572e + 01	6.087702e + 01
	max.	5.418445e + 01	6.373918e + 01	6.300756e + 01	6.136216e + 01
	std.	2.676687e + 00	6.383630e - 04	4.406603e - 02	1.047959e - 01
<b>7D</b>	med.	7.369087e + 01	1.277495e + 02	1.243542e + 02	1.202736e + 02
	avg.	7.331790e + 01	1.277494e + 02	1.243409e + 02	1.202181e + 02
	min.	6.399837e + 01	1.277452e + 02	1.239831e + 02	1.191725e + 02
	max.	9.515689e + 01	1.277524e + 02	1.246175e + 02	1.208634e + 02
	std.	9.594815e + 00	1.488020e - 03	1.332872e - 01	3.333696e - 01
<b>8D</b>	med.	2.029789e + 02	2.558168e + 02	—	2.427635e + 02
	avg.	1.919274e + 02	2.558166e + 02	—	2.427059e + 02
	min.	1.280000e + 02	2.558101e + 02	—	2.412057e + 02
	max.	2.149819e + 02	2.558202e + 02	—	2.441188e + 02
	std.	2.695106e + 01	1.946191e - 03	—	7.250297e - 01
<b>9D</b>	med.	3.163810e + 02	5.118538e + 02	—	4.882282e + 02
	avg.	3.114370e + 02	5.118524e + 02	—	4.881489e + 02
	min.	2.560000e + 02	5.118347e + 02	—	4.847595e + 02
	max.	3.859636e + 02	5.118626e + 02	—	4.912488e + 02
	std.	3.621628e + 01	5.955315e - 03	—	1.310149e + 00
<b>10D</b>	med.	6.894120e + 02	1.023840e + 03	—	9.775475e + 02
	avg.	6.332740e + 02	1.023840e + 03	—	9.773270e + 02
	min.	5.120000e + 02	1.023774e + 03	—	9.661528e + 02
	max.	8.042799e + 02	1.023872e + 03	—	9.845345e + 02
	std.	1.152292e + 02	1.830065e - 02	—	3.206300e + 00

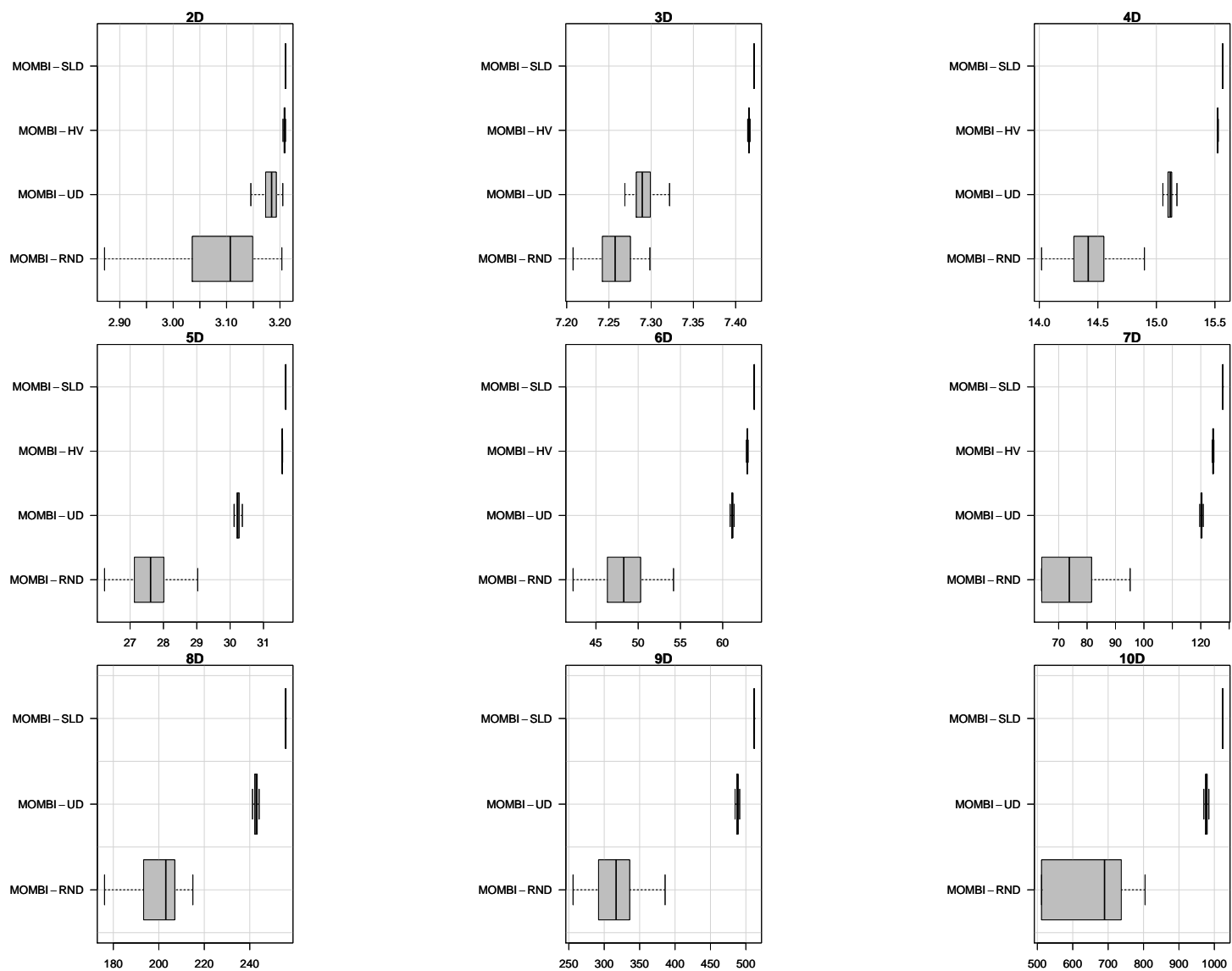


Figure A.423: Box-plot of the hypervolume indicator values for different optimizers on the DTLZ2 test problem.

Table A.182: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ2 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $ph$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
2D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e - 34$	–	$5.38e - 33$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.32e - 34$
MOMBI-UD	$3.68e - 22$	> 0.05	> 0.05	–
3D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e - 34$	–	$1.28e - 34$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.28e - 34$
MOMBI-UD	$9.92e - 25$	> 0.05	> 0.05	–
4D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.27e - 34$	–	$1.27e - 34$	$1.27e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.28e - 34$
MOMBI-UD	$1.28e - 34$	> 0.05	> 0.05	–
5D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e - 34$	–	$1.28e - 34$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.28e - 34$
MOMBI-UD	$1.28e - 34$	> 0.05	> 0.05	–
6D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e - 34$	–	$1.28e - 34$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.28e - 34$
MOMBI-UD	$1.28e - 34$	> 0.05	> 0.05	–
7D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.27e - 34$	–	$1.27e - 34$	$1.27e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.28e - 34$
MOMBI-UD	$1.28e - 34$	> 0.05	> 0.05	–
8D				
MOMBI-RND	–	> 0.05		> 0.05
MOMBI-SLD	$1.24e - 34$	–		$1.27e - 34$
MOMBI-HV			–	
MOMBI-UD	$1.25e - 34$	> 0.05		–
9D				
MOMBI-RND	–	> 0.05		> 0.05
MOMBI-SLD	$1.23e - 34$	–		$1.28e - 34$
MOMBI-HV			–	
MOMBI-UD	$1.23e - 34$	> 0.05		–
10D				
MOMBI-RND	–	> 0.05		> 0.05
MOMBI-SLD	$5.05e - 35$	–		$1.27e - 34$
MOMBI-HV			–	
MOMBI-UD	$5.08e - 35$	> 0.05		–

Table A.183: Comparison of R2 indicator values for different optimizers on the DTLZ2 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	2.233908e-01	2.180290e-01	2.180660e-01	2.186092e-01
	avg.	2.288037e-01	2.180293e-01	2.180713e-01	2.186863e-01
	min.	2.190060e-01	2.180262e-01	2.180392e-01	2.183384e-01
	max.	3.001496e-01	2.180386e-01	2.181568e-01	2.198870e-01
	std.	1.464236e-02	1.862875e-06	2.417210e-05	2.883526e-04
<b>3D</b>	med.	9.695817e-02	9.375511e-02	9.389725e-02	9.592836e-02
	avg.	9.697053e-02	9.375530e-02	9.389736e-02	9.594980e-02
	min.	9.644392e-02	9.374857e-02	9.388826e-02	9.568426e-02
	max.	9.770602e-02	9.376382e-02	9.390788e-02	9.635178e-02
	std.	2.382538e-04	3.420045e-06	4.175702e-06	1.308499e-04
<b>4D</b>	med.	6.701888e-02	5.308324e-02	5.372960e-02	5.809893e-02
	avg.	6.716436e-02	5.308394e-02	5.372918e-02	5.811004e-02
	min.	6.303777e-02	5.306685e-02	5.371004e-02	5.790773e-02
	max.	7.479549e-02	5.309920e-02	5.374982e-02	5.827752e-02
	std.	2.096796e-03	4.819623e-06	8.189068e-06	7.738825e-05
<b>5D</b>	med.	6.566286e-02	3.525855e-02	3.647213e-02	4.568251e-02
	avg.	6.625231e-02	3.525890e-02	3.647256e-02	4.568592e-02
	min.	5.825550e-02	3.524769e-02	3.643139e-02	4.553473e-02
	max.	7.604490e-02	3.527389e-02	3.653590e-02	4.587366e-02
	std.	3.919686e-03	5.126757e-06	1.964297e-05	6.969103e-05
<b>6D</b>	med.	7.863682e-02	2.432597e-02	2.947065e-02	3.591221e-02
	avg.	7.864516e-02	2.432653e-02	2.947103e-02	3.591386e-02
	min.	5.581171e-02	2.431796e-02	2.936529e-02	3.569577e-02
	max.	1.084465e-01	2.433874e-02	2.957603e-02	3.614205e-02
	std.	1.180518e-02	4.309046e-06	4.219387e-05	8.196271e-05
<b>7D</b>	med.	1.173563e-01	1.840720e-02	2.873940e-02	3.279980e-02
	avg.	1.193631e-01	1.840731e-02	2.874315e-02	3.281481e-02
	min.	6.872779e-02	1.840358e-02	2.855540e-02	3.263599e-02
	max.	1.428467e-01	1.841103e-02	2.891449e-02	3.339810e-02
	std.	2.384881e-02	1.353676e-06	6.850603e-05	1.164278e-04
<b>8D</b>	med.	5.167598e-02	1.401778e-02	—	2.665821e-02
	avg.	6.292849e-02	1.401808e-02	—	2.664958e-02
	min.	4.725899e-02	1.401077e-02	—	2.627136e-02
	max.	1.258298e-01	1.402664e-02	—	2.693673e-02
	std.	2.522221e-02	2.959920e-06	—	1.196373e-04
<b>9D</b>	med.	8.079596e-02	1.103051e-02	—	2.337283e-02
	avg.	8.431605e-02	1.103131e-02	—	2.336677e-02
	min.	5.233502e-02	1.101932e-02	—	2.304160e-02
	max.	1.119269e-01	1.104846e-02	—	2.381227e-02
	std.	1.742434e-02	5.515626e-06	—	1.442366e-04
<b>10D</b>	med.	6.079306e-02	8.880083e-03	—	2.089712e-02
	avg.	7.497897e-02	8.881076e-03	—	2.091650e-02
	min.	4.349609e-02	8.864819e-03	—	2.055747e-02
	max.	1.011008e-01	8.901353e-03	—	2.128932e-02
	std.	2.447303e-02	8.314531e-06	—	1.648323e-04

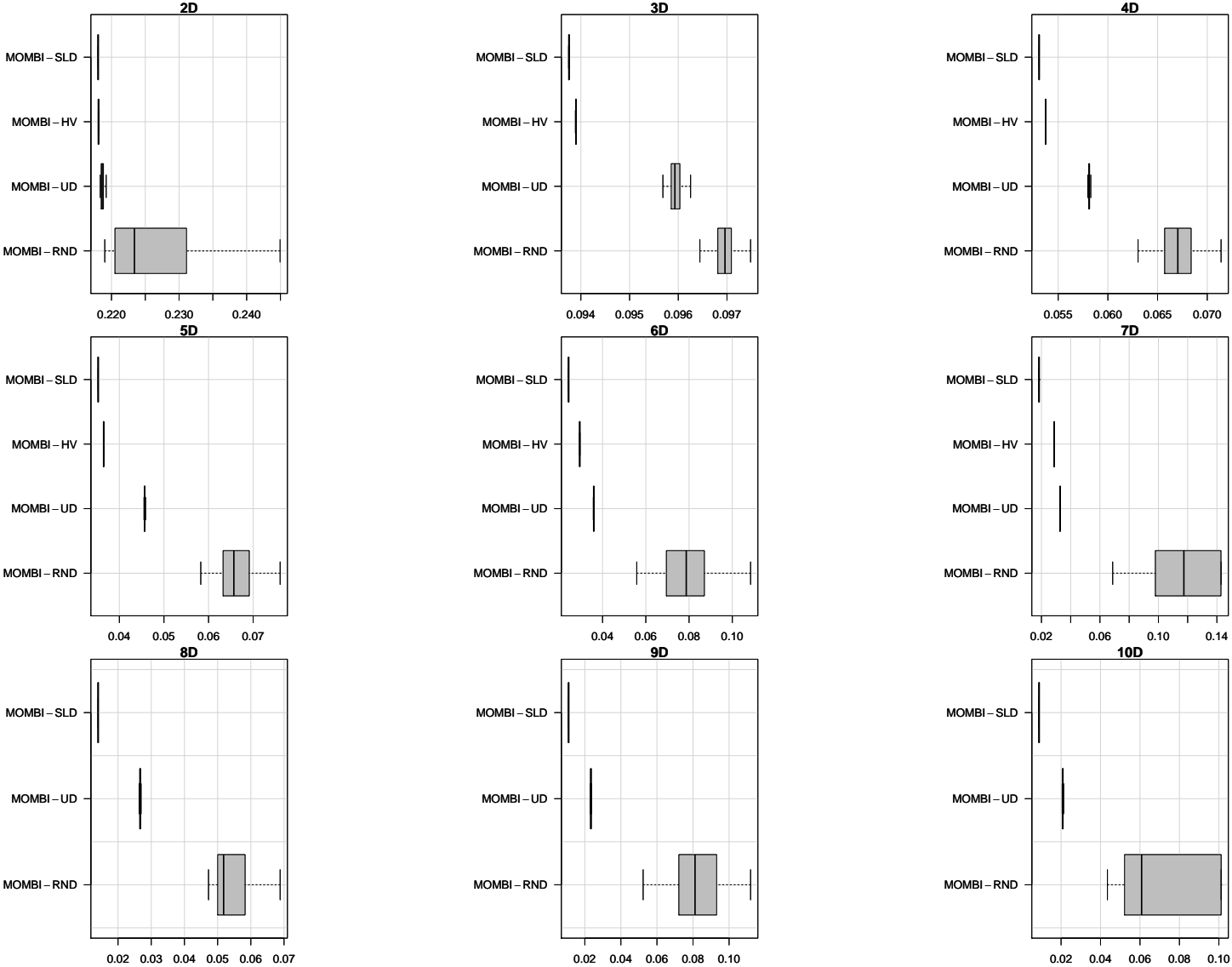


Figure A.424: Box-plot of the R2 indicator values for different optimizers on the DTLZ2 test problem.

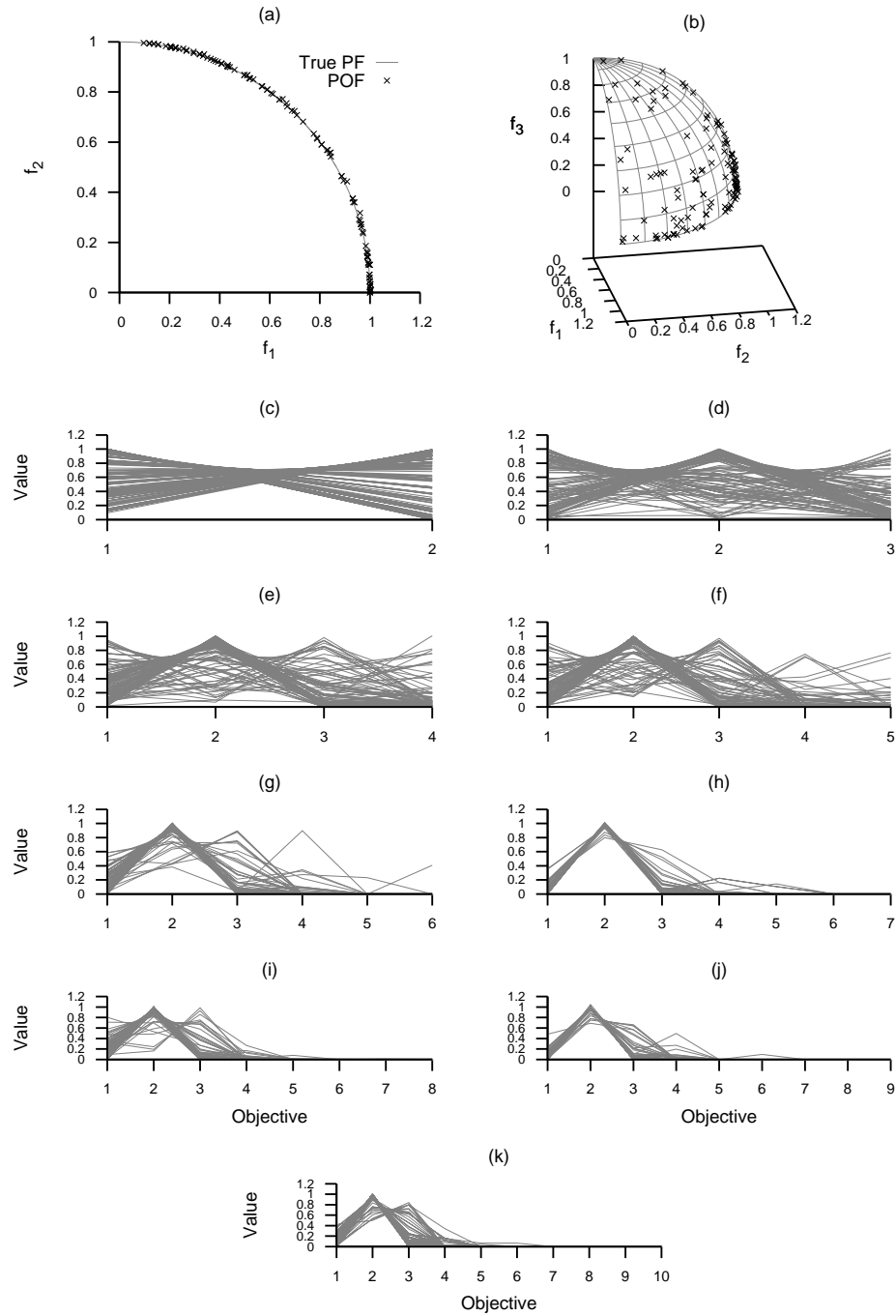


Figure A.425: Plots of the approximations obtained by MOMBI-RND from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

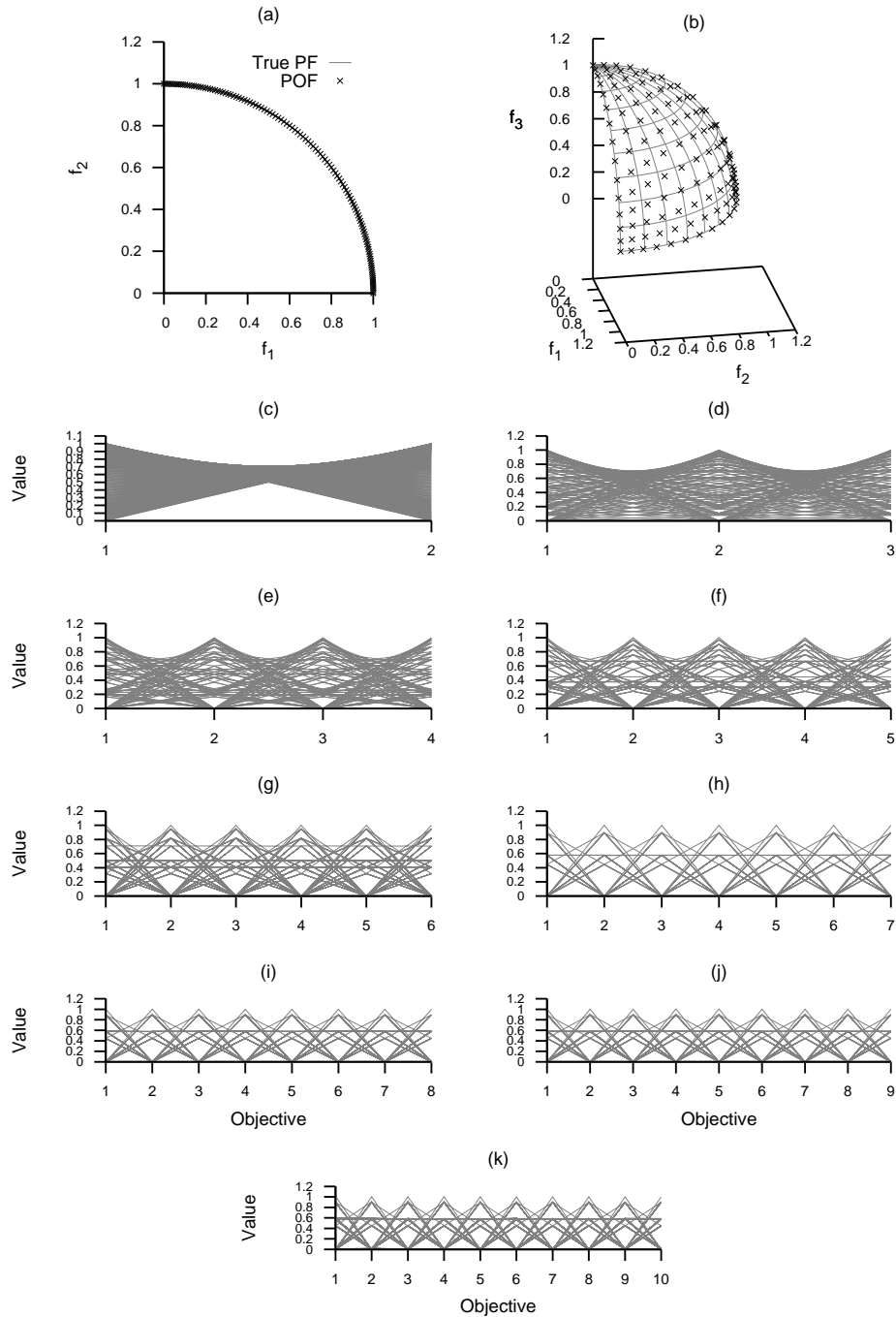


Figure A.426: Plots of the approximations obtained by MOMBI-SLD from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



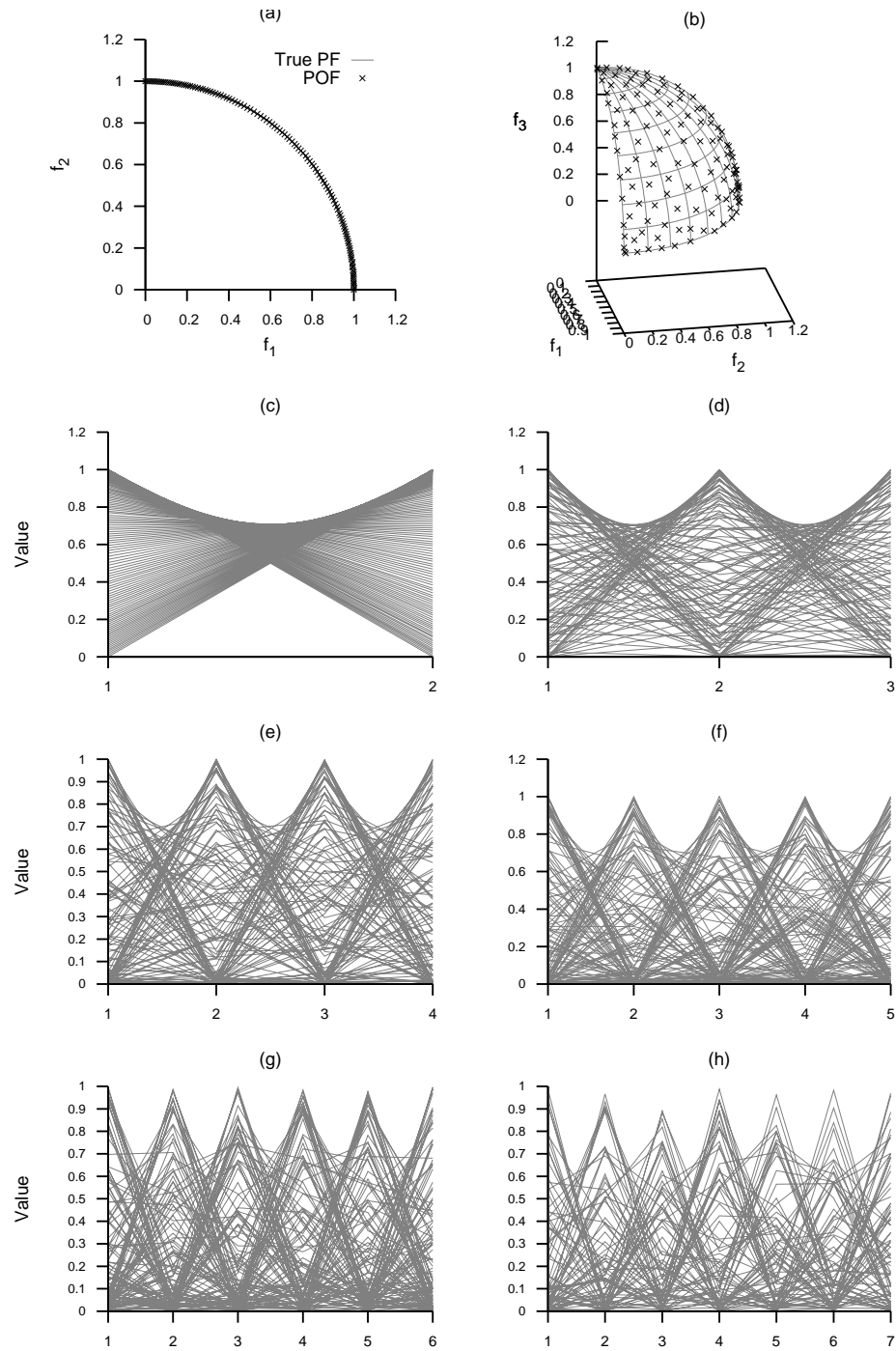


Figure A.427: Plots of the approximations obtained by MOMBI-HV from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

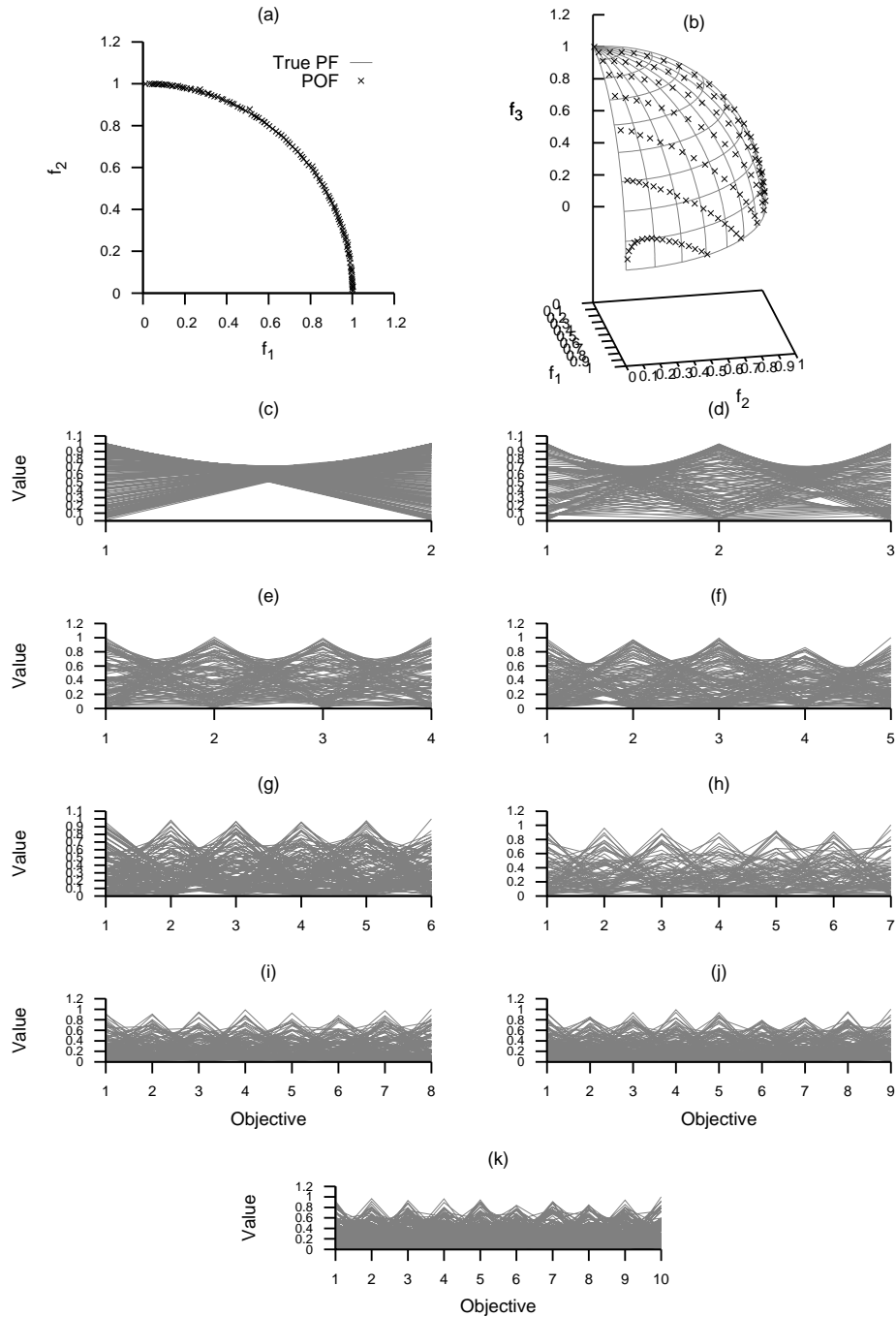


Figure A.428: Plots of the approximations obtained by MOMBI-UD from 2 to 10 objectives on the DTLZ2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.184: Comparison of hypervolume indicator values for different optimizers on the DTLZ3 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	4.200339e + 01	4.819584e + 01	4.818721e + 01	4.803198e + 01
	avg.	4.350326e + 01	4.819398e + 01	4.816212e + 01	4.799113e + 01
	min.	4.189037e + 01	4.816219e + 01	4.603123e + 01	4.547346e + 01
	max.	4.810353e + 01	4.820783e + 01	4.820319e + 01	4.817745e + 01
	std.	1.959696e + 00	9.266758e - 03	2.157709e - 01	2.691321e - 01
<b>3D</b>	med.	3.408360e + 02	3.423689e + 02	3.423266e + 02	3.412282e + 02
	avg.	3.408157e + 02	3.422074e + 02	3.423161e + 02	3.411700e + 02
	min.	3.396222e + 02	3.267967e + 02	3.421410e + 02	3.394248e + 02
	max.	3.419398e + 02	3.424076e + 02	3.423864e + 02	3.417708e + 02
	std.	4.271214e - 01	1.556943e + 00	4.455164e - 02	3.480678e - 01
<b>4D</b>	med.	2.366473e + 03	2.400368e + 03	2.399558e + 03	2.390500e + 03
	avg.	2.366046e + 03	2.399890e + 03	2.399290e + 03	2.388178e + 03
	min.	2.327716e + 03	2.390430e + 03	2.391747e + 03	2.248838e + 03
	max.	2.389676e + 03	2.400534e + 03	2.400050e + 03	2.396720e + 03
	std.	1.170514e + 01	1.811712e + 00	1.248270e + 00	1.482165e + 01
<b>5D</b>	med.	1.630784e + 04	1.680568e + 04	1.679981e + 04	1.672813e + 04
	avg.	1.629580e + 04	1.679563e + 04	1.674721e + 04	1.671749e + 04
	min.	1.440334e + 04	1.636562e + 04	1.176000e + 04	1.654291e + 04
	max.	1.668375e + 04	1.680651e + 04	1.680398e + 04	1.676883e + 04
	std.	2.527718e + 02	5.486492e + 01	5.038764e + 02	3.949392e + 01
<b>6D</b>	med.	1.089347e + 05	1.176446e + 05	1.174503e + 05	1.170864e + 05
	avg.	1.070208e + 05	1.176153e + 05	1.173415e + 05	1.168824e + 05
	min.	1.004815e + 05	1.157134e + 05	1.137850e + 05	1.139004e + 05
	max.	1.151672e + 05	1.176480e + 05	1.175476e + 05	1.173672e + 05
	std.	5.507946e + 03	1.948678e + 02	4.556406e + 02	5.950967e + 02
<b>7D</b>	med.	7.054786e + 05	8.235322e + 05	8.202260e + 05	8.168468e + 05
	avg.	7.082958e + 05	8.234707e + 05	8.177224e + 05	8.067111e + 05
	min.	7.019144e + 05	8.200723e + 05	7.291345e + 05	4.479151e + 05
	max.	7.872985e + 05	8.235409e + 05	8.218622e + 05	8.201158e + 05
	std.	1.447534e + 04	3.531972e + 02	1.199667e + 04	4.333352e + 04
<b>8D</b>	med.	4.939716e + 06	5.764670e + 06	—	5.729401e + 06
	avg.	5.124978e + 06	5.763876e + 06	—	5.651911e + 06
	min.	4.930544e + 06	5.727432e + 06	—	3.574066e + 06
	max.	5.662810e + 06	5.764786e + 06	—	5.745009e + 06
	std.	2.953902e + 05	4.057216e + 03	—	3.138665e + 05
<b>9D</b>	med.	3.456984e + 07	4.035238e + 07	—	4.012593e + 07
	avg.	3.488265e + 07	4.021284e + 07	—	3.996060e + 07
	min.	3.451191e + 07	3.051585e + 07	—	3.457828e + 07
	max.	3.874183e + 07	4.035332e + 07	—	4.022793e + 07
	std.	9.870304e + 05	1.053015e + 06	—	7.980569e + 05
<b>10D</b>	med.	2.419979e + 08	2.824600e + 08	—	2.806405e + 08
	avg.	2.420530e + 08	2.823987e + 08	—	2.800089e + 08
	min.	2.017159e + 08	2.800137e + 08	—	2.408476e + 08
	max.	2.695452e + 08	2.824737e + 08	—	2.814744e + 08
	std.	5.564176e + 06	2.928386e + 05	—	4.082925e + 06

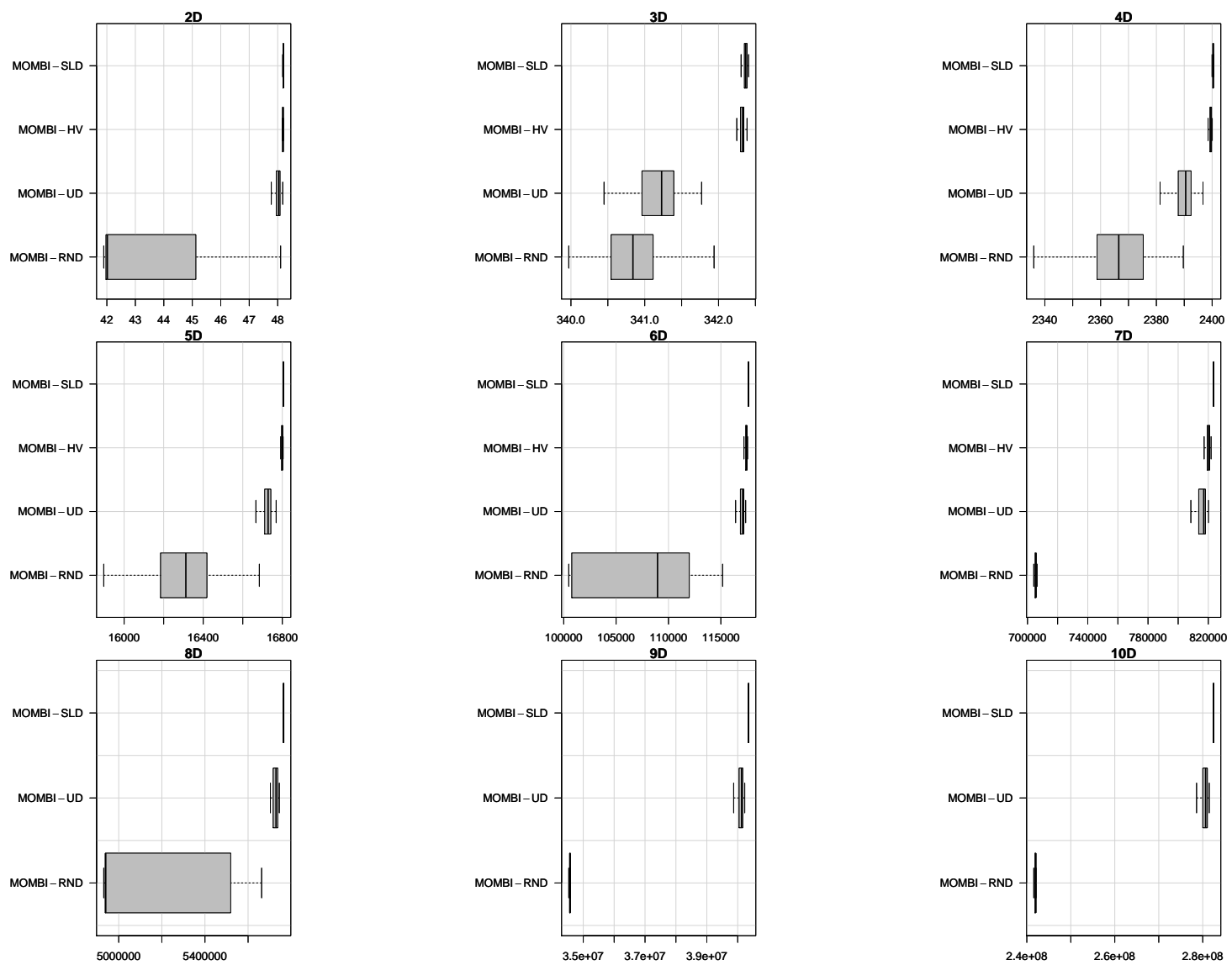


Figure A.429: Box-plot of the hypervolume indicator values for different optimizers on the DTLZ3 test problem.

Table A.185: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ3 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $ph$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
2D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e-34$	—	$8.89e-10$	$1.53e-34$
MOMBI-HV	$2.07e-34$	$> 0.05$	—	$9.80e-33$
MOMBI-UD	$8.98e-33$	$> 0.05$	$> 0.05$	—
3D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$2.51e-33$	—	$6.55e-16$	$2.51e-33$
MOMBI-HV	$1.28e-34$	$> 0.05$	—	$1.28e-34$
MOMBI-UD	$2.10e-10$	$> 0.05$	$> 0.05$	—
4D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e-34$	—	$1.48e-24$	$6.60e-33$
MOMBI-HV	$1.28e-34$	$> 0.05$	—	$6.83e-34$
MOMBI-UD	$5.42e-31$	$> 0.05$	$> 0.05$	—
5D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$5.88e-34$	—	$1.08e-24$	$6.24e-31$
MOMBI-HV	$2.51e-33$	$> 0.05$	—	$2.83e-32$
MOMBI-UD	$5.22e-34$	$> 0.05$	$> 0.05$	—
6D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e-34$	—	$2.31e-32$	$2.16e-33$
MOMBI-HV	$1.73e-34$	$> 0.05$	—	$6.33e-26$
MOMBI-UD	$2.01e-34$	$> 0.05$	$> 0.05$	—
7D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e-34$	—	$6.61e-34$	$1.32e-34$
MOMBI-HV	$1.58e-34$	$> 0.05$	—	$6.49e-22$
MOMBI-UD	$1.12e-29$	$> 0.05$	$> 0.05$	—
8D				
MOMBI-RND	—	$> 0.05$		$> 0.05$
MOMBI-SLD	$1.28e-34$	—		$6.62e-34$
MOMBI-HV			—	
MOMBI-UD	$6.51e-28$	$> 0.05$		—
9D				
MOMBI-RND	—	$> 0.05$		$> 0.05$
MOMBI-SLD	$3.27e-33$	—		$4.37e-32$
MOMBI-HV			—	
MOMBI-UD	$6.43e-34$	$> 0.05$		—
10D				
MOMBI-RND	—	$> 0.05$		$> 0.05$
MOMBI-SLD	$1.28e-34$	—		$2.74e-33$
MOMBI-HV			—	
MOMBI-UD	$2.37e-33$	$> 0.05$		—

Table A.186: Comparison of R2 indicator values for different optimizers on the DTLZ3 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	4.979178e-01	2.194980e-01	2.194797e-01	2.199547e-01
	avg.	4.149231e-01	2.198147e-01	2.218371e-01	2.223276e-01
	min.	2.204181e-01	2.182078e-01	2.183113e-01	2.182585e-01
	max.	5.078285e-01	2.245961e-01	4.203199e-01	4.380664e-01
	std.	1.051167e-01	1.127588e-03	2.008217e-02	2.182118e-02
<b>3D</b>	med.	9.855110e-02	9.563807e-02	9.581096e-02	9.805526e-02
	avg.	9.876663e-02	9.777978e-02	9.605257e-02	9.831564e-02
	min.	9.682925e-02	9.413437e-02	9.426669e-02	9.632500e-02
	max.	1.023623e-01	2.844945e-01	1.012244e-01	1.066219e-01
	std.	1.075774e-03	1.889792e-02	1.316989e-03	1.480840e-03
<b>4D</b>	med.	6.887673e-02	5.492468e-02	5.547992e-02	5.991323e-02
	avg.	7.028952e-02	5.775834e-02	5.631882e-02	6.254119e-02
	min.	6.480152e-02	5.345849e-02	5.408551e-02	5.822580e-02
	max.	1.337865e-01	1.122926e-01	1.051421e-01	1.903369e-01
	std.	8.745485e-03	1.174072e-02	5.194394e-03	1.490152e-02
<b>5D</b>	med.	7.152482e-02	3.677496e-02	3.791708e-02	4.809061e-02
	avg.	7.366824e-02	4.160361e-02	4.098105e-02	4.963687e-02
	min.	6.106187e-02	3.560064e-02	3.673029e-02	4.606071e-02
	max.	1.996832e-01	1.470714e-01	2.438578e-01	9.684344e-02
	std.	1.431860e-02	1.785872e-02	2.144451e-02	7.684741e-03
<b>6D</b>	med.	9.420522e-02	2.536503e-02	3.077522e-02	3.820858e-02
	avg.	1.148125e-01	2.939271e-02	3.391868e-02	4.192455e-02
	min.	5.710481e-02	2.456127e-02	2.923059e-02	3.623087e-02
	max.	1.695328e-01	1.042964e-01	1.130391e-01	1.102816e-01
	std.	4.489919e-02	1.128649e-02	1.218269e-02	1.265956e-02
<b>7D</b>	med.	1.433848e-01	1.908341e-02	3.005055e-02	3.524297e-02
	avg.	1.406102e-01	2.242408e-02	3.510280e-02	4.412404e-02
	min.	6.422181e-02	1.849163e-02	2.859140e-02	3.356245e-02
	max.	1.476589e-01	7.095338e-02	1.378904e-01	2.329096e-01
	std.	1.422537e-02	8.943611e-03	1.794612e-02	3.104595e-02
<b>8D</b>	med.	1.242286e-01	1.472536e-02	—	2.903383e-02
	avg.	1.039998e-01	1.778820e-02	—	3.715061e-02
	min.	4.846014e-02	1.419690e-02	—	2.691483e-02
	max.	1.267898e-01	5.985066e-02	—	1.847591e-01
	std.	3.273114e-02	7.824706e-03	—	2.860500e-02
<b>9D</b>	med.	1.121380e-01	1.189735e-02	—	2.609104e-02
	avg.	1.067668e-01	1.561339e-02	—	2.889332e-02
	min.	4.833015e-02	1.115997e-02	—	2.389473e-02
	max.	1.136333e-01	1.081384e-01	—	1.121312e-01
	std.	1.585956e-02	1.306597e-02	—	1.309039e-02
<b>10D</b>	med.	1.013491e-01	1.022632e-02	—	2.367977e-02
	avg.	1.013287e-01	1.295981e-02	—	2.509057e-02
	min.	4.788426e-02	9.242596e-03	—	2.216763e-02
	max.	2.023322e-01	4.828192e-02	—	1.042926e-01
	std.	1.262428e-02	7.514411e-03	—	8.718618e-03

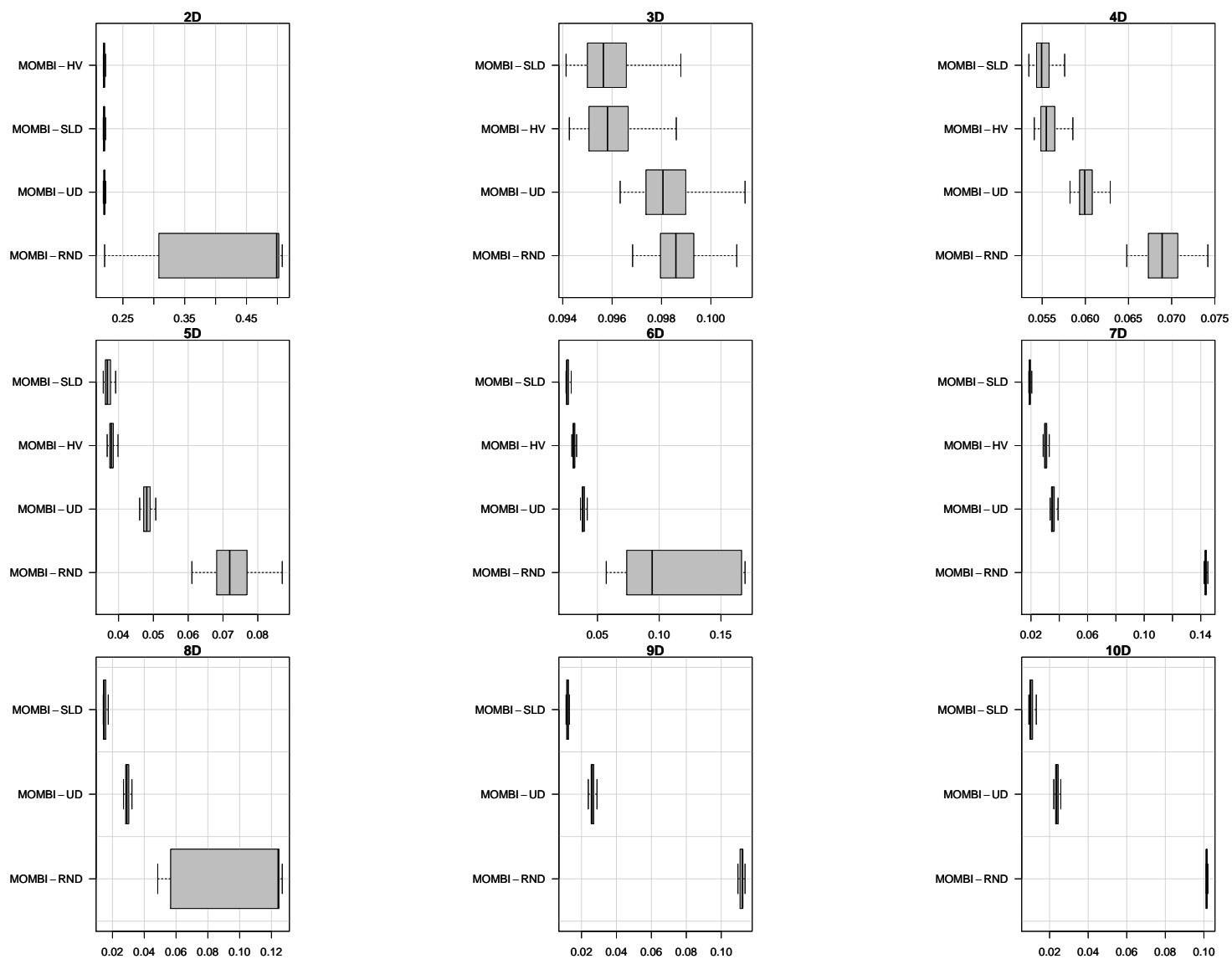


Figure A.430: Box-plot of the R2 indicator values for different optimizers on the DTLZ3 test problem.

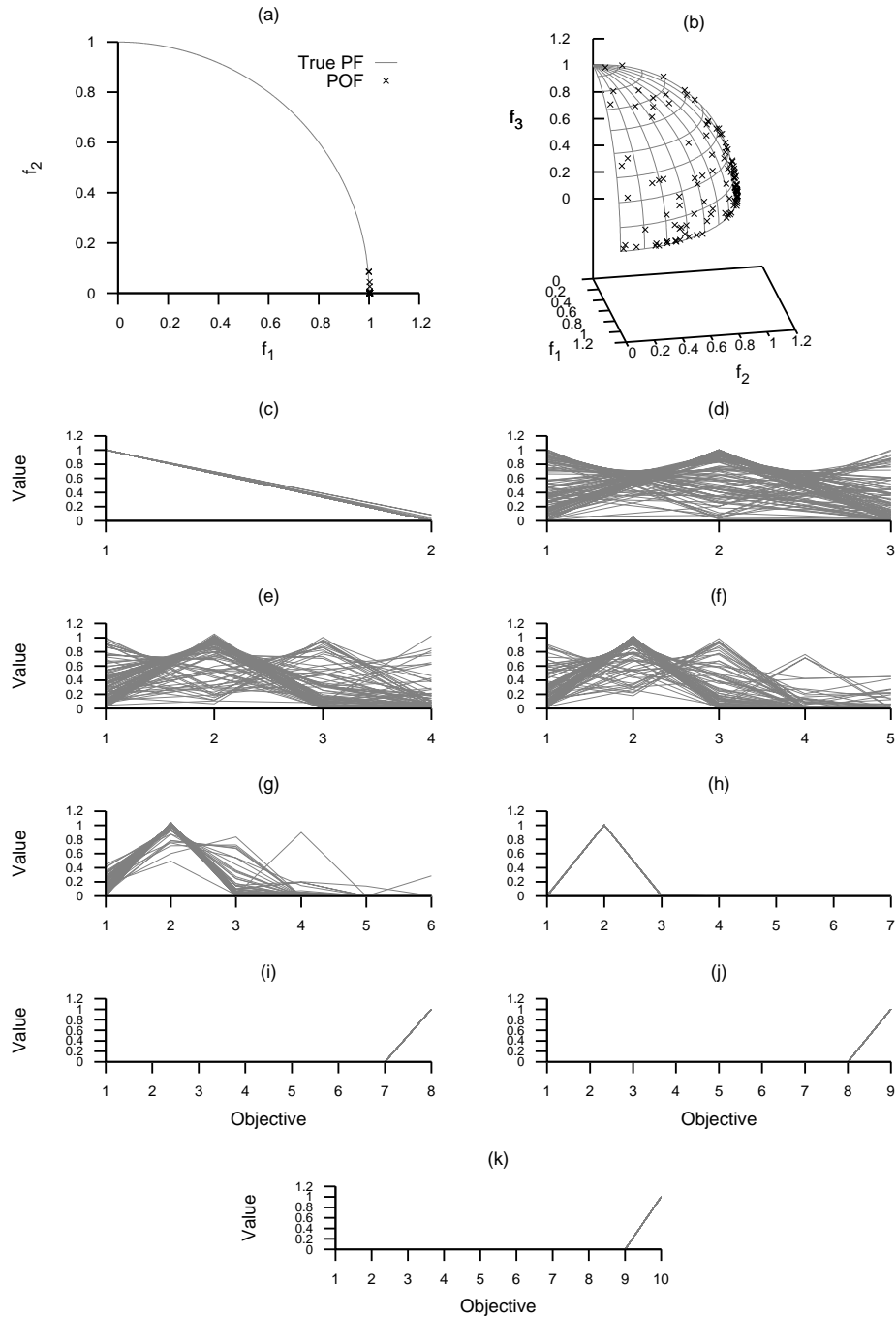


Figure A.431: Plots of the approximations obtained by MOMBI-RND from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



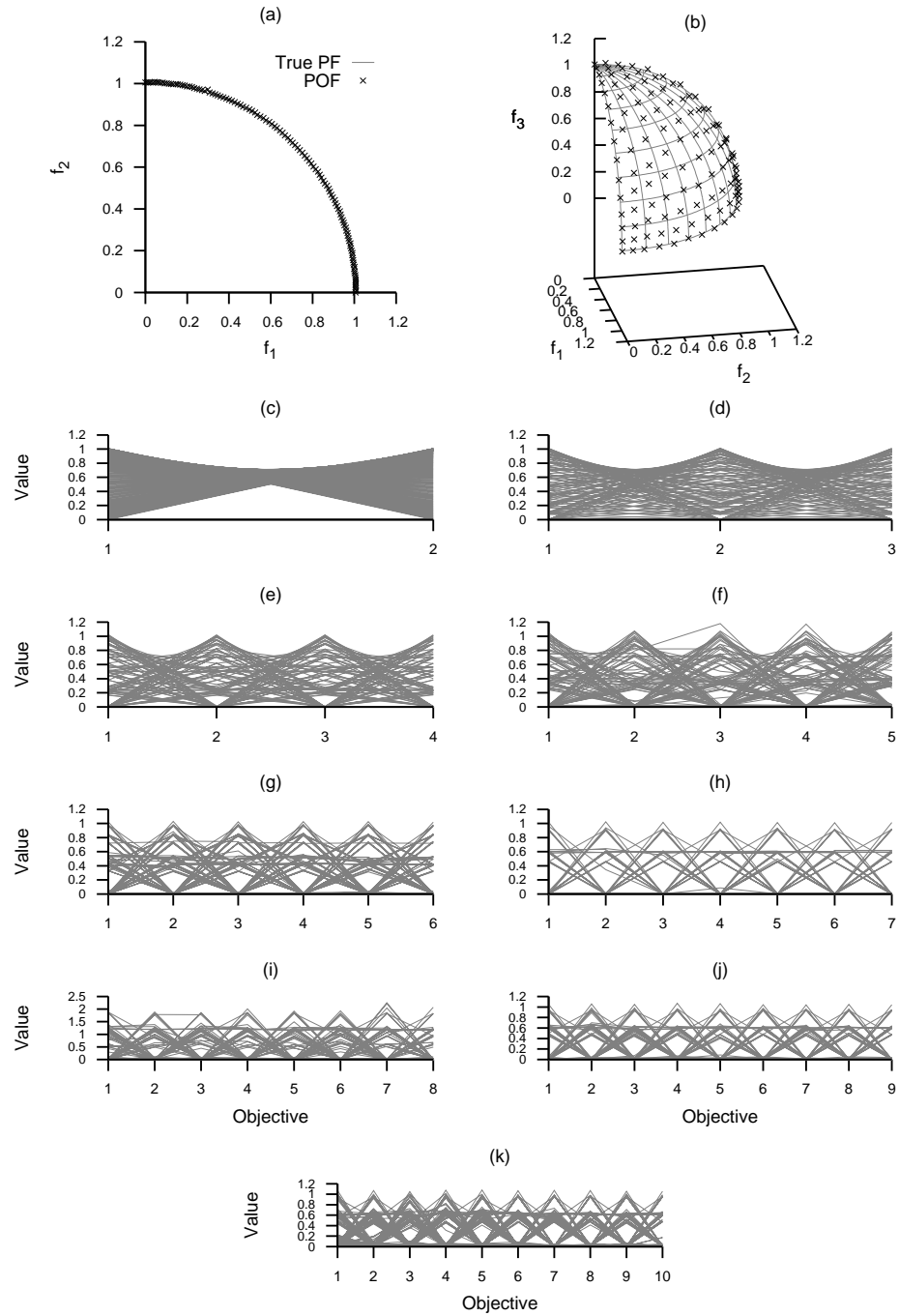


Figure A.432: Plots of the approximations obtained by MOMBI-SLD from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

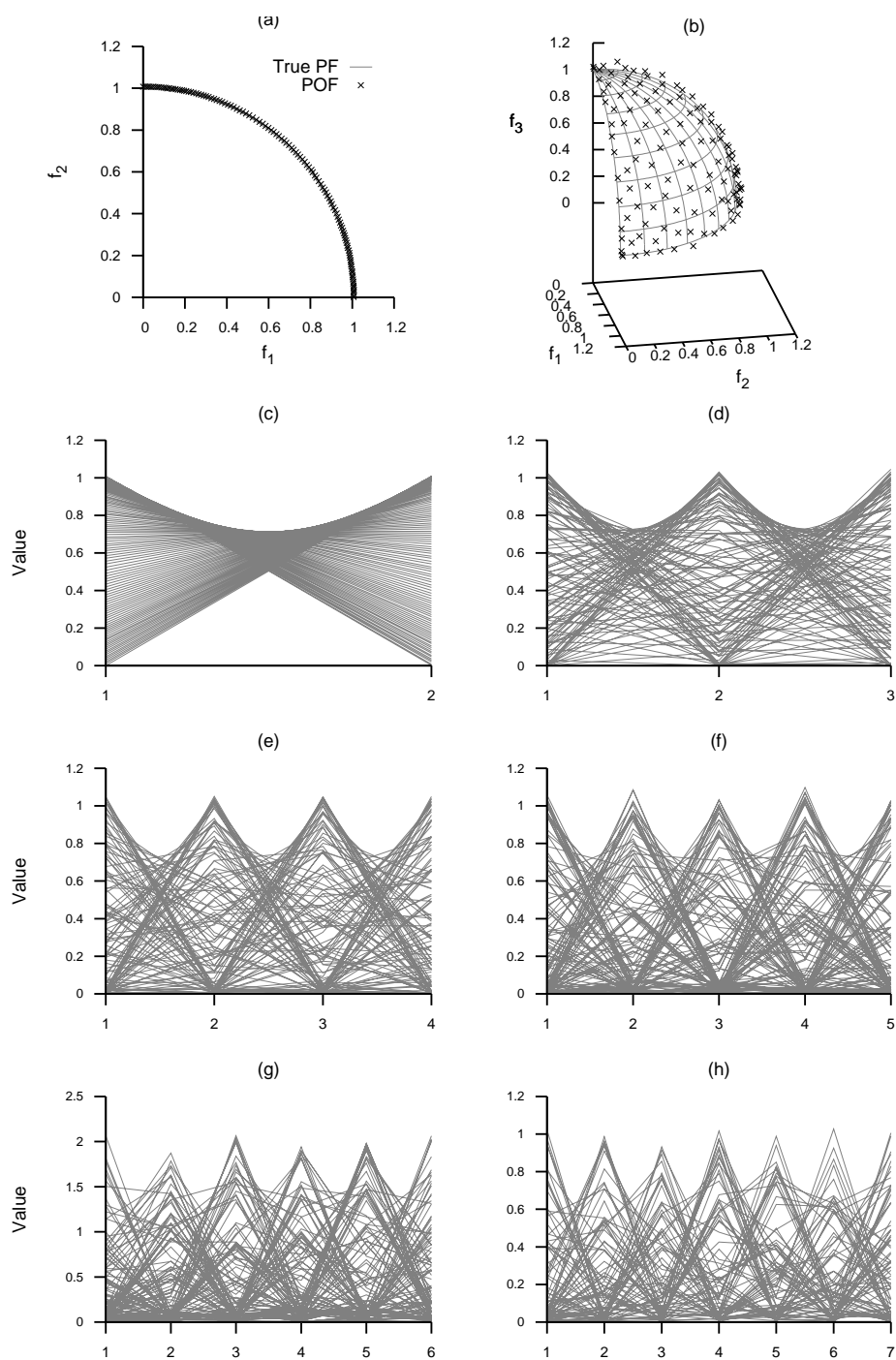


Figure A.433: Plots of the approximations obtained by MOMBI-HV from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

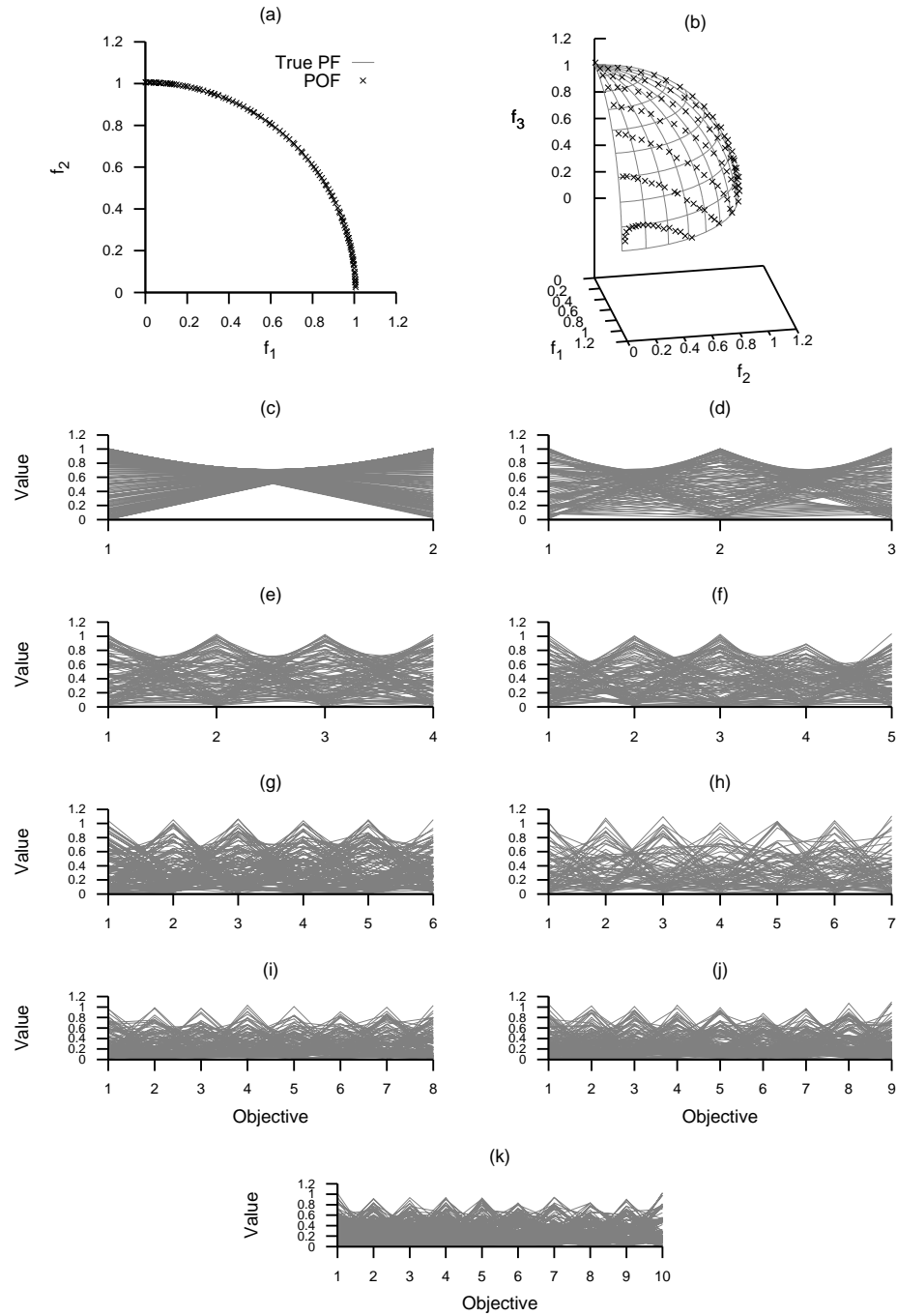


Figure A.434: Plots of the approximations obtained by MOMBI-UD from 2 to 10 objectives on the DTLZ3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.187: Comparison of hypervolume indicator values for different optimizers on the DTLZ4 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	3.156734e + 00	3.210584e + 00	3.209741e + 00	3.194348e + 00
	avg.	3.155259e + 00	3.210570e + 00	3.209708e + 00	3.192982e + 00
	min.	3.091281e + 00	3.210021e + 00	3.207960e + 00	3.170027e + 00
	max.	3.204243e + 00	3.210836e + 00	3.210752e + 00	3.208231e + 00
	std.	2.656221e - 02	1.428552e - 04	4.040477e - 04	7.989415e - 03
<b>3D</b>	med.	7.288032e + 00	7.421945e + 00	7.416789e + 00	7.306690e + 00
	avg.	7.288582e + 00	7.421937e + 00	7.416739e + 00	7.307336e + 00
	min.	7.268159e + 00	7.421639e + 00	7.415091e + 00	7.293740e + 00
	max.	7.313085e + 00	7.422105e + 00	7.417495e + 00	7.327278e + 00
	std.	9.432850e - 03	9.192183e - 05	4.861280e - 04	6.450697e - 03
<b>4D</b>	med.	1.474842e + 01	1.556772e + 01	1.553094e + 01	1.518911e + 01
	avg.	1.474563e + 01	1.556772e + 01	1.553092e + 01	1.517995e + 01
	min.	1.447297e + 01	1.556740e + 01	1.552681e + 01	1.471363e + 01
	max.	1.486077e + 01	1.556798e + 01	1.553414e + 01	1.523387e + 01
	std.	4.214897e - 02	1.121822e - 04	1.633019e - 03	6.846203e - 02
<b>5D</b>	med.	2.915548e + 01	3.166885e + 01	3.159364e + 01	3.040771e + 01
	avg.	2.916334e + 01	3.166885e + 01	3.159365e + 01	3.041200e + 01
	min.	2.853214e + 01	3.166841e + 01	3.158582e + 01	3.032465e + 01
	max.	2.941750e + 01	3.166932e + 01	3.160185e + 01	3.067167e + 01
	std.	1.261457e - 01	1.734797e - 04	2.698006e - 03	5.220064e - 02
<b>6D</b>	med.	5.651184e + 01	6.374105e + 01	6.319508e + 01	6.163220e + 01
	avg.	5.655963e + 01	6.373990e + 01	6.319173e + 01	6.164095e + 01
	min.	5.283937e + 01	6.370346e + 01	6.310376e + 01	6.116952e + 01
	max.	5.846403e + 01	6.374146e + 01	6.324554e + 01	6.237532e + 01
	std.	5.240758e - 01	4.725983e - 03	2.711932e - 02	1.193080e - 01
<b>7D</b>	med.	1.090023e + 02	1.277533e + 02	1.251959e + 02	1.218061e + 02
	avg.	1.091259e + 02	1.277448e + 02	1.252519e + 02	1.218621e + 02
	min.	1.026109e + 02	1.274352e + 02	1.249798e + 02	1.196336e + 02
	max.	1.248853e + 02	1.277543e + 02	1.265322e + 02	1.238385e + 02
	std.	2.370984e + 00	4.520091e - 02	2.346270e - 01	4.997533e - 01
<b>8D</b>	med.	2.141391e + 02	2.558149e + 02	—	2.472397e + 02
	avg.	2.134194e + 02	2.557670e + 02	—	2.472704e + 02
	min.	2.050186e + 02	2.549931e + 02	—	2.436284e + 02
	max.	2.165839e + 02	2.558255e + 02	—	2.505090e + 02
	std.	2.050332e + 00	1.059014e - 01	—	7.107601e - 01
<b>9D</b>	med.	4.329100e + 02	5.118762e + 02	—	4.996488e + 02
	avg.	4.327528e + 02	5.118761e + 02	—	4.995993e + 02
	min.	4.199398e + 02	5.118734e + 02	—	4.980345e + 02
	max.	4.388022e + 02	5.118773e + 02	—	5.011593e + 02
	std.	2.823934e + 00	6.949440e - 04	—	6.384595e - 01
<b>10D</b>	med.	8.341452e + 02	1.023911e + 03	—	1.007221e + 03
	avg.	8.346025e + 02	1.023911e + 03	—	1.007043e + 03
	min.	8.279763e + 02	1.023903e + 03	—	1.004419e + 03
	max.	8.466831e + 02	1.023913e + 03	—	1.009343e + 03
	std.	3.868409e + 00	1.860646e - 03	—	1.102268e + 00

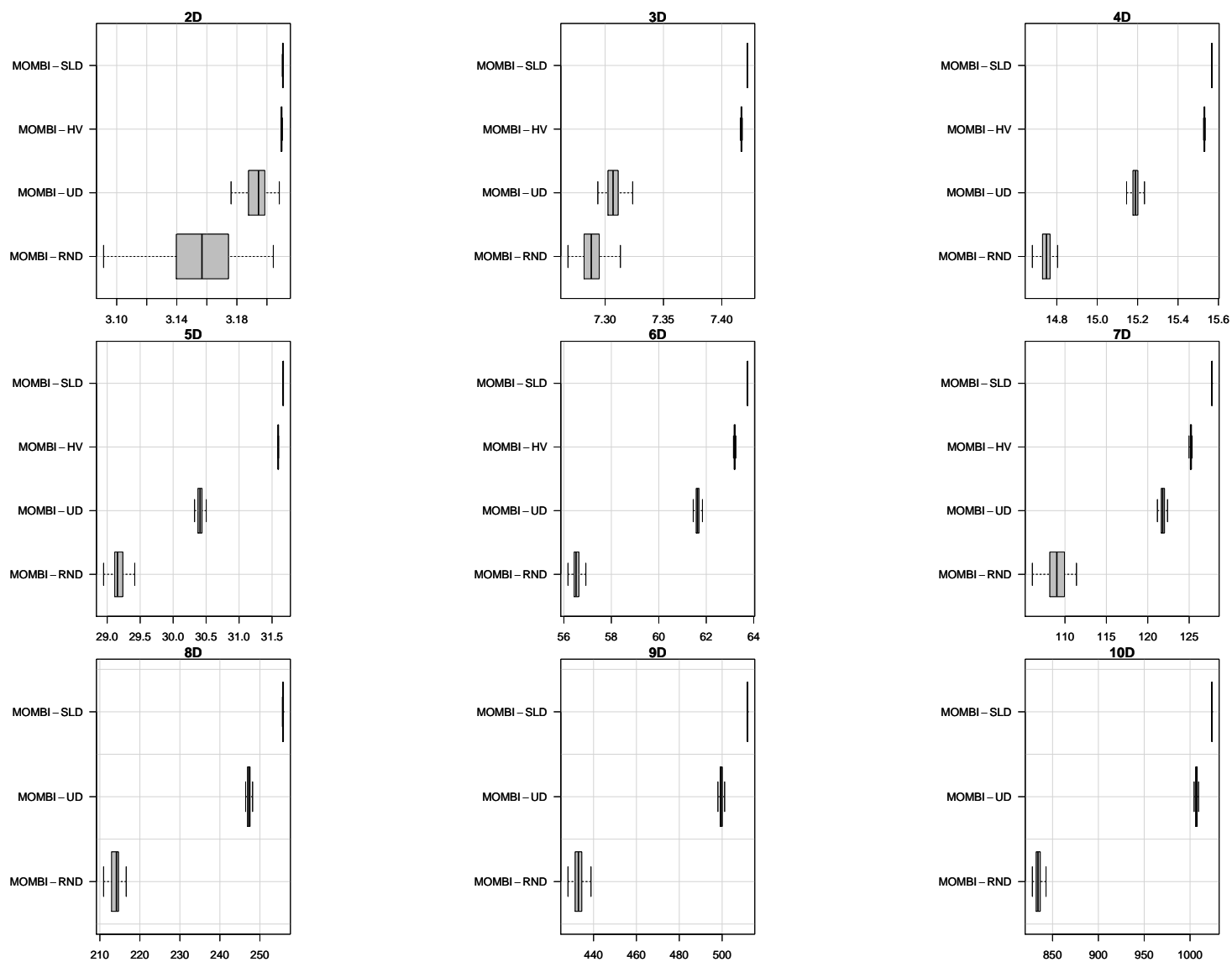


Figure A.435: Box-plot of the hypervolume indicator values for different optimizers on the DTLZ4 test problem.

Table A.188: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ4 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $ph$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
2D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e - 34$	–	$2.83e - 32$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.32e - 34$
MOMBI-UD	$8.97e - 25$	> 0.05	> 0.05	–
3D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e - 34$	–	$1.28e - 34$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.28e - 34$
MOMBI-UD	$2.75e - 28$	> 0.05	> 0.05	–
4D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.27e - 34$	–	$1.27e - 34$	$1.27e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.28e - 34$
MOMBI-UD	$1.68e - 32$	> 0.05	> 0.05	–
5D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.27e - 34$	–	$1.27e - 34$	$1.27e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.28e - 34$
MOMBI-UD	$1.28e - 34$	> 0.05	> 0.05	–
6D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e - 34$	–	$1.28e - 34$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.28e - 34$
MOMBI-UD	$1.28e - 34$	> 0.05	> 0.05	–
7D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.24e - 34$	–	$1.24e - 34$	$1.24e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.28e - 34$
MOMBI-UD	$2.51e - 33$	> 0.05	> 0.05	–
8D				
MOMBI-RND	–	> 0.05		> 0.05
MOMBI-SLD	$1.27e - 34$	–		$1.27e - 34$
MOMBI-HV			–	
MOMBI-UD	$1.28e - 34$	> 0.05		–
9D				
MOMBI-RND	–	> 0.05		> 0.05
MOMBI-SLD	$1.23e - 34$	–		$1.23e - 34$
MOMBI-HV			–	
MOMBI-UD	$1.28e - 34$	> 0.05		–
10D				
MOMBI-RND	–	> 0.05		> 0.05
MOMBI-SLD	$8.16e - 35$	–		$8.16e - 35$
MOMBI-HV			–	
MOMBI-UD	$1.28e - 34$	> 0.05		–

Table A.189: Comparison of R2 indicator values for different optimizers on the DTLZ4 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	2.201985e-01	2.180296e-01	2.180440e-01	2.184331e-01
	avg.	2.205196e-01	2.180297e-01	2.180455e-01	2.184676e-01
	min.	2.189293e-01	2.180256e-01	2.180314e-01	2.182245e-01
	max.	2.246986e-01	2.180370e-01	2.180651e-01	2.189584e-01
	std.	1.337825e-03	1.569955e-06	7.057712e-06	1.368853e-04
<b>3D</b>	med.	9.647363e-02	9.375453e-02	9.389731e-02	9.571412e-02
	avg.	9.648057e-02	9.375471e-02	9.389684e-02	9.571697e-02
	min.	9.630186e-02	9.374426e-02	9.388672e-02	9.562132e-02
	max.	9.681012e-02	9.377092e-02	9.391166e-02	9.588083e-02
	std.	9.727874e-05	4.075038e-06	4.422846e-06	3.679155e-05
<b>4D</b>	med.	6.359686e-02	5.307770e-02	5.372535e-02	5.785180e-02
	avg.	6.371856e-02	5.307795e-02	5.372653e-02	5.812106e-02
	min.	6.304339e-02	5.306466e-02	5.370391e-02	5.776338e-02
	max.	7.440944e-02	5.309459e-02	5.375351e-02	7.144174e-02
	std.	1.097622e-03	5.891424e-06	1.072904e-05	1.875553e-03
<b>5D</b>	med.	5.712249e-02	3.524549e-02	3.644678e-02	4.549493e-02
	avg.	5.727059e-02	3.524698e-02	3.645052e-02	4.547933e-02
	min.	5.660335e-02	3.522577e-02	3.638969e-02	4.386819e-02
	max.	6.081159e-02	3.527031e-02	3.650882e-02	4.573011e-02
	std.	6.296028e-04	1.014081e-05	2.657266e-05	1.791038e-04
<b>6D</b>	med.	5.287465e-02	2.430825e-02	2.941912e-02	3.575708e-02
	avg.	5.295884e-02	2.430860e-02	2.941605e-02	3.573014e-02
	min.	4.919005e-02	2.429536e-02	2.924702e-02	3.277102e-02
	max.	6.555317e-02	2.432824e-02	2.982365e-02	3.694997e-02
	std.	1.402393e-03	5.996833e-06	6.864157e-05	3.351744e-04
<b>7D</b>	med.	5.018076e-02	1.840356e-02	2.857602e-02	3.251296e-02
	avg.	5.028234e-02	1.845444e-02	2.841527e-02	3.242541e-02
	min.	2.763311e-02	1.839837e-02	2.409720e-02	2.955999e-02
	max.	5.441502e-02	2.095693e-02	2.874772e-02	3.529114e-02
	std.	2.876912e-03	3.576694e-04	6.744706e-04	6.265169e-04
<b>8D</b>	med.	4.739491e-02	1.400527e-02	—	2.646462e-02
	avg.	4.738820e-02	1.403864e-02	—	2.643041e-02
	min.	4.702912e-02	1.399780e-02	—	2.343376e-02
	max.	4.783538e-02	1.567318e-02	—	2.849879e-02
	std.	1.589170e-04	2.337120e-04	—	4.367933e-04
<b>9D</b>	med.	4.029763e-02	1.100217e-02	—	2.306952e-02
	avg.	4.045209e-02	1.100208e-02	—	2.305588e-02
	min.	3.949782e-02	1.098105e-02	—	2.261933e-02
	max.	4.476716e-02	1.101009e-02	—	2.361041e-02
	std.	7.348181e-04	4.213715e-06	—	2.083759e-04
<b>10D</b>	med.	4.099000e-02	8.835198e-03	—	2.027399e-02
	avg.	4.102415e-02	8.835717e-03	—	2.028650e-02
	min.	4.050824e-02	8.822666e-03	—	1.983195e-02
	max.	4.164343e-02	8.847600e-03	—	2.069621e-02
	std.	2.345604e-04	5.376546e-06	—	1.615792e-04

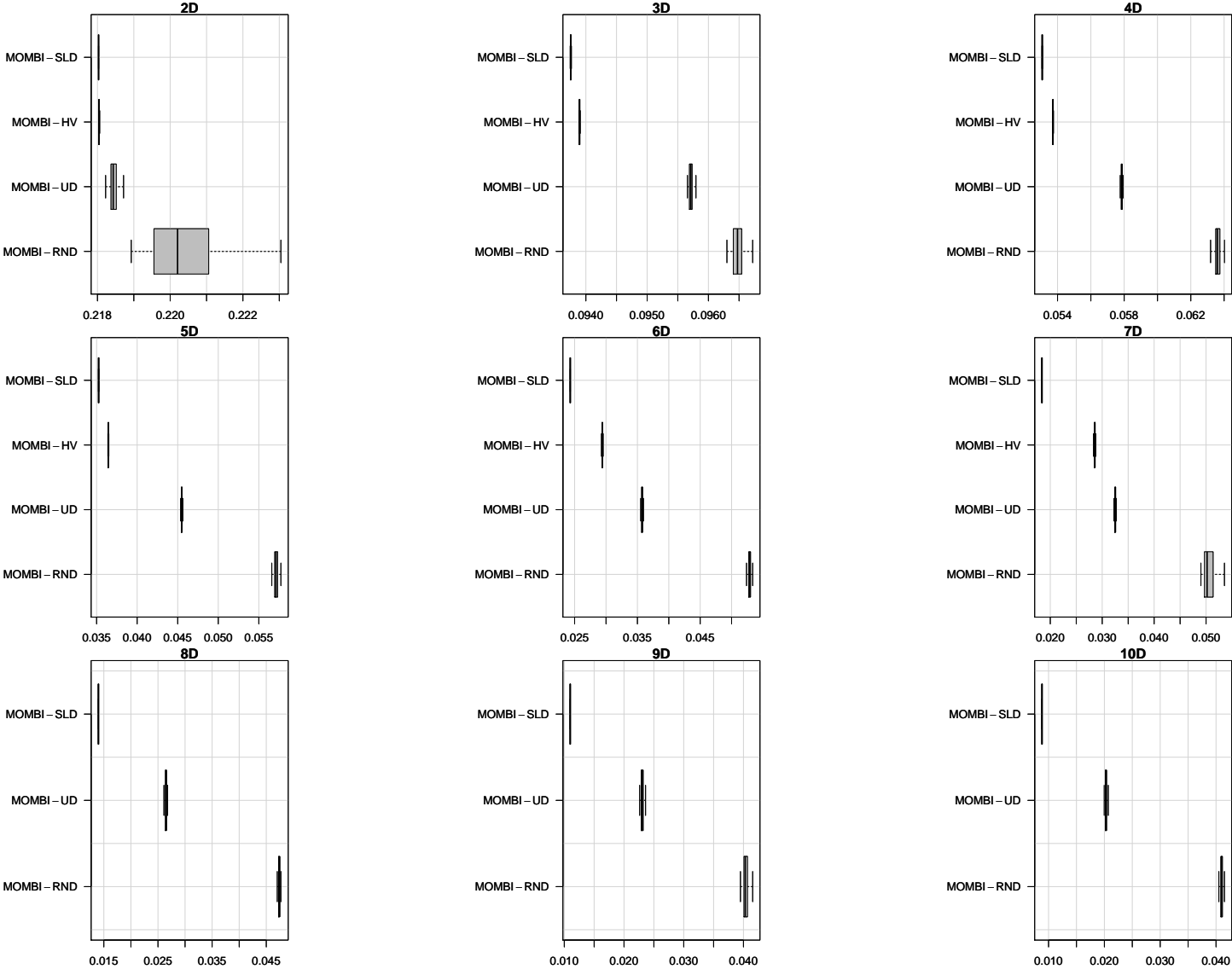


Figure A.436: Box-plot of the R2 indicator values for different optimizers on the DTLZ4 test problem.



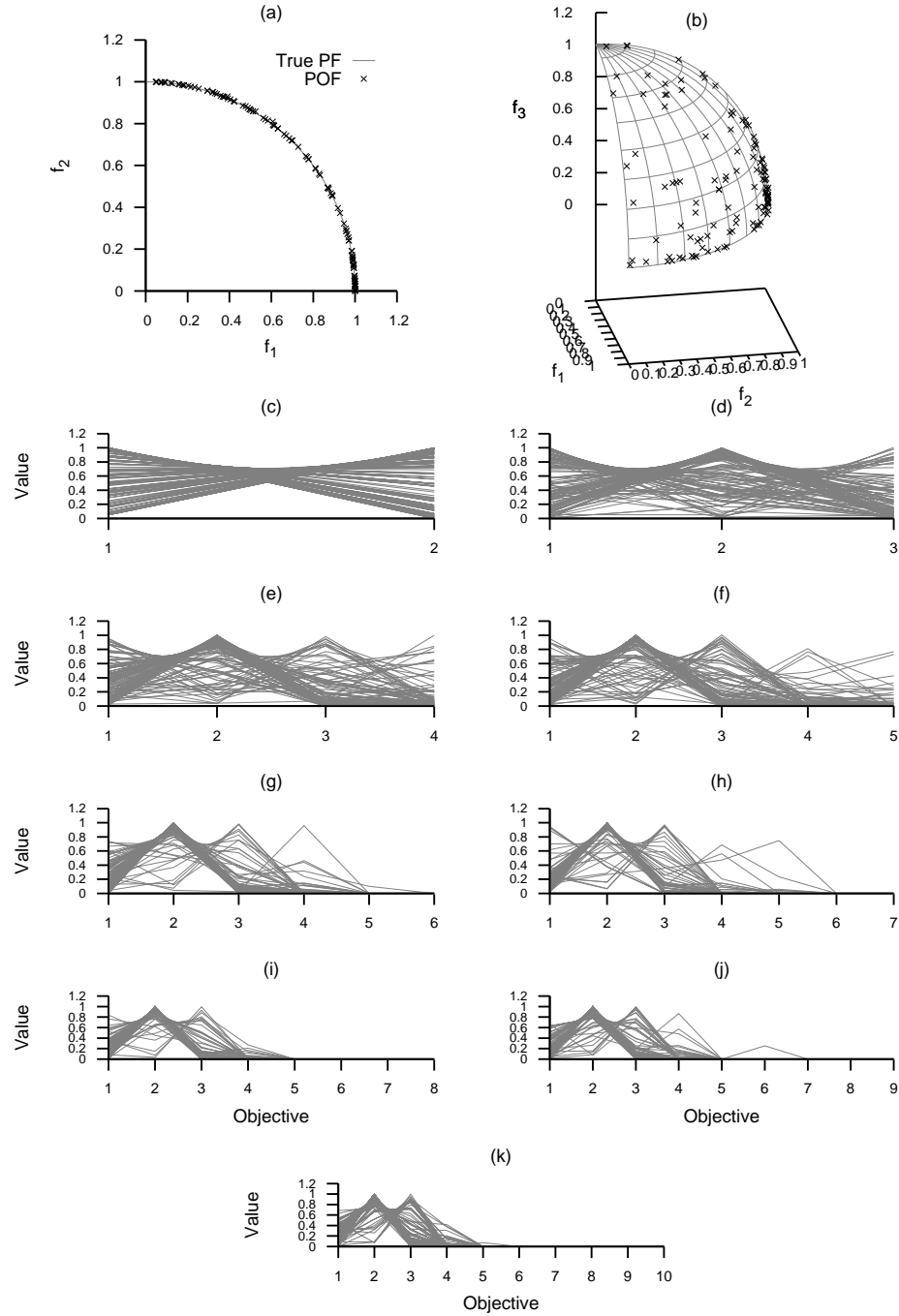


Figure A.437: Plots of the approximations obtained by MOMBI-RND from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

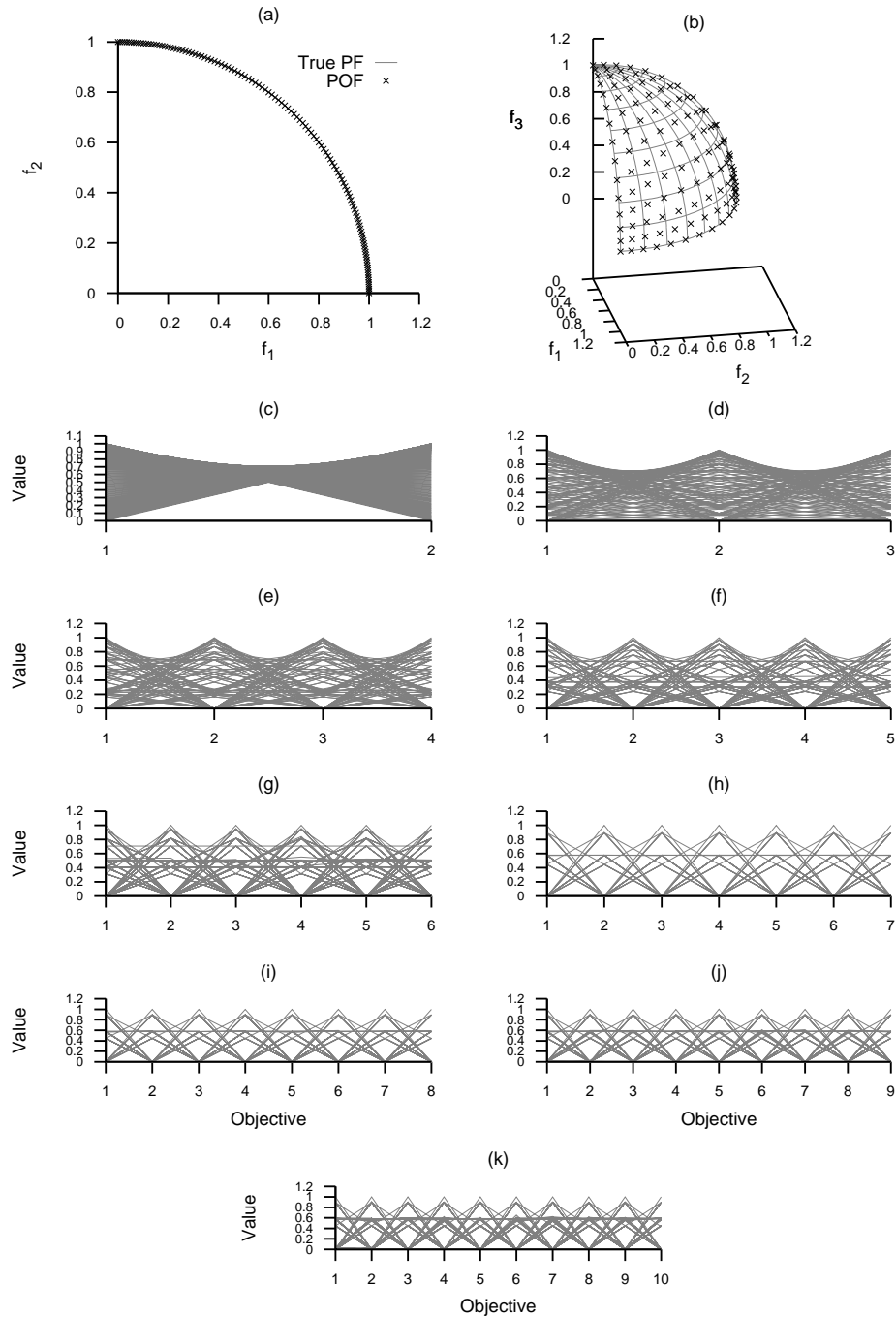


Figure A.438: Plots of the approximations obtained by MOMBI-SLD from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

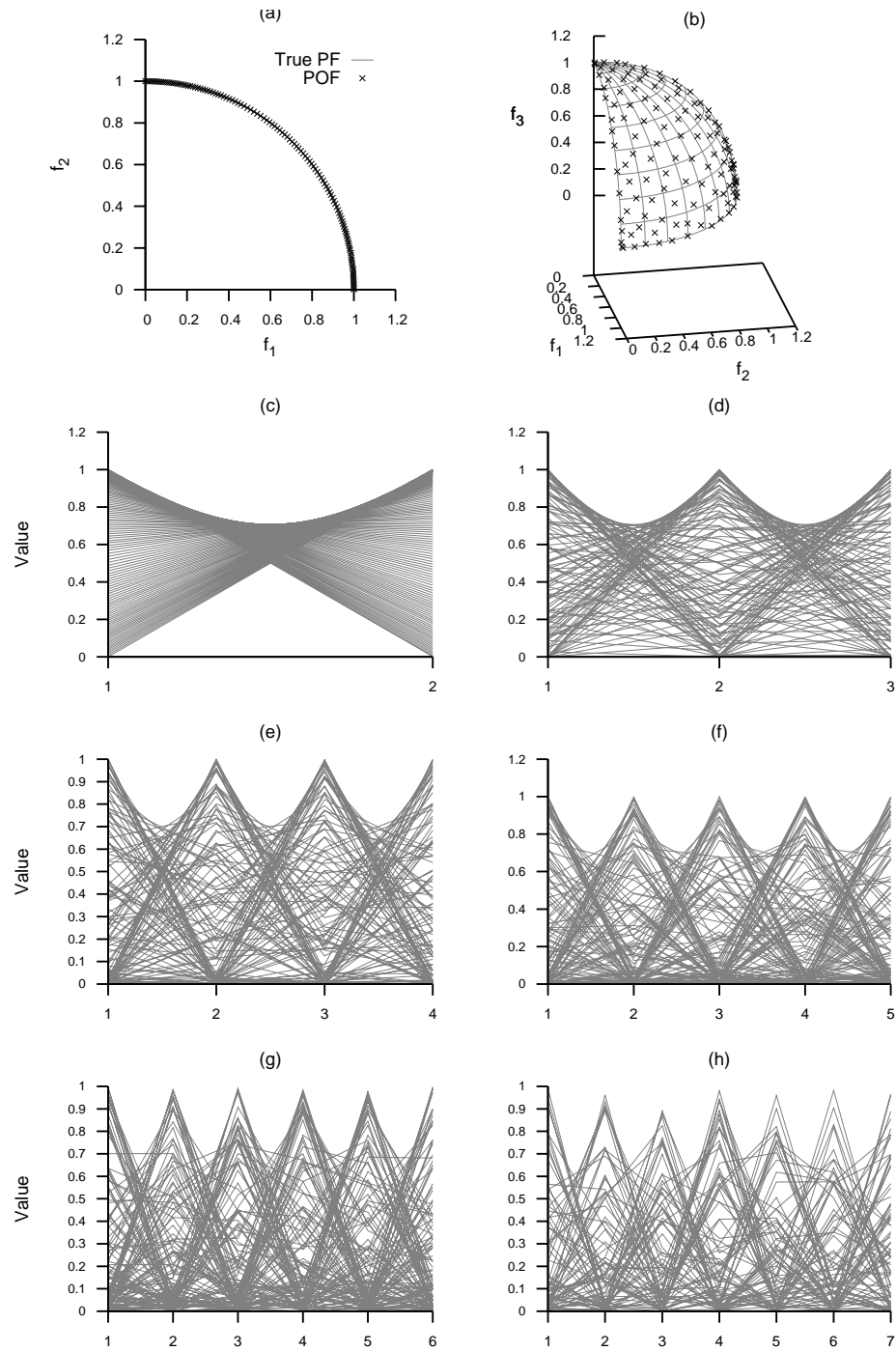


Figure A.439: Plots of the approximations obtained by MOMBI-HV from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

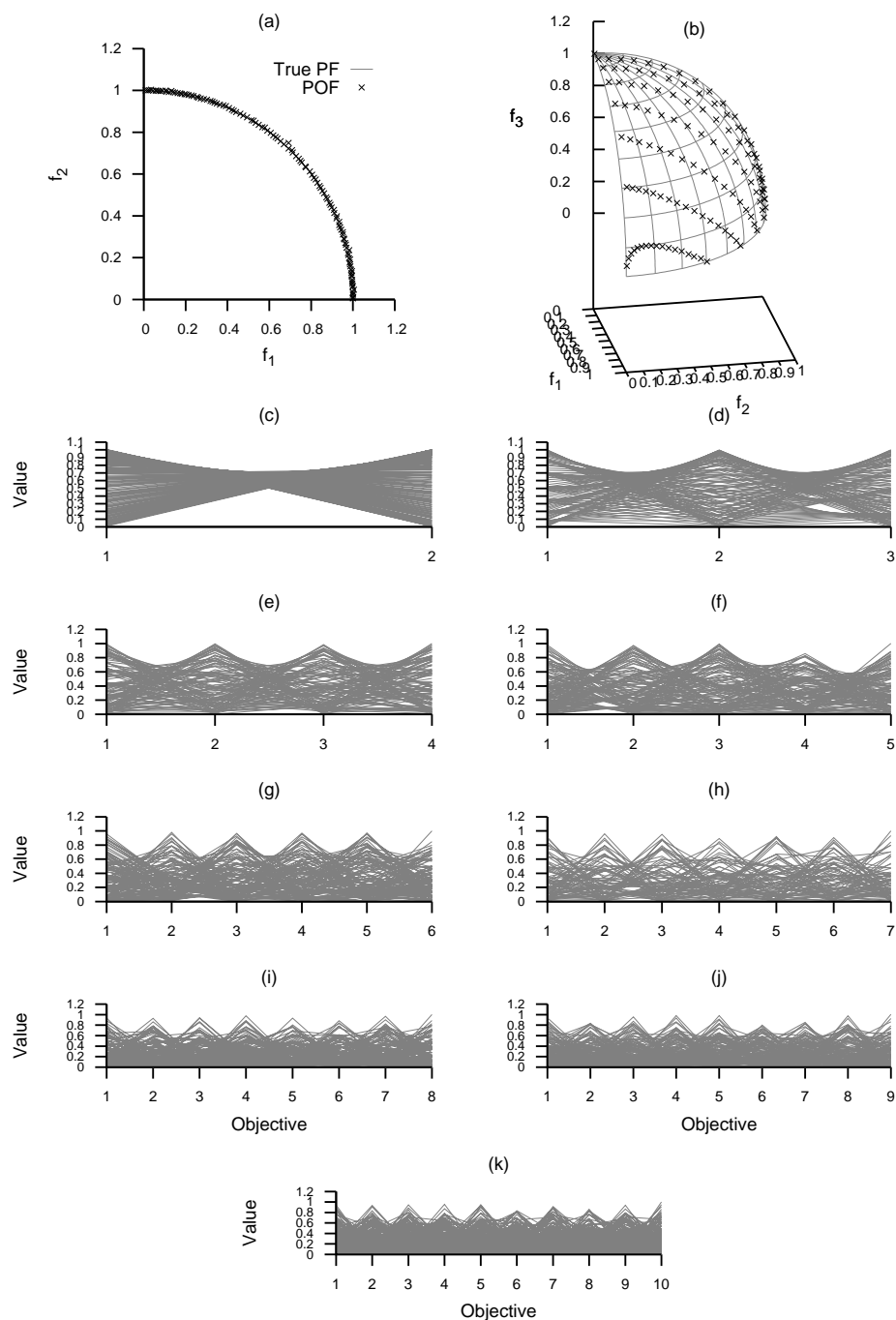


Figure A.440: Plots of the approximations obtained by MOMBI-UD from 2 to 10 objectives on the DTLZ4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.190: Comparison of hypervolume indicator values for different optimizers on the DTLZ5 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	1.484815e + 01	1.520986e + 01	1.520489e + 01	1.514443e + 01
	avg.	1.476039e + 01	1.520984e + 01	1.520488e + 01	1.513763e + 01
	min.	1.339191e + 01	1.520853e + 01	1.518979e + 01	1.502364e + 01
	max.	1.516982e + 01	1.521080e + 01	1.521038e + 01	1.520492e + 01
	std.	3.367195e - 01	4.525742e - 04	3.569061e - 03	4.276421e - 02
<b>3D</b>	med.	4.337341e + 01	5.973325e + 01	5.965785e + 01	5.960542e + 01
	avg.	4.338477e + 01	5.973338e + 01	5.962792e + 01	5.959819e + 01
	min.	4.337257e + 01	5.972217e + 01	5.924200e + 01	5.925533e + 01
	max.	4.367386e + 01	5.974040e + 01	5.981823e + 01	5.979164e + 01
	std.	3.701148e - 02	2.947675e - 03	1.414880e - 01	1.109965e - 01
<b>4D</b>	med.	1.617245e + 02	2.381430e + 02	1.830559e + 02	2.369956e + 02
	avg.	1.619964e + 02	2.381455e + 02	1.871269e + 02	2.368657e + 02
	min.	1.615107e + 02	2.380967e + 02	1.615493e + 02	2.350907e + 02
	max.	1.749233e + 02	2.381963e + 02	2.379903e + 02	2.383014e + 02
	std.	1.757726e + 00	1.799686e - 02	2.516043e + 01	7.767761e - 01
<b>5D</b>	med.	6.134039e + 02	9.471355e + 02	6.180067e + 02	9.345816e + 02
	avg.	6.134759e + 02	9.470561e + 02	6.205250e + 02	9.347564e + 02
	min.	6.129777e + 02	9.461896e + 02	6.138667e + 02	9.214029e + 02
	max.	6.151694e + 02	9.477242e + 02	7.885803e + 02	9.434493e + 02
	std.	3.756732e - 01	3.586048e - 01	1.759900e + 01	4.392181e + 00
<b>6D</b>	med.	2.363949e + 03	3.770220e + 03	2.400876e + 03	3.758165e + 03
	avg.	2.457072e + 03	3.770343e + 03	2.403698e + 03	3.757237e + 03
	min.	2.363927e + 03	3.762827e + 03	2.379169e + 03	3.707641e + 03
	max.	3.694327e + 03	3.785170e + 03	2.477872e + 03	3.779051e + 03
	std.	3.410363e + 02	2.962112e + 00	1.597242e + 01	1.280003e + 01
<b>7D</b>	med.	9.217297e + 03	1.493724e + 04	9.270711e + 03	1.228800e + 04
	avg.	1.019302e + 04	1.491233e + 04	9.270651e + 03	1.240414e + 04
	min.	9.209683e + 03	1.228800e + 04	9.216470e + 03	1.228800e + 04
	max.	1.459160e + 04	1.500855e + 04	9.335174e + 03	1.491053e + 04
	std.	2.046065e + 03	2.652903e + 02	2.398780e + 01	5.076963e + 02
<b>8D</b>	med.	4.915200e + 04	5.962531e + 04	—	5.917579e + 04
	avg.	4.446806e + 04	5.963591e + 04	—	5.885588e + 04
	min.	3.601025e + 04	5.957653e + 04	—	4.915200e + 04
	max.	4.915200e + 04	5.992570e + 04	—	5.966190e + 04
	std.	6.276789e + 03	4.580279e + 01	—	1.735792e + 03
<b>9D</b>	med.	1.966080e + 05	2.381610e + 05	—	2.359148e + 05
	avg.	1.836867e + 05	2.382079e + 05	—	2.347938e + 05
	min.	1.422700e + 05	2.379344e + 05	—	1.966428e + 05
	max.	1.966080e + 05	2.391610e + 05	—	2.379231e + 05
	std.	2.310968e + 04	2.052387e + 02	—	6.059691e + 03
<b>10D</b>	med.	7.864320e + 05	9.516820e + 05	—	9.481282e + 05
	avg.	7.511882e + 05	9.522355e + 05	—	9.435139e + 05
	min.	5.646656e + 05	9.507309e + 05	—	7.864321e + 05
	max.	7.864320e + 05	9.564849e + 05	—	9.516779e + 05
	std.	8.116106e + 04	1.291962e + 03	—	2.295054e + 04

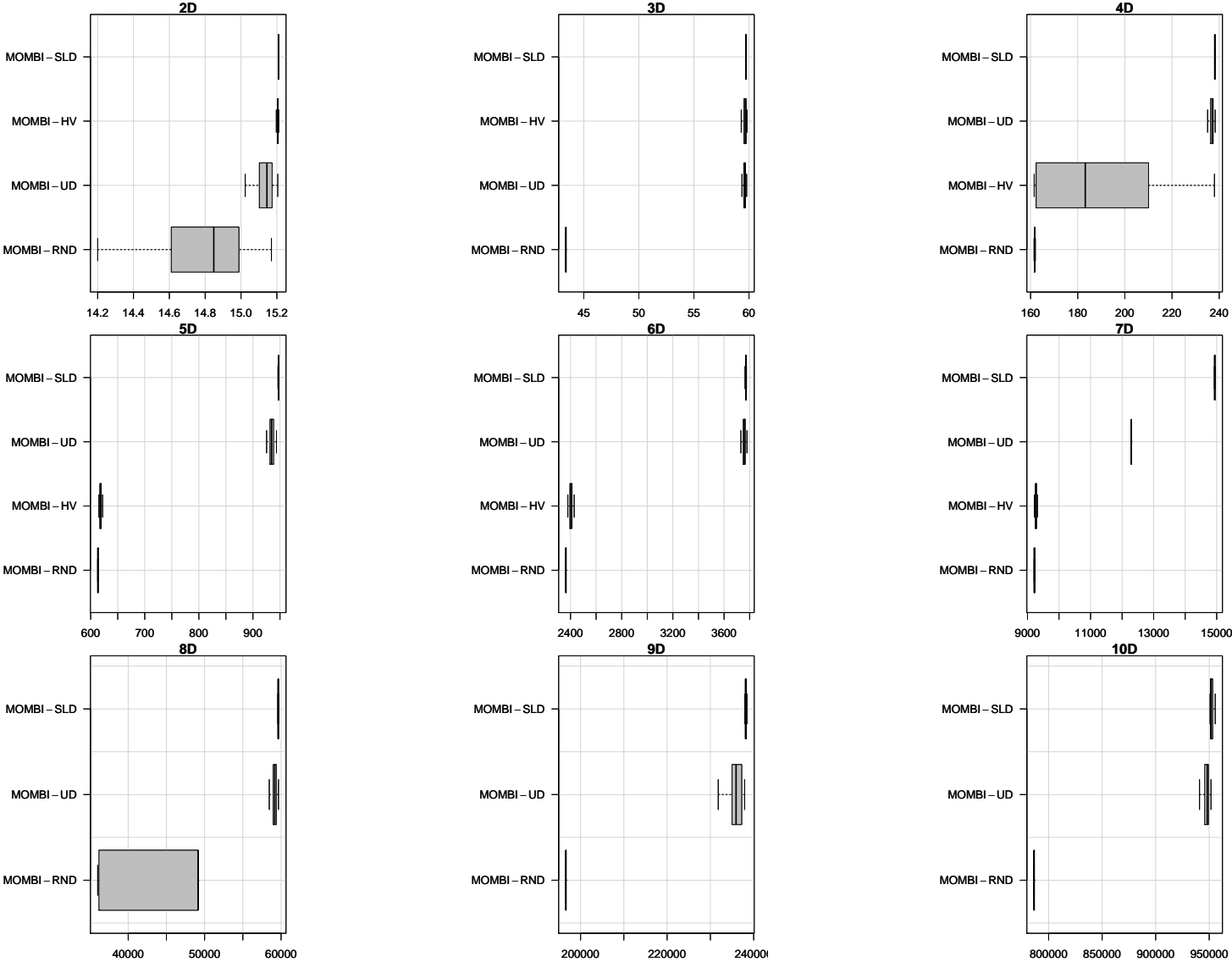


Figure A.441: Box-plot of the hypervolume indicator values for different optimizers on the DTLZ5 test problem.

Table A.191: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ5 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $ph$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
2D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e - 34$	—	$3.79e - 28$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	$> 0.05$	—	$1.03e - 33$
MOMBI-UD	$1.56e - 29$	$> 0.05$	$> 0.05$	—
3D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.27e - 34$	—	$6.27e - 10$	$8.30e - 22$
MOMBI-HV	$1.28e - 34$	$> 0.05$	—	$6.64e - 03$
MOMBI-UD	$1.28e - 34$	$> 0.05$	$> 0.05$	—
4D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e - 34$	—	$1.28e - 34$	$5.58e - 31$
MOMBI-HV	$7.08e - 30$	$> 0.05$	—	$> 0.05$
MOMBI-UD	$1.28e - 34$	$> 0.05$	$3.78e - 32$	—
5D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e - 34$	—	$1.28e - 34$	$1.28e - 34$
MOMBI-HV	$3.34e - 34$	$> 0.05$	—	$> 0.05$
MOMBI-UD	$1.28e - 34$	$> 0.05$	$1.28e - 34$	—
6D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.96e - 36$	—	$1.28e - 34$	$2.71e - 17$
MOMBI-HV	$1.84e - 27$	$> 0.05$	—	$> 0.05$
MOMBI-UD	$1.96e - 36$	$> 0.05$	$1.28e - 34$	—
7D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$2.13e - 34$	—	$1.28e - 34$	$3.41e - 35$
MOMBI-HV	$1.53e - 13$	$> 0.05$	—	$> 0.05$
MOMBI-UD	$1.63e - 16$	$> 0.05$	$5.13e - 36$	—
8D				
MOMBI-RND	—	$> 0.05$		$> 0.05$
MOMBI-SLD	$1.01e - 35$	—		$3.37e - 33$
MOMBI-HV			—	
MOMBI-UD	$5.66e - 35$	$> 0.05$		—
9D				
MOMBI-RND	—	$> 0.05$		$> 0.05$
MOMBI-SLD	$1.64e - 36$	—		$1.28e - 34$
MOMBI-HV			—	
MOMBI-UD	$1.64e - 36$	$> 0.05$		—
10D				
MOMBI-RND	—	$> 0.05$		$> 0.05$
MOMBI-SLD	$3.15e - 37$	—		$1.98e - 33$
MOMBI-HV			—	
MOMBI-UD	$3.15e - 37$	$> 0.05$		—

Table A.192: Comparison of R2 indicator values for different optimizers on the DTLZ5 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	2.252689e-01	2.180293e-01	2.180641e-01	2.185570e-01
	avg.	2.322155e-01	2.180295e-01	2.180692e-01	2.186279e-01
	min.	2.191240e-01	2.180250e-01	2.180307e-01	2.183013e-01
	max.	3.343816e-01	2.180362e-01	2.181557e-01	2.193668e-01
	std.	1.981644e-02	1.869108e-06	2.460950e-05	2.325466e-04
<b>3D</b>	med.	3.526432e-01	1.665141e-01	1.640870e-01	1.640580e-01
	avg.	3.524246e-01	1.665102e-01	1.640977e-01	1.640666e-01
	min.	3.468084e-01	1.664176e-01	1.638815e-01	1.637978e-01
	max.	3.526592e-01	1.665306e-01	1.645458e-01	1.644186e-01
	std.	7.168130e-04	1.793946e-05	1.386942e-04	1.002904e-04
<b>4D</b>	med.	2.626034e-01	1.228820e-01	1.980619e-01	1.210475e-01
	avg.	2.617969e-01	1.228703e-01	2.017311e-01	1.210792e-01
	min.	2.221352e-01	1.227258e-01	1.220801e-01	1.208360e-01
	max.	2.632240e-01	1.229519e-01	2.630971e-01	1.215249e-01
	std.	5.443831e-03	5.268034e-05	5.493722e-02	1.265915e-04
<b>5D</b>	med.	1.968028e-01	9.648736e-02	1.947604e-01	9.762363e-02
	avg.	1.967668e-01	9.644952e-02	1.935745e-01	9.764526e-02
	min.	1.957815e-01	9.589705e-02	1.194628e-01	9.717067e-02
	max.	1.970603e-01	9.676455e-02	1.965417e-01	9.836932e-02
	std.	2.192895e-04	1.703613e-04	7.922213e-03	2.286146e-04
<b>6D</b>	med.	1.703148e-01	8.180715e-02	1.689956e-01	8.021547e-02
	avg.	1.647455e-01	8.187495e-02	1.686729e-01	8.022703e-02
	min.	9.068944e-02	8.140893e-02	1.572111e-01	7.996043e-02
	max.	1.703843e-01	8.237041e-02	1.705589e-01	8.047682e-02
	std.	2.040396e-02	2.290449e-04	1.690945e-03	1.045279e-04
<b>7D</b>	med.	1.462455e-01	7.205915e-02	1.467668e-01	1.425000e-01
	avg.	1.353845e-01	7.276016e-02	1.467648e-01	1.389604e-01
	min.	8.266858e-02	7.134707e-02	1.457232e-01	7.094869e-02
	max.	1.466011e-01	1.425000e-01	1.478619e-01	1.425000e-01
	std.	2.397767e-02	7.046488e-03	4.084939e-04	1.537933e-02
<b>8D</b>	med.	1.239250e-01	6.374154e-02	—	6.290725e-02
	avg.	1.259469e-01	6.373029e-02	—	6.469403e-02
	min.	1.239250e-01	6.312910e-02	—	6.171342e-02
	max.	1.310494e-01	6.390906e-02	—	1.239250e-01
	std.	2.737710e-03	1.282735e-04	—	1.047230e-02
<b>9D</b>	med.	1.119269e-01	5.543011e-02	—	5.461778e-02
	avg.	1.122318e-01	5.543188e-02	—	5.608972e-02
	min.	1.119269e-01	5.495530e-02	—	5.439693e-02
	max.	1.143644e-01	5.562504e-02	—	1.118664e-01
	std.	5.988095e-04	1.117704e-04	—	8.225883e-03
<b>10D</b>	med.	1.011008e-01	5.024330e-02	—	4.943411e-02
	avg.	1.013937e-01	5.022162e-02	—	5.049348e-02
	min.	1.011008e-01	4.980550e-02	—	4.923024e-02
	max.	1.040216e-01	5.047363e-02	—	1.011007e-01
	std.	6.984585e-04	1.425866e-04	—	7.248543e-03



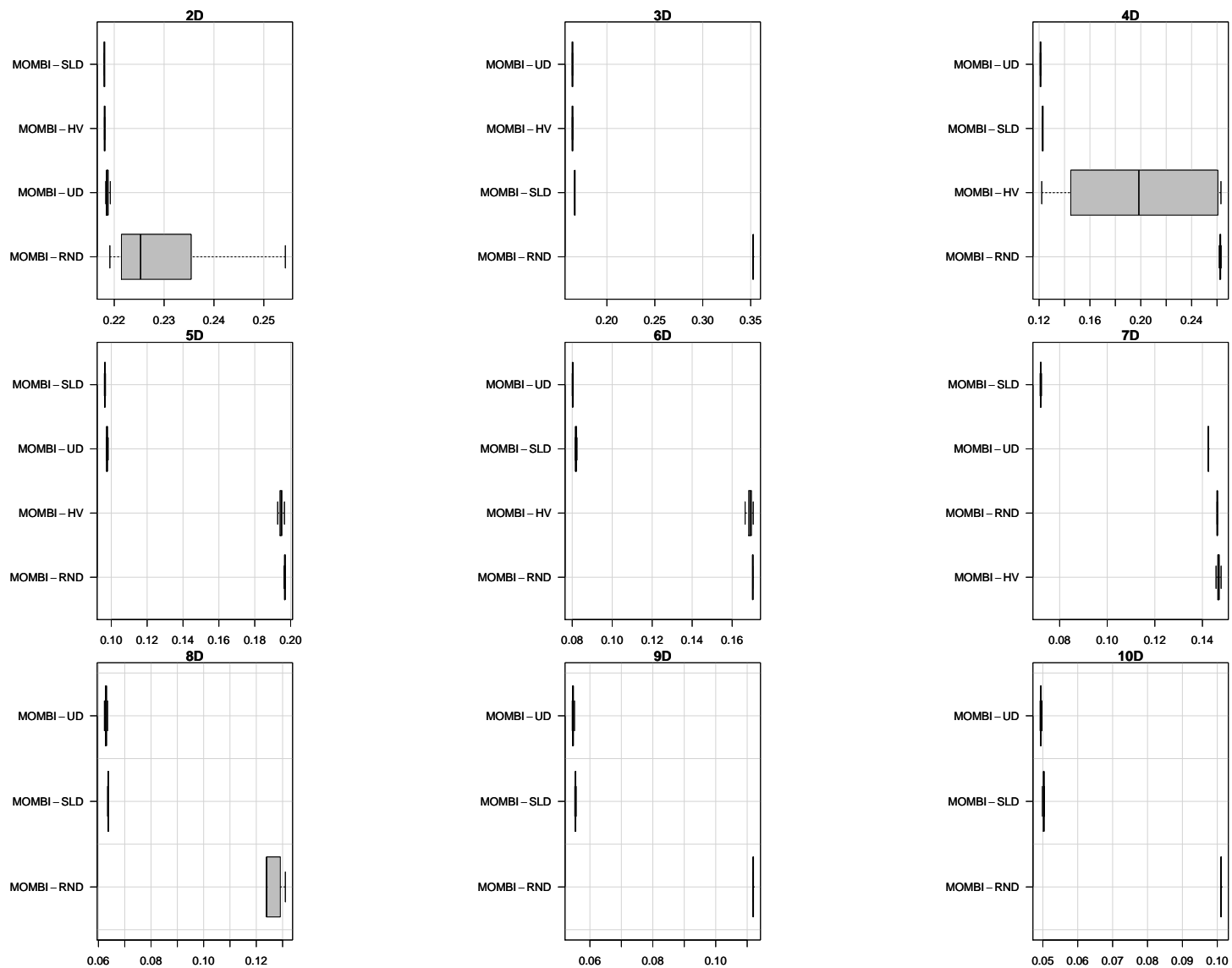


Figure A.442: Box-plot of the R2 indicator values for different optimizers on the DTLZ5 test problem.

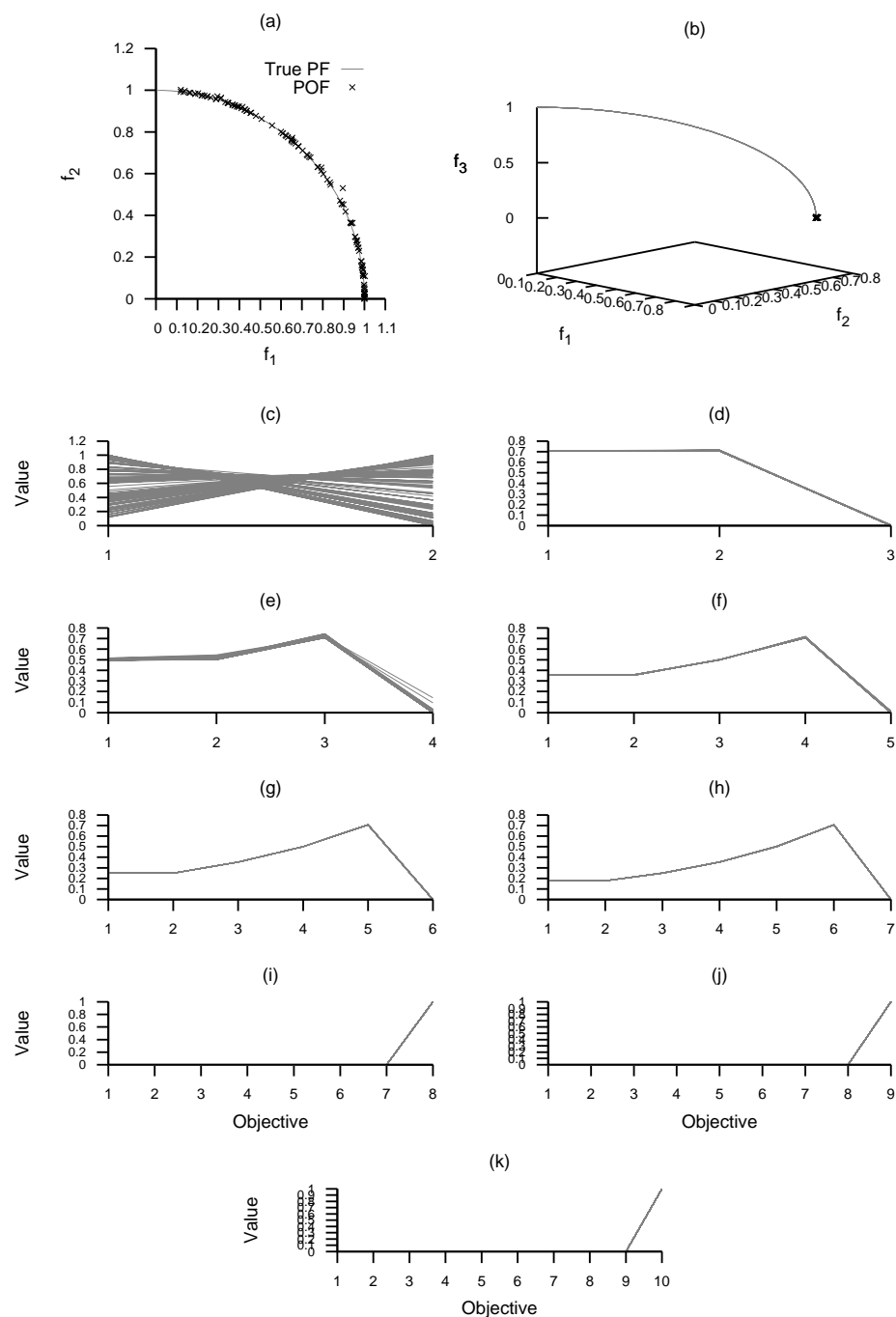


Figure A.443: Plots of the approximations obtained by MOMBI-RND from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

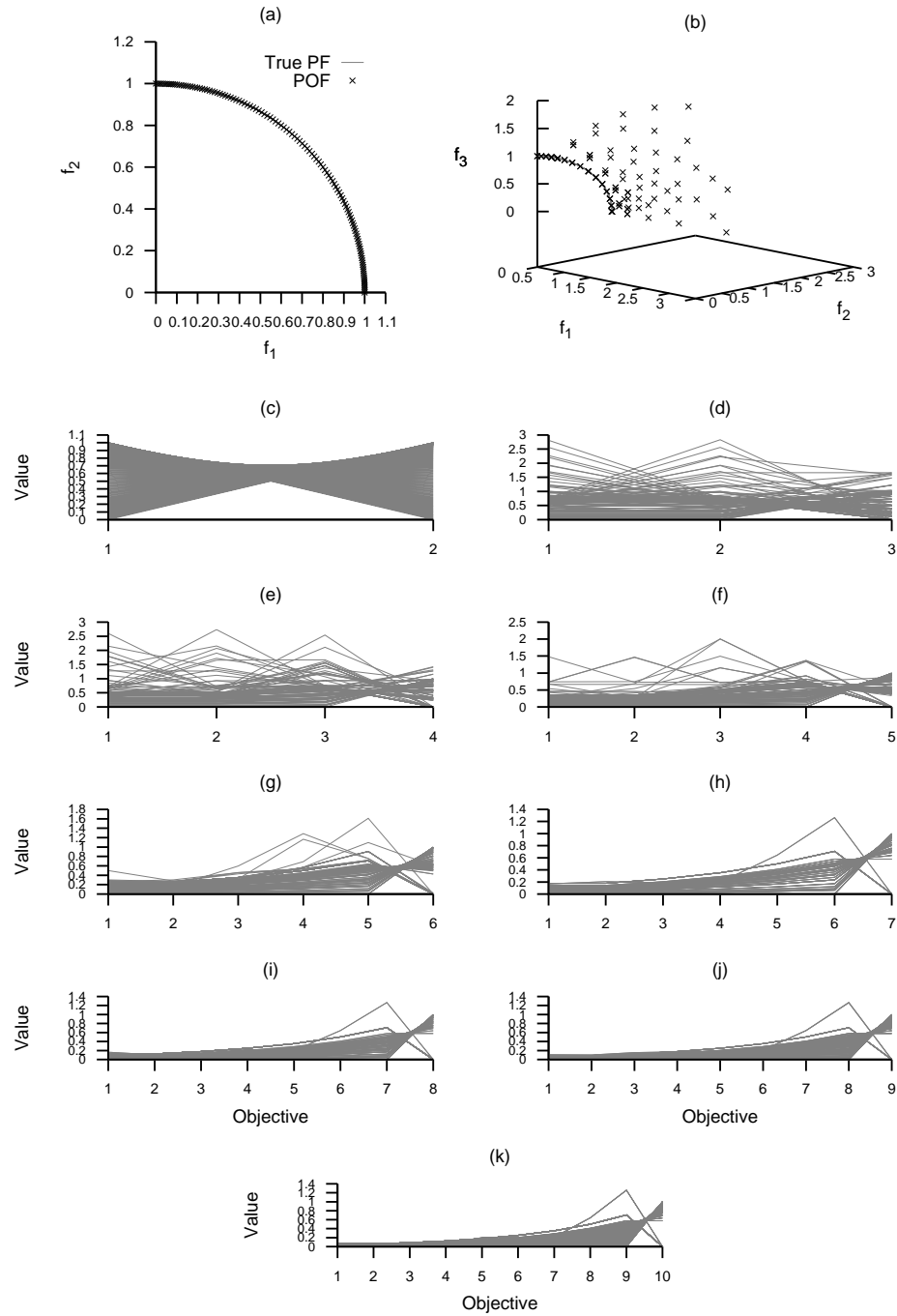


Figure A.444: Plots of the approximations obtained by MOMBI-SLD from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

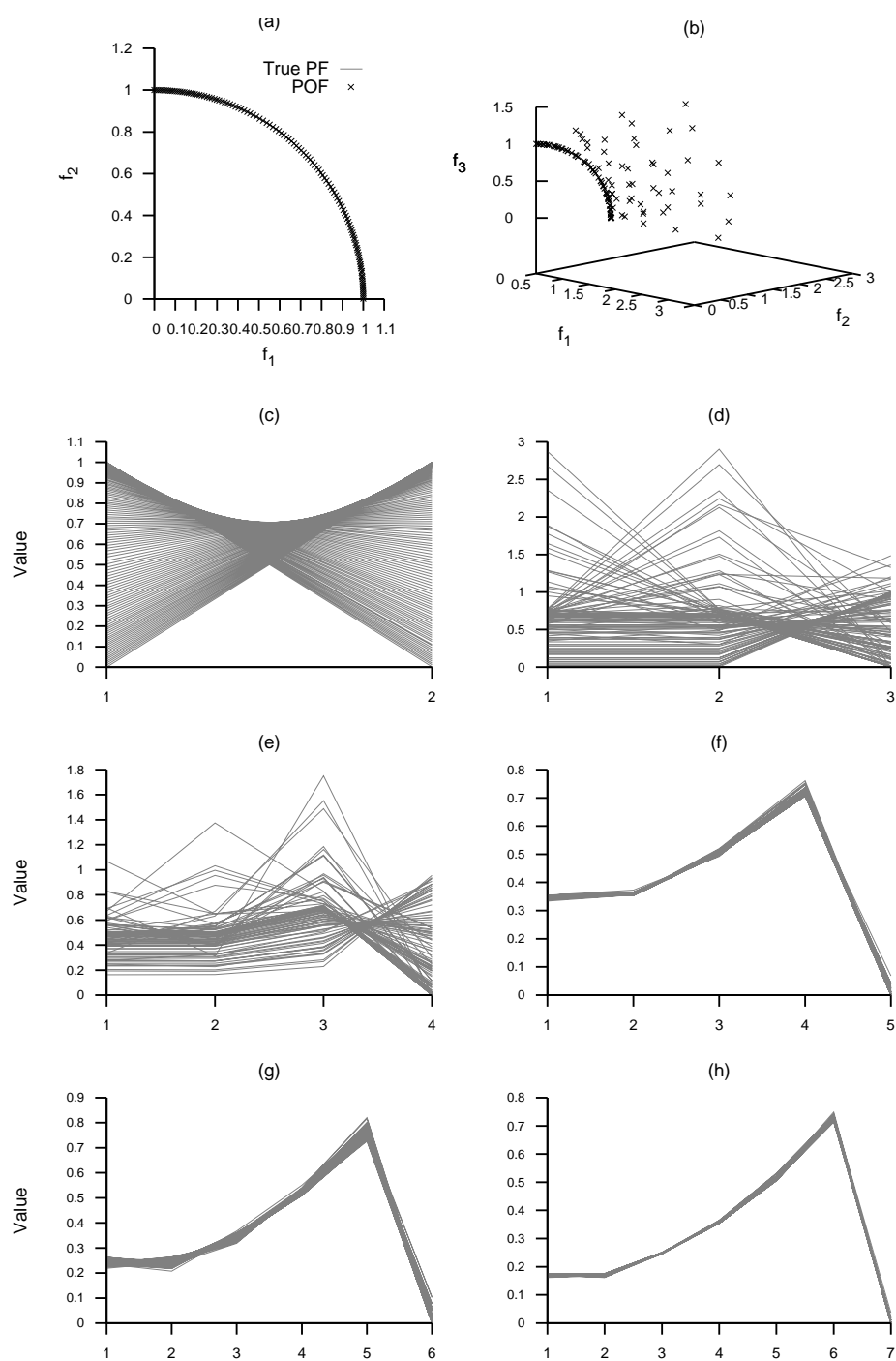


Figure A.445: Plots of the approximations obtained by MOMBI-HV from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

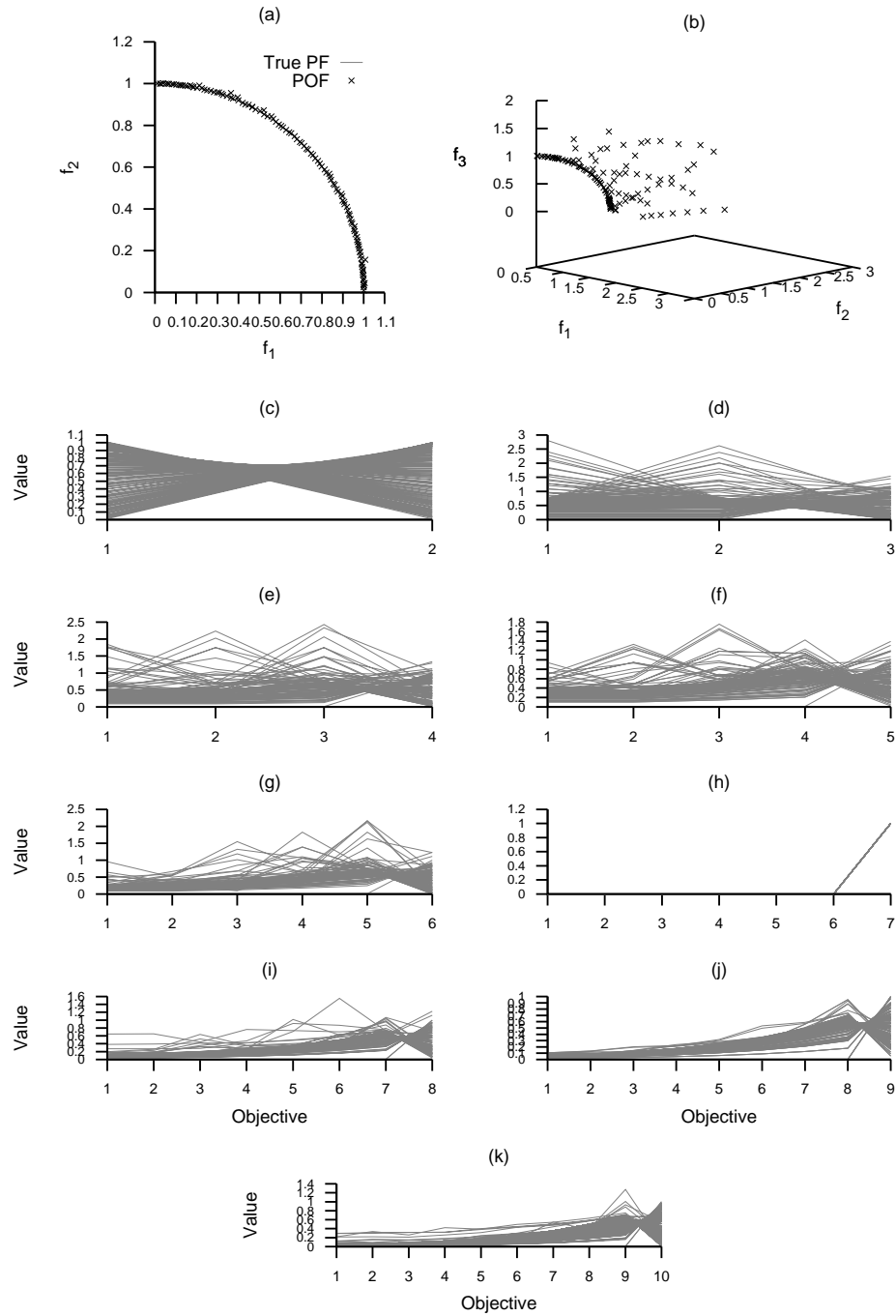


Figure A.446: Plots of the approximations obtained by MOMBI-UD from 2 to 10 objectives on the DTLZ5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.193: Comparison of hypervolume indicator values for different optimizers on the DTLZ6 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	1.173802e + 02	1.201141e + 02	1.200927e + 02	1.198232e + 02
	avg.	1.164246e + 02	1.201184e + 02	1.200954e + 02	1.198211e + 02
	min.	1.090658e + 02	1.200236e + 02	1.199832e + 02	1.194546e + 02
	max.	1.199700e + 02	1.202090e + 02	1.202056e + 02	1.201037e + 02
	std.	3.135927e + 00	4.214537e - 02	4.453042e - 02	1.368124e - 01
<b>3D</b>	med.	1.190492e + 03	1.313687e + 03	1.312678e + 03	1.310537e + 03
	avg.	1.195006e + 03	1.313639e + 03	1.312425e + 03	1.310407e + 03
	min.	1.163569e + 03	1.312197e + 03	1.307785e + 03	1.306021e + 03
	max.	1.276334e + 03	1.315808e + 03	1.314920e + 03	1.313005e + 03
	std.	2.771744e + 01	6.173216e - 01	1.587831e + 00	1.318367e + 00
<b>4D</b>	med.	1.366213e + 04	1.442441e + 04	1.422613e + 04	1.428520e + 04
	avg.	1.367279e + 04	1.442329e + 04	1.423048e + 04	1.428588e + 04
	min.	1.323225e + 04	1.438407e + 04	1.400411e + 04	1.420832e + 04
	max.	1.424303e + 04	1.444254e + 04	1.442271e + 04	1.435746e + 04
	std.	2.392196e + 02	1.004836e + 01	8.353596e + 01	3.462571e + 01
<b>5D</b>	med.	1.387634e + 05	1.586109e + 05	1.552026e + 05	1.543994e + 05
	avg.	1.386902e + 05	1.585925e + 05	1.553373e + 05	1.543727e + 05
	min.	1.374721e + 05	1.580406e + 05	1.539938e + 05	1.530696e + 05
	max.	1.395262e + 05	1.588501e + 05	1.581024e + 05	1.559687e + 05
	std.	4.449994e + 02	1.383448e + 02	7.920004e + 02	5.184095e + 02
<b>6D</b>	med.	1.461377e + 06	1.741087e + 06	1.668018e + 06	1.688329e + 06
	avg.	1.460983e + 06	1.740924e + 06	1.668374e + 06	1.687738e + 06
	min.	1.454123e + 06	1.735632e + 06	1.651265e + 06	1.669277e + 06
	max.	1.471105e + 06	1.745549e + 06	1.686978e + 06	1.698632e + 06
	std.	3.792740e + 03	2.069495e + 03	6.337929e + 03	5.875762e + 03
<b>7D</b>	med.	1.557868e + 07	1.921800e + 07	1.734737e + 07	1.847392e + 07
	avg.	1.558872e + 07	1.921647e + 07	1.735023e + 07	1.846641e + 07
	min.	1.526015e + 07	1.918984e + 07	1.717703e + 07	1.820084e + 07
	max.	1.593844e + 07	1.923109e + 07	1.753446e + 07	1.869646e + 07
	std.	1.285203e + 05	7.316100e + 03	7.865114e + 04	9.879192e + 04
<b>8D</b>	med.	1.706847e + 08	2.103508e + 08	—	1.970418e + 08
	avg.	1.722820e + 08	2.103304e + 08	—	1.970699e + 08
	min.	1.632175e + 08	2.092694e + 08	—	1.939031e + 08
	max.	1.944182e + 08	2.108674e + 08	—	2.000873e + 08
	std.	6.517372e + 06	2.798965e + 05	—	1.359234e + 06
<b>9D</b>	med.	1.873662e + 09	2.291250e + 09	—	2.083019e + 09
	avg.	1.903029e + 09	2.290574e + 09	—	2.084538e + 09
	min.	1.838675e + 09	2.269603e + 09	—	2.033491e + 09
	max.	2.134678e + 09	2.298838e + 09	—	2.132344e + 09
	std.	8.346457e + 07	5.271527e + 06	—	2.271299e + 07
<b>10D</b>	med.	2.057602e + 10	2.493592e + 10	—	2.215102e + 10
	avg.	2.123136e + 10	2.492915e + 10	—	2.213201e + 10
	min.	1.993850e + 10	2.473456e + 10	—	2.135511e + 10
	max.	2.352969e + 10	2.505609e + 10	—	2.281155e + 10
	std.	1.344823e + 09	6.235367e + 07	—	2.734691e + 08

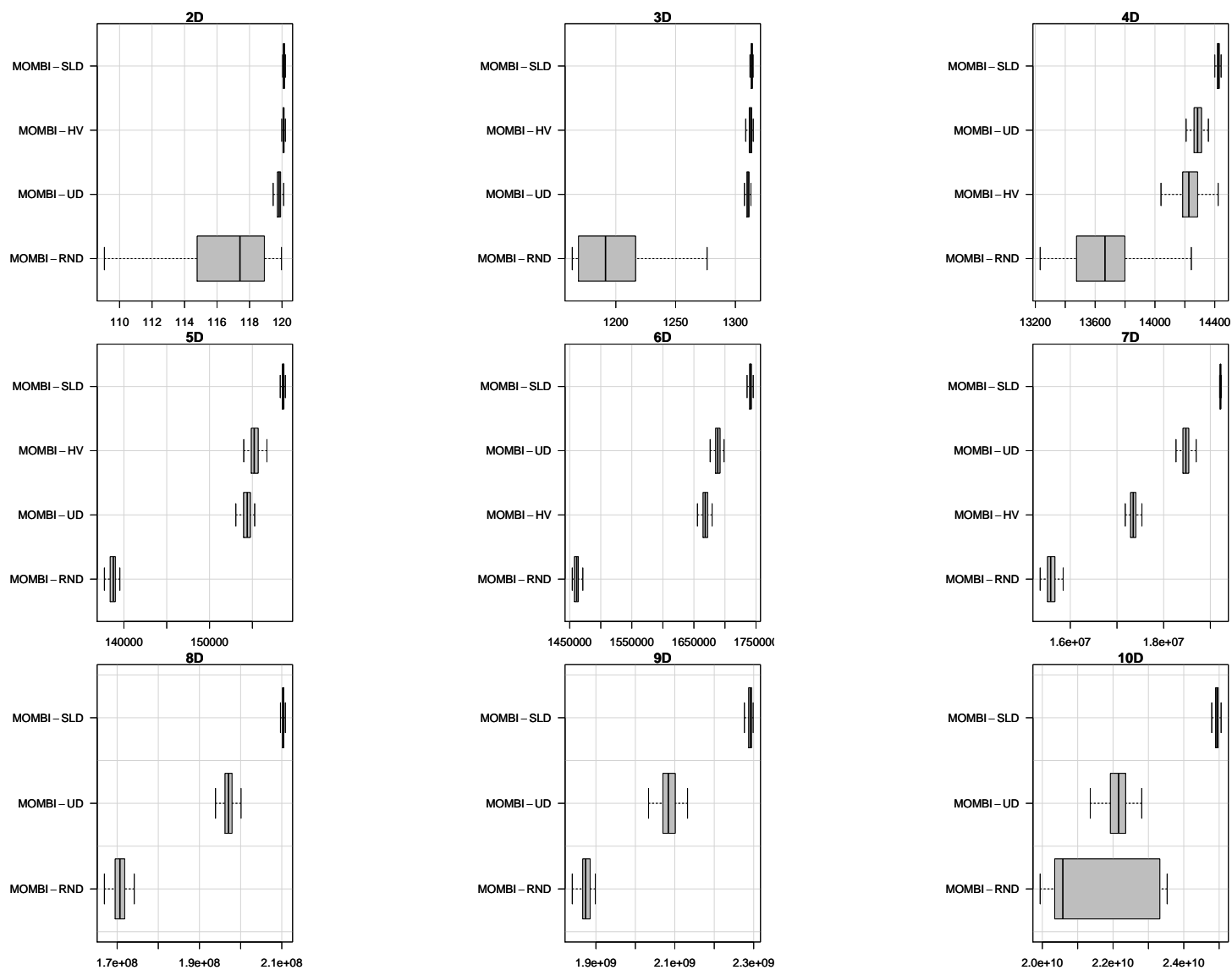


Figure A.447: Box-plot of the hypervolume indicator values for different optimizers on the DTLZ6 test problem.

Table A.194: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ6 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $ph$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
2D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e-34$	–	$9.91e-05$	$2.20e-33$
MOMBI-HV	$1.28e-34$	> 0.05	–	$3.57e-32$
MOMBI-UD	$1.06e-28$	> 0.05	> 0.05	–
3D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e-34$	–	$8.69e-10$	$6.24e-34$
MOMBI-HV	$1.28e-34$	> 0.05	–	$2.39e-17$
MOMBI-UD	$1.28e-34$	> 0.05	> 0.05	–
4D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e-34$	–	$1.24e-33$	$1.28e-34$
MOMBI-HV	$2.38e-32$	> 0.05	–	> 0.05
MOMBI-UD	$1.95e-34$	> 0.05	$3.66e-09$	–
5D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e-34$	–	$1.32e-34$	$1.28e-34$
MOMBI-HV	$1.28e-34$	> 0.05	–	$1.18e-19$
MOMBI-UD	$1.28e-34$	> 0.05	> 0.05	–
6D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e-34$	–	$1.28e-34$	$1.28e-34$
MOMBI-HV	$1.28e-34$	> 0.05	–	> 0.05
MOMBI-UD	$1.28e-34$	> 0.05	$3.37e-32$	–
7D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e-34$	–	$1.28e-34$	$1.28e-34$
MOMBI-HV	$1.28e-34$	> 0.05	–	> 0.05
MOMBI-UD	$1.28e-34$	> 0.05	$1.28e-34$	–
8D				
MOMBI-RND	–	> 0.05		> 0.05
MOMBI-SLD	$1.28e-34$	–		$1.28e-34$
MOMBI-HV			–	
MOMBI-UD	$1.49e-34$	> 0.05		–
9D				
MOMBI-RND	–	> 0.05		> 0.05
MOMBI-SLD	$1.28e-34$	–		$1.28e-34$
MOMBI-HV			–	
MOMBI-UD	$2.83e-21$	> 0.05		–
10D				
MOMBI-RND	–	> 0.05		> 0.05
MOMBI-SLD	$1.28e-34$	–		$1.28e-34$
MOMBI-HV			–	
MOMBI-UD	$9.63e-09$	> 0.05		–



Table A.195: Comparison of R2 indicator values for different optimizers on the DTLZ6 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	2.609845e-01	2.306198e-01	2.311215e-01	2.323154e-01
	avg.	3.000936e-01	2.297734e-01	2.304994e-01	2.316399e-01
	min.	2.190107e-01	2.180227e-01	2.180284e-01	2.185264e-01
	max.	5.424620e-01	2.418767e-01	2.428211e-01	2.418304e-01
	std.	8.729808e-02	5.528291e-03	5.198796e-03	5.086274e-03
<b>3D</b>	med.	3.025263e-01	1.964833e-01	1.923685e-01	2.023508e-01
	avg.	3.012250e-01	1.960103e-01	1.925752e-01	2.021724e-01
	min.	1.931911e-01	1.849071e-01	1.851564e-01	1.938558e-01
	max.	3.567755e-01	2.028018e-01	1.990818e-01	2.085221e-01
	std.	4.495708e-02	3.002515e-03	2.607660e-03	2.919059e-03
<b>4D</b>	med.	1.611568e-01	1.501110e-01	1.491212e-01	1.793900e-01
	avg.	1.627300e-01	1.499335e-01	1.491298e-01	1.795545e-01
	min.	1.387021e-01	1.445148e-01	1.413868e-01	1.689456e-01
	max.	1.911870e-01	1.559031e-01	1.565657e-01	1.904022e-01
	std.	1.319013e-02	2.435225e-03	3.073275e-03	4.904024e-03
<b>5D</b>	med.	1.806226e-01	1.141388e-01	1.251024e-01	1.831062e-01
	avg.	1.827446e-01	1.141551e-01	1.256727e-01	1.830991e-01
	min.	1.780477e-01	1.081304e-01	1.204025e-01	1.660263e-01
	max.	1.938621e-01	1.196324e-01	1.351612e-01	1.956008e-01
	std.	4.435107e-03	2.209391e-03	2.925530e-03	5.869596e-03
<b>6D</b>	med.	1.779879e-01	1.001913e-01	1.385454e-01	1.689574e-01
	avg.	1.776180e-01	1.003045e-01	1.391083e-01	1.689114e-01
	min.	1.689763e-01	9.421956e-02	1.265860e-01	1.607785e-01
	max.	1.841635e-01	1.071656e-01	1.546558e-01	1.808255e-01
	std.	3.279820e-03	2.804992e-03	5.214671e-03	3.995460e-03
<b>7D</b>	med.	1.562211e-01	7.464275e-02	1.392130e-01	1.382406e-01
	avg.	1.561920e-01	7.480907e-02	1.391111e-01	1.384856e-01
	min.	1.462467e-01	7.267178e-02	1.316348e-01	1.273736e-01
	max.	1.656886e-01	7.758847e-02	1.513013e-01	1.523659e-01
	std.	3.790442e-03	9.632929e-04	3.919896e-03	4.972467e-03
<b>8D</b>	med.	1.360009e-01	7.513981e-02	—	1.503096e-01
	avg.	1.366332e-01	7.536262e-02	—	1.499026e-01
	min.	1.268087e-01	6.887399e-02	—	1.394908e-01
	max.	1.786413e-01	8.187718e-02	—	1.627666e-01
	std.	5.525762e-03	2.064763e-03	—	4.365209e-03
<b>9D</b>	med.	1.190363e-01	8.412579e-02	—	1.537937e-01
	avg.	1.194695e-01	8.455333e-02	—	1.534585e-01
	min.	1.138745e-01	7.919593e-02	—	1.429086e-01
	max.	1.288905e-01	9.115897e-02	—	1.632760e-01
	std.	2.908422e-03	2.534970e-03	—	4.846012e-03
<b>10D</b>	med.	1.076273e-01	9.001985e-02	—	1.506220e-01
	avg.	1.091156e-01	9.013414e-02	—	1.515456e-01
	min.	1.032354e-01	8.414671e-02	—	1.398915e-01
	max.	1.534629e-01	9.508894e-02	—	1.615961e-01
	std.	7.860519e-03	2.126978e-03	—	4.471010e-03

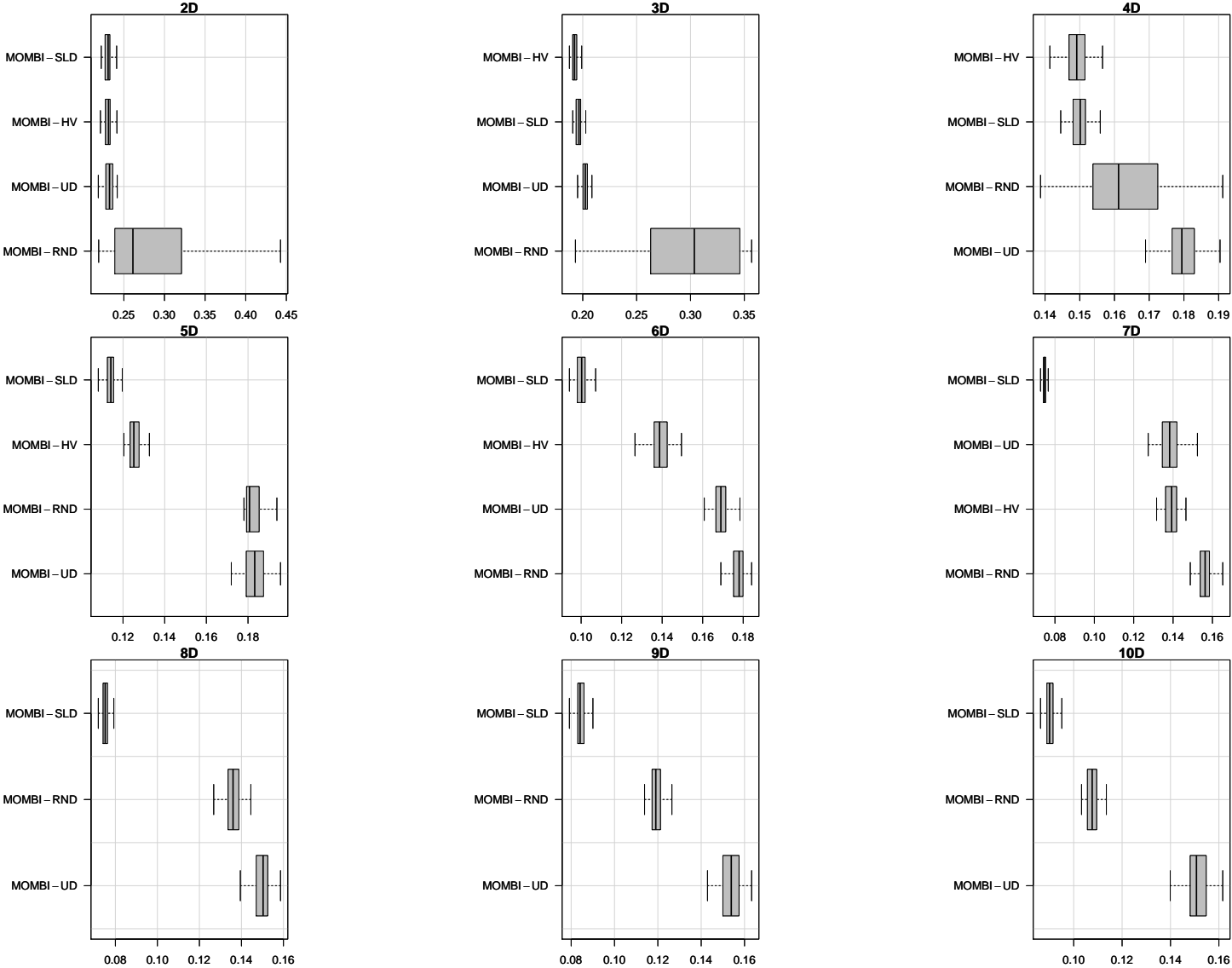


Figure A.448: Box-plot of the R2 indicator values for different optimizers on the DTLZ6 test problem.

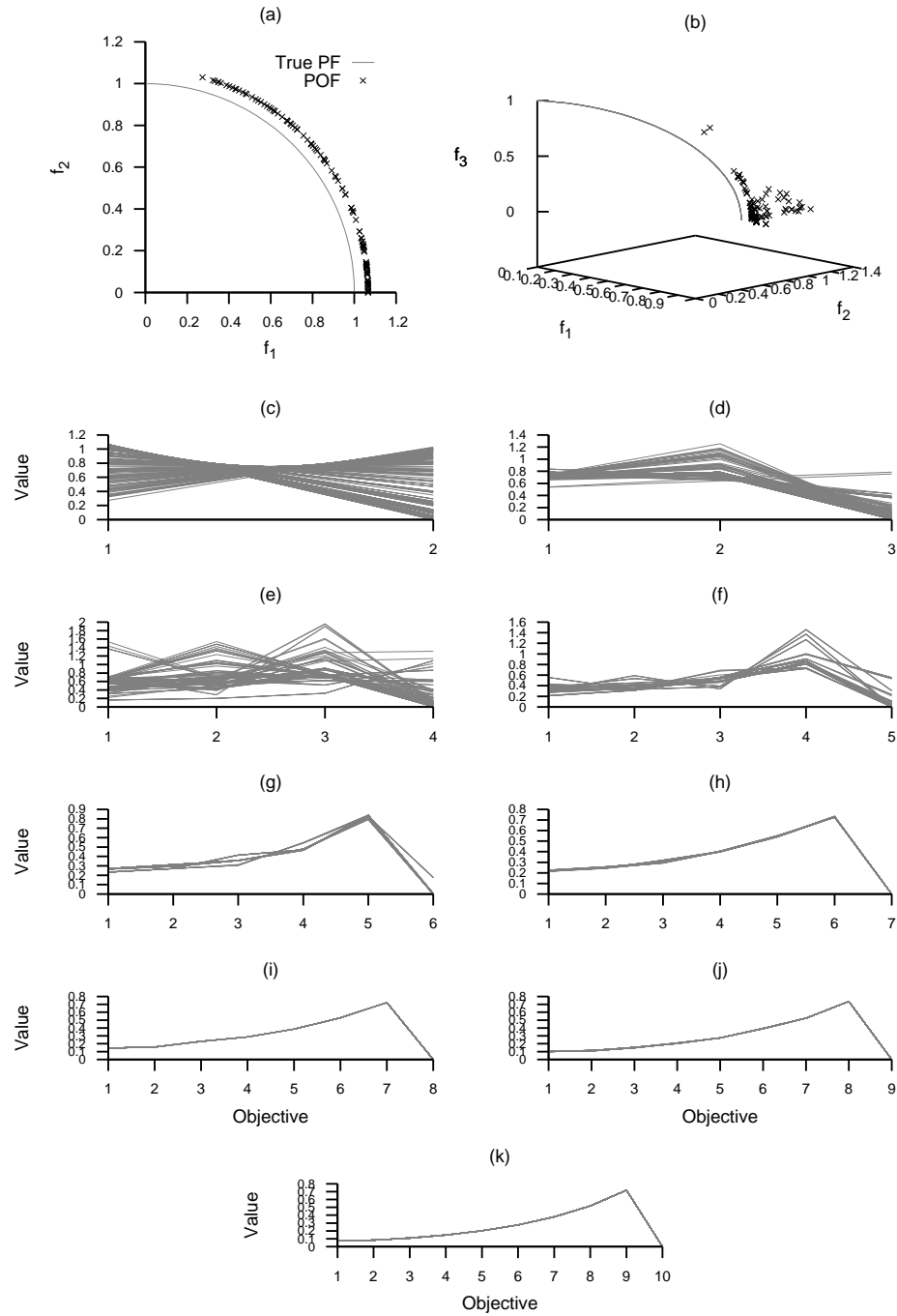


Figure A.449: Plots of the approximations obtained by MOMBI-RND from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

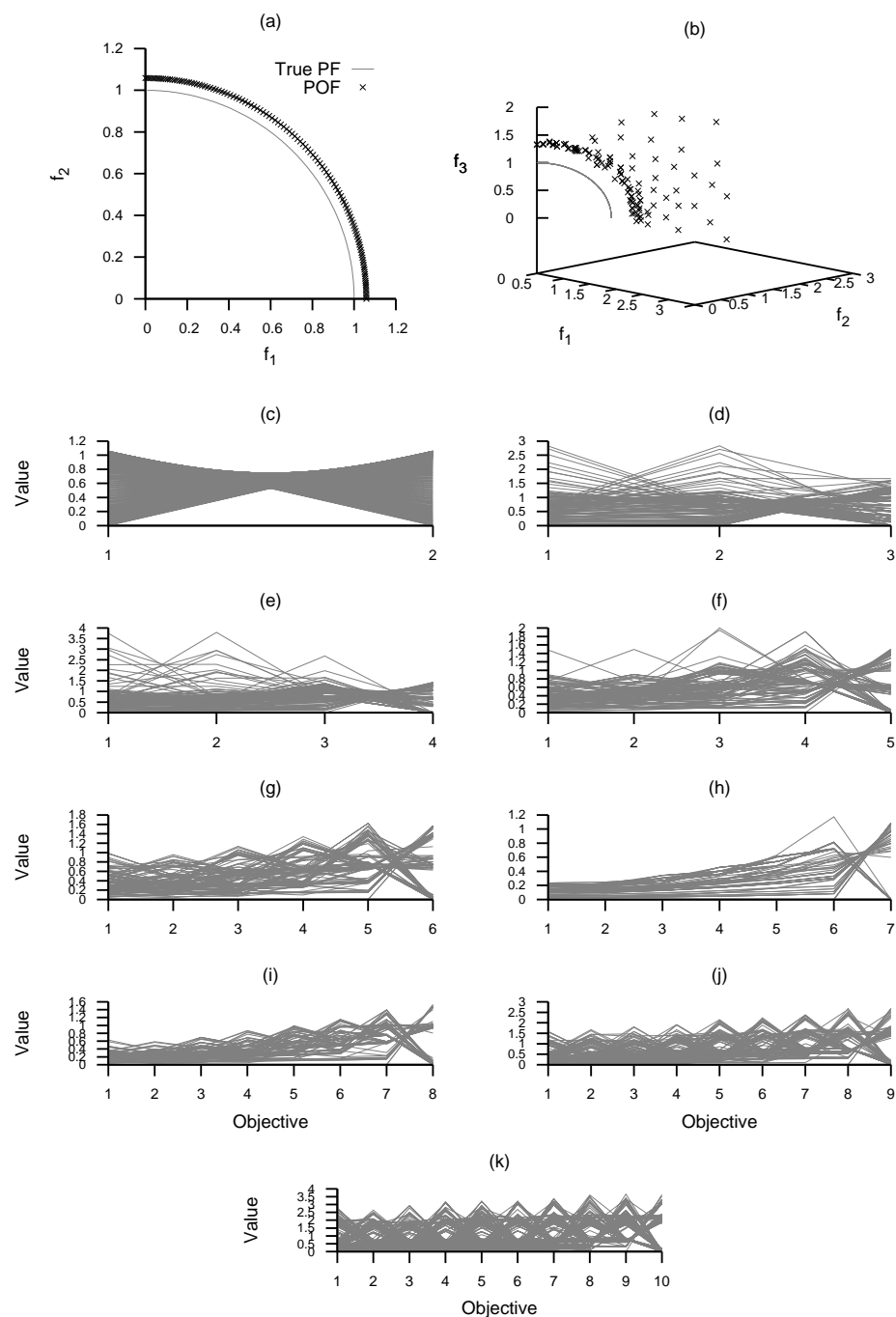


Figure A.450: Plots of the approximations obtained by MOMBI-SLD from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

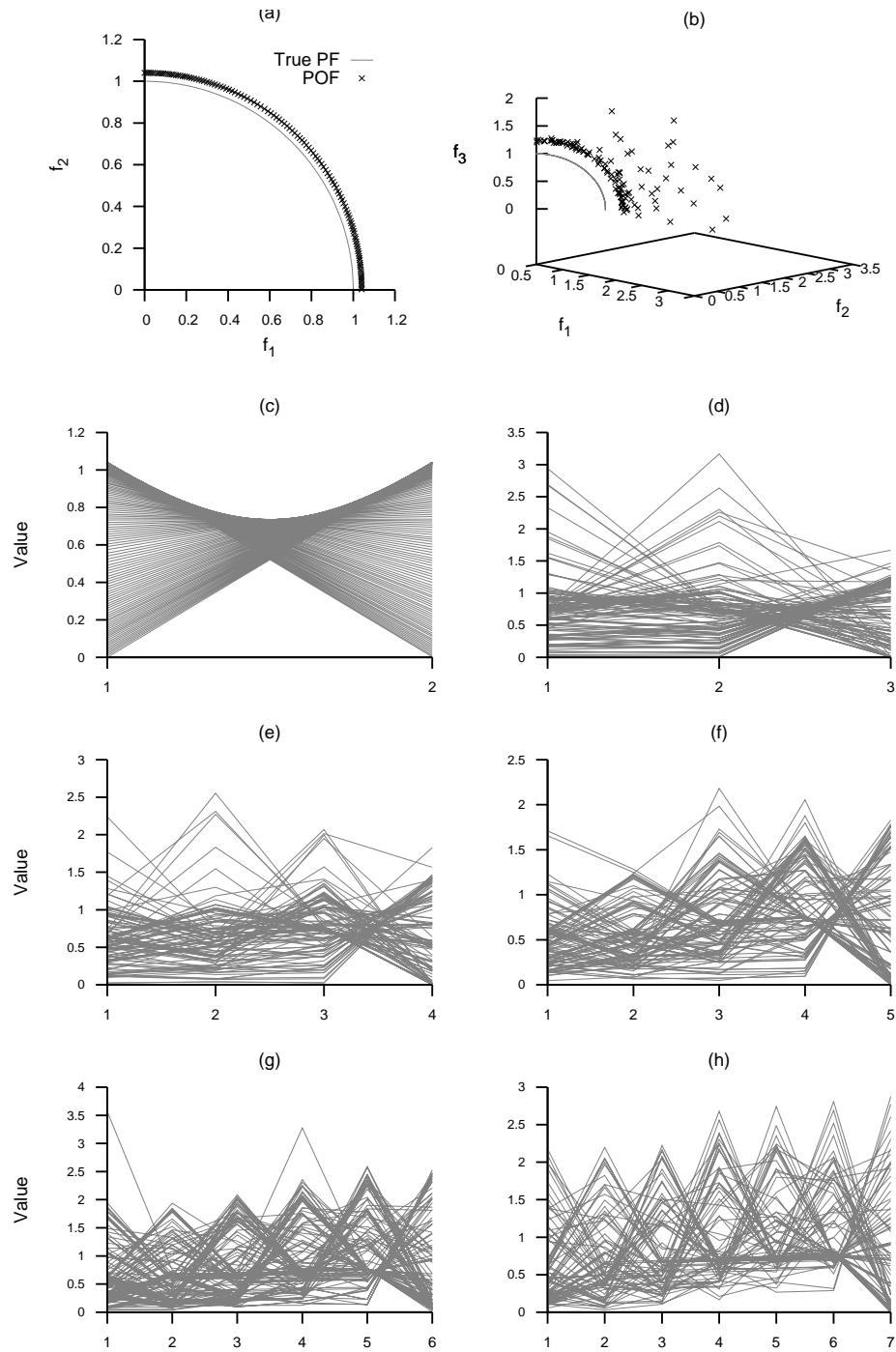


Figure A.451: Plots of the approximations obtained by MOMBI-HV from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

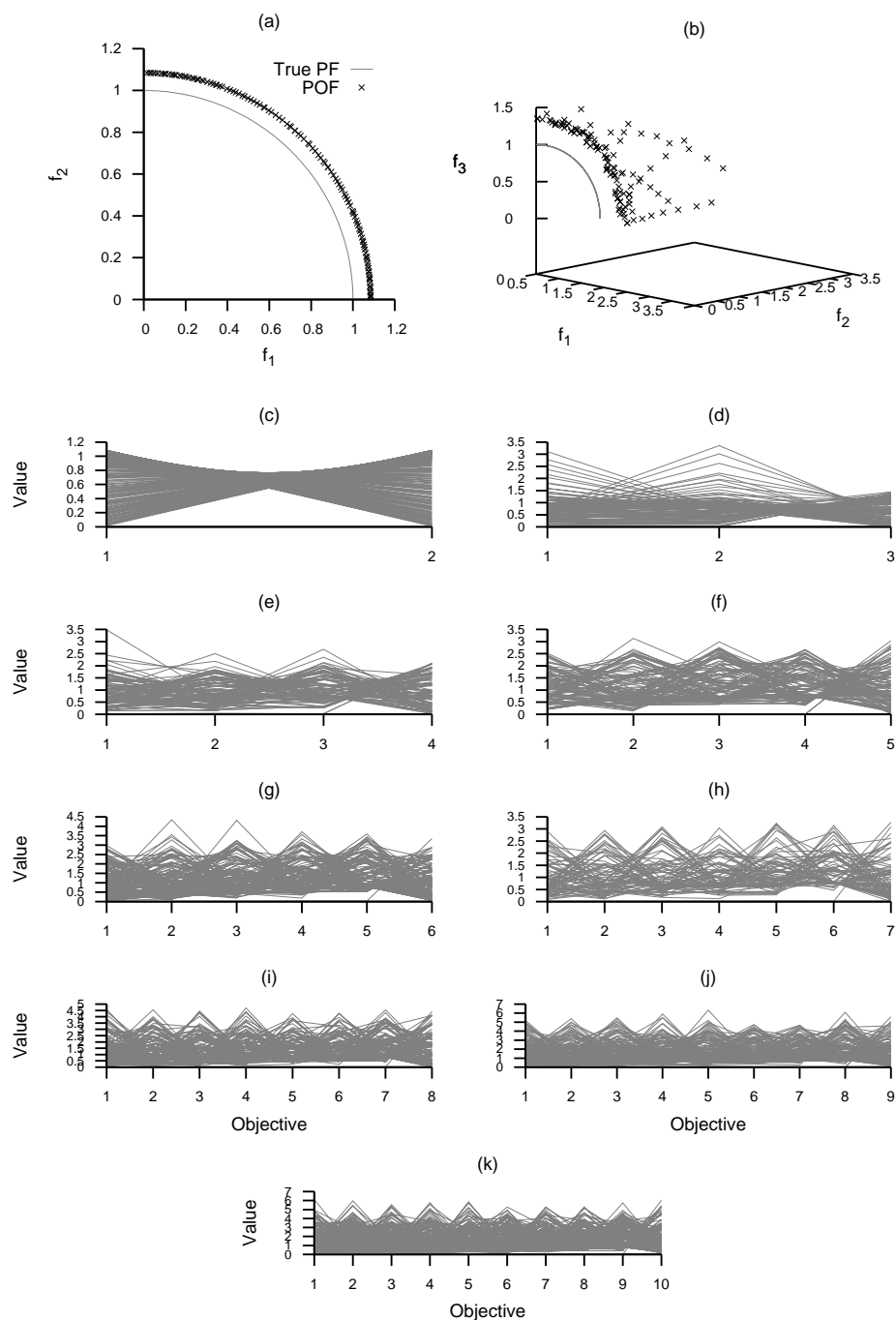


Figure A.452: Plots of the approximations obtained by MOMBI-UD from 2 to 10 objectives on the DTLZ6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.196: Comparison of hypervolume indicator values for different optimizers on the DTLZ7 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	4.217578e + 00	1.771479e + 01	1.771865e + 01	1.594218e + 01
	avg.	4.289827e + 00	1.771472e + 01	1.771782e + 01	1.589985e + 01
	min.	3.830904e + 00	1.770259e + 01	1.770742e + 01	1.365838e + 01
	max.	5.569846e + 00	1.772196e + 01	1.772189e + 01	1.764943e + 01
	std.	3.055069e - 01	4.494521e - 03	2.955984e - 03	1.177970e + 00
<b>3D</b>	med.	8.595908e + 00	1.624634e + 01	1.613667e + 01	1.287726e + 01
	avg.	8.100962e + 00	1.624572e + 01	1.613261e + 01	1.269213e + 01
	min.	2.324645e + 00	1.623277e + 01	1.606181e + 01	2.620570e + 00
	max.	1.128866e + 01	1.625675e + 01	1.617631e + 01	1.580816e + 01
	std.	2.056228e + 00	6.091022e - 03	2.414417e - 02	2.095431e + 00
<b>4D</b>	med.	1.317300e - 01	1.410370e + 01	1.057627e + 01	2.792129e + 00
	avg.	2.385810e - 01	1.410058e + 01	1.025876e + 01	5.403884e + 00
	min.	8.754830e - 02	1.399617e + 01	6.879746e + 00	3.131912e - 01
	max.	9.770391e - 01	1.414433e + 01	1.214337e + 01	1.161779e + 01
	std.	2.302297e - 01	2.571520e - 02	1.367797e + 00	3.687247e + 00
<b>5D</b>	med.	1.592767e - 02	8.209755e + 00	5.230590e + 00	3.072706e - 01
	avg.	1.626308e - 02	7.180385e + 00	4.826793e + 00	1.650196e + 00
	min.	1.294995e - 02	3.314862e - 01	7.970508e - 01	1.751797e - 02
	max.	2.409104e - 02	1.155211e + 01	7.565992e + 00	7.866020e + 00
	std.	2.369419e - 03	3.451955e + 00	1.743212e + 00	2.324595e + 00
<b>6D</b>	med.	2.173237e - 03	1.630909e - 01	7.195688e - 02	2.625938e - 02
	avg.	2.233892e - 03	1.912855e - 01	2.513574e - 01	7.810007e - 02
	min.	1.792256e - 03	3.482859e - 02	1.626990e - 02	2.887399e - 03
	max.	3.365766e - 03	1.696014e + 00	1.175718e + 00	1.084273e + 00
	std.	3.010848e - 04	2.539030e - 01	2.757068e - 01	1.823078e - 01
<b>7D</b>	med.	2.929255e - 04	5.595103e - 03	6.682353e - 04	3.503613e - 03
	avg.	3.025953e - 04	1.050742e - 02	7.926399e - 04	6.172799e - 03
	min.	2.553336e - 04	3.996743e - 04	3.853081e - 04	3.851478e - 04
	max.	1.203926e - 03	1.368809e - 01	3.748919e - 03	2.661368e - 02
	std.	9.320934e - 05	1.496899e - 02	6.068743e - 04	7.301560e - 03
<b>8D</b>	med.	5.411617e - 05	7.625796e - 04	—	2.203271e - 04
	avg.	5.825629e - 05	1.040001e - 03	—	5.213897e - 04
	min.	4.492400e - 05	1.030993e - 04	—	8.346673e - 05
	max.	2.675674e - 04	3.900317e - 03	—	2.989406e - 03
	std.	2.959479e - 05	9.206921e - 04	—	6.650307e - 04
<b>9D</b>	med.	9.059224e - 06	1.189006e - 04	—	2.486323e - 05
	avg.	9.226589e - 06	1.242922e - 04	—	6.327687e - 05
	min.	7.941575e - 06	1.529420e - 05	—	1.580757e - 05
	max.	1.199143e - 05	5.076035e - 04	—	5.223932e - 04
	std.	8.638603e - 07	7.245688e - 05	—	8.727582e - 05
<b>10D</b>	med.	1.610530e - 06	1.808261e - 05	—	3.814977e - 06
	avg.	1.632002e - 06	1.697813e - 05	—	8.231580e - 06
	min.	1.344195e - 06	2.753713e - 06	—	2.621521e - 06
	max.	1.976859e - 06	7.262282e - 05	—	5.546235e - 05
	std.	1.383755e - 07	1.064368e - 05	—	8.170808e - 06

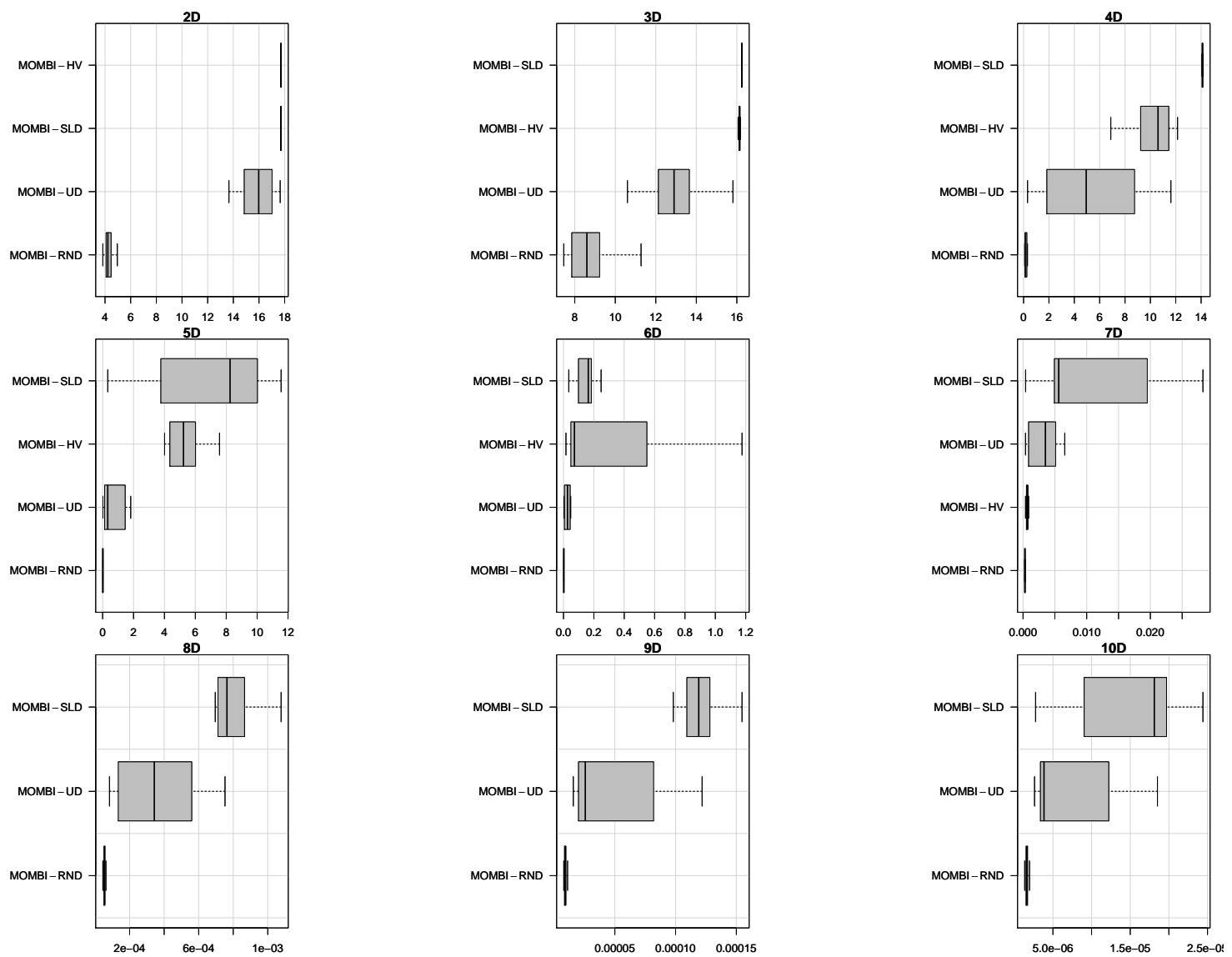


Figure A.453: Box-plot of the hypervolume indicator values for different optimizers on the DTLZ7 test problem.



Table A.197: Wilcoxon rank sum test applied to the hypervolume indicator values on the DTLZ7 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $ph$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
2D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e - 34$	—	$> 0.05$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	$1.07e - 07$	—	$1.28e - 34$
MOMBI-UD	$1.28e - 34$	$> 0.05$	$> 0.05$	—
3D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e - 34$	—	$1.28e - 34$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	$> 0.05$	—	$1.28e - 34$
MOMBI-UD	$1.42e - 30$	$> 0.05$	$> 0.05$	—
4D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e - 34$	—	$1.28e - 34$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	$> 0.05$	—	$3.33e - 21$
MOMBI-UD	$3.15e - 34$	$> 0.05$	$> 0.05$	—
5D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e - 34$	—	$3.75e - 11$	$1.33e - 26$
MOMBI-HV	$1.28e - 34$	$> 0.05$	—	$1.53e - 15$
MOMBI-UD	$3.65e - 34$	$> 0.05$	$> 0.05$	—
6D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e - 34$	—	$> 0.05$	$2.62e - 20$
MOMBI-HV	$1.28e - 34$	$> 0.05$	—	$6.95e - 14$
MOMBI-UD	$1.68e - 34$	$> 0.05$	$> 0.05$	—
7D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.32e - 34$	—	$2.00e - 32$	$1.94e - 10$
MOMBI-HV	$1.98e - 33$	$> 0.05$	—	$> 0.05$
MOMBI-UD	$3.65e - 34$	$> 0.05$	$1.14e - 25$	—
8D				
MOMBI-RND	—	$> 0.05$		$> 0.05$
MOMBI-SLD	$2.07e - 34$	—		$2.14e - 16$
MOMBI-HV			—	
MOMBI-UD	$2.51e - 33$	$> 0.05$		—
9D				
MOMBI-RND	—	$> 0.05$		$> 0.05$
MOMBI-SLD	$1.28e - 34$	—		$1.06e - 21$
MOMBI-HV			—	
MOMBI-UD	$1.28e - 34$	$> 0.05$		—
10D				
MOMBI-RND	—	$> 0.05$		$> 0.05$
MOMBI-SLD	$1.28e - 34$	—		$3.20e - 17$
MOMBI-HV			—	
MOMBI-UD	$1.28e - 34$	$> 0.05$		—

Table A.198: Comparison of R2 indicator values for different optimizers on the DTLZ7 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	1.251393e + 00	1.180573e + 00	1.181831e + 00	1.188027e + 00
	avg.	1.250750e + 00	1.180597e + 00	1.181820e + 00	1.188822e + 00
	min.	1.238107e + 00	1.180211e + 00	1.180282e + 00	1.181065e + 00
	max.	1.255837e + 00	1.181104e + 00	1.184763e + 00	1.201038e + 00
	std.	3.220865e - 03	2.285352e - 04	9.341892e - 04	4.521911e - 03
<b>3D</b>	med.	9.162457e - 01	9.024650e - 01	9.039877e - 01	9.180904e - 01
	avg.	9.184833e - 01	9.024707e - 01	9.039243e - 01	9.184497e - 01
	min.	9.079576e - 01	9.021539e - 01	9.029889e - 01	9.038272e - 01
	max.	9.467811e - 01	9.030330e - 01	9.046555e - 01	9.843893e - 01
	std.	8.483473e - 03	1.742986e - 04	3.639698e - 04	1.181497e - 02
<b>4D</b>	med.	8.375953e - 01	7.796026e - 01	7.806718e - 01	8.046982e - 01
	avg.	8.343637e - 01	7.790660e - 01	7.805692e - 01	8.050740e - 01
	min.	8.037944e - 01	7.713191e - 01	7.685838e - 01	7.801813e - 01
	max.	8.438179e - 01	7.832957e - 01	7.902733e - 01	8.356847e - 01
	std.	9.769489e - 03	2.797371e - 03	3.688277e - 03	1.357711e - 02
<b>5D</b>	med.	7.837828e - 01	7.500165e - 01	7.559506e - 01	7.816793e - 01
	avg.	7.838319e - 01	7.500688e - 01	7.556260e - 01	7.859959e - 01
	min.	7.779972e - 01	7.316444e - 01	7.406009e - 01	7.550067e - 01
	max.	7.888562e - 01	7.625244e - 01	7.682999e - 01	8.319696e - 01
	std.	2.240732e - 03	6.389577e - 03	5.244116e - 03	1.927833e - 02
<b>6D</b>	med.	6.851694e - 01	6.663450e - 01	6.671115e - 01	6.848384e - 01
	avg.	6.847583e - 01	6.651875e - 01	6.668039e - 01	6.842107e - 01
	min.	6.783777e - 01	6.373136e - 01	6.514049e - 01	6.683836e - 01
	max.	6.878141e - 01	6.726222e - 01	6.797808e - 01	7.005942e - 01
	std.	1.931721e - 03	5.416971e - 03	6.198798e - 03	6.709294e - 03
<b>7D</b>	med.	6.307099e - 01	6.221614e - 01	6.281307e - 01	6.381122e - 01
	avg.	6.323171e - 01	6.230698e - 01	6.340106e - 01	6.370708e - 01
	min.	6.283844e - 01	6.053582e - 01	6.201436e - 01	6.189951e - 01
	max.	7.877582e - 01	7.870757e - 01	7.909695e - 01	6.545679e - 01
	std.	1.573862e - 02	2.273020e - 02	2.771274e - 02	7.325040e - 03
<b>8D</b>	med.	5.943335e - 01	5.831100e - 01	—	5.929952e - 01
	avg.	5.970652e - 01	5.819380e - 01	—	5.937037e - 01
	min.	5.912287e - 01	5.727312e - 01	—	5.785827e - 01
	max.	7.356945e - 01	5.890586e - 01	—	7.368860e - 01
	std.	1.980093e - 02	3.650490e - 03	—	1.567082e - 02
<b>9D</b>	med.	5.720244e - 01	5.626002e - 01	—	5.692522e - 01
	avg.	5.720072e - 01	5.624158e - 01	—	5.687488e - 01
	min.	5.688434e - 01	5.563507e - 01	—	5.564812e - 01
	max.	5.737068e - 01	5.670006e - 01	—	5.797291e - 01
	std.	9.576361e - 04	1.942341e - 03	—	5.385329e - 03
<b>10D</b>	med.	5.519230e - 01	5.409959e - 01	—	5.465230e - 01
	avg.	5.518567e - 01	5.409024e - 01	—	5.468312e - 01
	min.	5.482888e - 01	5.348026e - 01	—	5.352019e - 01
	max.	5.543813e - 01	5.463542e - 01	—	5.544662e - 01
	std.	9.551607e - 04	1.727795e - 03	—	3.916913e - 03

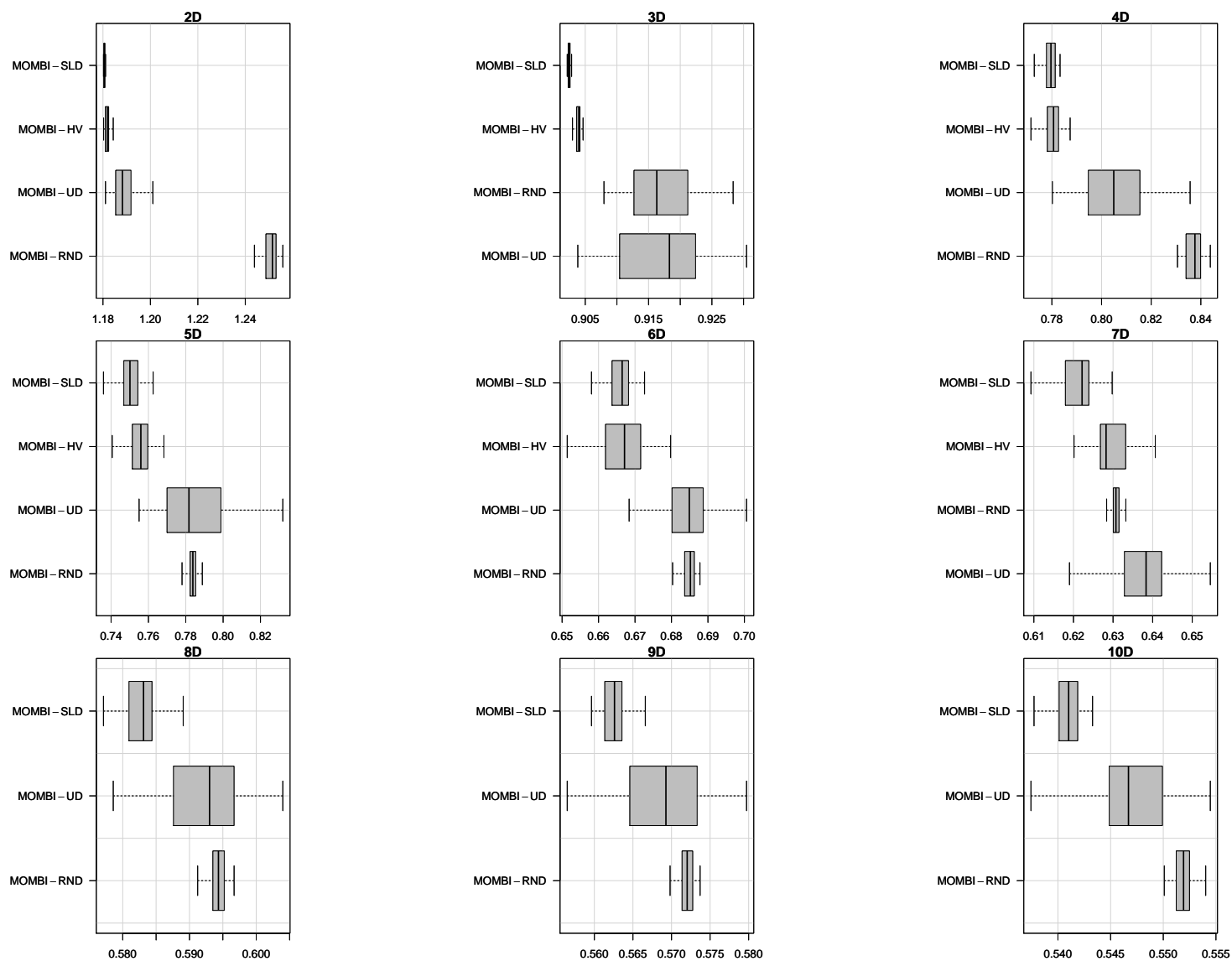


Figure A.454: Box-plot of the R2 indicator values for different optimizers on the DTLZ7 test problem.

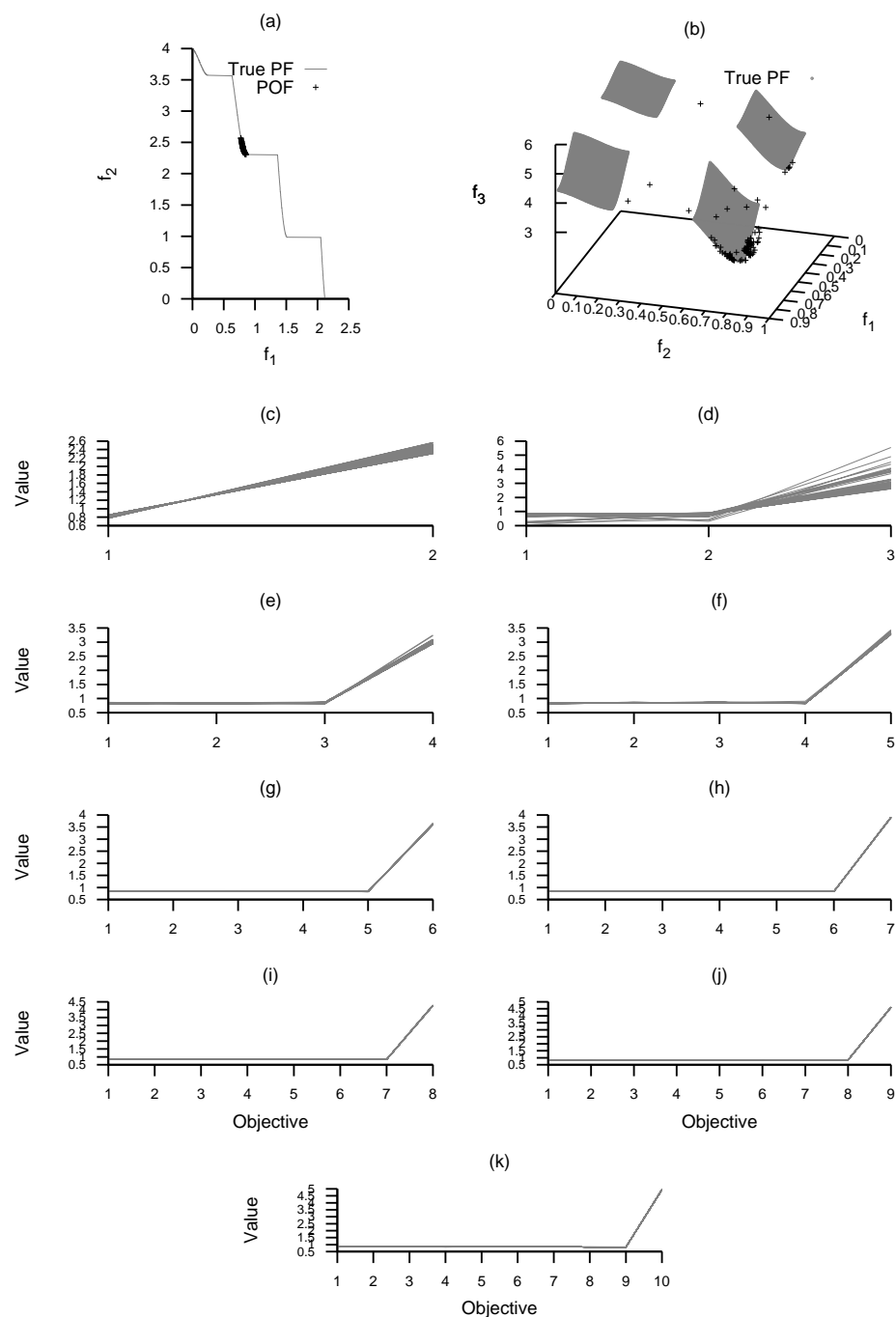


Figure A.455: Plots of the approximations obtained by MOMBI-RND from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

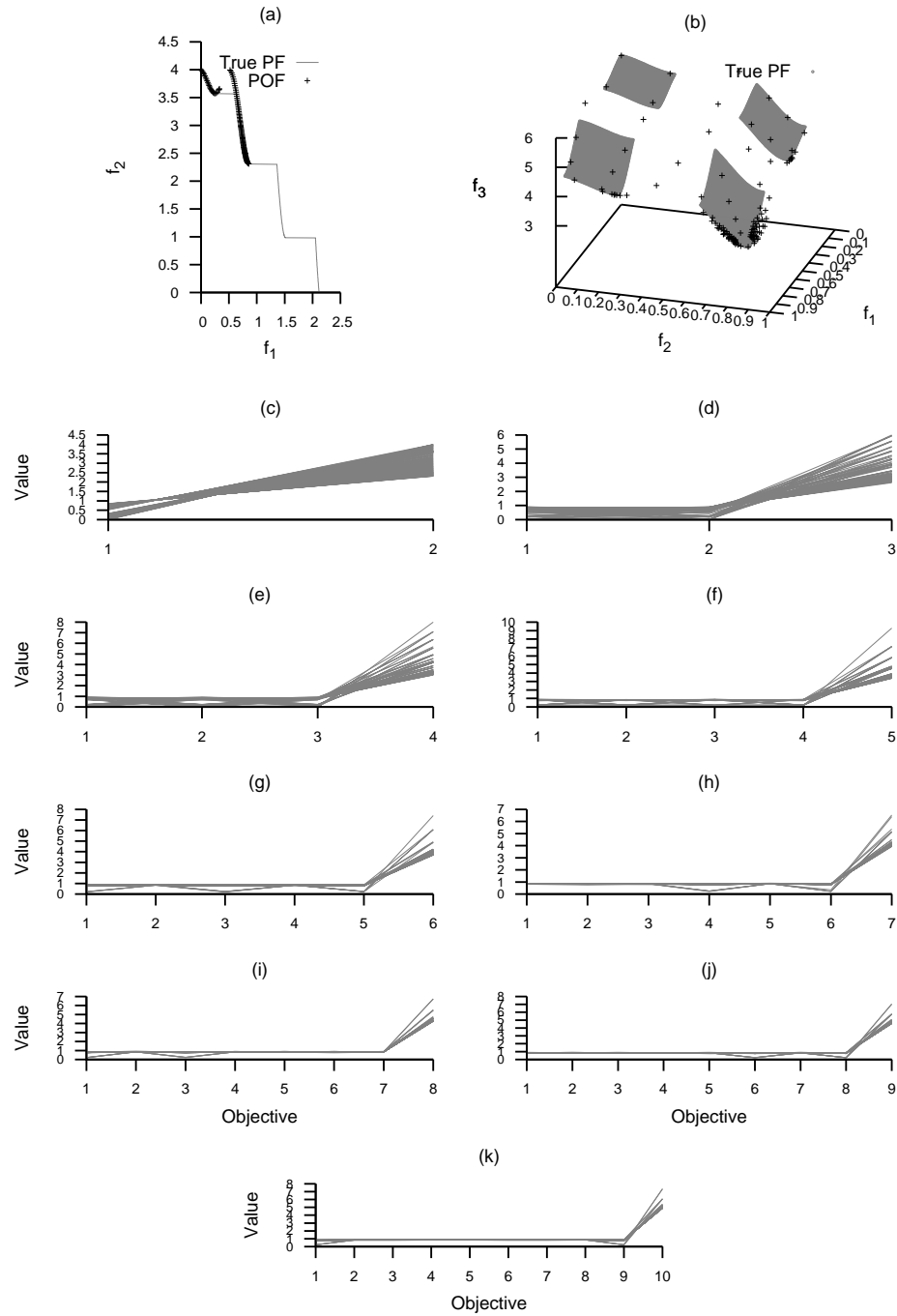


Figure A.456: Plots of the approximations obtained by MOMBI-SLD from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

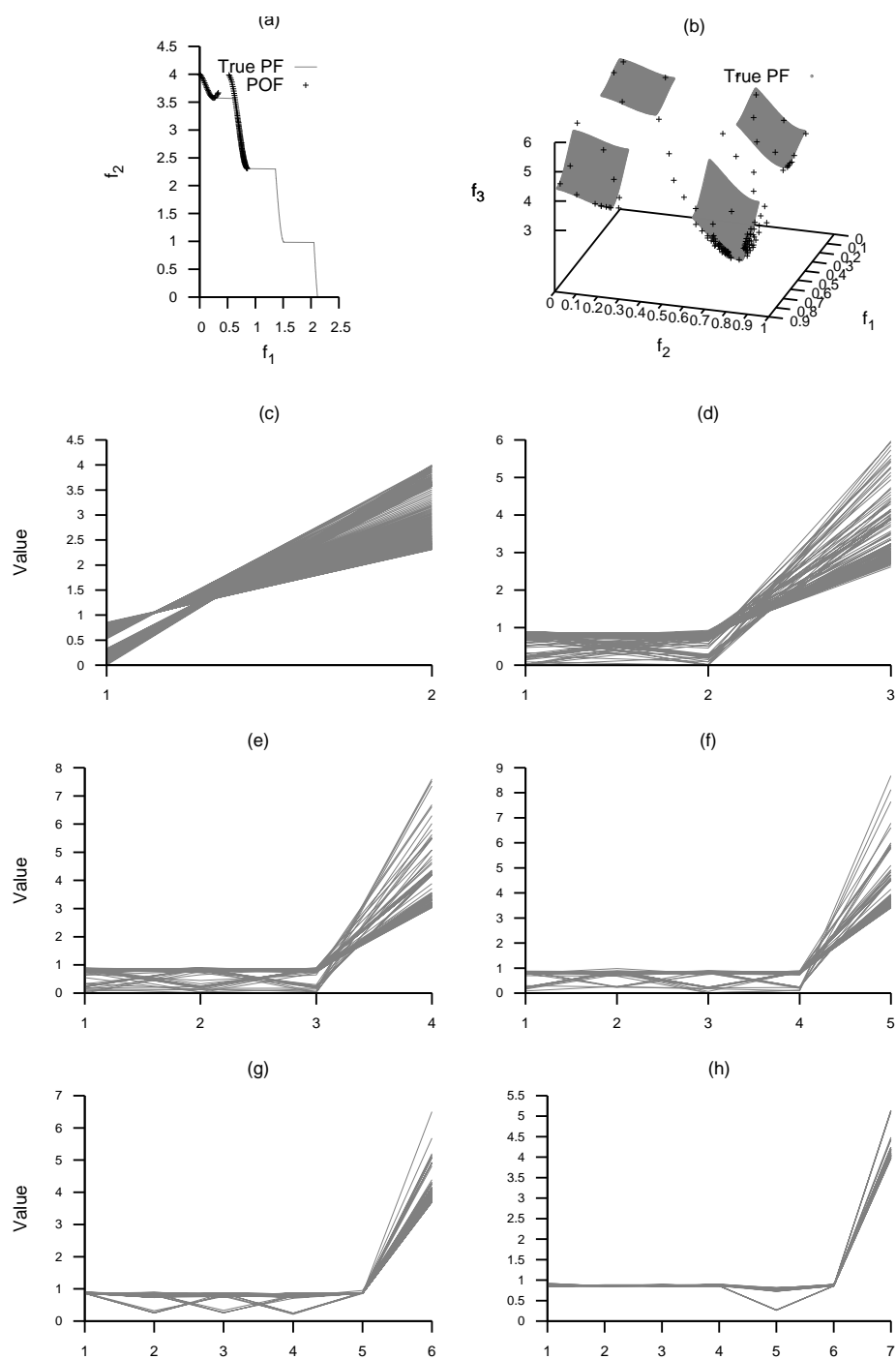


Figure A.457: Plots of the approximations obtained by MOMBI-HV from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

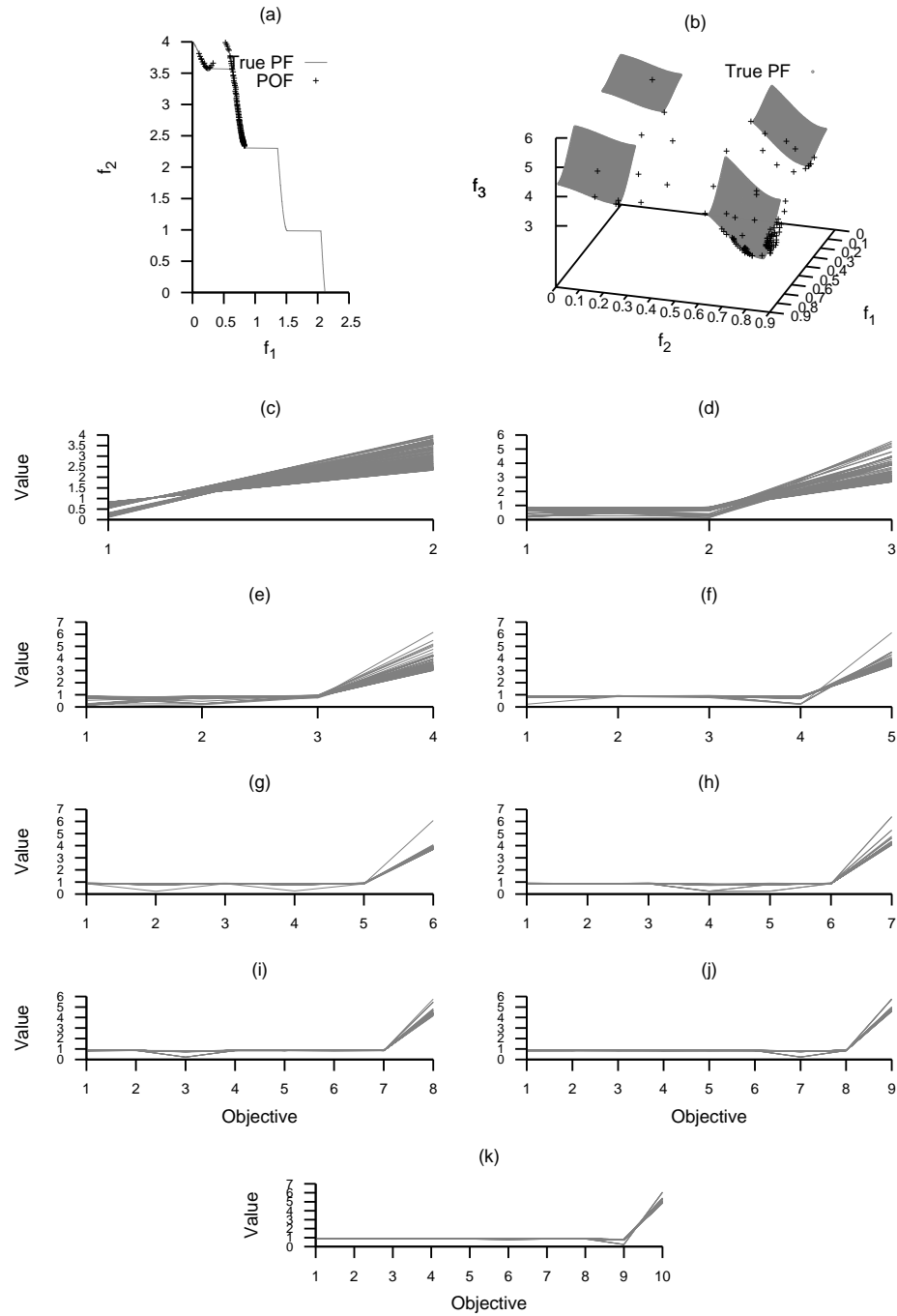


Figure A.458: Plots of the approximations obtained by MOMBI-UD from 2 to 10 objectives on the DTLZ7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.199: Comparison of hypervolume indicator values for different optimizers on the WFG1 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	3.305723e + 00	4.988857e + 00	4.954650e + 00	4.691035e + 00
	avg.	3.368751e + 00	4.984418e + 00	4.981717e + 00	4.688182e + 00
	min.	2.749734e + 00	4.445552e + 00	4.515806e + 00	3.956468e + 00
	max.	4.553368e + 00	5.527258e + 00	5.637887e + 00	5.254322e + 00
	std.	3.164915e - 01	2.198463e - 01	2.058030e - 01	2.541786e - 01
<b>3D</b>	med.	4.089924e + 01	4.914331e + 01	4.851307e + 01	3.767218e + 01
	avg.	4.251617e + 01	4.834862e + 01	4.765282e + 01	3.881441e + 01
	min.	3.758082e + 01	4.321601e + 01	4.210778e + 01	3.558766e + 01
	max.	4.886526e + 01	5.087687e + 01	5.105889e + 01	4.679009e + 01
	std.	3.250336e + 00	2.171869e + 00	2.436233e + 00	3.076249e + 00
<b>4D</b>	med.	3.651223e + 02	3.926037e + 02	3.703557e + 02	3.216677e + 02
	avg.	3.629534e + 02	3.883287e + 02	3.778953e + 02	3.231147e + 02
	min.	3.278479e + 02	3.578290e + 02	3.483811e + 02	3.062104e + 02
	max.	4.033946e + 02	4.184153e + 02	4.096835e + 02	3.739801e + 02
	std.	1.919803e + 01	1.850285e + 01	1.891759e + 01	1.029491e + 01
<b>5D</b>	med.	3.933256e + 03	3.722174e + 03	3.635974e + 03	3.437105e + 03
	avg.	3.910025e + 03	3.746024e + 03	3.665712e + 03	3.434356e + 03
	min.	3.548095e + 03	3.585311e + 03	3.460207e + 03	3.269744e + 03
	max.	4.146729e + 03	4.139490e + 03	4.080490e + 03	3.588186e + 03
	std.	1.339574e + 02	1.105658e + 02	1.158227e + 02	5.731464e + 01
<b>6D</b>	med.	4.649412e + 04	4.554310e + 04	4.331216e + 04	4.226709e + 04
	avg.	4.691174e + 04	4.551963e + 04	4.360388e + 04	4.225501e + 04
	min.	4.306454e + 04	4.401062e + 04	4.186799e + 04	4.054204e + 04
	max.	5.104506e + 04	4.959969e + 04	4.801090e + 04	4.439740e + 04
	std.	2.002018e + 03	7.652575e + 02	1.243307e + 03	7.235028e + 02
<b>7D</b>	med.	6.901212e + 05	6.829535e + 05	6.326743e + 05	6.254289e + 05
	avg.	6.887075e + 05	6.825417e + 05	6.347560e + 05	6.289899e + 05
	min.	6.151523e + 05	6.355952e + 05	5.967520e + 05	5.941108e + 05
	max.	7.310202e + 05	7.359148e + 05	7.062827e + 05	7.004084e + 05
	std.	2.249462e + 04	1.799570e + 04	1.779146e + 04	1.540721e + 04
<b>8D</b>	med.	1.147202e + 07	1.136502e + 07	—	1.042738e + 07
	avg.	1.148426e + 07	1.136923e + 07	—	1.043538e + 07
	min.	1.071276e + 07	1.055121e + 07	—	1.000253e + 07
	max.	1.215327e + 07	1.225547e + 07	—	1.085440e + 07
	std.	3.024182e + 05	2.622769e + 05	—	1.761885e + 05
<b>9D</b>	med.	2.062483e + 08	2.053557e + 08	—	1.895092e + 08
	avg.	2.056125e + 08	2.052052e + 08	—	1.894928e + 08
	min.	1.929038e + 08	1.957923e + 08	—	1.823042e + 08
	max.	2.176248e + 08	2.116231e + 08	—	1.958473e + 08
	std.	4.768403e + 06	2.928029e + 06	—	2.934750e + 06
<b>10D</b>	med.	4.027571e + 09	4.052464e + 09	—	3.790214e + 09
	avg.	4.028210e + 09	4.048627e + 09	—	3.793651e + 09
	min.	3.821664e + 09	3.943880e + 09	—	3.684412e + 09
	max.	4.250108e + 09	4.147987e + 09	—	3.894119e + 09
	std.	1.045133e + 08	4.769155e + 07	—	4.320626e + 07



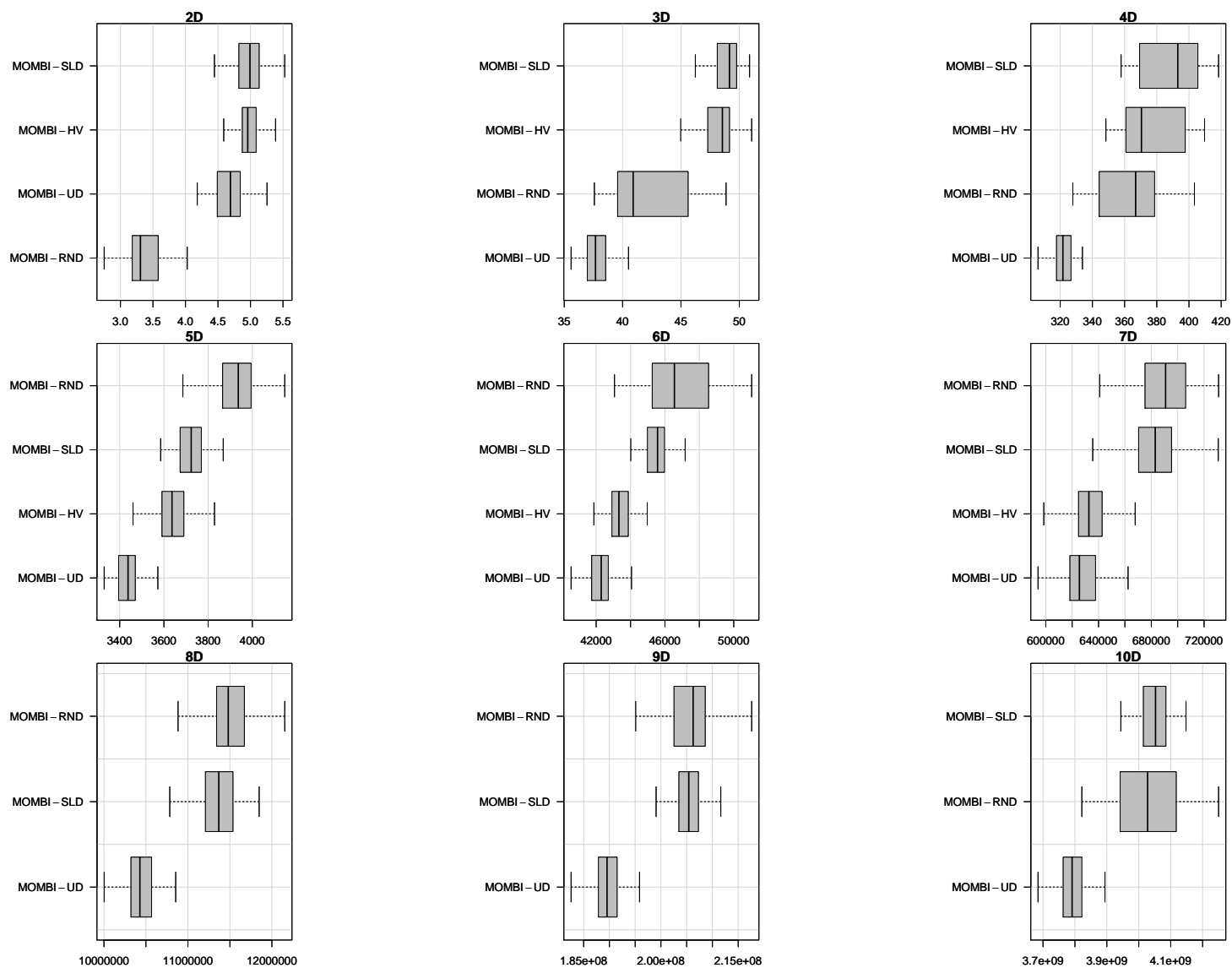


Figure A.459: Box-plot of the hypervolume indicator values for different optimizers on the WFG1 test problem.

Table A.200: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG1 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $ph$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
2D				
MOMBI-RND	–	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.40e - 34$	–	$> 0.05$	$1.51e - 14$
MOMBI-HV	$1.36e - 34$	$> 0.05$	–	$2.97e - 15$
MOMBI-UD	$3.65e - 34$	$> 0.05$	$> 0.05$	–
3D				
MOMBI-RND	–	$> 0.05$	$> 0.05$	$2.61e - 19$
MOMBI-SLD	$8.11e - 25$	–	$6.94e - 04$	$5.42e - 31$
MOMBI-HV	$1.09e - 22$	$> 0.05$	–	$3.78e - 30$
MOMBI-UD	$> 0.05$	$> 0.05$	$> 0.05$	–
4D				
MOMBI-RND	–	$> 0.05$	$> 0.05$	$3.78e - 32$
MOMBI-SLD	$6.16e - 13$	–	$2.61e - 06$	$9.75e - 34$
MOMBI-HV	$3.11e - 06$	$> 0.05$	–	$7.88e - 33$
MOMBI-UD	$> 0.05$	$> 0.05$	$> 0.05$	–
5D				
MOMBI-RND	–	$5.88e - 14$	$3.04e - 21$	$1.36e - 34$
MOMBI-SLD	$> 0.05$	–	$9.05e - 12$	$1.32e - 34$
MOMBI-HV	$> 0.05$	$> 0.05$	–	$8.40e - 34$
MOMBI-UD	$> 0.05$	$> 0.05$	$> 0.05$	–
6D				
MOMBI-RND	–	$1.21e - 06$	$4.78e - 27$	$2.96e - 34$
MOMBI-SLD	$> 0.05$	–	$1.90e - 24$	$1.63e - 34$
MOMBI-HV	$> 0.05$	$> 0.05$	–	$1.31e - 20$
MOMBI-UD	$> 0.05$	$> 0.05$	$> 0.05$	–
7D				
MOMBI-RND	–	$3.12e - 03$	$6.51e - 29$	$4.98e - 31$
MOMBI-SLD	$> 0.05$	–	$5.00e - 30$	$1.73e - 32$
MOMBI-HV	$> 0.05$	$> 0.05$	–	$3.77e - 03$
MOMBI-UD	$> 0.05$	$> 0.05$	$> 0.05$	–
8D				
MOMBI-RND	–	$7.12e - 04$		$1.89e - 34$
MOMBI-SLD	$> 0.05$	–		$3.24e - 34$
MOMBI-HV			–	
MOMBI-UD	$> 0.05$	$> 0.05$		–
9D				
MOMBI-RND	–	$> 0.05$		$1.84e - 34$
MOMBI-SLD	$> 0.05$	–		$1.32e - 34$
MOMBI-HV			–	
MOMBI-UD	$> 0.05$	$> 0.05$		–
10D				
MOMBI-RND	–	$> 0.05$		$1.31e - 33$
MOMBI-SLD	$> 0.05$	–		$1.28e - 34$
MOMBI-HV			–	
MOMBI-UD	$> 0.05$	$> 0.05$		–

Table A.201: Comparison of R2 indicator values for different optimizers on the WFG1 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	1.182000e + 00	9.924861e - 01	9.954845e - 01	1.027989e + 00
	avg.	1.175695e + 00	9.937779e - 01	9.937626e - 01	1.027993e + 00
	min.	1.036328e + 00	9.294975e - 01	9.195173e - 01	9.655009e - 01
	max.	1.255198e + 00	1.060322e + 00	1.050156e + 00	1.112416e + 00
	std.	3.840169e - 02	2.638958e - 02	2.393278e - 02	2.921206e - 02
<b>3D</b>	med.	6.354891e - 01	5.635336e - 01	5.700776e - 01	6.772161e - 01
	avg.	6.229639e - 01	5.697188e - 01	5.778727e - 01	6.674434e - 01
	min.	5.558364e - 01	5.448656e - 01	5.455515e - 01	5.937037e - 01
	max.	6.719495e - 01	6.176042e - 01	6.335955e - 01	7.047949e - 01
	std.	3.099298e - 02	1.963859e - 02	2.251778e - 02	2.913256e - 02
<b>4D</b>	med.	5.070970e - 01	4.834046e - 01	5.005103e - 01	5.486577e - 01
	avg.	5.078159e - 01	4.866824e - 01	4.972196e - 01	5.467628e - 01
	min.	4.726556e - 01	4.614504e - 01	4.722383e - 01	5.114189e - 01
	max.	5.383088e - 01	5.106357e - 01	5.207507e - 01	5.644587e - 01
	std.	1.446945e - 02	1.405155e - 02	1.384022e - 02	9.344089e - 03
<b>5D</b>	med.	4.214614e - 01	4.348081e - 01	4.412576e - 01	4.587342e - 01
	avg.	4.222126e - 01	4.341394e - 01	4.403724e - 01	4.591850e - 01
	min.	4.057666e - 01	4.125514e - 01	4.161051e - 01	4.476982e - 01
	max.	4.430788e - 01	4.482280e - 01	4.600824e - 01	4.719880e - 01
	std.	7.718685e - 03	6.856290e - 03	7.009649e - 03	4.976867e - 03
<b>6D</b>	med.	3.690671e - 01	3.822745e - 01	4.004604e - 01	4.081731e - 01
	avg.	3.686770e - 01	3.829238e - 01	3.999220e - 01	4.082419e - 01
	min.	3.513657e - 01	3.703416e - 01	3.835230e - 01	3.943529e - 01
	max.	3.851892e - 01	3.921545e - 01	4.102493e - 01	4.231516e - 01
	std.	7.125544e - 03	4.401813e - 03	5.543670e - 03	5.271102e - 03
<b>7D</b>	med.	3.291559e - 01	3.266166e - 01	3.510619e - 01	3.513956e - 01
	avg.	3.297084e - 01	3.267369e - 01	3.509381e - 01	3.497137e - 01
	min.	3.158012e - 01	3.064098e - 01	3.345049e - 01	3.350362e - 01
	max.	3.488926e - 01	3.455388e - 01	3.685923e - 01	3.635110e - 01
	std.	6.829224e - 03	7.383690e - 03	6.990384e - 03	6.200237e - 03
<b>8D</b>	med.	2.942230e - 01	2.923742e - 01	—	3.138620e - 01
	avg.	2.940150e - 01	2.923719e - 01	—	3.139489e - 01
	min.	2.822397e - 01	2.793135e - 01	—	3.043969e - 01
	max.	3.062026e - 01	3.104500e - 01	—	3.240880e - 01
	std.	4.553674e - 03	5.427049e - 03	—	4.232673e - 03
<b>9D</b>	med.	2.774123e - 01	2.729884e - 01	—	2.921348e - 01
	avg.	2.770893e - 01	2.729022e - 01	—	2.923097e - 01
	min.	2.676971e - 01	2.644673e - 01	—	2.859492e - 01
	max.	2.846715e - 01	2.833979e - 01	—	3.023027e - 01
	std.	3.013851e - 03	3.100178e - 03	—	3.381656e - 03
<b>10D</b>	med.	2.618948e - 01	2.586732e - 01	—	2.743212e - 01
	avg.	2.620428e - 01	2.591151e - 01	—	2.741735e - 01
	min.	2.563981e - 01	2.551416e - 01	—	2.697056e - 01
	max.	2.676514e - 01	2.638784e - 01	—	2.801339e - 01
	std.	2.468736e - 03	2.085934e - 03	—	2.086912e - 03

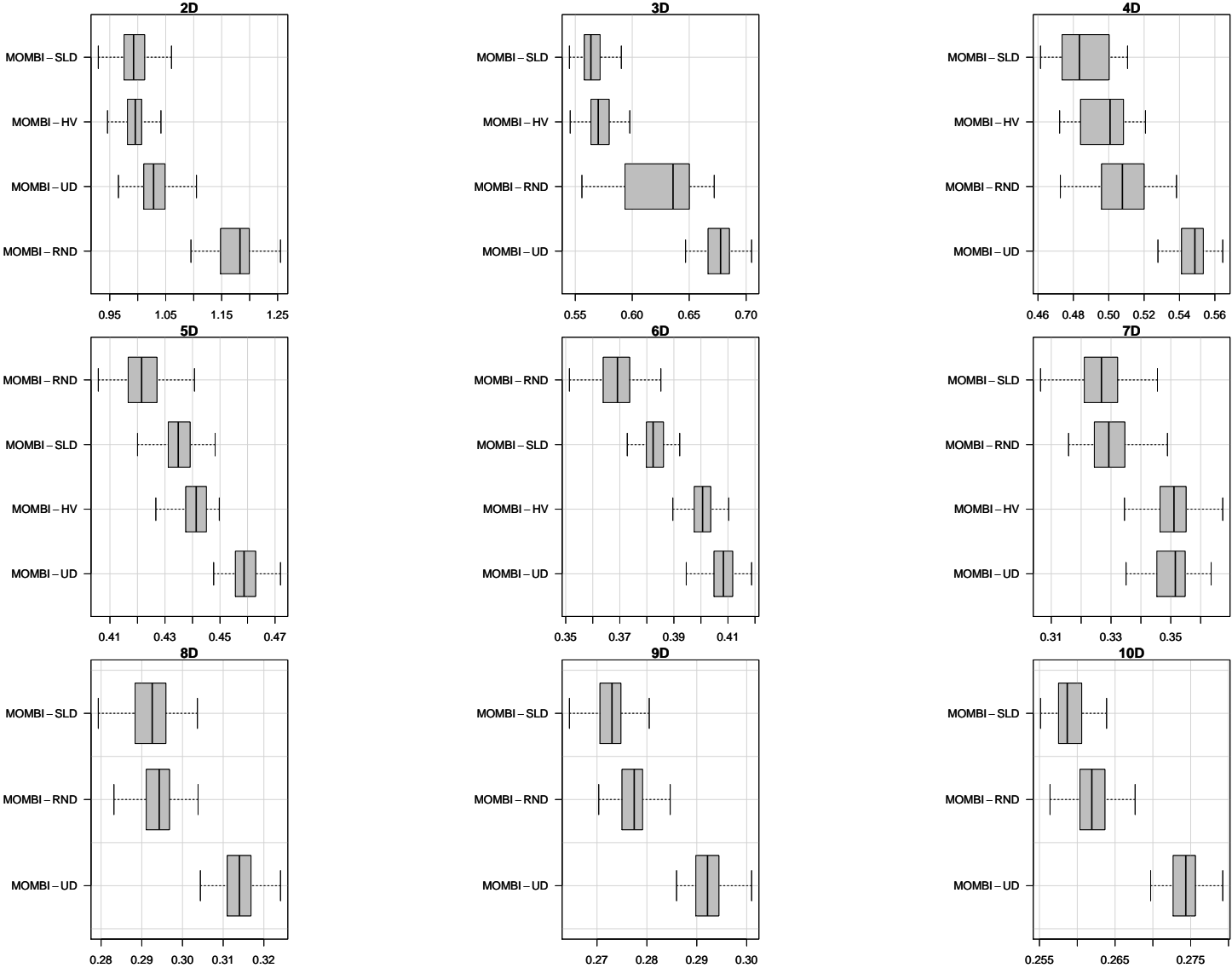


Figure A.460: Box-plot of the R2 indicator values for different optimizers on the WFG1 test problem.

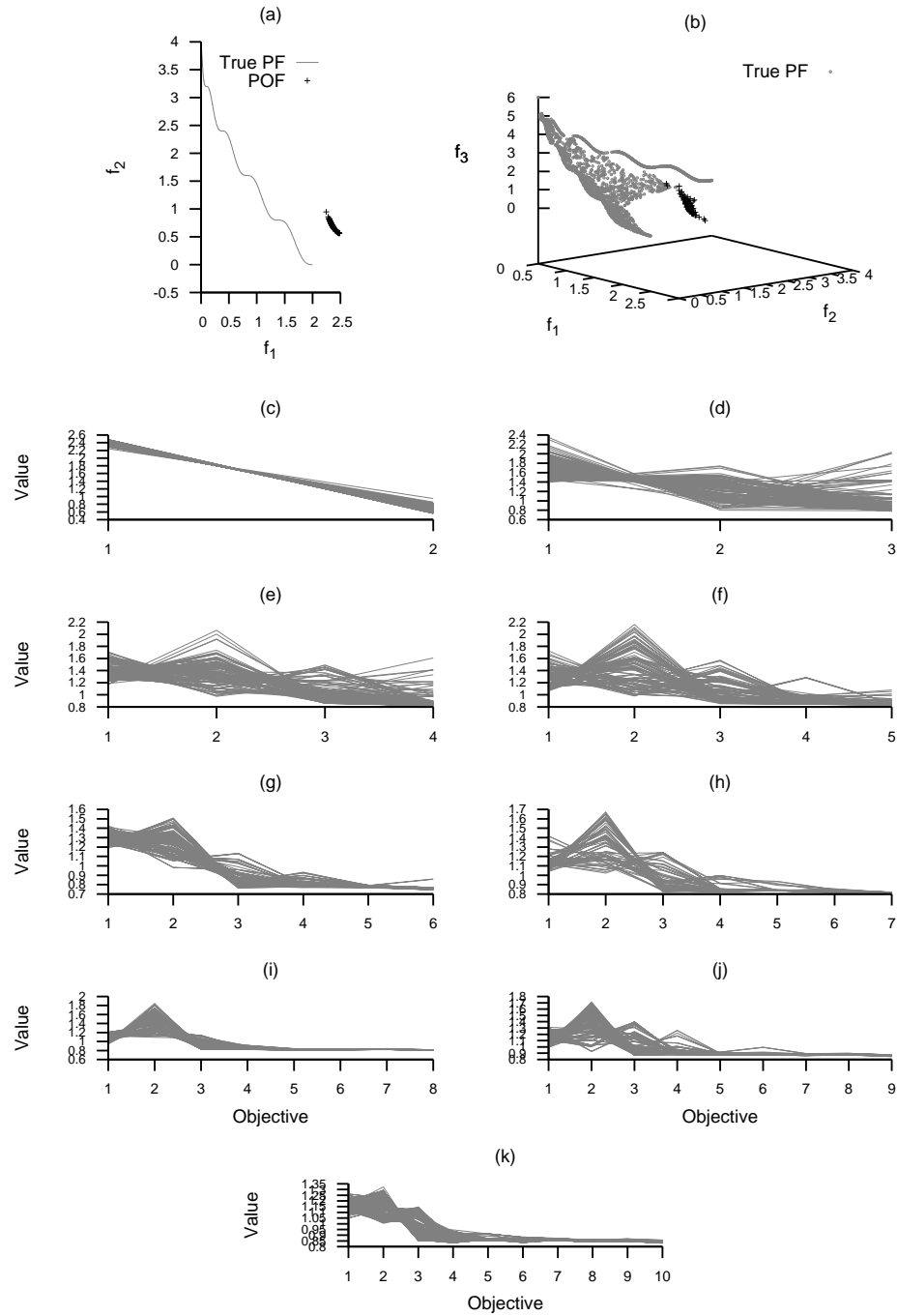


Figure A.461: Plots of the approximations obtained by MOMBI-RND from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

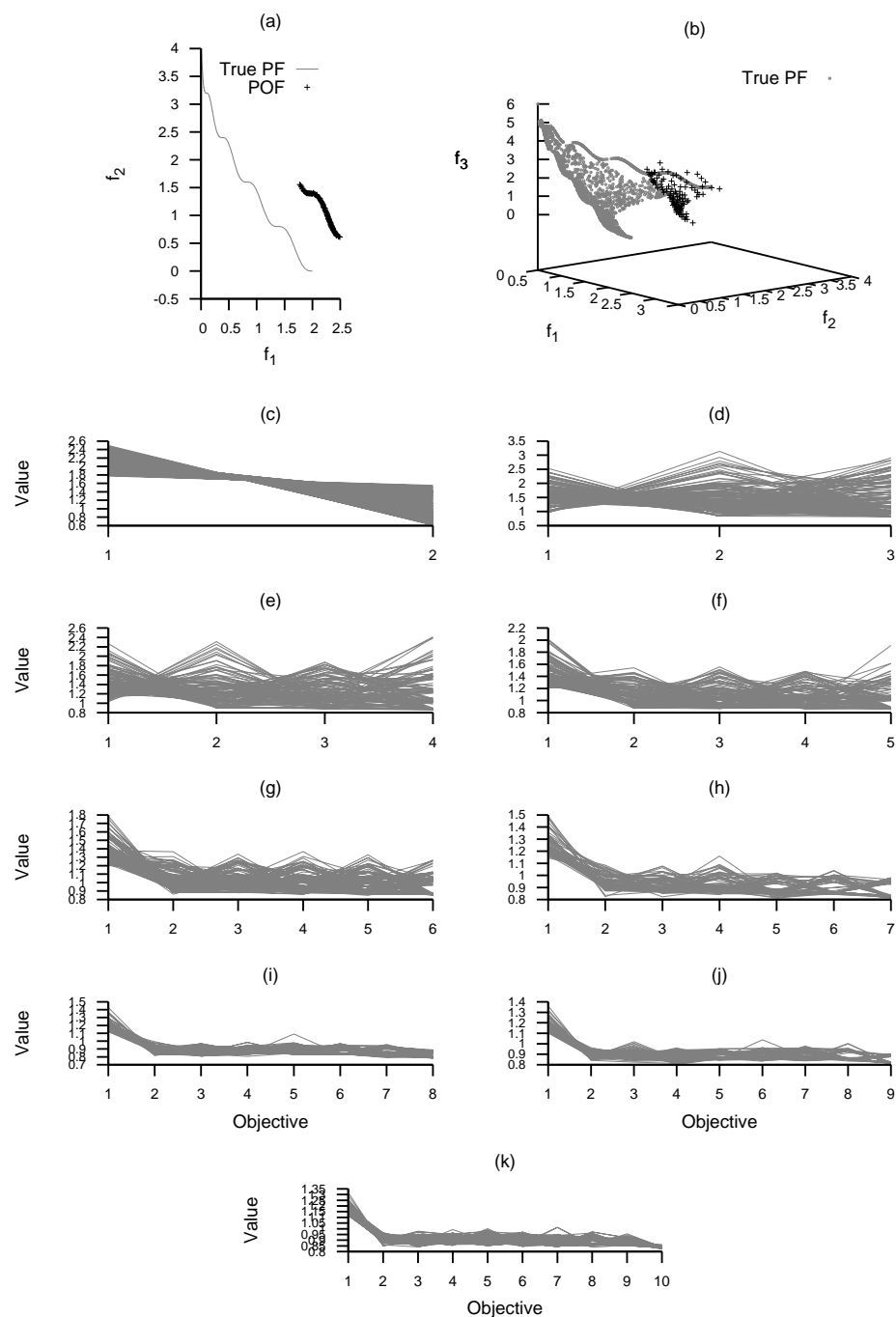


Figure A.462: Plots of the approximations obtained by MOMBI-SLD from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

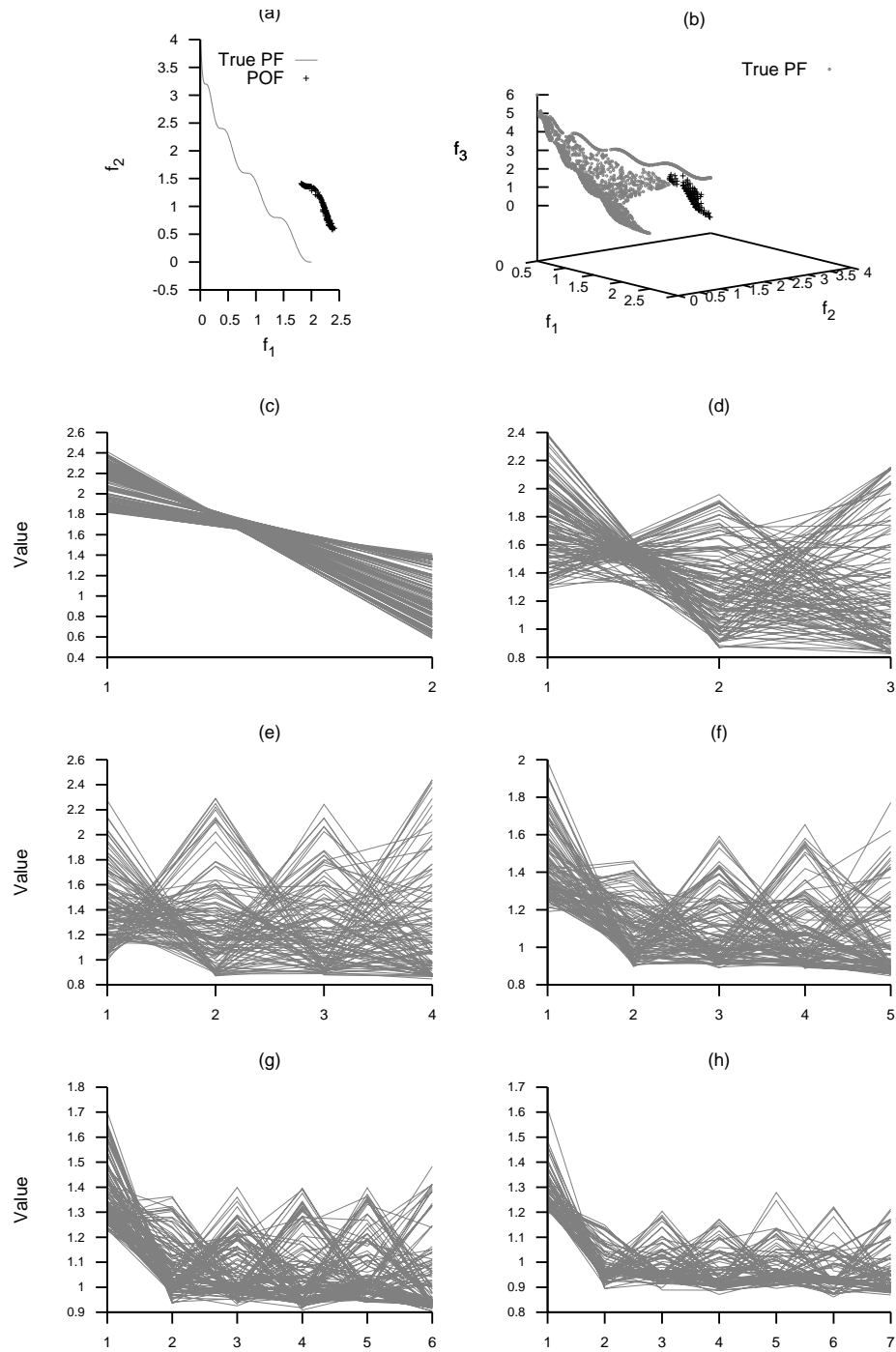


Figure A.463: Plots of the approximations obtained by MOMBI-HV from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

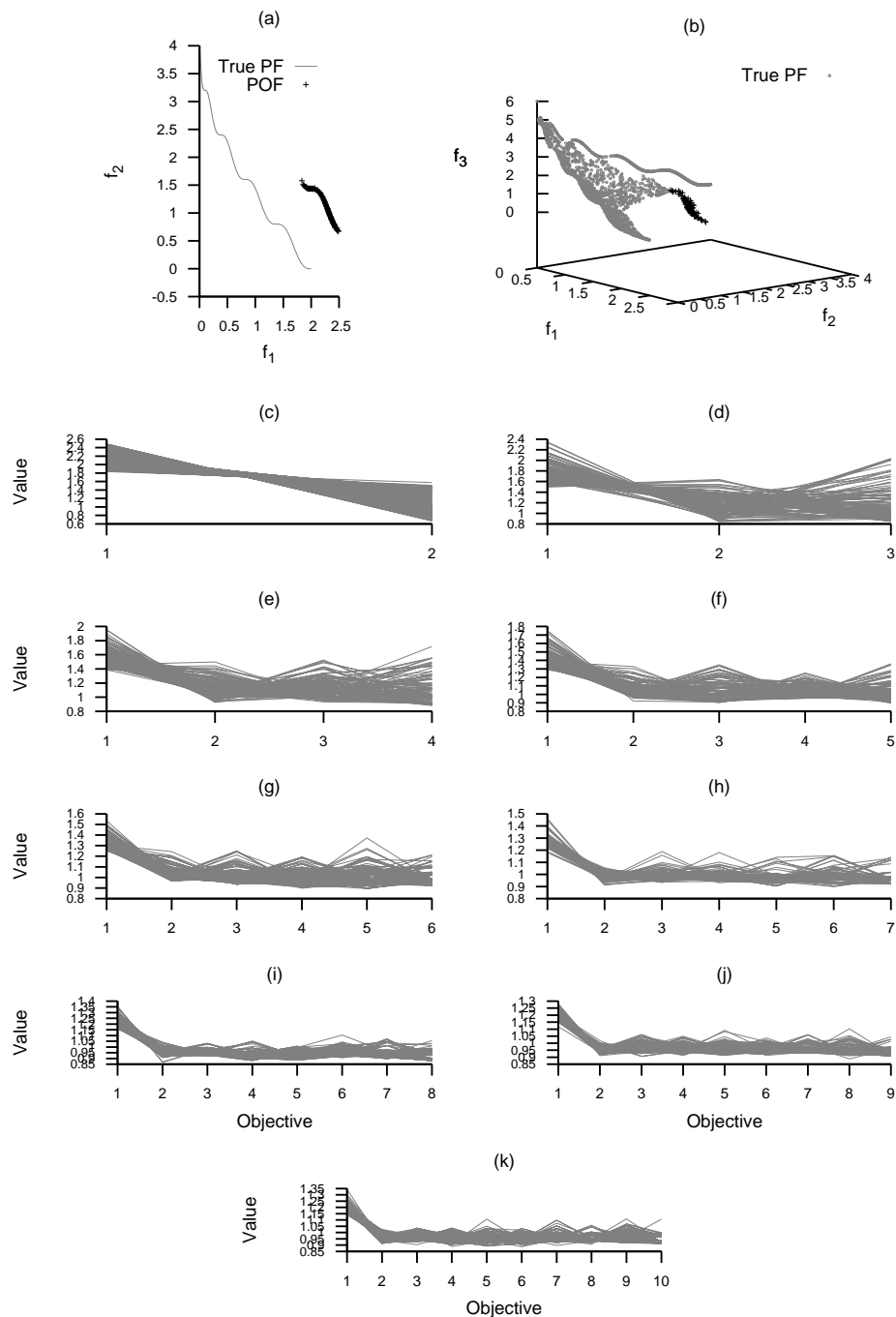


Figure A.464: Plots of the approximations obtained by MOMBI-UD from 2 to 10 objectives on the WFG1 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



Table A.202: Comparison of hypervolume indicator values for different optimizers on the WFG2 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
2D	med.	9.358377e + 00	1.037124e + 01	1.036506e + 01	9.841332e + 00
	avg.	9.183573e + 00	1.025424e + 01	1.023516e + 01	9.975464e + 00
	min.	8.271679e + 00	9.818665e + 00	9.666549e + 00	9.672033e + 00
	max.	1.043259e + 01	1.116540e + 01	1.108053e + 01	1.105021e + 01
	std.	5.597698e - 01	2.413692e - 01	2.526106e - 01	2.503246e - 01
3D	med.	8.835536e + 01	9.704777e + 01	9.603062e + 01	8.953644e + 01
	avg.	8.615570e + 01	9.325344e + 01	9.281015e + 01	8.658674e + 01
	min.	7.707746e + 01	8.053266e + 01	8.114858e + 01	7.618728e + 01
	max.	9.185186e + 01	9.867995e + 01	9.793079e + 01	9.325551e + 01
	std.	5.127793e + 00	6.620676e + 00	6.426721e + 00	5.520529e + 00
4D	med.	7.779520e + 02	8.606343e + 02	8.551518e + 02	7.784578e + 02
	avg.	7.479130e + 02	8.188486e + 02	8.158154e + 02	7.541407e + 02
	min.	6.562312e + 02	7.162765e + 02	7.090694e + 02	6.604900e + 02
	max.	8.273786e + 02	8.944190e + 02	8.907893e + 02	8.218820e + 02
	std.	5.243917e + 01	6.662676e + 01	6.835987e + 01	5.199472e + 01
5D	med.	8.473410e + 03	9.223856e + 03	9.065747e + 03	8.423626e + 03
	avg.	8.263829e + 03	8.799602e + 03	8.632771e + 03	8.146527e + 03
	min.	7.138034e + 03	7.526088e + 03	7.537197e + 03	7.061388e + 03
	max.	9.292290e + 03	9.686114e + 03	9.566732e + 03	9.089285e + 03
	std.	6.149522e + 02	7.493304e + 02	7.283424e + 02	5.822098e + 02
6D	med.	1.013431e + 05	1.153269e + 05	1.115049e + 05	1.057525e + 05
	avg.	9.898765e + 04	1.105043e + 05	1.069588e + 05	1.022729e + 05
	min.	8.493017e + 04	9.214910e + 04	9.147734e + 04	8.580347e + 04
	max.	1.121130e + 05	1.229746e + 05	1.195494e + 05	1.140891e + 05
	std.	8.110586e + 03	8.962970e + 03	8.831173e + 03	7.496347e + 03
7D	med.	1.323937e + 06	1.425057e + 06	1.370461e + 06	1.347381e + 06
	avg.	1.370032e + 06	1.502247e + 06	1.430362e + 06	1.392150e + 06
	min.	1.185403e + 06	1.315510e + 06	1.260175e + 06	1.196738e + 06
	max.	1.618256e + 06	1.780936e + 06	1.644835e + 06	1.628588e + 06
	std.	1.106883e + 05	1.367359e + 05	1.203335e + 05	1.111361e + 05
8D	med.	2.268738e + 07	2.689436e + 07	—	2.472228e + 07
	avg.	2.332252e + 07	2.607620e + 07	—	2.420978e + 07
	min.	2.055951e + 07	2.239932e + 07	—	2.063952e + 07
	max.	2.801932e + 07	2.998732e + 07	—	2.764049e + 07
	std.	1.807110e + 06	2.224662e + 06	—	1.955278e + 06
9D	med.	4.437476e + 08	5.173363e + 08	—	4.775646e + 08
	avg.	4.588312e + 08	4.980084e + 08	—	4.663718e + 08
	min.	3.886393e + 08	4.184567e + 08	—	4.098154e + 08
	max.	5.412695e + 08	5.567429e + 08	—	5.194294e + 08
	std.	3.909928e + 07	4.087591e + 07	—	3.715828e + 07
10D	med.	9.950645e + 09	1.096050e + 10	—	1.042259e + 10
	avg.	9.732409e + 09	1.072770e + 10	—	1.013441e + 10
	min.	8.432584e + 09	8.847950e + 09	—	8.469318e + 09
	max.	1.134669e + 10	1.141683e + 10	—	1.119517e + 10
	std.	7.732074e + 08	6.909596e + 08	—	7.404472e + 08

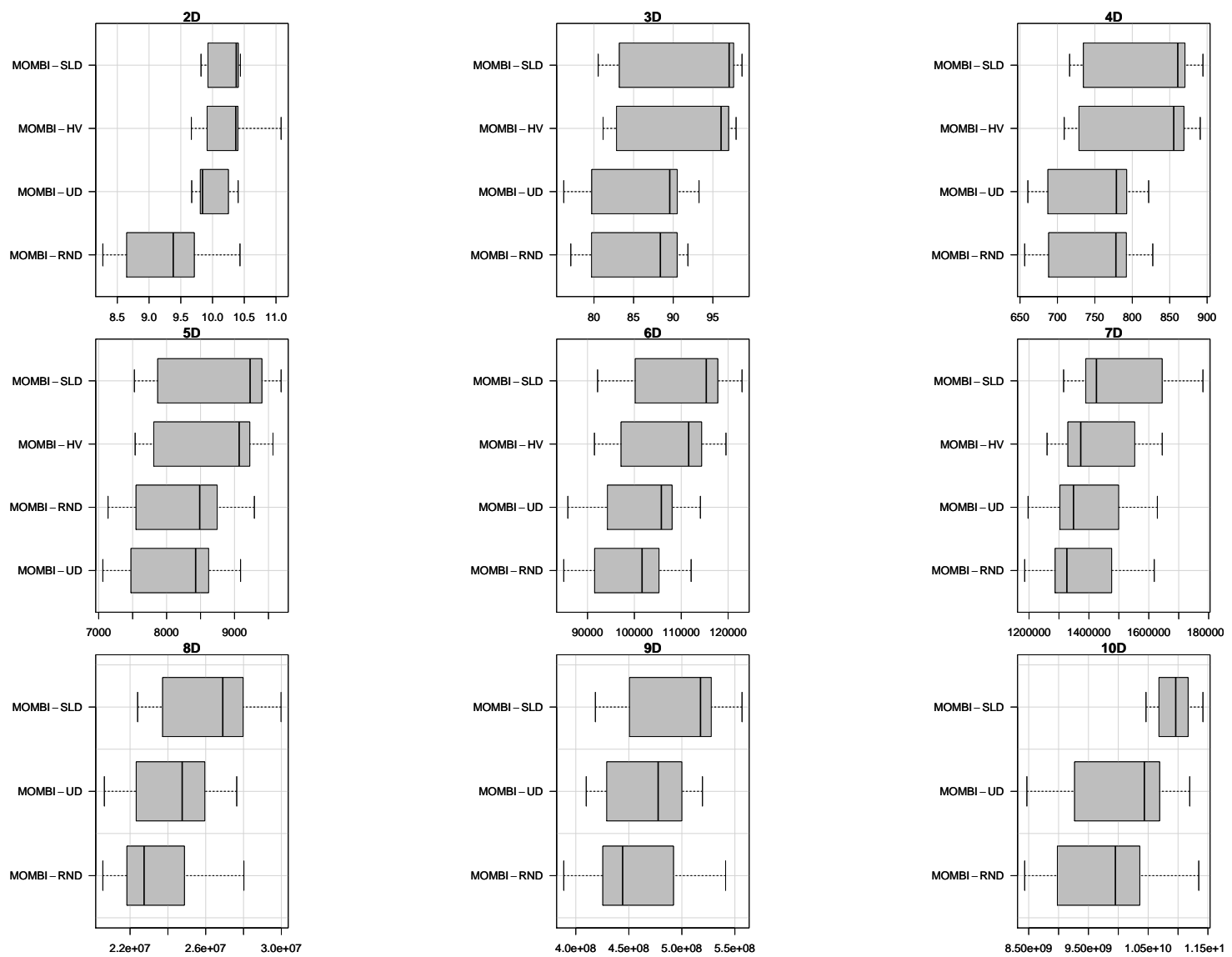


Figure A.465: Box-plot of the hypervolume indicator values for different optimizers on the WFG2 test problem.

Table A.203: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG2 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $ph$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
2D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.73e - 32$	—	$> 0.05$	$3.17e - 20$
MOMBI-HV	$7.14e - 32$	$> 0.05$	—	$1.39e - 17$
MOMBI-UD	$4.08e - 26$	$> 0.05$	$> 0.05$	—
3D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.90e - 15$	—	$1.85e - 04$	$1.51e - 14$
MOMBI-HV	$5.17e - 16$	$> 0.05$	—	$3.96e - 15$
MOMBI-UD	$> 0.05$	$> 0.05$	$> 0.05$	—
4D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$2.32e - 12$	—	$> 0.05$	$1.35e - 10$
MOMBI-HV	$2.36e - 12$	$> 0.05$	—	$1.35e - 10$
MOMBI-UD	$> 0.05$	$> 0.05$	$> 0.05$	—
5D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$1.67e - 02$
MOMBI-SLD	$8.80e - 11$	—	$1.07e - 03$	$1.14e - 11$
MOMBI-HV	$3.09e - 08$	$> 0.05$	—	$1.67e - 09$
MOMBI-UD	$> 0.05$	$> 0.05$	$> 0.05$	—
6D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$2.74e - 15$	—	$1.50e - 06$	$1.47e - 11$
MOMBI-HV	$1.59e - 12$	$> 0.05$	—	$9.79e - 08$
MOMBI-UD	$2.37e - 04$	$> 0.05$	$> 0.05$	—
7D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$5.10e - 12$	—	$1.29e - 07$	$1.94e - 10$
MOMBI-HV	$7.26e - 06$	$> 0.05$	—	$3.64e - 03$
MOMBI-UD	$3.44e - 02$	$> 0.05$	$> 0.05$	—
8D				
MOMBI-RND	—	$> 0.05$		$> 0.05$
MOMBI-SLD	$8.59e - 17$	—		$1.03e - 10$
MOMBI-HV			—	
MOMBI-UD	$4.36e - 04$	$> 0.05$		—
9D				
MOMBI-RND	—	$> 0.05$		$> 0.05$
MOMBI-SLD	$2.17e - 12$	—		$1.62e - 11$
MOMBI-HV			—	
MOMBI-UD	$> 0.05$	$> 0.05$		—
10D				
MOMBI-RND	—	$> 0.05$		$> 0.05$
MOMBI-SLD	$5.88e - 19$	—		$9.93e - 14$
MOMBI-HV			—	
MOMBI-UD	$2.70e - 06$	$> 0.05$		—

Table A.204: Comparison of R2 indicator values for different optimizers on the WFG2 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	6.027349e-01	5.587838e-01	5.594040e-01	5.786773e-01
	avg.	6.216203e-01	5.612979e-01	5.621043e-01	5.727209e-01
	min.	4.729905e-01	4.412949e-01	4.449314e-01	4.437009e-01
	max.	7.007988e-01	5.817906e-01	5.967717e-01	5.929199e-01
	std.	4.343848e-02	1.470532e-02	1.835300e-02	1.893537e-02
<b>3D</b>	med.	1.909552e-01	1.663383e-01	1.668897e-01	1.841094e-01
	avg.	2.678619e-01	2.358213e-01	2.338293e-01	2.579594e-01
	min.	1.746087e-01	1.592678e-01	1.608496e-01	1.752607e-01
	max.	4.389833e-01	4.369007e-01	4.308062e-01	4.389958e-01
	std.	1.168970e-01	1.161002e-01	1.146607e-01	1.152040e-01
<b>4D</b>	med.	1.418204e-01	1.006252e-01	1.016642e-01	1.322881e-01
	avg.	2.440875e-01	2.097087e-01	2.102811e-01	2.195169e-01
	min.	1.140855e-01	8.891387e-02	9.150664e-02	1.149793e-01
	max.	4.361581e-01	4.220366e-01	4.250455e-01	4.349986e-01
	std.	1.398348e-01	1.523861e-01	1.531254e-01	1.376795e-01
<b>5D</b>	med.	1.086732e-01	7.737798e-02	8.207604e-02	1.107174e-01
	avg.	2.219661e-01	2.039864e-01	2.242001e-01	2.289727e-01
	min.	7.505164e-02	6.457072e-02	6.428883e-02	8.593649e-02
	max.	4.612570e-01	4.554648e-01	4.524364e-01	4.707560e-01
	std.	1.671378e-01	1.781311e-01	1.816063e-01	1.683602e-01
<b>6D</b>	med.	1.167430e-01	7.150911e-02	8.152921e-02	9.945713e-02
	avg.	2.562769e-01	1.863585e-01	2.038546e-01	2.128684e-01
	min.	8.204506e-02	5.091374e-02	6.269047e-02	7.655589e-02
	max.	4.399476e-01	4.324078e-01	4.301418e-01	4.398947e-01
	std.	1.623727e-01	1.666749e-01	1.671783e-01	1.594649e-01
<b>7D</b>	med.	4.207087e-01	4.128676e-01	4.175495e-01	4.174090e-01
	avg.	3.070373e-01	2.745071e-01	2.906847e-01	3.036348e-01
	min.	8.089696e-02	5.345952e-02	7.176361e-02	7.228069e-02
	max.	4.413283e-01	4.260265e-01	4.323758e-01	4.416483e-01
	std.	1.555843e-01	1.714996e-01	1.649788e-01	1.592964e-01
<b>8D</b>	med.	4.128328e-01	7.056286e-02	—	9.023420e-02
	avg.	3.022995e-01	2.025783e-01	—	2.372020e-01
	min.	6.480312e-02	4.633196e-02	—	6.529018e-02
	max.	4.303162e-01	4.176754e-01	—	4.275616e-01
	std.	1.571996e-01	1.711065e-01	—	1.688936e-01
<b>9D</b>	med.	4.139854e-01	6.033524e-02	—	7.755170e-02
	avg.	2.563942e-01	1.780422e-01	—	2.246202e-01
	min.	5.547808e-02	4.218352e-02	—	5.644671e-02
	max.	4.296680e-01	4.200970e-01	—	4.214846e-01
	std.	1.733880e-01	1.704896e-01	—	1.748556e-01
<b>10D</b>	med.	7.415980e-02	4.997953e-02	—	6.040165e-02
	avg.	2.108498e-01	1.119555e-01	—	1.651698e-01
	min.	4.979504e-02	4.193152e-02	—	4.573292e-02
	max.	4.233724e-01	4.182713e-01	—	4.232878e-01
	std.	1.734744e-01	1.374986e-01	—	1.648524e-01

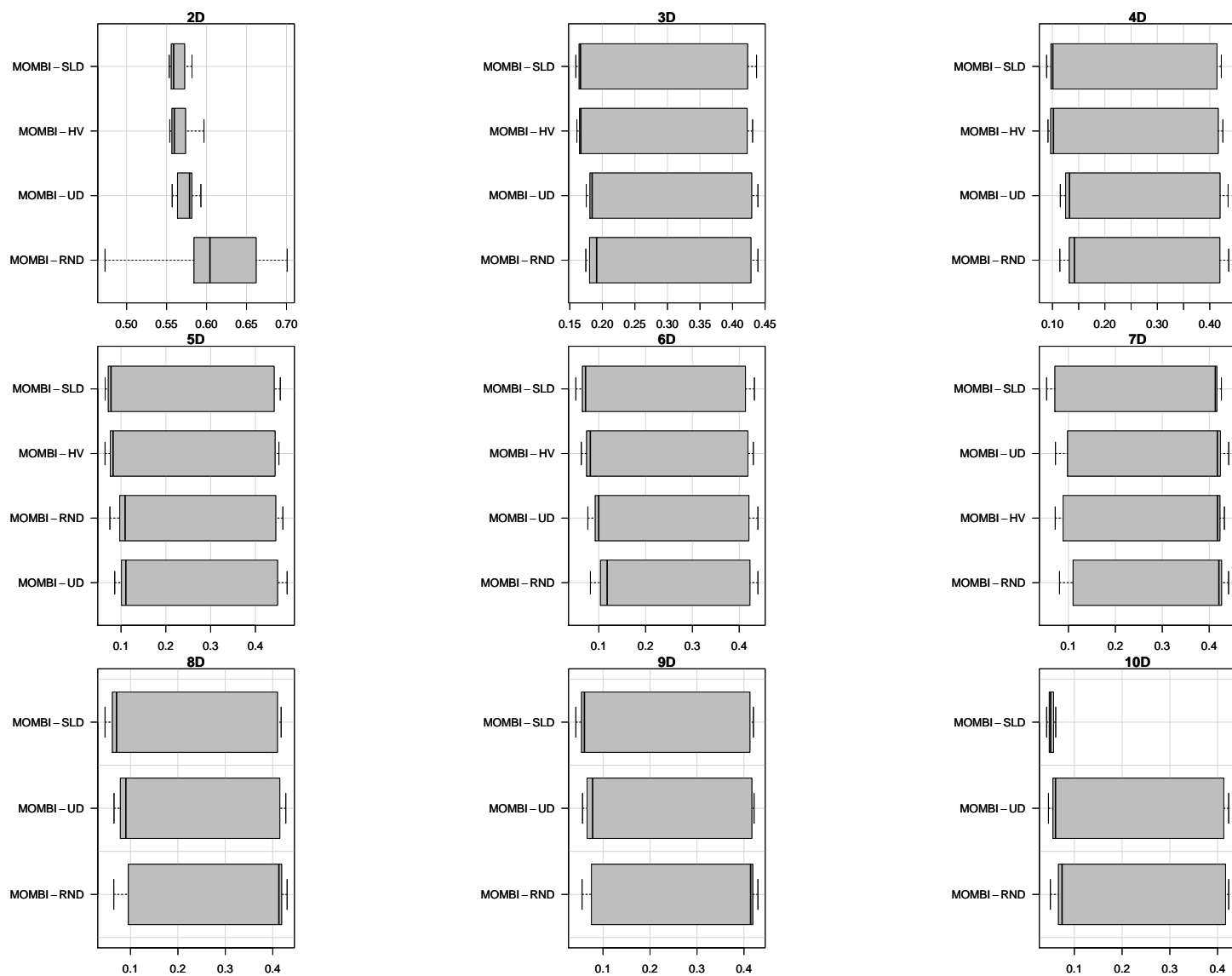


Figure A.466: Box-plot of the R2 indicator values for different optimizers on the WFG2 test problem.

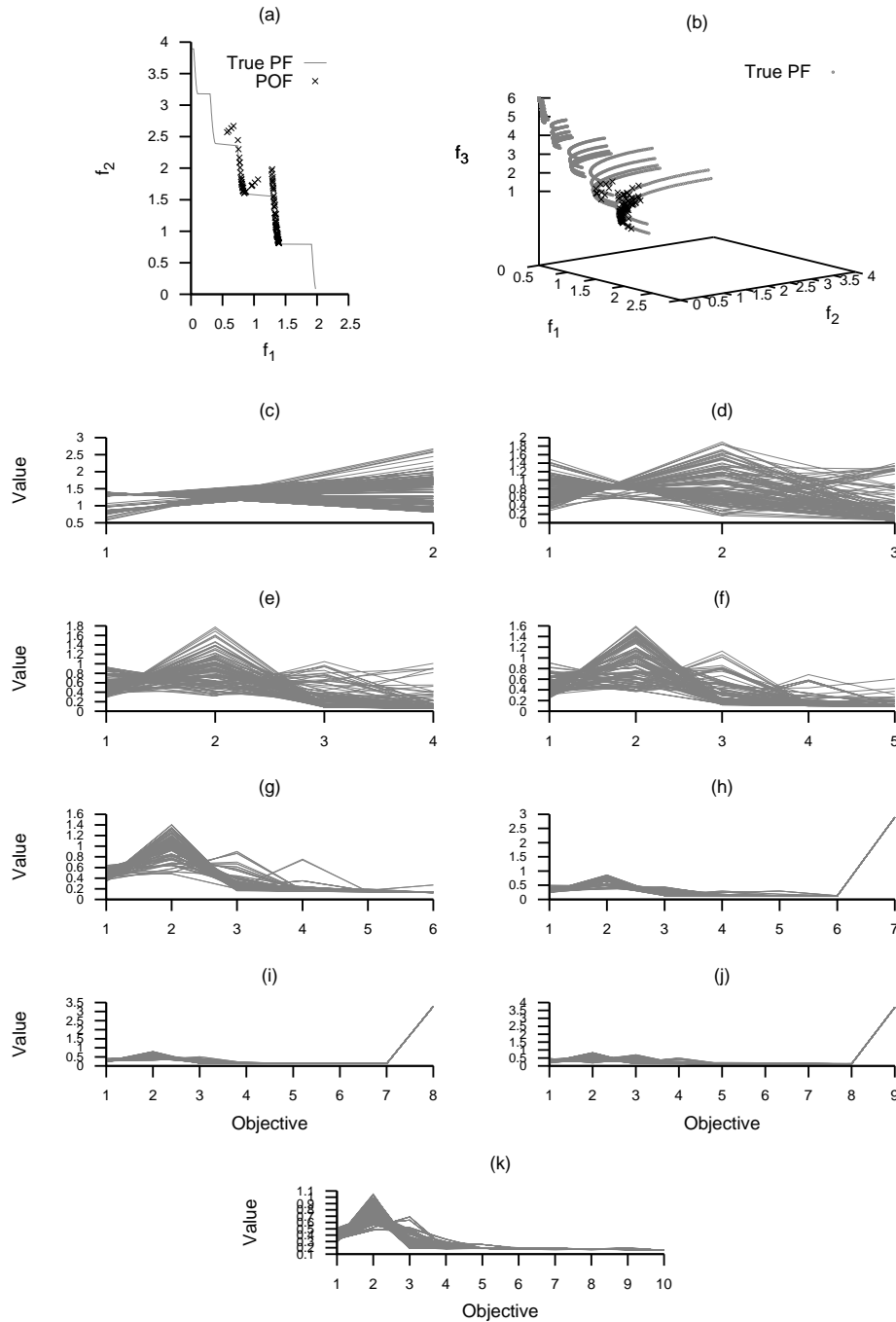


Figure A.467: Plots of the approximations obtained by MOMBI-RND from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

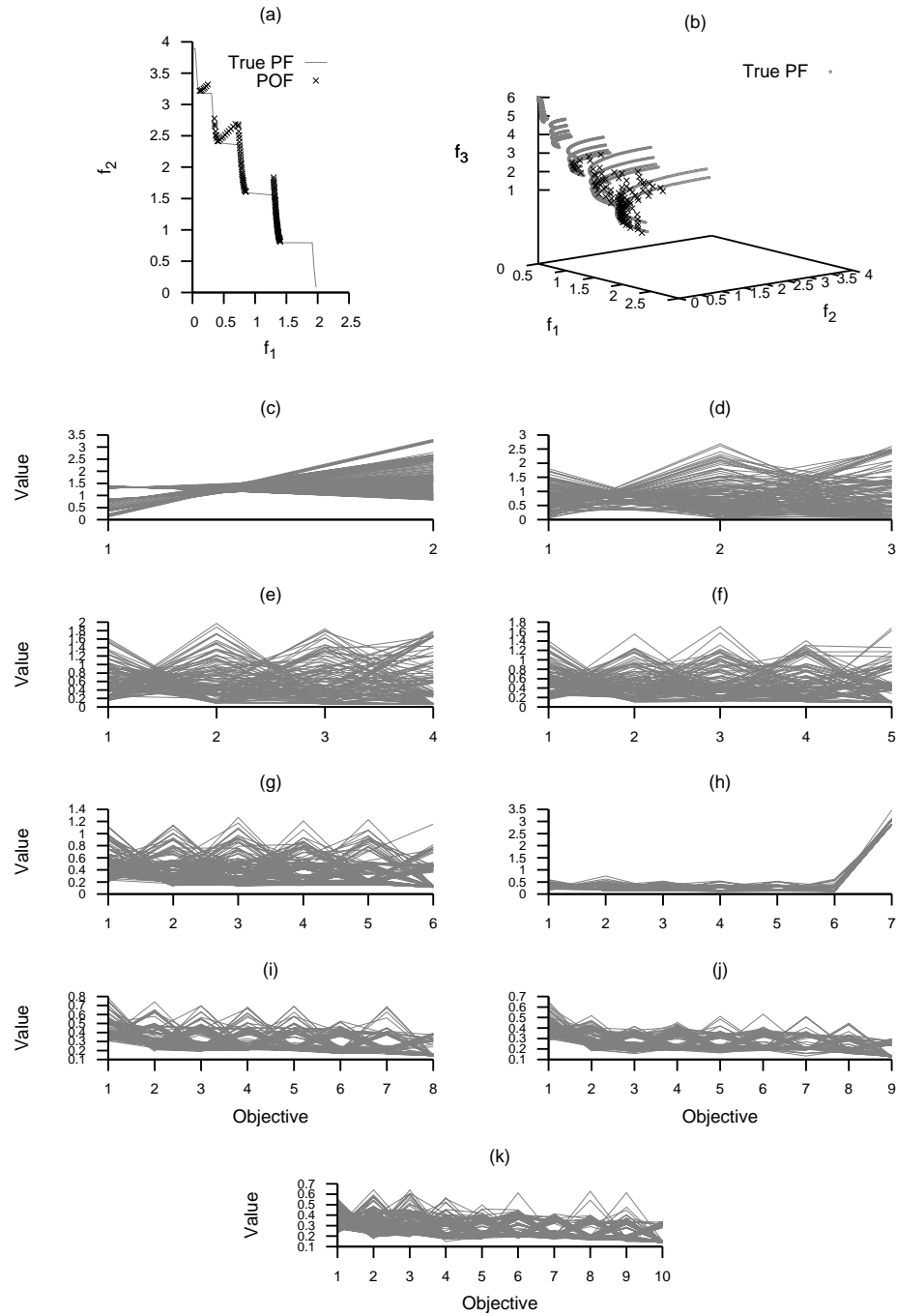


Figure A.468: Plots of the approximations obtained by MOMBI-SLD from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

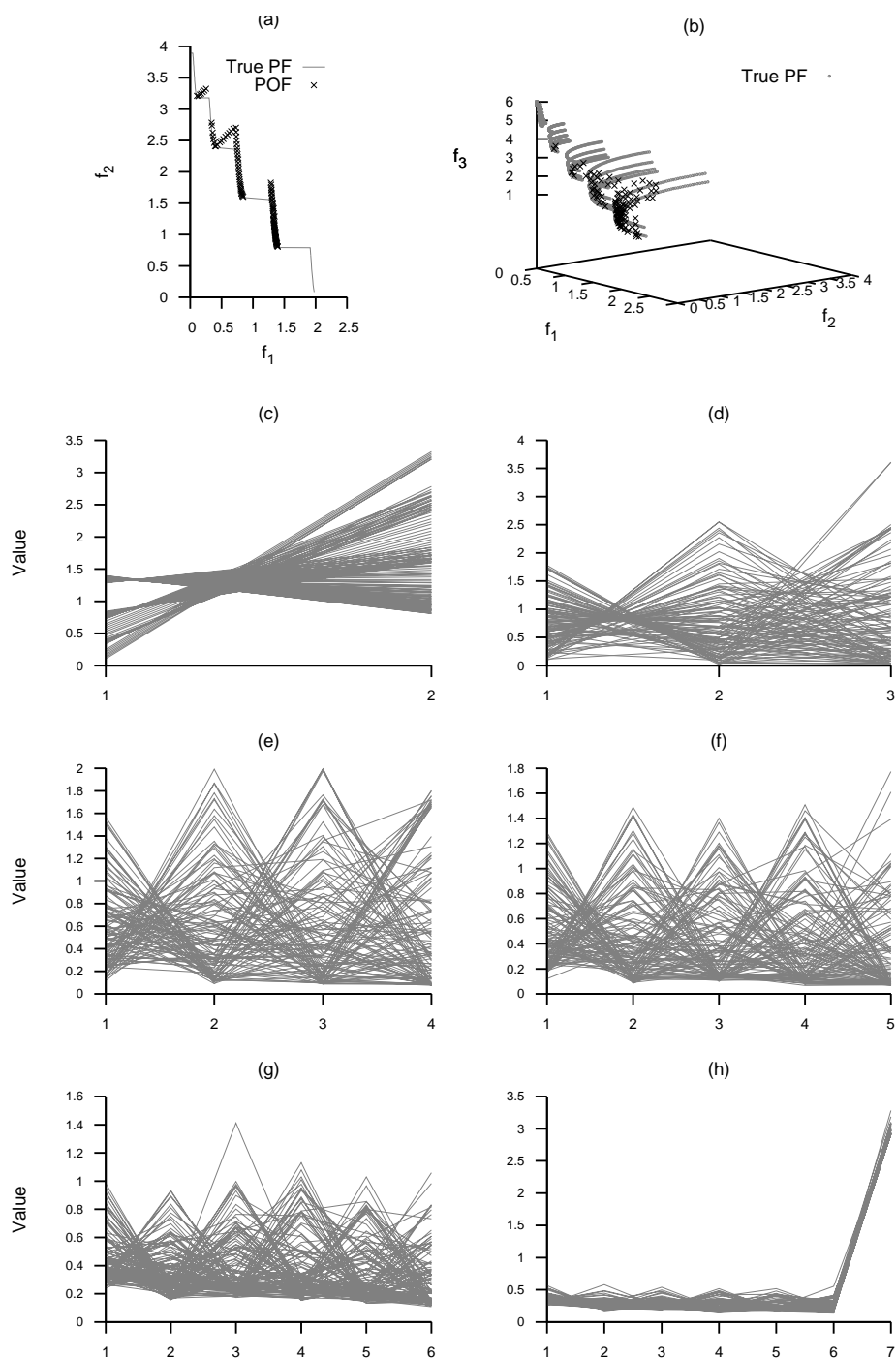


Figure A.469: Plots of the approximations obtained by MOMBI-HV from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



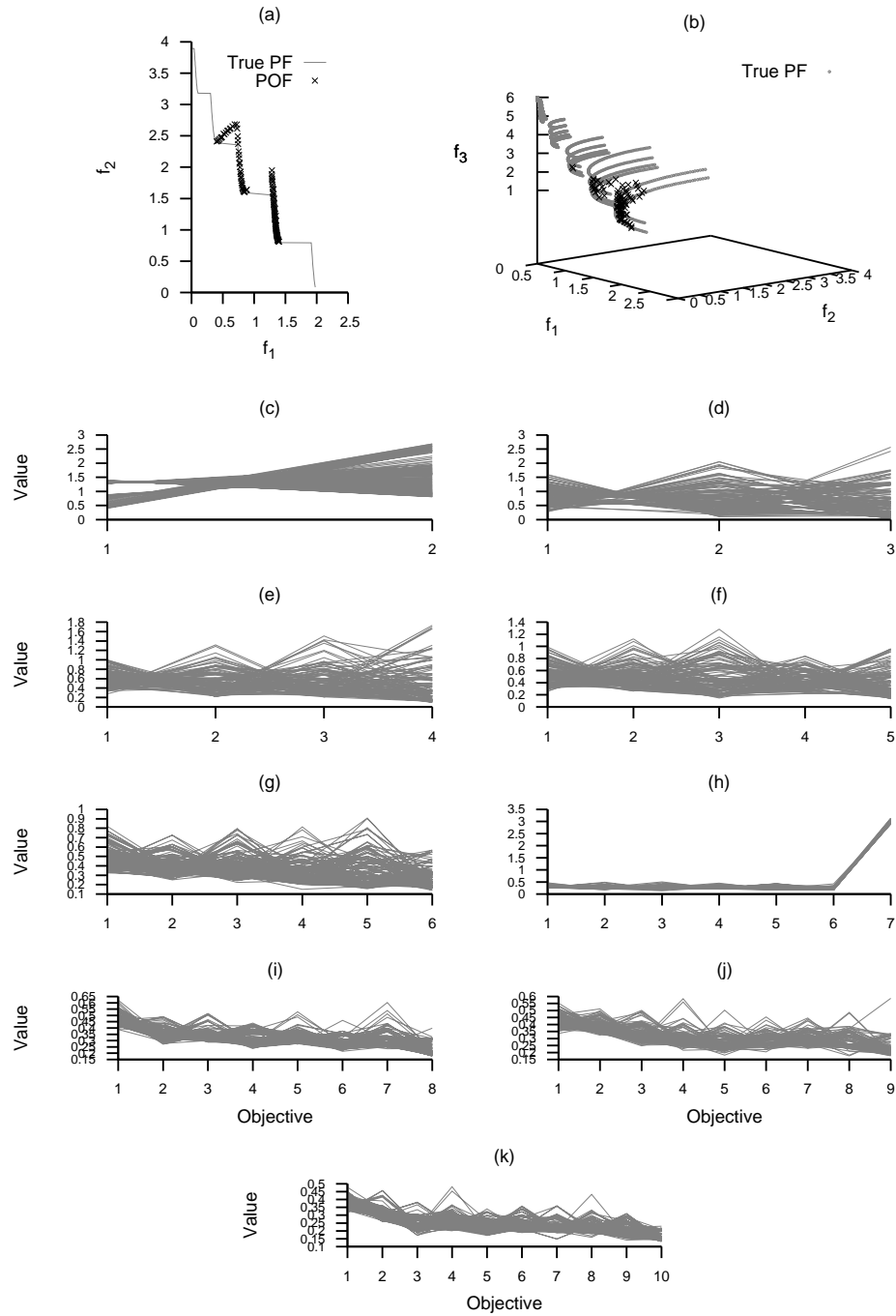


Figure A.470: Plots of the approximations obtained by MOMBI-UD from 2 to 10 objectives on the WFG2 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.205: Comparison of hypervolume indicator values for different optimizers on the WFG3 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	8.038833e + 00	1.082196e + 01	1.081190e + 01	1.017628e + 01
	avg.	8.094405e + 00	1.080816e + 01	1.079524e + 01	1.010678e + 01
	min.	6.749937e + 00	1.060830e + 01	1.055181e + 01	9.218961e + 00
	max.	9.692677e + 00	1.087529e + 01	1.086012e + 01	1.066392e + 01
	std.	6.666962e - 01	4.794573e - 02	5.106023e - 02	3.260838e - 01
<b>3D</b>	med.	4.769312e + 01	7.302527e + 01	6.792690e + 01	6.761543e + 01
	avg.	4.782636e + 01	7.281425e + 01	6.794127e + 01	6.717389e + 01
	min.	4.585715e + 01	7.028626e + 01	6.395449e + 01	6.036303e + 01
	max.	4.965557e + 01	7.363234e + 01	7.196554e + 01	7.139811e + 01
	std.	8.545983e - 01	6.442233e - 01	1.649902e + 00	2.374912e + 00
<b>4D</b>	med.	3.700312e + 02	5.998299e + 02	4.981987e + 02	5.356674e + 02
	avg.	3.710728e + 02	5.994755e + 02	5.002928e + 02	5.359054e + 02
	min.	3.538582e + 02	5.808513e + 02	4.780734e + 02	4.890007e + 02
	max.	3.954641e + 02	6.125754e + 02	5.329430e + 02	5.640088e + 02
	std.	7.042894e + 00	7.121537e + 00	1.148059e + 01	1.562659e + 01
<b>5D</b>	med.	3.123600e + 03	5.833894e + 03	4.718625e + 03	4.360846e + 03
	avg.	3.118949e + 03	5.846922e + 03	4.752023e + 03	4.313215e + 03
	min.	2.803987e + 03	5.564728e + 03	4.433291e + 03	3.442650e + 03
	max.	3.372607e + 03	6.124097e + 03	5.445136e + 03	4.643029e + 03
	std.	1.030986e + 02	1.175068e + 02	1.570872e + 02	2.369207e + 02
<b>6D</b>	med.	2.579805e + 04	6.801635e + 04	4.666617e + 04	4.564863e + 04
	avg.	2.574982e + 04	6.788002e + 04	4.658138e + 04	4.814150e + 04
	min.	2.139711e + 04	6.504757e + 04	4.330049e + 04	4.053996e + 04
	max.	3.033380e + 04	7.022240e + 04	4.991684e + 04	6.154738e + 04
	std.	1.901976e + 03	1.225171e + 03	1.420696e + 03	5.839642e + 03
<b>7D</b>	med.	3.118317e + 05	9.235848e + 05	5.368744e + 05	4.730584e + 05
	avg.	3.117024e + 05	8.926488e + 05	5.356282e + 05	4.730085e + 05
	min.	2.234597e + 05	5.228600e + 05	4.635962e + 05	4.043872e + 05
	max.	4.025298e + 05	9.872385e + 05	5.898162e + 05	5.291144e + 05
	std.	3.678636e + 04	7.970684e + 04	2.434806e + 04	2.613431e + 04
<b>8D</b>	med.	5.277907e + 06	1.189775e + 07	—	6.307309e + 06
	avg.	5.223616e + 06	1.192168e + 07	—	6.418023e + 06
	min.	3.397866e + 06	8.147875e + 06	—	5.062653e + 06
	max.	6.477294e + 06	1.528290e + 07	—	8.282884e + 06
	std.	6.182673e + 05	1.094443e + 06	—	6.807016e + 05
<b>9D</b>	med.	1.055774e + 08	2.079841e + 08	—	1.096823e + 08
	avg.	1.049425e + 08	2.076498e + 08	—	1.108354e + 08
	min.	8.614681e + 07	1.207398e + 08	—	8.751576e + 07
	max.	1.227183e + 08	2.545567e + 08	—	1.437871e + 08
	std.	8.624720e + 06	2.158269e + 07	—	1.106581e + 07
<b>10D</b>	med.	1.991860e + 09	4.082743e + 09	—	2.074359e + 09
	avg.	2.034788e + 09	3.879076e + 09	—	2.101675e + 09
	min.	1.719620e + 09	2.338422e + 09	—	1.566269e + 09
	max.	2.463183e + 09	5.062126e + 09	—	3.214316e + 09
	std.	1.913416e + 08	5.734642e + 08	—	2.641555e + 08

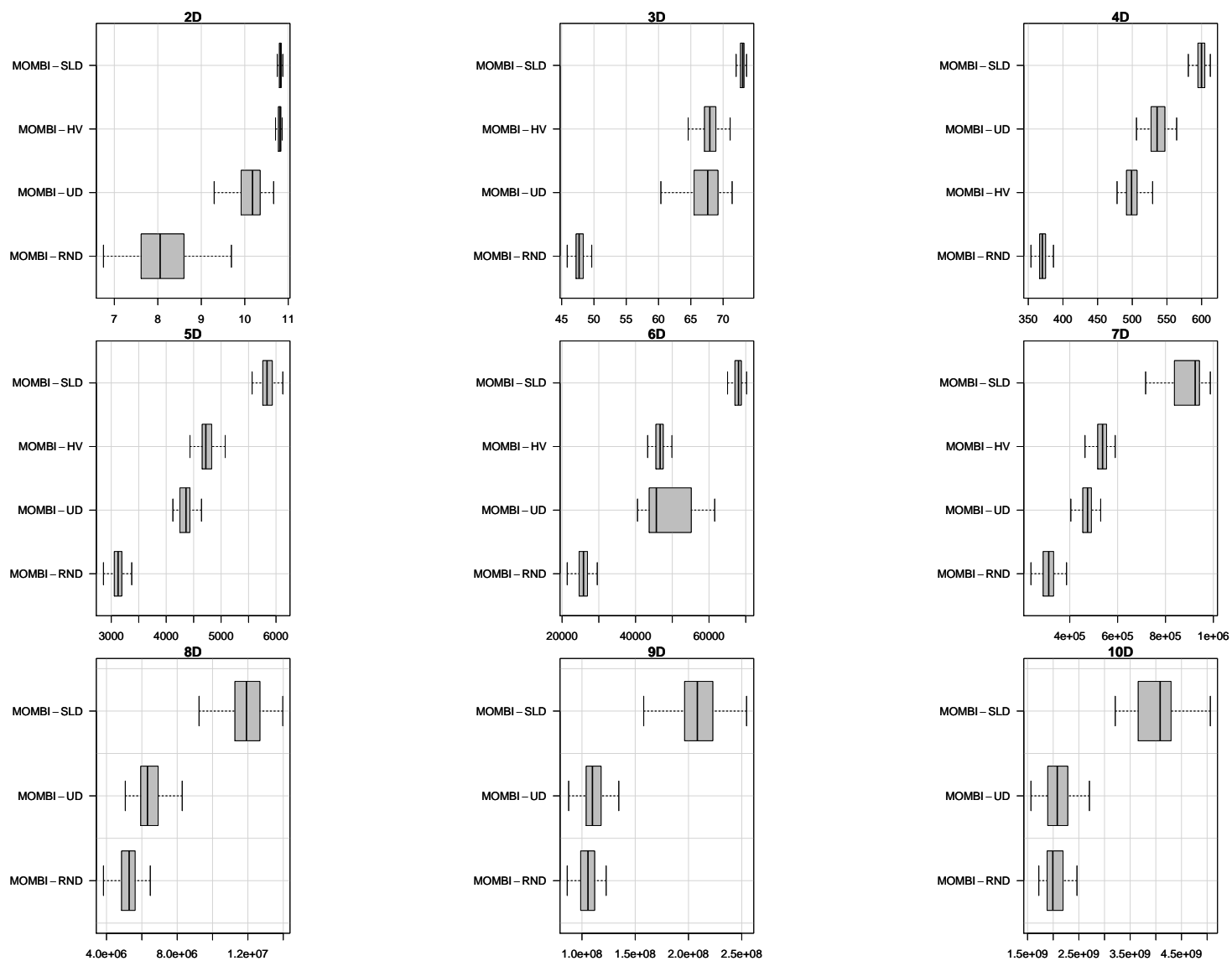


Figure A.471: Box-plot of the hypervolume indicator values for different optimizers on the WFG3 test problem.

Table A.206: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG3 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $ph$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
2D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e-34$	–	$5.43e-03$	$1.53e-34$
MOMBI-HV	$1.28e-34$	> 0.05	–	$1.78e-34$
MOMBI-UD	$3.15e-34$	> 0.05	> 0.05	–
3D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e-34$	–	$2.63e-34$	$1.73e-34$
MOMBI-HV	$1.28e-34$	> 0.05	–	$3.07e-02$
MOMBI-UD	$1.28e-34$	> 0.05	> 0.05	–
4D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e-34$	–	$1.28e-34$	$1.28e-34$
MOMBI-HV	$1.28e-34$	> 0.05	–	> 0.05
MOMBI-UD	$1.28e-34$	> 0.05	$3.77e-29$	–
5D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e-34$	–	$1.28e-34$	$1.28e-34$
MOMBI-HV	$1.28e-34$	> 0.05	–	$1.12e-32$
MOMBI-UD	$1.28e-34$	> 0.05	> 0.05	–
6D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e-34$	–	$1.28e-34$	$1.28e-34$
MOMBI-HV	$1.28e-34$	> 0.05	–	> 0.05
MOMBI-UD	$1.28e-34$	> 0.05	> 0.05	–
7D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e-34$	–	$1.10e-33$	$1.36e-34$
MOMBI-HV	$1.28e-34$	> 0.05	–	$1.44e-29$
MOMBI-UD	$1.28e-34$	> 0.05	> 0.05	–
8D				
MOMBI-RND	–	> 0.05		> 0.05
MOMBI-SLD	$1.28e-34$	–		$1.32e-34$
MOMBI-HV			–	
MOMBI-UD	$3.55e-24$	> 0.05		–
9D				
MOMBI-RND	–	> 0.05		> 0.05
MOMBI-SLD	$1.40e-34$	–		$2.33e-34$
MOMBI-HV			–	
MOMBI-UD	$8.61e-05$	> 0.05		–
10D				
MOMBI-RND	–	> 0.05		> 0.05
MOMBI-SLD	$2.40e-34$	–		$1.27e-33$
MOMBI-HV			–	
MOMBI-UD	> 0.05	> 0.05		–

Table A.207: Comparison of R2 indicator values for different optimizers on the WFG3 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	6.786398e-01	4.616910e-01	4.619044e-01	4.883542e-01
	avg.	6.772797e-01	4.620420e-01	4.623633e-01	4.953840e-01
	min.	5.249414e-01	4.598820e-01	4.599275e-01	4.653684e-01
	max.	8.161947e-01	4.673842e-01	4.699693e-01	5.673227e-01
	std.	6.664991e-02	1.285155e-03	1.544337e-03	2.285514e-02
<b>3D</b>	med.	6.726853e-01	3.916074e-01	4.221382e-01	4.261702e-01
	avg.	6.709683e-01	3.926373e-01	4.235590e-01	4.301408e-01
	min.	6.307383e-01	3.894458e-01	3.964070e-01	3.989388e-01
	max.	7.020743e-01	4.053946e-01	4.556046e-01	4.992334e-01
	std.	1.451369e-02	2.897555e-03	1.300532e-02	2.161250e-02
<b>4D</b>	med.	6.801205e-01	3.623409e-01	4.558466e-01	4.048016e-01
	avg.	6.805908e-01	3.633086e-01	4.561022e-01	4.093752e-01
	min.	6.547337e-01	3.578921e-01	4.313781e-01	3.753279e-01
	max.	7.075562e-01	3.797331e-01	4.748513e-01	4.804861e-01
	std.	1.252796e-02	3.775932e-03	7.991587e-03	2.184134e-02
<b>5D</b>	med.	5.694903e-01	3.402282e-01	4.308081e-01	4.175996e-01
	avg.	5.696369e-01	3.399978e-01	4.295106e-01	4.248393e-01
	min.	5.364382e-01	3.290633e-01	3.948307e-01	4.007959e-01
	max.	5.920319e-01	3.534111e-01	4.475654e-01	5.254530e-01
	std.	9.062539e-03	4.736119e-03	1.010447e-02	2.527300e-02
<b>6D</b>	med.	5.335408e-01	3.381273e-01	4.840612e-01	4.776786e-01
	avg.	5.380061e-01	3.385245e-01	4.845359e-01	4.551217e-01
	min.	4.791903e-01	3.318811e-01	4.488933e-01	3.732226e-01
	max.	6.145109e-01	3.475980e-01	5.100110e-01	5.122080e-01
	std.	2.754407e-02	3.272936e-03	1.050802e-02	4.583735e-02
<b>7D</b>	med.	5.249014e-01	3.239390e-01	5.105287e-01	5.286035e-01
	avg.	5.257802e-01	3.367551e-01	5.105081e-01	5.287550e-01
	min.	4.523407e-01	3.141847e-01	4.799851e-01	5.041253e-01
	max.	6.084066e-01	4.524069e-01	5.364161e-01	5.500045e-01
	std.	3.394090e-02	2.618315e-02	1.045458e-02	9.313802e-03
<b>8D</b>	med.	4.554098e-01	3.726532e-01	—	4.908634e-01
	avg.	4.570229e-01	3.753808e-01	—	4.924511e-01
	min.	3.884901e-01	3.106899e-01	—	4.743559e-01
	max.	5.506980e-01	4.253907e-01	—	5.155393e-01
	std.	3.261803e-02	1.793206e-02	—	8.315494e-03
<b>9D</b>	med.	3.959284e-01	3.664454e-01	—	4.518542e-01
	avg.	3.975795e-01	3.684000e-01	—	4.530568e-01
	min.	3.600346e-01	3.413168e-01	—	4.329411e-01
	max.	4.410559e-01	4.165913e-01	—	4.835157e-01
	std.	1.884098e-02	1.704467e-02	—	1.117487e-02
<b>10D</b>	med.	3.701608e-01	3.607337e-01	—	4.253448e-01
	avg.	3.694640e-01	3.626536e-01	—	4.270261e-01
	min.	3.374014e-01	3.370712e-01	—	4.059638e-01
	max.	4.032061e-01	3.913810e-01	—	4.619819e-01
	std.	1.619305e-02	1.363811e-02	—	1.114669e-02

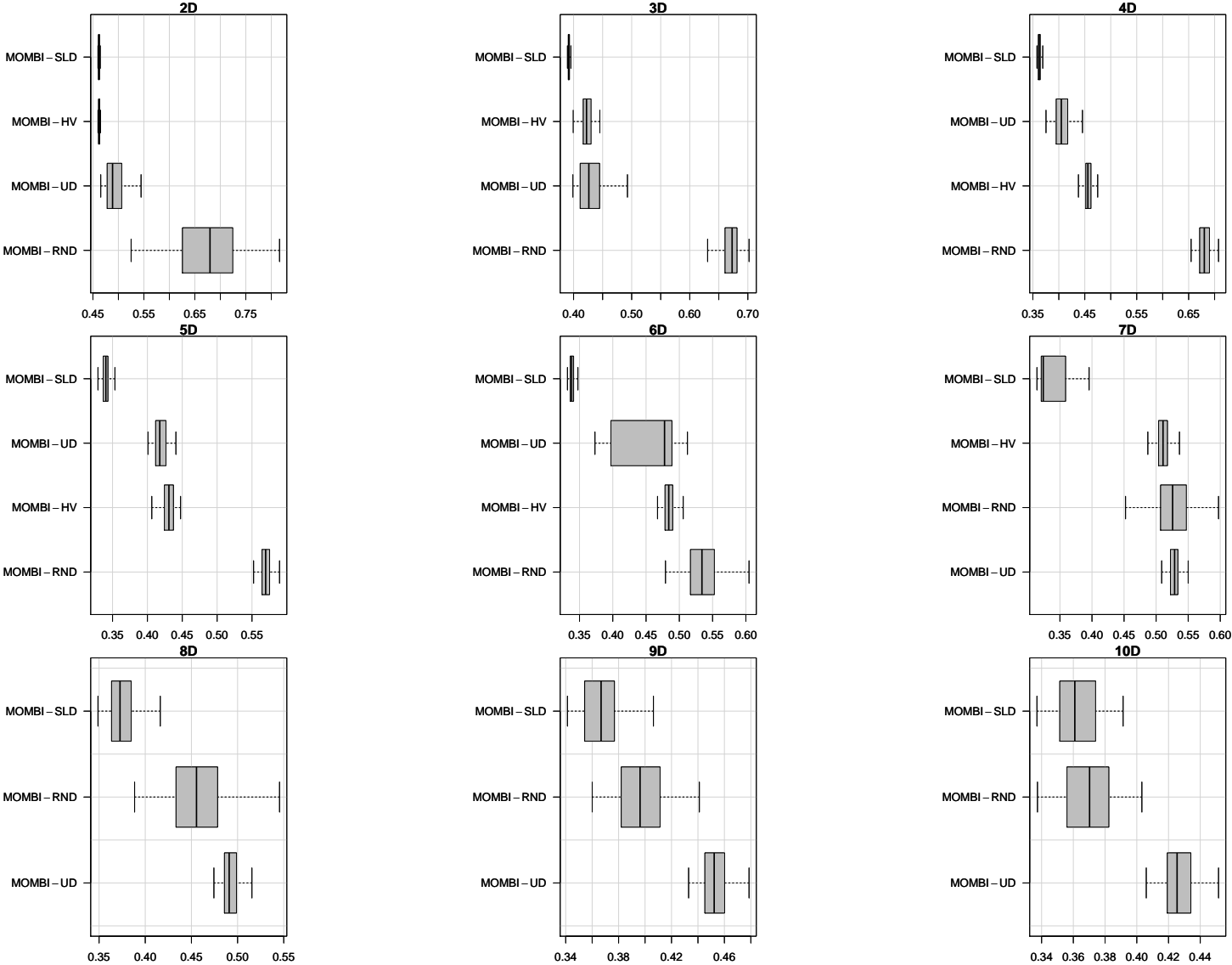


Figure A.472: Box-plot of the R2 indicator values for different optimizers on the WFG3 test problem.

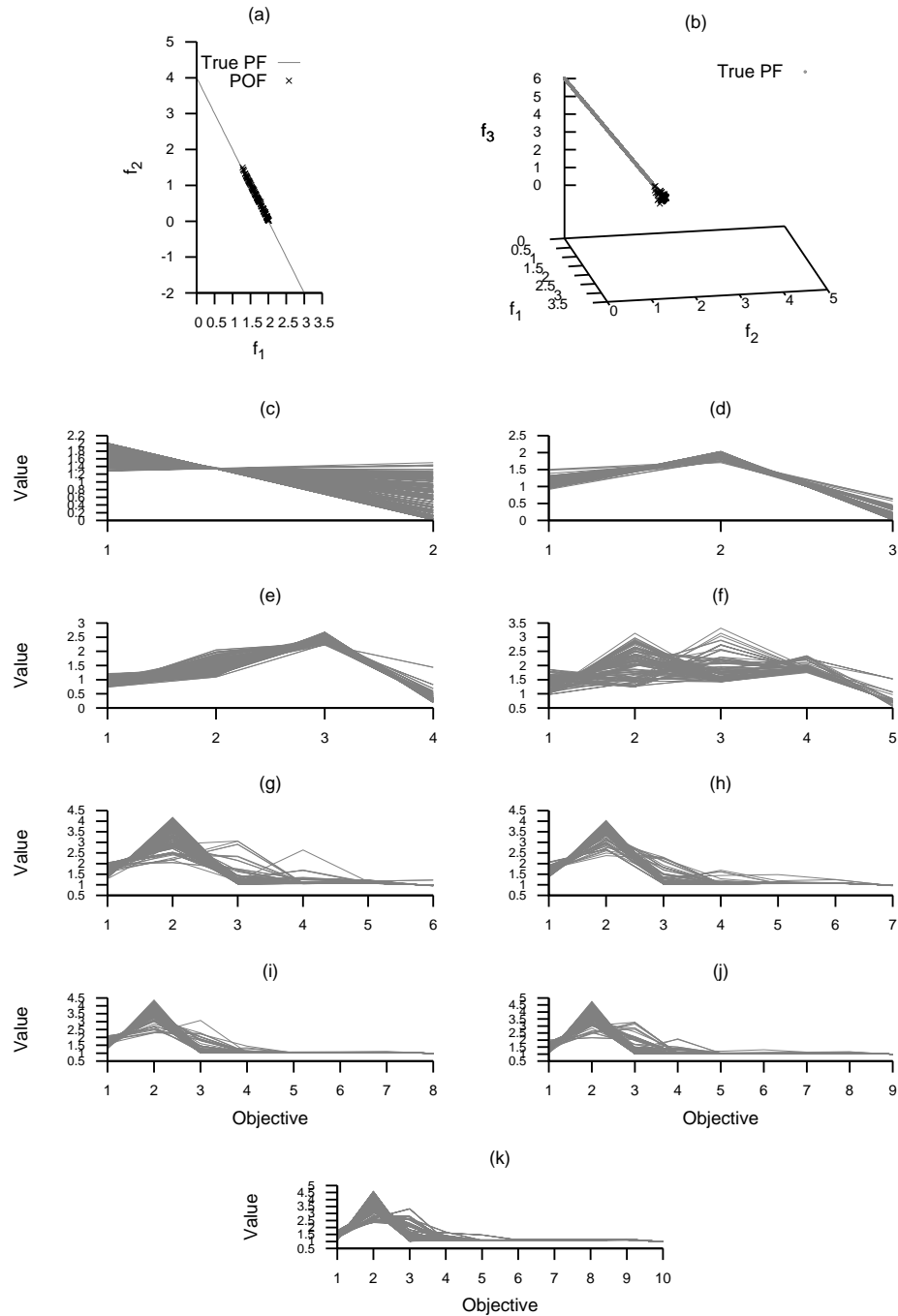


Figure A.473: Plots of the approximations obtained by MOMBI-RND from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

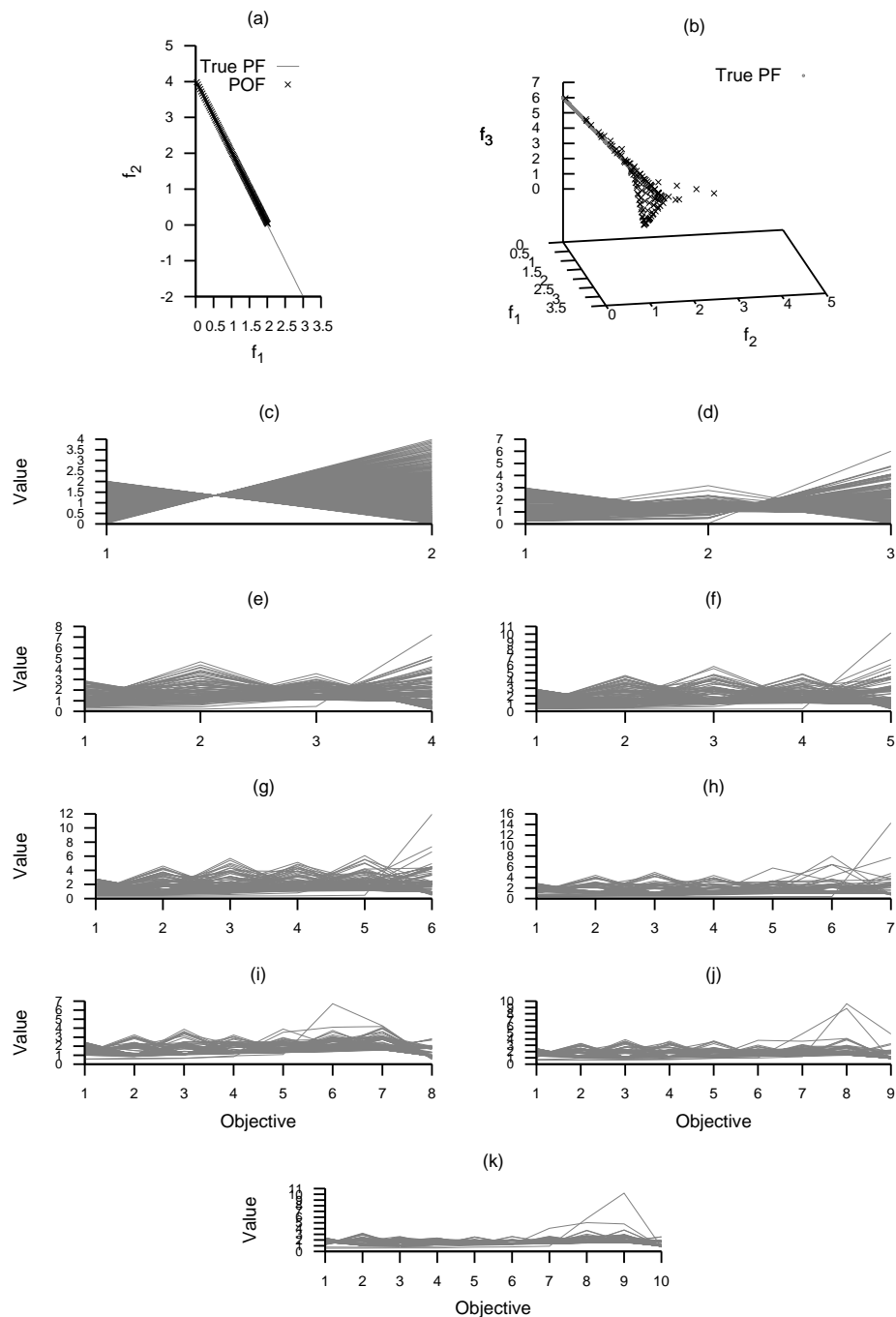


Figure A.474: Plots of the approximations obtained by MOMBI-SLD from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



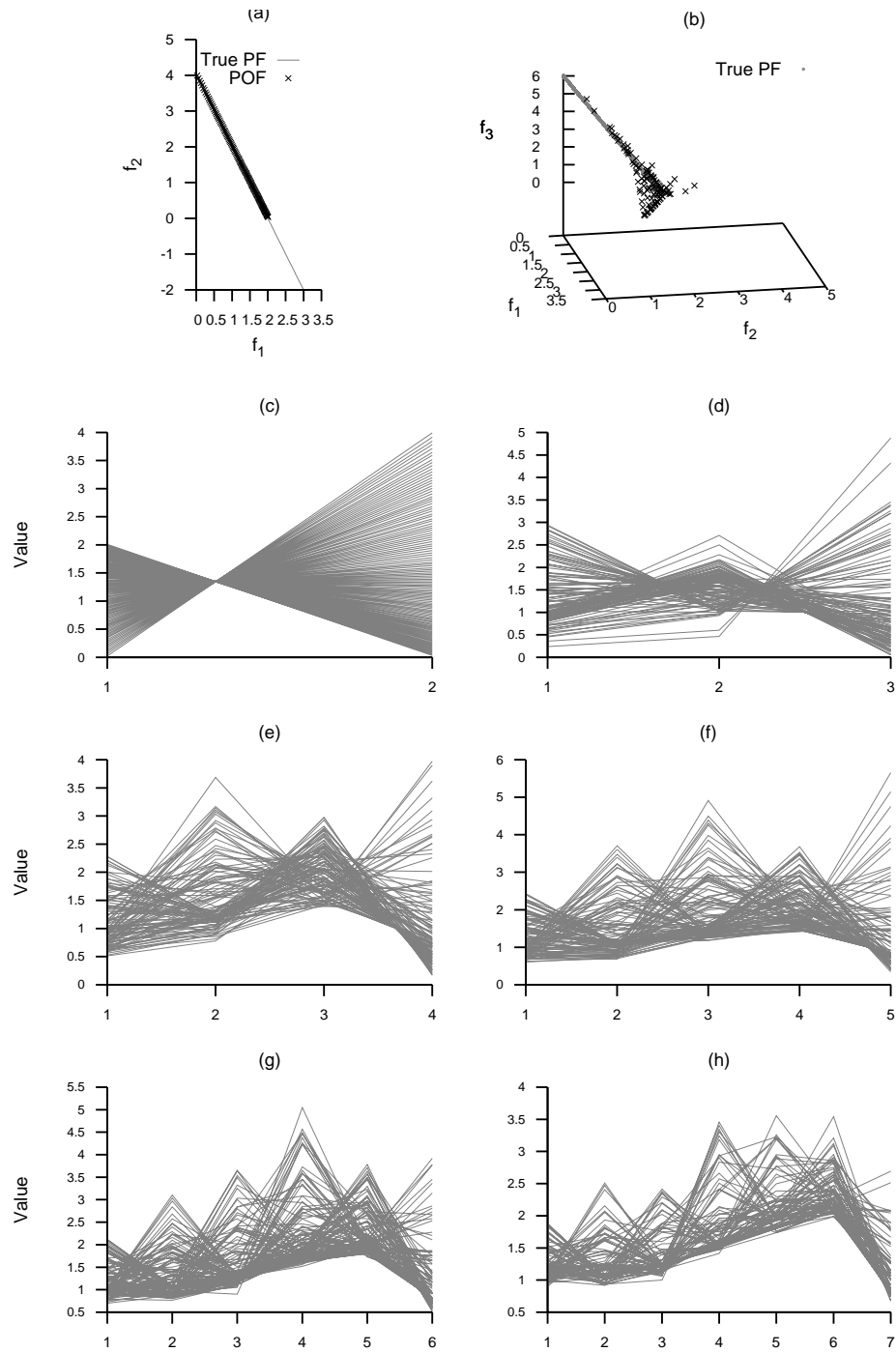


Figure A.475: Plots of the approximations obtained by MOMBI-HV from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

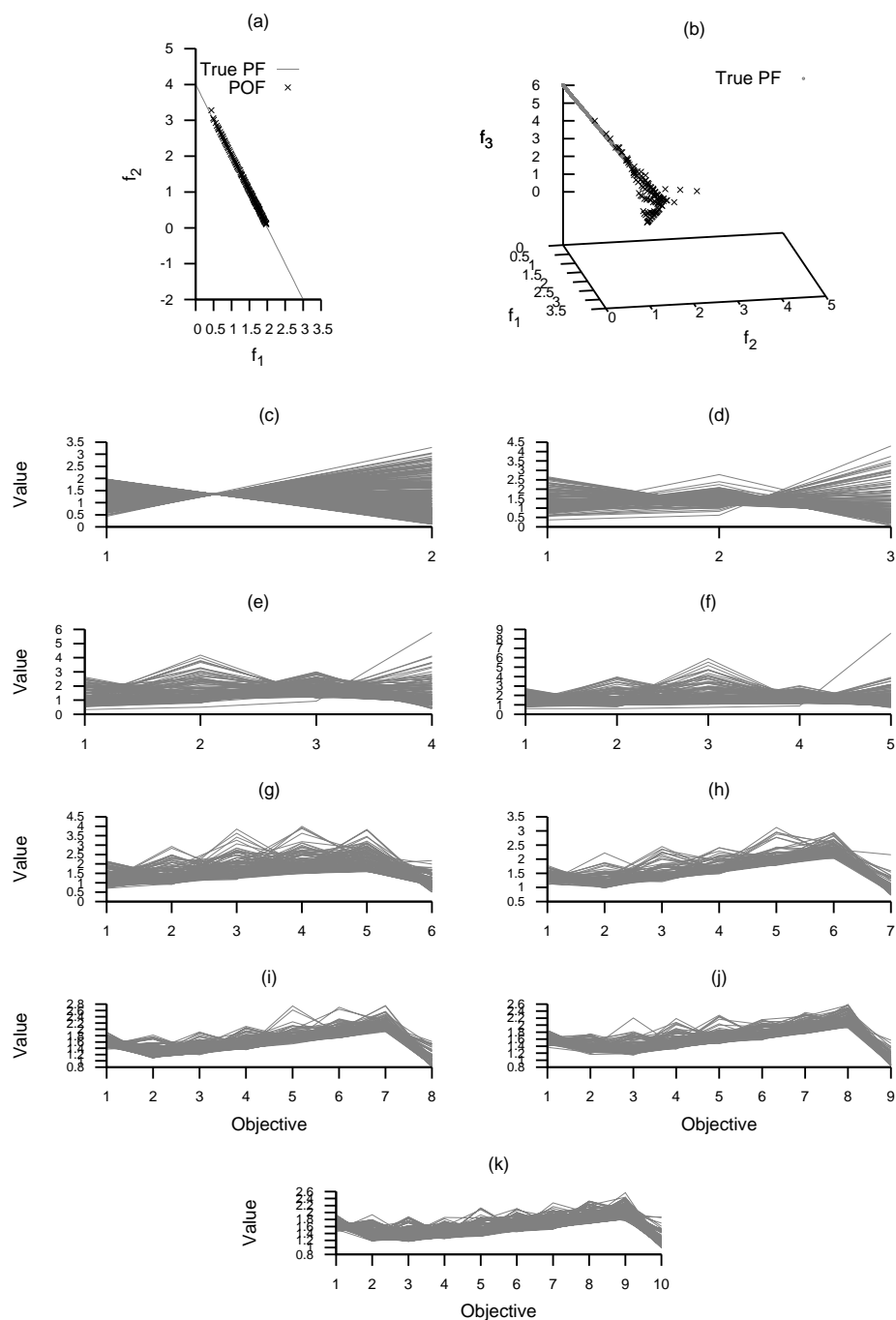


Figure A.476: Plots of the approximations obtained by MOMBI-UD from 2 to 10 objectives on the WFG3 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.208: Comparison of hypervolume indicator values for different optimizers on the WFG4 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	4.998116e + 00	8.512189e + 00	8.502855e + 00	8.224601e + 00
	avg.	5.004378e + 00	8.510736e + 00	8.499258e + 00	8.220359e + 00
	min.	4.978073e + 00	8.424609e + 00	8.402435e + 00	7.911521e + 00
	max.	5.083392e + 00	8.587181e + 00	8.554336e + 00	8.420553e + 00
	std.	1.569416e - 02	2.828531e - 02	2.829586e - 02	9.074351e - 02
<b>3D</b>	med.	6.630660e + 01	7.325775e + 01	7.254976e + 01	6.756666e + 01
	avg.	6.638146e + 01	7.328489e + 01	7.255806e + 01	6.751443e + 01
	min.	6.500193e + 01	7.252755e + 01	7.177932e + 01	6.580141e + 01
	max.	6.790273e + 01	7.391064e + 01	7.334450e + 01	6.888465e + 01
	std.	5.822367e - 01	3.022272e - 01	3.298881e - 01	5.981821e - 01
<b>4D</b>	med.	5.834977e + 02	7.111190e + 02	6.797374e + 02	6.040558e + 02
	avg.	5.804280e + 02	7.115377e + 02	6.795649e + 02	6.040336e + 02
	min.	5.421020e + 02	6.999201e + 02	6.613594e + 02	5.848166e + 02
	max.	6.081551e + 02	7.223507e + 02	6.911705e + 02	6.243967e + 02
	std.	1.504235e + 01	4.103053e + 00	5.431343e + 00	8.834593e + 00
<b>5D</b>	med.	6.482647e + 03	7.744822e + 03	7.383132e + 03	5.712558e + 03
	avg.	6.486036e + 03	7.745332e + 03	7.388516e + 03	5.705702e + 03
	min.	6.029635e + 03	7.464303e + 03	7.046159e + 03	5.383359e + 03
	max.	6.798456e + 03	7.947847e + 03	7.629638e + 03	5.936485e + 03
	std.	1.558656e + 02	8.912868e + 01	1.076498e + 02	1.221367e + 02
<b>6D</b>	med.	6.391948e + 04	9.294382e + 04	8.310709e + 04	4.398775e + 04
	avg.	5.827368e + 04	9.296114e + 04	8.213072e + 04	4.639293e + 04
	min.	2.632410e + 04	8.089046e + 04	6.484938e + 04	4.040624e + 04
	max.	7.589702e + 04	1.010195e + 05	8.919791e + 04	7.135038e + 04
	std.	1.465232e + 04	3.268383e + 03	4.375501e + 03	7.314525e + 03
<b>7D</b>	med.	4.109422e + 05	1.306889e + 06	6.658926e + 05	6.622757e + 05
	avg.	5.202542e + 05	1.159801e + 06	7.732589e + 05	6.675251e + 05
	min.	3.862948e + 05	6.639146e + 05	6.584410e + 05	6.547249e + 05
	max.	1.231820e + 06	1.477194e + 06	1.130171e + 06	8.744367e + 05
	std.	2.448392e + 05	2.880657e + 05	1.628121e + 05	3.117250e + 04
<b>8D</b>	med.	1.274991e + 07	2.026773e + 07	—	1.102482e + 07
	avg.	1.128956e + 07	1.877321e + 07	—	1.101785e + 07
	min.	2.011789e + 06	1.098810e + 07	—	1.067165e + 07
	max.	1.841004e + 07	2.378899e + 07	—	1.115434e + 07
	std.	5.342090e + 06	4.289259e + 06	—	7.839728e + 04
<b>9D</b>	med.	2.771688e + 08	3.483119e + 08	—	2.036890e + 08
	avg.	2.261079e + 08	3.138514e + 08	—	2.035431e + 08
	min.	3.387436e + 07	2.020754e + 08	—	1.964331e + 08
	max.	3.957559e + 08	4.227263e + 08	—	2.075518e + 08
	std.	1.162802e + 08	8.589725e + 07	—	2.254154e + 06
<b>10D</b>	med.	5.123542e + 09	6.695075e + 09	—	4.154093e + 09
	avg.	3.912232e + 09	6.329458e + 09	—	4.150918e + 09
	min.	6.373930e + 08	4.055430e + 09	—	4.000695e + 09
	max.	7.588749e + 09	8.321443e + 09	—	4.311239e + 09
	std.	2.462856e + 09	1.445235e + 09	—	5.755867e + 07

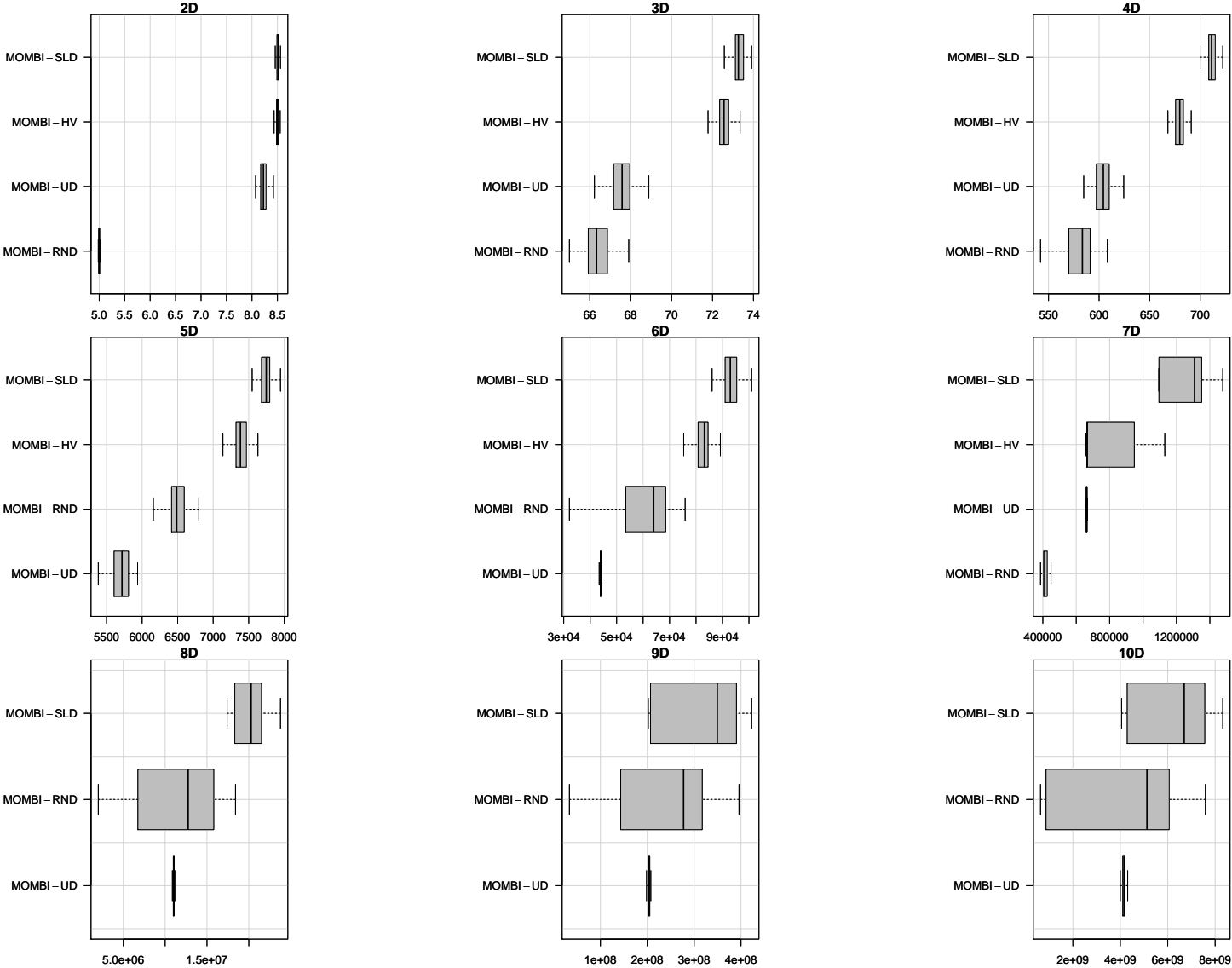


Figure A.477: Box-plot of the hypervolume indicator values for different optimizers on the WFG4 test problem.

Table A.209: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG4 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $ph$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
2D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e-34$	—	$4.11e-03$	$1.28e-34$
MOMBI-HV	$1.28e-34$	$> 0.05$	—	$1.36e-34$
MOMBI-UD	$1.28e-34$	$> 0.05$	$> 0.05$	—
3D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e-34$	—	$9.72e-28$	$1.28e-34$
MOMBI-HV	$1.28e-34$	$> 0.05$	—	$1.28e-34$
MOMBI-UD	$5.28e-24$	$> 0.05$	$> 0.05$	—
4D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e-34$	—	$1.28e-34$	$1.28e-34$
MOMBI-HV	$1.28e-34$	$> 0.05$	—	$1.28e-34$
MOMBI-UD	$7.02e-26$	$> 0.05$	$> 0.05$	—
5D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$1.28e-34$
MOMBI-SLD	$1.28e-34$	—	$4.37e-34$	$1.28e-34$
MOMBI-HV	$1.28e-34$	$> 0.05$	—	$1.28e-34$
MOMBI-UD	$> 0.05$	$> 0.05$	$> 0.05$	—
6D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$4.24e-13$
MOMBI-SLD	$1.28e-34$	—	$8.35e-33$	$1.28e-34$
MOMBI-HV	$1.33e-32$	$> 0.05$	—	$2.63e-34$
MOMBI-UD	$> 0.05$	$> 0.05$	$> 0.05$	—
7D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$4.84e-28$	—	$2.25e-21$	$1.55e-31$
MOMBI-HV	$9.71e-17$	$> 0.05$	—	$4.02e-13$
MOMBI-UD	$7.07e-14$	$> 0.05$	$> 0.05$	—
8D				
MOMBI-RND	—	$> 0.05$		$2.32e-03$
MOMBI-SLD	$1.15e-19$	—		$6.16e-29$
MOMBI-HV			—	
MOMBI-UD	$> 0.05$	$> 0.05$		—
9D				
MOMBI-RND	—	$> 0.05$		$1.11e-04$
MOMBI-SLD	$6.37e-09$	—		$7.89e-27$
MOMBI-HV			—	
MOMBI-UD	$> 0.05$	$> 0.05$		—
10D				
MOMBI-RND	—	$> 0.05$		$2.67e-02$
MOMBI-SLD	$1.37e-13$	—		$7.15e-25$
MOMBI-HV			—	
MOMBI-UD	$> 0.05$	$> 0.05$		—

Table A.210: Comparison of R2 indicator values for different optimizers on the WFG4 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	1.000076e + 00	5.931164e - 01	5.935576e - 01	6.034608e - 01
	avg.	9.994257e - 01	5.932498e - 01	5.935658e - 01	6.045308e - 01
	min.	9.910791e - 01	5.913263e - 01	5.918518e - 01	5.964546e - 01
	max.	1.001703e + 00	5.956028e - 01	5.958976e - 01	6.292437e - 01
	std.	1.637089e - 03	7.991419e - 04	7.826691e - 04	5.550868e - 03
<b>3D</b>	med.	3.439367e - 01	3.214853e - 01	3.235663e - 01	3.399016e - 01
	avg.	3.439338e - 01	3.213694e - 01	3.236343e - 01	3.400055e - 01
	min.	3.385185e - 01	3.196981e - 01	3.213913e - 01	3.348051e - 01
	max.	3.499643e - 01	3.239349e - 01	3.262030e - 01	3.464214e - 01
	std.	2.189799e - 03	8.557186e - 04	9.552453e - 04	2.308077e - 03
<b>4D</b>	med.	2.722849e - 01	2.184237e - 01	2.292193e - 01	2.666870e - 01
	avg.	2.786791e - 01	2.182900e - 01	2.294110e - 01	2.666573e - 01
	min.	2.621919e - 01	2.158972e - 01	2.265173e - 01	2.579988e - 01
	max.	3.197708e - 01	2.212490e - 01	2.343992e - 01	2.798292e - 01
	std.	1.502136e - 02	9.735673e - 04	1.580481e - 03	3.956290e - 03
<b>5D</b>	med.	2.149064e - 01	1.847100e - 01	1.927692e - 01	2.496930e - 01
	avg.	2.173301e - 01	1.847490e - 01	1.931858e - 01	2.506948e - 01
	min.	2.039061e - 01	1.798692e - 01	1.881475e - 01	2.388041e - 01
	max.	2.560025e - 01	1.905848e - 01	2.000243e - 01	2.643023e - 01
	std.	1.132956e - 02	1.650164e - 03	2.489784e - 03	5.962137e - 03
<b>6D</b>	med.	3.246720e - 01	1.633801e - 01	1.839677e - 01	3.353411e - 01
	avg.	3.788931e - 01	1.614015e - 01	1.859636e - 01	3.241528e - 01
	min.	2.278414e - 01	1.452887e - 01	1.744816e - 01	2.180309e - 01
	max.	6.655007e - 01	1.939032e - 01	2.284572e - 01	3.448526e - 01
	std.	1.311472e - 01	9.679089e - 03	9.461465e - 03	3.303510e - 02
<b>7D</b>	med.	5.667703e - 01	1.453797e - 01	2.867976e - 01	2.871194e - 01
	avg.	4.995106e - 01	1.811782e - 01	2.562246e - 01	2.852891e - 01
	min.	1.579060e - 01	1.281570e - 01	1.774071e - 01	2.144034e - 01
	max.	5.735342e - 01	2.871385e - 01	2.878019e - 01	2.880219e - 01
	std.	1.376137e - 01	6.078380e - 02	4.514136e - 02	1.089310e - 02
<b>8D</b>	med.	2.737049e - 01	1.452392e - 01	—	2.526949e - 01
	avg.	5.465741e - 01	1.615249e - 01	—	2.527262e - 01
	min.	1.829330e - 01	1.152037e - 01	—	2.519119e - 01
	max.	1.983216e + 00	2.532140e - 01	—	2.550349e - 01
	std.	5.696651e - 01	4.953163e - 02	—	5.219368e - 04
<b>9D</b>	med.	2.432751e - 01	1.380783e - 01	—	2.264293e - 01
	avg.	6.203667e - 01	1.615653e - 01	—	2.264615e - 01
	min.	1.228156e - 01	1.071724e - 01	—	2.243151e - 01
	max.	2.015517e + 00	2.272361e - 01	—	2.291171e - 01
	std.	7.128252e - 01	5.040569e - 02	—	8.143835e - 04
<b>10D</b>	med.	2.541193e - 01	1.303714e - 01	—	2.054767e - 01
	avg.	7.561792e - 01	1.443584e - 01	—	2.056246e - 01
	min.	1.193650e - 01	1.017764e - 01	—	2.032079e - 01
	max.	2.023275e + 00	2.073630e - 01	—	2.078750e - 01
	std.	7.616766e - 01	3.903566e - 02	—	8.982000e - 04

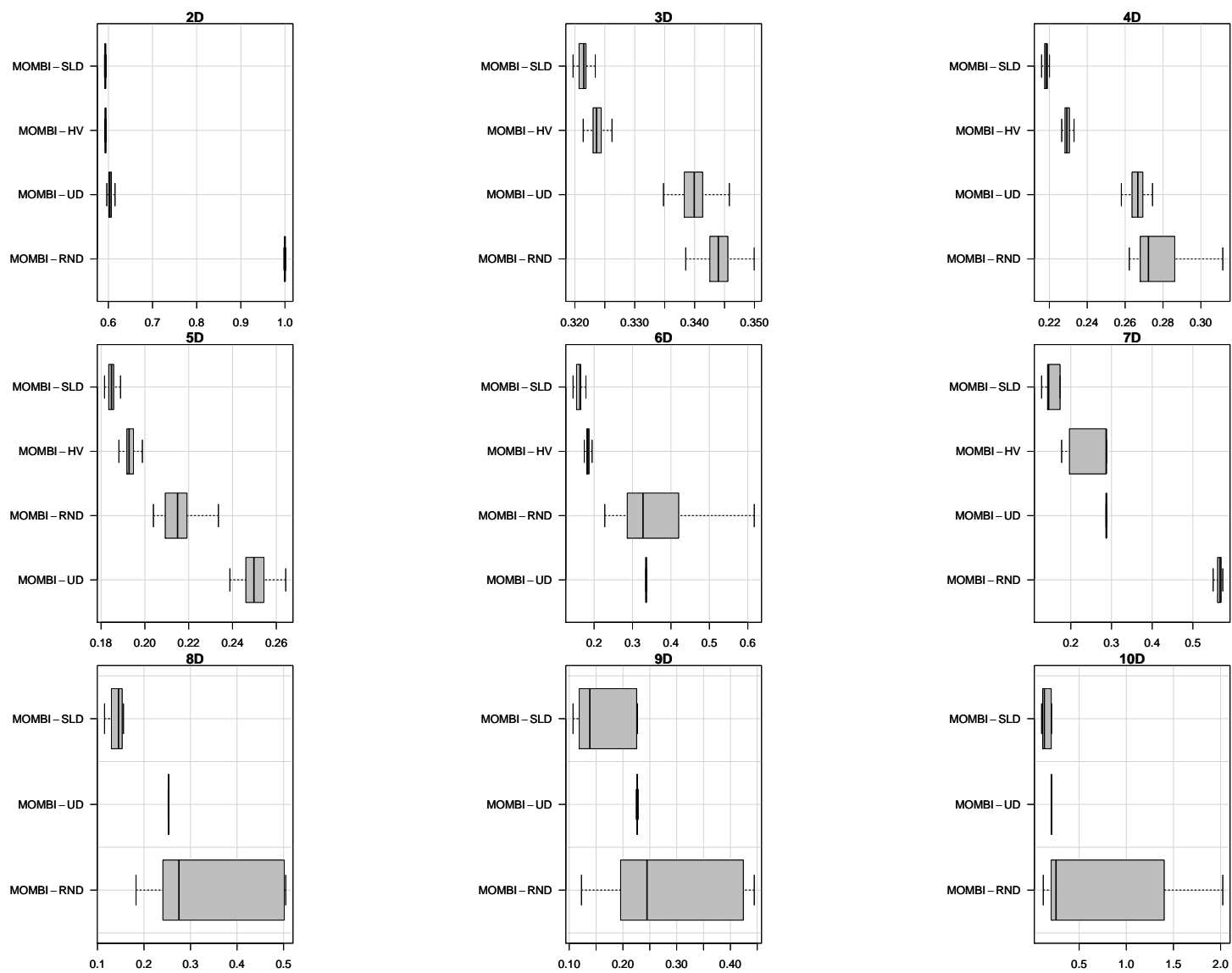


Figure A.478: Box-plot of the R2 indicator values for different optimizers on the WFG4 test problem.

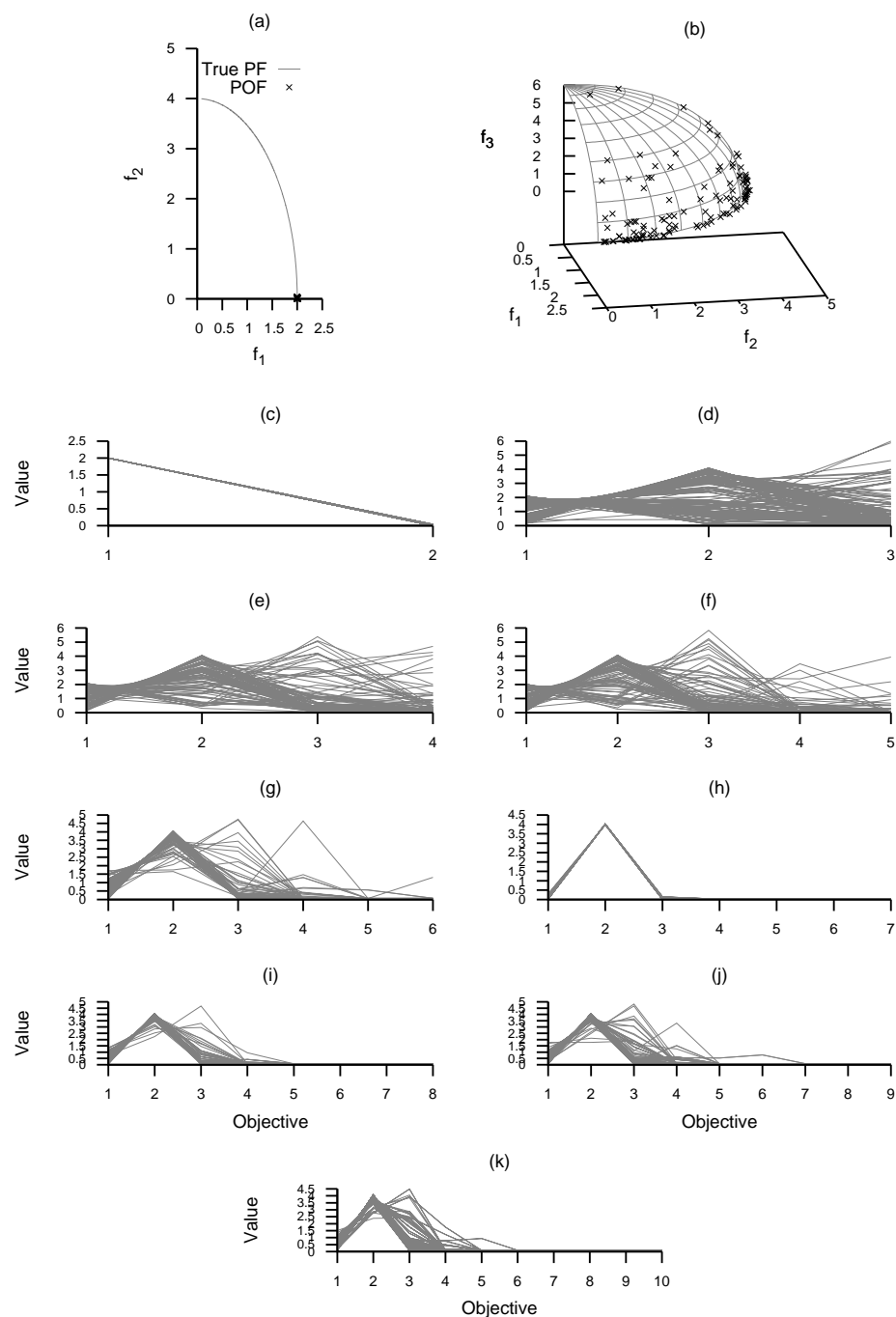


Figure A.479: Plots of the approximations obtained by MOMBI-RND from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.



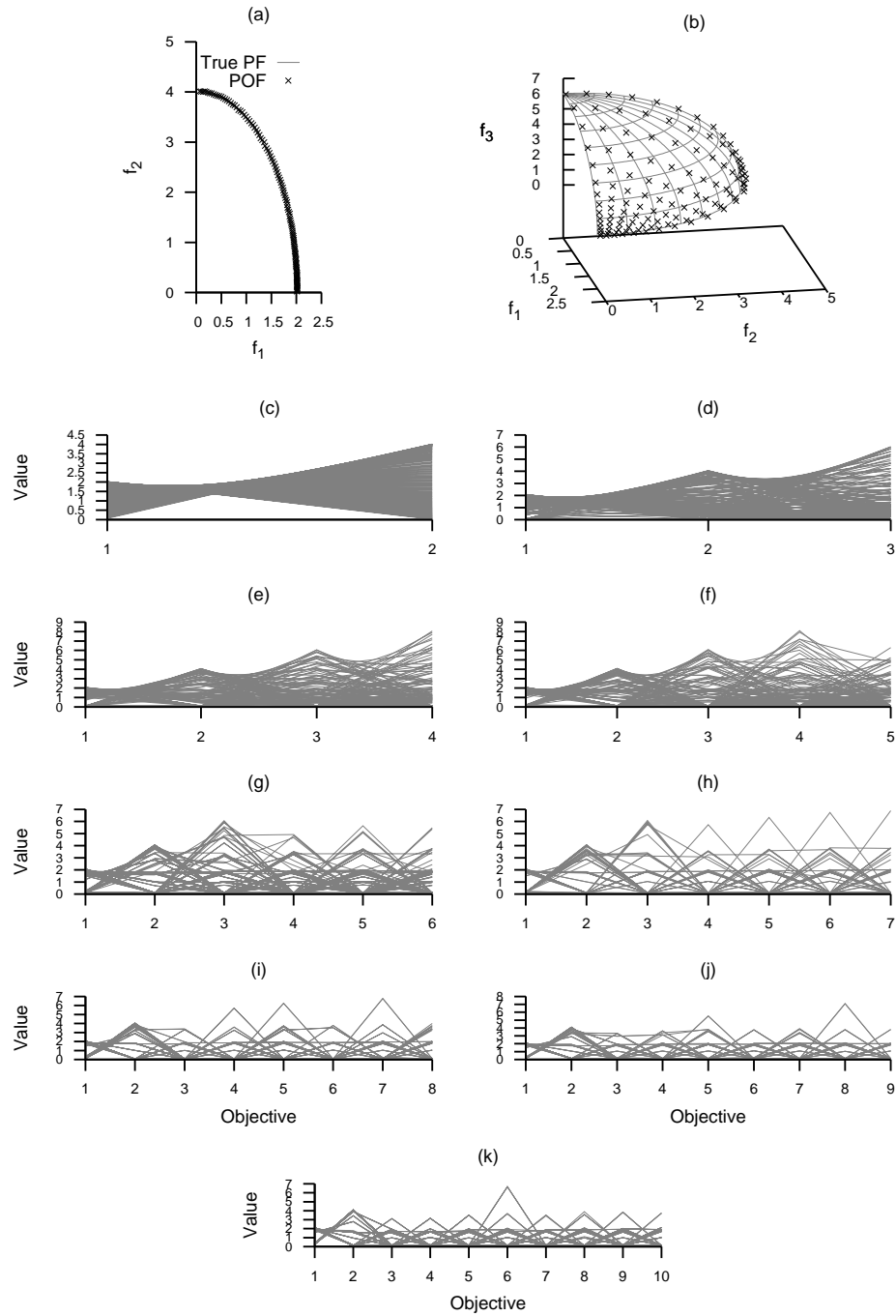


Figure A.480: Plots of the approximations obtained by MOMBI-SLD from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

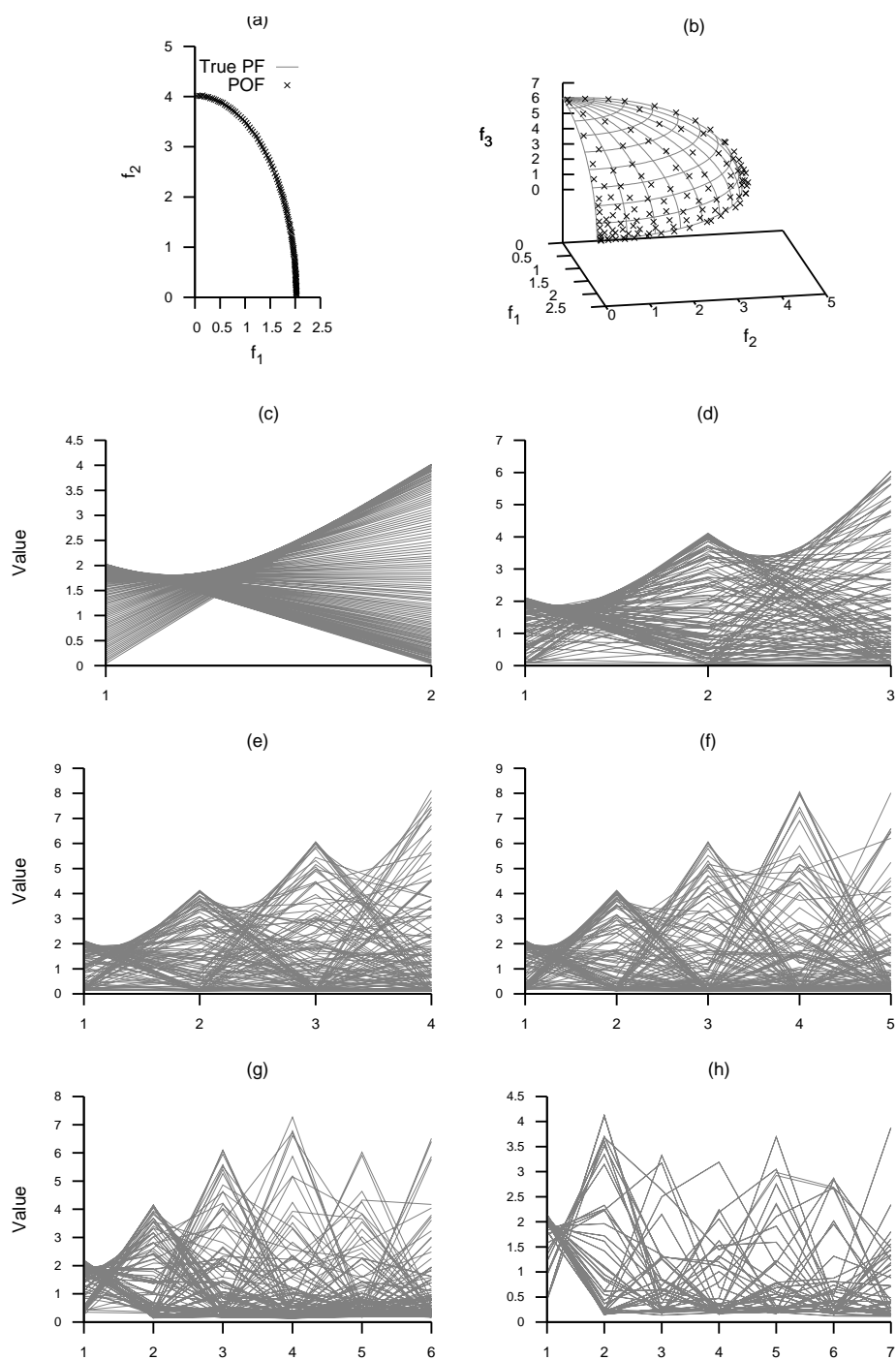


Figure A.481: Plots of the approximations obtained by MOMBI-HV from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

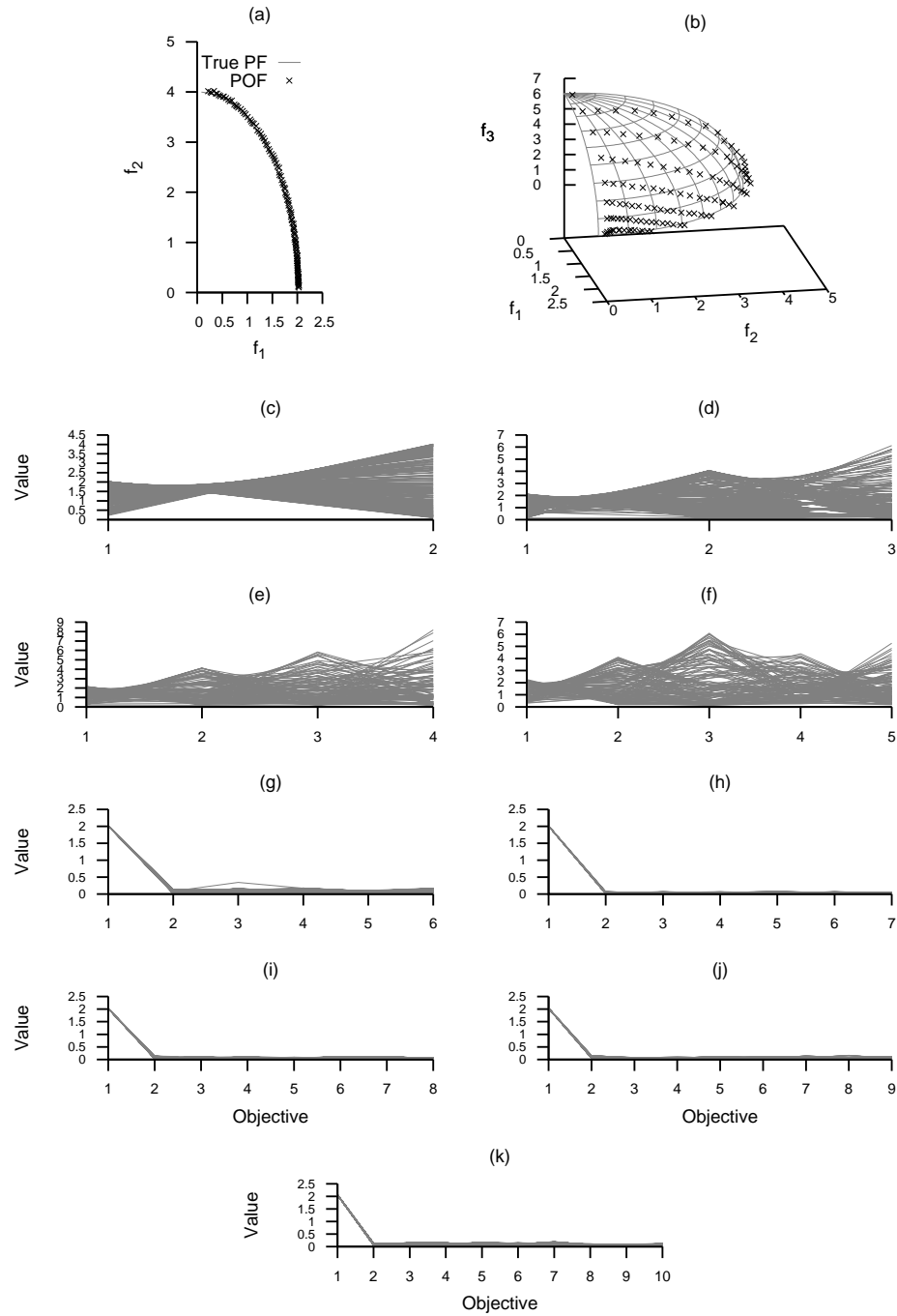


Figure A.482: Plots of the approximations obtained by MOMBI-UD from 2 to 10 objectives on the WFG4 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.211: Comparison of hypervolume indicator values for different optimizers on the WFG5 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	4.831089e + 00	8.096190e + 00	8.082172e + 00	7.857217e + 00
	avg.	4.983343e + 00	8.101683e + 00	8.084205e + 00	7.844554e + 00
	min.	4.708275e + 00	8.071070e + 00	8.053056e + 00	7.559744e + 00
	max.	7.607272e + 00	8.185343e + 00	8.135598e + 00	7.989219e + 00
	std.	5.124626e - 01	2.112125e - 02	1.628228e - 02	8.246838e - 02
<b>3D</b>	med.	6.422555e + 01	7.077768e + 01	7.007490e + 01	6.488644e + 01
	avg.	6.432002e + 01	7.077819e + 01	7.008892e + 01	6.490005e + 01
	min.	6.294553e + 01	6.994676e + 01	6.925547e + 01	6.350152e + 01
	max.	6.590015e + 01	7.160496e + 01	7.084712e + 01	6.647370e + 01
	std.	6.175197e - 01	3.317563e - 01	3.158238e - 01	7.315665e - 01
<b>4D</b>	med.	5.621781e + 02	6.895545e + 02	6.645925e + 02	5.695436e + 02
	avg.	5.598454e + 02	6.895115e + 02	6.648538e + 02	5.685794e + 02
	min.	5.268793e + 02	6.826949e + 02	6.550137e + 02	5.195969e + 02
	max.	5.798953e + 02	6.985266e + 02	6.761515e + 02	6.020211e + 02
	std.	1.215121e + 01	2.989744e + 00	3.929475e + 00	1.310441e + 01
<b>5D</b>	med.	6.307307e + 03	7.544042e + 03	7.171497e + 03	5.418597e + 03
	avg.	6.275521e + 03	7.526131e + 03	7.166274e + 03	5.297701e + 03
	min.	5.771715e + 03	7.149590e + 03	6.820366e + 03	3.251733e + 03
	max.	6.411844e + 03	7.728910e + 03	7.404054e + 03	5.710192e + 03
	std.	1.161161e + 02	1.136257e + 02	1.012321e + 02	5.089266e + 02
<b>6D</b>	med.	6.877245e + 04	9.337986e + 04	7.471001e + 04	4.197530e + 04
	avg.	6.883155e + 04	9.333034e + 04	7.034835e + 04	4.260481e + 04
	min.	4.895205e + 04	7.868465e + 04	4.143439e + 04	3.871262e + 04
	max.	8.083910e + 04	9.910495e + 04	9.256889e + 04	6.593260e + 04
	std.	7.000884e + 03	2.983705e + 03	1.219069e + 04	3.183947e + 03
<b>7D</b>	med.	1.161712e + 06	1.323323e + 06	6.302278e + 05	6.179801e + 05
	avg.	9.076581e + 05	1.279240e + 06	6.654317e + 05	6.188666e + 05
	min.	3.549547e + 05	6.325560e + 05	6.173433e + 05	6.146174e + 05
	max.	1.230011e + 06	1.447013e + 06	9.413851e + 05	6.320977e + 05
	std.	3.496109e + 05	1.629864e + 05	8.997689e + 04	2.909561e + 03
<b>8D</b>	med.	1.756521e + 07	2.050195e + 07	—	1.040800e + 07
	avg.	1.724478e + 07	1.891145e + 07	—	1.042289e + 07
	min.	1.322921e + 07	1.059388e + 07	—	1.030610e + 07
	max.	1.971617e + 07	2.419014e + 07	—	1.068635e + 07
	std.	1.564458e + 06	4.210443e + 06	—	6.112145e + 04
<b>9D</b>	med.	3.156684e + 08	2.609648e + 08	—	1.944542e + 08
	avg.	3.167109e + 08	2.826456e + 08	—	1.947088e + 08
	min.	2.338592e + 08	1.896652e + 08	—	1.916203e + 08
	max.	3.695348e + 08	4.150215e + 08	—	2.034429e + 08
	std.	3.148973e + 07	8.479607e + 07	—	1.642094e + 06
<b>10D</b>	med.	6.431547e + 09	4.057034e + 09	—	3.946301e + 09
	avg.	6.467081e + 09	4.831120e + 09	—	3.933916e + 09
	min.	4.838139e + 09	3.689389e + 09	—	3.159853e + 09
	max.	8.036291e + 09	8.054476e + 09	—	4.026577e + 09
	std.	6.354413e + 08	1.314662e + 09	—	9.060048e + 07

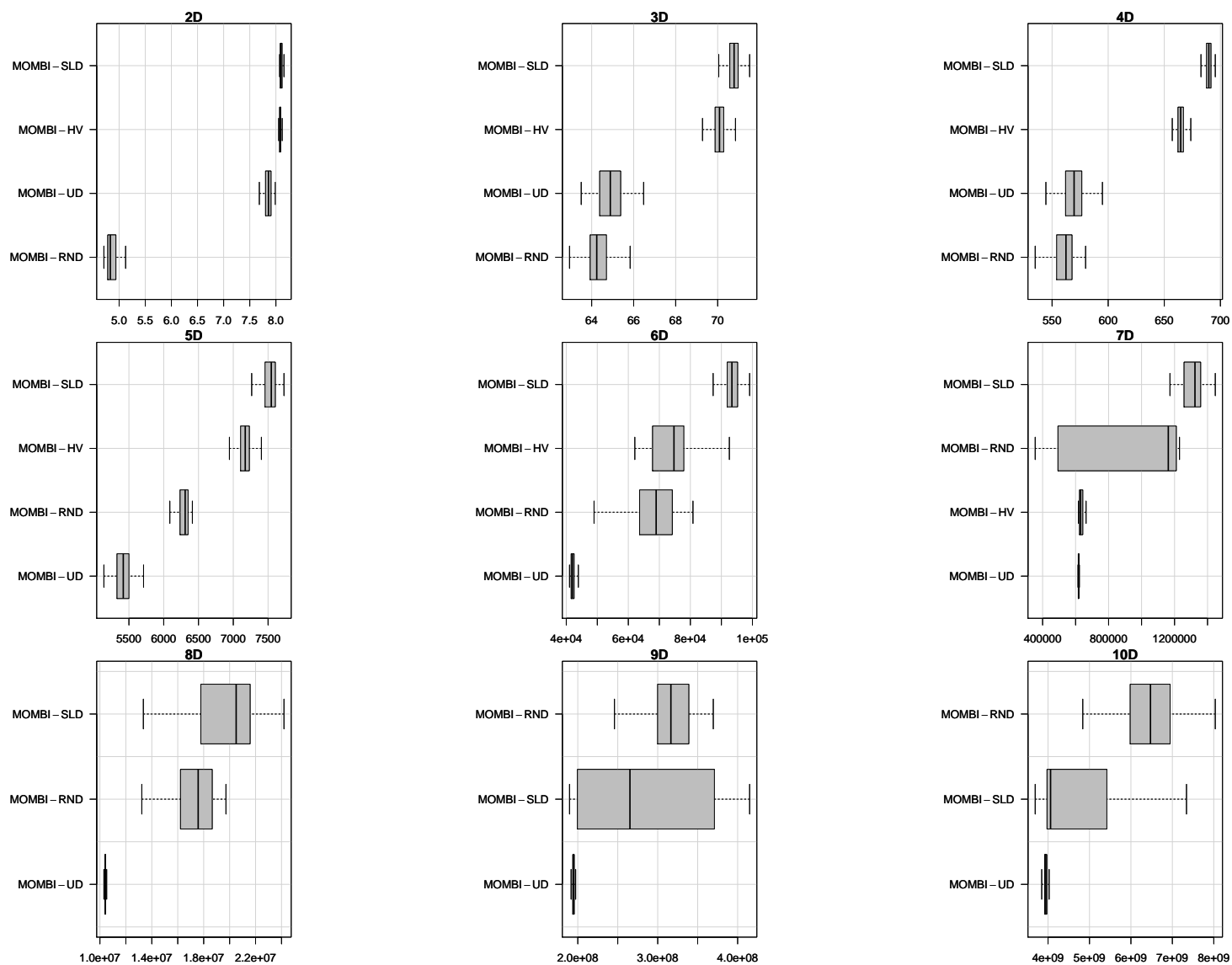


Figure A.483: Box-plot of the hypervolume indicator values for different optimizers on the WFG5 test problem.

Table A.212: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG5 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $ph$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
2D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e - 34$	–	$1.74e - 09$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.28e - 34$
MOMBI-UD	$1.44e - 34$	> 0.05	> 0.05	–
3D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e - 34$	–	$1.17e - 26$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.28e - 34$
MOMBI-UD	$3.90e - 08$	> 0.05	> 0.05	–
4D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e - 34$	–	$1.28e - 34$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.28e - 34$
MOMBI-UD	$7.24e - 07$	> 0.05	> 0.05	–
5D				
MOMBI-RND	–	> 0.05	> 0.05	$1.28e - 34$
MOMBI-SLD	$1.28e - 34$	–	$5.71e - 33$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.28e - 34$
MOMBI-UD	> 0.05	> 0.05	> 0.05	–
6D				
MOMBI-RND	–	> 0.05	> 0.05	$5.38e - 34$
MOMBI-SLD	$1.68e - 34$	–	$7.03e - 34$	$1.28e - 34$
MOMBI-HV	$4.84e - 04$	> 0.05	–	$6.24e - 30$
MOMBI-UD	> 0.05	> 0.05	> 0.05	–
7D				
MOMBI-RND	–	> 0.05	$1.18e - 06$	$1.45e - 07$
MOMBI-SLD	$7.99e - 26$	–	$5.88e - 33$	$1.28e - 34$
MOMBI-HV	> 0.05	> 0.05	–	$1.20e - 22$
MOMBI-UD	> 0.05	> 0.05	> 0.05	–
8D				
MOMBI-RND	–	> 0.05		$1.28e - 34$
MOMBI-SLD	$3.96e - 10$	–	–	$1.63e - 34$
MOMBI-HV			–	
MOMBI-UD	> 0.05	> 0.05		–
9D				
MOMBI-RND	–	$1.85e - 02$		$1.28e - 34$
MOMBI-SLD	> 0.05	–		$2.27e - 28$
MOMBI-HV			–	
MOMBI-UD	> 0.05	> 0.05		–
10D				
MOMBI-RND	–	$4.28e - 15$		$1.28e - 34$
MOMBI-SLD	> 0.05	–		$6.43e - 16$
MOMBI-HV			–	
MOMBI-UD	> 0.05	> 0.05		–

Table A.213: Comparison of R2 indicator values for different optimizers on the WFG5 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	1.011756e + 00	6.104407e - 01	6.109642e - 01	6.252675e - 01
	avg.	9.928605e - 01	6.101976e - 01	6.109062e - 01	6.269589e - 01
	min.	6.532467e - 01	6.068879e - 01	6.084719e - 01	6.161805e - 01
	max.	1.024964e + 00	6.116674e - 01	6.122945e - 01	6.577884e - 01
	std.	6.550938e - 02	9.749281e - 04	7.659341e - 04	7.706846e - 03
<b>3D</b>	med.	3.530719e - 01	3.285524e - 01	3.311433e - 01	3.510930e - 01
	avg.	3.528453e - 01	3.287080e - 01	3.311775e - 01	3.509318e - 01
	min.	3.467214e - 01	3.259216e - 01	3.289925e - 01	3.435875e - 01
	max.	3.598371e - 01	3.318075e - 01	3.345306e - 01	3.606921e - 01
	std.	2.860525e - 03	1.190821e - 03	1.149992e - 03	3.736584e - 03
<b>4D</b>	med.	2.788343e - 01	2.239873e - 01	2.338526e - 01	2.847138e - 01
	avg.	2.831439e - 01	2.241520e - 01	2.337672e - 01	2.850265e - 01
	min.	2.708754e - 01	2.212574e - 01	2.298931e - 01	2.662949e - 01
	max.	3.263366e - 01	2.320541e - 01	2.369321e - 01	3.103573e - 01
	std.	1.194843e - 02	1.583670e - 03	1.255433e - 03	7.685479e - 03
<b>5D</b>	med.	2.147473e - 01	1.879079e - 01	1.978173e - 01	2.624095e - 01
	avg.	2.174145e - 01	1.885043e - 01	1.978681e - 01	2.709773e - 01
	min.	2.110866e - 01	1.827837e - 01	1.920636e - 01	2.532007e - 01
	max.	2.617001e - 01	1.972543e - 01	2.078692e - 01	4.060124e - 01
	std.	7.806148e - 03	2.957109e - 03	2.621317e - 03	3.339564e - 02
<b>6D</b>	med.	2.636011e - 01	1.623719e - 01	1.996735e - 01	3.391635e - 01
	avg.	2.741908e - 01	1.618825e - 01	2.199277e - 01	3.364720e - 01
	min.	2.123869e - 01	1.449213e - 01	1.693646e - 01	2.418210e - 01
	max.	4.539511e - 01	1.952131e - 01	3.418195e - 01	3.498634e - 01
	std.	4.833740e - 02	7.474787e - 03	4.860497e - 02	1.361909e - 02
<b>7D</b>	med.	1.644477e - 01	1.447412e - 01	2.906617e - 01	2.935536e - 01
	avg.	3.003405e - 01	1.554510e - 01	2.794508e - 01	2.933219e - 01
	min.	1.578354e - 01	1.284536e - 01	1.992802e - 01	2.902640e - 01
	max.	5.771192e - 01	2.904291e - 01	2.937595e - 01	2.938319e - 01
	std.	1.731325e - 01	3.139537e - 02	2.898719e - 02	6.140383e - 04
<b>8D</b>	med.	1.872487e - 01	1.438928e - 01	—	2.572483e - 01
	avg.	2.003948e - 01	1.603361e - 01	—	2.570034e - 01
	min.	1.598605e - 01	1.135556e - 01	—	2.537839e - 01
	max.	3.035284e - 01	2.553263e - 01	—	2.579441e - 01
	std.	3.316736e - 02	4.534088e - 02	—	6.962330e - 04
<b>9D</b>	med.	1.878747e - 01	1.693970e - 01	—	2.296762e - 01
	avg.	1.927416e - 01	1.780467e - 01	—	2.294149e - 01
	min.	1.374138e - 01	1.139166e - 01	—	2.239753e - 01
	max.	2.876614e - 01	2.307614e - 01	—	2.308390e - 01
	std.	3.119064e - 02	4.721571e - 02	—	8.805152e - 04
<b>10D</b>	med.	1.708849e - 01	2.056309e - 01	—	2.087167e - 01
	avg.	1.740713e - 01	1.839151e - 01	—	2.090360e - 01
	min.	1.091268e - 01	1.097689e - 01	—	2.067617e - 01
	max.	2.572954e - 01	2.131419e - 01	—	2.279641e - 01
	std.	3.019733e - 02	3.468076e - 02	—	2.105975e - 03

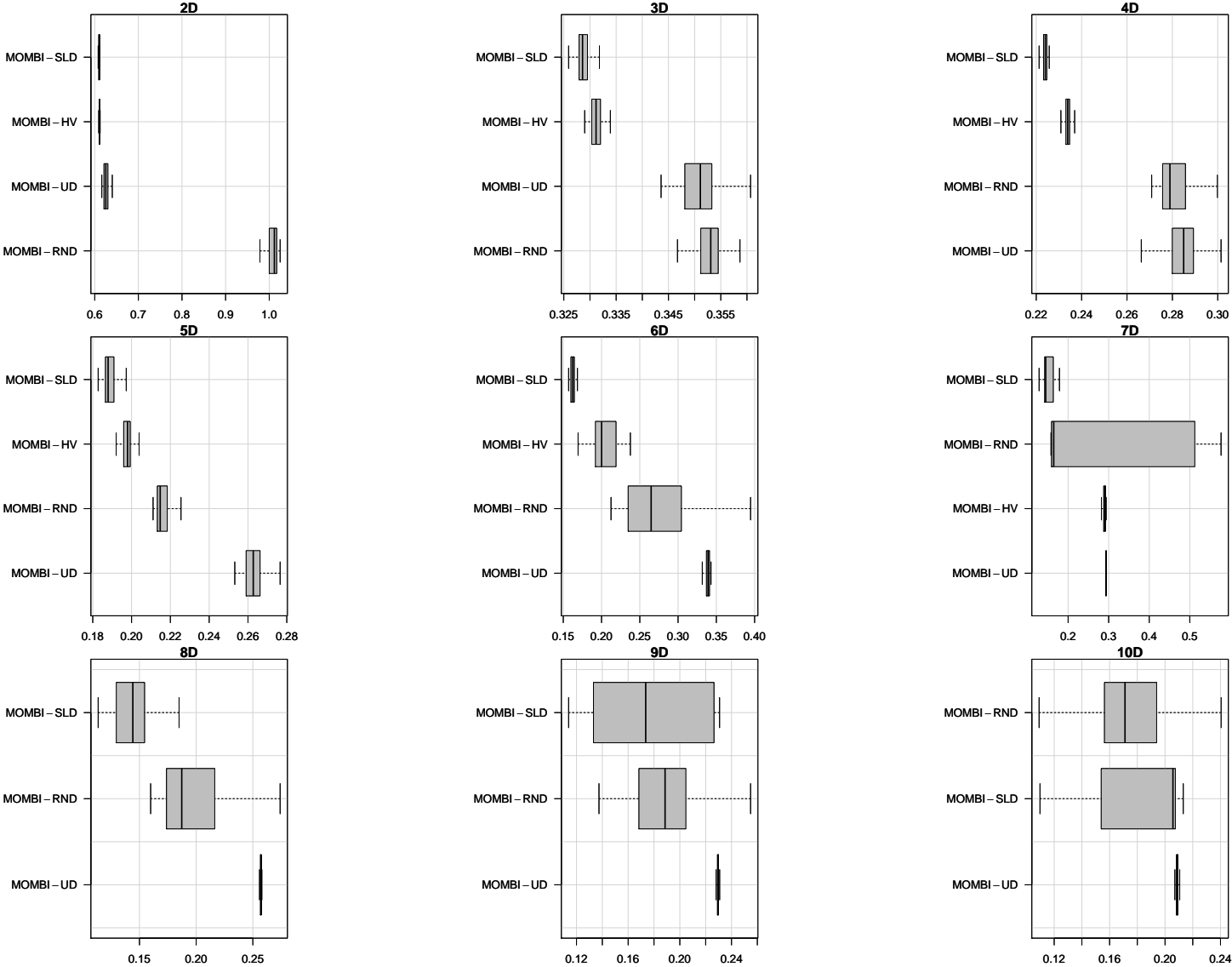


Figure A.484: Box-plot of the R2 indicator values for different optimizers on the WFG5 test problem.



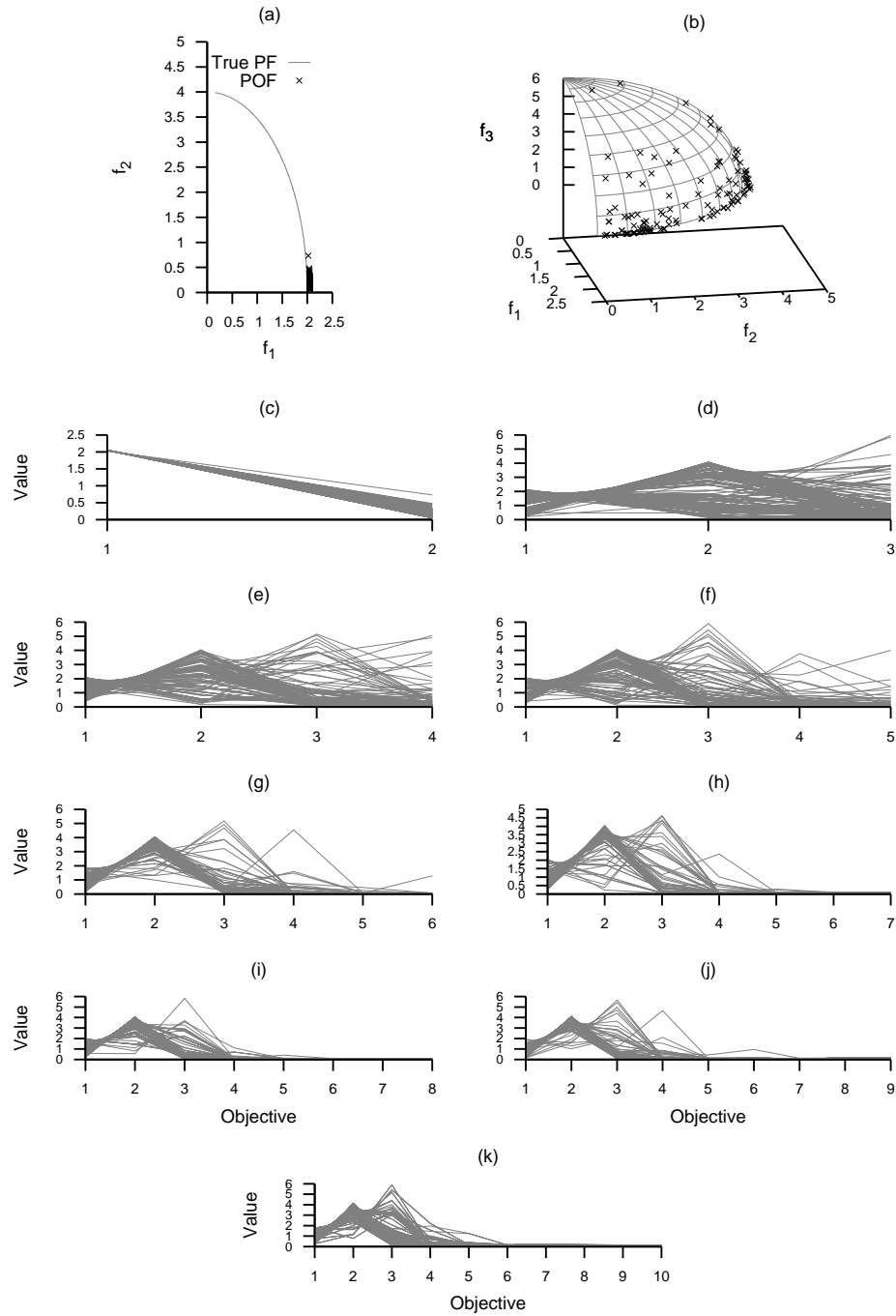


Figure A.485: Plots of the approximations obtained by MOMBI-RND from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

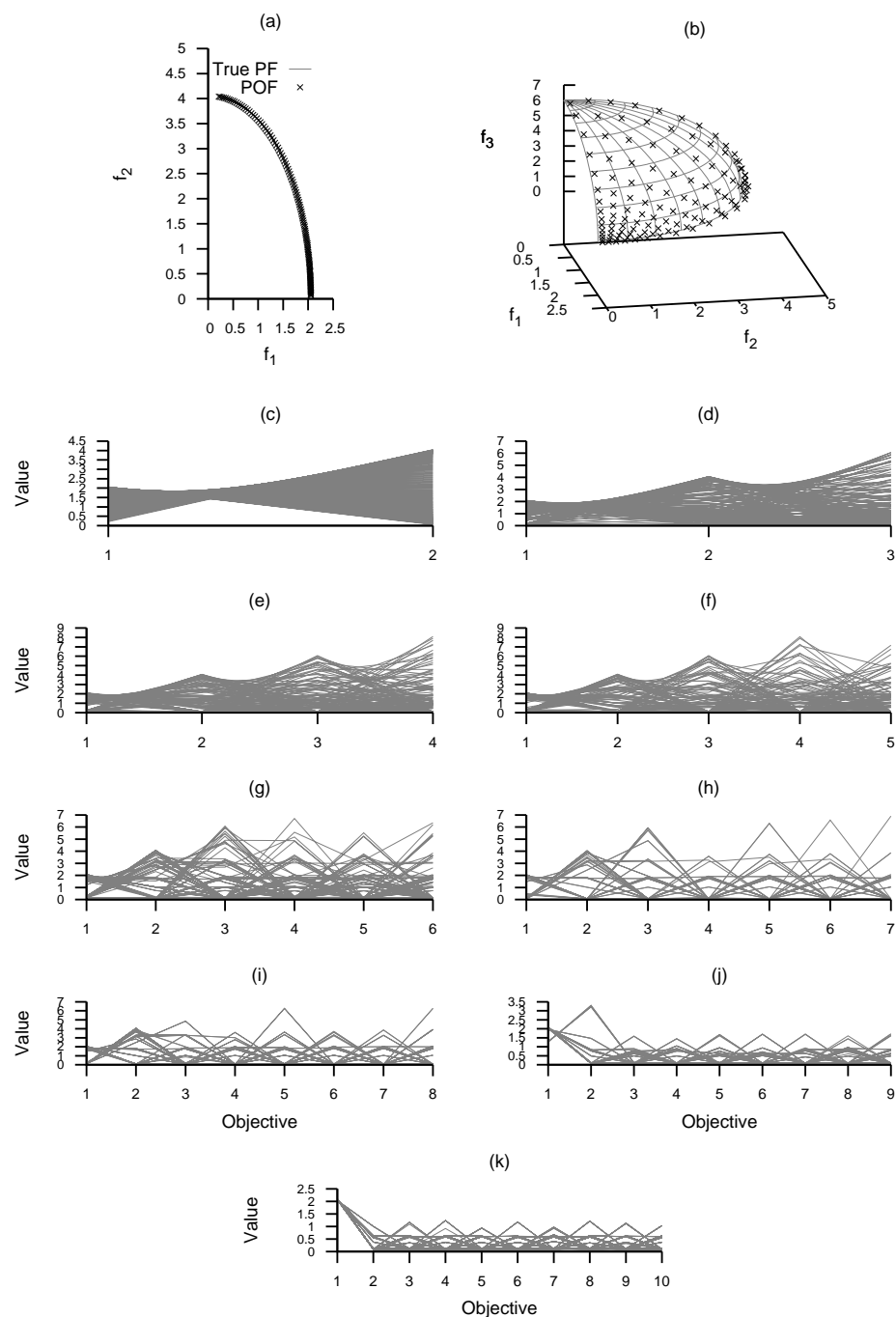


Figure A.486: Plots of the approximations obtained by MOMBI-SLD from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

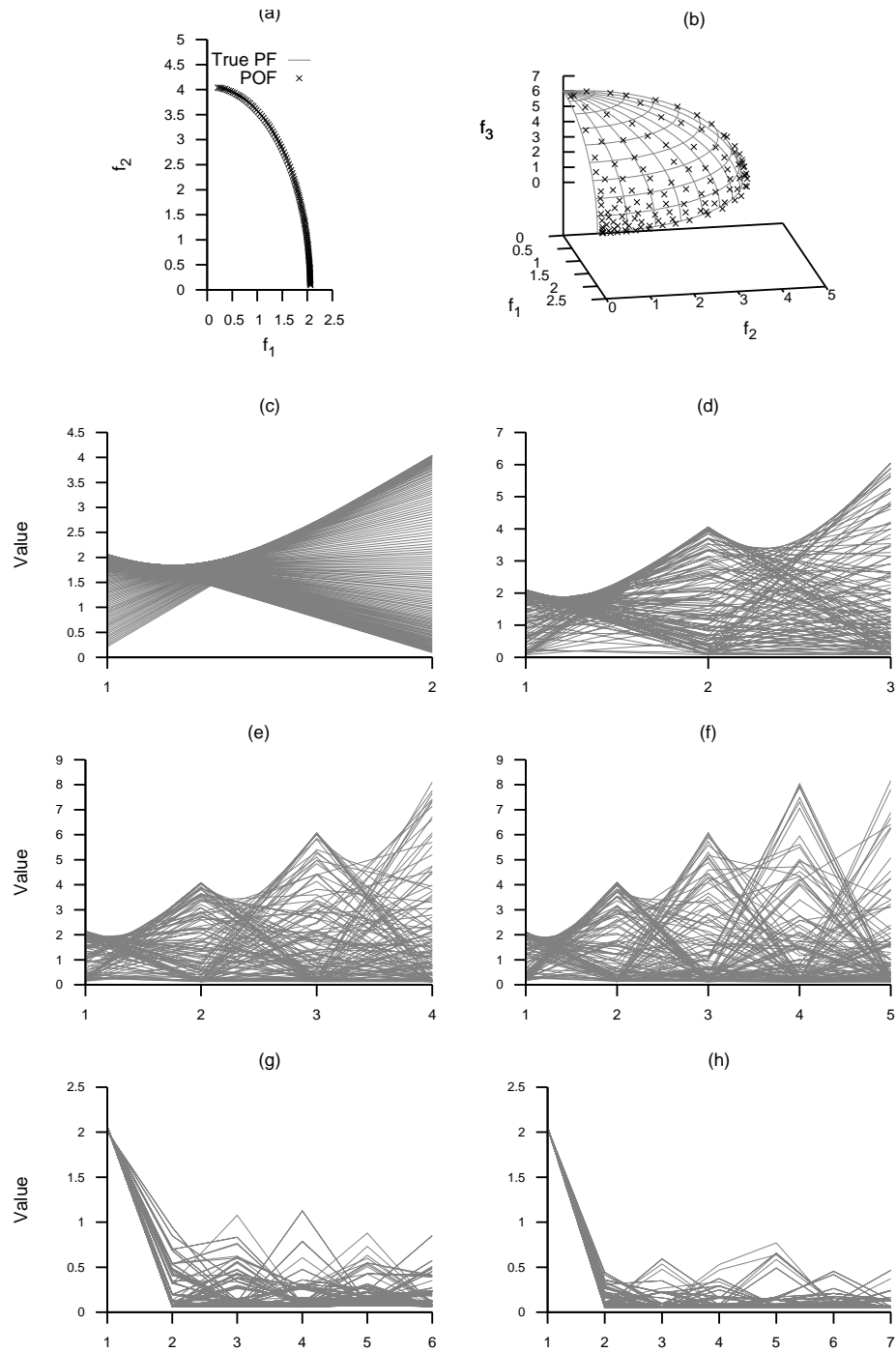


Figure A.487: Plots of the approximations obtained by MOMBI-HV from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

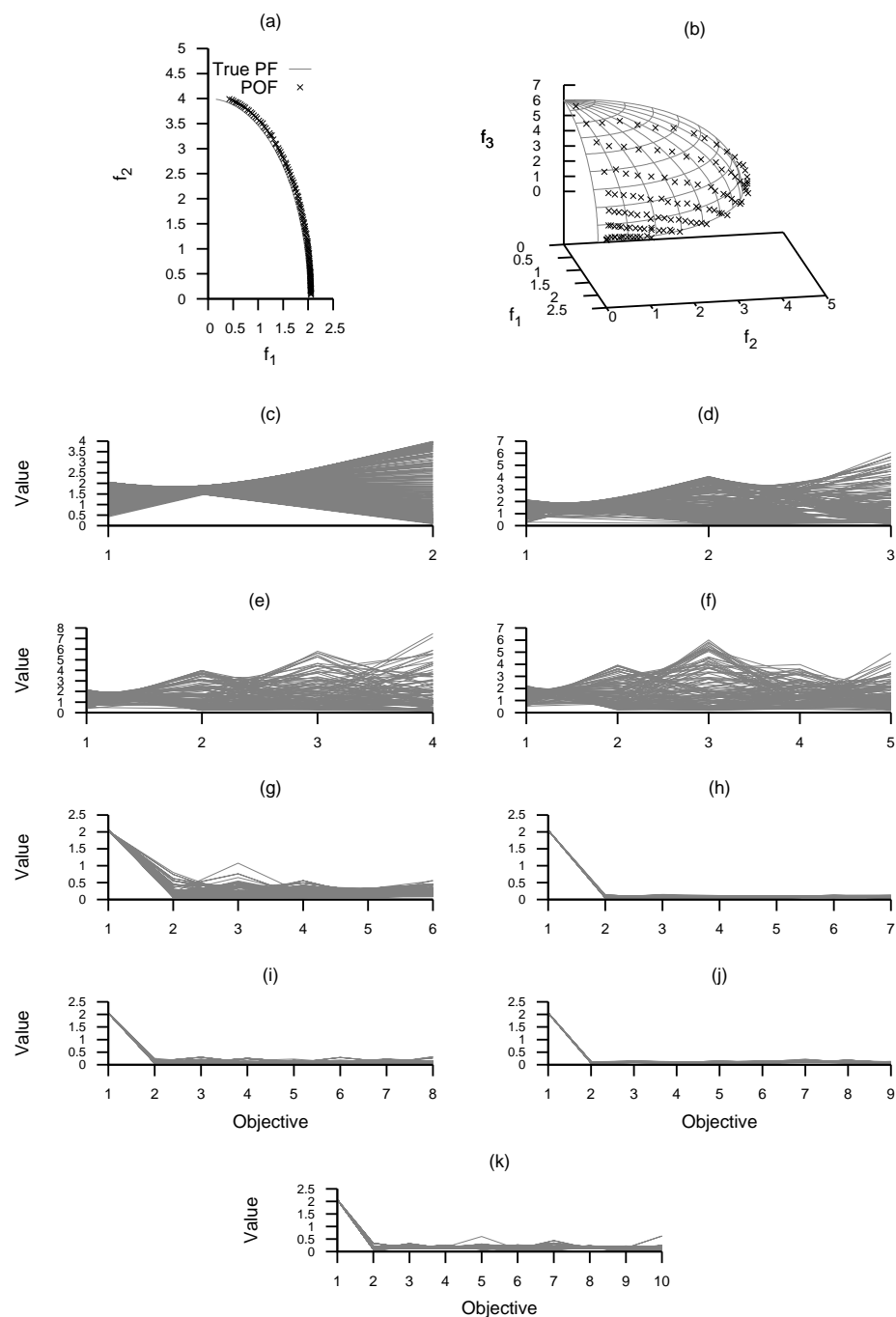


Figure A.488: Plots of the approximations obtained by MOMBI-UD from 2 to 10 objectives on the WFG5 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.214: Comparison of hypervolume indicator values for different optimizers on the WFG6 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	4.765822e + 00	8.214838e + 00	8.201714e + 00	7.967249e + 00
	avg.	5.360251e + 00	8.213655e + 00	8.202254e + 00	7.971234e + 00
	min.	4.630720e + 00	7.828557e + 00	8.074626e + 00	7.670626e + 00
	max.	7.913880e + 00	8.324155e + 00	8.313406e + 00	8.164630e + 00
	std.	1.076814e + 00	6.857822e - 02	4.881344e - 02	9.610386e - 02
<b>3D</b>	med.	6.418613e + 01	7.062954e + 01	7.005591e + 01	6.502219e + 01
	avg.	6.422785e + 01	7.069707e + 01	7.001918e + 01	6.503265e + 01
	min.	6.171990e + 01	6.904781e + 01	6.848351e + 01	6.287222e + 01
	max.	6.587204e + 01	7.279869e + 01	7.172254e + 01	6.723486e + 01
	std.	8.188556e - 01	6.636563e - 01	6.008288e - 01	8.724831e - 01
<b>4D</b>	med.	5.700880e + 02	7.005694e + 02	6.638828e + 02	5.641079e + 02
	avg.	5.707931e + 02	7.006819e + 02	6.630824e + 02	5.661763e + 02
	min.	5.307422e + 02	6.864898e + 02	6.333224e + 02	5.478244e + 02
	max.	6.122780e + 02	7.117735e + 02	6.806917e + 02	5.897819e + 02
	std.	1.334417e + 01	5.805109e + 00	7.811399e + 00	8.890695e + 00
<b>5D</b>	med.	6.536222e + 03	7.742878e + 03	7.248910e + 03	5.159130e + 03
	avg.	6.534270e + 03	7.738214e + 03	7.245044e + 03	5.157299e + 03
	min.	6.293282e + 03	7.417048e + 03	6.960991e + 03	4.775030e + 03
	max.	6.711890e + 03	8.046896e + 03	7.474042e + 03	5.659901e + 03
	std.	8.640344e + 01	1.040464e + 02	1.075780e + 02	1.648635e + 02
<b>6D</b>	med.	7.291720e + 04	9.548506e + 04	8.132599e + 04	6.210073e + 04
	avg.	7.210514e + 04	9.531730e + 04	8.116579e + 04	6.062090e + 04
	min.	4.345068e + 04	8.487883e + 04	7.573824e + 04	3.768320e + 04
	max.	8.135747e + 04	1.000632e + 05	8.595360e + 04	6.833244e + 04
	std.	5.613559e + 03	3.145860e + 03	1.866959e + 03	5.582609e + 03
<b>7D</b>	med.	6.551939e + 05	1.266489e + 06	9.552559e + 05	6.559056e + 05
	avg.	7.546748e + 05	1.177279e + 06	8.897636e + 05	7.543625e + 05
	min.	3.768879e + 05	6.499539e + 05	6.491116e + 05	6.078523e + 05
	max.	1.257142e + 06	1.459954e + 06	1.060856e + 06	9.644599e + 05
	std.	3.363467e + 05	2.667357e + 05	1.457081e + 05	1.238592e + 05
<b>8D</b>	med.	1.572252e + 07	2.038014e + 07	—	1.115412e + 07
	avg.	1.476989e + 07	1.901188e + 07	—	1.205509e + 07
	min.	6.378499e + 06	1.101115e + 07	—	1.012143e + 07
	max.	2.018497e + 07	2.422043e + 07	—	1.459048e + 07
	std.	3.829927e + 06	4.470176e + 06	—	1.166897e + 06
<b>9D</b>	med.	3.794836e + 08	3.510523e + 08	—	2.087020e + 08
	avg.	3.502847e + 08	3.383941e + 08	—	2.114834e + 08
	min.	2.081610e + 08	2.083865e + 08	—	1.725308e + 08
	max.	4.126784e + 08	4.321593e + 08	—	2.485529e + 08
	std.	6.702460e + 07	6.520412e + 07	—	1.089106e + 07
<b>10D</b>	med.	6.760538e + 09	6.741791e + 09	—	4.329476e + 09
	avg.	6.465514e + 09	6.488256e + 09	—	4.381170e + 09
	min.	3.742212e + 09	4.297342e + 09	—	2.881162e + 09
	max.	8.315464e + 09	8.528813e + 09	—	5.074074e + 09
	std.	1.258858e + 09	1.201614e + 09	—	2.586710e + 08

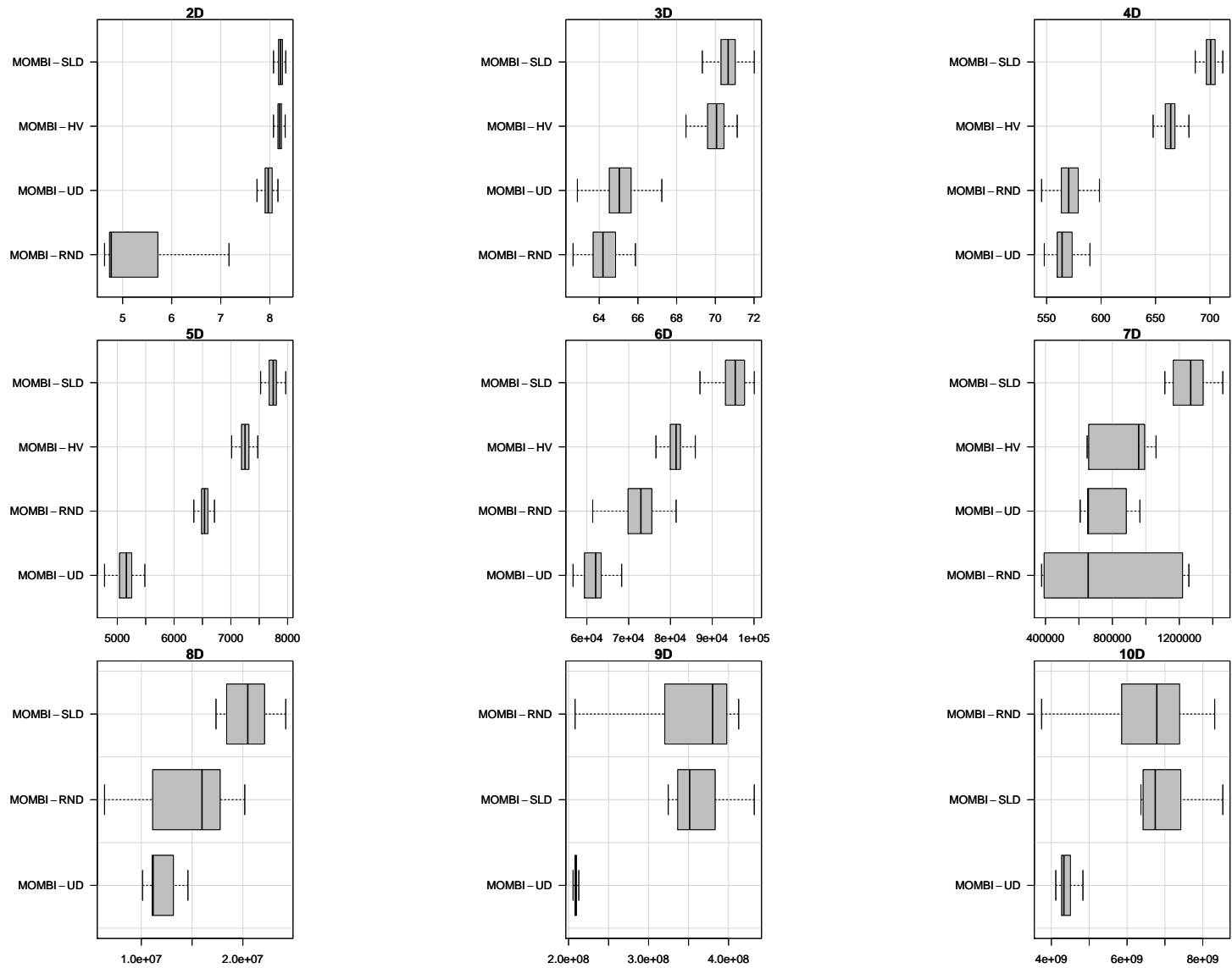


Figure A.489: Box-plot of the hypervolume indicator values for different optimizers on the WFG6 test problem.

Table A.215: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG6 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $ph$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
2D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.32e-34$	—	$2.06e-02$	$2.38e-32$
MOMBI-HV	$1.28e-34$	$> 0.05$	—	$2.17e-33$
MOMBI-UD	$2.96e-34$	$> 0.05$	$> 0.05$	—
3D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e-34$	—	$5.84e-12$	$1.28e-34$
MOMBI-HV	$1.28e-34$	$> 0.05$	—	$1.28e-34$
MOMBI-UD	$4.69e-10$	$> 0.05$	$> 0.05$	—
4D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$6.00e-04$
MOMBI-SLD	$1.28e-34$	—	$1.28e-34$	$1.28e-34$
MOMBI-HV	$1.28e-34$	$> 0.05$	—	$1.28e-34$
MOMBI-UD	$> 0.05$	$> 0.05$	$> 0.05$	—
5D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$1.28e-34$
MOMBI-SLD	$1.28e-34$	—	$1.44e-34$	$1.28e-34$
MOMBI-HV	$1.28e-34$	$> 0.05$	—	$1.28e-34$
MOMBI-UD	$> 0.05$	$> 0.05$	$> 0.05$	—
6D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$6.17e-29$
MOMBI-SLD	$1.28e-34$	—	$1.49e-34$	$1.28e-34$
MOMBI-HV	$5.35e-32$	$> 0.05$	—	$1.28e-34$
MOMBI-UD	$> 0.05$	$> 0.05$	$> 0.05$	—
7D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.71e-16$	—	$1.03e-16$	$1.63e-20$
MOMBI-HV	$1.10e-03$	$> 0.05$	—	$2.19e-13$
MOMBI-UD	$> 0.05$	$> 0.05$	$> 0.05$	—
8D				
MOMBI-RND	—	$> 0.05$		$2.02e-09$
MOMBI-SLD	$7.48e-14$	—		$6.70e-19$
MOMBI-HV			—	
MOMBI-UD	$> 0.05$	$> 0.05$		—
9D				
MOMBI-RND	—	$9.28e-03$		$3.60e-28$
MOMBI-SLD	$> 0.05$	—		$1.75e-27$
MOMBI-HV			—	
MOMBI-UD	$> 0.05$	$> 0.05$		—
10D				
MOMBI-RND	—	$> 0.05$		$3.74e-21$
MOMBI-SLD	$> 0.05$	—		$6.72e-23$
MOMBI-HV			—	
MOMBI-UD	$> 0.05$	$> 0.05$		—

Table A.216: Comparison of R2 indicator values for different optimizers on the WFG6 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	1.019740e + 00	6.062742e - 01	6.068379e - 01	6.167592e - 01
	avg.	9.435947e - 01	6.065013e - 01	6.068639e - 01	6.172558e - 01
	min.	6.225647e - 01	6.017449e - 01	6.022835e - 01	6.069383e - 01
	max.	1.031898e + 00	6.338241e - 01	6.125711e - 01	6.415767e - 01
	std.	1.377567e - 01	3.674016e - 03	2.100684e - 03	5.932679e - 03
<b>3D</b>	med.	3.529784e - 01	3.294873e - 01	3.316512e - 01	3.519230e - 01
	avg.	3.529231e - 01	3.295095e - 01	3.319077e - 01	3.515070e - 01
	min.	3.457851e - 01	3.226565e - 01	3.266417e - 01	3.412458e - 01
	max.	3.645198e - 01	3.350196e - 01	3.368681e - 01	3.601481e - 01
	std.	3.465024e - 03	2.224510e - 03	2.012873e - 03	4.046978e - 03
<b>4D</b>	med.	2.841059e - 01	2.212509e - 01	2.348063e - 01	2.853646e - 01
	avg.	2.857516e - 01	2.213915e - 01	2.352536e - 01	2.848628e - 01
	min.	2.607104e - 01	2.187577e - 01	2.295624e - 01	2.731129e - 01
	max.	3.427305e - 01	2.256610e - 01	2.468471e - 01	2.956516e - 01
	std.	1.387472e - 02	1.523754e - 03	2.746193e - 03	4.689495e - 03
<b>5D</b>	med.	2.103745e - 01	1.852959e - 01	1.967515e - 01	2.754283e - 01
	avg.	2.116235e - 01	1.852543e - 01	1.971324e - 01	2.746811e - 01
	min.	2.058575e - 01	1.785591e - 01	1.916757e - 01	2.514023e - 01
	max.	2.347819e - 01	1.921555e - 01	2.048582e - 01	2.951745e - 01
	std.	4.877020e - 03	2.318865e - 03	2.823477e - 03	9.022924e - 03
<b>6D</b>	med.	2.468415e - 01	1.502418e - 01	1.892290e - 01	2.474510e - 01
	avg.	2.563188e - 01	1.552348e - 01	1.891168e - 01	2.550380e - 01
	min.	1.881884e - 01	1.447426e - 01	1.803101e - 01	2.322381e - 01
	max.	4.338789e - 01	1.890472e - 01	2.015019e - 01	3.534020e - 01
	std.	3.899344e - 02	9.958428e - 03	4.021680e - 03	2.535560e - 02
<b>7D</b>	med.	2.884749e - 01	1.509977e - 01	1.957255e - 01	2.882425e - 01
	avg.	3.424792e - 01	1.770405e - 01	2.189145e - 01	2.567830e - 01
	min.	1.556240e - 01	1.276518e - 01	1.837389e - 01	2.005236e - 01
	max.	5.759221e - 01	2.890829e - 01	2.891933e - 01	2.951620e - 01
	std.	1.677425e - 01	5.576070e - 02	4.179740e - 02	3.765192e - 02
<b>8D</b>	med.	2.376944e - 01	1.443598e - 01	—	2.519745e - 01
	avg.	2.453341e - 01	1.605850e - 01	—	2.311916e - 01
	min.	1.433912e - 01	1.130613e - 01	—	1.920434e - 01
	max.	5.055514e - 01	2.531042e - 01	—	2.599027e - 01
	std.	8.646940e - 02	5.030490e - 02	—	2.341963e - 02
<b>9D</b>	med.	1.269832e - 01	1.364088e - 01	—	2.248469e - 01
	avg.	1.538402e - 01	1.471378e - 01	—	2.183249e - 01
	min.	1.152007e - 01	1.034148e - 01	—	1.875389e - 01
	max.	2.251985e - 01	2.251514e - 01	—	2.393228e - 01
	std.	4.153001e - 02	3.769432e - 02	—	1.209474e - 02
<b>10D</b>	med.	1.707652e - 01	1.289928e - 01	—	1.857168e - 01
	avg.	1.711545e - 01	1.397297e - 01	—	1.901253e - 01
	min.	1.068619e - 01	1.000761e - 01	—	1.658372e - 01
	max.	3.356292e - 01	2.036378e - 01	—	2.350418e - 01
	std.	4.461873e - 02	3.381057e - 02	—	1.332312e - 02



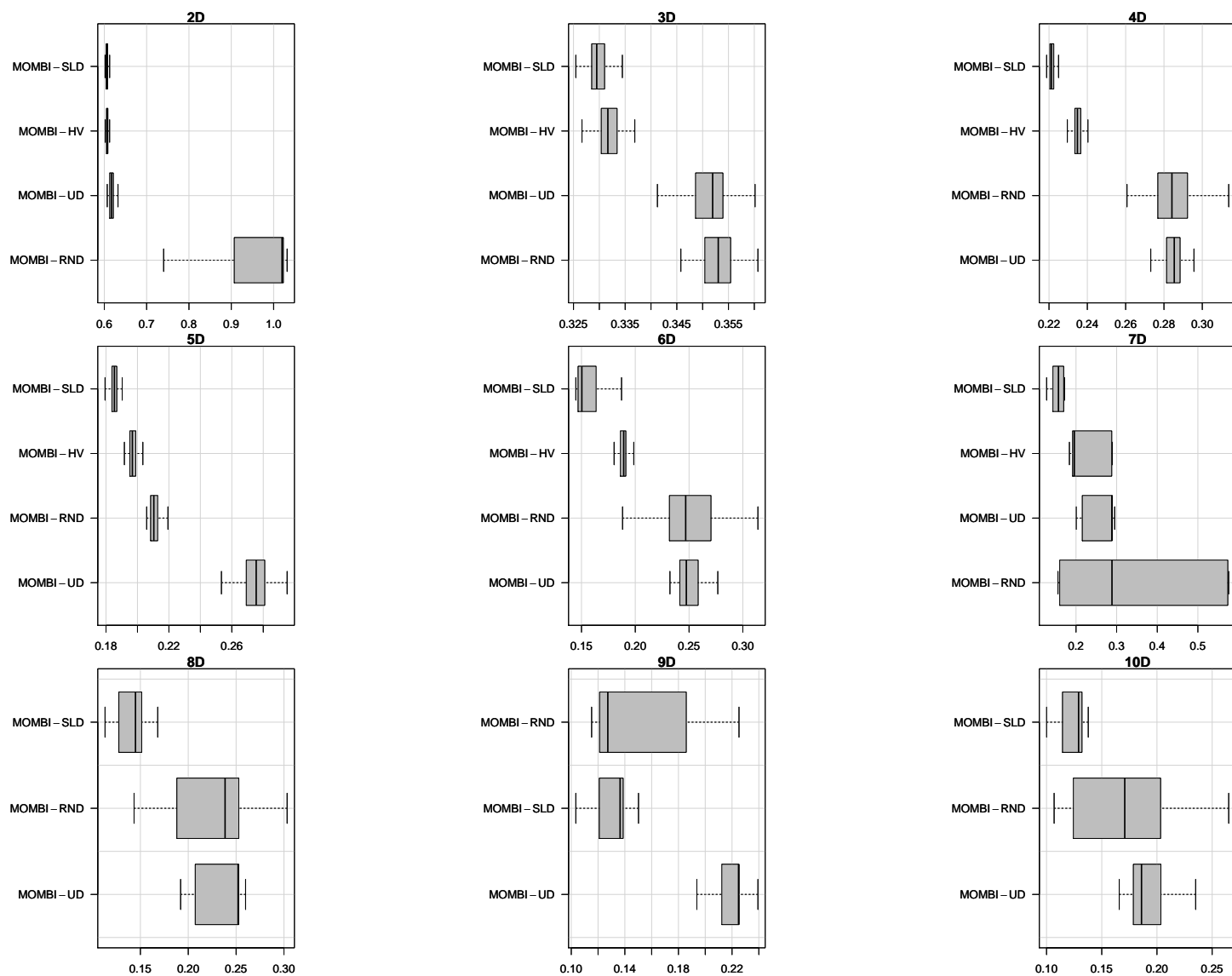


Figure A.490: Box-plot of the R2 indicator values for different optimizers on the WFG6 test problem.

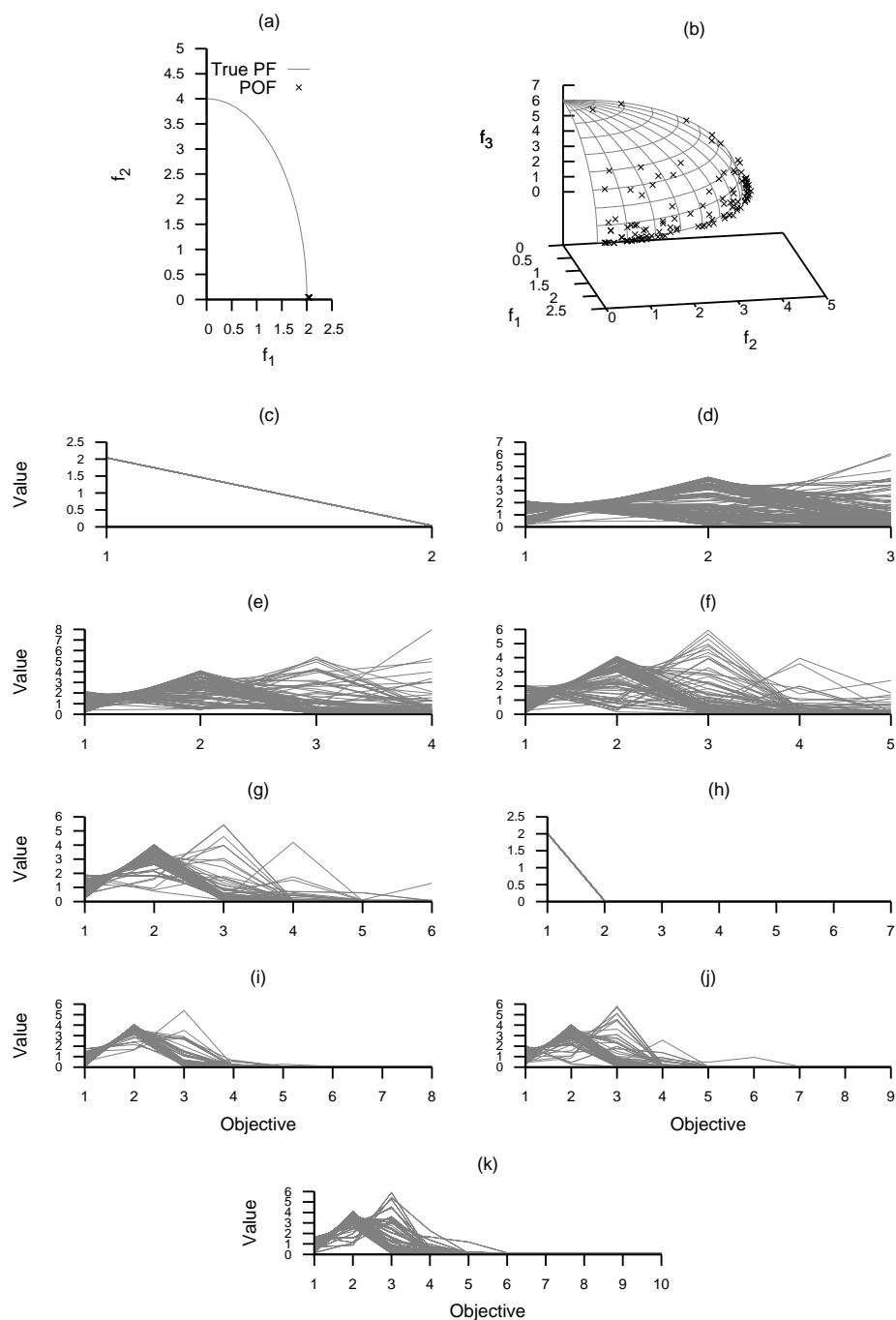


Figure A.491: Plots of the approximations obtained by MOMBI-RND from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

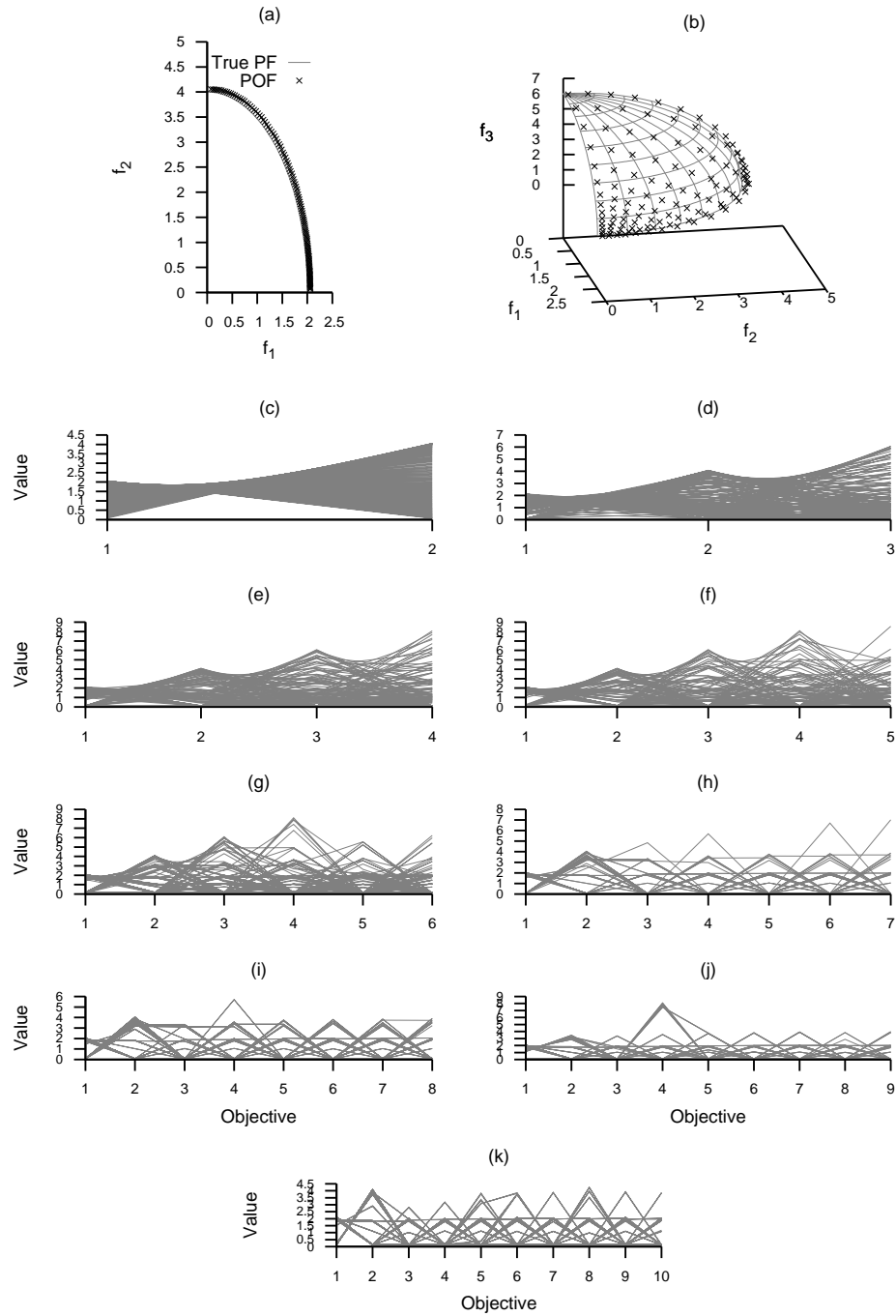


Figure A.492: Plots of the approximations obtained by MOMBI-SLD from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

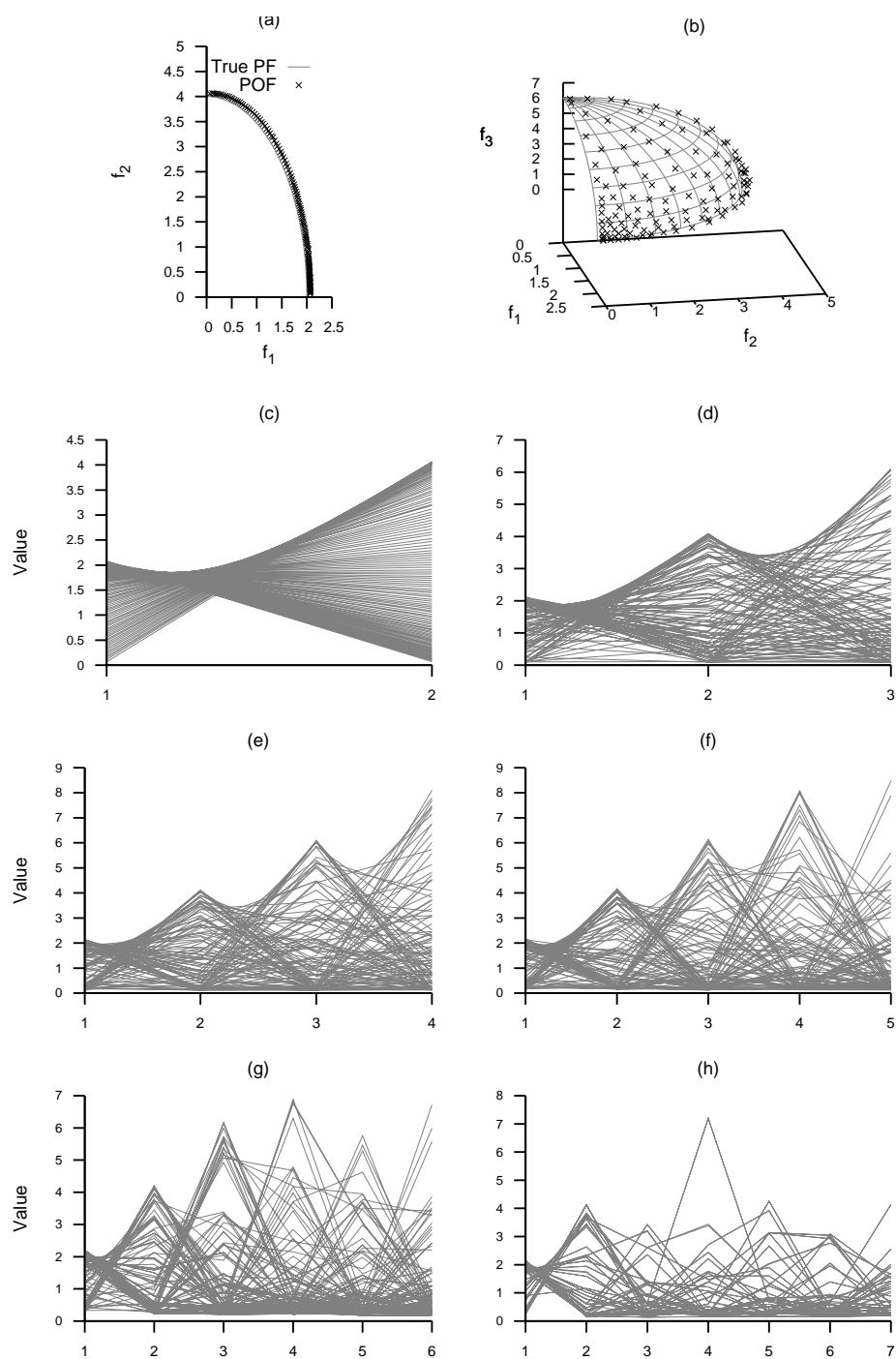


Figure A.493: Plots of the approximations obtained by MOMBI-HV from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

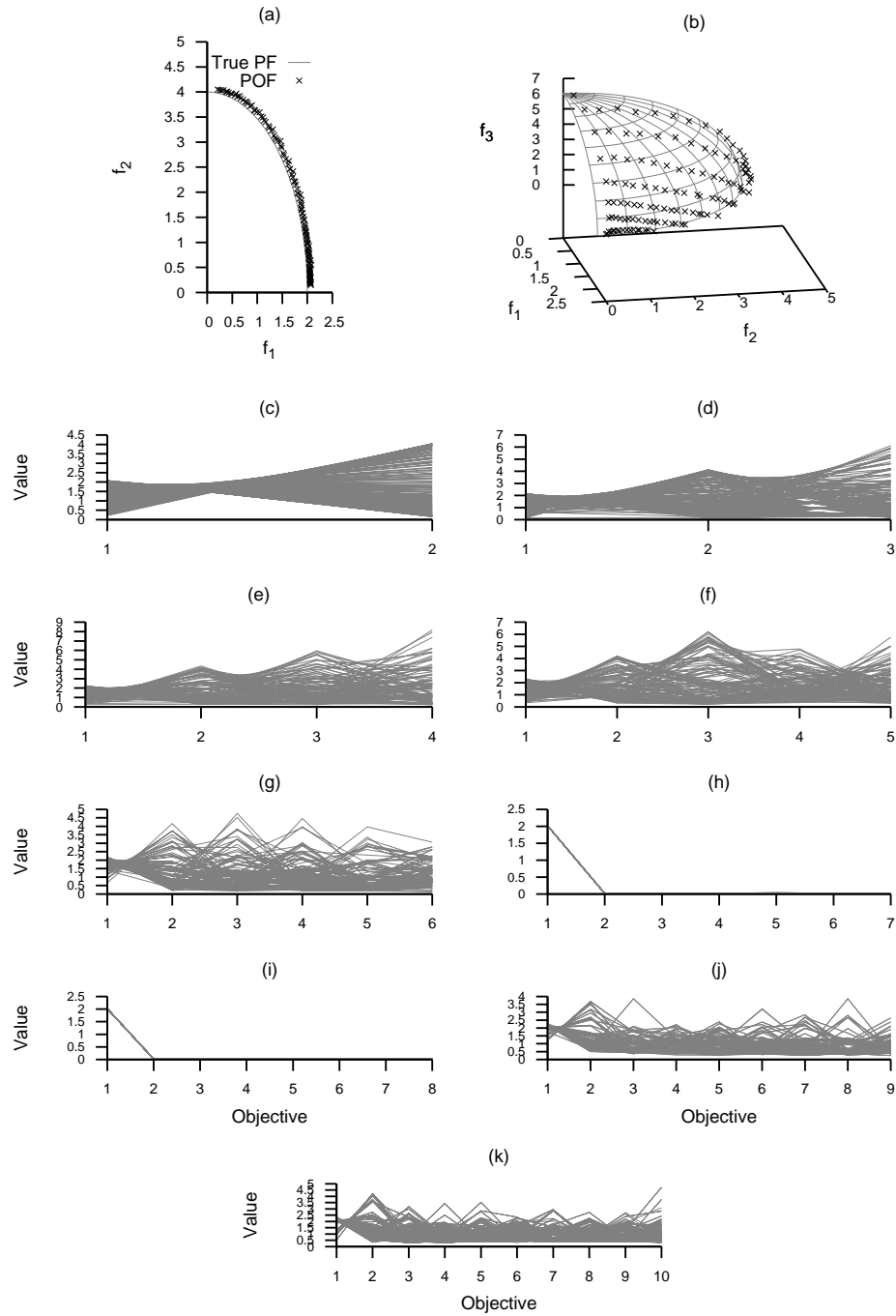


Figure A.494: Plots of the approximations obtained by MOMBI-UD from 2 to 10 objectives on the WFG6 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.217: Comparison of hypervolume indicator values for different optimizers on the WFG7 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	4.999617e + 00	8.573867e + 00	8.549099e + 00	8.155045e + 00
	avg.	4.999590e + 00	8.569835e + 00	8.549247e + 00	8.132556e + 00
	min.	4.998858e + 00	8.494432e + 00	8.511405e + 00	7.828253e + 00
	max.	4.999852e + 00	8.610363e + 00	8.588356e + 00	8.395904e + 00
	std.	1.490397e - 04	1.890122e - 02	1.630341e - 02	1.381030e - 01
<b>3D</b>	med.	6.618943e + 01	7.408364e + 01	7.316522e + 01	6.765510e + 01
	avg.	6.631525e + 01	7.406293e + 01	7.302792e + 01	6.769084e + 01
	min.	6.351309e + 01	7.241197e + 01	6.843605e + 01	6.526865e + 01
	max.	6.879146e + 01	7.471598e + 01	7.405769e + 01	6.955522e + 01
	std.	1.046104e + 00	3.833948e - 01	7.406652e - 01	9.025930e - 01
<b>4D</b>	med.	5.871243e + 02	7.271328e + 02	6.884843e + 02	5.799307e + 02
	avg.	5.811171e + 02	7.263309e + 02	6.884661e + 02	5.816443e + 02
	min.	5.136311e + 02	7.125332e + 02	6.670999e + 02	5.593868e + 02
	max.	6.147734e + 02	7.396641e + 02	7.042385e + 02	6.116950e + 02
	std.	2.323596e + 01	5.649822e + 00	7.557352e + 00	1.113334e + 01
<b>5D</b>	med.	6.403750e + 03	7.823396e + 03	7.473972e + 03	5.588031e + 03
	avg.	6.306173e + 03	7.815714e + 03	7.482069e + 03	5.595919e + 03
	min.	5.073228e + 03	7.285838e + 03	7.165642e + 03	5.235231e + 03
	max.	6.891774e + 03	8.213618e + 03	7.744603e + 03	5.981116e + 03
	std.	4.679320e + 02	1.729500e + 02	1.097871e + 02	1.372211e + 02
<b>6D</b>	med.	6.620132e + 04	9.334681e + 04	8.181715e + 04	4.496504e + 04
	avg.	6.258830e + 04	9.067198e + 04	7.491327e + 04	4.517425e + 04
	min.	2.655393e + 04	4.496612e + 04	4.472384e + 04	4.409903e + 04
	max.	8.446466e + 04	1.009094e + 05	8.762171e + 04	6.762984e + 04
	std.	1.367332e + 04	9.392935e + 03	1.422181e + 04	2.270448e + 03
<b>7D</b>	med.	4.053245e + 05	6.748471e + 05	6.748054e + 05	6.746559e + 05
	avg.	5.815227e + 05	8.899296e + 05	7.301773e + 05	6.764188e + 05
	min.	4.003351e + 05	6.744606e + 05	6.743947e + 05	6.133344e + 05
	max.	1.293223e + 06	1.497407e + 06	1.076010e + 06	9.127159e + 05
	std.	2.399512e + 05	2.898248e + 05	1.238189e + 05	2.464382e + 04
<b>8D</b>	med.	1.145629e + 07	1.145888e + 07	—	1.145012e + 07
	avg.	1.150697e + 07	1.472573e + 07	—	1.144985e + 07
	min.	6.503519e + 06	1.144620e + 07	—	1.143748e + 07
	max.	1.940591e + 07	2.316115e + 07	—	1.146066e + 07
	std.	4.229456e + 06	4.569918e + 06	—	4.605490e + 03
<b>9D</b>	med.	2.768806e + 08	2.172364e + 08	—	2.169283e + 08
	avg.	2.881325e + 08	2.604671e + 08	—	2.169122e + 08
	min.	1.652640e + 08	2.168521e + 08	—	2.161478e + 08
	max.	4.241227e + 08	4.494773e + 08	—	2.172854e + 08
	std.	6.978854e + 07	6.851723e + 07	—	1.796862e + 05
<b>10D</b>	med.	5.625648e + 09	4.539080e + 09	—	4.525853e + 09
	avg.	5.423155e + 09	5.148193e + 09	—	4.524888e + 09
	min.	2.607280e + 09	4.512678e + 09	—	4.500925e + 09
	max.	8.364770e + 09	8.465318e + 09	—	4.546577e + 09
	std.	1.446243e + 09	1.153782e + 09	—	8.770297e + 06

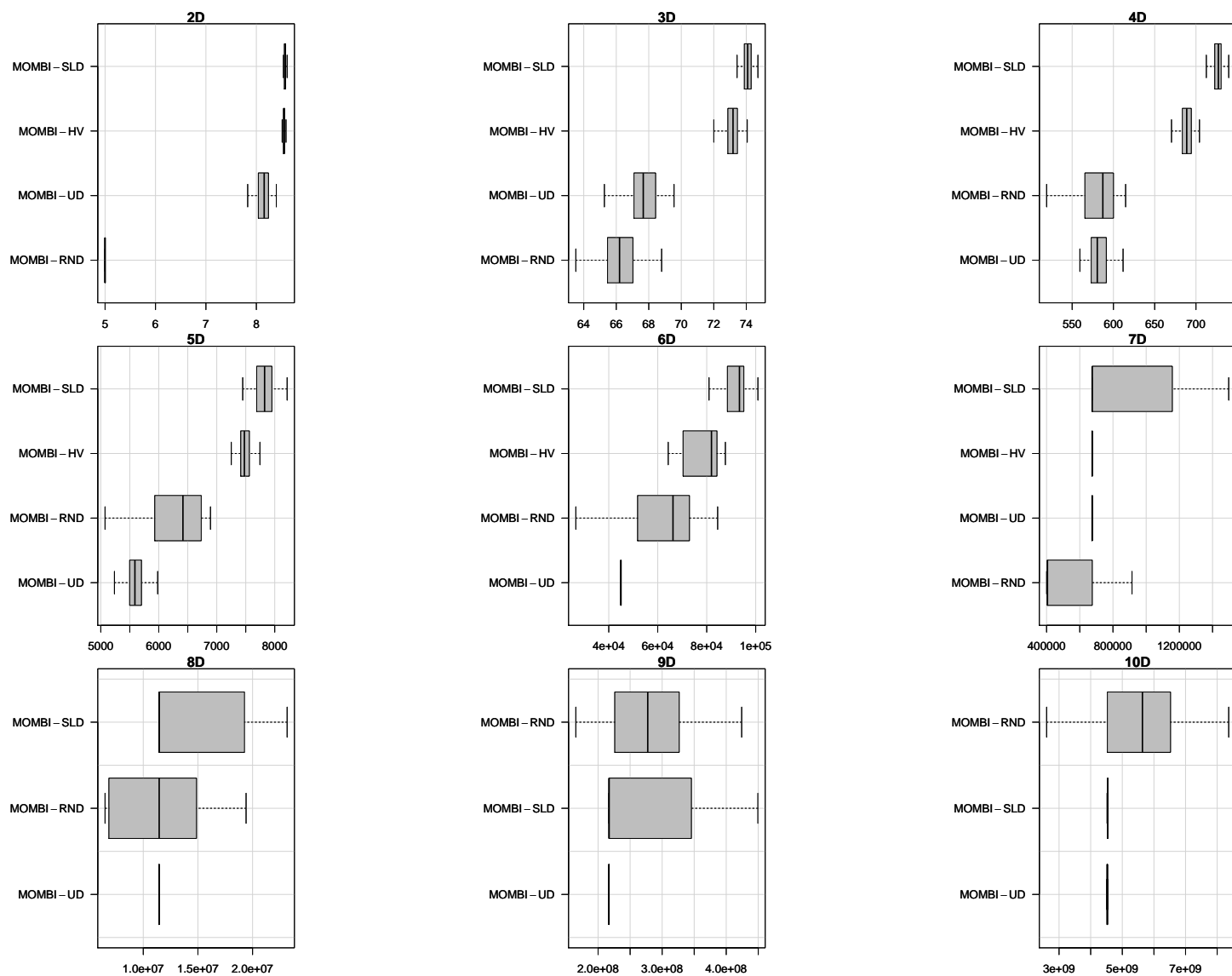


Figure A.495: Box-plot of the hypervolume indicator values for different optimizers on the WFG7 test problem.

Table A.218: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG7 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $ph$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
2D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e - 34$	–	$8.05e - 14$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.28e - 34$
MOMBI-UD	$1.28e - 34$	> 0.05	> 0.05	–
3D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e - 34$	–	$3.28e - 29$	$1.28e - 34$
MOMBI-HV	$1.36e - 34$	> 0.05	–	$2.63e - 34$
MOMBI-UD	$1.78e - 16$	> 0.05	> 0.05	–
4D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e - 34$	–	$1.28e - 34$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.28e - 34$
MOMBI-UD	> 0.05	> 0.05	> 0.05	–
5D				
MOMBI-RND	–	> 0.05	> 0.05	$1.81e - 23$
MOMBI-SLD	$1.28e - 34$	–	$6.51e - 28$	$1.28e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.28e - 34$
MOMBI-UD	> 0.05	> 0.05	> 0.05	–
6D				
MOMBI-RND	–	> 0.05	> 0.05	$8.02e - 22$
MOMBI-SLD	$3.54e - 31$	–	$2.16e - 24$	$6.63e - 34$
MOMBI-HV	$6.38e - 13$	> 0.05	–	$8.56e - 29$
MOMBI-UD	> 0.05	> 0.05	> 0.05	–
7D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$2.71e - 12$	–	$9.04e - 04$	$1.16e - 14$
MOMBI-HV	$2.12e - 08$	> 0.05	–	$5.90e - 11$
MOMBI-UD	$1.58e - 03$	> 0.05	> 0.05	–
8D				
MOMBI-RND	–	> 0.05		> 0.05
MOMBI-SLD	$1.01e - 05$	–		$1.58e - 23$
MOMBI-HV			–	
MOMBI-UD	> 0.05	> 0.05		–
9D				
MOMBI-RND	–	$9.77e - 04$		$7.98e - 19$
MOMBI-SLD	> 0.05	–		$6.13e - 24$
MOMBI-HV			–	
MOMBI-UD	> 0.05	> 0.05		–
10D				
MOMBI-RND	–	$4.89e - 02$		$1.79e - 10$
MOMBI-SLD	> 0.05	–		$1.67e - 22$
MOMBI-HV			–	
MOMBI-UD	> 0.05	> 0.05		–



Table A.219: Comparison of R2 indicator values for different optimizers on the WFG7 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	1.000031e + 00	5.910565e - 01	5.915724e - 01	6.082370e - 01
	avg.	1.000033e + 00	5.911039e - 01	5.915902e - 01	6.120987e - 01
	min.	1.000012e + 00	5.901199e - 01	5.907002e - 01	5.955843e - 01
	max.	1.000093e + 00	5.928264e - 01	5.924721e - 01	6.420387e - 01
	std.	1.217288e - 05	4.558899e - 04	3.943507e - 04	1.186698e - 02
<b>3D</b>	med.	3.426076e - 01	3.193996e - 01	3.220174e - 01	3.386323e - 01
	avg.	3.425867e - 01	3.195935e - 01	3.225458e - 01	3.387055e - 01
	min.	3.350007e - 01	3.178257e - 01	3.195864e - 01	3.329876e - 01
	max.	3.551891e - 01	3.252477e - 01	3.362091e - 01	3.491833e - 01
	std.	3.349016e - 03	1.256144e - 03	2.155994e - 03	3.312115e - 03
<b>4D</b>	med.	2.751870e - 01	2.151848e - 01	2.258795e - 01	2.753260e - 01
	avg.	2.869606e - 01	2.155969e - 01	2.258594e - 01	2.746847e - 01
	min.	2.579819e - 01	2.126508e - 01	2.205427e - 01	2.621139e - 01
	max.	3.878682e - 01	2.242755e - 01	2.318917e - 01	2.874024e - 01
	std.	2.901478e - 02	1.935123e - 03	1.916256e - 03	5.246530e - 03
<b>5D</b>	med.	2.302715e - 01	1.838932e - 01	1.897110e - 01	2.550376e - 01
	avg.	2.501300e - 01	1.839008e - 01	1.896778e - 01	2.554507e - 01
	min.	2.018772e - 01	1.772740e - 01	1.838704e - 01	2.408112e - 01
	max.	4.045861e - 01	1.987704e - 01	1.957734e - 01	2.709491e - 01
	std.	5.114385e - 02	3.294624e - 03	2.239625e - 03	6.589945e - 03
<b>6D</b>	med.	3.284271e - 01	1.611439e - 01	1.828753e - 01	3.334565e - 01
	avg.	3.381552e - 01	1.716946e - 01	2.114359e - 01	3.324271e - 01
	min.	1.874884e - 01	1.430852e - 01	1.731886e - 01	2.265616e - 01
	max.	6.652799e - 01	3.334676e - 01	3.340007e - 01	3.354082e - 01
	std.	1.035314e - 01	3.097731e - 02	5.552356e - 02	1.069582e - 02
<b>7D</b>	med.	5.711853e - 01	2.857778e - 01	2.857787e - 01	2.857925e - 01
	avg.	4.397214e - 01	2.387537e - 01	2.694734e - 01	2.851104e - 01
	min.	1.537048e - 01	1.268211e - 01	1.821254e - 01	2.091384e - 01
	max.	5.723084e - 01	2.858262e - 01	2.858316e - 01	2.940949e - 01
	std.	1.501508e - 01	6.228102e - 02	3.625734e - 02	7.718726e - 03
<b>8D</b>	med.	3.047274e - 01	2.501343e - 01	—	2.501741e - 01
	avg.	3.549182e - 01	2.113120e - 01	—	2.501748e - 01
	min.	1.817651e - 01	1.242018e - 01	—	2.501123e - 01
	max.	5.027878e - 01	2.502130e - 01	—	2.502414e - 01
	std.	1.229634e - 01	5.353220e - 02	—	2.704653e - 05
<b>9D</b>	med.	2.400756e - 01	2.224701e - 01	—	2.225359e - 01
	avg.	2.454535e - 01	1.953848e - 01	—	2.225430e - 01
	min.	1.141183e - 01	1.011597e - 01	—	2.224419e - 01
	max.	4.038177e - 01	2.225735e - 01	—	2.227473e - 01
	std.	7.603961e - 02	4.207767e - 02	—	5.093647e - 05
<b>10D</b>	med.	2.154632e - 01	2.004785e - 01	—	2.006160e - 01
	avg.	2.432635e - 01	1.824933e - 01	—	2.006316e - 01
	min.	1.118232e - 01	1.043229e - 01	—	2.003780e - 01
	max.	4.020155e - 01	2.007860e - 01	—	2.009252e - 01
	std.	7.367377e - 02	3.335168e - 02	—	1.070198e - 04

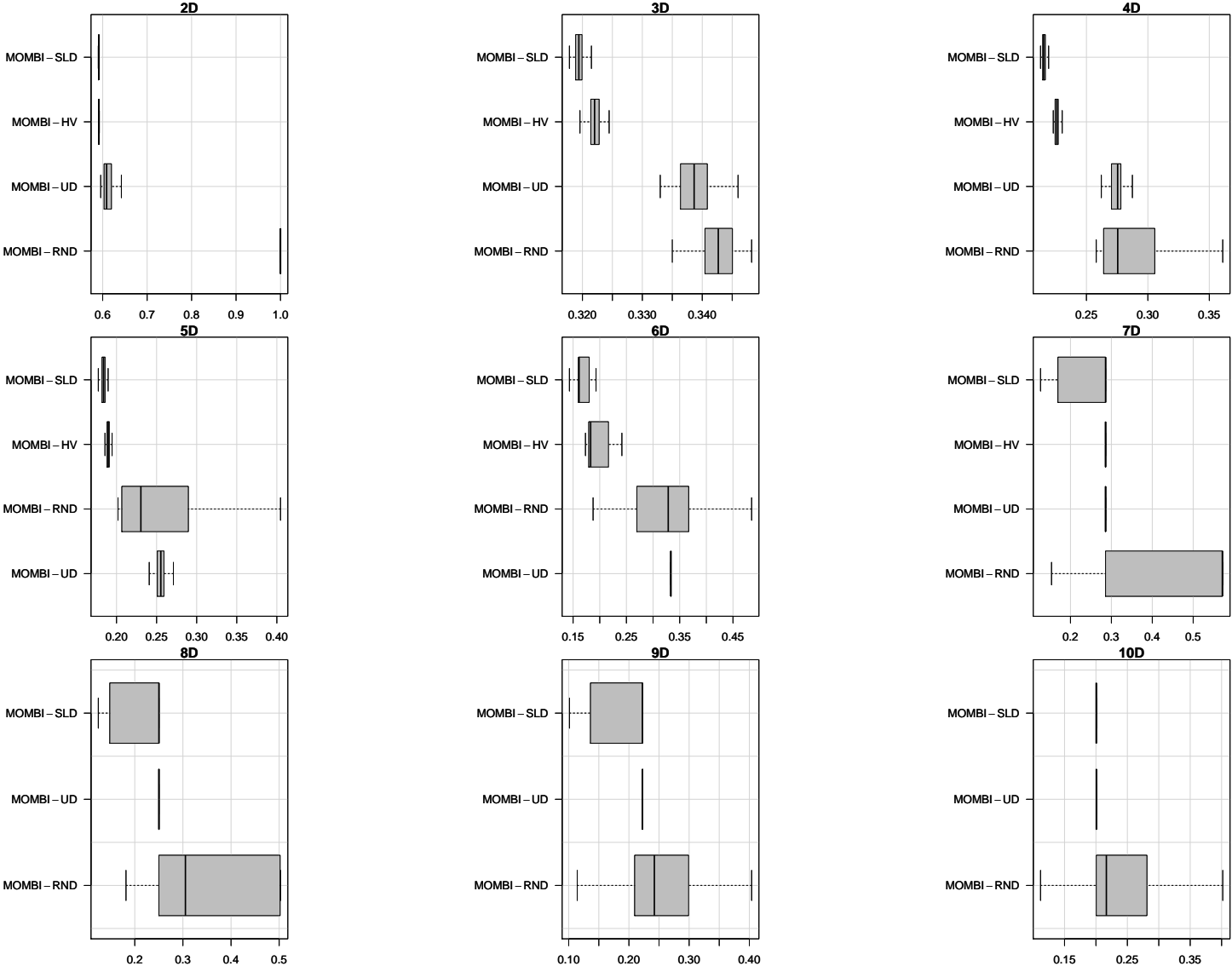


Figure A.496: Box-plot of the R2 indicator values for different optimizers on the WFG7 test problem.

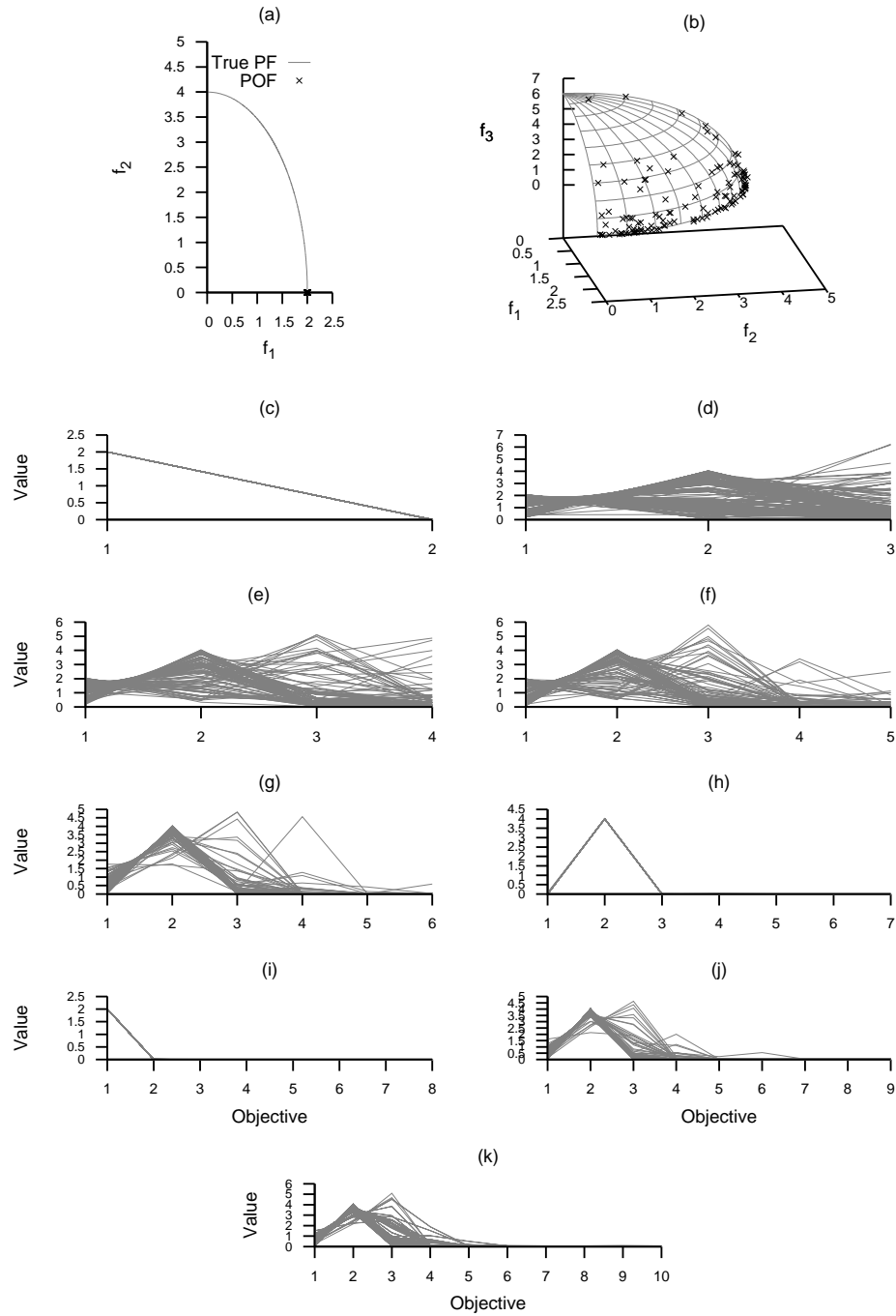


Figure A.497: Plots of the approximations obtained by MOMBI-RND from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

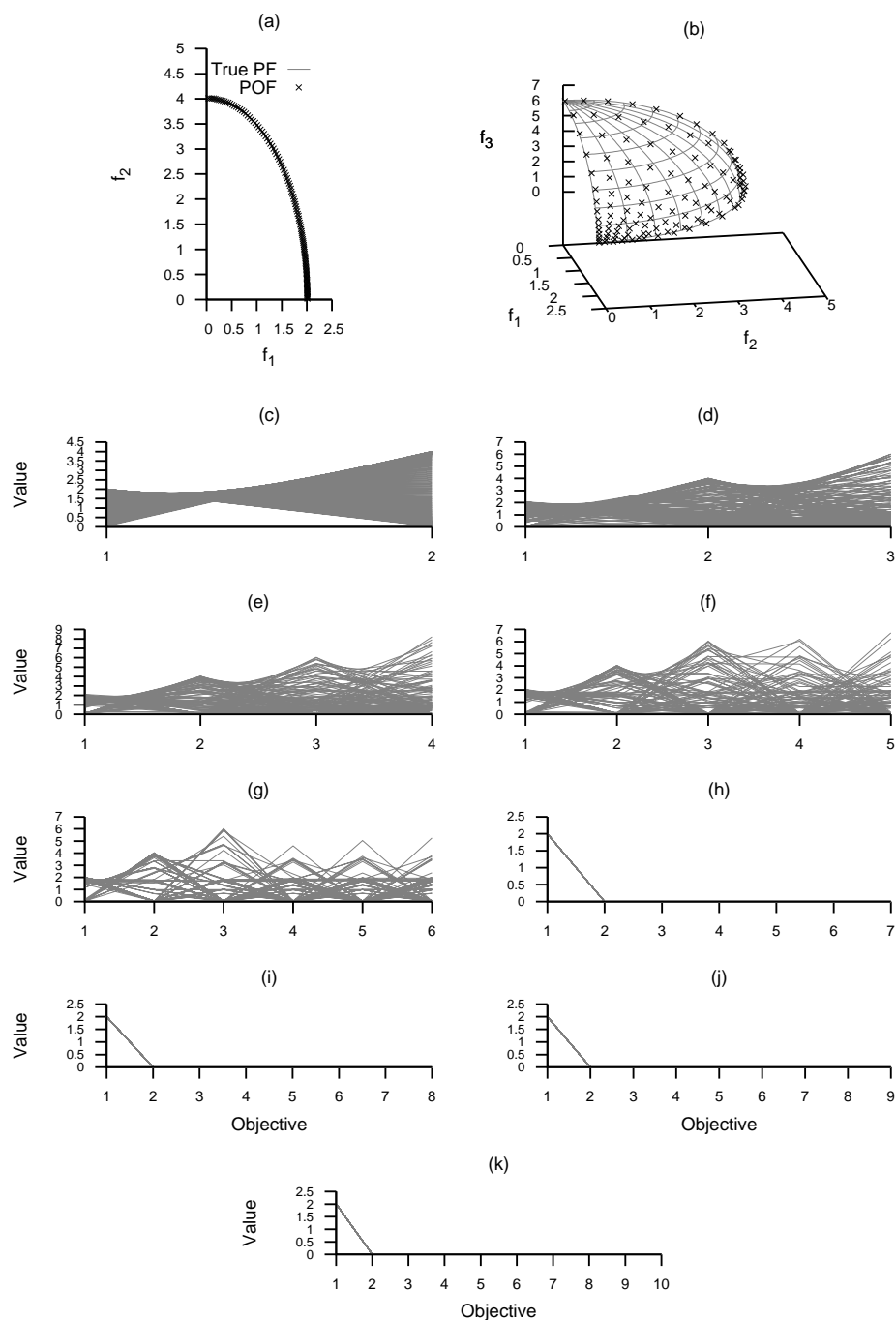


Figure A.498: Plots of the approximations obtained by MOMBI-SLD from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

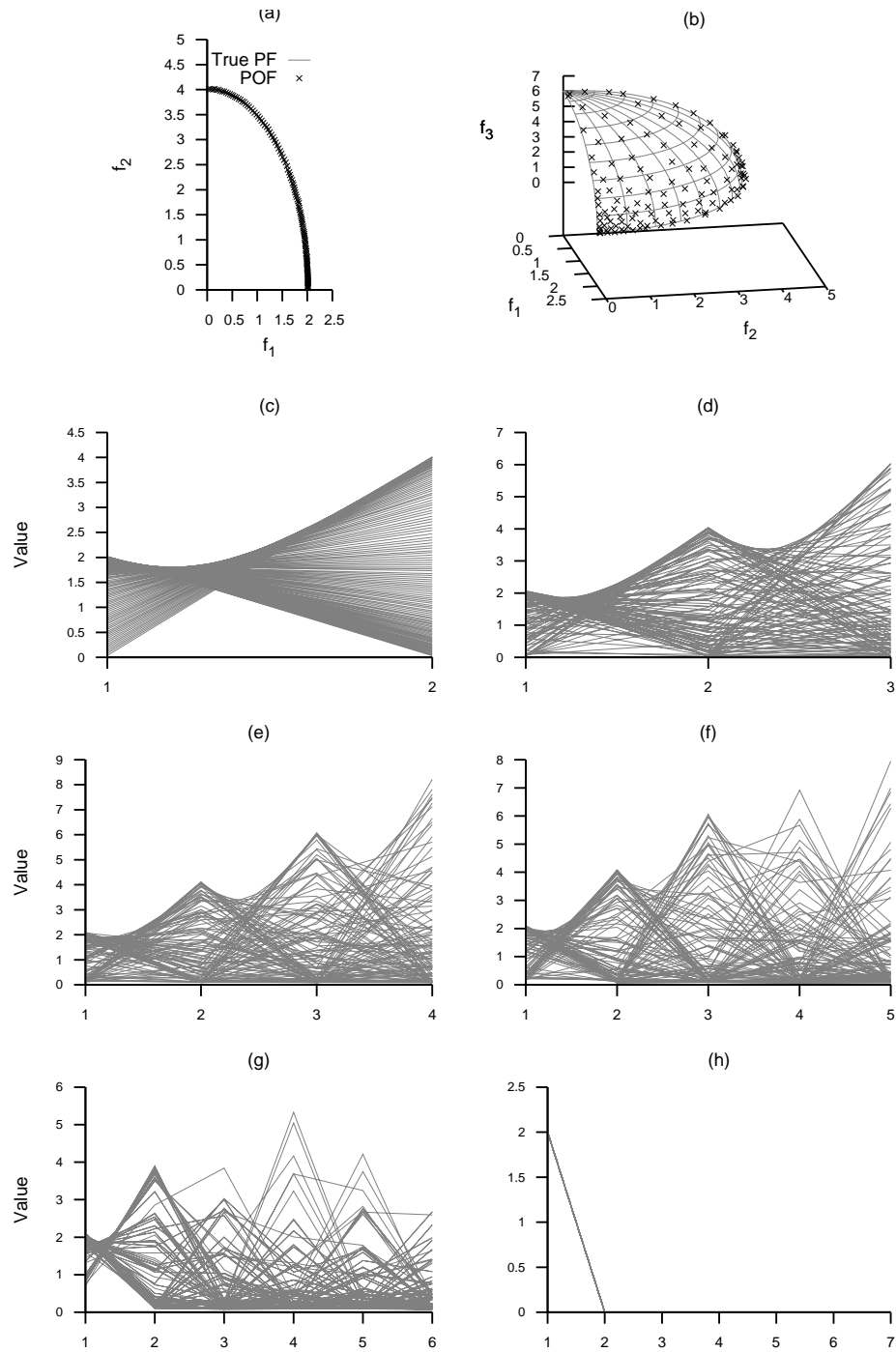


Figure A.499: Plots of the approximations obtained by MOMBI-HV from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

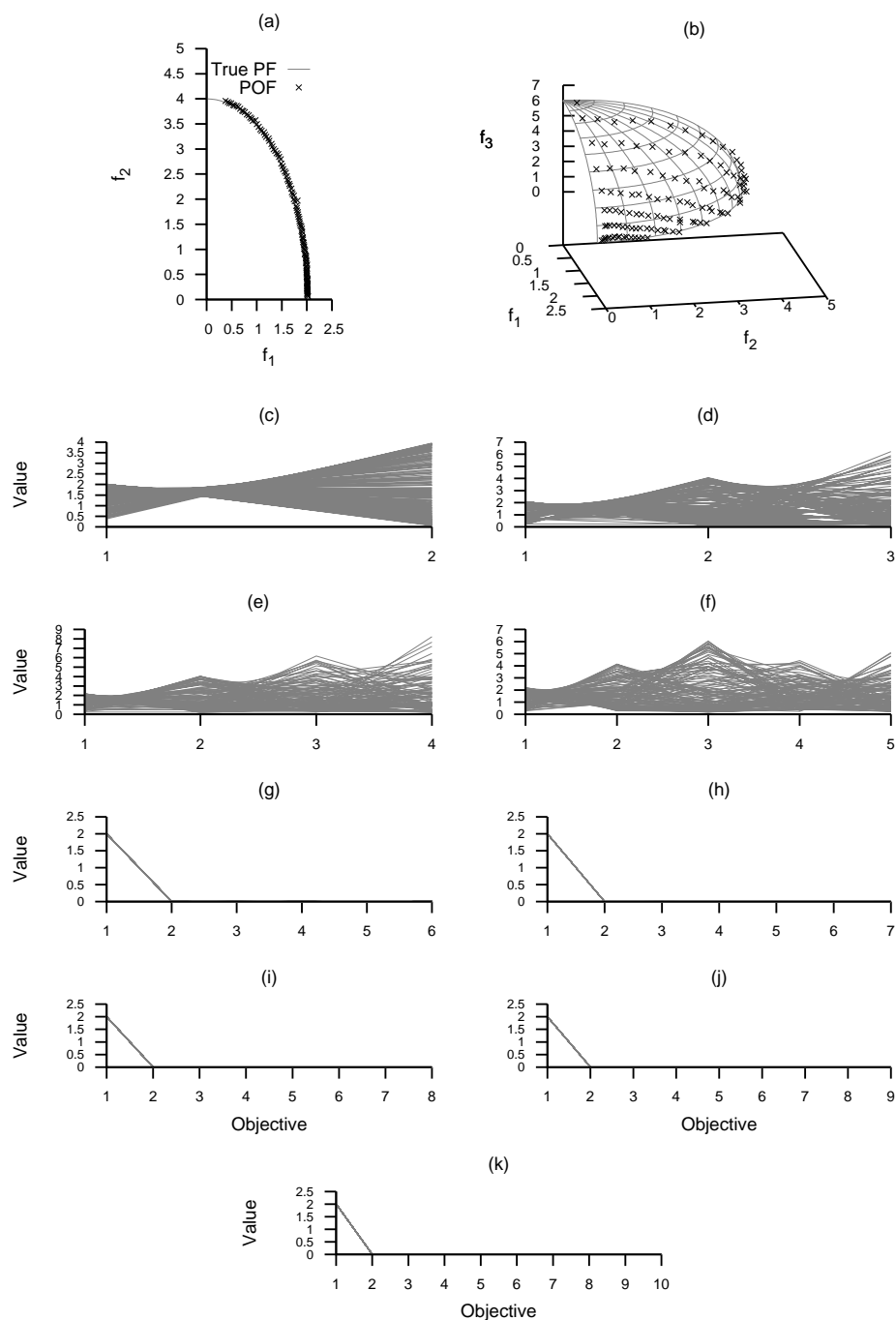


Figure A.500: Plots of the approximations obtained by MOMBI-UD from 2 to 10 objectives on the WFG7 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.220: Comparison of hypervolume indicator values for different optimizers on the WFG8 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	3.839255e + 00	7.415300e + 00	7.393378e + 00	6.990973e + 00
	avg.	3.830912e + 00	7.404612e + 00	7.389346e + 00	6.818410e + 00
	min.	3.561466e + 00	7.229432e + 00	7.209893e + 00	3.832094e + 00
	max.	3.839698e + 00	7.483943e + 00	7.493334e + 00	7.315785e + 00
	std.	4.757165e - 02	4.815030e - 02	4.978307e - 02	5.505061e - 01
<b>3D</b>	med.	5.879424e + 01	6.732307e + 01	6.684230e + 01	6.083121e + 01
	avg.	5.872986e + 01	6.730195e + 01	6.680732e + 01	6.080851e + 01
	min.	5.653321e + 01	6.618387e + 01	6.500867e + 01	5.830696e + 01
	max.	6.068833e + 01	6.814175e + 01	6.778081e + 01	6.267942e + 01
	std.	8.988024e - 01	3.818398e - 01	4.277220e - 01	9.108727e - 01
<b>4D</b>	med.	4.821589e + 02	6.604468e + 02	6.240707e + 02	5.343707e + 02
	avg.	4.815798e + 02	6.606315e + 02	6.243981e + 02	5.334410e + 02
	min.	4.326517e + 02	6.419560e + 02	6.066603e + 02	5.113654e + 02
	max.	5.159935e + 02	6.730066e + 02	6.428426e + 02	5.620283e + 02
	std.	1.707529e + 01	6.289356e + 00	7.196176e + 00	9.883807e + 00
<b>5D</b>	med.	4.991668e + 03	7.317141e + 03	6.660688e + 03	4.920372e + 03
	avg.	4.974862e + 03	7.306952e + 03	6.660335e + 03	4.891489e + 03
	min.	4.101097e + 03	7.057350e + 03	6.446245e + 03	4.500819e + 03
	max.	5.471618e + 03	7.501375e + 03	6.891763e + 03	5.233177e + 03
	std.	2.736147e + 02	9.593642e + 01	9.315846e + 01	1.471097e + 02
<b>6D</b>	med.	4.831317e + 04	8.628496e + 04	7.067739e + 04	5.360828e + 04
	avg.	4.316378e + 04	8.600283e + 04	7.042913e + 04	4.953002e + 04
	min.	1.205617e + 04	7.928437e + 04	6.588550e + 04	2.639398e + 04
	max.	6.138742e + 04	9.073195e + 04	7.468774e + 04	6.175608e + 04
	std.	1.390583e + 04	2.448978e + 03	1.924400e + 03	1.011982e + 04
<b>7D</b>	med.	2.791645e + 05	1.168244e + 06	6.710251e + 05	4.316710e + 05
	avg.	2.942487e + 05	1.143551e + 06	6.211852e + 05	4.436030e + 05
	min.	1.181823e + 05	4.189283e + 05	2.474033e + 05	2.144886e + 05
	max.	7.599772e + 05	1.345682e + 06	9.269428e + 05	7.461265e + 05
	std.	1.027141e + 05	1.284598e + 05	1.799006e + 05	1.066803e + 05
<b>8D</b>	med.	8.494309e + 06	1.798081e + 07	—	7.086961e + 06
	avg.	8.288801e + 06	1.736660e + 07	—	6.829818e + 06
	min.	2.211212e + 06	6.889212e + 06	—	3.744002e + 06
	max.	1.451800e + 07	2.046882e + 07	—	1.028539e + 07
	std.	3.574619e + 06	2.570384e + 06	—	1.055435e + 06
<b>9D</b>	med.	1.786773e + 08	3.182940e + 08	—	1.296086e + 08
	avg.	1.743896e + 08	3.056650e + 08	—	1.227496e + 08
	min.	7.279057e + 07	1.291625e + 08	—	6.789260e + 07
	max.	2.755227e + 08	3.556082e + 08	—	1.749032e + 08
	std.	4.612770e + 07	4.377479e + 07	—	2.216543e + 07
<b>10D</b>	med.	4.058729e + 09	6.080968e + 09	—	2.707306e + 09
	avg.	3.918933e + 09	5.885601e + 09	—	2.592517e + 09
	min.	1.320265e + 09	2.663726e + 09	—	1.579378e + 09
	max.	5.052207e + 09	7.308873e + 09	—	3.291198e + 09
	std.	7.604440e + 08	8.880641e + 08	—	3.697279e + 08

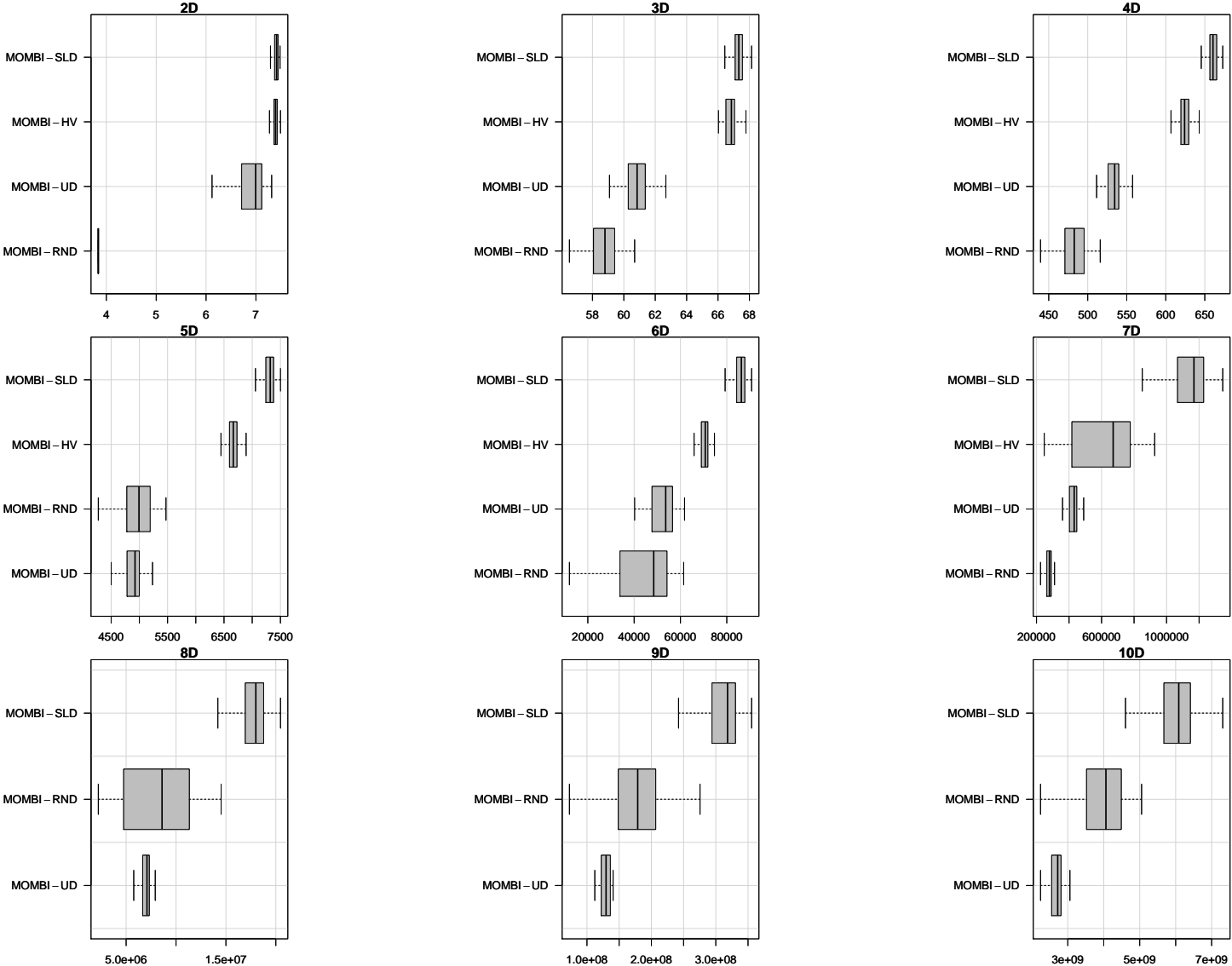


Figure A.501: Box-plot of the hypervolume indicator values for different optimizers on the WFG8 test problem.



Table A.221: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG8 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $ph$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
2D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e-34$	—	$5.86e-03$	$1.58e-34$
MOMBI-HV	$1.28e-34$	$> 0.05$	—	$2.20e-34$
MOMBI-UD	$3.89e-32$	$> 0.05$	$> 0.05$	—
3D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e-34$	—	$5.60e-15$	$1.28e-34$
MOMBI-HV	$1.28e-34$	$> 0.05$	—	$1.28e-34$
MOMBI-UD	$4.35e-28$	$> 0.05$	$> 0.05$	—
4D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e-34$	—	$1.32e-34$	$1.28e-34$
MOMBI-HV	$1.28e-34$	$> 0.05$	—	$1.28e-34$
MOMBI-UD	$1.73e-34$	$> 0.05$	$> 0.05$	—
5D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$9.61e-04$
MOMBI-SLD	$1.28e-34$	—	$1.28e-34$	$1.28e-34$
MOMBI-HV	$1.28e-34$	$> 0.05$	—	$1.28e-34$
MOMBI-UD	$> 0.05$	$> 0.05$	$> 0.05$	—
6D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.28e-34$	—	$1.28e-34$	$1.28e-34$
MOMBI-HV	$1.28e-34$	$> 0.05$	—	$1.28e-34$
MOMBI-UD	$1.28e-04$	$> 0.05$	$> 0.05$	—
7D				
MOMBI-RND	—	$> 0.05$	$> 0.05$	$> 0.05$
MOMBI-SLD	$1.58e-34$	—	$1.48e-33$	$6.63e-34$
MOMBI-HV	$2.34e-28$	$> 0.05$	—	$8.39e-11$
MOMBI-UD	$6.96e-22$	$> 0.05$	$> 0.05$	—
8D				
MOMBI-RND	—	$> 0.05$		$9.59e-03$
MOMBI-SLD	$1.64e-31$	—		$1.57e-33$
MOMBI-HV			—	
MOMBI-UD	$> 0.05$	$> 0.05$		—
9D				
MOMBI-RND	—	$> 0.05$		$9.94e-18$
MOMBI-SLD	$4.86e-30$	—		$5.88e-33$
MOMBI-HV			—	
MOMBI-UD	$> 0.05$	$> 0.05$		—
10D				
MOMBI-RND	—	$> 0.05$		$1.77e-25$
MOMBI-SLD	$6.86e-28$	—		$5.83e-32$
MOMBI-HV			—	
MOMBI-UD	$> 0.05$	$> 0.05$		—

Table A.222: Comparison of R2 indicator values for different optimizers on the WFG8 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	1.108407e + 00	6.491344e - 01	6.510954e - 01	6.960171e - 01
	avg.	1.109284e + 00	6.501943e - 01	6.517625e - 01	7.225641e - 01
	min.	1.108361e + 00	6.459399e - 01	6.460458e - 01	6.588987e - 01
	max.	1.137611e + 00	6.665888e - 01	6.655925e - 01	1.109081e + 00
	std.	5.001088e - 03	3.452557e - 03	3.531631e - 03	7.438397e - 02
<b>3D</b>	med.	3.850748e - 01	3.489428e - 01	3.505320e - 01	3.804507e - 01
	avg.	3.856359e - 01	3.492997e - 01	3.506795e - 01	3.809851e - 01
	min.	3.767509e - 01	3.466299e - 01	3.473383e - 01	3.725695e - 01
	max.	3.972609e - 01	3.537493e - 01	3.584859e - 01	3.967022e - 01
	std.	5.007170e - 03	1.448587e - 03	1.542350e - 03	4.614748e - 03
<b>4D</b>	med.	3.597283e - 01	2.416382e - 01	2.558031e - 01	3.057578e - 01
	avg.	3.618146e - 01	2.420415e - 01	2.557388e - 01	3.061274e - 01
	min.	3.102095e - 01	2.382962e - 01	2.481788e - 01	2.898480e - 01
	max.	4.634684e - 01	2.489727e - 01	2.624830e - 01	3.214395e - 01
	std.	2.755294e - 02	2.014673e - 03	2.681880e - 03	5.485944e - 03
<b>5D</b>	med.	3.154050e - 01	1.967060e - 01	2.226419e - 01	2.839208e - 01
	avg.	3.211432e - 01	1.990010e - 01	2.225376e - 01	2.865593e - 01
	min.	2.555494e - 01	1.919024e - 01	2.135907e - 01	2.679216e - 01
	max.	4.662092e - 01	2.115387e - 01	2.301310e - 01	3.112599e - 01
	std.	4.143475e - 02	5.575923e - 03	3.862692e - 03	9.540557e - 03
<b>6D</b>	med.	4.115814e - 01	1.751534e - 01	2.250413e - 01	2.917415e - 01
	avg.	4.588743e - 01	1.744277e - 01	2.248316e - 01	3.084295e - 01
	min.	2.619848e - 01	1.618051e - 01	2.151955e - 01	2.681253e - 01
	max.	7.473657e - 01	1.942356e - 01	2.383452e - 01	4.011409e - 01
	std.	1.412062e - 01	5.731743e - 03	4.772097e - 03	3.827909e - 02
<b>7D</b>	med.	6.030698e - 01	1.598130e - 01	2.501352e - 01	3.329882e - 01
	avg.	5.744006e - 01	1.656294e - 01	2.764713e - 01	3.304886e - 01
	min.	2.773755e - 01	1.436072e - 01	2.140450e - 01	2.491948e - 01
	max.	6.772943e - 01	3.364620e - 01	3.956008e - 01	4.103234e - 01
	std.	8.650608e - 02	2.278006e - 02	5.123600e - 02	3.091698e - 02
<b>8D</b>	med.	3.614098e - 01	1.514398e - 01	—	2.942613e - 01
	avg.	3.898986e - 01	1.599561e - 01	—	2.991701e - 01
	min.	2.135479e - 01	1.373200e - 01	—	2.468585e - 01
	max.	5.876896e - 01	2.971850e - 01	—	3.571870e - 01
	std.	1.185854e - 01	2.726032e - 02	—	1.797642e - 02
<b>9D</b>	med.	3.236579e - 01	1.437835e - 01	—	2.647884e - 01
	avg.	3.292433e - 01	1.512731e - 01	—	2.714421e - 01
	min.	1.612699e - 01	1.318365e - 01	—	2.345189e - 01
	max.	4.761780e - 01	2.651562e - 01	—	3.221411e - 01
	std.	7.057813e - 02	2.456303e - 02	—	1.903115e - 02
<b>10D</b>	med.	2.714488e - 01	1.406353e - 01	—	2.374749e - 01
	avg.	2.744009e - 01	1.470685e - 01	—	2.428590e - 01
	min.	1.707942e - 01	1.277756e - 01	—	2.230815e - 01
	max.	4.436128e - 01	2.396232e - 01	—	2.822349e - 01
	std.	4.849048e - 02	2.075978e - 02	—	1.352962e - 02

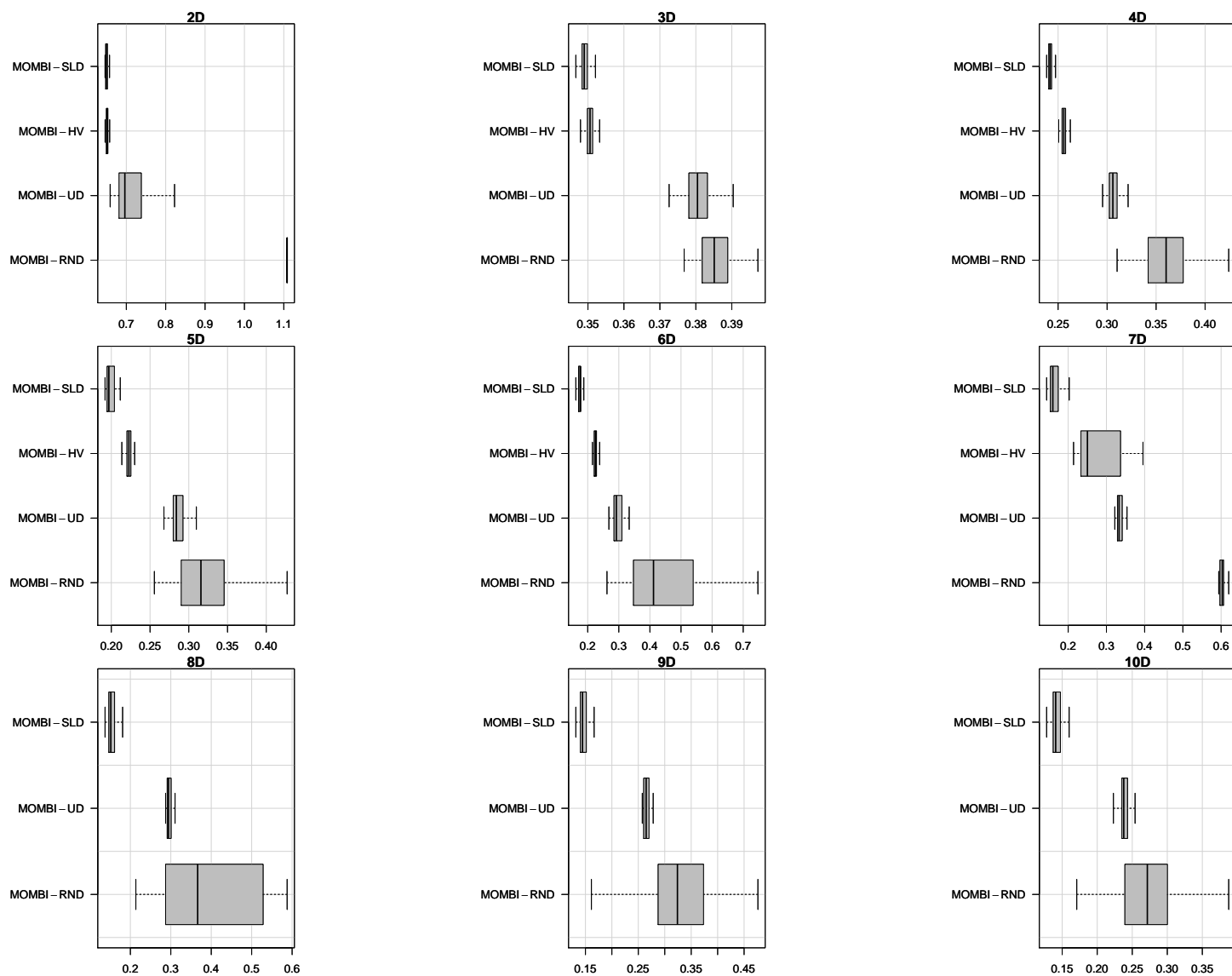


Figure A.502: Box-plot of the R2 indicator values for different optimizers on the WFG8 test problem.

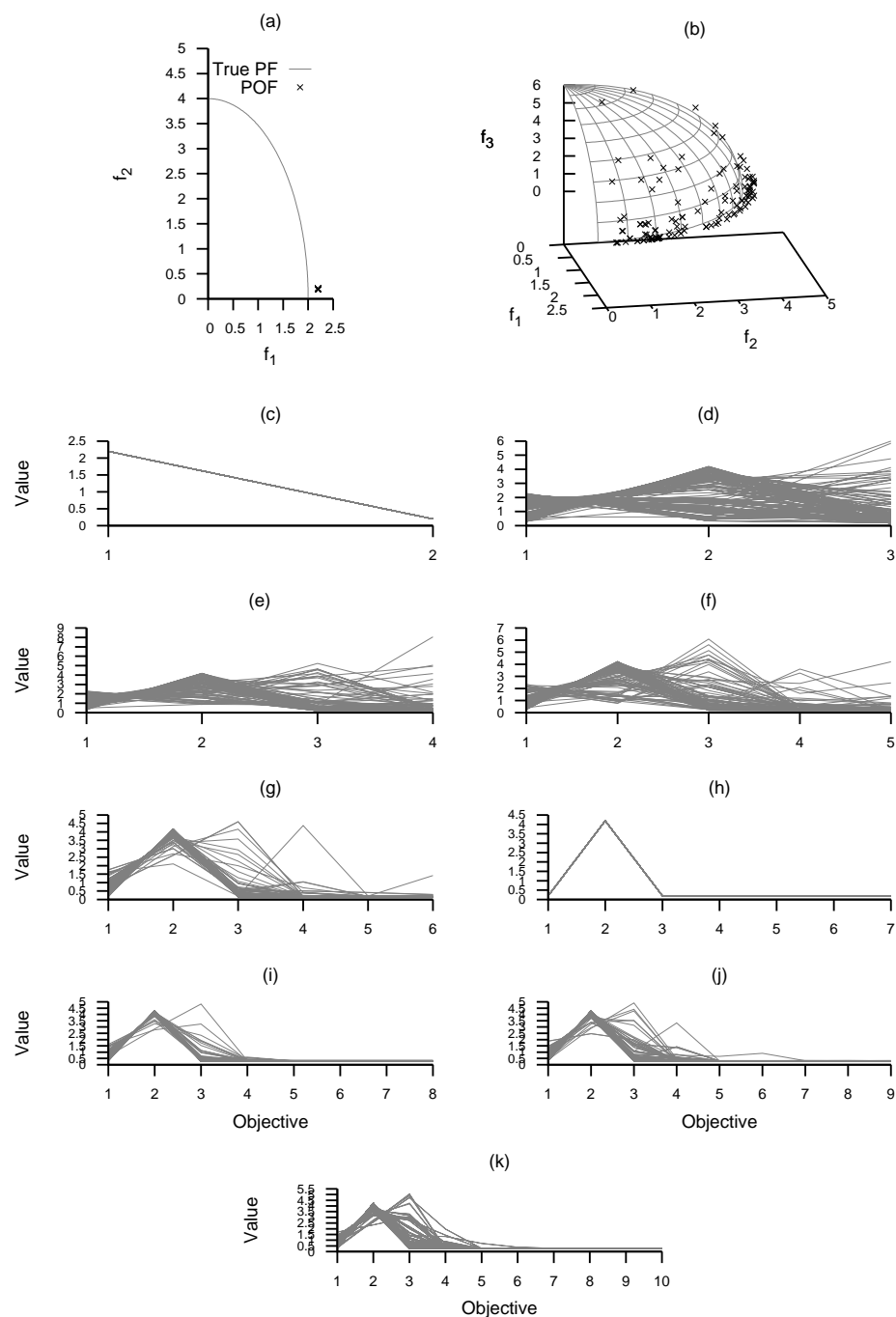


Figure A.503: Plots of the approximations obtained by MOMBI-RND from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

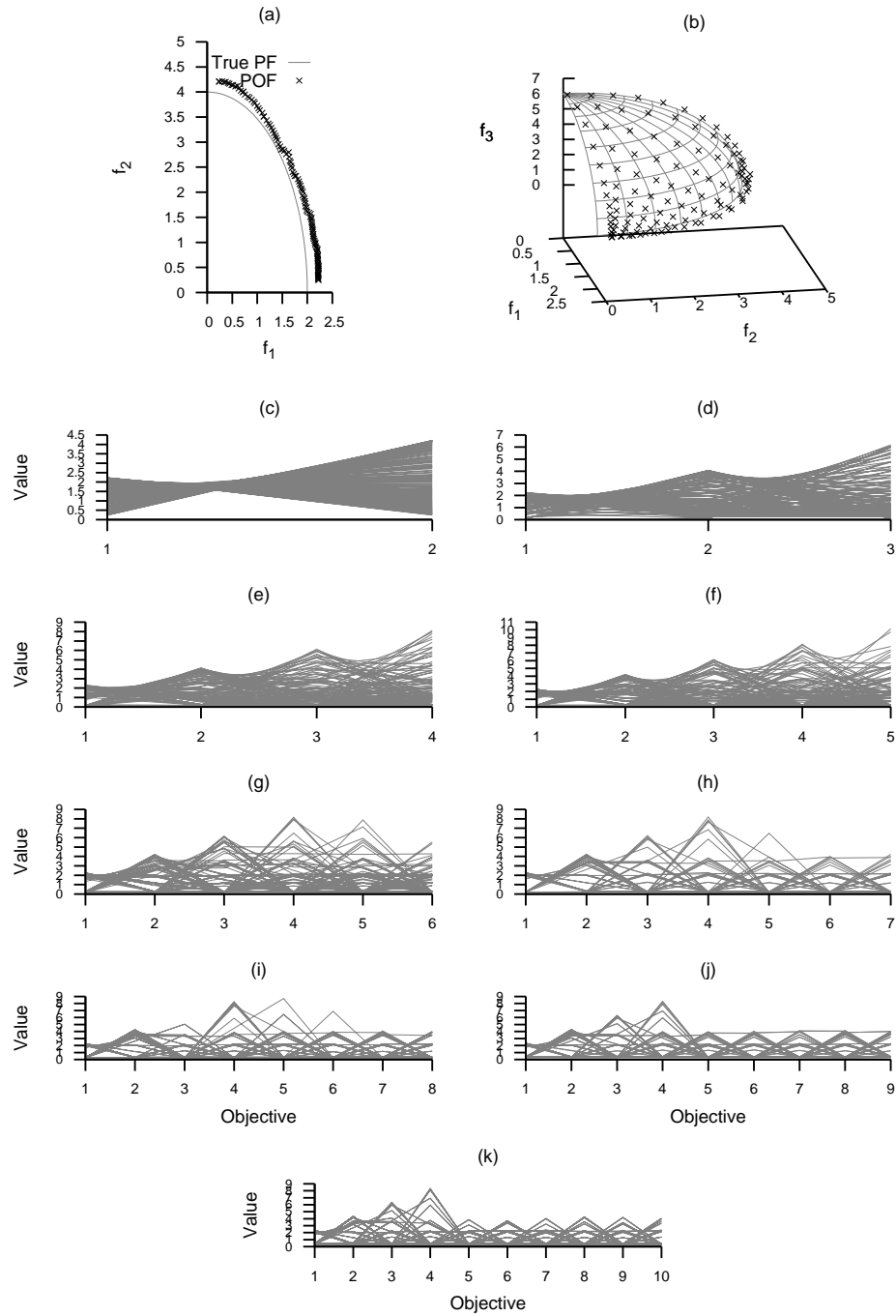


Figure A.504: Plots of the approximations obtained by MOMBI-SLD from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

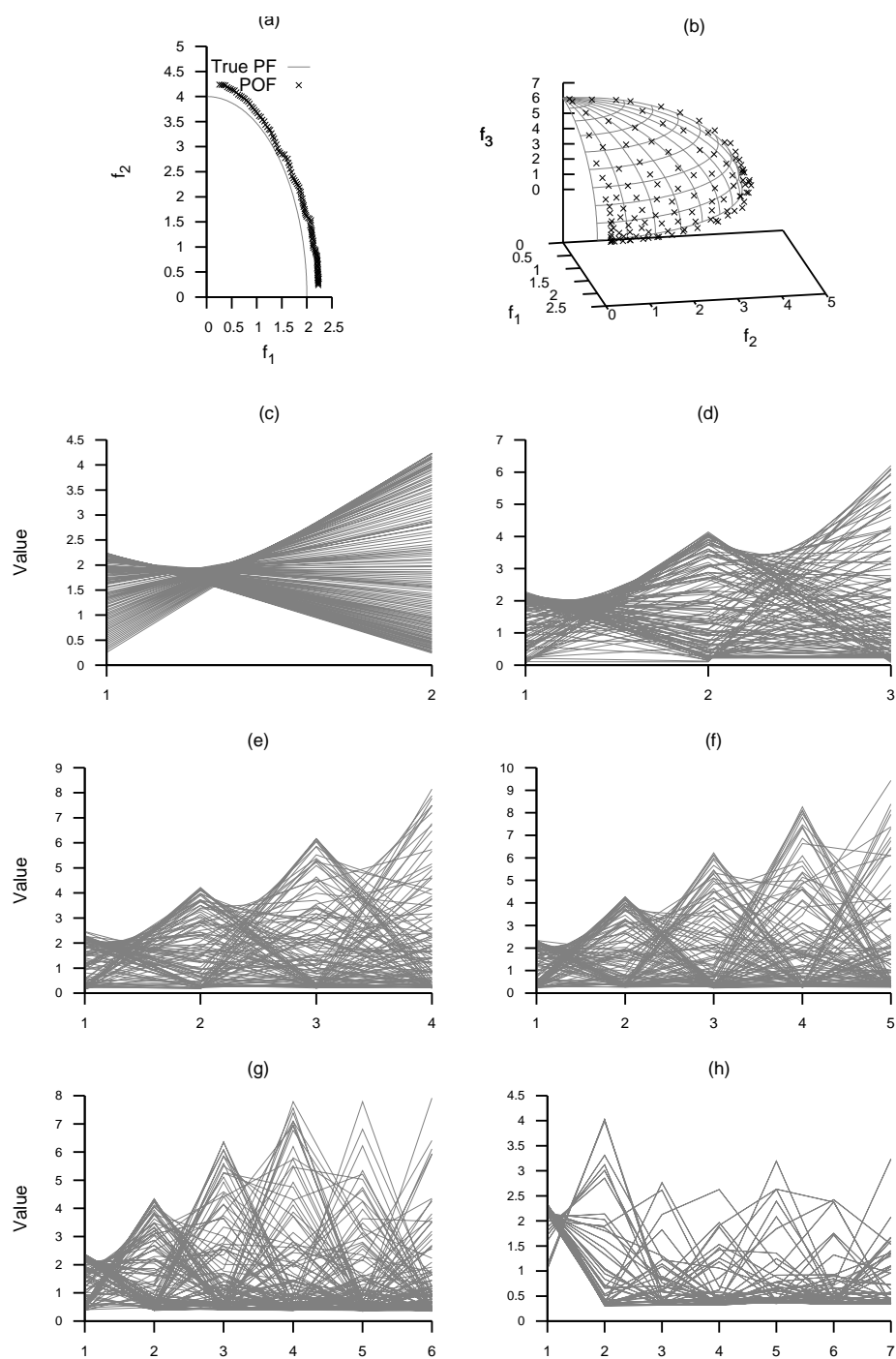


Figure A.505: Plots of the approximations obtained by MOMBI-HV from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

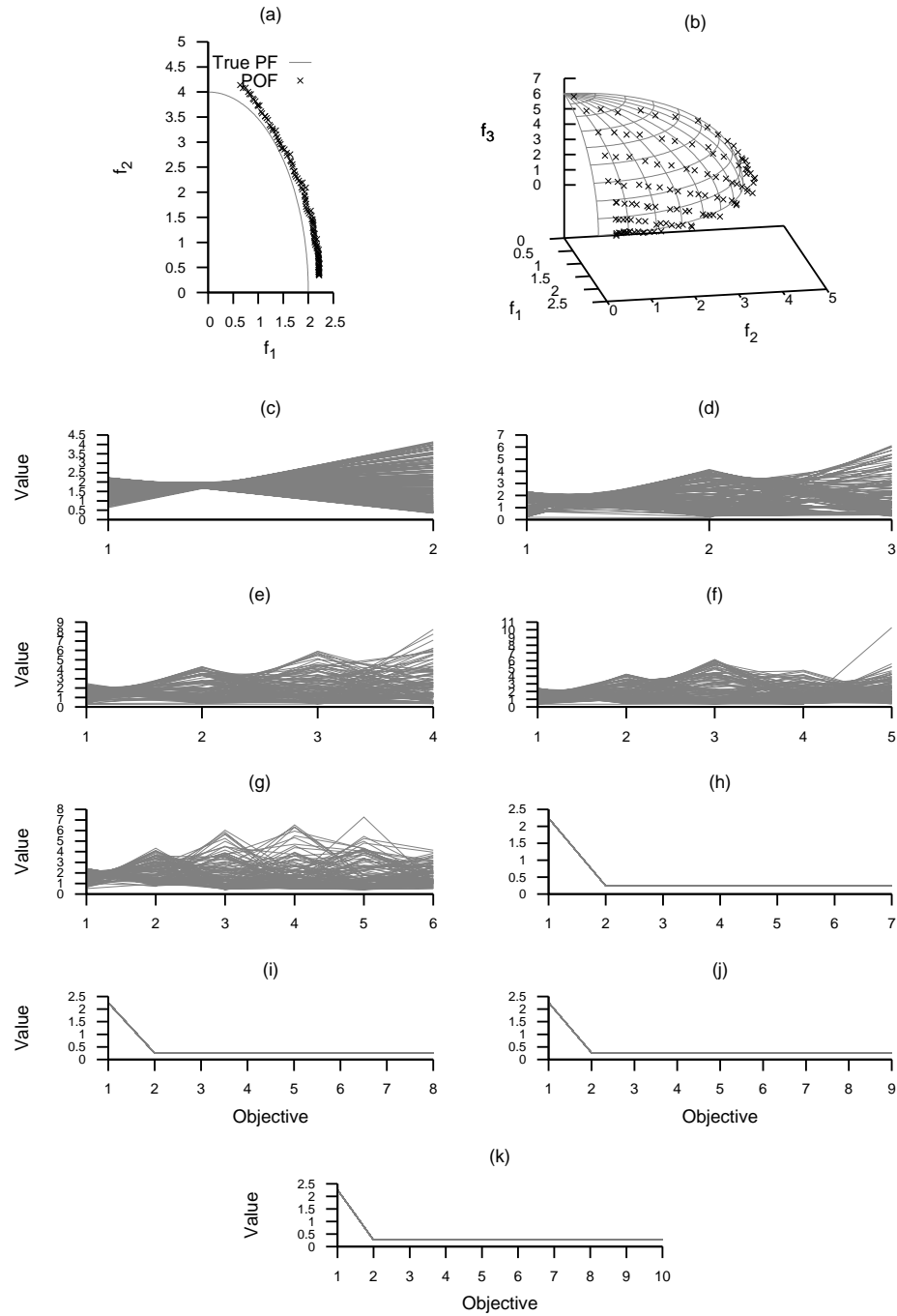


Figure A.506: Plots of the approximations obtained by MOMBI-UD from 2 to 10 objectives on the WFG8 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

Table A.223: Comparison of hypervolume indicator values for different optimizers on the WFG9 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	7.421680e + 00	7.763813e + 00	7.690804e + 00	7.635657e + 00
	avg.	7.451667e + 00	7.958253e + 00	7.865788e + 00	7.775184e + 00
	min.	6.808029e + 00	7.673182e + 00	7.679072e + 00	7.471218e + 00
	max.	8.148914e + 00	8.280153e + 00	8.337209e + 00	8.158200e + 00
	std.	3.066971e - 01	2.706047e - 01	2.552316e - 01	2.298658e - 01
<b>3D</b>	med.	6.412773e + 01	6.695943e + 01	6.664956e + 01	6.453466e + 01
	avg.	6.387571e + 01	6.758651e + 01	6.702182e + 01	6.481902e + 01
	min.	6.014424e + 01	6.612982e + 01	6.546093e + 01	6.309815e + 01
	max.	6.663993e + 01	7.100642e + 01	7.068154e + 01	6.723498e + 01
	std.	1.392957e + 00	1.265612e + 00	1.077804e + 00	1.095146e + 00
<b>4D</b>	med.	5.424442e + 02	6.490009e + 02	6.185663e + 02	5.466279e + 02
	avg.	5.408133e + 02	6.442410e + 02	6.196736e + 02	5.461104e + 02
	min.	4.862310e + 02	5.810762e + 02	5.819108e + 02	4.922038e + 02
	max.	5.641310e + 02	6.711816e + 02	6.563694e + 02	5.976273e + 02
	std.	1.419310e + 01	1.812689e + 01	1.731734e + 01	1.976797e + 01
<b>5D</b>	med.	5.472339e + 03	6.694550e + 03	6.450145e + 03	4.890495e + 03
	avg.	5.450493e + 03	6.686312e + 03	6.484101e + 03	4.888137e + 03
	min.	4.601209e + 03	5.877548e + 03	5.915849e + 03	4.386513e + 03
	max.	5.887838e + 03	7.430043e + 03	7.101498e + 03	6.080184e + 03
	std.	2.235197e + 02	2.754041e + 02	2.193323e + 02	2.465998e + 02
<b>6D</b>	med.	6.371206e + 04	7.730573e + 04	7.220851e + 04	5.729336e + 04
	avg.	6.346378e + 04	7.729574e + 04	7.189178e + 04	5.726259e + 04
	min.	5.399127e + 04	6.996034e + 04	5.391489e + 04	4.902736e + 04
	max.	6.931570e + 04	8.479190e + 04	7.911099e + 04	6.468553e + 04
	std.	2.841685e + 03	3.388114e + 03	3.830019e + 03	2.763100e + 03
<b>7D</b>	med.	8.845880e + 05	9.724089e + 05	8.309673e + 05	7.735122e + 05
	avg.	8.819631e + 05	9.718787e + 05	8.259121e + 05	7.656859e + 05
	min.	3.698860e + 05	7.607911e + 05	6.582010e + 05	6.145521e + 05
	max.	1.051834e + 06	1.101527e + 06	9.436681e + 05	8.819133e + 05
	std.	8.197159e + 04	7.254651e + 04	5.903550e + 04	4.705686e + 04
<b>8D</b>	med.	1.382712e + 07	1.436106e + 07	—	1.236252e + 07
	avg.	1.384979e + 07	1.427624e + 07	—	1.233564e + 07
	min.	1.142679e + 07	1.132028e + 07	—	1.021482e + 07
	max.	1.648237e + 07	1.685952e + 07	—	1.400455e + 07
	std.	1.061457e + 06	1.152160e + 06	—	9.111105e + 05
<b>9D</b>	med.	2.521438e + 08	2.377493e + 08	—	2.124896e + 08
	avg.	2.501635e + 08	2.377532e + 08	—	2.135867e + 08
	min.	1.975919e + 08	1.569287e + 08	—	1.835712e + 08
	max.	2.937220e + 08	2.998451e + 08	—	2.527112e + 08
	std.	1.867341e + 07	2.230777e + 07	—	1.482883e + 07
<b>10D</b>	med.	4.736617e + 09	4.048924e + 09	—	4.222015e + 09
	avg.	4.751559e + 09	4.104797e + 09	—	4.193478e + 09
	min.	3.688317e + 09	2.982501e + 09	—	3.290364e + 09
	max.	5.919523e + 09	5.554424e + 09	—	4.882871e + 09
	std.	4.541620e + 08	4.424891e + 08	—	2.863325e + 08



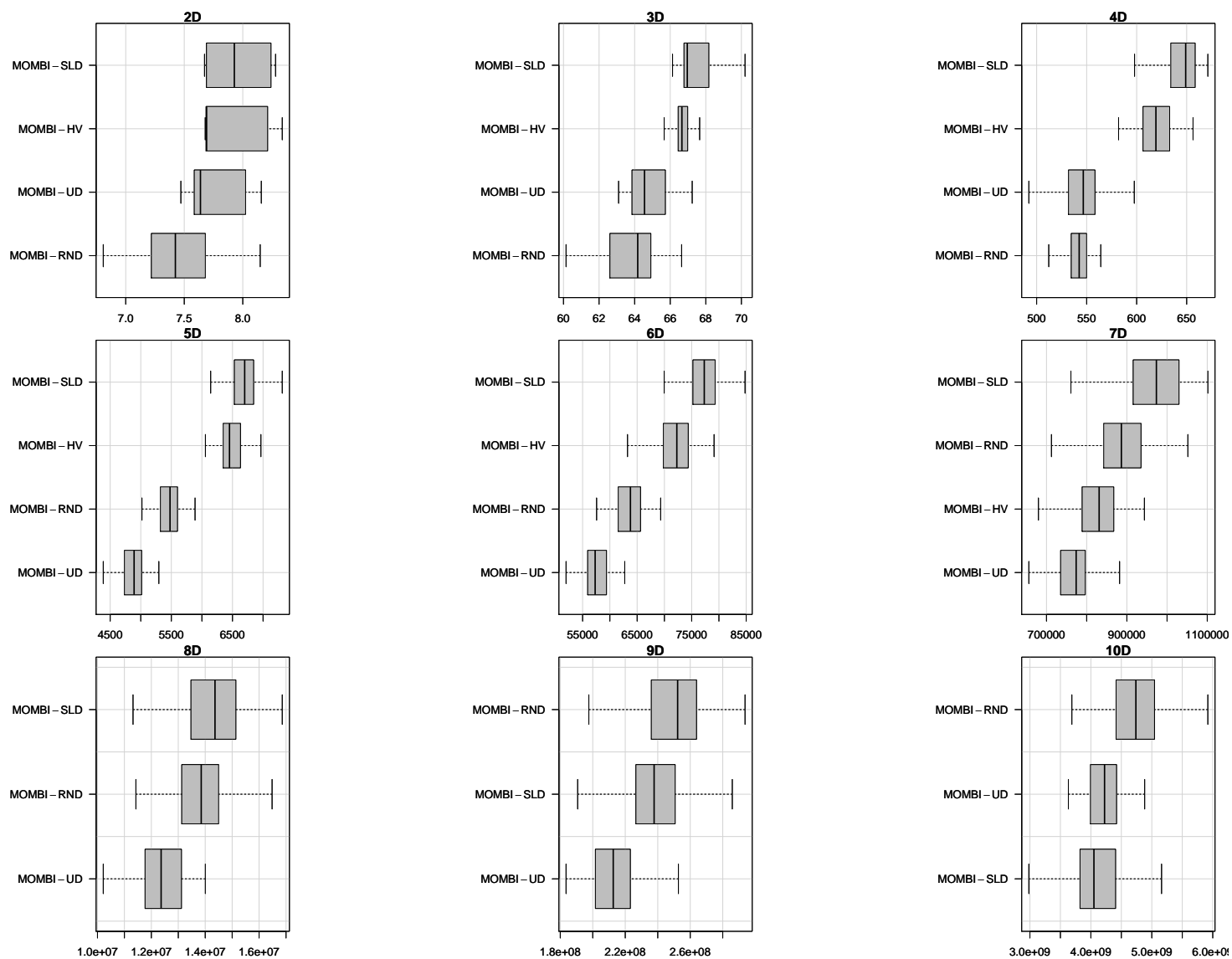


Figure A.507: Box-plot of the hypervolume indicator values for different optimizers on the WFG9 test problem.

Table A.224: Wilcoxon rank sum test applied to the hypervolume indicator values on the WFG9 test problem. The table contains for each pair of optimizers  $O_R$  (row) and  $O_C$  (column) the  $ph$ -values with respect to the alternative hypothesis that the indicator values for  $O_R$  are significantly better than those for  $O_C$ .

Optimizer	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
2D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.18e - 20$	–	$4.39e - 04$	$4.17e - 13$
MOMBI-HV	$1.26e - 16$	> 0.05	–	$2.11e - 08$
MOMBI-UD	$1.32e - 14$	> 0.05	> 0.05	–
3D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.73e - 34$	–	$6.58e - 08$	$3.54e - 31$
MOMBI-HV	$1.16e - 33$	> 0.05	–	$7.85e - 28$
MOMBI-UD	$1.75e - 05$	> 0.05	> 0.05	–
4D				
MOMBI-RND	–	> 0.05	> 0.05	> 0.05
MOMBI-SLD	$1.28e - 34$	–	$9.71e - 17$	$1.49e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$5.22e - 34$
MOMBI-UD	$3.79e - 02$	> 0.05	> 0.05	–
5D				
MOMBI-RND	–	> 0.05	> 0.05	$4.20e - 29$
MOMBI-SLD	$1.32e - 34$	–	$1.28e - 08$	$1.36e - 34$
MOMBI-HV	$1.28e - 34$	> 0.05	–	$1.40e - 34$
MOMBI-UD	> 0.05	> 0.05	> 0.05	–
6D				
MOMBI-RND	–	> 0.05	> 0.05	$9.72e - 28$
MOMBI-SLD	$1.28e - 34$	–	$6.01e - 19$	$1.28e - 34$
MOMBI-HV	$1.27e - 30$	> 0.05	–	$1.81e - 33$
MOMBI-UD	> 0.05	> 0.05	> 0.05	–
7D				
MOMBI-RND	–	> 0.05	$6.09e - 10$	$3.32e - 26$
MOMBI-SLD	$2.60e - 14$	–	$1.27e - 27$	$1.98e - 33$
MOMBI-HV	> 0.05	> 0.05	–	$3.43e - 13$
MOMBI-UD	> 0.05	> 0.05	> 0.05	–
8D				
MOMBI-RND	–	> 0.05		$6.70e - 19$
MOMBI-SLD	$1.77e - 03$	–		$4.35e - 23$
MOMBI-HV			–	
MOMBI-UD	> 0.05	> 0.05		–
9D				
MOMBI-RND	–	$1.01e - 05$		$4.19e - 26$
MOMBI-SLD	> 0.05	–		$7.83e - 16$
MOMBI-HV			–	
MOMBI-UD	> 0.05	> 0.05		–
10D				
MOMBI-RND	–	$1.06e - 17$		$1.42e - 17$
MOMBI-SLD	> 0.05	–		> 0.05
MOMBI-HV			–	
MOMBI-UD	> 0.05	$1.76e - 02$		–

Table A.225: Comparison of R2 indicator values for different optimizers on the WFG9 test problem.

Dim.	Stat.	MOMBI-RND	MOMBI-SLD	MOMBI-HV	MOMBI-UD
<b>2D</b>	med.	6.591293e-01	6.136919e-01	6.374784e-01	6.416881e-01
	avg.	6.636102e-01	6.222102e-01	6.276707e-01	6.338646e-01
	min.	6.089413e-01	6.037629e-01	5.995737e-01	6.105114e-01
	max.	7.399146e-01	6.397733e-01	6.391835e-01	6.534045e-01
	std.	2.980038e-02	1.540261e-02	1.454884e-02	1.387895e-02
<b>3D</b>	med.	3.585252e-01	3.503658e-01	3.515692e-01	3.617517e-01
	avg.	3.609077e-01	3.473487e-01	3.497226e-01	3.609862e-01
	min.	3.491428e-01	3.320356e-01	3.337868e-01	3.474925e-01
	max.	3.783890e-01	3.542988e-01	3.575863e-01	3.710720e-01
	std.	7.590854e-03	5.958497e-03	5.043063e-03	5.686697e-03
<b>4D</b>	med.	2.998623e-01	2.502732e-01	2.630077e-01	3.045470e-01
	avg.	3.007954e-01	2.524841e-01	2.640926e-01	3.053939e-01
	min.	2.836432e-01	2.402292e-01	2.482790e-01	2.768252e-01
	max.	3.304656e-01	2.773804e-01	2.818230e-01	3.395240e-01
	std.	8.936018e-03	8.078241e-03	8.356263e-03	1.240944e-02
<b>5D</b>	med.	2.596414e-01	2.314780e-01	2.393433e-01	3.097552e-01
	avg.	2.611702e-01	2.321040e-01	2.391842e-01	3.093070e-01
	min.	2.424236e-01	2.034542e-01	2.165764e-01	2.469978e-01
	max.	3.107743e-01	2.652567e-01	2.599226e-01	3.439042e-01
	std.	1.111036e-02	1.035637e-02	8.676372e-03	1.516239e-02
<b>6D</b>	med.	2.599860e-01	2.098583e-01	2.324855e-01	2.850997e-01
	avg.	2.618789e-01	2.096416e-01	2.330089e-01	2.852767e-01
	min.	2.421852e-01	1.895418e-01	2.120032e-01	2.489400e-01
	max.	2.961154e-01	2.287591e-01	2.998391e-01	3.208699e-01
	std.	9.574646e-03	8.856355e-03	1.244792e-02	1.384860e-02
<b>7D</b>	med.	2.253815e-01	2.111602e-01	2.431706e-01	2.592444e-01
	avg.	2.300697e-01	2.113285e-01	2.439395e-01	2.609733e-01
	min.	1.983270e-01	1.847129e-01	2.184733e-01	2.293893e-01
	max.	4.232053e-01	2.850295e-01	2.858322e-01	2.960374e-01
	std.	2.542592e-02	1.485329e-02	1.216955e-02	1.214205e-02
<b>8D</b>	med.	2.206166e-01	2.093978e-01	—	2.379009e-01
	avg.	2.220703e-01	2.114941e-01	—	2.385074e-01
	min.	1.902823e-01	1.844625e-01	—	2.175046e-01
	max.	2.524181e-01	2.504337e-01	—	2.717026e-01
	std.	1.275353e-02	1.266981e-02	—	1.208631e-02
<b>9D</b>	med.	2.160377e-01	2.060629e-01	—	2.309643e-01
	avg.	2.176433e-01	2.079558e-01	—	2.320011e-01
	min.	1.863081e-01	1.781492e-01	—	2.101399e-01
	max.	2.573487e-01	2.595335e-01	—	2.614910e-01
	std.	1.254939e-02	1.178300e-02	—	9.624247e-03
<b>10D</b>	med.	2.117363e-01	2.148813e-01	—	2.181469e-01
	avg.	2.129680e-01	2.140962e-01	—	2.192865e-01
	min.	1.776163e-01	1.806739e-01	—	2.037169e-01
	max.	2.550430e-01	2.651083e-01	—	2.507356e-01
	std.	1.577462e-02	1.326187e-02	—	8.736312e-03

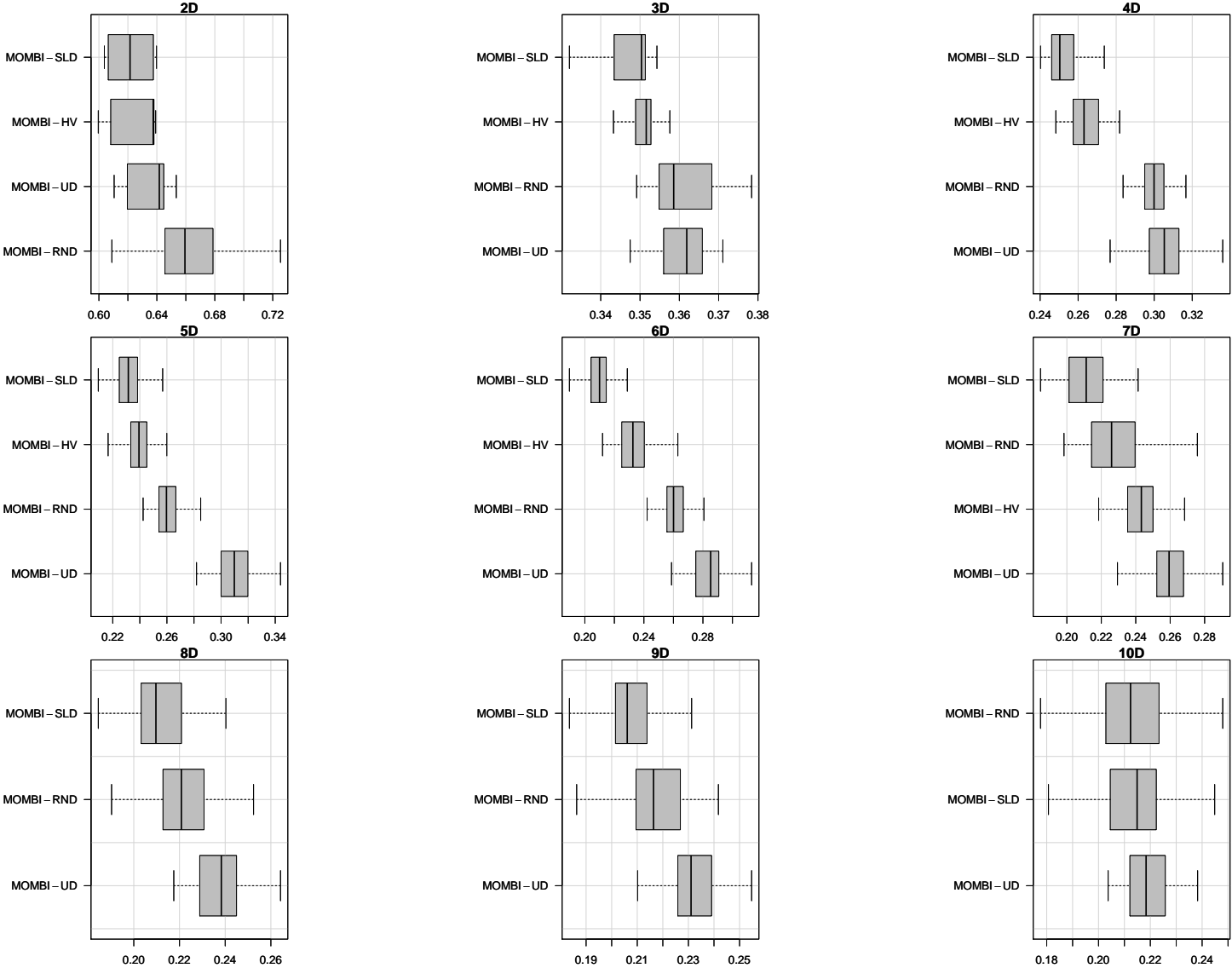


Figure A.508: Box-plot of the R2 indicator values for different optimizers on the WFG9 test problem.

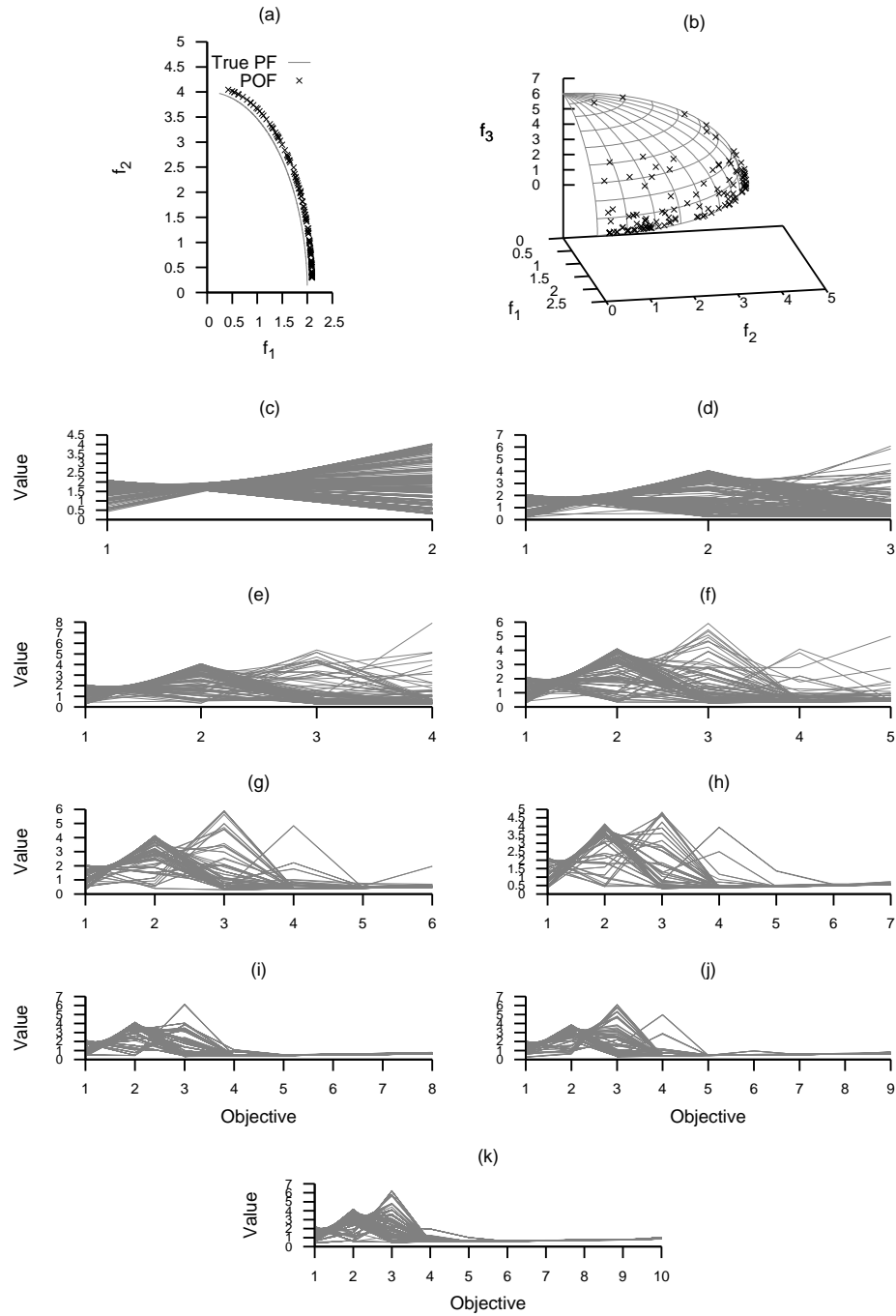


Figure A.509: Plots of the approximations obtained by MOMBI-RND from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

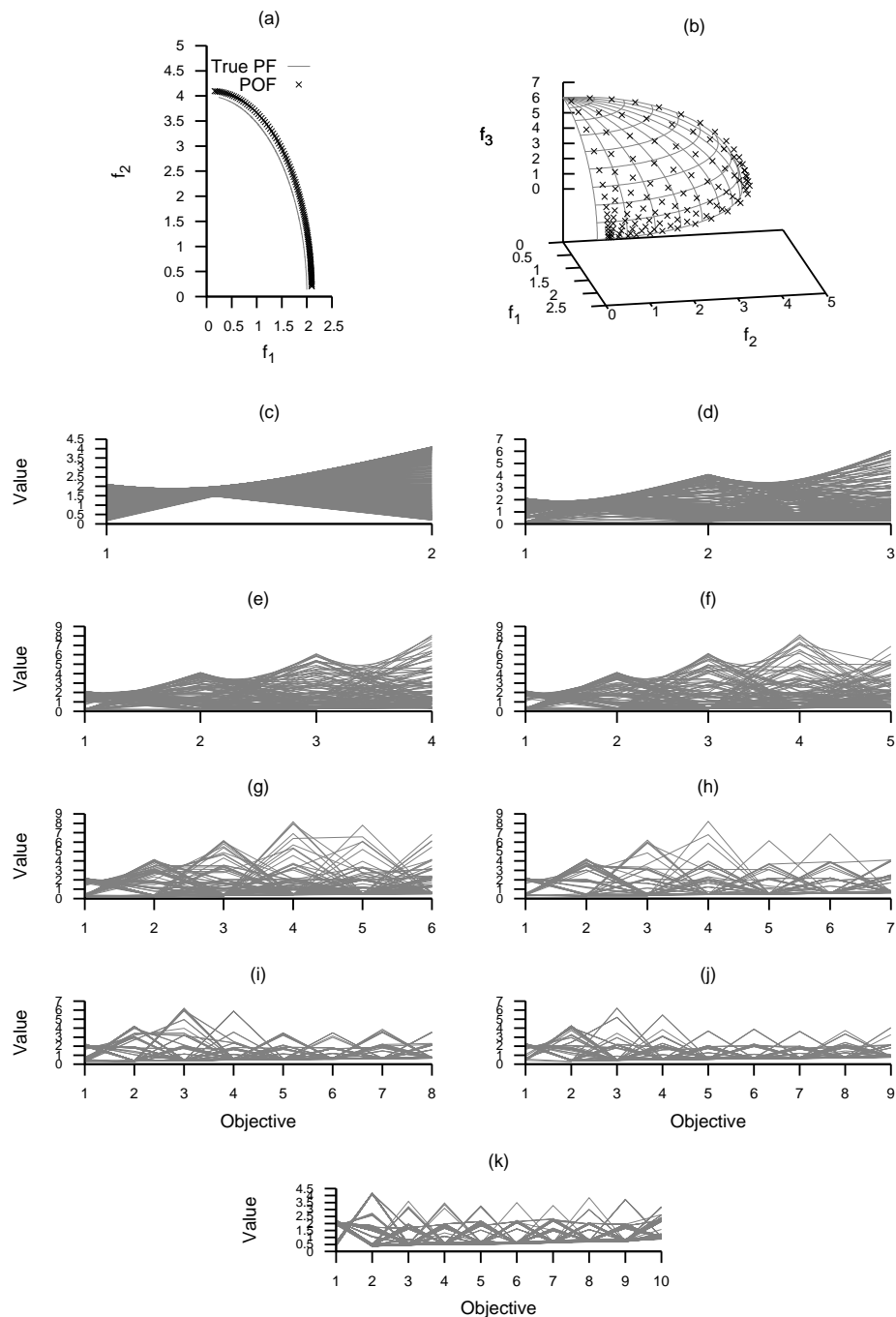


Figure A.510: Plots of the approximations obtained by MOMBI-SLD from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

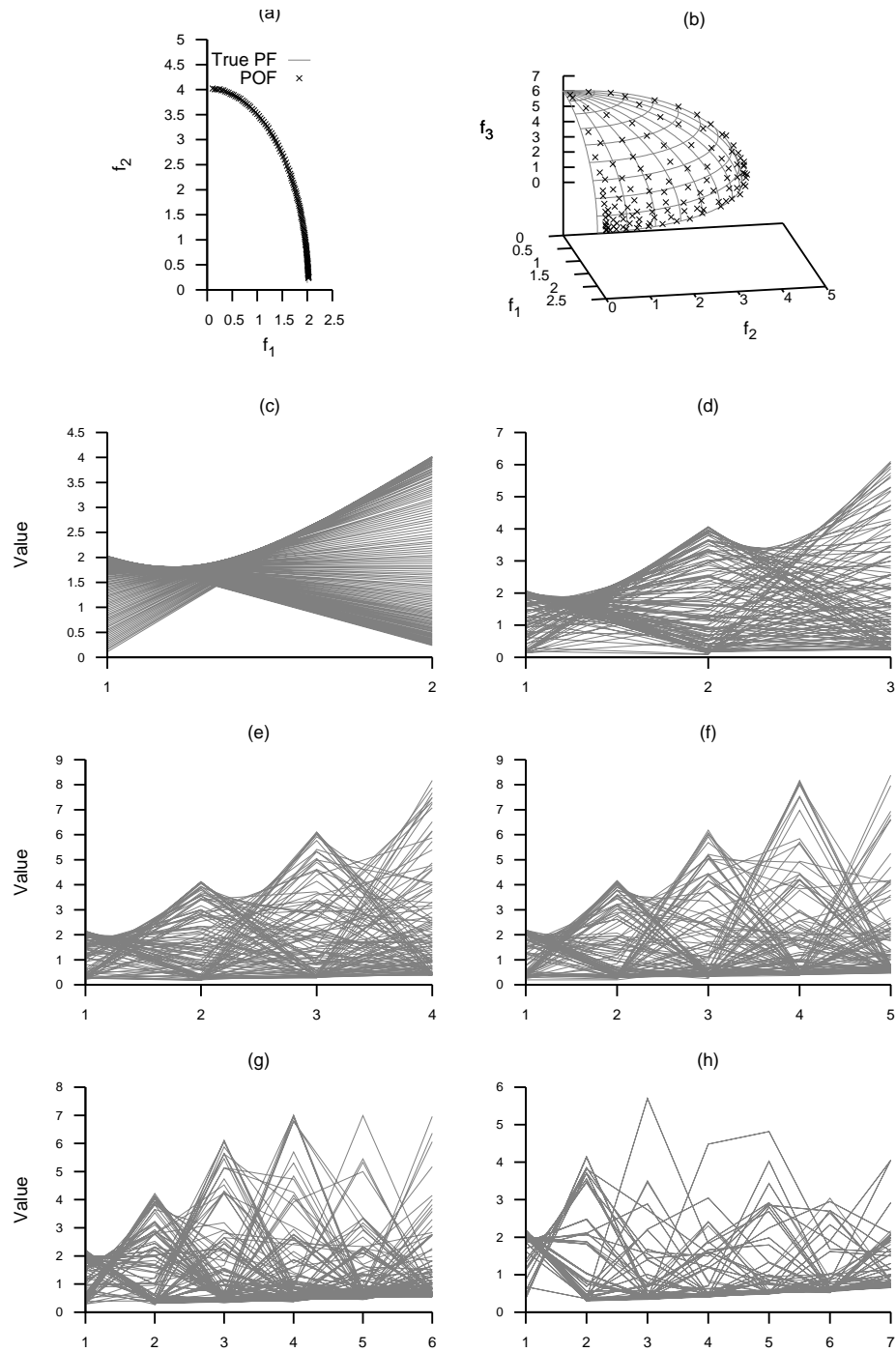


Figure A.511: Plots of the approximations obtained by MOMBI-HV from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.

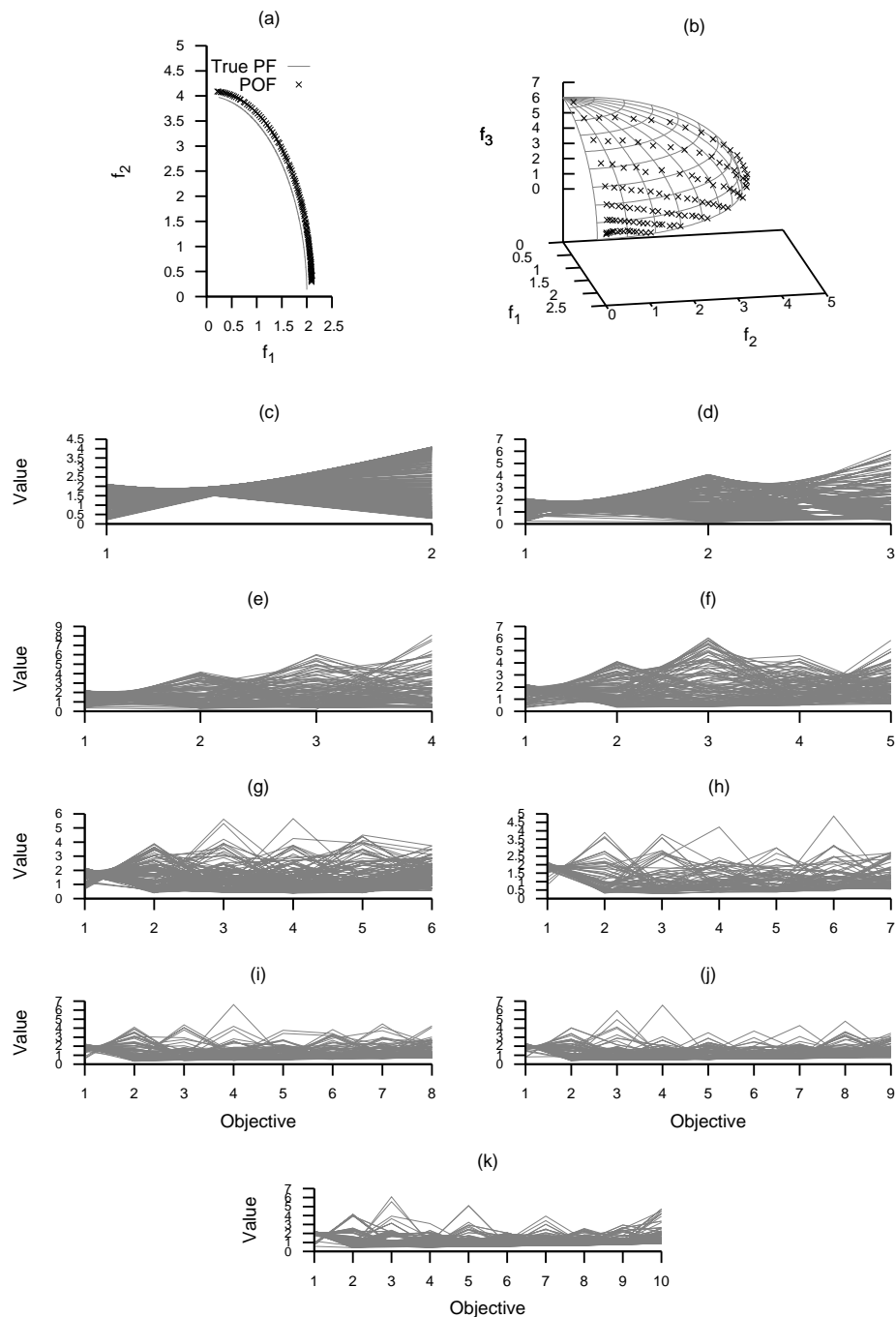


Figure A.512: Plots of the approximations obtained by MOMBI-UD from 2 to 10 objectives on the WFG9 test problem. These plots correspond to the median hypervolume value for 100 independent runs.